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ELECTROSTATIC AND MAGNETIC IMAGING WITHOUT IMAGE ROTATION

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ABSTRACT

A method is discussed for designing electrostatic and magnetic imaging systems without image rotation. Tsukkerman's condition for an imaging system free of rotation does not appear to be applicable to practical designs. We have modified his theory to obtain two other conditions for the normalized axial electric potential V(z), the normalized axial magnetic induction G(z), and a lens strength parameter λ which can be interpreted as an eigen-value. Some analytical examples and numerical results are given.

Keywords: Electron optics, magnetic electron lenses, cathode lenses, imaging systems, imaging sections

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INTRODUCTION

Tsukkerman's theory [2-6] can be outlined as follows:

1. The vector of position $\mathbf{r}(z)=\mathbf{r}_*(z)+\mathbf{p}(z)$ of an electron trajectory is written as the sum of that of a principal trajectory $\mathbf{r}_*(z)$ and an increment $\mathbf{p}(z)$ for which simpler equations are obtained.

2. The axial potential V(z) and the magnetic field distribution G(z) are subject to conditions simplifying the "paraxial" ray equation. For a given object plane $z=z_0$ and a given image plane $z=z_i$ the simplified ray equation can be treated as an eigen-value problem.

Tsukkerman's theory does not appear to be applicable to the case where $\phi(z_0)=0$, i.e. where the object plane is the cathode surface. Zhou[7] has modified Tsukkerman's theory to make it applicable to that case. In the present paper we have applied this modified theory to establish two other conditions for V(z), G(z) and λ.

FUNDAMENTAL THEORY OF THE IMAGING SYSTEM

For an axisymmetric cathode lens we express the complex coordinates $\mathbf{r}_*(z)$ of the principal ray and $\mathbf{p}(z)$ of the increment by their moduli and arguments:

$$\mathbf{r}_*(z)=r_*(z)\,e^{i\theta_*(z)},$$

$$\mathbf{p}(z)=p(z)\,e^{i\beta(z)}. \qquad (1)$$

Their sum

$$\mathbf{r}(z)=\mathbf{r}_*(z)+\mathbf{p}(z) \qquad (2)$$

describes a ray neighboring to the principal ray (cf. Fig.1).

LIST OF SYMBOLS

$B(z)$ = Axial magnetic induction

B_0 = Magnetic induction at the cathode surface

e = Electron charge

$e\varepsilon_0$ = Initial energy of emission electrons, $e\varepsilon_0 = (m_0/2)v_0^2$

$e\varepsilon_r^*$ = Relative transversal initial energy of emission electrons, $e\varepsilon_r^* = e\varepsilon_0 \sin^2\alpha_0/\phi_i$

$e\varepsilon_Z^*$ = Relative axial initial energy of emission electrons, $e\varepsilon_Z^* = e\varepsilon_0 \cos^2\alpha_0/\phi_i$

$e\varepsilon_{Z1}^*$ = Relative axial initial energy of emission electrons, which can be focused ideally at the image plane

$G(z)$ = Relative axial magnetic induction, $G(z) = B(z)/B_0$

l = Axial distance from cathode surface to image plane

$\mathbf{M}$ = Magnification vector

M = Magnification

m_0 = Electron mass

m = Integer number, $0 < m \leqslant n$

n = Total of loops

$\mathbf{p}(z)$ = Vector of increment of electron ray

$p(z)$ = Module of $\mathbf{p}(z)$

$\mathbf{q}(z)$ = Vector of increment of electron ray in a twisted coordinate system (x, y)

$\mathbf{r}(z)$ = Vector of electron ray

$r(z)$ = Module of $\mathbf{r}(z)$

r_0 = Off-axis height from axis z at the cathode surface

$\mathbf{r}_*(z)$ = Vector of principal electron ray

$r_*(z)$ = Module of $\mathbf{r}_*(z)$

$\mathbf{u}(z)$ = Vector of electron ray in a twisted coordinate system (x, y)

