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The Rules of the Game: A Review of Hintikka's Proposals for Game Theory Semantics

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Jaakko Hintikka and his associates have recently been applying a new sort of semantics to some recalcitrant natural language phenomena. The semantics is called game-theoretical semantics because it assigns truth values to sentences according to properties of the play of a formal game. Hintikka _et al_ have claimed that the game-theoretical approach treats the following problems:

> The semantics of sentences which Hintikka _et al_ claim do not have a first-order predicate calculus reading such as
>
> (0) Every writer likes his first book almost as much as every critic dislikes the latest book he reviewed.
>
> The semantics of sentences with crossing pronominalization, such as
>
> (1) The boy who was fooling her kissed the girl who loved him.

The semantics of sentences with "pronouns of laziness", such as it in

(2) The man who gives his paycheck to his wife envies the man who gives it to his mistress.

The semantics of "donkey-beating" sentences, such as

(3) Any man who owns a donkey beats it.

All of these treatments are carried out in an intuitively pleasing system employing notation which closely resembles natural language itself. No wonder, then, that game-theoretical semantics has received attention.

This paper will be devoted to an exposition of the game-theoretical semantics as it has been employed thus far, and to an assessment of its success in treating the phenomena above. The paper is divided into two parts, corresponding to the two quite different sorts of game-theoretical semantics which have been proposed by Hintikka et al.

1. Pick and Choose Semantics.

I have dubbed the sort of semantics appearing in Hintikka (1974), Hintikka and Saarinen (1975), Hintikka (1975) and Carlson and Hintikka (unpublished), "Pick and Choose Semantics"(hereafter, "P&C Semantics"), for reasons which should become clear presently. The account of it given below represents the themes common to these papers.

The metaphor on which P&C Semantics is based is a simple and appealing one. We are to imagine ourselves as The Speaker of a sentence S who is attempting to make S turn out to be true of the world. Opposing our efforts is a cunning and malicious Nature who attempts to falsify S. This battle of wits between Nature and us as The Speaker is formalized as a game played on the sentence S. Each move of the game is intended to yield a new sentence S' which eliminates a logical operator from S. After a finite number of moves, the play yields an atomic sentence. The truth or falsehood of the atomic sentence is determined and we as The Speaker or Nature wins the game on S.

Let us illustrate the rules and the play of the game by considering a simplified version of the game, one eliminating only propositional calculus operators. Here are the rules for a propositional P&C Semantics game:

The Set-Up. The game has two players, The Speaker and Nature. There are two game roles, that of the Verifier and that of the Falsifier. The Speaker begins each game in the role of the Verifier and Nature begins each game in the role of the Falsifier. These roles are characterized by the move rules and the winning conditions.

The Game. A game on S is carried out move by move with respect to a world w. Each move puts a new sentence into play and the game proceeds with respect to it. The move rules are as follows:

(G.or) If S is of the form ⌜s or p⌝, the Verifier chooses either s or p and play proceeds with respect to the sentence chosen.

(G.and) If S is of the form ⌜s and p⌝, the Falsifier chooses either s or p and play proceeds with respect to the sentence chosen.

(G.but) Same as (G.and) with "but" for "and".

(G.neg) If S is of the form ⌜neg p⌝, the players switch roles and play p.

[Note: "neg p" is an expression referring to the negation of p. My inclination is to think of it as "It is not the case that p", but Hintikka <u>et al</u> hint at transformations which will accomplish a more natural negation.]

(G.if) If S is of the form ⌜s if p⌝, the Verifier chooses either ⌜neg p⌝ or s and play proceeds with respect to the sentence chosen.

These move rules are ordered by the following rule:

(O.Comm) If an operator 01 commands but is not commanded by an operator 02, a game rule must not be applied to 02 before one has been applied to 01. (01 commands 02 $=_{df}$ 01 occurs in a higher clause than 02).

[Note: parentheses will be added to sentences in order to display the command relations among operators when confusion is possible. For example, the sentence "John runs and Mary walks or Elaine reads" will be written either as "(John runs and Mary walks) or Elaine reads" or as "John runs and (Mary walks or Elaine reads)"].

<u>Definitions</u>. An <u>operator</u> is any of the following: <u>or</u>, <u>and</u>, <u>but</u>, <u>if</u>, <u>neg</u>.

An <u>atomic sentence</u> is a sentence containing no operators.

