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OPTICS OF WIDE ELECTRON BEAM FOCUSING

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ABSTRACT

In the present paper, we have further studied an approach to the theoretical treatment of wide electron beam focusing, based on the application of the theory of focusing fields with arbitrary curved optical axis, considering the initial condition emitted from the cathode surface. It is assumed that the magnetic field vector **B** will be used instead of the magnetic scalar potential Ω and the electrostatic potential φ and the magnetic field vector **B** are known functions either by computation or experimentally in the laboratory system of coordinates. The new approach to the mathematical treatment appears to be a useful method for solving problems of wide electron beam focusing with arbitrary electrostatic and magnetic fields.

The theoretical treatment presented here can be extended to solve problems of deflection defocusing in cathode-ray tubes at large deflection angles.

Key words: Electron optics, electron beams, electron lenses, cathode lenses, imaging systems, deflection systems

INTRODUCTION

In electron optical systems involving cathode lenses, where the photoemitter is used as the object surface, the cathode is situated in both magnetic and electrostatic fields and electrons leave the cathode with small velocity and large inclination. The theory of conventional narrow electron beam focusing in a curvilinear coordinate system cannot be applied to deal with such cases involving wide electron beam focusing with large cathode surface.

An approach to the theoretical treatment of wide electron beam focusing has been derived [9], based on the application of the theory of focusing fields with arbitrary curved optical axes, considering the initial conditions emitted from the cathode surface. In that paper[9], equations of the principal trajectory and "paraxial" trajectories are deduced. The orthogonal condition of a curvilinear "paraxial" system and the electron optical properties of an orthogonal system have been studied, some practical examples of wide electron beam focusing satisfying the orthogonal condition have been given. The results of computation showed that the theoretical treatment is efficient and suitable for the electrostatic cathode lenses, particularly for the determination of field curvature and astigmatism.

It has been found that in the above mentioned article the magnetic field is expressed by magnetic scalar potential Ω, but it is not suitable for the practical computation. In the present paper, the magnetic field vector **B** and the electrostatic potential φ are known functions either experimentally or by computation in the laboratory system of coordinates. The generalized theory of wide electron beam focusing is further studied. The new approach to the mathematical treatment appears to be a useful method for solving problems of wide electron beam focusing with arbitrary electrostatic and magnetic fields.

The theoretical treatment presented here can be exten-

LIST OF SYMBOLS

A^l, C^l	=Parameters in eqs.(16)
$a_{11}, a_{12}, a_{21}, a_{22}$	=Parameters in eqs.(72)
$\mathbf{B}$	=Vector of magnetic induction
B_1, B_2, B_3	=Components of the vector **B** in the Frenet local coordinate system(x^1, x^2, x^3)
B_X, B_Y, B_Z	=Components of the vector **B** in the laboratory Cartesian coordinate system(x, y, z)
B_n, B_{n2}, B_{n3}; B_b, B_{b2}, B_{b3}; B_t, B_{t2}, B_{t3}	=Parameters in eqs.(32) and (35)
B_{X2}, B_{X3}; B_{Y2}, B_{Y3}; B_{Z2}, B_{Z3}	=Parameters in eqs.(33)
C	=Constant in eqs.(95a,b)
ds	=Element of arc length of the principal trajectory
ds^*	=Element of arc length of the neighboring trajectory
$\mathbf{E}$	=Vector of electric field intensity
e	=Electron charge
$e\varepsilon_0$	=Initial energy of electrons emitted from the cathode surface
$e\varepsilon_s$	=Initial energy of principal electrons emitted from the cathode surface
$e\varepsilon_{s1}$	=Initial energy of principal electron which can be focused ideally at the image foci
$e\varepsilon_z$	=Axial initial energy of emission electrons emitted from the cathode surface
$e\varepsilon_{z1}$	=Axial initial energy of emission electron which can be focused ideally at the image plane
F_1, G_1, F_2, G_2	=Parameters in eqs.(92)
f_1, f_2; g_1, g_2; h_1, h_2	=Parameters in eqs.(66)
g	=Determinant of the components g_{ij}
g_{ij}	=Components of the covariant metric tensor
g^{ij}	=Components of the contravariant metric tensor
$G(i,j)$	=Sub-determinant of the components g_{ij}
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	=Unit vectors of the laboratory Cartesian coordinate
M	=Module of magnification
M_u, M_v	=Module of magnification in two directions
m_0	=Electron mass
n, K, F, R, N, Q	=Parameters in eqs.(47)
n_X, n_Y, n_Z; b_X, b_Y, b_Z; t_X, t_Y, t_Z	=Parameters in eqs.(40)
$\mathbf{p}(s)$	=Vector of increment of electron trajectory from point N to point N*
$\mathbf{r}_N$	=Vector of space curved axis at point N
$\mathbf{r}^*$	=Vector of the neighboring trajectory at point N*
$\mathbf{t}, \mathbf{n}, \mathbf{b}$	=Unit vectors in the tangent, principal normal and binormal directions respectively
u, v	=Variables in a twisted coordinate system
$\mathbf{u}$	$=u+iv$
$u_\alpha, u_\beta; v_\alpha, v_\beta$	=Two sets of special solutions of eqs.(74a,b)
U, V	=Parameters in eqs.(75)
$\mathbf{v}$	=Vector of velocity of principal electron
v	=Module of **v**
v^*	=Velocity of electron at the neighboring trajectory
V_0	=Constant electrostatic potential
$w_1^{(i)}, w_2^{(i)}, w_3^{(i)}, w_4^{(i)}$	=A sets of solutions of eqs.(65)
x_N, y_N, z_N	=Components of the space curve $\mathbf{r}_N$ in the laboratory coordinate system
(x, y, z)	=Laboratory Cartesian coordinate system
(x^1, x^2, x^3)	=Frenet local coordinate system, $(x^1, x^2, x^3)=(s, p_2, p_3)$
x^1	=Arc length along the curved axis from the initial point A, $x^1=s$
z	=Axial coordinate
Γ_{ij}^l	=Christoffel symbols of the second kind
$1/\rho$	=Curvature of the principal trajectory
$1/\tau$	=Torsion of the principal trajectory
φ	=Space electrostatic potential
φ_*	=Normalized electrostatic potential
φ_i, φ_{ij}	=Parameters in eqs.(25)
$\phi(s)$	=Electrostatic potential along the curved axis
ϕ_X, ϕ_Y, ϕ_Z	$=\phi_X=\partial\phi/\partial x, \phi_Y=\partial\phi/\partial y, \phi_Z=\partial\phi/\partial z$
χ	=Angle of rotation of the twisted coordinate system (u,v) with respect to the fixed coordinate system (p_2, p_3)
Ω	=Scalar magnetic potential

ded to solve problems of deflection defocusing in cathode-ray tubes at large deflection angles.

ELECTRON MOTION EQUATION AND ELECTRON TRAJECTORY EQUATION IN CURVILINEAR COORDINATE SYSTEM

Let $\mathbf{r}_N=\mathbf{r}_N(x^1)$ be a natural representation of a space curve, consisting of the curved axis of an electron beam (principal trajectory), emitted from a point A at the object (cathode) surface, where x^1 is the arc length along the curved axis from the initial point A. The normal plane through an arbitrary point N on the curved axis intersects the neighboring curvilinear trajectory at point N^*. It follows that the position of N^* can be determined by a vector $\mathbf{p}$ from N to N^*. If the expressions of $\mathbf{p}=\mathbf{p}(x^1)$ are known, then the curvilinear neighboring trajectory will be completely defined.

Introduce Frenet local coordinate system (x^1,x^2,x^3) (indices 1,2,3 are here used as superscripts, not exponents) and denote the unit vectors at the point N by $\mathbf{t}$, $\mathbf{n}$, $\mathbf{b}$ for the tangent, principal normal and binormal directions respectively. The curvature radius and torsion radius of a space curve (principal trajectory) $\mathbf{r}_N$ are expressed by $\rho=\rho(x^1)$ and $\tau=\tau(x^1)$. As mentioned above, the vector $\mathbf{p}$ is placed in the normal plane orthogonal to the tangent of the principal trajectory, therefore its components at the principal normal and binormal directions are x^2 and x^3 respectively, as shown in Fig.1.

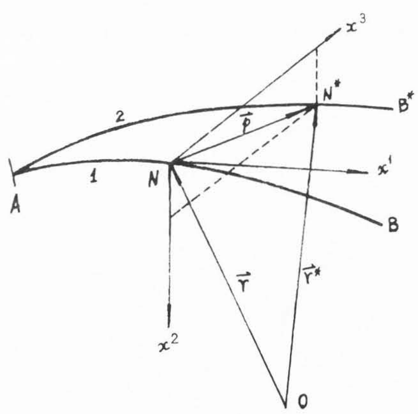

Fig.1 The Frenet local curvilinear coordinate system, principal trajectory and its neighboring trajectories
1——principal trajectory
2——neighboring trajectory

Suppose the principal trajectory (central trajectory) in a laboratory Cartesian coordinate system can be expressed as the following vector form:

$$\mathbf{r}_N=\mathbf{r}(x^1)=x_N(x^1)\mathbf{i}+y_N(x^1)\mathbf{j}+z_N(x^1)\mathbf{k}, \tag{1}$$

where $\mathbf{i},\mathbf{j},\mathbf{k}$ are the unit vectors in the laboratory Cartesian coordinate system.

Curvature and torsion of the space curve of principal trajectory (1) are given as:

$$\frac{1}{\rho}=\frac{|\mathbf{r}_N'\times\mathbf{r}_N''|}{|\mathbf{r}_N'|^3}, \tag{2}$$

$$\frac{1}{\tau}=\frac{|\mathbf{r}_N'\times\mathbf{r}_N''|\cdot\mathbf{r}_N'''}{|\mathbf{r}_N'\times\mathbf{r}_N''|^2}. \tag{3}$$

It can be written in a scalar form:

$$\frac{1}{\rho}=[(y_N'z_N''-z_N'y_N'')^2+(z_N'x_N''-x_N'z_N'')^2 +(x_N'y_N''-y_N'x_N'')^2]^{1/2}, \tag{4}$$

$$\frac{1}{\tau}=\left\{(y_N'z_N''-z_N'y_N'')x_N'''+(z_N'x_N''-x_N'z_N'')y_N''' +(x_N'y_N''-y_N'x_N'')z_N'''\right\}\Big/\left\{(y_N'z_N''-z_N'y_N'')^2 +(z_N'x_N''-x_N'z_N'')^2+(x_N'y_N''-y_N'x_N'')^2\right\}. \tag{5}$$

here and below the prime indicates derivatives with respect to arc length x^1.

