Synthesis of Electron Lenses

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Abstract

This paper is a review of different approaches to one of the most ambitious goals of Electron and Ion Optics: to produce elements and systems with prescribed first-order properties and minimal aberrations. Synthesis of such elements is usually done in two steps: the first is a search for a field distribution with the given properties and the second is the reconstruction of electrodes (pole pieces) that would produce this field distribution. The first problem can be solved by the use of different techniques: Calculus of Variations, Dynamic Programming or Function Minimization. The second one is more complicated and requires a lot of ingenuity. Our novel approach takes a different course of action. It combines the two steps into one. Low-aberration field distributions are sought by dynamic programming or function minimization procedures in the form of continuous curves constructed of cubic splines. A very simple algorithm is used for the reconstruction of the electrodes or pole pieces. This approach combines the widely recognized advantages of our optimization techniques with a built-in accurate and effective reconstruction procedure. The final design is simplified on the basis of the requirements of manufacturability. High-quality electrostatic lenses have been designed by the use of this technique. Thus, electron and ion optical synthesis has been transformed from a dream to reality.

Key words: aberrations, chromatic aberration, electron lens, electrostatic lens, field distribution, ion lens, optimization, potential distribution, spherical aberration, synthesis.

Introduction

There has been a struggle to overcome the aberrations of electron lenses since the very beginnings of Electron and Ion Optics. The reduction of aberrations is of specific importance now because of the great interest in particle beam technologies. Electron and ion beam lithography and especially focused ion beams offer new possibilities for integrated circuit fabrication as well as new microanalytical capabilities.

Ion beams must be focused by electrostatic lenses to ensure the independence of the refraction of the charge-to-mass ratio of the ions. However, due to the fact that electrostatic fields can not be concentrated to narrow regions like magnetic fields, there is a widespread belief that the aberrations of electrostatic lenses are intrinsically much higher than those of magnetic lenses.

Scherzer (1936b) showed that it is impossible to correct all of the aberrations of axially symmetric electron lenses by another system of similar symmetry but it was he again who proposed several other ways to compensate them by introducing additional features: discontinuities in the field distributions, space charge, high-frequency fields or other types of symmetry (Septier, 1966). In this paper we shall not be concerned with compensation. Our topic is optimization i. e. search for such electron and ion lenses that would provide themselves (without additional compensating elements) the required optical properties with minimum aberrations.

There are two totally different approaches to optimization: analysis and synthesis.

If the method of analysis is used, the designer starts with a given simple set of electrodes or pole pieces. The design is gradually improved by analyzing the optical properties and changing the geometrical dimensions as well as the electric and magnetic parameters of the system. This process is repeated until it converges to an acceptable solution. Due to the infinite number of possible configurations, the procedure is extremely slow and tedious. It can yield the results quickly only if a reasonable guess of the answer is already available before the work starts or if the constraints are so severe that there is not very much choice left. The iteration procedure can be facilitated by automation (Chu and Munro, 1981).

Optimization by synthesis is based on the fact that the imaging field, its optical properties and aberrations are totally determined by the axial distribution of the field. Only the axial distribution and its derivatives appear in the paraxial ray equation and in the expressions of the aberration coefficients. Then, instead of analyzing a vast amount of different electrode and pole piece configurations, we can start with the criteria defining an optimum system as initial conditions and try to find the imaging field distribution (and subsequently the electrodes or pole pieces) that would produce it.

This is a complicated problem. Its solution has been keeping many researchers interested for an extended period of time. Before we start to review their work, let us first show the difference between the analytic and synthetic approaches by using a simple example.

This example is the ideal quadrupole lens. It consists of four identical infinite hyperbolic surfaces held at alternate positive and negative potentials. Since infinite surfaces can not be realized in practice, different approaches have been used to approximate the ideal quadrupole field. The analytic approach is to compensate the missing parts of the hyperbolic surfaces by changing the shapes of the remaining electrodes. The synthetic approach (Szilagyi, 1978b) starts with the ideal field distribution and tries to reproduce it by recognizing the fact that the most important characteristic of a quadrupole lens is not the number of its electrodes but the presence of exactly two mutually perpendicular symmetry planes. The solution then follows quite easily: a high number of simple electrodes held at suitably chosen potentials produces a much better approximation of the ideal quadrupole lens than any system of four sophisticated electrodes.

Let us now see how the problem of aberration reduction can be attacked by the synthetic approach.

Is Aberrationless Electron Optics Possible?

Contrary to a widespread belief, Scherzer's theorem does not exclude the possibility of aberrationless electron lenses. It only states that the spherical aberration coefficient can not change sign i. e. it can not be compensated by the spherical aberration of another axially symmetric electron lens. The situation is similar for the chromatic aberration. It is feasible, therefore, to search for a lens with zero aberrations.