$\mathbf{u}_*(z)$ = Vector of principal electron ray in a twisted coordinate system (x, y)

u = Variable in eq.(24)

$V(z)$ = Relative axial electric potential, $V(z) = \phi(z)/\phi_i$

$v(z), w(z)$ = Two independent special solutions of the "paraxial" ray equation

v_0 = Initial velocity of electrons

(X, Y, z) = Fixed Cartesian coordinate system

(x, y, z) = Twisted Cartesian coordinate system

z = Axial coordinate measured in unit of the distance l between cathode surface and image plane

z_0, z_i = Coordinates of cathode surface and image plane respectively

α_0 = Emission angle of electrons

$\beta(z)$ = Angle of $\mathbf{p}(z)$ turned around to the axis X

β_0 = Initial value of $\beta(z)$

γ = Parameter in eq.(46)

$\theta(z)$ = Angle of $\mathbf{r}(z)$ turned around to the axis X

$\theta_*(z)$ = Angle of $\mathbf{r}_*(z)$ turned around to the axis X

θ_0 = Initial value of $\theta_*(z)$

λ = Dimensionless lens strength parameter

$\phi(z)$ = Axial electric potential

ϕ_i = Electric potential at the image plane

$\chi(z)$ = Angle of rotation of the twisted coordinate system (x, y) with respect to the fixed coordinate system (X, Y)

χ_0 = Initial value of $\chi(z)$

ψ = Variable in eq.(25)

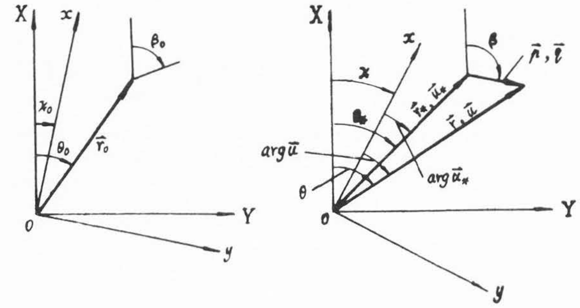

Fig.1 The principal trajectory and its increment in fixed Cartesian coordinate system (X,Y) and twisted Cartesian coordinate system (x,y)

From the "paraxial" ray equation for $r(z)$ two equations for the principal ray

$$\sqrt{V(z)+\varepsilon_Z^*}\,\frac{d}{dz}\left(\sqrt{V(z)+\varepsilon_Z^*}\,\frac{dr_*}{dz}\right)$$

$$+\left\{\frac{1}{4}V''(z)+\lambda\left[G^2(z)-\frac{r_0^4}{r_*^4}\right]\right\}r_*=0, \tag{3}$$

$$\sqrt{V(z)+\varepsilon_Z^*}\,\frac{d\theta_*}{dz}=\sqrt{\lambda}\left(G(z)-\frac{r_0^2}{r_*^2}\right); \tag{4}$$

and two others for the increment

$$\sqrt{V(z)+\varepsilon_Z^*}\,\frac{d}{dz}\left(\sqrt{V(z)+\varepsilon_Z^*}\,\frac{dp}{dz}\right)$$

$$+\left[\frac{1}{4}V''(z)+\lambda G^2(z)\right]p=0, \tag{5}$$

$$\frac{d\beta}{dz}=\sqrt{\lambda}\,\frac{G(z)}{\sqrt{V(z)+\varepsilon_Z^*}} \tag{6}$$

are derived [7]. z is the axial coordinate measured in unit of the distance l between cathode and image plane, i.e. $z_0=0$, $z_i=1$. Primes denote differentiation with respect to z, $\phi(z)$ is the axial electric potential, $\phi(0)=0$, $\phi_i=\phi(1)$, $B_0=B(0)$, $V(z)=\phi(z)/\phi_i$, $G(z)=B(z)/B_0$ $r_0=r(0)$, $e\varepsilon_0=(m_0/2)v_0^2$ is the initial energy of the electron, α_0 the angle between its initial tangent and the z-axis, $\varepsilon_z^*=\varepsilon_0\cos^2\alpha_0/\phi_i$, $\varepsilon_r^*=\varepsilon_0\sin^2\alpha_0/\phi_i$ and

$$\lambda=\frac{e}{8m_0}\frac{B_0^2 l^2}{\phi_i} \tag{7}$$

is a dimensionless lens strength parameter.