<u>Winning Conditions</u>. The game ends when an atomic sentence S' is produced. S' is checked against the world w. If S' is true of w, the player in the role of the Verifier wins the game on S. If S' is false of w, the player in the role of the Falsifier wins the game on S.

The intuition behind these rules is appealing. In (G.and), for example, if The Speaker utters a sentence of the form ⌜s and p⌝, where p and s are atomic sentences, then both of Nature's possible choices will issue in a true sentence. If the sentence uttered is false, then at least one of the two conjuncts is false and a cunning Nature will be able to choose that conjunct and win the game on S.

Similarly for (G.or). If The Speaker has uttered a true sentence of the form ⌜s or p⌝, then he has asserted that one (at least) of the two disjuncts is true, and a careful choice by him will result in its being the next sentence in play. If The Speaker has uttered a false sentence of this form, no amount of care will allow him to choose a true sentence as the next sentence in play.

(G.neg) is the only rule creating a rule reversal. The idea behind the rule is simple. If the sentence uttered is of the form ⌜neg p⌝, then The Speaker has asserted that p is false. In that case, The Speaker should be able to falsify p. Thus the rule calls for the roles to reverse and for p to be the next sentence in play.

Here is a demonstration propositional P&C Semantics game played with the rules given above. The Speaker and Nature are seated in the centre of a large stadium filled with people. Between them is a pedestal supporting a world of this description:

The World Bill walks. Helen does not read.

The game begins (as does every game) with The Speaker in the role of the Verifier and Nature in the role of the Falsifier. A hush falls over the crowd as The Speaker utters (4) and the game begins. The scoreboard display is:

IN PLAY	(4)	(Bill walks and Helen reads) or neg Helen reads.

Since "or" commands "and" and is not commanded by it, (O.Comm) requires that (G.or) apply first. The Speaker may choose "Bill walks and Helen reads", or "neg Helen reads". Acting on a hunch, he chooses the first sentence, and the scoreboard display is:

IN PLAY	(5)	Bill walks and Helen reads.

The game rule which now applies is (G.and). It is Nature's move. She may choose "Bill walks" or "Helen reads" to be the next sentence in play. She reasons as follows. If I choose "Bill walks", the game will be over. "Bill walks" is true of the world and I, as Falsifier, will lose. If I choose "Helen reads" the game will also be over. But "Helen reads" is false of the world and I, as Falsifier, will win. Therefore I will choose "Helen reads".

Nature acts on the basis of this reasoning and the scoreboard display is:

IN PLAY	(6)	Helen reads.

(6) is atomic. The game ends and, as Nature foresaw, (6) is declared false of the world. The scoreboard reads:

FALSIFIER (NATURE) WINS. NATURE 1 :: SPEAKER 0

The crowd cheers, and spectators are seen to wave Nature pennants.

After the excitement has died down the players begin another game. On the advice of his coach, The Speaker utters (4) again. The crowd is amazed, but The Speaker has reasoned as follows. The first move is mine, by (G.or). Last game I lost by choosing (5). I have a strong hunch, though, that "neg Helen reads" is a winner. Therefore this time I will choose "neg Helen reads" instead of (5).

The Speaker acts on the basis of this reasoning and the scoreboard after his opening play reads:

IN PLAY (7) neg Helen reads.

The next rule to apply is (G.neg). There is no choice involved here. The players reverse roles so that The Speaker is now the Falsifier and Nature the Verifier. The scoreboard reads:

IN PLAY (8) Helen reads

(8) is atomic and play ends. It is ascertained that (8) is false of the world. The scoreboard reads:

FALSIFIER (SPEAKER) WINS. NATURE 1 :: SPEAKER 1

The crowd goes wild.

Let us leave the stadium at this point to reflect on what has happened. We see that The Speaker has lost a game on (4) and then won a game on (4). The second game seems to reflect his best play, in that <u>if The Speaker plays as he did in the second game, Nature cannot win the game on (4)</u>. This observation leads us to the semantical aspect of the theory.

The game itself is lively, diverting and of semantic value. Hintikka <u>et al</u> propose the following definition based on it:

<u>A sentence S is true in a world w iff The Speaker has a winning strategy for the play of S in w.</u>

Another way to say this is to say that The Speaker can win a play of S in w no matter what Nature does iff S is true in w. It can be proved that this semantics assigns truth values to sentences just as the propositional calculus does. The proof is simple, but lengthy, and has been relegated to Part 1 of the Appendix. Included there are rules for translating surface sentences into their equivalents in propositional calculus notation.