Glaser (1940) calculated first the magnetic field distribution necessary to give zero spherical aberration. Unfortunately, the resulting field is not strong enough to produce a real image. It was established, however, that for the reduction of spherical aberration the second derivative of the axial distribution of the magnetic flux density must be positive throughout the lens. This led to a substantial improvement of beta-spectrographs (Siegbahn, 1946).

Although proofs of zero limit of the spherical aberration have been published (Kas'yankov, 1955), all the attempts to calculate a practically realizable field distribution with zero spherical aberration have failed (Crewe, 1977). Recknagel (1941) showed that electrostatic lenses free of aberrations can not form a real image either. Finally, Tretner (1959) calculated the lower limits of the spherical and chromatic aberration coefficients for both electrostatic and magnetic lenses.

Since the practical realization of aberrationless electron lenses seems to be impossible, our efforts must be concentrated on the reduction of aberrations.

Early Attempts of Synthesis

Synthesis of electron lenses with minimum aberrations has been attempted since the first decade of Electron Optics. Scherzer (1936a) and Glaser (1938) calculated the axial electrostatic and magnetic weak lens distributions, respectively, with the least spherical aberration.

The synthesis of real thick lenses is much more complicated. Kas'yankov (1952) was the first to try his wits at this formidable problem. He derived a set of high-order nonlinear differential equations the solution of which would minimize certain aberration integrals. Unfortunately, the initial conditions for these equations as well as practical methods of their numerical solution remained unclear.

Burfoot (1953) calculated a special set of quadrupole lenses of extremely complicated form to obtain a potential distribution for which the spherical aberration is corrected.

Septier (1966) mentions two other early approaches based on the general trajectory equation (P. Lapostolle) and the approximation of the ideal hyperbolic lens (R. Rudenberg and A. Septier), respectively. They produced comparatively weak lenses with not very low aberrations.

Calculus of Variations

Tretner (1954, 1959) realized that the Calculus of Variations is a natural approach to the problem of synthesis since the aberration coefficients can always be expressed as definite integrals of the form

$$C = \int_{z_o}^{z_i} F[r_1(z), r_1'(z), r_2(z), r_2'(z), V(z), V'(z), V''(z)]\,dz$$

$$= \int_{z_o}^{z_i} F(z)\,dz \qquad (1)$$

where F(z) may represent the integrand of a combination of aberration coefficients or just one coefficient, z_o and z_i are the object and image coordinates, respectively. The Calculus of Variations is well suited to minimize integrals. The difficulty is that the integrand F(z) is a complicated function of not only the unknown electrostatic and magnetic field (or potential) distributions [V(z)] and their derivatives [V'(z) and V"(z)] but also of two linearly independent solutions $r_1(z)$ and $r_2(z)$ of the paraxial ray equation

$$f[r(z),r'(z),r''(z),V(z),V'(z),V''(z)] = f(z) = 0 \qquad (2)$$

which is a second-order differential equation with coefficients depending on the same unknown field distributions and their derivatives. Though Tretner used the Calculus of Variations to find the limits of the chromatic and spherical aberration coefficients of electrostatic and magnetic lenses, he left the question of how to achieve these limits open. For special cases he managed to simplify the mathematical problem by a series of variable transformations but in a general case these simplifications usually do not work.

The first mathematically justified approach to synthesis with practical results was that of Moses. He first used Calculus of Variations for the optimization of quadrupoles (1970, 1971a, 1971b) but later successfully applied it to the design of axially symmetric magnetic lenses, too [(Moses, 1973) and (Rose and Moses, 1973)].

In order to determine the unknown functions V(z) and r(z) so that they minimize several aberration integrals simultaneously and at the same time satisfy the differential equation (2) together with additional constraints we first define the Lagrange multiplier $\mu(z)$ so that

$$\delta \int_{z_o}^{z_i} [F(z) + \mu(z)\, f(z)]\, dz = 0 \qquad (3)$$

where δ means the first variation of the integral. Carrying out the variations of the integrand and integrating by parts we obtain two high-order nonlinear differential equations (the Euler-Lagrange-Poisson equations). Combining them with the paraxial ray equation (2), one has a system of three coupled nonlinear differential equations. An additional equation appears when the residues obtained from the partial integration are equated to zero. The initial conditions are determined by the constraints. As they may be given at different (even a priori unknown) points and the differential equations are usually very complicated, special numerical procedures are needed for their solution. Fortunately, for magnetic lenses the procedure yields a system of second-order differential equations and Moses was able to solve it. He actually managed to design a coma-free magnetic lens with low spherical aberration (Moses, 1973).

For electrostatic lenses the situation is much more complicated. The Euler-Lagrange-Poisson equations yield a system of fourth-order differential equations with complicated initial conditions.

