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E. F. Ritz Jr.
Tektronix, Inc.

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PROGRESS IN THE THEORY OF ELECTRON-BEAM DEFLECTION

E.F. Ritz, Jr.

Tektronix, Inc.
P.O. Box 500
Mail Station 50-493
Beaverton, Oregon 97077
Phone: (503) 627-5435

Abstract

Analysis, simulation, and design of electron-beam-deflection systems are reviewed in light of the current state of theoretical understanding. A brief review of the physical principles is followed by a detailed discussion of electrostatic, magnetostatic, mixed-field, traveling-wave, and scan-expansion systems. Each methodology is examined from a triple perspective: calculation of electromagnetic fields, calculation of electron trajectories, and calculation of the ensemble of trajectories forming the beam. Applications discussed include deflectors for television displays, lithography, scanning microscopes, and CRT oscillography. Developments of the last ten years are stressed, thereby supplementing and updating the author's previous review on this subject.

In field calculation, recent developments in the use of numerical methods on computers dominate. These methods include finite-difference, finite-element, and charge-density or integral-equation techniques. In trajectory calculations, increasing use of numerical integration as well as improvements and extensions of the aberration theory are found. In treatment of the beam bundle, the growing sophistication of numerical deflected-beam models has lead to increased use of aberration figures, current-density plots, and phase-space methods.

Key Words: electron-beam deflection, electrostatic deflection, magnetostatic deflection, mixed-field deflection, traveling-wave deflection, scan-expansion lenses, numerical field calculation, electron-trajectory calculation, description of electron beams, aberration theory of deflection.

Introduction

Scope and Organization

In this review, we shall consider the present theoretical understanding of electron-beam deflection, concentrating on the work of the last ten years. Thus, the discussion overlaps and updates the coverage of this author's previous review (Ritz, 1979). Hutter (1974) also reviews progress in electron-beam deflection, covering about 20 years prior to 1974. Both of these previous reviews have extensive lists of references and are recommended reading as an introduction to the field. No textbook exists that adequately serves this purpose.

In the theory of electron-beam deflection, we are faced with three fundamental problems. First, we must solve the Maxwell equations for the electric and magnetic fields produced by the deflection system. Second, we must solve the Lorentz equation of motion for the trajectories of the individual electrons in these fields. Third, we must use the solutions to the first and second problems to calculate the collective behavior of the ensemble of electrons forming the beam. In general, these problems cannot be solved analytically. Therefore, simplified analytical models, approximations such as perturbation analysis, or numerical simulation on a computer are used. The various subfields of our subject differ greatly in the degree of success obtained with each of these methods.

We shall consider our subject under the headings of electrostatic, magnetostatic, mixed-field, traveling-wave, and scan-expansion systems. However, for future reference, we shall first briefly review the physical principles and basic viewpoints needed for all of these topics.

Calculation of Electromagnetic Fields

We now review the relevant portions of electromagnetic theory, using MKSA formulas and units. Our problem is to solve the Maxwell equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{2}$$

$$\nabla \cdot \mathbf{D} = \varrho \tag{3}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{4}$$

for electric intensity **E**, electric displacement **D**, magnetic induction **B**, and magnetic intensity **H**, given the physical structure of the deflector and the electrical sources that give

rise to the electric current density **J** and the electric charge density ϱ. In addition, **J** and ϱ must satisfy the equation of continuity

$$\nabla\cdot\mathbf{J} + \frac{\partial\varrho}{\partial t} = 0 \tag{5}$$

Finally, we shall generally assume isotropic, homogeneous media with

$$\mathbf{D} = \epsilon\mathbf{E} \tag{6}$$

$$\mathbf{B} = \mu\mathbf{H} \tag{7}$$

in which the permittivity ϵ and the permeability μ are constant within a material (with vacuum values ϵ_0 and μ_0 as usual).

It is frequently convenient to employ the electric scalar potential Φ and the magnetic vector potential **A** in place of the field vectors **E**, **B** to which they are related by

$$\mathbf{E} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t} \tag{8}$$

$$\mathbf{B} = \nabla\times\mathbf{A} \tag{9}$$

In the general time-dependent case, the electromagnetic potentials satisfy the coupled wave equations

$$\nabla^2\mathbf{A} - \frac{1}{v_p^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = \mu\mathbf{J} \tag{10}$$

$$\nabla^2\Phi - \frac{1}{v_p^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{\varrho}{\epsilon} \tag{11}$$

in which v_p is the phase velocity

$$v_p = \frac{1}{\sqrt{\mu\epsilon}} \tag{12}$$

and the Lorentz gauge condition

$$\frac{1}{v_p^2}\frac{\partial\Phi}{\partial t} + \nabla\cdot\mathbf{A} = 0 \tag{13}$$

applies. The solutions of the wave equations represent waves of **E** and **B** propagating with phase velocity v_p.

In the static-field case, eqs. (10) - (11) reduce to

$$\nabla^2\mathbf{A} = -\mu\mathbf{J} \tag{14}$$

$$\nabla^2\Phi = -\frac{\varrho}{\epsilon} \tag{15}$$

If in addition we are in a source-free region, then **J** and ϱ vanish so that

$$\nabla^2\Phi = 0 \tag{16}$$

$$\mathbf{E} = -\nabla\Phi \tag{17}$$

$$\nabla^2\Psi = 0 \tag{18}$$

$$\mathbf{H} = -\nabla\Psi \tag{19}$$

where Ψ is the magnetic scalar potential now permitted by the vanishing of $\nabla\times\mathbf{H}$. This Laplace formulation of the problem is particularly useful when the potentials or currents on conductors are given and can be used to obtain boundary conditions on Φ or Ψ.

In most problems of electron-beam deflection, we can neglect the wave aspects of the fields and solve the simpler static Laplace equations, (16) and (18), rather than the full wave equations. This *static-field* approximation assumes, in effect, that the driving fields travel from the input terminals to all parts of the deflector in a time that is short compared with the time in which the input signal changes appreciably. That is, for a sinusoidal input of frequency f, we must have

$$2\pi f t_w << 1 \tag{20}$$

in which t_w is the transit time required for the wave to reach the farthest part of the deflector. This condition is violated in traveling-wave deflectors for high-speed deflection. Note that in vacuum, t_w will be about 33 psec for every centimeter of travel.