Now that we have an idea of the way the semantics works we can consider the rest of the P&C Semantics rules. The effect of the remaining rules is to make P&C Semantics operate as the predicate calculus does. The innovations have to do with quantification, but the idea is much as before. In the propositional game, it was Nature's move when a conjunction was in play and it was The Speaker's move when a disjunction was in play. Similarly, the new rules specify that Nature will choose an individual when a universal quantifier is being eliminated, and that The Speaker will choose an individual when an existential quantifier is being eliminated. The rules of this (the complete P&C Semantics game) are as follows:

The Set-Up. Same as before.

The Game. Same as before, except that play is carried out with respect to a world *and* a domain of individuals.

Some new game rules are also added:

(G.some) If S is of the form ⌜X some Y who Z W⌝, the Verifier chooses an individual from the domain and gives that individual a name b not occurring in S. Play proceeds with respect to the sentence ⌜X b W, and b is a Y, and b Z⌝.

[Note: in structural descriptions of the sort given above, either X or W may be null. Further, the clause "who Z" is optional. If ⌜who Z⌝ does not appear in S then an application of (G.some), for example, will place ⌜X b W and b is a Y⌝ in play.]

(G.an) Same as (G.some) with "a" or "an" for "some".

(G.every) If S is of the form ⌜X every Y who Z W⌝, the Falsifier chooses an individual from the domain and gives that individual a name b. Play then proceeds with respect to the sentence ⌜X b W, if (b is a Y and b Z)⌝.

(G.any) Same as (G.every), with "any" for "every".

(G.Sing.the) If S is of the form ⌜X the Y who Z W⌝, the Verifier chooses an individual from the domain and gives that individual a name b which does not occur in S. The Falsifier chooses another individual from the domain and gives that individual a name c not occurring in S and which is not b. Play proceeds with respect to the sentence ⌜X b W and b is a Y and b Z and c is not a Y who Z⌝.

(G.the only) Same as (G.Sing.the) with "the only" for "the".

(G.only) If S is of the form ⌜X only A W⌝ and if A is a proper name, then the Falsifier chooses a different individual from the domain and gives that individual a name b which does not occur in S. Play proceeds with respect to the sentence ⌜X A W, but neg X b W⌝.

(G.gen) If S is of the form ⌜X Y's Z W⌝ and Y is a proper name or a pronoun pronominalizing a proper name, then the Verifier chooses an individual and gives that individual a name b which does not occur in S, and play proceeds with respect to the sentence, ⌜X b W, and b is the Z of Y⌝. (Where Y is replaced by its antecedent if it pronominalized a proper name.)

These move rules are ordered by (O.Comm) and also by the following rules:

(O.gen) (G.gen) normally has to be applied before the propositional rules.

(O.any) (G.any) takes precedence over (G.neg) and (G.if) and (normally) (O.L-R) and even (O.Comm).

(O.every) (G.every) does not have priority over propositional rules.

(O.L-R) If an operator O1 precedes an operator O2 in left-right order, a game rule must not be applied to O2 before one has applied to O1.

[Note: (O.Comm) takes precedence over (O.L-R).]

These rules are further restricted by the following clause:

The Pronominalization Clause.
(G.or), (G.and), (G.but) and (G.if) may only apply to a sentence S to yield a sentence P if

i. all pronouns in P which pronominalized phrases outside P pronominalized proper names; and

ii. their antecedents were substituted for the pronouns in order to derive P as the game rule was applied.

Definitions. An *operator* is any of the following: *or*, *and*, *but*, *if*, *neg*, *some*, *an*, *every*, *any*, *the*, *the only*, *only*, *'s*.
An *atomic sentence* is a sentence containing no operators.

The game termination and winning conditions are the same as before.

Before we consider the play of some interesting sentences with these rules, some remarks are in order.

1. The game rules are defined on surface sentences of English, with one addition. The Pronominalization Clause and the (G.gen) rule will not make sense unless we can determine which words are the antecedents of pronouns. This fact is never discussed by Hintikka et al, and, as we shall see, it leads to some difficulties. It is necessary to assume, for the purposes of our discussion, that the game rules are determined on surface sentences of English for which the pronominalization relations are known. Thus (9) will not be ambiguous in P&C Semantics.

9. Mary told Helen that she will be a doctor.

Somehow, along with (9) the players will receive the information that "she" pronominalizes "Mary" or that "she" pronominalizes "Helen".