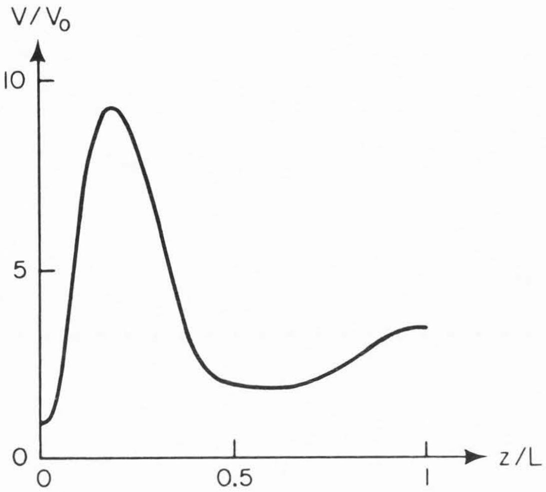

Fig. 1. Axial potential distribution for a high-performance electrostatic lens.

The solution of such a system is numerically intractable. Simpler procedures are needed for practical design.

<u>A Simple Systematic Approach</u>

If a mathematically justified approach yields such a complicated procedure, it is natural to try something much easier. The easiest way to search for field distributions that provide small aberrations is to investigate classes of <u>parametrized analytical functions</u> or simply try to produce certain distributions by the use of different <u>curve fitting techniques</u> (Szilagyi, 1983a). Using this simple approach in a systematic way we were able to find many electrostatic axial potential distributions that satisfy practical requirements for probe forming lenses and have very low spherical and chromatic aberrations. One of these distributions is shown in Fig. 1. The potential distribution and the axial dimensions are related to the object-side potential V_o and the effective length L of the lens, respectively. This distribution corresponds to a four-electrode electrostatic lens that has a spherical aberration coefficient C_{so} for infinite magnification, referred to the object space and related to the object-side focal length f_o equal to $C_{so}/f_o = 0.82$ and chromatic aberration coefficient for the same case equal to $C_{co}/f_o = 0.46$. For a six-electrode lens we could obtain even better results: $C_{so}/f_o = 0.63$ and $C_{co}/f_o = 0.29$, respectively.

If one compares these data with the aberrations of the best available electrostatic lenses, it is easy to see that a systematic search for good axial potential distributions can provide much better results than any analytical design. Even if we include the difficulties of the electrode reconstruction (see later), we can safely declare that general statements about the intrinsic inferiority of electrostatic lenses so common in the literature are certainly not justified.

This simple approach is very powerful because it produces good results without complex mathematical operations. One can find excellent imaging field distributions even with a personal computer.

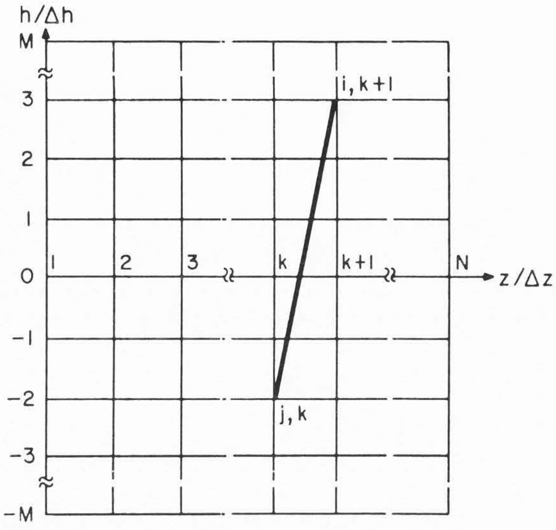

Fig. 2. Computational grid for the dynamic programming procedure.

However, if one really wishes to explore the vast universe of all feasible field distributions, mathematically justified and practically realizable optimization techniques are needed. Since the Calculus of Variations is too complicated for practical use, other approaches had to be considered.

Dynamic Programming

Our problem is to find the distribution V(z) that minimizes the aberration integral (1) simultaneously satisfying the differential equation (2) and the constraints imposed by practical requirements. The possible distributions constitute an infinite set even if the length of the lens as well as the maximum allowable field strength are both limited. We can reduce this infinite set to a finite but extremely large one in many different ways. For example, one can seek V(z) in the form of an n-degree polynomial or another parametrized function. In this case we have to determine the coefficients of the polynomial or the parameters of the function. As it was shown in the preceding section, it is a feasible alternative but without any hope to explore even a reasonable fraction of the different possibilities. Indeed, even if each coefficient of the polynomial can take only m different values, the number of possible variations is m^n which is an astronomical number for any pair of practical values of m and n. This brute force approach, therefore, does not satisfy our requirement of being mathematically justified. Is there any hope at all if e. g. n=10 and m=100?