In order to solve the equations (3) to (6) we introduce a twisted coordinate system by using the complex coordinate $u=r\exp(-i\chi(z))$ where

$$\chi(z)=\sqrt{\lambda}\int_0^z\frac{G(z)}{\sqrt{V(z)+\varepsilon_z^*}}dz+\chi_0. \tag{8}$$

In the twisted coordinate system the differential equations for the principal ray and the increment assume the same form [7]:

$$[V(z)+\varepsilon_z^*]u''+\frac{1}{2}V'(z)u'+[\frac{1}{4}V''(z)+\lambda G^2(z)]u=0. \tag{9}$$

Their solutions can be expressed using two linearly independent solutions v(z) and w(z) of this ray equation satisfying the initial conditions:

$$\begin{aligned} &v(z_0=0)=0, \quad v'(z_0=0)\sqrt{\varepsilon_z^*}=1;\\ &w(z_0=0)=1, \quad w'(z_0=0)\sqrt{\varepsilon_z^*}=0. \end{aligned} \tag{10}$$

For electrons emitted from the cathode surface with the initial conditions:

$$\mathbf{r}_0=r_0e^{i\theta_0}, \qquad \mathbf{r}_0'=l\cdot\tan\alpha_0 e^{i\beta_0}; \tag{11}$$

the solution of the principal trajectory equation can be written as

$$r_\cdot(z)=r_0\sqrt{w^2+\lambda v^2}, \tag{12}$$

$$\theta_\cdot(z)=\theta_0+\sqrt{\lambda}\int_0^z\frac{G(z)}{\sqrt{V(z)+\varepsilon_z^*}}dz - \text{Arctan}\frac{\sqrt{\lambda}v}{w}, \tag{13}$$

whereas the solution for the increment becomes

$$p(z)=\sqrt{\varepsilon_r^*}\, l\,|v|, \tag{14}$$

$$\beta(z)=\sqrt{\lambda}\int_0^z\frac{G(z)}{\sqrt{V(z)+\varepsilon_z^*}}dz+\beta_0+(m-1)\pi \tag{15}$$

with $\theta_0=\theta_\cdot(z_0=0)$, $\beta_0=\beta(z_0=0)$. The integer number m which depends on z is defined by the inequality:

$$(m-1)\pi<\sqrt{\lambda}\int_0^z\frac{G(z)}{\sqrt{V(z)+\varepsilon_z^*}}dz\leqslant m\pi,$$

its value for $z=1$ is $m=n$. If

$$v(z_i,\varepsilon_{z1}^*)=0, \tag{16}$$

then all electrons having the same initial axial velocity component, i.e. the same value of $\varepsilon_z^*=\varepsilon_{z1}^*$, are ideally focused at the image plane $z=1$. The complex magnification becomes

$$\mathbf{M}=\frac{\mathbf{r}_\cdot(z_i,\varepsilon_{z1}^*)}{\mathbf{r}_0}=w(z_i,\varepsilon_{z1}^*)e^{i(\chi_i-\chi_0)}=\frac{1}{v'(z_i,\varepsilon_{z1}^*)\sqrt{1+\varepsilon_{z1}^*}}e^{i(\chi_i-\chi_0)} \tag{17}$$

and its module

$$M=|w(z_i,\varepsilon_{z1}^*)|.$$

For the angle of image rotation we obtain

$$\theta_\cdot(z_i)-\theta_0=\chi_i-\chi_0-n\pi \tag{18}$$

where $\chi_i=\chi(1)$ follows from eq.(8) for $z=1$. The system is free of image rotation if $\theta_\cdot(z_i)=\theta_0$, i.e.