So far, we have formulated the field problem in terms of partial differential equations relating the fields and their sources. One can also express the field as an integral over the sources. In the static case of electric charges on the surfaces of conductors with specified potentials, the integral in question is

$$\Phi(\mathbf{x}_2) = \frac{1}{4\pi\epsilon_0}\iint_S \frac{\sigma(\mathbf{x}_1)}{R}\,dS_1 \tag{21}$$

in which σ is the density of surface charge, S represents the surface (or surfaces) of the conductor(s), and $R = |\mathbf{x}_2 - \mathbf{x}_1|$. When the point of observation $\mathbf{x}_2$ lies on one of the conductors, then Φ must be the constant potential of that conductor. Thus, (21) can be regarded as an integral equation for the unknown charge density $\sigma(\mathbf{x}_1)$ on the conductors. Once this density is determined, the potential at any point can be calculated from (21). For the general static case, we must add to (21) integrals over the volume density ϱ and over the surface dipole layers (e.g., Stratton, 1941).

The general time-dependent problem can be treated similarly. If we assume a sinusoidal dependence $\exp(i\omega t)$ for the fields, currents, and charges, then

$$\mathbf{A}(\mathbf{x}_2) = \frac{\mu}{4\pi}\iiint_V \frac{\mathbf{J}(\mathbf{x}_1)e^{-ikR}}{R}\,dV_1 \tag{22}$$

$$\Phi(\mathbf{x}_2) = \frac{1}{4\pi\epsilon}\iint_S \frac{\sigma(\mathbf{x}_1)e^{-ikR}}{R}\,dS_1 \tag{23}$$

in which V is the volume containing the currents and $k = \omega(\mu\epsilon)^{1/2} = 2\pi/\lambda$, with λ the wavelength. We shall not pursue this formulation here because it has yet to be applied in numerical field calculations for traveling-wave deflectors.

We can give a third formulation of the field problem: the variational formulation. We define a functional F to describe the fields. For static electric or magnetic fields with scalar potentials Φ or Ψ, the functionals are

$$F = \iiint \frac{1}{2}\epsilon(\nabla\Phi)^2 dV$$
$$F = \iiint \frac{1}{2}\mu(\nabla\Psi)^2 dV \qquad (24)$$

Equations (24) are the energies stored in the fields, expressed as integrals over the volume V containing the fields.

The potentials Φ and Ψ are determined by the condition that the variation of F vanish with respect to variations of the fields, thus minimizing the energy:

$$\delta F = 0 \qquad (25)$$

The resulting Euler-Lagrange equations reproduce (16) and (18).

Each of the three formulations of the field problem is the basis of a corresponding technique for numerical solution of that problem on a computer. The partial differential equations (10) - (11) lead to the finite-difference method (FDM). The integral equations (21) - (23) lead to the integral-equation or boundary-element method (IEM). The variational equations (24) - (25) lead to the finite-element method (FEM). Rather than discuss these methods here, we shall introduce them as appropriate in later sections. For a general view of these methods, see Kasper (1982) on magnetic-field calculation. Much of his discussion also applies to electrostatic fields. Schaefer (1983) gives a helpful comparison of these numerical techniques as well as methods of combining numerical and analytical solutions.

Calculation of Electron Trajectories

Electron trajectories in an electromagnetic field are governed by the Lorentz equation of motion for the position vector $\mathbf{r}(t)$:

$$\frac{d}{dt}\left[\frac{1}{\sqrt{1-\beta^2}}\frac{d\mathbf{r}}{dt}\right] = -\eta\left[\mathbf{E} + \frac{d\mathbf{r}}{dt}\times\mathbf{B}\right] \qquad (26)$$

in which $\beta = |d\mathbf{r}/dt|/c$, where c is the speed of light in vacuum and η is the ratio of electronic charge e to rest mass m. In the literature, the nonrelativistic limit

$$\ddot{\mathbf{r}} = -\eta[\mathbf{E} + \dot{\mathbf{r}}\times\mathbf{B}] \qquad (27)$$

is usually employed, and we shall also do so for simplicity. It is sometimes convenient to replace the independent variable t with arc length s along the trajectory or with z, the coordinate in the direction of the beam.

For numerical calculation, it is usually convenient to replace (27) with two equivalent first-order equations:

$$\mathbf{v} = \dot{\mathbf{r}} \qquad (28)$$

$$\dot{\mathbf{v}} = -\eta[\mathbf{E} + \mathbf{v}\times\mathbf{B}] \qquad (29)$$

in which we have defined the second variable $\mathbf{v}$. It is also useful to think of the electron as moving in the *phase space* defined by the six-vector $(\mathbf{r},m\mathbf{v})$ or, less rigorously, $(\mathbf{r},\mathbf{v})$.

In principle, the fields $\mathbf{E}$ and $\mathbf{B}$ should include both the fields of the deflector as discussed in *Calculation of Electromagnetic Fields* and the fields due to the electrons in the beam. However, the latter fields have not as yet been incorporated successfully into calculations of beam deflection, although there has been much progress in the study of such effects in electron guns using the average-field or space-charge approximation (e.g., Hauke, 1977). Therefore, we shall not discuss the self-fields of the beam.

The trajectory equations (26) or (27) or (28) - (29) must be solved subject to initial conditions $\mathbf{r}(0) = \mathbf{r}_0$, $\dot{\mathbf{r}}(0) = \dot{\mathbf{r}}_0$ on the starting position and velocity of the electron in question. Each electron in the beam has a different set of initial parameters, although only slightly different. In what follows, we shall assume that the beam when undeflected is directed along the z axis of a rectangular coordinate system and travels in the positive-z direction. Thus, all electrons in the beam will have approximately the same value of $\dot{z}_0$. We take the term "beam" to mean that the transverse velocities $\dot{x}_0$ and $\dot{y}_0$ are small compared with $\dot{z}_0$.

In general, the trajectory equations cannot be solved analytically. To circumvent this difficulty, perturbation methods called *aberration theory* have been developed. In the classical formulation (e.g., Glaser, 1949,1952; Haantjes and Lubben, 1957, 1959; and Kaashoek, 1968), the Lagrangian function is expanded in a four-dimensional power series with dimensions x, y, x', y'. The initial conditions are specified by giving the total energy (or beam potential), the position and slope x_s, y_s, x_s', x_s' *at the screen* for the undeflected ray, and the position $z = z_0$ of the starting plane. Here, primes indicate z derivatives. The deflecting fields and certain of their derivatives must be supplied along the z axis.