From analytical geometry, the system of unit vectors in the Frenet triad is defined by

$$\begin{aligned} \mathbf{t}&=\mathbf{r}_N', &&\text{(tangent)}\\ \mathbf{n}&=\rho\mathbf{r}_N'', &&\text{(normal)}\\ \mathbf{b}&=\rho(\mathbf{r}_N'\times\mathbf{r}_N''). &&\text{(binormal)} \end{aligned} \tag{6}$$

Frenet-Serret's formulae are

$$\begin{aligned} \frac{d\mathbf{t}}{dx^1}&=\frac{1}{\rho}\mathbf{n},\\ \frac{d\mathbf{n}}{dx^1}&=-\frac{1}{\rho}\mathbf{t}+\frac{1}{\tau}\mathbf{b},\\ \frac{d\mathbf{b}}{dx^1}&=-\frac{1}{\tau}\mathbf{n}. \end{aligned} \tag{7}$$

The vector $\mathbf{r}^*$ at point N^* on the neighboring trajectory can be expressed by

$$\mathbf{r}^*=\mathbf{r}_N+\mathbf{p}=\mathbf{r}_N+x^2\mathbf{n}+x^3\mathbf{b}. \tag{8}$$

Now let us calculate the element of arc length ds^* in the selected curvilinear coordinate system. Differentiating eq.(8), using Frenet-Serret's formulae (7), considering that $(ds^*)^2=(d\mathbf{r}^*)^2$ and expressing it in tensor form, we have[5]

$$(ds^*)^2=g_{ij}dx^idx^j, \tag{9}$$

where g_{ij} are the components of the covariant metric tensor, which can be written as follows:

$$(g_{ij})=\begin{pmatrix} (1-\frac{1}{\rho}x^2)^2+(\frac{1}{\tau}x^2)^2+(\frac{1}{\tau}x^3)^2 & -\frac{1}{\tau}x^3 & \frac{1}{\tau}x^2 \\ -\frac{1}{\tau}x^3 & 1 & 0 \\ \frac{1}{\tau}x^2 & 0 & 1 \end{pmatrix} \quad (10)$$

In the non-relativistic case, the contravariant components of Lorentz's equation have the form [4]:

$$m_0(\ddot{x}^l+\Gamma_{ij}{}^l\dot{x}^i\dot{x}^j)=e(A^l-C^l),\quad (l=1,2,3) \quad (11)$$

where e is the magnitude of the electron charge, m_0, the electron mass. $\Gamma_{ij}{}^l$, the Christoffel symbols of the second kind, can be written as

$$\Gamma_{ij}{}^l=\frac{1}{2}g^{kl}\left(\frac{\partial g_{kj}}{\partial x^i}+\frac{\partial g_{ik}}{\partial x^j}-\frac{\partial g_{ij}}{\partial x^k}\right), \quad (12)$$

where g^{ij} are the components of the contravariant metric tensor, i.e.

$$g^{ij}=\frac{G(i,j)}{g}, \quad (13)$$

In formula (13), g and G(i,j) denote the determinant and sub-determinant of the components g_{ij}. From (10), we have

$$g=\det|g_{ij}|=(1-\frac{1}{\rho}x^2)^2, \quad (14)$$

$$(g^{ij})=\begin{pmatrix} \frac{1}{g} & \frac{\frac{1}{\tau}x^3}{g} & \frac{-\frac{1}{\tau}x^2}{g} \\ \frac{\frac{1}{\tau}x^3}{g} & \frac{(1-\frac{1}{\rho}x^2)^2+(\frac{1}{\tau}x^3)^2}{g} & \frac{-(\frac{1}{\tau})^2x^2x^3}{g} \\ \frac{-\frac{1}{\tau}x^2}{g} & \frac{-(\frac{1}{\tau})^2x^2x^3}{g} & \frac{(1-\frac{1}{\rho}x^2)^2+(\frac{1}{\tau}x^2)^2}{g} \end{pmatrix} \quad (15)$$

Using formulae (10) (14) and (15), considering that $\Gamma_{ij}{}^l=\Gamma_{ji}{}^l$, we can get each value of $\Gamma_{ij}{}^l$. A^l, C^l $(l=1,2,3,)$ in (11) have the form:

$$A^l=g^{li}\frac{\partial\varphi}{\partial x^i},$$

$$C^1=\frac{1}{\sqrt{g}}(\dot{x}_2B_3-\dot{x}_3B_2),$$

$$C^2=\frac{1}{\sqrt{g}}(\dot{x}_3B_1-\dot{x}_1B_3),\quad C^3=\frac{1}{\sqrt{g}}(\dot{x}_1B_2-\dot{x}_2B_1), \quad (16)$$

where

$$\dot{x}_i=g_{ij}\dot{x}^j, \quad (17)$$

and φ is the potential of the electrostatic field. The magnetic field vector **B** can be defined by three components B_1, B_2, B_3 at the Frenet local coordinate system.

For convenience, we can use symbols (s, p_2, p_3) instead of (x^1, x^2, x^3). From (11), we have the electron motion equations in curvilinear coordinate system (s, p_2, p_3):

$$\frac{m_0}{e}\left\{\ddot{s}+\frac{\frac{1}{\rho\tau}p_3+\frac{1}{\rho^2}\rho'p_2}{1-\frac{1}{\rho}p_2}\dot{s}^2-\frac{\frac{2}{\rho}}{1-\frac{1}{\rho}p_2}\dot{s}\dot{p}_2\right\}$$

$$=\frac{1}{(1-\frac{1}{\rho}p_2)^2}\left\{\frac{\partial\varphi}{\partial s}+\frac{1}{\tau}p_3\frac{\partial\varphi}{\partial p_2}-\frac{1}{\tau}p_2\frac{\partial\varphi}{\partial p_3}\right\}$$

$$-\frac{1}{1-\frac{1}{\rho}p_2}\left\{(\dot{p}_2-\frac{1}{\tau}p_3\dot{s})B_3-(\dot{p}_3+\frac{1}{\tau}p_2\dot{s})B_2\right\},$$

$$\frac{m_0}{e}\left\{\ddot{p}_2+(\frac{1}{\rho}-\frac{1}{\rho^2}p_2-\frac{1}{\tau^2}p_2+\frac{1}{\tau^2}\tau'p_3\right.$$

$$+\frac{\frac{1}{\rho\tau^2}p_3^2+\frac{1}{\rho^2\tau}\rho'p_2p_3}{1-\frac{1}{\rho}p_2})\dot{s}^2-\frac{2\frac{1}{\rho\tau}p_3}{1-\frac{1}{\rho}p_2}\dot{s}\dot{p}_2$$

$$\left.-\frac{2}{\tau}\dot{s}\dot{p}_3\right\}=\frac{1}{(1-\frac{1}{\rho}p_2)^2}\left\{\frac{1}{\tau}p_3\frac{\partial\varphi}{\partial s}\right.$$

$$\left.+\left[(1-\frac{1}{\rho}p_2)^2+\frac{1}{\tau^2}p_3^2\right]\frac{\partial\varphi}{\partial p_2}-\frac{1}{\tau^2}p_2p_3\frac{\partial\varphi}{\partial p_3}\right\}$$

$$-\frac{1}{1-\frac{1}{\rho}p_2}\left\{(\dot{p}_3+\frac{1}{\tau}p_2\dot{s})B_1-\left\{\left[(1-\frac{1}{\rho}p_2)^2\right.\right.\right.$$

$$\left.\left.\left.+\frac{1}{\tau^2}p_2^2+\frac{1}{\tau^2}p_3^2\right]\dot{s}-\frac{1}{\tau}p_3\dot{p}_2+\frac{1}{\tau}p_2\dot{p}_3\right\}B_3\right\},$$

$$\frac{m_0}{e}\left\{\ddot{p}_3+\left[-\frac{1}{\tau^2}\tau' p_2-\frac{1}{\tau^2}p_3-\frac{1}{1-\frac{1}{\rho}p_2}\left(\frac{1}{\tau\rho^2}\rho' p_2{}^2\right.\right.\right.$$

$$\left.\left.\left.+\frac{1}{\tau^2\rho}p_2p_3\right)\right]\dot{s}^2+\frac{2\frac{1}{\tau}\dot{s}\dot{p}_2}{1-\frac{1}{\rho}p_2}\right\}=\frac{1}{(1-\frac{1}{\rho}p_2)^2}\left\{-\frac{1}{\tau}p_2\frac{\partial\varphi}{\partial s}\right.$$

$$\left.-\frac{1}{\tau^2}p_2p_3\frac{\partial\varphi}{\partial p_2}+\left[(1-\frac{1}{\rho}p_2)^2+\frac{1}{\tau^2}p_2{}^2\right]\frac{\partial\varphi}{\partial p_3}\right\}$$

$$-\frac{1}{1-\frac{1}{\rho}p_2}\left\{\left\{\left[(1-\frac{1}{\rho}p_2)^2+\frac{1}{\tau^2}p_2{}^2+\frac{1}{\tau^2}p_3{}^2\right]\dot{s}\right.\right.$$

$$\left.\left.-\frac{1}{\tau}p_3\dot{p}_2+\frac{1}{\tau}p_2\dot{p}_3\right\}B_2-(\dot{p}_2-\frac{1}{\tau}\dot{s}\,p_3)B_1\right\}.$$

(18a,b,c)

(Note that, here the upper indices denote exponents.)

To transform the general equations (18a, b, c) of electron motion trajectory equations, let $v^*=ds^*/dt$, the velocity of electrons at the neighboring trajectory. From the standpoint of energy, we have

$$ds^*/dt=\sqrt{(2e/m_0)\varphi_*}\,, \qquad (19)$$

where $\varphi_*=\varphi+\varepsilon_0$, $e\varepsilon_0$ is the initial energy of electrons emitted from the cathode surface, φ_* is called the normalized potential. The element of arc length ds of the principal trajectory and ds^* of its neighboring trajectory are so related such that:

$$(\frac{ds^*}{ds})^2=(1-\frac{1}{\rho}p_2)^2+\frac{1}{\tau^2}(p_2{}^2+p_3{}^2)-2\frac{1}{\tau}p_3p_2'$$

$$+2\frac{1}{\tau}p_2p_3'+p_2'{}^2+p_3'{}^2. \qquad (20)$$

Transforming each of the parameters $\dot{s}$, $\dot{p}_2$, $\ddot{p}_2$, we have

$$\dot{s}=\frac{ds}{ds^*}\frac{ds^*}{dt}=\sqrt{\frac{2e}{m_0}\varphi_*}\,\frac{ds}{ds^*}.$$

$$\dot{p}_2=\sqrt{\frac{2e}{m_0}\varphi_*}\,\frac{ds}{ds^*}p_2', \qquad (21)$$

$$\ddot{p}_2=\frac{e}{m_0}\left\{2\varphi_*(\frac{ds}{ds^*})^2p_2''+p_2'\frac{d}{ds}\left[\varphi_*(\frac{ds}{ds^*})^2\right]\right\}$$