$$\chi_i-\chi_0=n\pi, \tag{19}$$

The module of the magnification becomes

$$M=(-1)^n w(z_i,\varepsilon_{z1}^*)=(-1)^n\frac{1}{v'(z_i,\varepsilon_{z1}^*)\sqrt{1+\varepsilon_{z1}^*}} \tag{20}$$

Eqs.(17) and (20) can also be applied to the case $V(0)=0$ and $\varepsilon_{z1}^*=0$.

A NEW APPROACH TO REALIZE AN IMAGING SYSTEM WITHOUT IMAGE ROTATION

Tsukkerman has pointed out that a system without image rotation can be realized if the condition

$$G(z)=\frac{r_0^2}{r_\cdot^2(z)} \tag{21}$$

is satisfied. Then eq.(3) assumes the form of the "paraxial" ray equation for a purely electrostatic field. It can be solved with the initial conditions $r_\cdot(0)=r_0$, $r'_\cdot(0)=$

0, and the magnification becomes $M=r_*(1)/r_0$. The principal trajectory is then a meridional ray following a magnetic field line.

Since we think that the condition (21) can hardly be realized, we suggest another solution of the problem. In order to solve the "paraxial" ray equation in the twisted coordinate system

$$\frac{d}{dz}\left(\sqrt{V(z)+\varepsilon_{z1}^{*}}\frac{dv}{dz}\right)+\left(\frac{V''(z)}{4\sqrt{V(z)+\varepsilon_{z1}^{*}}}+\lambda\frac{G^2(z)}{\sqrt{V(z)+\varepsilon_{z1}^{*}}}\right)v=0, \tag{22}$$

for $v(z)$ with the boundary condition

$$v(z_0=0)=0,\quad v(z_i=1,\ \varepsilon_{z1}^{*})=0; \tag{23}$$

we introduce the new coordinates

$$u=\sqrt{G}\,v \tag{24}$$

and

$$\psi=\int_0^z\frac{G(z)}{\sqrt{V(z)+\varepsilon_{z1}^{*}}}dz. \tag{25}$$

The ray equation in terms of the new coordinates can be written as

$$\frac{d^2u}{d\psi^2}+\lambda u+\frac{u}{4G^2}\left\{\left(\frac{dG}{d\psi}\right)^2-2G\frac{d^2G}{d\psi^2}+\frac{G}{V+\varepsilon_{z1}^{*}}\left[G\frac{d^2V}{d\psi^2}+\frac{dG}{d\psi}\frac{dV}{d\psi}-\frac{G}{2[V+\varepsilon_{z1}^{*}]}\left(\frac{dV}{d\psi}\right)^2\right]\right\}=0 \tag{26}$$

and

$$u(0)=0,\quad u(\psi_i,\ \varepsilon_{z1}^{*})=0; \tag{27}$$

where

$$\psi_i=\psi(z_i=1).$$

If the contents of the curved brackets in eq.(26) vanish, eq.(26) is reduced to the Sturm-Liouville equation:

$$\frac{d^2u}{d\psi^2}+\lambda u=0. \tag{28}$$

The solution which satisfies the condition (27) is

$$u=\sin(\sqrt{\lambda}\,\psi), \tag{29}$$

where

$$\sin(\sqrt{\lambda}\,\psi_i)=0. \tag{30}$$

Equation (30) is equivalent to the condition (19). It allows to define the n-th eigen-value λ_n for which $\sqrt{\lambda_n}\,\psi_i=n\pi$. For $\lambda=\lambda_n$, there are $(n-1)$ intermediate foci between the cathode surface $z_0=0$ and the image plane $z_i=1$.