2. For rules with a structural description containing the phrase "Y who Z", it is assumed that "who" functions as the subject of the phrase Z.

3. As written, (G.an) may give rise to an infinite series of moves which result in no atomic sentence. Hintikka in Hintikka, 1975, p 12, says something which suggests that he is aware of this but he does not change the rule.

4. When (G.Sing.the), (G.the only) and (G.only) are applied, the resultant sentences have a final uniqueness clause. In those clauses, any pronoun which referred to the phrase being eliminated now refer to c (for the first two rules) or b (for (G.only)). This condition is different in Hintikka and Carlson (unpub) but I believe that their treatment in that paper was a false step.

5. Some of the ordering principles which Hintikka et al give are omitted, primarily because they are characterized as "mildly preferential". We shall discuss this point again when we consider the treatment of paycheck sentences.

6. I have departed from the original exposition in three important ways. First, Hintikka et al refer to the two players as "Nature" and "Myself". In a class exposition of this system, this nomenclature engendered a good deal of confusion when we played some demonstration games with the class in the role of Myself and myself commenting on Myself's strategy (which was opposed to that of mine). The notation employed here seems faithful to the spirit of the game and avoids such confusion.

Second, I have departed from the original notation in naming the roles "the Verifier" and "the Falsifier". Hintikka et al refer to them as "Myself" and "Nature", but this creates a good deal of unnecessary confusion once a role-reversal has taken place.

Third, I have eliminated the P&C Semantics rules dealing with epistemic operators, because those rules are irrelevant to the issues we are concerned with, and because they require the presence of many more worlds in an already crowded stadium.

7. We note that The Speaker may utter a true sentence without being able to win a game played on it. Sentences are judged true in P&C Semantics iff there is a winning strategy, regardless of whether or not The Speaker knows it. Henceforth in playing out games on sentences we shall play The Speaker's and Nature's best moves. In so doing, we shall be able to decide who has a winning strategy.

8. Finally, it may be proved that P&C Semantics gives readings to sentences equivalent to those readings given by the first-order predicate calculus. This proof is sketched in Part II of the Appendix. In Part II rules are also given for deriving the predicate calculus equivalents of the P&C Semantics readings.

The reader may wonder why P&C Semantics has been proposed, if it is equivalent in effect to the workings of the predicate calculus. There are two sorts of reply to this question. One is that Hintikka et al do not demonstrate an awareness of the theorem proved in Part II of the Appendix to this paper. Another reply involves considering some problems discussed in the literature.

In Hintikka (1974) and Hintikka and Saarinen (1975) the claim is advanced that certain English sentences have readings which require representation in the second-order predicate calculus. This point is made by them in two ways. Sometimes the authors write as though certain English sentences have readings which must be represented by branching quantifiers, and that some of these readings cannot be represented in the first-order predicate calculus. At other times, the authors write of Skolem functions, second-order functions which are capable of being used to produce readings of formulas employing branching quantifiers. Both these claims are made in Hintikka (1974) with respect to

10. Every writer likes a book of his almost as much as every critic dislikes some book he has reviewed.

Hintikka provides the following as the second-order translation of (10), where f and g are Skolem functions, "Wx" is "x is a writer", "Cx" is "x is a critic","Axy" is "x wrote y", "Bx" is "x is a book", and "Rxy" is "x reviewed y" :

11. $(\exists f)(\exists g)(x)(z)((Wx \,\&\, Cz) \rightarrow (Bf(x) \,\&\, Ax, f(x) \,\&\, Bg(z) \,\&\, Rz,g(z)$ $\&$ x likes $f(x)$ almost as much as z dislikes $g(z)))$.

Hintikka contrasts (11) with (12) which he proposes as the most likely translation of (10) into the first-order predicate calculus:

12. $(x)(Wx \rightarrow (\exists y)(By \; \& \; Axy \; \& \; (z)(Cz \rightarrow (\exists u)(Bu \; \& \; Rzu \; \& \; x \text{ likes } y$ $\text{almost as much as } z \text{ dislikes } u))))$.

According to Hintikka, (11) is stronger than (12) in that there are worlds in which (12) is true while (11) is false. One such world is described by (13):

13. Every writer likes his latest book almost as much as every critic dislikes the first book by that writer he had to review.