The resulting equations of motion are solved by successive approximations. The result is a power series for the landing position of a ray at the screen or target plane. The first correction to the landing point of the undeflected ray is the Gaussian deflection, which is proportional to the deflection potential or deflection current. The next correction consists of the so-called third-order aberrations, which fall into three classes. Terms that are cubic in the x and y Gaussian deflections but independent of x_s, y_s, x_s', y_s' produce *raster distortion*. Terms that are quadratic in the Gaussian deflections but linear in the ray parameters produce *curvature of the field* and *astigmatism*. Terms that are linear in the Gaussian deflection but quadratic in the ray parameters produce *coma*. Higher corrections are rarely used because of their number and complexity. In all cases, the coefficients of the power series are complicated integrals of Gaussian deflections and field functions on the z axis.

The aberration theory has recently been improved (e.g., Chu and Munro, 1982a) in connection with deflection systems for scanning microscopy and electron-beam lithography. We shall consider these improvements under *Mixed-Field Deflection*.

It is important to note that the aberration theory of deflection is a *narrow-angle theory*. In the third-order approximation, the theory is limited to deflection angles of 20° or 25°. The series expansions employed in the derivation of the theory fail altogether to converge for angles of 45° or greater. To avoid this limit, some authors have employed curved optical axes for their perturbation expansions. However, such theories are even more unwieldy than the narrow-angle versions. The paper of Hutter (1970) expounds this method and also has a good review of earlier work. In all cases, these perturbation approximations apply only to static fields.

The failure to obtain analytical solutions and the limitations of aberration theories have led to the development of numerical methods for use on computers. The problem is the solution of the three coupled second-order equations (27) or the six first-order equations (28) - (29). These are linear ordinary differential equations for which several methods of solution exist. A recent review of this topic appears in Kasper (1982).

In the discussion above, we have tacitly assumed that we can neglect any changes in the deflection fields that occur during the passage of a particular electron through the deflector. For this assumption to be valid, a condition like (20) must apply. That is, for a sinusoidal input to the deflector, we must have

$$2\pi f t_e << 1 \tag{30}$$

in which f is the frequency and t_e is the transit time for a single deflector. If this is true, we can use the instantaneous values of the fields at the time the electron enters to calculate the entire trajectory. Thus, *two* conditions, (20) and (30), must be satisfied for the static-deflection approximation to hold.

Calculation of Beam Properties

After we have calculated a single trajectory in the deflecting fields, we are far from knowing how the entire beam behaves. If we think of each electron as a point in the phase space $(x, y, z, \dot{x}, \dot{y}, \dot{z})$, then the beam is represented at any instant by a cloud of points in the same six-dimensional space (we assume non-interacting electrons). It is clearly impossible to calculate the paths of all these electrons through phase space, hence the importance of the methods used to determine the properties of the beam and of the final spot by less drastic means. Since most of these methods involve small subsets of the electrons forming the beam, it is essential to remember the full complexity of the distribution in phase space; otherwise, it is easy to draw unjustified conclusions.

Under the conditions of static deflection, a cloud of points representing a group of electrons moves with time so that the volume of phase space occupied remains constant (Liouville's theorem); however, the shape of that volume can alter significantly. Thus, if the spread in velocity increases, the spread in position must decrease and vice versa. Consequently, considerable knowledge of the beam can be obtained by following the evolution of the *boundary* of the volume. To aid in visualization, the volume can be projected onto the three phase planes $(x,\dot{x})$, $(y,\dot{y})$, $(z,\dot{z})$. Where one or more of these phase planes are uncoupled in the equations of motion, Liouville invariance can be applied to the projection of the total volume onto the uncoupled plane or planes and the remaining volume. See the reviews by Lejeune and Aubert (1980) and by Crawford and Brody (1966) for an introduction to these ideas.

If the beam is approximately monoenergetic, a useful description of the beam is the amount of charge passing through the point (x,y) in the transverse plane z = constant per unit time and per unit volume $dxdydx'dy'$ in the transverse *trace space* (x,y,x',y'). This quantity is variously named and symbolized. We shall call it *brightness* and denote it by $R(x,y,x',y',z)$ after the original German term *Richtstrahlfunktion*. Note that Liouville invariance does not generally apply in trace space.

The brightness function can be measured (e.g., Lejeune and Aubert, 1980; Lauer, 1982) or simulated on a computer (e.g., Hauke, 1977) for some electron guns. Then the brightness in the exit plane $z = z_0$ of the gun can be transformed into the target or screen plane $z = z_1$ by using the transformation functions $x_1 = x_1(x_0,y_0,x_0',y_0')$, etc. that connect the initial with the final ray parameters. These functions are obtained from trajectory calculations for the deflectors. The initial and final brightnesses are obtained from the relation

$$R(x_1,y_1,x_1',y_1',z_1)dx_1dy_1dx_1'dy_1' = R(x_0,y_0,x_0',y_0',z_0)Jdx_0dy_0dx_0'dy_0' \tag{31}$$

in which J is the Jacobian determinant of the parameter transformations.

The current density in any plane $z = z_1$ can now be found by integrating the left side of (31) over x_1' and y_1'. This has actually been done by Wang (1967a) for some special cases using the full Jacobian formalism. This was possible because Wang obtained analytical transformation functions using aberration theory from which the Jacobian was calculated analytically.

However, for computer simulations this cannot be done directly. Consequently, it is more common to calculate a large number of trajectories by numerical integration and to plot their landing points to give an impression of the deflected spot. The initial parameters of these trajectories are chosen to represent the density distribution of electrons in phase space. Examples of this approach include Wang (1967b), Lucchesi and Carpenter (1979), Kanaya and Baba (1980), and Baba and Kanaya (1981). (Calculations of this kind require careful interpretation because the actual density in phase space is frequently approximated using more or less drastic assumptions.)

A simpler and much older method of characterizing the deflected spot uses *aberration figures*. In this method, the undeflected beam is represented by a hollow cone having base radius r and height L (in the z direction). This cone is traced out by a rotating ray starting at an azimuthal angle φ in the base. The aberration figure is the closed curve described at the screen by the landing point of the deflected generating ray. The values of r and L can be varied to explore the behavior of various parts of the brightness distribution. These aberration figures can be calculated (e.g., Wang, 1967b; Kasper and Scherle, 1982; Kanaya and Baba, 1980) and also measured using suitable apparatus (e.g., Friend, 1951). A disadvantage is the limitation to meridional rays (point-focused beam).

Electrostatic Deflection

Introduction

Most of the published work on this topic after the review of Ritz (1979) appears to be confined to electrostatic multipole deflectors. This review, together with that of Hutter (1974), covers earlier work. Great improvements in the calculation of electrostatic fields in these systems have been made since 1979. The level of sophistication now approaches that already attained in the study of magnetostatic deflecting fields. The new work has been done in connection with narrow-angle deflectors for electron-beam lithography and scanning electron microscopy.