If the contents of the curved brackets in eq. (26) are expressed as a function of z and set equal to zero, we obtain:

$$\frac{3[V+\varepsilon_{z1}^{*}]}{G^2}\left(\frac{dG}{dz}\right)^2-\frac{2[V+\varepsilon_{z1}^{*}]}{G}\frac{d^2G}{dz^2}-\frac{1}{G}\frac{dv}{dz}\frac{dG}{dz}+\frac{d^2V}{dz^2}=0. \tag{31}$$

The n-th eigen-value must satisfy the condition:

$$\lambda_n=\frac{e}{8m_0}\frac{B_0^2 l^2}{\phi_i}=\frac{n^2\pi^2}{\left[\int_0^1\frac{G(z)}{\sqrt{V(z)+\varepsilon_{z1}^{*}}}dz\right]^2}. \tag{32}$$

If these two conditions are satisfied, the cathode surface $z_0=0$ is imaged onto the image plane $z_i=1$ without image rotation. Our conditions are more general than Tsukkerman's condition (21) since they do not imply that $\theta_*(z)=\theta_0$ for the principal ray but only that $\theta_*(z_i)=\theta_0$ in the image plane.

From eqs.(24) and (28) together with the initial conditious (10) it follows that

$$v(z,\varepsilon_{z1}^{*})=\frac{1}{\sqrt{\lambda G(z)}}\sin\left(\sqrt{\lambda}\int_0^z\frac{G(z)}{\sqrt{V(z)+\varepsilon_{z1}^{*}}}dz\right), \tag{33}$$

$$w(z,\varepsilon_{z1}^{*})=-\sqrt{\varepsilon_{z1}^{*}}\,v(z,\varepsilon_{z1}^{*})\left[\frac{d}{dz}\frac{1}{\sqrt{G(z)}}\right]_{z=0}+\frac{1}{\sqrt{G(z)}}\cos\left(\sqrt{\lambda}\int_0^z\frac{G(z)}{\sqrt{V(z)+\varepsilon_{z1}^{*}}}dz\right). \tag{34}$$

For $z=z_i$ we find the magnification

$$M=\frac{1}{\sqrt{G(z_i)}}=\sqrt{B_0/B(z_i)}. \tag{35}$$

The same result has been obtained by Tsukkerman [3,4,6] but we think that our treatment is more general and more realistic.

DESIGN OF AN IMAGING SYSTEM WITHOUT IMAGE ROTATION

The design procedure is as follows:

1. $V(z)$ or $G(z)$ is given;
2. Use condition (31) to determine $G(z)$ from given $V(z)$ or $V(z)$ from given $G(z)$. V and G must satisfy the conditions:

$$V(z_0=0)=0, \qquad V(z_i=1)=1,$$

$$G(z_0=0)=1, \qquad G(z_i=1)=\frac{1}{M^2};$$

3. Determine M;
4. Choose suitable values of ϕ_i, B_0, l and n which must be in agreement with eq.(32), and evaluate $\lambda=\lambda_n$;
5. The solutions v and w of eq.(22) are then given by eqs.(33) and (34);
6. Determine $r(z)=r_c(z)+p(z)$ using the solutions $v(z)$ and $w(z)$;
7. Design electrodes and magnetic circuit in a way that they generate the fields $V(z)$ and $G(z)$ with the constants V_i and B_0.

We think that our method is more general and flexible than Tsukkerman's since his method is restricted to the case that $V(z)$ is given and that $G'(0)=0$, and it yields only one solution $G(z)$ and one value for the magnification for a given $V(z)$. On the other hand, we can, for a given $V(z)$, choose an arbitrary value for the magnification M, and then solve eq.(31) for $G(z)$ with the boundary conditions $G(0)=1$; $G(1)=1/M^2$.