Similar expressions can be obtained for $\dot{p}_3$ and $\ddot{p}_3$. Substituting these into eqs.(18b,c), we can derive the trajectory equations in curvilinear coordinate system, convenient for the study of wide electron beam focusing and its aberrations.

$$\frac{2\varphi_*}{(1-\frac{1}{\rho}p^2)^2+(\frac{1}{\tau}p_2)^2+(\frac{1}{\tau}p_3)^2-2\frac{1}{\tau}p_3p_2'+2\frac{1}{\tau}p_2p_3'+p_2'{}^2+p_3'{}^2}p_2''$$

$$+p_2'\frac{d}{ds}\left[\frac{\varphi_*}{(1-\frac{1}{\rho}p_2)^2+(\frac{1}{\tau}p_2)^2+(\frac{1}{\tau}p_3)^2-2\frac{1}{\tau}p_3p_2'+2\frac{1}{\tau}p_2p_3'+p_2'{}^2+p_3'{}^2}\right]$$

$$+\frac{2\varphi_*}{(1-\frac{1}{\rho}p_2)^2+(\frac{1}{\tau}p_2)^2+(\frac{1}{\tau}p_3)^2-2\frac{1}{\tau}p_3p_2'+2\frac{1}{\tau}p_2p_3'+p_2'{}^2+p_3'{}^2}\left(\frac{1}{\rho}\right.$$

$$\left.-\frac{1}{\rho^2}p_2-\frac{1}{\tau^2}p_2+\frac{\tau'}{\tau^2}p_3+\frac{\frac{1}{\rho\tau^2}p_3{}^2+\frac{1}{\rho^2\tau}\rho' p_2p_3-2\frac{1}{\rho\tau}p_3p_2'}{1-\frac{1}{\rho}p_2}-2\frac{1}{\tau}p_3'\right)$$

$$=\frac{1}{(1-\frac{1}{\rho}p_2)^2}\left\{\frac{1}{\tau}p_3\frac{\partial\varphi}{\partial s}+\left[(\frac{1}{\tau}p_3)^2+(1-\frac{1}{\rho}p_2)^2\right]\frac{\partial\varphi}{\partial p_2}-\frac{1}{\tau^2}p_2p_3\frac{\partial\varphi}{\partial p_3}\right\}$$

$$-\frac{\sqrt{\frac{2e}{m_0}}}{(1-\frac{1}{\rho}p_2)}\left(\frac{\varphi_*}{(1-\frac{1}{\rho}p_2)^2+(\frac{1}{\tau}p_2)^2+(\frac{1}{\tau}p_3)^2-2\frac{1}{\tau}p_3p_2'+2\frac{1}{\tau}p_2p_3'+p_2'^2+p_3'^2}\right)^{\frac{1}{2}}$$

$$\times\left\{(p_3'+\frac{1}{\tau}p_2)B_1-\left\{\left[(1-\frac{1}{\rho}p_2)^2+(\frac{1}{\tau}p_2)^2+(\frac{1}{\tau}p_3)^2\right]-\frac{1}{\tau}p_3p_2'+\frac{1}{\tau}p_2p_3'\right\}B_3\right\},$$

$$\frac{2\varphi_*}{(1-\frac{1}{\rho}p_2)^2+(\frac{1}{\tau}p_2)^2+(\frac{1}{\tau}p_3)^2-2\frac{1}{\tau}p_3p_2'+2\frac{1}{\tau}p_2p_3'+p_2'^2+p_3'^2}p_3''$$

$$+p_3'\frac{d}{ds}\left[\frac{\varphi_*}{(1-\frac{1}{\rho}p_2)^2+(\frac{1}{\tau}p_2)^2+(\frac{1}{\tau}p_3)^2-2\frac{1}{\tau}p_3p_2'+2\frac{1}{\tau}p_2p_3'+p_2'^2+p_3'^2}\right]$$

$$+\frac{2\varphi_*}{(1-\frac{1}{\rho}p_2)^2+(\frac{1}{\tau}p_2)^2+(\frac{1}{\tau}p_3)^2-2\frac{1}{\tau}p_3p_2'+2\frac{1}{\tau}p_2p_3'+p_2'^2+p_3'^2}\left(-\frac{\tau'}{\tau^2}p_2\right.$$

$$\left.-\frac{1}{\tau^2}p_3-\frac{\frac{1}{\tau\rho^2}\rho'p_2^2+\frac{1}{\tau^2\rho}p_2p_3}{1-\frac{1}{\rho}p_2}+\frac{2\frac{1}{\tau}p_2'}{1-\frac{1}{\rho}p_2}\right)$$

$$=\frac{1}{(1-\frac{1}{\rho}p_2)^2}\left\{-\frac{1}{\tau}p_2\frac{\partial\varphi}{\partial s}-\frac{1}{\tau^2}p_2p_3\frac{\partial\varphi}{\partial p_2}+\left[(1-\frac{1}{\rho}p_2)^2+(\frac{1}{\tau}p_2)^2\right]\frac{\partial\varphi}{\partial p_3}\right\}$$

$$-\frac{1}{1-\frac{1}{\rho}p_2}\sqrt{\frac{2e}{m_0}}\left(\frac{\varphi_*}{(1-\frac{1}{\rho}p_2)^2+(\frac{1}{\tau}p_2)^2+(\frac{1}{\tau}p_3)^2-2\frac{1}{\tau}p_3p_2'+2\frac{1}{\tau}p_2p_3'+p_2'^2+p_3'^2}\right)^{\frac{1}{2}}$$

$$\times\left\{\left\{\left[(1-\frac{1}{\rho}p_2)^2+(\frac{1}{\tau}p_2)^2+(\frac{1}{\tau}p_3)^2\right]-\frac{1}{\tau}p_3p_2'+\frac{1}{\tau}p_2p_3'\right\}B_2-(p_2'-\frac{1}{\tau}p_3)B_1\right\}. \qquad (22a,b)$$

If the magnetic field is defined by scalar magnetic potential Ω, then $-\frac{\partial\Omega}{\partial x^i}$ $(x^1=s, x^2=p_2, x^3=p_3)$ can be used instead of $B_i\,(i=1,2,3)$ in the above equations, we may obtain those equations derived in ref.[9].Eqs.(22a,b) are convenient for studying wide electron beam focusing and narrow electron beam focusing and their aberrations in the curvilinear coordinate system.

ELECTROSTATIC AND MAGNETIC FIELD EXPANSIONS ALONG THE CURVED OPTICAL AXIS

In what follows mathematical expressions are going to be derived in terms of the electrostatic potential and magnetic induction along curved optical axis.

As is known, the electrostatic potential must satisfy Laplace equation:

$$\nabla^2\varphi=0. \qquad (23)$$

we assume that there is no singular point near to the optical axis, the electrostatic potential $\varphi(s,p_2,p_3)$ is an analytical function.

Expanding electrostatic potential φ in a power series of p_2 and p_3 along the curved optical axis with coefficients that are functions of s, we have

$$\varphi(s,p_2,p_3)=\phi(s)+p_2\varphi_2(s)+\frac{1}{2}p_2^2\varphi_{22}(s)+$$

$$+p_2p_3\varphi_{23}(s)+\frac{1}{2}p_3^2\varphi_{33}(s)+\cdots\cdots \quad (24)$$

where the symbols introduced are:

$$\varphi\Big|_{\substack{p_2=0\\p_3=0}}=\varphi(s,0,0)=\phi(s),$$

$$\frac{\partial\varphi}{\partial p_i}\Big|_{\substack{p_2=0\\p_3=0}}=\varphi_i,\quad \frac{\partial^2\varphi}{\partial p_i\partial p_j}\Big|_{\substack{p_2=0\\p_3=0}}=\varphi_{ij}; \quad (25)$$

here

$$\varphi_{\cdot}=\varphi(s,p_2,p_3)+\varepsilon_0,\quad \varphi(0,0,0)=\phi(0)=0,\varphi_1=\frac{d\phi}{ds}.$$

The quantities of φ_i, φ_{ij} in (24) can be expressed by the known partial derivatives in the laboratory Cartesian coordinate system:

$$\varphi_2=\left[n_x\frac{\partial\varphi}{\partial x}+n_Y\frac{\partial\varphi}{\partial y}+n_z\frac{\partial\varphi}{\partial z}\right]_{x=x_N,\ y=y_N,\ z=z_N}$$

$$\varphi_3=\left[b_x\frac{\partial\varphi}{\partial x}+b_Y\frac{\partial\varphi}{\partial y}+b_z\frac{\partial\varphi}{\partial z}\right]_{x=x_N,\ y=y_N,\ z=z_N}$$

$$\varphi_{22}=\left[n_x^2\frac{\partial^2\varphi}{\partial y^2}+n_Y^2\frac{\partial^2\varphi}{\partial y^2}+n_z^2\frac{\partial^2\phi}{\partial z^2}+2n_xn_Y\frac{\partial^2\varphi}{\partial x\partial y}\right.$$

$$\left.+2n_xn_Z\frac{\partial^2\varphi}{\partial x\partial z}+2n_Yn_Z\frac{\partial^2\varphi}{\partial y\partial z}\right]_{x=x_N,\ y=y_N,\ z=z_N}$$

$$\varphi_{23}=\left[n_x\left(\frac{\partial^2\varphi}{\partial x^2}b_x+\frac{\partial^2\varphi}{\partial x\partial y}b_Y+\frac{\partial^2\varphi}{\partial x\partial z}b_Z\right)\right.$$

$$+n_Y\left(\frac{\partial^2\varphi}{\partial x\partial y}b_x+\frac{\partial^2\varphi}{\partial y^2}b_Y+\frac{\partial^2\varphi}{\partial y\partial z}b_Z\right)$$

$$\left.+n_Z\left(\frac{\partial^2\varphi}{\partial x\partial z}b_x+\frac{\partial^2\varphi}{\partial y\partial z}by+\frac{\partial^2\varphi}{\partial z^2}b_Z\right)\right]_{\substack{x=x_N,\\y=y_N,\\z=z_N}}$$

$$\varphi_{33}=\left[b_x^2\frac{\partial^2\varphi}{\partial x^2}+b_Y^2\frac{\partial^2\varphi}{\partial y^2}+b_Z^2\frac{\partial^2\varphi}{\partial z^2}+2b_xb_Y\frac{\partial^2\varphi}{\partial x\partial y}\right.$$

$$\left.+2b_xb_Z\frac{\partial^2\varphi}{\partial x\partial z}+2b_Yb_Z\frac{\partial^2\varphi}{\partial y\partial z}\right]_{x=x_N,y=y_N,z=z_N}. \quad (26)$$

Substituting (24) into (23) yields the relations between some of the coefficients

$$-\left[\varphi_{22}+\varphi_{33}\right]+\frac{1}{\rho}\varphi_2=\frac{d^2\phi}{ds^2}. \quad (27)$$

Because the direction of curved optical axis at point N coincides with unit vector **t**, we have

$$\varphi_1=\phi'=\left(\frac{d\varphi}{ds}\right)_N=\text{grad}\varphi\cdot\mathbf{t}.$$

Differentiating it,

$$\frac{d^2\phi}{ds^2}=\frac{d}{ds}(\text{grad}\varphi)\cdot\mathbf{t}+\text{grad}\varphi\cdot\frac{d\mathbf{t}}{ds},$$

and using Frenet-Serret's formulae (7), we obtain

$$\frac{d^2\phi}{ds^2}=\varphi_{11}+\frac{1}{\rho}\varphi_2. \quad (28)$$

From(27) and (28), we have

$$\varphi_{11}+\varphi_{22}+\varphi_{33}=0. \quad (29)$$

For the magnetic field, similar to the treatment in ref.[3], we assume that the components of vector **B** are known in the laboratory Cartesian coordinate system:

$$\mathbf{B}(x,y,z)=B_x(x,y,z)\mathbf{i}+B_Y(x,y,z)\mathbf{j}+B_Z(x,y,z)\mathbf{k}. \quad (30)$$

It means that at the principal trajectory, $\mathbf{B}(s)=\mathbf{B}(0,0,s)=\mathbf{B}(x_N,y_N,z_N)$ is known.