I disagree with Hintikka's intuitions regarding the falsehood of (10) in such a world, as do most of the native speakers I have queried. Gilles Fauconnier has argued persuasively against Hintikka's claims here, in Fauconnier (1975). But the question is not entirely a matter of conflicting intuitions. Hintikka has not said enough about his Skolem functions for us to tell what his intended reading for (10) should be, but he has said enough for us to demonstrate that his claims about that reading are in conflict.

Let us follow the play of a game on (10), with the assumption that the world is as described in (13) and that the world contains critics and writers. We return to the stadium to find the players studying this scoreboard display:

IN PLAY	(10)	Every writer likes a book of his almost as much as every critic dislikes some book which he has reviewed.

According to (O.L-R), the first rule to apply is (G.every). Nature chooses an individual and names him Bill. The scoreboard display is:

IN PLAY	(14)	Bill likes a book of his almost as much as every critic dislikes some book which he has reviewed, if Bill is a writer.

(O.Comm) requires that (G.if) apply next. We assume that Nature has played well and that Bill is a writer. (If not, The Speaker could choose the negation of the second part of (14) and win quickly.) Hence The Speaker chooses the first part of (14) and the scoreboard display is:

IN PLAY	(15)	Bill likes a book of his almost as much as every critic dislikes some book which he has reviewed.

Now (O.L-R) calls for an application of (G.an). The Speaker chooses Bill's latest book and names it Tribes. The scoreboard reads:

IN PLAY	(16)	Bill likes Tribes almost as much as every critic dislikes some book which he has reviewed, and Tribes is a book.

(O.L-R) calls for an application of (G.and) and Nature adroitly chooses the first part of (16). Now (O.L-R) calls for an application of (G.every). Nature chooses an individual and names him Mark. The scoreboard reads:

IN PLAY	(17)	Bill likes Tribes almost as much as Mark dislikes some book which he has reviewed if Mark is a critic.

The next rule to apply is (G.if). Assuming Nature has done well and chosen a critic, The Speaker should choose the first part of (17). Then (O.L-R) demands the application of (G.some). The Speaker chooses the first book by Bill which Mark had to review and names it Vibes. The scoreboard reads:

IN PLAY	(18)	Bill likes Tribes almost as much as Mark dislikes Vibes, and Vibes is a book, and Vibes was reviewed by Mark.

Now comes the point of the game. Nature must choose one of the three conjuncts of (18) through (G.and). We assumed that (13) is true of the world. That assumption guarantees the truth of each of the three conjuncts of (18). The game has been fought long and hard by Nature, but she will lose immediately no matter what choice she makes.

Let us reflect for a moment on The Speaker's victory. The game just described was won in a way which is easily generalizable. In fact, it is easy for The Speaker to win no matter what Nature does, if (13) is true. The Speaker simply chooses the latest book of whatever writer Nature selects, and then chooses the first book written by that writer which was reviewed by whatever critic Nature chooses. We have just described a winning strategy for The Speaker. But this means, according to P&C Semantics, that (10) is true in the world of (13).

This fact creates some difficulties for Hintikka's position. No matter what the best rendering of (10) is, we have shown that the truth of (13) guarantees the truth of (10) in P&C Semantics. And in fact we have shown more. The Appendix results guarantee that there is a first-order reading of (10) equivalent to that which P&C Semantics assigns to (10). By following the translation procedure given in the Appendix, we see that it is

19. $(x_1)(\exists x_2)(x_3)((\exists x_4)(x_1 \text{ likes } x_2 \text{ almost as much as } x_3 \text{ dislikes } x_4 \mathbin{\&} Bx_2 \mathbin{\&} Ax_1x_2 \mathbin{\&} Rx_3x_4 \mathbin{\&} Bx_4) \vee \sim Cx_3) \vee \sim Wx_1)$.

But (19) is equivalent to (12). Thus it is a consequence of the results in the Appendix that (12) is true in the first-order predicate calculus iff (10) is true in P&C Semantics.

Hintikka appears to be caught in a dilemma. If he is correct in maintaining that (12) is not an adequate rendering of (10), then he has argued for the inadequacy of his semantics, since the P&C Semantics reading of (10) is provably equivalent to (12). On the other hand, if he is wrong about the correct representation of (10), then he has given a bad argument for the claim that English quantification can not be represented in first-order logic.

This seems to me to be a serious dilemma, and I find no evidence that Hintikka appreciates it. The argument just given and one other (to be considered next) are the only arguments we find for the claim that first-order logics can not represent English quantification adequately. But both arguments are problematic. Let us consider the other one briefly.