There is nothing new of interest in the calculation of trajectories by numerical integration; however, the aberration theory is being applied extensively, and several improvements have been made. As these improvements are general enough to apply to systems with mixed electric and magnetic fields for both focusing and deflecting the beam, they will be considered in the section on mixed-field deflection.

In the study of deflection aberrations of the beam, accurate determination of the fields has permitted wider use of the coefficients from the aberration theory. These have been used to construct aberration figures and spot profiles generated from a multitude of individual landing points of deflected rays. Also, progress involving heavy use of the aberration theory has been reported in the dynamic correction of beam aberrations with stigmators.

Calculation of the Electrostatic Field in Multipole Deflectors

Multipole deflectors are of two types, which we shall call surface multipoles and rod multipoles. Both types consist of longitudinal electrodes arranged on a surface of revolution, usually a cylinder. However, surface electrodes are thin and conform to the surface of revolution, while rod electrodes are usually cylindrical in cross section with their *axes* lying on the surface of revolution. Appropriate deflection potentials are applied to each electrode through a resistor chain or by separate voltage supplies. Such deflectors may provide one or two axes of deflection. If both x and y deflection are provided, the deflector is often referred to as an electrostatic *yoke*. In the following account of field calculations for these deflectors, we follow the treatment of Munro and Chu (1982b).

The potential Φ for surface multipoles can be written as a Fourier series

$$\Phi(r,\varphi,z) = \sum_{m=1}^{\infty} [U_m(r,z)\cos m\varphi + V_m(r,z)\sin m\varphi], \quad m \text{ odd} \tag{32}$$

in the azimuthal angle φ (cylindrical coordinates r,φ,z). The terms in U_m give the horizontal deflection and those in V_m give the vertical deflection. When substituted into the Laplace equation (16), this Fourier expansion gives the *reduced* Laplace equation

$$\frac{\partial^2 U_m}{\partial r^2} + \frac{1}{r}\frac{\partial U_m}{\partial r} + \frac{\partial^2 U_m}{\partial z^2} - \frac{m^2 U_m}{r^2} = 0 \tag{33}$$

which is satisfied by both U_m and V_m.

On the surface $r = R_d(z)$ of the deflector, U_m and V_m take the values

$$U_m(R_d,z) = \frac{1}{\pi}\int_0^{2\pi} \Phi(R_d,\varphi,z)\cos m\varphi d\varphi \tag{34}$$

$$V_m(R_d,z) = \frac{1}{\pi}\int_0^{2\pi} \Phi(R_d,\varphi,z)\sin m\varphi d\varphi \tag{35}$$

which give the required Dirichlet conditions for the solution of (33) for U_m and V_m. (This assumes that the variation of potential in the gaps between electrodes can be neglected or approximated in some manner, e.g., linearly.)

The *multipole potentials* U_m and V_m can be expanded about the z axis in power series

$$U_1(r,z) = -f_1(z)r + \frac{1}{8}f_1''(z)r^3 - \ldots \tag{36}$$

$$U_3(r,z) = -f_3(z)r^3 + \frac{1}{16}f_3''(z)r^5 - \ldots \tag{37}$$

and so on. Consequently, all of the U_m and V_m vanish for $r=0$ and vary as r^m near the axis. That is, near the axis we can neglect the terms beyond some finite value of m as being small corrections and truncate the infinite series (32). At large distances from the deflector, the multipole potentials must remain finite. Thus, the three-dimensional problem is reduced to a finite sum of two-dimensional problems.

In general, the reduced Laplace equation must be solved by numerical computation. Use of the finite-element method has been reported by Munro and Chu (1982b). As in magnetic deflection, one could also use the finite-difference method. We shall later discuss such a solution for the full three-dimensional field. The functional to be minimized has the form

$$F_m = \iint \frac{1}{2}\epsilon_0\left[\left(\frac{\partial U_m}{\partial r}\right)^2 + \left(\frac{\partial U_m}{\partial z}\right)^2 + \left(\frac{mU_m}{r}\right)^2\right]\pi r dr dz \tag{38}$$

for U_m with a similar expression for V_m. The integration is over the part of the (r,z) plane with $r \geq 0$ and enclosed within an outer boundary (at a large distance from the deflector) on which $U_m = V_m = 0$. References describing the details of the finite-element minimization procedure were given under *Calculation of Electromagnetic Fields*, above.

The method just described is inappropriate for rod-multipole deflectors because the boundary potentials are not defined on a surface of revolution about the z axis. Consequently, the calculation remains a true three-dimensional problem. To solve it, Munro and Chu (1982b) introduce the "charge-density method," a special case of the integral-equation approach.

Each electrode surface is subdivided into many small patches or subelectrodes, the j-th one of which carries a uniform surface-charge density σ_j. The potential is then expressed as the integral (21) over the surfaces of the electrodes. This integral becomes a summation over the subelectrodes. The potential at the center of subelectrode i resulting from all the N σ_j is

$$\Phi_i = \sum_{j=1}^{N} P_{ij}\sigma_j \tag{39}$$

in which the coefficients P_{ij} are

$$P_{ij} = \frac{1}{4\pi\epsilon_0}\iint_{S_j}\frac{dS_j}{R_{ij}} \tag{40}$$

where $R_{ij} = |\mathbf{x}_i - \mathbf{x}_j|$ and S_j is the area of subelectrode j. Equation (39) represents N equations in N unknowns for the charge densities σ_j. The Φ_i are all known from the

given electrode potentials and the P_{ij} are calculated from the integral (40). These equations are solved numerically by any suitable standard method. Once the σ_j are known, the potential anywhere can be calculated by performing the integral (21) for each subelectrode and then summing over all the subelectrodes.

Munro and Chu (1982b) also describe the calculation of an effective capacitance for these deflectors using both the finite-element and integral-equation results. The calculation rests on the relationship between stored energy and capacitance.

Baba and Kanaya (1981) have calculated the field of a quadrupole deflector having noncylindrical rod electrodes of finite length. They use the finite-difference method with successive over-relaxation to calculate the full three-dimensional field. The computational mesh is defined by horizontal and vertical lines in the (r,z) planes and by circles and radial lines in the (r,φ) planes.