Yes, there is not only hope but there are also practical ways of solution [(Szilagyi, 1977a), (Szilagyi, Yakowitz and Duff, 1984), (Szilagyi,1984b)].

Let us first see the Dynamic Programming approach (Szilagyi, 1977a). We start with a rectangular computational grid (Fig. 2) that defines the domain of existence for the field distribution. The field is limited by its maximum allowable value. Its axial extension is defined by the given effective length L of the lens. The effective length is divided into N equal regions. The unknown distribution V(z) or one of its derivatives is approximated by a straight line in each region. We shall denote this piecewise linear function by h(z). For a magnetic lens h(z) is the axial flux density distribution, for an electrostatic lens it is the electric field distribution. If one wishes to drastically reduce the amount of calculations, even a step-function model can be used where the field is constant in each region and its derivatives are calculated as differences at neighbouring regions. As we shall see later, the most effective approach is to use the highest derivative appearing in the aberration integral for h(z). In any case h(z) can take only 2M+1 different values at the boundaries of the regions (see Fig.2). Thus, our problem is reduced to that of finding N*(2M+1) intersection points of the computational grid that will provide the linear segments of the optimized field distribution.

The paraxial ray equation can be solved in each interval analytically or numerically. The continuity of the solution requires that the initial values of r and r' for each region be taken as equal to their terminal values in the preceding region. If N is a sufficiently large number, the aberration integral can be approximated by a finite sum and we can easily calculate the contribution of each region to this sum.

Let us now denote the initial and terminal values of h(z) in the kth interval by

$$h_k(z_k) = j\,\Delta h \tag{4}$$

and

$$h_k(z_{k+1}) = i\,\Delta h \tag{5}$$

respectively where Δh is the minimum amount of change in the value of the field distribution (see Fig. 2). Evidently, the solution of the paraxial ray equation as well as the contribution to the aberration sum will depend on the values of i, j and k.

Of course, we still have the astronomical number of $(2M+1)^N$ different possibilities. We must find a way to effectively search for the best possible solution.

Our multidimensional problem can be reduced to a one-dimensional multistage decision problem which can be solved step by step. Let us suppose that we have been able to find a potential distribution for which the value of the aberration integral between z_0 and z_k is minimal. We shall denote this optimized intermediate value of the integral by G_{jk}. How to proceed further? If F_{ijk} is the contribution of the kth region to the integral, then the minimized value $G_{i(k+1)}$ of the aberration integral between z_0 and z_{k+1} is given by the recursive relationship

$$G_{i(k+1)} = \min_j \left(G_{jk} + F_{ijk}\right) \tag{6}$$

where j is a variable and the optimization procedure is aimed at finding that particular optimal value of j for which the value of the sum in parentheses is minimal. As we have already calculated everything needed to obtain F_{ijk} for the given value of i the procedure yields both the optimized value of j and the new value of $G_{i(k+1)}$. Since we started with the assumption that the optimum distribution leading to the point (j,k) was known, now one further portion of the optimum distribution has been found.

We start at k=1 with $G_{j1}=0$ which expresses the fact that in the field-free region beyond the lens the contribution of the aberration integral is zero. Therefore, the search in the first region is reduced to the comparison of the different F_{ijk} values. For each i we find the corresponding j_{opt} value that minimizes F_{ijk}. We proceed region by region recursively toward the object space using Eq. (6) for the determination of the optimum values of j for each k and i until we reach the end of the computational grid (k=N+1). Note that at each region we only have to evaluate $(2M+1)^2$ possibilities. As a result of $N*(2M+1)^2$ evaluations we shall have 2M+1 different optimized axial field distributions (with different terminal values of i) together with the corresponding particle trajectories. Each distribution can uniquely be traced back by following the optimum values of j for all the pairs of the i and k values. One chooses the particular distribution that best satisfies the practical requirements of the given design. For example, if the field must vanish at the boundary of the lens and h(z) means the field distribution, then naturally i=0 must be chosen at the terminal point. When this is not required but we would like the image to be located inside the lens, we can make N a variable and stop the calculations at the pair of values i and N where r=0.

The most important advantages of the dynamic programming procedure are as follows:

1. No initial guess of the result is required. The algorithm searches through the entire problem space in an efficient way without the requirement of any initial assumptions.

2. A global search is provided with only $N*(2M+1)^2$ evaluations.

3. In general, rich patterns of optimized distributions are produced. They contain vast numbers of distributions as sub-solutions of the original problem with different initial and terminal conditions.

4. The procedure is simplified by any additional constraint that reduces the number of possible choices at a particular stage.