In Hintikka and Saarinen (1975) the authors treat Bach-Peters sentences (sentences of a type containing crossing pronominalization). One frequently encounters sentences of this sort which concern maneouvers employed in aerial warfare, such as

20. The pilot who shot at it hit the Mig which was chasing him.

Hintikka and Saarinen settle on more down-to-earth examples such as

21. The boy who was fooling her kissed the girl who loved him.

The authors note that there has been a good deal of controversy concerning the correct reading on (21), but the majority of opinions seem to be along the lines of a reading suggested by Karttunen in Karttunen (1971). (Actually this is only one of two readings Karttunen proposes for (21), but the point is made with either one).

22. The boy who was fooling the girl who loved him kissed her.

The authors disagree with such readings because, according to them, there are worlds in which (21) is true when (22) is false. Such a world is one of this description:

<u>The World</u>.	Bill was fooling with Betty	:	Bill kissed Betty
	Betty loved Bill	:	Jim was fooling Jan
	Jim kissed Jan	:	Jan loved Jim

The reader can verify that (21) will turn out true in this world but that (22) will not, according to P&C Semantics. The authors claim that this is the correct result, and that (21) is true in worlds with any number of pairs of individuals who kiss, love and fool only the other member of the pair. On the other hand, (22) is false in any world containing more than one loving, fooling and kissing pair. The argument is that this fact demonstrates the error in the (22)-style representation of (21). And (22), of course, has a first-order predicate calculus representation.

The authors' claim seems to me to be highly doubtful, although the case is surely a marginal one concerning unclear data. But the authors go on to assert some more things about their version of the reading of (21). They claim that

> "... if any of these pairs that satisfy the uniqueness clauses is such that the boy in question did not kiss the girl in question, then the sentence (10) is false in this model. (This is due to the fact that Nature may choose any one of the pairs and hence will of course select the pair that yields a falsity if there is such)," (p 6).

It is not clear here whether the authors are describing their intuitive reading of (21) or whether they are letting their intuitions be determined by what they think P&C Semantics will do with (21). In either case, they have made a mistake. Let us consider (21) played with respect to a world of this description:

The World.
Fred was fooling Fay: Fred kissed Fay: Fay loved Fred
Bill was fooling Betty: Bill kissed Betty: Betty loved Bill
Jim was fooling Jan : Jim did not kiss Jan: Jan loved Jim

P&C Semantics declares (22) false in this world, but it rules (21) true there. (If The Speaker chooses Fred and Fay, or Bill and Betty, he wins despite Nature's best defense.) In the passage quoted above, Hintikka and Saarinen seem to have gotten their players mixed up. According to their rules The Speaker, not Nature, chooses when (G.Sing.the) is applied.

From all this we may conclude once again that the authors are caught in a dilemma. If the correct reading of (21) is a non-first-order reading of the sort that they hint at, the P&C Semantics will not express it. P&C Semantics rules (21) true in any world containing at least one pair who satisfy the uniqueness conditions and who kiss, regardless of what anyone else does or does not do. Furthermore, the reading P&C Semantics gives to (21) will be equivalent to this first-order formula derived by following the translation procedure in the Appendix:

23. $(\exists x_1)(\exists x_3)(x_1$ kissed x_3 & x_1 was fooling x_3 & $(x_2)(x_1$ is distinct from $x_2 \rightarrow x_2$ is not a boy who was fooling $x_3)$ & $(x_4)(x_3$ is distinct from $x_4 \rightarrow x_4$ is not a girl who loved $x_1)$ & x_3 loved $x_1)$.

The other horn of the dilemma is that the authors are wrong about the correct reading of (21) and are left with no argument that their talk of branching quantifiers and Skolem functions is relevant to the cases they consider. It seems to me that the truth lies here. And after all, it wouldn't be so bad if the second-order talk were dropped. At least, the system would then be much more accessible to non-logicians.

The arguments that P&C Semantics does more than predicate calculus semantics can do have been judged problematic. We are left with the question, why use P&C Semantics if it does nothing new ? The question is not discussed explicitly in published papers by Hintikka et al, but in an unpublished paper, Hintikka and Lauri Carlson employ P&C Semantics in a fashion which predicate calculus semantics may not be able to duplicate. The paper is called "Pronouns of laziness in Game-Theoretical Semantics" and it concerns the semantics of sentences such as (24).

24. Any man who spends his paycheck envies any man who saves it.

Here it seems to stand for the phrase "his paycheck". There is pronominalization going on, but on the natural reading of (24), the thing referred to by "his paycheck" is not the same thing referred to by it.