AN ANALYTIC EXAMPLE FOR THE DESIGN OF AN IMAGING SYSTEM WITHOUT IMAGE ROTATION

If we choose

$$G(z)=\frac{1}{[1-(1-M)z]^2}, \tag{36}$$

and

$$V(z)=\frac{Mz}{1-(1-M)z}, \tag{37}$$

then we have

$$G\frac{d^2G}{dz^2}-\frac{3}{2}\left(\frac{dG}{dz}\right)^2=0,$$

$$G\frac{d^2V}{dz^2}-\frac{dV}{dz}\frac{dG}{dz}=0, \tag{38}$$

i.e. eq.(31) is satisfied [1,7].

From eqs.(33) and (34) we obtain

$$v(z)=\frac{1}{\sqrt{\lambda}}[1-(1-M)z]\sin(\chi-\chi_0), \tag{39}$$

$$w(z)=[1-(1-M)z]\{\cos(\chi-\chi_0)+\sqrt{\frac{\varepsilon_z^*}{\lambda}}(1-M)\sin(\chi-\chi_0)\}; \tag{40}$$

where

$$\chi=\frac{2\sqrt{\lambda}}{M}\sqrt{V(z)+\varepsilon_z^*},\qquad \chi_0=\frac{2\sqrt{\lambda}}{M}\sqrt{\varepsilon_z^*}; \tag{41}$$

with v and w from eqs. (39) and (40) we obtain from eqs.(12) to (15)

$$r_c(z)=r_0[1-(1-M)z]\left\{1+(1-M)\sqrt{\frac{\varepsilon_z^*}{\lambda}}\sin2(\chi-\chi_0)+(1-M)^2\frac{\varepsilon_z^*}{\lambda}\sin^2(\chi-\chi_0)\right\}^{1/2}, \tag{42}$$

$$\theta_c(z)=\theta_0+\chi-\chi_0-\text{Arctan}\frac{\sin(\chi-\chi_0)}{\cos(\chi-\chi_0)+(1-M)\sqrt{\frac{\varepsilon_z^*}{\lambda}}\sin(\chi-\chi_0)}, \tag{43}$$

$$p(z)=\sqrt{\frac{\varepsilon_r^*}{\lambda}}\, l[1-(1-M)]\,|\sin(\chi-\chi_0)|, \tag{44}$$

$$\beta(z)=\chi-\chi_0+\beta_0+(m-1)\pi. \tag{45}$$

For $M=1$, the example describes the well-known system using parallel homogeneous electric and magnetic fields. Fig.2 shows graphs for the case $M=1.4$, $n=1$. In Fig.3, $V(z)$ and $G(z)$ are plotted for a number of different magnifications in the range $0.6\leqslant M\leqslant2.5$.

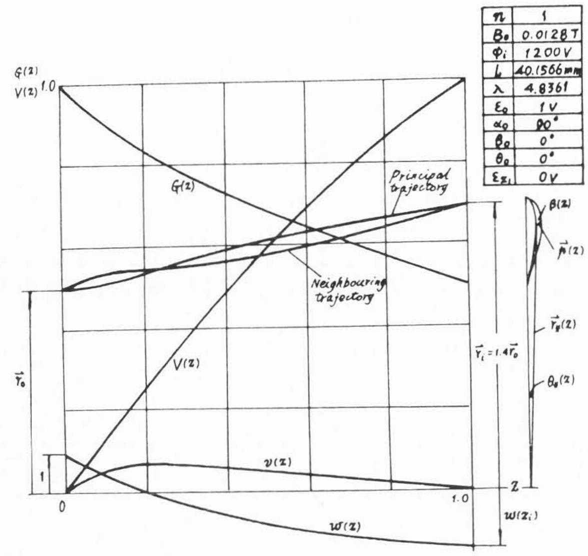

Fig.2 An analytic example of imaging system with M=1.4, n=1 (graphs of G(z), V(z), v(z), w(z), $r_{\cdot}(z)$, $\theta_{\cdot}(z)$, p(z), β(z))

A COMPUTATIONAL EXAMPLE FOR THE DESIGN OF AN IMAGING SYSTEM WITH VARIABLE MAGNIFICATION

For a given V(z), an arbitrary value for the magnification M may be chosen, and then eq.(31) may be solved numerically for G(z) with the boundary conditions G(0)=1, G(1)=1/M². We have worked out a computer program to do this. As an example, Fig.4 shows for the given function

$$V(z) = \frac{\gamma z}{1-(1-\gamma)z} \tag{46}$$

with $\gamma = 0.6$ a number of solutions G(z) for different values of the given magnification in the range $0.6 \leq M \leq 2.5$.