In comparing the finite-element, finite-difference, and integral-equation methods, it is well to note that, according to Kasper (1982), the finite-element method suffers from a loss of accuracy in rotationally symmetric systems near the axis. This is a result of the linear approximation to the potential that is ordinarily employed in each finite element. The problem can be corrected by using a higher-order approximation, but at the expense of increased complexity. Although a study of this problem for deflection fields has not been published, it appears that the finite-element method should be used cautiously.

Calculation of Electron Trajectories and Beam Properties

These calculations have been carried out in the framework of aberration theory in the papers seen. Numerical integration of trajectories has not been used. No doubt this is because the applications for these deflectors are in narrow-angle systems, for which the third-order theory is generally adequate. Aberration theory also has the advantage of giving the landing positions as power series in the initial ray parameters, which permits economical calculation of numerous landing points to simulate the deflected spot.

Chu and Munro (1982a) formulate extensive improvements of third-order theory to permit calculations for any combination of electric or magnetic, focusing or deflection fields. Kanaya and Baba (1980) and Baba and Kanaya (1981) display numerous examples of both aberration figures and spot profiles constructed from many individual landing points. (The first of these two papers deals with a sequential system of parallel-plate deflectors.) Kanaya and Baba also describe the use of dynamic focusing and an octupole stigmator to correct for deflection aberrations.

In closing, note that the multipole potentials in $m = 1$ and $m = 3$ completely characterize the fields for purposes of third-order aberration theory. These first two harmonics contain all terms depending on r or r^3. These are precisely the terms needed to specify the field profiles on axis that appear in the integrals for the aberration coefficients.

Magnetostatic Deflection

Introduction

As was the case in the earlier review (Ritz, 1979), the bulk of the work in the theory of deflection continues to be done in magnetostatic deflection, and the largest portion of that effort relates to color television. However, the work in magnetostatic deflection for electron-beam lithography is increasing.

In field calculation, the finite-difference methods that were previously the standard have been supplemented with finite-element and integral-equation calculations. In the latter method, the requirement of infinite permeability for the core has been relaxed, permitting investigation of the effects of finite permeability on the deflecting fields. Also, the field calculations have been used to calculate the stored energy and inductance for magnetic yokes.

In trajectory calculation, both numerical integration and aberration theory continue to supply useful results. Substantial advances in the theory of self-converging inline color TV have been reported using both techniques, including a new method for reducing the residual misconvergence in these systems. The improvements in aberration theory alluded to above apply also to magnetostatic deflection, although we again defer discussion to *Mixed-Field Deflection*.

In the calculation of beam and spot properties, detailed simulations have been carried out for examples of self-converging inline systems using fields calculated by finite differences and trajectory end points determined by a combination of numerical integration and interpolation. The results agree very well with experiment. Several studies have produced aberration figures for deflection in magnetic yokes. A method for combining a minimum number of directly integrated trajectories with interpolating functions of a theoretically desirable form has been described. This technique permits large numbers of landing points to be calculated from a small number of suitably chosen sample trajectories.

Finally, some progress has been reported on the problem of synthesis in the design of magnetostatic deflectors. As opposed to the problem of analysis of given structures to determine their behavior, the problem of synthesis requires the determination of structures that will produce a desired behavior. In general, such a structure may not exist. The problem of synthesis is then one of optimization. The reported work addresses both the existence and the optimization problems.

Calculation of Fields

In the work reviewed by Ritz (1979), the technique of finite differences dominated the numerical calculation of fields in magnetostatic deflectors when permeable cores were present. The finite-element method was just being introduced. In recent years, the fashion has shifted somewhat in the direction of integral-equation methods, beginning with Fye (1979) and Ţugulea *et al.* (1979). These two papers illustrate divergent approaches, however. Fye reduces the dimension of the problem from three to two by Fourier analysis in the azimuthal angle φ as above, expressing the density of surface current in a Fourier series. Ţugulea and his co-workers solve the full three-dimensional problem, as do Munro and Chu (1982b) in the analogous electrostatic case. Kasper and Scherle (1982) follow Fye in this respect, but employ a variant procedure in which rings of magnetic charge replace the permeable surfaces and currents. In Scherle (1983a,b), the permeability of the core need not be infinite. He provides numerous field profiles for saddle and toroidal yokes having values of μ/μ_0 ranging from 10^5 to 1. Above 10^3, he finds the effects of further increases hardly noticeable.

The finite-difference method continues to be used.

Yokota *et al.* (1979) apply the FDM to a yoke for 110°, 18-inch diagonal deflection for a color-TV application. This yoke has toroidal vertical and saddle horizontal windings. The toroid is wound on the surface of the core while the saddle is detached. They employ the method of Nomura (1971) in dealing with this detached winding. Equipotential plots, field profiles, and computer-program organization are given. However, they encounter unexplained difficulties with some of their trajectory calculations. Ximen and Chen (1980) also employ the FDM in the simulation of a toroidal yoke on a cylinder. In all cases, these authors first decompose the scalar potential Ψ and the current distribution into Fourier series in azimuth φ as described, for example, in the review by Ritz (1979) and then solve the resulting two-dimensional problems. Thus, nothing fundamentally new is being reported here.

Munro and Chu (1982a) describe the application of the finite-element method to the fields of saddle yokes with or without the presence of permeable elements. As with the FDM, two-dimensional multipole potentials are calculated. The FEM permits the use of finite permeability for the core. A valuable feature of this paper is a comparison of the finite-element solution for a particular yoke with the analytic solution as calculated by the Biot-Savart law from the currents (no core present). An accuracy of 1% to 2% is reported.

An interesting analytic method is described by Dasgupta (1983b). He assumes that in each cross section z = constant, the fields behave as do those of an infinite cylindrical yoke of the same cross section. The effect of the end turns in saddle and toroidal yokes is more difficult to calculate. Dasgupta ignores the effect of the core on the end turns in saddle windings and neglects toroidal end turns altogether. The resulting fields are checked against fields calculated by the method of Fye (1979). There is a good resemblance between the two but errors are substantial, on the order of 20% in field strength. This accuracy is probably inadequate for most purposes. Also, the basic calculation uses the Biot-Savart law and is quite involved, requiring the use of hypergeometric and beta functions.

In another paper, Dasgupta (1983a) analyzes the effect of finite winding thickness on the field. This is an important question, because ordinarily it is assumed that the windings can be represented by sheet currents. He uses analytical calculations for infinite-cylinder yokes to show that for thick windings the higher harmonics of the field have smaller amplitudes than in the thin-winding case. Also, since the size of the error depends on m, there is no "average" position at which the approximate sheet current can be placed. The effect is more severe for detached than for surface windings.