5. The procedure can be directly applied to any symmetry.

This method has been successfully used for electron lens design. Numerous interesting configurations have been found both for magnetic (Szilagyi, 1977b) and electrostatic (Szilagyi, 1978a, 1978c and 1983b) lenses. A typical case is shown in Fig. 3. This is a 50*60 computational grid for the design of accelerating electrostatic immersion lenses (Szilagyi, 1978a). Here h(z) is proportional to the axial electrostatic field distribution. All the acceptable solutions start and end with approximately zero fields and represent two-cylinder immersion lenses with the second cylinder having a smaller diameter and a higher potential.

As usual, there are also some limitations:

1. The procedure is aimed at subsequent nodes of the computational grid and not at the a priori unknown terminal node. As a consequence, it may happen that a "non-optimal" distribution provides a solution sooner than the "optimal" one.

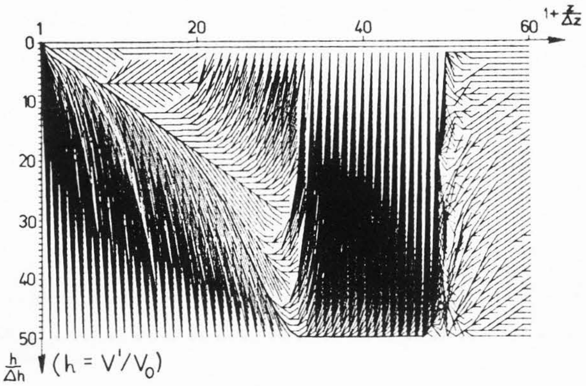

Fig. 3. Optimized electrostatic field distributions obtained by dynamic programming.

2. Due to the discrete nature of the procedure, storage and manipulation of large arrays of data are required.

The procedure can be improved by any of the following suggestions:

1. Aim the optimization procedure at the given terminal condition. Then for an arbitrary node of the computational grid the predecessor node is chosen to minimize the value of the aberration integral not at the given node but at an a priori chosen terminating point.

2. Maintain the original algorithm but make the value of Δz a variable to ensure a smooth distribution even for small grids.

3. Replace the computational grid in the h-z plane by a grid in the phase plane r-r' of the paraxial ray.

4. Avoid the difficulties connected with the discrete character of the method by the utilization of differential dynamic programming.

5. Try something totally different. This will be discussed in the next section.

Function Minimization

A very effective optimization procedure was recently proposed by Szilagyi, Yakowitz and Duff (1984). We again divide the axial length of the lens into N regions and in each region represent the unknown axial distribution V(z) by a simple polynomial expression. The simpler this expression is, the faster and easier the procedure becomes. We require the continuity of V(z) and its lower derivatives. This requirement imposes some relationships between the coefficients of the polynomials at neighbouring regions. We always define the requirements of continuity in such a way that the set of coefficients at the highest-degree terms remain free. Then our problem is reduced to that of finding N coefficients satisfying the paraxial ray equation (2) and the constraints of the problem and minimizing the aberration integral (1).

Let us start with an arbitrary set of the free coefficients. The paraxial equation is solved with these coefficients for each interval and the corresponding aberration coefficients are evaluated numerically. Naturally, the aberration coefficients will be too high and the constraints will not be met. Now we add a penalty reflecting the violation of the constraints to the value of the objective function constructed from the aberration coefficients. This sum is the new objective function we are trying to minimize.

The minimization can be done simply by applying some nonlinear programming technique (e.g. a quasi-Newton algorithm) to the new objective function of the N coefficients (function minimization). The result is a set of coefficients that minimize the aberrations with the simultaneous satisfaction of the constraints.

Though this procedure requires an initial set of coefficients to be chosen by the user, it is extremely fast and effective. Almost any combination of the initial data converges to a good solution in seconds of CPU time. We were able to design some very good electrostatic lenses by the use of this technique.

Reconstruction of the Electrodes (Pole Pieces)

When the optimized axial electrostatic or magnetic scalar potential distribution has been found, our next task is to find the electrode (pole piece) configuration that will produce the optimized distribution. Since the potential produced by an axially symmetric system at an arbitrary point in space is uniquely determined by its axial distribution, from theoretical point of view the electrode reconstruction does not constitute a problem.

The power series expansion of the potential, however, requires the axial distribution to be a 2(n-1) times differentiable function of the axial coordinate z where n is the number of terms used in the power series. Unfortunately, for a good convergence a large value of n is needed. If the axial distribution is given as a numerical data set, or even if it is given in the form of a complicated analytical function, the higher derivatives must be produced by numerical techniques which are extremely inaccurate. This difficulty can be avoided by using the charge density method (Hawkes, 1981) but it requires excessive computer time for the reconstruction of the potential distribution at a considerable distance from the axis.

The axial potential distribution given as a discrete data set can be replaced by a continuous, n times differentiable function with minimum interpolation noise (Szilagyi, 1980). The computational feasibility of this approach, however, is also questionable.