Hintikka and Carlson call "it" in (24) a "pronoun of laziness", after Barbara Partee in Partee (1972). The term seems to have originated with Geach, but he used it to refer to a different sort of phenomenon. According to Hintikka and Carlson, the phenomenon, or something like it, has also been termed "sloppy identity". We shall dodge a decision on the correct name for the phenomenon. In the present discussion we shall term it "paycheckery", a name which is certain not to confuse the issue by catching on.

The authors illustrate the virtues of their approach by following the play of a game on a sentence like (24). Let us do the same. Imagine once again that we are in the stadium and that the world is one in which the natural reading of (24) is false:

The World.
John does not envy Bill, John gets check J, John spends check J.
Bill gets check B, Bill saves check B, John and Bill are men.

(24) is in play and (G.any) applies first. Nature picks John if she is on her toes, and the scoreboard reads:

IN PLAY	(25)	John envies any man who saves it, if (John is a man and John spends his paycheck).

(G.any) applies again and Nature picks Bill if she is alert, yielding

IN PLAY	(26)	(John envies Bill, if (John is a man and John spends his paycheck)), if(Bill is a man and Bill saves it).

Now (O.Comm) would normally require an application of (G.if), but (O.gen) overrides this requirement and (G.gen) is applied. The Speaker chooses check J if he is in form and the scoreboard reads:

IN PLAY	(27)	((John envies Bill, if (John is a man and John spends check J)), if (Bill is a man and Bill saves it)), and check J is the paycheck of John.

Now (G.and) applies. Nature chooses the first conjunct if she is wise and the scoreboard reads:

IN PLAY	(28)	(John envies Bill, if (John is a man and John spends check J)), if (Bill is a man and Bill saves it).

(G.if) allows The Speaker to choose one of the disjuncts. He sees that he will lose against good play if he chooses the first. Hence, he chooses the negation of the second and the scoreboard display is:

IN PLAY	(29)	neg (Bill is a man and Bill saves check J)

(Note that the pronoun is replaced by the proper name as required by the Pronominalization Clause)
Now (G.neg) reverses the players' roles and the game stands as follows:

IN PLAY	(30)	Bill is a man and Bill saves check J

The Speaker chooses the second conjunct (since he is now the Falsifier) and the scoreboard reads:

IN PLAY	(31)	Bill saves check J.

(31) is atomic and false. The Speaker as Falsifier wins. But those Nature supporters in the crowd who have read the program protest, for they know that The Speaker has won the play of a <u>false</u> sentence against Nature's best play. This means that the semantics will judge (24) <u>true</u> of the world. What has gone wrong ?

Here is the point at which Hintikka and Carlson appear to have resources available to them which are lacking in a predicate calculus semantics. They say with respect to a sentence similar to (26),

> "In order to obtain the intended reading on which Bill [saves] <u>his</u> (Bill's) paycheck, ... we have to apply (G.if) to the last if-clause of (26) <u>in violation of the Pronominalization Clause</u>," (p 20)

To illustrate the point, let us follow the play of another game, just like the preceding one up to the point at which (26) is in play. Now the Pronominalization Clause and (O.Gen) are ignored. The Speaker chooses the negation of the last major clause through (G.if) and the scoreboard reads:

IN PLAY (32) neg (Bill is a man and Bill saves his paycheck).

The game is on the right track now, but there is a problem here which the authors do not discuss. We recall that the sentences to be played come with antecedents assigned to their pronouns. In (26), "his" pronominalized "John". With respect to (32) the authors assert that "his" pronominalizes "Bill", but they do not explain how the reference of "his" has gotten switched. Perhaps what they intend is a rule for establishing the reference of pronouns with severed antecedents. Such a rule might go like this:

> <u>Severed Antecedent Clause</u>. If the Pronominalization Clause is violated in such a way as to yield a sentence S with pronouns that do not pronominalize any phrase, such pronouns are considered to pronominalize the first noun phrase in S.

With this rule, the game works out nicely. The players switch roles by an application of (G.neg) and the scoreboard reads:

IN PLAY (33) Bill is a man and Bill saves his paycheck.

Both conjuncts of (33) are true and The Speaker, as Falsifier, must make a losing choice. Against Nature's best play, the game has been lost by The Speaker, which is as it should be. The Nature fans are placated.