Calculation of Energy Storage and Inductance

Ritz (1980) describes the calculation of energy storage and inductance for magnetic yokes, using fields calculated by the FDM for the angular harmonics. He shows that each harmonic of the field contributes independently to the stored energy and inductance. The calculation rests on the relation

$$W = \frac{1}{2} LI^2 \tag{41}$$

amongst inductance L, current I, and energy W stored in the field of the yoke. The stored energy is calculated by integrating the energy density $\frac{1}{2}\mu|\mathbf{H}|^2$ over the entire volume containing the field. This leads to two methods: a surface integral in the (r,z) plane and a contour integral in that plane. The second follows by use of Green's first identity of potential theory. The theory is used to compare energy storage in saddle and toroidal yokes. [Note that the sign of his Eq. (7) is reversed.]

Dasgupta (1982) calculates the inductance directly by means of flux linkages for saddle yokes only. His result is a special case of Ritz's second (contour integral) method. The fields used are calculated by the FDM.

Scherle (1983b) also calculates the stored energy but uses fields calculated by the IEM. His method permits the core permeability to be finite, and he gives a very instructive graph showing the effect of permeability on the total stored energy for saddle and toroidal yokes. The energy increases until $\mu/\mu_0 \approx 500$, after which little change occurs.

Calculation of Trajectories

Both numerical-integration and aberration-theory calculations of trajectories are well established in magnetostatic deflection. Consequently, applications rather than newly introduced methods are currently interesting. The important areas of study are misconvergence in self-converging color inline CRTs and deflection aberrations in systems for electron-beam lithography. As noted previously, the latter topic is considered under *Mixed-Field Deflection*. Good accounts of the misconvergence problem, based on third-order aberration theory, are given by Heijnemans *et al.* (1980) and by Hutter (1979). We summarize them here.

An inline color CRT contains three electron guns: red, blue, and green. The green gun is aimed along the z axis while the other two lie at equal distances either side of green in the horizontal (x,z) plane. The two outer beams are aimed so that all three undeflected beams land at the center of the screen. The common landing point must also be maintained as the beams are deflected. Departures from this condition are termed *misconvergence*. In *self-convergent* inline CRTs, convergence is achieved by proper design of the yoke, gun, and sometimes magnetic pole pieces rather than with auxiliary convergence coils driven by dynamic currents that depend on the deflection currents, as in delta-gun CRTs.

Self-convergence is quite difficult to achieve. Ordinary deflection yokes with so-called homogeneous fields cause the three beams to cross and diverge before reaching the screen. This defect is partially corrected by giving the horizontal deflecting field a pin-cushion shape and the vertical field a barrel shape. This is done by introducing a large third-harmonic component into the angular distribution of winding current for each axis of deflection. The strengths are adjusted to remove *horizontal* red-blue misconvergence at the ends of the x and y axes on the screen.

In the third-order aberration theory, this also removes *horizontal* red-blue misconvergence in the corners but leaves a *vertical* red-blue misconvergence due to *astigmatism* error. There is also *coma* error, which causes the green beam to land inward from the average position of the red and blue spots.

Astigmatism is affected most strongly by the amount of third harmonic at the yoke *exit* while coma is affected most strongly at the *entrance* and with the opposite sign. Consequently, by having the amount of third harmonic in the windings change sign from front to rear, both astigmatic and comatic misconvergence can be removed at the ends

of the x and y axes, although the vertical astigmatic misconvergence between red and blue remains in the corners. The desired winding distribution is achieved with nonradial (nonmeridional) turns. All of these conclusions follow from use of the third-order aberration theory.

There is another widely used method of correcting for the coma error. Permeable pole pieces called *shunts* are placed at the exits of the two outer guns (red and blue) to reduce their deflection slightly, and another pole piece called an *enhancer* is placed at the exit of the center (green) gun to increase its deflection slightly. Because the effect depends linearly on the strength of the yoke field, it exactly corrects everywhere for coma error, which depends linearly on the yoke currents. Formerly, the pole pieces were designed empirically, but recently Fye and Grinberg (1980) have calculated the effects of these field controllers using the integral-equation method.

Ando *et al.* (1977) and Nakamura *et al.* (1982) have asserted and demonstrated, respectively, that the residual vertical red-blue misconvergence in the corners can be reduced or even eliminated by displacing the horizontal winding toward the gun with respect to the vertical winding. The demonstration is accomplished with the third-order aberration theory as formulated by Kaashoek (1968).

Calculation of Beam Properties

Severe aberration of the deflected spot is an unwanted consequence of self-convergent deflection fields. The yoke produces a vertical line focus everywhere on the screen for a cone of rays focused at the center of the screen. The diameter of the base of the cone is the distance between the two outer beams at the exit of the gun. The deflected image of the smaller cone approximating the green (center) beam should also be a (shorter) vertical line. However, because practical beams have spherical aberration and hence are focused (undeflected) for the circle of least confusion, the resultant spot is actually quite different. There is a bright horizontal line or cigar with triangular regions of flare above and below and with a large elliptical halo surrounding the core region. (See Yoshida *et al.*, 1974 and Hutter, 1979 for qualitative explanations of these phenomena.)

Lucchesi and Carpenter (1979) have simulated both the undeflected and deflected spots by computer for a self-converging inline yoke. This is the most elaborate and complete spot simulation in electron optics that is known to this reviewer. They begin with a computer program that computes positions and slopes for 123 trajectories at the exit of the electron gun, including the effects of space charge and thermal velocities. These trajectories, calculated originally in the (r,z) plane, are repeated at 100 equally spaced angles in the (r,φ) transverse plane. The resulting 12,300 rays are projected through the yoke fields with the aid of numerical integration.

However, Lucchesi and Carpenter do not integrate *all* these rays, which would be impossibly expensive and time consuming. Instead, they calculate a much smaller number, taking twelve different azimuths, four different radii, and six different slopes, plus the central ray, for a total of 289 trajectories. The landing points for all of the other rays are found by linear interpolation between the directly calculated rays.

The result is a picture of the deflected spot that agrees remarkably well with photographs of actual spots. The main discrepancy is a reduction of size, which the authors ascribe to the neglect of space charge in the deflecting fields. This reviewer suggests that their apparent neglect of skew rays may also contribute to the discrepancy.