The difficulties of electrode (pole piece) reconstruction, therefore, constituted a common weakness for all methods of electron and ion optical synthesis. Some researchers have even expressed scepticism about the very possibility that the problem of electron optical synthesis can ever be solved (Kasper, 1981).

We were able to achieve quite acceptable accuracy of reconstruction by fitting the numerical data sets with smooth cubic spline curves (Szilagyi, 1983a). Of course, inside the regions the cubic splines do not have any derivatives higher than the third. However, the difficulty is

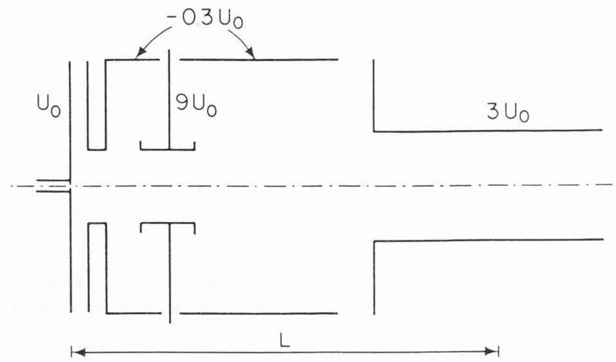

Fig. 4. Electrostatic lens designed on the basis of the potential distribution shown in Fig. 1.

that the higher derivatives are undefined at the boundaries of the regions. Nevertheless, with some insight and experience one can always reconstruct the electrodes. Fortunately, in most cases the complicated curved equipotential surfaces can be replaced by simpler ones with straight boundaries. As a result we were able to design lenses with electron optical properties very close to those of the optimized axial potential distributions. A four-electrode electrostatic lens based on the axial potential distribution given in Fig.1 is shown in Fig. 4. This lens has very good optical properties in a wide range of the parameters. Its aberrations are quite low even at large working distances. If the electrodes are held at the relative potentials shown in the figure, the spherical aberration coefficient is equal to $C_{so}/f_o=0.92$ and the chromatic aberration coefficient has a value of $C_{co}/f_o=0.55$.

The success of utilization of the cubic spline curve fitting technique has led us to the solution of the synthesis problem.

Solution of the Problem of Electron Lens Synthesis

The difficulties of electrode (pole piece) reconstruction were connected with the fact that the reconstruction procedure was totally separated from the optimization algorithm. When we search for an optimum axial field distribution we do not care about the problem of off-axis expansion. This is wrong: <u>the reconstruction process must be an integral part of the optimization procedure</u>. Instead of replacing the discrete axial distribution data sequence by a continuous curve in a complicated way or trying to accomplish the off-axis expansion by some other sophisticated techniques, <u>we must seek the solution directly in the form of a continuous piecewise cubic spline function</u>. The simple expression

$$v(r,z) = V(z) - \frac{r^2}{4} V''(z) \qquad (7)$$

will then be used for the potential distribution in the entire space. The reconstruction of the electrodes or pole pieces thus becomes an almost trivial task (Szilagyi, 1984a).

This approach allows us to combine the evident advantages of both the dynamic programming and the function minimization procedures with an easy, fast and accurate reconstruction technique.

In the following we shall briefly outline our method of electron optical synthesis (Szilagyi, 1984b).

The axial electrostatic or magnetic scalar potential distribution will be represented by a piecewise cubic function. As usual, the axial length of the lens is divided into N equal regions. The unknown distribution V(z) is sought in the form of

$$V_k(z) = A_k + B_k(z - z_k) + C_k(z - z_k)^2 + D_k(z - z_k)^3 \quad (8)$$

for each region (k=1,2,...,N) where z_k is the axial coordinate of the starting point of the kth region. The coefficients A_k, B_k, C_k and D_k are different for each region, therefore V(z), V'(z) and V"(z) have different expressions for each region. It is very easy, however, to ensure the continuity of these functions by requiring the satisfaction of the following relationships between the coefficients:

$$A_{k+1} = A_k + B_k\Delta z + C_k(\Delta z)^2 + D_k(\Delta z)^3 \quad (9)$$

$$B_{k+1} = B_k + 2C_k\Delta z + 3D_k(\Delta z)^2 \quad (10)$$

and

$$C_{k+1} = C_k + 3D_k\Delta z \quad (11)$$

where

$$\Delta z = L/N = z_{k+1} - z_k \quad (12)$$

Let us formulate the constraints now. Naturally, the fields must be practically realizable. Therefore, the magnitudes of the potential and its derivatives must be limited:

$$V_I \le V(z) \le V_{II} \quad (13)$$

$$|V'(z)| \le V'_I \quad (14)$$

and

$$|V''(z)| \le V''_I \quad (15)$$

where V_I, V_{II}, V'_I and V''_I are a priori given numbers. It is usually required that the fields vanish at both the object and the image:

$$V'(z_o) = V'(z_i) = 0 \quad (16)$$

The particle trajectories must be focused toward the optical axis but should not cross it inside the optical element. In addition, we must provide some working distance beyond the lens. Therefore, we must require

$$r(z) > 0 \quad (17)$$

and

$$r'(z_o) > 0 \quad (18)$$

Depending on the particular problem, the constraints may be different from these.