For the study of the motion of electrons near the curved optical axis, we must expand the field vector $\mathbf{B}(p_2,p_3,s)$ in a power series of p_2 and p_3 with coefficients that are functions of s. Since we investigate the focusing properties of a system with wide electron beam focusing, so we limit our analysis to first order terms, then we have

$$\mathbf{B}(p_2,p_3,s)=\mathbf{B}(s)+p_2(\mathbf{n}\cdot\nabla)\mathbf{B}+p_3(\mathbf{b}\cdot\nabla)\mathbf{B}+\cdots\cdots \quad (31)$$

where $\nabla=\frac{\partial}{\partial x}\mathbf{i}+\frac{\partial}{\partial y}\mathbf{j}+\frac{\partial}{\partial z}\mathbf{k}$,

and all the derivatives are calculated at the point (x_N, y_N, z_N) of the principal trajectory.

From (31) we can get the component of the field vector **B** in the direction of unit vector **n**:

$$B_2=\mathbf{B}(s)\mathbf{n}+p_2\mathbf{n}(\mathbf{n}\cdot\nabla)\mathbf{B}+p_3\mathbf{n}(\mathbf{b}\cdot\nabla)\mathbf{B}$$

$$=B_n(s)+B_{n2}p_2+B_{n3}p_3; \quad (32)$$

where

$$B_n(s)=n_xB_x+n_YB_Y+n_ZB_Z,$$

$$B_{n2}=n_xB_{x2}+n_YB_{Y2}+n_ZB_{Z2},$$

$$B_{n3}=n_xB_{x3}+n_YB_{Y3}+n_ZB_{Z3}; \quad (33)$$

and

$$B_x=B_x(x,y,z)\Big|_{x=x_N,\ y=y_N,\ z=z_N},$$

$$B_{x2}=\left[n_x\frac{\partial B_x}{\partial x}+n_Y\frac{\partial B_Y}{\partial y}+n_Z\frac{\partial B_Z}{\partial z}\right]_{x=x_N,y=y_N,z=z_N},$$

$$B_{x3}=\left[b_x\frac{\partial B_x}{\partial x}+b_Y\frac{\partial B_Y}{\partial y}+b_Z\frac{\partial B_Z}{\partial z}\right]_{x=x_N,y=y_N,z=z_N}. \quad (34)$$

Similarly, the other coefficients B_Y, B_{Y2}, B_{Y3} ; B_Z B_{Z2} , B_{Z3} will be obtained if in (34) B_x is replaced by B_Y, B_Z.

Similar to (32), we can get the components of the field vector **B** in the direction of unit vectors **t** and **b**.

$$B_1 = B_t(s) + B_{t2}p_2 + B_{t3}p_3,$$

$$B_3 = B_b(s) + B_{b2}p_2 + B_{b3}p_3; \qquad (35)$$

where $B_t(s)$, B_{t2}, B_{t3} and $B_b(s)$, B_{b2}, B_{b3} can be obtained if we use t_x, t_y, t_z or b_x, b_Y, b_z instead of n_x, n_Y, n_Z in (33).

Since the magnetic field vector B must satisfy the relations:

$$\nabla \cdot \mathbf{B} = 0, \qquad (36)$$

$$\nabla \times \mathbf{B} = 0; \qquad (37)$$

substituting (31) into (36) and (37) yields the relations between some of the coefficients:

$$B_t'(s) + B_{n3} + B_{b3} - \frac{1}{\rho} B_n(s) = 0; \qquad (38)$$

$$B_{n3} - B_{b2} = 0,$$

$$B_{t3} - B_b'(s) = 0,$$

$$B_{t2} - B_n'(s) = 0. \qquad (39)$$

In the above mentioned expressions n_x, n_y, n_z; b_x, b_y, b_z and t_x, t_Y, t_Z can be obtained by the following formulae:

$$t_x = x_N', \quad t_Y = y_N', \quad t_Z = z_N';$$

$$n_x = \rho x_N'', \quad n_Y = \rho y_N'', \quad n_Z = \rho z_N'';$$

$$b_x = \rho(y_N' z_N'' - z_N' y_N''),$$

$$b_Y = \rho(z_N' x_N'' - x_N' z_N''),$$

$$b_Z = o(x' y_N'' - y_N' x_N''); \qquad (40)$$

where the prime denotes derivatives with respect to the arc length s.

EQUATIONS OF PRINCIPAL TRAJECTORY AND ITS NEIGHBORING "PARAXIAL" TRAJECTORIES

In an electron-optical system with wide electron beam focusing, we consider the cathode with null electrostatic potential to be where the electron has small initial velocity, as mentioned above.

Similar to the cathode lenses with axial symmetry[8, 10], we assume that in the system with wide electron beam focusing, the curvilinear trajectories, being adjacent to the principal trajectory, satisfy the following "curved paraxial conditions" everywhere:

$$p^2(s) \approx 0 \ , \ p(s) \ll \rho(s) \ , \ p(s) \ll \tau(s);$$

$$\dot{p}^2(s) \ll 1 \qquad (41)$$

and $\varphi(0) = 0$. It is not necessary to assume that the condition $p'^2(s) \ll 1$ is satisfied everywhere.

We introduce the "curved paraxial conditions" (41) for wide electron beam focusing into (22a,b), regarding p_2, p_3 and $\dot{p}_2$, $\dot{p}_3$ as first order terms and using the method given in refs.[8,10], investigate each of the coefficients in (22a,b), we can thus arrive at equations for principal trajectory and its neighboring "paraxial" trajectories.

Now study the common coefficient of first terms on the right hand of equations (22a,b), which is related to the tangent velocity $\dot{s}$. It may be seen that this coefficient at the initial point can be written as

$$\left\{ \varphi \Big/ \left[(1 - \frac{1}{\rho}p_2)^2 + (\frac{1}{\tau}p_2)^2 + (\frac{1}{\tau}p_3)^2 - 2\frac{1}{\tau}p_3 p_2' + 2\frac{1}{\tau}p_2 p_3' + p_2'^2 + p_3'^2 \right] \right\}_{s=0,\,p_2=0,\,p_3=0}$$

$$= \left(\frac{\varepsilon_0}{1 + p_2'^2 + p_3'^2} \right) = \varepsilon_s. \qquad (42)$$

Therefore, the 0+1 order approximation can be expressed by

$$\left\{ \varphi \Big/ \left[(1 - \frac{1}{\rho}p_2)^2 + (\frac{1}{\tau}p_2)^2 + (\frac{1}{\tau}p_3)^2 - 2\frac{1}{\tau}p_3 p_2' + 2\frac{1}{\tau}p_2 p_3' + p_2'^2 + p_3'^2 \right] \right\}_{0+1}$$

$$= \varepsilon_s + \phi + p_2\varphi_2 + p_3\varphi_3 + (\varepsilon_s + \phi)\frac{2}{\rho}p_2. \qquad (43)$$

From the 0+1+2 order approximation of this coefficient, we may obtain the condition, by which the formula was set up.

$$p^2 \approx 0, \quad \frac{\frac{m_0}{2e}(\dot{p}^2 - \dot{p}_0^2)}{\phi + \varepsilon_s} \ll 1 \qquad (44)$$

This is called the "paraxial" condition. Therefore, when investigating the system with wide electron beam focusing, we regard p and $\sqrt{\frac{m_0}{2e}}\,\dot{p} \Big/ \sqrt{\phi + \varepsilon_s}$ as first order terms and the products as second order terms, which can be neglected.

Similar to those discussed above and for other coefficients in (22a,b), after a series of rather complicated manipulations, we have the trajectory equation of zero order approximation:

$$\frac{2}{\rho}(\phi+\varepsilon_s)=\varphi_2+\sqrt{\frac{2e}{m_0}}\sqrt{\phi+\varepsilon_s}\,B_b(s),$$

$$\varphi_3=\sqrt{\frac{2e}{m_0}}\sqrt{\phi+\varepsilon_s}\,B_n(s); \qquad (45a,b)$$

and the trajectory equations of first-order approximation:

$$\frac{d}{ds}\left(n\frac{dp_2}{ds}\right)=Fp_2+Np_3+2K\frac{dp_3}{ds},$$

$$\frac{d}{ds}\left(n\frac{dp_3}{ds}\right)=Rp_3-Qp_2-2K\frac{dp_2}{ds}; \qquad (46a,b)$$

where n, K, F, R, N and Q express the following functions at the curve of principal trajectory:

$$n=\sqrt{\phi+\varepsilon_s}, \qquad K=\frac{\sqrt{\phi+\varepsilon_s}}{\tau}-\sqrt{\frac{e}{8m_0}}B_t(s),$$

$$F=\sqrt{\phi+\varepsilon_s}\left(\frac{1}{\tau^2}-\frac{1}{\rho^2}\right)+\frac{\varphi_{22}}{2\sqrt{\phi+\varepsilon_s}}-\frac{\varphi_2}{\rho\sqrt{\phi+\varepsilon_s}}$$

$$-\sqrt{\frac{e}{2m_0}}\left(\frac{1}{\tau}B_t(s)-B_{b2}-\frac{\varphi_2 B_b(s)}{2(\phi+\varepsilon_s)}\right),$$

$$R=\frac{1}{\tau^2}\sqrt{\phi+\varepsilon_s}+\frac{\varphi_{33}}{2\sqrt{\phi+\varepsilon_s}}-\sqrt{\frac{e}{2m_0}}\left(\frac{1}{\tau}B_t(s)\right.$$

$$\left.+B_{b2}+\frac{\varphi_3 B_n(s)}{2(\phi+\varepsilon_s)}\right),$$

$$N=\frac{\varphi_1}{2\tau\sqrt{\phi+\varepsilon_s}}-\frac{\sqrt{\phi+\varepsilon_s}}{\tau^2}\frac{d\tau}{ds}-\frac{\varphi_3}{\rho\sqrt{\phi+\varepsilon_s}}$$

$$+\frac{\varphi_{23}}{2\sqrt{\phi+\varepsilon_s}}+\sqrt{\frac{e}{2m_0}}\left(B_{b3}+\frac{\varphi_3 B_b(s)}{2(\phi+\varepsilon_s)}\right),$$

$$Q=\frac{\varphi_1}{2\tau\sqrt{\phi+\varepsilon_s}}-\frac{\sqrt{\phi+\varepsilon_s}}{\tau^2}\frac{d\tau}{ds}-\frac{\varphi_{23}}{2\sqrt{\phi+\varepsilon_s}}$$

$$+\sqrt{\frac{e}{2m_0}}\left(B_{n2}+\frac{\varphi_2 B_n(s)}{2(\phi+\varepsilon_s)}\right). \qquad (47)$$

It must be pointed out that the following relationship among coefficients N, Q and K in (47) holds[7]:

$$\frac{N+Q}{2}=\frac{dK}{ds}. \qquad (48)$$

(45a,b) are called equations for the principal trajectory. The coordinate curve s derived from eqs. (45a,b) when $p_2=p_3=0$ thus represents one of the electron trajectories——principal trajectory (it might be the axis of the system).