The principal defect of their method is the still large number of trajectories to be calculated. This number can be reduced by an order of magnitude using a technique described by Ritz (1981). The x and y positions at the screen are expressed as Fourier series in the azimuthal angle φ for a ray lying on the surface of a cone having base radius r and length L (undeflected). The ordinary aberrations of astigmatism and coma require only the terms in φ and 2φ for their representation. Ritz shows that only 16 trajectories with various values of r, φ, and L need be calculated to determine all the necessary Fourier coefficients. The angular dependence is fitted using the fast Fourier transform. The dependences on r and L are fitted by least squares.

Once the coefficients are known, any landing point can be interpolated between the sample values. In this way, landing points become easy, fast, and inexpensive to calculate in great numbers. Thus, aberration figures and the current distribution in the spot become much more accessible.

This method is actually a special case derived from a six-dimensional power series expansion in a phase-space perturbation theory of wide-angle deflection. Typically, powers of order 2 or 3 (Fourier components through 2φ or 3φ) suffice for excellent accuracy of fitting. If the power series formulation were used, least-squares fitting to the samples in r and L would be unnecessary and the number of samples required would be reduced further for a given accuracy.

The Problem of Synthesis

Suppose that we ask what *winding density* is required to produce a desired set of *harmonics* of the winding, determined for example by the procedure described above for achieving a self-convergent inline yoke. At first glance, this question appears to be a trivial matter of adding up the individual harmonics of the winding according to their required amplitudes.

However, in practical yokes the mechanical constraints on the placement of the wires may make it impossible to produce the harmonics required, and only those harmonics. We must therefore synthesize a winding that comes as close as possible to the desired one. This inverse problem is exceedingly difficult and has not been addressed previously in any effective way.

The three papers of Vassell (1981a,b,c) address the question of windability for winding densities with a finite number of adjustable parameters, discrete densities, and densities with an infinite number of parameters. These papers are difficult though elegant. In essence, Vassell transforms the winding variable from azimuthal angle φ to $\xi = \cos^2\varphi$ and then applies some results of the branch of mathematics concerned with the theory of power moments of distributions.

Each winding density is characterized by its normalized harmonic ratios

$$h_{2p+1} = \frac{A_{2p+1}}{A_1} \tag{42}$$

in which A_{2p+1}, $p = 1, 2, 3, \ldots$ is the amplitude of the $2p+1$ harmonic of the winding. For a given set of constraints on the winding, the set of feasible windings is represented by a subregion of the space with coordinates

(h_3,h_5,h_7, . . .). These regions of feasibility are determined using the moment theory. Since in practice, one specifies a small finite number of harmonic amplitudes, the space in question is usually of low dimensionality. In any case, these papers deserve wider notice than they have received so far.

Dasgupta (1983c) addresses the same problem as Vassell, but limits his study to toroidal yokes having radial (meridional) windings. Like Vassell, Dasgupta derives conditions on the values of the harmonics permitted by the winding constraints, but his approach and results are far less general. He also proposes to circumvent these restrictions by adding turns at properly chosen angles.

Another approach to determining a structure that will have the desired electron-optical behavior is taken by Chu and Munro (1982b). They consider a set of m functions f_i depending on a set of n parameters. Each function is the product of a weight and an individual aberration. A *defect function* or *merit function* equal to the sum of the squares of the m functions f_i is to be minimized by varying the n parameters.

This is done using the *damped least squares* method, according to which the partial derivatives of the f_i^2 with respect to the parameters are used to obtain an improved value of the defect function. A damping factor is used at each step to stabilize the process and to speed convergence. The calculations are done by computer. As an example, they optimize a pure magnetic focusing and deflection system suitable for electron-beam lithography, although a pure deflection system could presumably be treated in the same manner.

Mixed-Field Deflection

Introduction

Under this topic, we shall consider deflecting systems composed of electric or magnetic fields for focusing or deflection in any number or combination. Historically, mixed-field devices have been used typically in imaging tubes such as vidicons and in such devices as scan converters. However, the current field of greatest activity is in electron-beam lithography and scanning microscopy.

This section is organized somewhat differently from the previous sections. Because we have already discussed the calculation of electrostatic and magnetostatic deflecting fields individually, we need not do so again here. Likewise, as the calculation of fields for focusing lenses is widely discussed in the literature of electron optics, there is no point in repeating that discussion here. Consequently, we shall proceed under the combined topic of trajectory and beam calculation.

Calculation of Trajectories and Beam Properties

Because the typical electron-beam lithography or scanning-microscope deflecting system is a narrow-angle system, the aberration theory of deflection is appropriate and is heavily used. However, the previous formulations of aberration theory are intended for single- or double-axis deflectors and not for multiple sequential deflectors or for deflectors with overlapping focusing fields.

This circumstance has prompted many recent improvements of the third-order aberration theory. The most comprehensive and accessible treatment is that of Chu and Munro (1982a). We summarize here the principal features of their formulation and compare it with the traditional formulation.

Chu and Munro permit any combination of focusing and deflecting fields; these may overlap. The electron source may have a finite size. Both electric and magnetic fields can be used.

The deflection currents are expressed in complex form $I = I_x + iI_y$ and the deflection potentials as $V = V_x + iV_y$, with complex conjugates $\bar{I}$ and $\bar{V}$. The same currents and potentials are applied to all deflectors in a multiple deflector. The complex current and complex potential appear explicitly in the ultimate aberration expansions. In the conventional theory, they are concealed in the Gaussian deflections.

The theory uses the complex position $w = x + iy$ and complex slope $w' = x' + iy'$ as well as their complex conjugates $\bar{w}$ and $\bar{w}'$. The ray parameters are expressed as $w = w_0$, $w' = s_0$, I, V in some initial plane $z = z_0$. Contrast this with the conventional method of specifying position and slope of the undeflected ray *at the screen.*

The fields and trajectories are expanded in powers of w, $\bar{w}$, w', $\bar{w}'$, I, $\bar{I}$, V, and $\bar{V}$ as appropriate. However, instead of using these expansions to construct the Lagrangian and Euler-Lagrange equations, Chu and Munro use the Lorentz equation (27) directly (after changing variables from t to z). The resulting equations of motion are solved by successive approximations. In first order, the two Gaussian deflections in x and y are replaced by four principal rays. Each principal ray is specified by w_0, s_0, I, and V, of which one parameter is unity and the rest zero for a given ray. These principal rays are then used to obtain third-order corrections.

The final aberrations are expressed in terms of the following complex quantities at the image plane: aperture angle s_i, Gaussian spot size w_{ig}, magnetic deflection vector w_{im}, and electrostatic deflection vector w_{ie}. These are simply related to w_0, s_0, I, and V by the four principal rays. In general, there are 59 complex coefficients. When the Gaussian spot size w_{ig} can be neglected, there are only 27.