If we relate the potential distribution to V_o then

$$A_1 = 1 \quad (19)$$

Conditions (16) yield

$$B_1 = 0 \quad (20)$$

and

$$B_N + 2C_N\Delta z + 3D_N(\Delta z)^2 = 0 \quad (21)$$

The above constraints leave only the coefficients C_1 and D_k (k=1,2,...,N-1) free. For a given set of coefficients the paraxial ray equation can easily be solved and the aberration integrals evaluated numerically. Therefore, the problem of searching for a function in the infinite set of different possibilities is reduced to that of finding these N coefficients so that they satisfy the paraxial equation and our constraints and at the same time minimize the aberration integrals.

This is an N-dimensional constrained optimization problem. We can solve it either by the dynamic programming or the function minimization approach with the fundamental advantage over the original versions of these methods that the optimized potential distribution will now be available in the form of a continuous curve with only three derivatives instead of a digital data set.

If the dynamic programming procedure is used, we shall utilize the fact that for the piecewise cubic lens model the second derivative of the axial potential distribution is a linear function of the axial coordinate within each region:

$$V''(z) = 2C_k + 6D_k(z - z_k) \quad (22)$$

The distribution of the second derivative V"(z) is then given by a series of continuous linear segments. We simplify the search by restricting these linear segments to those connecting the nodes of the discrete computational grid i. e. we only allow 2M+1 different discrete values of the second derivative at the boundaries of the regions. We have to substitute V" for h in equations (4) and (5). Then the coefficients are expressed through i and j by

$$C_k = j\,\Delta V''/2 \quad (23)$$

and

$$D_k = \frac{i-j}{6\Delta z}\,\Delta V'' \quad (24)$$

with

$$\Delta V'' = V''/M_I \quad (25)$$

The rest of the procedure is identical to that outlined above in the description of the dynamic programming method.

We have modified our function minimization procedure, too, to provide a continuous, piecewise cubic function for the optimized axial potential distribution in the form of Eq. (8), the coefficients of which are determined by the procedure.

Dynamic programming and function minimization very well complement each other: while the first provides a global search in a discrete domain the second is very fast and accurate. Both give quite satisfactory results but in extraordinarily difficult cases the synthesis procedure may combine them by starting with dynamic programming and refining the optimized axial potential distributions by the function minimization procedure.

The reconstruction of the electrodes (pole pieces) is extremely simple now because we already know not only the axial distribution of the potential but also its continuous second derivative. The third derivative is not continuous but it does not appear anywhere so it cannot cause any problem. The fourth and higher derivatives are zero everywhere except the boundaries of the regions where they are still undefined. However, we are not using the spline function for curve fitting now. Therefore, it is justified to assume that these higher derivatives not appearing anywhere in the expressions of the focusing properties and aberrations will not affect the potential distribution either. This is why the simple expression (7) can be used for the potential distribution in the entire space. By the use of this formula we are able to reproduce the equipotential surfaces that will provide the same functions V(z), V'(z) and V"(z) and thus the same focusing properties and aberrations as the original theoretical distribution obtained from the optimization procedure.

First we examine the potential distribution and determine its inflection points. The number of electrodes (pole pieces) is always one more than the number of inflection points that separate the neighbouring electrodes. Then we substitute the suitably chosen values of the electrostatic or magnetic scalar equipotentials into Eq. (7) for each electrode and obtain simple relationships r=r(z) for the shapes of the electrodes or pole pieces. If the value of V" is negative between two inflection points, the electrode (pole piece) potential is chosen slightly higher than the maximum value of V(z) in the given region. If V" is positive, the electrode (pole piece) potential is lower than the minimum value of V(z) in the region.

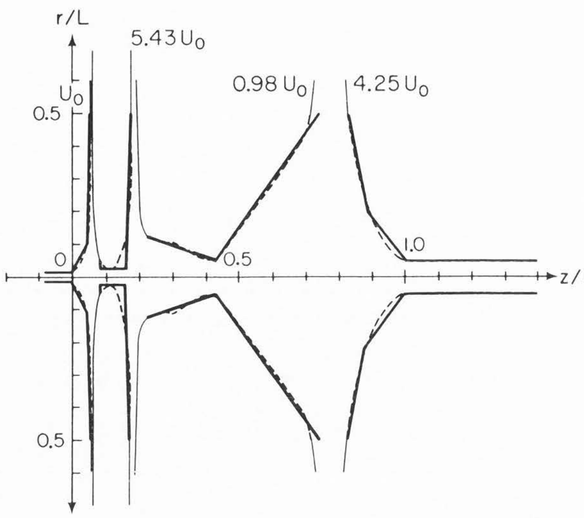

Fig. 5. Four-electrode optimized electrostatic lens designed by the author's new method of synthesis. The electrode shapes are gradually simplified to plane surfaces.