Eqs.(46a,b) can be used to describe the neighboring trajectories with rather large initial slope, emitting from the cathode surface with potential $\phi(0)=0$. we will call them curvilinear "paraxial" trajectory equations.

It is to be noted that the value of coefficients n,F, N, R, Q and K must be taken from the principal trajectory when solving eqs.(46a,b). Thus, we usually solve eqs.(45a,b) and (46a,b) simultaneously. Besides, either in the principal trajectory equations (45a,b) or in the curvilinear "paraxial" trajectory equations (46a,b), the same value for ε_s must be taken.

In what follows we will further discuss the physical meaning of the principal trajectory equations expressed in the form of (45a,b). From the fundamental equation of electron motion

$$\frac{d}{dt}(m_0\mathbf{v})=-e\mathbf{E}-e(\mathbf{v}\times\mathbf{B}), \qquad (49)$$

we investigate the projection of motion equation of principal trajectory on the Frenet local coordinate system. For the motion of principal electron, $\mathbf{v}=v\mathbf{t}$, $v=ds/dt$, we have

$$\frac{d}{dt}(m_0\mathbf{v})=m_0 v\left(\frac{dv}{ds}\mathbf{t}+v\frac{d\mathbf{t}}{ds}\right). \qquad (50)$$

Using Frenet-Serret's formulae (7), equation (49) can be transformed to

$$m_0 v\frac{dv}{ds}+m_0\frac{v^2}{\rho}\mathbf{n}=e\,\mathrm{grad}\varphi-e(v\mathbf{t}\times\mathbf{B}). \qquad (51)$$

Multiply eq.(51) by **t**, **n**, **b** as scalar multiplication, we obtain

$$m_0 v\frac{dv}{ds}=e\frac{d\varphi}{ds}, \qquad (52)$$

$$\frac{m_0 v^2}{\rho}=e(\mathrm{grad}\varphi\cdot\mathbf{n})+ev(\mathbf{B}\cdot\mathbf{b}), \qquad (53)$$

$$\mathrm{grad}\varphi\cdot\mathbf{b}=v(\mathbf{B}\cdot\mathbf{n}). \qquad (54)$$

Integrate (52) and consider the initial energy $e\varepsilon_s$ of principal electron emitted from the cathode surface and $\varphi(0,0,s)=\phi(s)$, $\phi(s=0)=0$, we have the velocity of principal electron

$$v=\sqrt{\frac{2e}{m_0}(\phi+\varepsilon_s)}. \tag{55}$$

Differentiate eqs.(53) and (54) with respect to s, we have

$$\frac{d}{ds}\left(\frac{m_0 v^2}{\rho}\right)=e\left\{\frac{d}{ds}\mathrm{grad}\varphi\cdot\mathbf{n}-\frac{1}{\rho}\mathrm{grad}\varphi\cdot\mathbf{t}+\frac{1}{\tau}\mathrm{grad}\varphi\cdot\mathbf{b}\right.$$

$$\left.+v\left(\frac{d}{ds}\mathbf{B}\cdot\mathbf{b}-\frac{1}{\tau}\mathbf{B}\cdot\mathbf{n}\right)+\frac{e}{m_0 v}\frac{d\varphi}{ds}\mathbf{B}\cdot\mathbf{b}\right\} \tag{56}$$

$$\frac{d}{ds}\mathrm{grad}\varphi\cdot\mathbf{b}-\frac{1}{\tau}\mathrm{grad}\varphi\cdot\mathbf{n}=v\left(\frac{d}{ds}\mathbf{B}\cdot\mathbf{n}-\frac{1}{\rho}\mathbf{B}\cdot\mathbf{t}+\frac{1}{\tau}\mathbf{B}\cdot\mathbf{b}\right)$$

$$+\frac{e}{m_0 v}\frac{d\varphi}{ds}\mathbf{B}\cdot\mathbf{n}. \tag{57}$$

Using (55), eqs.(53) (54) (56) (57) can be written as

$$\frac{2(\phi+\varepsilon_s)}{\rho}=\varphi_2+\sqrt{\frac{2e}{m_0}}\sqrt{\phi+\varepsilon_s}\,B_b(s), \tag{58}$$

$$\varphi_3=\sqrt{\frac{2e}{m_0}}\sqrt{\phi+\varepsilon_s}\,B_n(s), \tag{59}$$

$$\frac{d}{ds}\left(\frac{2(\phi+\varepsilon_s)}{\rho}\right)=\varphi_{12}-\frac{1}{\rho}\varphi_1+\frac{1}{\tau}\varphi_3+\sqrt{\frac{2e}{m_0}}\sqrt{\phi+\varepsilon_s}\,(B_{t3}$$

$$-\frac{1}{\tau}B_n(s))+\sqrt{\frac{e}{2m_0}}\frac{1}{\sqrt{\phi+\varepsilon_s}}\varphi_1 B_b(s), \tag{60}$$

$$\varphi_{13}-\frac{1}{\tau}\varphi_2=\sqrt{\frac{2e}{m_0}}\sqrt{\phi+\varepsilon_s}\left(B_{t2}-\frac{1}{\rho}B_t(s)\right.$$

$$\left.-\frac{1}{\tau}B_b(s)\right)+\sqrt{\frac{e}{2m_0}}\frac{1}{\sqrt{\phi+\varepsilon_s}}\varphi_1 B_n(s). \tag{61}$$

It will be seen that the four equations (58)~(61) are derived from the motion equation, and equations (58) and (59) are just the same equations (45a,b) derived above.

Problems of wide electron beam focusing can be put forward in two ways. Usually, given the electrostatic and magnetic fields, the electron trajectories can be calculated through their differential equations, thus determining the focusing properties. At that time, eq.(45a) or (58) indicates the equation of principal trajectory, and the other three equations (59)(60)(61) will be automatically satisfied. But, on the contrary, the field distributions may also be defined from the given beams or trajectories. At this moment, (58) (59) (60) (61) will be the four complemental equations to be satisfied, which show the relations between the coefficients of electrostatic and magnetic fields, that are not independent of each other.

In general, the principal trajectory equation (45a), that are functions of arc length s, is not convenient to solve here, we transform it into a form using the laboratory Cartesian coordinate system. Because of

$$\varphi_2=n_x\phi_x+n_y\phi_y+n_z\phi_z,$$

$$B_b(s)=b_xB_x+b_yB_y+b_zB_z;$$

where $\phi_x=\frac{\partial\phi}{\partial x}$, $\phi_y=\frac{\partial\phi}{\partial y}$, $\phi_z=\frac{\partial\phi}{\partial z}$;

let b_x, n_x be expressed by (40), ρ by (4) and transform each derivative with respect to s into the derivative with respect to z, using

$$ds=\sqrt{1+[x'(z)]^2+[y'(z)]^2}\,dz, \tag{62}$$

then the principal trajectory equation (45a) can be written as

$$\frac{d}{dz}\left(\frac{\sqrt{\phi+\varepsilon_s}}{\sqrt{1+x'^2+y'^2}}x'\right)=\frac{\sqrt{1+x'^2+y'^2}}{2\sqrt{\phi+\varepsilon_s}}\frac{\partial\phi}{\partial x}$$

$$+\sqrt{\frac{e}{2m_0}}(B_y-y'B_z),$$

$$\frac{d}{dz}\left(\frac{\sqrt{\phi+\varepsilon_s}}{\sqrt{1+x'^2+y'^2}}y'\right)=\frac{\sqrt{1+x'^2+y'^2}}{2\sqrt{\phi+\varepsilon_s}}\frac{\partial\phi}{\partial y}$$

$$+\sqrt{\frac{e}{2m_0}}(x'B_z-B_x)\,. \tag{63a,b}$$

where the prime indicates derivatives with respect to z.

If eqs. (63a,b) are set up, suppose

$$\varphi_3=b_x\phi_x+b_y\phi_y+b_z\phi_z,$$

$$B_n(s)=n_xB_x+n_yB_y+n_zB_z,$$

substitute them into (45b), and through a series of transformations, we can prove that the equation (45b) is automatically satisfied.

For equations (46a,b) of neighboring "paraxial" trajectories, it is convenient to transform them into equations that are functions of z. By using (62), (46a,b) will have the form:

$$\frac{d}{dz}\left(\sqrt{\phi+\varepsilon_s}\,\frac{dp_2}{dz}\right)=(1+x'^2+y'^2)(Fp_2+Np_3)$$

$$+2K\sqrt{1+x'^2+y'^2}\,\frac{dp_3}{dz}+\sqrt{\phi+\varepsilon_s}\,\frac{x'x''+y'y''}{1+x'^2+y'^2}\,\frac{dp_2}{dz},$$

$$\frac{d}{dz}\left(\sqrt{\phi+\varepsilon_s}\,\frac{dp_3}{dz}\right)=(1+x'^2+y'^2)(Rp_3-Qp_2)$$

$$-2K\sqrt{1+x'^2+y'^2}\,\frac{dp_2}{dz}+\sqrt{\phi+\varepsilon_s}\,\frac{x'x''+y'y''}{1+x'^2+y'^2}\,\frac{dp_3}{dz}. \tag{64a,b}$$

It is to be noted that the value of coefficients n, K, F, R, N and Q and x', y', x'', y'' must be taken from the principal trajectory when solving eqs.(64a,b).