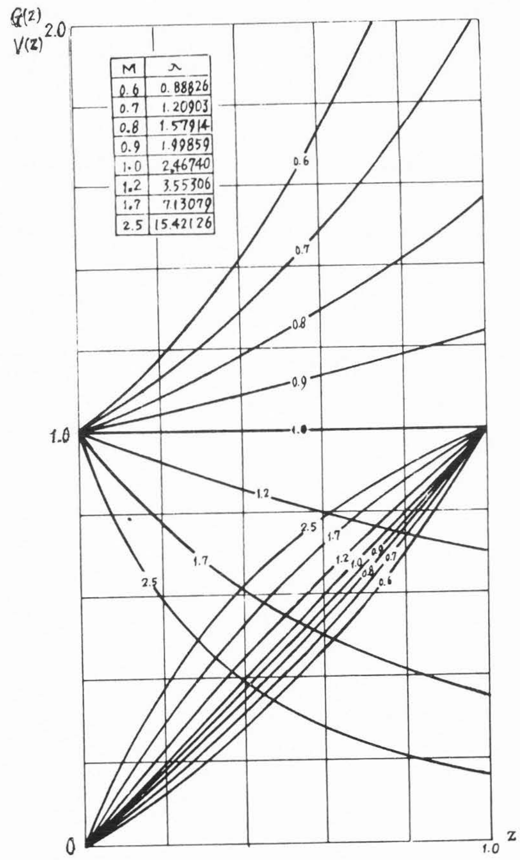

Fig.3 Some analytic curves V(z) and G(z) for different magnification M

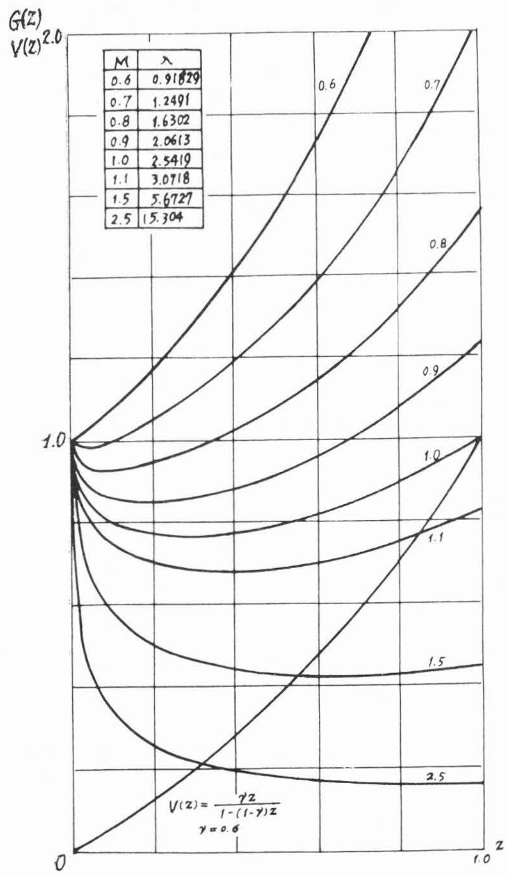

Fig.4 A computational example for a system with variable magnification

SUMMARY

Tsukkerman's method of finding combined electrostatic and magnetic imaging systems without image rotation is generalized in order to make it applicable to cathode lenses where the electric potential in the cathode surface vanishes and to magnetic fields which are not necessarily perpendicular to the cathode surface. A few analytical and numerical examples are given.

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