The maximum value of r must be limited to a realistic size, usually to half of the length of the system. Since the value of r rapidly approaches infinity at the inflection points, parts of the electrodes (pole pieces) must be omitted. This automatically ensures a certain minimum distance between them which is necessary to avoid electric breakdown in case of electrostatic lenses.

A very important remark is due here. When the final electrode shapes are chosen, we actually depart from the optimized potential distribution by omitting parts of the electrodes. During this process we can also simplify the electrode shapes and replace the complicated curved boundaries by easily manufacturable simple straight surfaces. The accuracy of the reconstruction will only slightly be affected by these actions but on the other hand the practical value of the method is tremendously increased by them.

As an example, the schematics of a four-electrode electrostatic lens designed by the function minimization method of synthesis is shown in Fig. 5. The dimensions of the lens are given in units of the effective length L (the distance between the axial points beyond which the potentials are practically constant at both sides of the lens). The electrode potentials are related to the potential of the first electrode at the object side. The optimization procedure first yields (1) the infinitely long electrodes, parts of which are shown as thin lines in the figure. As the next step (2), the electrodes are cut to reasonable sizes and parts of them are totally omitted in order to avoid breakdown (broken lines). The final simple electrode shapes (3) are shown in thick continuous lines.

The axial potential distributions of the reconstructed lenses were re-calculated by the charge density method and the optical properties (locations of the cardinal elements and values of the aberration coefficients) were determined by ray tracing and numerical integration. The results for the three systems are compared in the following table:

System #	Focal plane z_o/L	Principal plane P_o/L	Focal length f_o/L	Spherical aberr. C_{so}/L	Chromatic aberr. C_{co}/L
1.	-0.029	0.182	0.212	0.190	0.169
2.	-0.025	0.187	0.212	0.224	0.175
3.	-0.018	0.194	0.212	0.235	0.177

As we can see, even the optical properties of the simplest lens are excellent and not very much different from those of the optimized theoretical potential distribution. At a very low maximum-to-minimum electrode potential ratio (U_{max}/U_{min}=5.54) we have C_{so}/f_o= 1.11 and C_{co}/f_o = 0.83. The negative value of the location of the focal plane provides comfortable working distances when the lens is used in the probe forming mode (the zero point coincides with the center of the entrance aperture).

We were also able to design a three-electrode electrostatic lens with U_{max}/U_{min}= 8.86, C_{so}/f_o = 0.95 and C_{co}/f_o = 0.72 (Szilagyi, 1984a) as well as a five-electrode einzel-lens with U_{max}/U_{min}=6.75, C_{so}/f_o= 1.03 and C_{co}/f_o= 0.76 (Szilagyi, 1984b).

Conclusion

Electron and ion optical synthesis is a very real problem of Particle Beam Optics. The final goal is to be able to design an entire optical column automatically on the basis of given properties. Many years of work by a number of different researchers has eventually culminated in our solution outlined above. It provides a powerful practical tool for the synthesis of electron and ion lenses. The reconstruction of electrodes (pole pieces) is built in the optimization procedure and does not require any extra effort. Our design examples not only prove the effectiveness of this approach but also give encouragement to the possibility of building very high performance electrostatic lenses so much needed for ion beam lithography.

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Discussion at the Conference

This work has generated an unusually great interest at the Conference. Very active and lengthy discussions took place both during its presentation and at the special workshop on electron lens design. Although it is not possible to describe the entire debate, a summary of the most important items is presented in the following:

1. Synthesis is necessary because different applications require different designs. Therefore, it is not sufficient to catalogue some simple cases but a general design method is needed.

2. The author's method must be used with additional simplification of the electrodes (pole pieces). The strength of the method is that it is able to find the locations and potentials of the electrodes. Their shapes do not influence the optical properties very much.

3. The simplification of the electrode shapes naturally introduces higher derivatives into the axial potential distribution. However, this does not constitute any real problem since our aim is not an exercise in pure mathematics but practical design.

4. To obtain more general designs it is convenient to define the electrode shapes and potentials in dimensionless units. The unit of potential is determined by the given energy of the particles either in the object or the image space. Then the minimum value of the effective length is chosen from the requirement that in order to avoid breakdown the electric field must not exceed a certain maximum value.