Eqs.(64a,b) are two coupled, linear, second-order differential equations which may be transformed to a system of four first order equations. Let

$$w_1=p_2,\quad w_2=p_2',\quad w_3=p_3,\quad w_4=p_3';$$

then eqs.(64a,b) become

$$w_1'=w_2,$$
$$w_2'=g_1w_1+g_2w_3+f_1w_2+f_2w_4,$$
$$w_3'=w_4,$$
$$w_4'=h_1w_1+h_2w_3+f_1w_4-f_2w_2; \tag{65}$$

where

$$g_1=\left(\frac{1+x'^2+y'^2}{\sqrt{\phi+\varepsilon_s}}F\right)_{x=x_N,\ y=y_N,\ z=z_N}$$

$$g_2=\left(\frac{1+x'^2+y'^2}{\sqrt{\phi+\varepsilon_s}}N\right)_{x=x_N,\ y=y_N,\ z=z_N}$$

$$h_1=\left(\frac{1+x'^2+y'^2}{\sqrt{\phi+\varepsilon_s}}Q\right)_{x=x_N,\ y=y_N,\ z=z_N}$$

$$h_2=-\left(\frac{1+x'^2+y'^2}{\sqrt{\phi+\varepsilon_s}}R\right)_{x=x_N,\ y=y_N,\ z=z_N}$$

$$f_1=\left(\frac{x'x''+y'y''}{1+x'^2+y'^2}-\frac{1}{2(\phi+\varepsilon_s)}\frac{d\phi}{dz}\right)_{x=x_N,\ y=y_N,\ z=z_N}$$

$$f_2=\left(2\frac{\sqrt{1+x'^2+y'^2}}{\sqrt{\phi+\varepsilon_s}}K\right)_{x=x_N,\ y=y_N,\ z=z_N} \tag{66}$$

By numerical method, one may determine a set of solutions of (65):

$$w_1^{(i)},\ w_2^{(i)},\ w_3^{(i)},\ w_4^{(i)};\ (i=1,2,3,4)$$

which satisfy the following initial conditions:

i	1	2	3	4
$w_1^{(i)}=p_2^{(i)}$	1	0	0	0
$w_2^{(i)}=(p_2^{(i)})'$	0	$1/\sqrt{\varepsilon_s}$	0	0
$w_3^{(i)}=p_3^{(i)}$	0	0	1	0
$w_4^{(i)}=(p_3^{(i)})'$	0	0	0	$1/\sqrt{\varepsilon_s}$

(67)

The solutions of (64a,b) may then be written as

$$p_2(z)=p_{20}w_1^{(1)}+\sqrt{\varepsilon_s}p_{20}'w_1^{(2)}+p_{30}w_1^{(3)}+\sqrt{\varepsilon_s}\,p_{30}'\,w_1^{(4)},$$

$$p_3(z)=p_{20}\,w_3^{(1)}+\sqrt{\varepsilon_s}\,p_{20}'\,w_3^{(2)}+p_{30}w_3^{(3)}+\sqrt{\varepsilon_s}\,p_{30}'\,w_3^{(4)}; \tag{68a,b}$$

and these satisfy the initial conditions:

$$p_2(z_0)=p_{20},\qquad p_2'(z_0)=p_{20}';$$
$$p_3(z_0)=p_{30},\qquad p_3'(z_0)=p_{30}'.$$

THE ORTHOGONAL CONDITION OF SYSTEMS WITH WIDE ELECTRON BEAM FOCUSING

The orthogonal condition of systems with narrow electron beam focusing was first given by Sturrock [6]. We will try here to derive the orthogonal condition of system with wide electron beam focusing. It is necessary first of all to simplify eqs.(46a,b), in order that the term p_3' does not appear in eq.(46a) and p_2' in eq.(46b).

Introducing coordinate transformation

$$p_2+ip_3=(u+iv)e^{i\chi} \tag{69}$$

to eqs.(46a,b), and supposing that

$$\chi'=\frac{d\chi}{ds}=-\frac{K}{n}, \tag{70}$$

we can get the curvilinear "paraxial" trajectory equations in a rotating coordinate system (u, v):

$$\frac{d}{ds}\left(n\frac{du}{ds}\right)=a_{11}u+a_{12}v,$$

$$\frac{d}{ds}\left(n\frac{dv}{ds}\right)=a_{21}u+a_{22}v; \tag{71a,b}$$

where

$$a_{11}=F\cos^2\chi+R\sin^2\chi+(N-Q)\sin\chi\cos\chi-\frac{K^2}{n},$$

$$a_{12}=N\cos^2\chi+Q\sin^2\chi-(F-R)\sin\chi\cos\chi-K',$$

$$a_{21}=-N\sin^2\chi-Q\cos^2\chi-(F-R)\sin\chi\cos\chi+K'.$$

$$a_{22}=F\sin^2\chi+R\cos^2\chi-(N-Q)\sin\chi\cos\chi-\frac{K^2}{n}. \tag{72}$$

It may be seen from (71a,b) that the variables of these two equations are still not separated. But it can be proved from (72) and (48) that $a_{12}=a_{21}$. It means that (71a,b) are self-conjugate equations.

Apparently, in case the variables are completely separated, eqs.(71a,b) have to satisfy the following condition: $a_{12}=a_{21}=0$. If so, it can be written as $a_{12}+a_{21}=0$. Then we have

$$\tan 2\chi=\frac{N-Q}{F-R}. \tag{73}$$

Transforming the expressions of a_{11} and a_{22} in (72) by using (73), and let $U=a_{11}$, $V=a_{22}$, the we get

$$\frac{d}{ds}\left(n\frac{du}{ds}\right)=Uu,$$

$$\frac{d}{ds}\left(n\frac{dv}{ds}\right)=Vv; \tag{74a,b}$$

where

$$U=\frac{1}{2}(F+R)+\frac{1}{2}\sqrt{(F-R)^2+(N-Q)^2}-\frac{K^2}{n},$$

$$V=\frac{1}{2}(F+R)-\frac{1}{2}\sqrt{(F-R)^2+(N-Q)^2}-\frac{K^2}{n}. \tag{75}$$

From (70) and (73), we can derive the conditional relationship, which the coefficients n, K, F, R, N and Q must satisfy, when the variables are separated:

$$n[(N-Q)(F'-R')-(F-R)(N'-Q')]$$

$$=2K[(F-R)^2+(N-Q)^2]. \tag{76}$$

This is the orthogonal condition for systems with wide electron beam focusing. Systems, which satisfy the orthogonal condition (76), will be named orthogonal curvilinear "paraxial" systems.

For curvilinear electron optical systems satisfying the orthogonal condition, the image with point focusing or line focusing can be formed, as it has been pointed out by Sturrock [6].

(74a,b) are second order homogeneous linear differential equations of u and v respectively. Although in these equations a singular point exists at the cathode surface when $\varepsilon_S=0$. There are no essential difficulties in solving eqs.(74a,b), compared with the cathode lenses with axial symmetry. Thus, the general solutions u and v of eqs. (74a,b) can be expressed by two linear independent special solutions. Let u_α, u_β and v_α, v_β be the two sets of special solutions of eqs. (74a, b), which satisfy the following conditions:

$$u_\alpha(0)=v_\alpha(0)=0,\qquad \sqrt{\varepsilon_S}u_\alpha'(0)=\sqrt{\varepsilon_S}v_\alpha'(0)=1;$$

$$u_\beta(0)=v_\beta(0)=1,\qquad \sqrt{\varepsilon_S}u_\beta'(0)=\sqrt{\varepsilon_S}v_\beta'(0)=0. \tag{77}$$

In order to obtain an image of point focusing (stigmatic imaging), it is obviously necessary to make U and V in eqs.(74a,b) equal to each other. In that case, it becomes possible only when

$$F-R=N-Q=0. \tag{78}$$

Then at the image plane $s=s_i$, which corresponds to $\varepsilon_S=\varepsilon_{S1}$, we have

$$u_\alpha(\varepsilon_{S1},s_i)=v_\alpha(\varepsilon_{S1},s_i)=0,$$

$$u_\alpha'(\varepsilon_{S1},s_i)=v_\alpha'(\varepsilon_{S1},s_i). \tag{79}$$

The modulus of its magnification can be written as

$$M_u=M_v=M=\left|\frac{1}{u_\alpha'(\varepsilon_{S1},s_i)\sqrt{\phi(s_i)+\varepsilon_{S1}}}\right|. \tag{80}$$

It can be seen from (74a,b) and (76) that systems satisfying condition (78) will certainly satisfy the orthogonal condition given above. Condition (78) corresponds to the two relationships that follows:

$$\frac{\sqrt{\phi+\varepsilon_S}}{\rho^2}+\frac{\varphi_{33}-\varphi_{22}}{2\sqrt{\phi+\varepsilon_S}}+\frac{\varphi_2}{\rho\sqrt{\phi+\varepsilon_S}}-\sqrt{\frac{2e}{m_0}}\Bigg(B_{n3}$$

$$+\frac{B_b(s)\varphi_2+B_n(s)\varphi_3}{4\sqrt{\phi+\varepsilon_S}}\Bigg)=0,$$

$$\frac{\varphi_{23}}{\sqrt{\phi+\varepsilon_S}}-\frac{\varphi_3}{\rho\sqrt{\phi+\varepsilon_S}}-\sqrt{\frac{e}{2m_0}}\Bigg(B_{n2}-B_{b3}$$

$$-\frac{B_b(s)\varphi_3-B_n(s)\varphi_2}{2\sqrt{\phi+\varepsilon_S}}\Bigg)=0. \tag{81}$$

Condition (78) or (81) shows that the electron beam emitted from the cathode surface with $\varepsilon_S=\varepsilon_{S1}$ will have an ideal point focusing (stigmatic imaging), therefore the system will have properties similar to those of the electron optical system with axial symmetry.

If the system with wide electron beam focusing only satisfies the orthogonal condition (76), but does not satisfy condition (78), thus $U \neq V$, and the special solutions u_α, u_β and v_α, v_β are different. This means anisotropy in magnification, and its moduli in two directions in case of $\varepsilon_S = \varepsilon_{S1}$ will be

$$M_u = \left| \frac{1}{u_\alpha'(\varepsilon_{S1}, s_{1i}) \sqrt{\phi(s_{1i}) + \varepsilon_{S1}}} \right|, \tag{82a}$$

$$M_v = \left| \frac{1}{v_\alpha'(\varepsilon_{S1}, s_{2i}) \sqrt{\phi(s_{2i}) + \varepsilon_{S1}}} \right|. \tag{82b}$$

The positions s_{1i} and s_{2i} of focusing image for the electron beam do not coincide with each other even when the initial energy $e\varepsilon_S = e\varepsilon_{S1}$ is the same. It means that two ideal focusing images corresponding to $\varepsilon_S = \varepsilon_{S1}$ will be possible in a given orthogonal curvilinear system. A focusing segment in the binormal direction **b** at the location $s = s_{1i}$ is formed, and another focusing segment in the normal direction **n** at the location $s = s_{2i}$ at the same time. Thus, the difference between the two locations can be defined as the astigmatism of the system with wide electron beam focusing.