The 27 aberration coefficients for the latter case are given in terms of two general integration functions $F(w_1, w_2, \bar{w}_3)$ and $G(\bar{w}_1, \bar{w}_2, \bar{w}_3)$ in which w_1, w_2, and w_3 are dummy arguments. These functions are complicated integrals that require computer evaluation.

The effect of these improvements is to increase generality, simplify the use of symmetry arguments in eliminating terms, and simplify algebraic bookkeeping and computer programing.

Many other authors have applied similar ideas to various systems too numerous to mention here individually. These include Goto and Soma (1977); Ohiwa (1978, 1979); Kuroda (1980); Lencova (1981); Hosokawa *et al.* (1981); Ximen and Li (1982); Li and Ximen (1982); and Chen *et al.* (1983). The paper of Chu and Munro (1982b) on optimization was reviewed above under *Magnetostatic Deflection.*

Traveling-Wave Deflection

Assessment

There are no recent publications reporting anything new of particular note on the theory of traveling-wave deflectors since the review by Ritz (1979). This writer still retains his opinion that further progress requires use of computer simulations of the full wave fields as described in the *Introduction* of the present paper. Such simulations

have been done for other high-frequency structures but not yet for traveling-wave deflectors.

Scan Expansion

Introduction

Only a few papers have appeared on the theory of scan expansion since the previous review by Ritz (1979). These papers fall into three categories: mesh scan expansion; meshless expansion with rotationally symmetrical lenses; and meshless expansion with quadrupole lenses. The techniques of analysis used include geometrical optics, field calculations in two and three dimensions using numerical methods, and third-order aberration theory.

Calculation of Fields

The FDM is now being used in the calculation of fields for scan-expansion systems. Hawes and Nelson (1978) employ this technique to calculate the field for a rotationally symmetric mesh post-deflection lens. This is a two-dimensional calculation in the (r,z) plane. Franzen (1983) describes a program that combines three-dimensional FDM (Liebman) calculations with Fourier expansion methods to calculate the fields in a complex, nonrotationally symmetric meshless lens of quadrupole type. The Fourier-Bessel expansion is used to provide boundary values for the FDM relaxation calculation. Further details are given in Janko *et al.* (1983) and in Hawken *et al.* (1983).

Haley *et al.* (1979) describe the calculation of fields for a rotationally symmetric expansion lens using the method of moments, which is an integral-equation technique. Their work appears to be two-dimensional, although this is not explicitly stated. Their lens is used as a projection lens to magnify a storage mesh illuminated by a flood-beam of electrons, and hence is not a true scan-expansion lens operating on the deflected rays.

Calculation of Trajectories and Beam Properties

Hawes and Nelson (1978) have calculated rays for their mesh lens using both a thin-lens geometrical-optics method and numerical ray-tracing in a field determined by the FDM. The calculated magnifications agreed within 2%.

Franzen (1983) calculates trajectories in his meshless quadrupole lens numerically and uses the results to study and characterize the distortions of the raster. His program uses gradients of potentials calculated both numerically and from the Fourier series he uses near the outer boundary of his fields. Franzen also mentions simulations of beam defocusing, although no details are given.

Lenz (1979) presents a third-order theory of distortion in rotationally symmetric mesh post-deflection expansion lenses. The potential near the axis is obtained analytically. Then the equations of motion are solved successively in first- and third-order approximations to find the landing points at the viewing screen. No account is taken of the effects of individual mesh holes.

Conclusion

In his review of 1979, the present author predicted successfully that application of the numerical methods then being used in magnetostatic deflection to problems in electrostatic deflection and scan expansion would produce useful advances in those areas.

However, having perhaps exaggerated the relative merits of numerical simulations over aberration theory, he failed to perceive the renascence of aberration theory that was even then occurring in electron-beam lithography and scanning microscopy. Ironically, this new interest is partly due to the accurate field calculations made possible by numerical methods and computers. Adding embarrassment, the classical aberration theory again displayed its power of explanation in the theory of self-convergent inline deflection, even though exact calculations eluded its grasp.

When his previous review went to press, the author was unsure whether numerical simulations were sufficiently accurate to calculate beam properties. That question has now been answered resoundingly in the affirmative, and we can expect wider use of such calculations.

Regrettably, he was all too correct when asserting that further progress in traveling-wave deflection must await the application of numerical methods. Those methods have not been applied, and we are still waiting.

In the study of scan expansion, conventional methods continue in use for computer simulation of rotationally symmetrical expansion systems. For complicated systems of the quadrupole type, angular Fourier expansions have been combined with three-dimensional finite-difference methods to deal with an intrinsically three-dimensional problem. This parallels the earlier developments in magnetic-yoke theory discussed in the author's last review.

An aberration theory has been developed for scan expansion, but has not yet been applied. As many scan-expansion systems have small angles of deflection, the combination of aberration theory with numerically calculated fields might prove fruitful.

With the exception of traveling-wave deflection, the work described in this review generally shows increasingly sophisticated computer-aided simulation of increasingly complex deflection systems. Aberration theory and numerical calculation of fields and trajectories have been improved. Full three-dimensional field calculations are becoming feasible. The output of space-charge models of electron guns has been combined with deflector models to produce excellent depictions of deflected spots, although space-charge effects in the deflectors themselves have yet to be treated successfully. Thus, many of the long-standing theoretical problems of electron-beam deflection have either been solved or have solutions in view.

With this increasing power of *analytical* techniques and understanding (including numerical analysis), attention is beginning to shift to the *synthetic* or design problem in electron-beam deflection. This reviewer believes that several influences tend in this direction.

First, the power and accuracy of numerical simulations are usually gained at the expense of analytical and intuitive understanding. Each simulation is a special case, much like a single experiment, and inference of general behavior requires many special cases to be studied in a manner almost empirical.

Second, although in some cases the use of aberration theory provides analytical insight, this theory is very complicated even for simple deflection systems. This complication greatly reduces its usefulness in aiding understanding.

Third, the complicated mixed focus and deflection systems now used in lithography and scanning microscopy have so many parameters that it is difficult to comprehend the design possibilities and limitations of a given system.

There are thus two challenges for future work in this field. First, momentum must be maintained in attacking the remaining analytical problems. Second, means must

be found to assimilate and use effectively the mass of results being obtained as a result of successes in analytical theory and computer simulation.

Several of the papers reviewed show some progress in meeting the second challenge, but much remains to be done to prevent our current successes from burying us.

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