SOME PRACTICAL EXAMPLES FOR SYSTEMS WITH WIDE ELECTRON BEAM FOCUSING

1. System with Wide Electron Beam Focusing, in which the Principal Trajectory is a Linear Axis of Symmetry

When the principal trajectory is a linear axis of symmetry, $\frac{1}{\rho} = \frac{1}{\tau} = 0$. In consideration of axially symmetric fields, their potential on the axis takes the extreme value, and there is no difference between the normal and binormal directions.

Let $\mathbf{p} = p_2 + ip_3$, $s = z$, $\varepsilon_s = \varepsilon_z$, $\phi(s) = \phi(z)$, $B_t(s) = B(z)$, we obtain

$$n = \sqrt{\phi(z) + \varepsilon_z}, \quad K = -\sqrt{\frac{e}{8m_0}} B(z), \quad N - Q = 0,$$

$$F = R = -\frac{1}{4} \frac{\phi''}{\sqrt{\phi(z) + \varepsilon_z}}. \tag{83}$$

Then

$$U = V = -\frac{1}{4} \frac{\phi''}{\sqrt{\phi(z) + \varepsilon_z}} - \frac{e}{8m_0} \frac{B^2(z)}{\sqrt{\phi(z) + \varepsilon_z}}. \tag{84}$$

It is obvious that the system of wide electron beam focusing with a linear axis of symmetry not only satisfies condition (76), but also satisfies condition (78). Thus, this system has the properties of ideal focusing (point focusing).

Let

$$\mathbf{u} = u + iv = pe^{-i\chi}, \tag{85}$$

where

$$\chi' = \sqrt{\frac{e}{8m_0}} \frac{B(z)}{\sqrt{\phi(z) + \varepsilon_z}}.$$

eqs. (74a,b) will then have the following form:

$$[\phi(z) + \varepsilon_z] \mathbf{u}'' + \frac{1}{2} \phi'(z) \mathbf{u}' + \frac{1}{4} [(\phi''(z) + \frac{e}{2m_0} B^2(z)] \mathbf{u} = 0. \tag{86}$$

It is just the linear differential equation of a system with wide electron beam focusing [8], where the principal trajectory is a straight axis having axial symmetry. Using eq. (86), we have solved a concentric spherical system of combined electrostatic and magnetic fields with wide electron beam focusing [1].

2. Electrostatic Cathode Lenses with Wide Electron Beam Focusing

For electrostatic cathode lenses with axial symmetry, the off-axis principal trajectory emitted from the cathode surface will be a plane curve which is normal to the cathode surface. Thus,

$$B = 0, \quad \frac{1}{\tau} = 0, \quad \varphi_{23} = 0,$$

then from (47) we have

$$n = \sqrt{\phi(s) + \varepsilon_s}, \qquad K = 0,$$

$$F = -\frac{3}{\rho^2} \sqrt{\phi(s) + \varepsilon_s} + \frac{\varphi_{22}}{2\sqrt{\phi(s) + \varepsilon_s}},$$

$$R = \frac{\varphi_{33}}{2\sqrt{\phi(s) + \varepsilon_s}}, \qquad N = Q = 0. \tag{87}$$

The coefficients in expressions (87) also satisfy the orthogonal condition (76). It is obvious that the variables u and v are also separated.

Because $\chi=0$, suppose that $u=p_2$, $v=p_3$; then the principal trajectory equation can be written as

$$\varphi_2=\frac{2[\phi(s)+\varepsilon_s]}{\rho}. \tag{88}$$

The curvilinear "paraxial" trajectory equations would be

$$\frac{d}{ds}\left(\sqrt{\phi(s)+\varepsilon_s}\frac{dp_2}{ds}\right)=\left\{-\frac{3\sqrt{\phi(s)+\varepsilon_s}}{\rho^2}+\frac{\varphi_{22}}{2\sqrt{\phi(s)+\varepsilon_s}}\right\}p_2,$$

$$\frac{d}{ds}\left(\sqrt{\phi(s)+\varepsilon_s}\frac{dp_3}{ds}\right)=\frac{\varphi_{33}}{2\sqrt{\phi(s)+\varepsilon_3}}p_3.$$

(89a,b)

The arc length s, which defines a coordinate curve of the principal trajectory, can be transformed into a form with cylindrical coordinates (r,z). From (88) and(89), we have the principal trajectory equation:

$$r''=\frac{1+r'^2}{2[\varphi(z,r)+\varepsilon_s]}\left(\frac{\partial\varphi}{\partial r}-r'\frac{\partial\varphi}{\partial z}\right), \tag{90}$$

and the curvilinear neighboring trajectory equation:

$$p_2''+F_1p_2'+F_2p_2=0,$$

$$p_3''+G_1p_3'+G_2p_3=0; \tag{91a,b}$$

where

$$F_1=G_1=\frac{1+r'^2}{2[\varphi(z,r)+\varepsilon_s]}\frac{\partial\varphi}{\partial z},$$

$$G_2=\frac{-(1+r'^2)}{2[\varphi(z,r)+\varepsilon_s]}\frac{1}{r}\frac{\partial\varphi}{\partial r},$$

$$F_2=\frac{3r''^2}{(1+r'^2)^2}+\frac{1}{2[\varphi(z,r)+\varepsilon_s]}\left(-\frac{\partial^2\varphi}{\partial r^2}+2r'\frac{\partial^2\varphi}{\partial r\partial z}-r'^2\frac{\partial^2\varphi}{\partial z^2}\right). \tag{92}$$

It is necessary to remind that the terms in the coefficients F_1, G_1, F_2, G_2, should be taken to be the values on the principal trajectory, and that the prime notations in the eqs.(90), (91a,b) and (92) denote derivatives with respect to the axial coordinate z.

For the electron beam emitted from the axial point of cathode surface, because the principal trajectory is actually the rotating symmetrical axis, we will have $r=r'=r''=0$, and

$$\left(\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)_{r=0}=\left(\frac{\partial^2\varphi}{\partial r^2}\right)_{r=0}=-\frac{1}{2}\phi''(z). \tag{93}$$

Let $p=p_2+ip_3$, $\varepsilon_s=\varepsilon_z$, then eqs.(91a,b) take the following form:

$$p''+\frac{1}{2}\frac{\phi'}{\phi(z)+\varepsilon_z}p'+\frac{1}{4}\frac{\phi''}{\phi(z)+\varepsilon_z}p=0. \tag{94}$$

It is just the well-known "paraxial" equation in the case of electrostatic cathode lenses.

3. Two Dimensional Field of Plane Symmetry with Wide Electron Beam Focusing

Let the principal trajectory be an axis of symmetry for a two dimensional field with plane symmetry, then the orthogonal coordinate system (p_2,p_3,s) can be replaced by the Cartesian coordinate system (x,y,z), and the coordinate z, the axis of symmetry for the system, coincides with the principal trajectory. Therefore,

$$\frac{1}{\rho}=\frac{1}{\tau}=0.$$

For the two dimensional field, which is independent of the coordinate x, let $B_t(z)=B$, $\varepsilon_s=\varepsilon_z$, the "paraxial" trajectory equations (46a,b) can be written as

$$x'=-\sqrt{\frac{e}{2m_0}}\frac{1}{\sqrt{\phi+\varepsilon_z}}By+\frac{C}{\sqrt{\phi+\varepsilon_z}},$$

$$y''+\frac{\phi'}{2[\phi+\varepsilon_z]}y'+\left(\frac{\phi''}{2[\phi+\varepsilon_z]}+\frac{eB^2}{2m_0[\phi+\varepsilon_z]}\right)y=\sqrt{\frac{e}{2m_0}}C\frac{B}{\phi+\varepsilon_z}; \tag{95a,b}$$

where

$$C=\sqrt{\varepsilon_z}x_0'+\sqrt{\frac{e}{2m_0}}B_0y_0.$$

It will be seen from (95a,b) that this system does not satisfy the orthogonal condition.

For the case of the electrostatic field of plane symmetry with wide electron beam focusing, we obtain

$$x'=\frac{x_0'\sqrt{\varepsilon_z}}{\sqrt{\phi(z)+\varepsilon_z}},$$

$$y''+\frac{\phi'}{2[\phi(z)+\varepsilon_z]}y'+\frac{\phi''}{2[\phi(z)+\varepsilon_z]}y=0. \tag{96a,b}$$

It is obvious that the variables x and y are also separated. It is enough to prove that this kind of focusing system, which is actually a so-called cylindrical cathode lens, will focus only the plane electron beam located in the plane (y, z). In the direction perpendicular to the plane (y, z), there is no applied force of electric field. Thus, the solid electron beam emitted from the axial point of cathode surface, will be affected only in the direction perpendicular to the plane of symmetry. It will not have a point focusing as in the field with axial symmetry, but will have a line focusing in the direction perpendicular to the plane (y,z).

AN APPROACH TO THE TREATMENT OF DEFLECTION DEFOCUSING EFFECTS IN CATHODE-RAY TUBES USING THEORY OF WIDE ELECTRON BEAM FOCUSING

To describe the convergence defects of deflection fields at angles larger than 45°, Hutter [3] put forward a method to the theoretical treatment, based on the application of the theory of "focusing fields with arbitrary curved optical axes". In his paper, a rotating system of coordinates u, v, w has been introduced, which rotates with respect to the Frenet local coordinate system, the potential φ and field vector **B** are expanded in a power series of u and v with coefficients that are functions of arc length w, the zero-order path conditions and the paraxial equations of pure electrostatic and pure magnetic deflection systems have been derived, and the aberrations of system have also been discussed. Hutter's paper presented a new approach to solve the theory of the deflection system at large deflection angles.

It must be pointed out that from the beginning of Hutter's paper the introduction of a rotating coordinate system with respect to Frenet local coordinate system seems to complicate the problem, and there is no clear relation in his paper between the zero-order path equation derived from the rotating coordinate system (see eq. (57) in ref. [3].) and the equation of central trajectory derived from the laboratory Cartesian coordinate system (see eq. (7) in ref. [3]).

In fact, one may extend the principal trajectory equations (45a,b) and the "paraxial" trajectory equations (46a,b) derived above to solve the problem of large angle deflection, if we assume $\varphi(s=0)\neq 0$, $\varepsilon_s=\varepsilon_0$, $\phi(s)\gg\varepsilon_0$. Therefore, problem with focusing of deflection system at large deflecting angle (in general, it is treated as a problem with narrow electron beam focusing [2, 4].) in a curvilinear coordinate system can be regarded as a special case thereof.

Now we illustrate with example for a pure magnetic deflection system with large angle deflection given by Hutter. In that case, $\phi=V_0=\text{const}$. From (45a,b)(46a,b), we have the principal trajectory equations:

$$\frac{1}{\rho}=\sqrt{\frac{e}{2m_0V_0}}\,B_b(s),$$

$$B_n(s)=0; \qquad (97a,b)$$

and the "paraxial" trajectory equations:

$$p_2''(s)-2\left(\frac{1}{\tau}-\sqrt{\frac{e}{8m_0V_0}}\,B_t(s)\right)p_3'(s)-\left[\left(\frac{1}{\tau^2}-\frac{1}{\rho^2}\right)-\sqrt{\frac{e}{2m_0V_0}}\left(\frac{1}{\tau}B_t(s)-B_{n3}\right)\right]p_2(s)$$

$$+\left(\frac{1}{\tau^2}\frac{d\tau}{ds}-\sqrt{\frac{e}{2m_0V_0}}\,B_{b3}\right)p_3(s)=0,$$

$$p_3''(s)+2\left(\frac{1}{\tau}-\sqrt{\frac{e}{8m_0V_0}}\,B_t(s)\right)p_2'(s)-\left[\frac{1}{\tau^2}-\sqrt{\frac{e}{2m_0V_0}}\left(\frac{1}{\tau}B_t(s)+B_{n3}\right)\right]p_3(s)$$

$$-\left(\frac{1}{\tau^2}\frac{d\tau}{ds}-\sqrt{\frac{e}{2m_0V_0}}\,B_{n2}\right)p_2(s)=0. \qquad (98a,b)$$

Transforming eq. (97a) into a form expressed by laboratory coordinate system, we obtain

$$\frac{d}{dz}\left(\frac{x'}{\sqrt{1+x'^2+y'^2}}\right)=\sqrt{\frac{e}{2m_0V_0}}\,(B_Y-y'Bz),$$

$$\frac{d}{dz}\left(\frac{y'}{\sqrt{1+x'^2+y'^2}}\right)=\sqrt{\frac{e}{2m_0V_0}}\,(x'B_z-B_x).$$

$$(99a,b)$$

It is enough to prove that eq. (97b) is automatically satisfied if eqs. (99a,b) is set up.

Similarly, for a pure electrostatic deflection system with large angle deflection, we have the principal trajectory equations:

$$\frac{1}{\rho}=\frac{\varphi_2}{2\phi},$$

$$\varphi_3=0; \qquad (100a,b)$$

and the paraxial trajectory equations:

$$p_2''+\frac{1}{2}\frac{\phi'}{\phi}p_2'-2\frac{1}{\tau}p_3'+\Big[\Big(\frac{1}{\rho^2}-\frac{1}{\tau^2}\Big)+\frac{\varphi_2}{\rho\phi}-\frac{\varphi_{22}}{2\phi}\Big]p_2+\Big(\frac{1}{\tau^2}\tau'-\frac{\phi'}{2\tau\phi}-\frac{\varphi_{23}}{2\phi}\Big)p_3=0,$$

$$p_3''+\frac{1}{2}\frac{\phi'}{\phi}p_3'+2\frac{1}{\tau}p_2'-\Big(\frac{1}{\tau^2}+\frac{\varphi_{33}}{2\phi}\Big)p_3-\Big(\frac{1}{\tau^2}\tau'-\frac{\phi'}{2\tau\phi}-\frac{\phi_{23}}{2\phi}\Big)p_2=0. \quad (101a,b)$$

Under the circumstances of the laboratory Cartesian coordinate system, the principal trajectory equation (100a) corresponds to the following equations:

$$\frac{d}{dz}\Big(\frac{\sqrt{\phi}}{\sqrt{1+x'^2+y'^2}}x'\Big)=\frac{\sqrt{1+x'^2+y'^2}}{2\sqrt{\phi}}\frac{\partial\phi}{\partial x},$$

$$\frac{d}{dz}\Big(\frac{\sqrt{\phi}}{\sqrt{1+x'^2+y'^2}}y'\Big)=\frac{\sqrt{1+x'^2+y'^2}}{2\sqrt{\phi}}\frac{\partial\phi}{\partial y}. \quad (102a,b)$$

We can also prove that eq.(100b) is automatically satisfied if eqs.(102a,b) are set up.

For computation, it is convenient to transform (98a,b) or (101a,b) into equations that are functions of z by using

$$\frac{dp}{ds}=\frac{dp}{dz}\frac{1}{\sqrt{1+x'^2+y'^2}},$$

$$\frac{d^2p}{ds^2}=\frac{d^2p}{dz^2}\frac{1}{1+x'^2+y'^2}-\frac{dp}{dz}\frac{x'x''+y'y''}{(1+x'^2+y'^2)^2}. \quad (103)$$

For example, transforming (98a,b) by (103) into a system of four first order equations, similar to eq.(65), where coefficients g_1, g_2, h_1, h_2, f_1, f_2, can be written as

$$g_1=\Big\{(1+x'^2+y'^2)\Big[\frac{1}{\tau^2}-\frac{1}{\rho^2}-\sqrt{\frac{e}{2m_0V_0}}\Big(\frac{1}{\tau}B_t(s)-B_{n3}\Big)\Big]\Big\}_{x=x_N,\ y=y_N,\ z=z_N}$$

$$g_2=\Big\{-(1+x'^2+y'^2)\Big(\frac{1}{\tau^2}\frac{d\tau}{ds}-\sqrt{\frac{e}{2m_0V_0}}B_{b3}\Big)\Big\}_{x=x_N,y=y_N,z=z_N}$$

$$h_1=\Big\{(1+x'^2+y'^2)\Big(\frac{1}{\tau^2}\frac{d\tau}{ds}-\sqrt{\frac{e}{2m_0V_0}}B_{n2}\Big)\Big\}_{x=x_N,\ y=y_N,\ z=z_N}$$

$$h_2=\Big\{(1+x'^2+y'^2)\Big[\frac{1}{\tau^2}-\sqrt{\frac{e}{2m_0V_0}}\Big(\frac{1}{\tau}B_t(s)+B_{n3}\Big)\Big]\Big\}_{x=x_N,\ y=y_N,\ z=z_N}$$

$$f_1=\Big(\frac{x'x''+y'y''}{1+x'^2+y'^2}\Big)_{x=x_N,\ y=y_N,\ z=z_N}$$

$$f_2=\Big\{\sqrt{1+x'^2+y'^2}\Big(\frac{2}{\tau}-\sqrt{\frac{e}{2m_0V_0}}B_t(s)\Big)\Big\}_{x=x_N,\ y=y_N,\ z=z_N} \quad (104)$$

Similarly, for eqs.(101a,b), we have

$$g_1=\Big\{-(1+x'^2+y'^2)\Big[\frac{1}{\rho^2}-\frac{1}{\tau^2}+\frac{\varphi_2}{\rho\phi}-\frac{\varphi_{22}}{2\phi}\Big]\Big\}_{x=x_N,\ y=y_N,\ z=z_N}$$

$$g_2=\Big\{-(1+x'^2+y'^2)\Big(\frac{1}{\tau^2}\tau'-\frac{\phi'}{2\tau\phi}-\frac{\varphi_{23}}{2\phi}\Big)\Big\}_{x=x_N,\ y=y_N,\ z=z_N}$$

$$h_1=-g_2,$$

$$h_2=\Big\{(1+x'^2+y'^2)\Big(\frac{1}{\tau^2}+\frac{\varphi_{33}}{2\phi}\Big)\Big\}_{x=x_N,\ y=y_N,\ z=z_N}$$

$$f_1=\Big\{\frac{x'x''+y'y''}{1+x'^2+y'^2}-\frac{1}{2}\frac{\phi'}{\phi}\sqrt{1+x'^2+y'^2}\Big\}_{x=x_N,\ y=y_N,\ z=z_N}$$

$$f_2=\Big\{2\frac{1}{\tau}\sqrt{1+x'^2+y'^2}\Big\}_{x=x_N,\ y=y_N,\ z=z_N} \quad (105)$$

By numerical method, one may determine a set of solutions of (65):

$$w_1^{(i)},\quad w_2^{(i)},\quad w_3^{(i)},\quad w_4^{(i)}\qquad (i=1,2,3,4)$$

which satisfies the following conditions:

i	1	2	3	4
$w_1^{(i)}=p_2^{(i)}$	1	0	0	0
$w_2^{(i)}=(p_2^{(i)})'$	0	1	0	0
$w_3^{(i)}=p_3^{(i)}$	0	0	1	0
$w_4^{(i)}=(p_3^{(i)})'$	0	0	0	1

(106)

The solutions of p_2, p_3 may be expressed by

$$p_2(z)=p_{20}w_1^{(1)}+p_{20}'w_1^{(2)}+p_{30}w_1^{(3)}+p_{30}'w_1^{(4)}$$

$$p_3(z)=p_{20}w_3^{(1)}+p_{20}'w_3^{(2)}+p_{30}w_3^{(3)}+p_{30}'w_3^{(4)} \qquad (107a,b)$$

where the prime denotes the derivatives to z, and

$$p_2(z_0)=p_{20},\quad p_2'(z_0)=p_{20}',$$

$$p_3(z_0)=p_{30},\quad p_3'(z_0)=p_{30}'.$$

It will be seen from (67) and (106) that a great difference exists in the assumption of the initial conditions for solving the two kinds of curvilinear paraxial trajectory equations.

SUMMARY

1. In the present paper, the author tries to develop his previous work. Based on the assumption that the electrostatic potential φ and magnetic field vector **B** are known functions in the laboratory Cartesian coordinate system, we have deduced the principal trajectory equation and the "paraxial" trajectory equations of systems with wide electron beam focusing by given mathematical expressions of fields along curved optical axis, we have also derived the orthogonal condition of a curvilinear "paraxial" system with wide electron beam focusing, Some practical examples of systems with wide' electron beam focusing satisfying the orthogonal condition are also given.

2. It has been found in the present paper that the zero order equations of trajectory derived above are just the same equations of principal trajectory deduced from the fundamental equation of electron motion. It seems that the principal trajectory is determined by two differential equations. But in fact, if we investigate the focusing of wide electron beam from a given electrostatic potential and magnetic field, only one of these two equations is required for determining the principal trajectory, the other equation is automatically satisfied.

3. For problems concerned with wide electron beam focusing, no matter whether it is a system with axial symmetry or a curvilinear "paraxial" system satisfying orthogonal condition, the principal trajectory as well as the curvilinear "paraxial" neighboring trajectories must take the same value for $e\varepsilon_s$, which is the initial energy of principal trajectory along the tangential direction. Only under such a condition will the system have properties of point focusing or line focusing.

4. This paper has given as a set of equations for solving the principal trajectory and "paraxial" trajectories with derivatives that are functions of laboratory Cartesian coordinate z, so that the given equations are convenient for computation.

5. The result of this paper can be extended to solve deflection systems with deflection angles larger than 45°, and the equations obtained for solving the principal trajectory equation and paraxial trajectories in the deflection systems appear to be simpler and more convenient than predecessors' work.

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