The authors say about this game just concluded:

> "First and foremost, we can now see that in a perfectly good sense the game-theoretical semantics gives us a uniform treatment of pronouns of the sort missed, eg, by Partee (1972, pp 434-436). Both pronouns of laziness and the 'normal' pronouns are dealt with by means of the same game rules. The only distinction between the two is a difference in the order in which the several game rules are applied," (p 49).

Later on in the paper, the authors state that their "main thesis" is that "Typical pronouns of laziness are characterized precisely by a violation of the Pronominalization Clause," (p 21). But neither the claim that we now have a "uniform treatment" of paycheckery nor the claim that the authors have characterized paycheck readings seems to be true.

With respect to the characterization, we should note that the Pronominalization Clause is not the only rule that has been violated. (O.gen) has been violated as well. Further, pronouns have been severed from their antecedents, and we have had to establish new pronominalization relations with the aid of a rule which the authors do not give. The rule as stated above does not seem to work in all cases. Let us consider another sentence involving paycheckery:

35. Any woman who loves a man who spends his paycheck envies any woman who doesn't let him have it.

After two moves by Nature, the scoreboard will look something like this:

IN PLAY	(35)	(Jan envies Jill, if Jan is a woman who loves a man who spends his paycheck), if Jill is a woman who doesn't let him have it.

As before, we apply (G.if) and The Speaker chooses (37a) or (37b):

37. a. Jan envies Jill, if Jan is a woman who loves a man who spends his paycheck.
    b. neg Jill is a woman who doesn't let a man who spends his paycheck have his paycheck.

The problems concern (37b). "It" in (35) pronominalized "his paycheck". "He" in "his" pronominalized "a man who spends his paycheck". That phrase is lost, and "his" in (37b) is a pronoun with a severed antecedent. The rule for re-establishing pronominal relations will assign "Jill" as the antecedent of "he" in "his", and this is incorrect.

We should note that even if this problem may be solved by a different form of the Severed Antecedent Clause, the truth conditions embodied in (37b) are inappropriate. The paycheck reading of (34) concerns envying women who love a man who spends his paycheck and do not let him have it. The reading embodied in (37b) and in (37a) does not contain the information that the women being discussed love the men in question.

The following example illustrates the point. Suppose (34) is to be played with respect toa world of this description:

The World.
Jan envies Joan. Jan does not envy Jill. Jan loves a man who spends his paycheck. Joan loves a man who spends his paycheck and she does not let him have it. Jill loves a man who spends his paycheck and she lets him have it. Jill's father spends his paycheck and she doesn't let him have it. Jill does not love her father.

We see on reflection that the paycheck reading of (35) is true of this world, since (a) Jan is the only woman who loves a man who spends his paycheck, and (b) Jan envies Joan, the only woman who loves a man who spends his paycheck and does not let him have it. Jill is irrelevant to the reading since she does not love her father. Jill's presence, however, will cause difficulties for the P&C Semantics, because Nature has a winning strategy for the play of (35).

The game begins with two applications of (G.any). On her first move, Nature chooses Jan. On her second move, Nature chooses Jill rather than Joan. The scoreboard displays (36). (G.if) applies and The Speaker may choose (37a) or (37b). He reasons as follows. If I choose (37a), it will be my move again, and I will have to choose "Jan envies Jill" or "neg Jan is a woman who loves a man who spends his paycheck". I lose immediately if I make the first choice, and I lose in a more protracted fashion if I make the second choice. I can't do worse to choose (37b), and maybe I will do better.

On the basis of this reasoning, The Speaker chooses (37b). The players reverse roles by (G.neg) and the scoreboard reads:

IN PLAY	(37)	Jill is a woman who doesn't let a man who spends his paycheck have his paycheck.

The Speaker will lose the play of (37) eventually against Nature's best play. Nature has only to choose Jill as the woman and Jill's father as the man in order to win. This means that Nature has a winning strategy for the play of (35) and, hence, that P&C Semantics rules (35) false in this world (but it is true).

The situation, then, is a good deal more complicated than we are led to believe by the claims that the authors make. Furthermore, even if the problems just discussed are soluble, the semantics will run afoul of another difficulty which I believe forces a radical reconstruction of it.

Crudely put, the difficulty is that the solution to the paycheckery phenomenon is really no solution at all. The "perfectly good sense" in which P&C Semantics generates paycheck readings is the sense in which it does not generate them. As we have seen, P&C Semantics does not generate the paycheck reading of (24). For it to do so would require a change in the ordering rules and the Pronominalization Clause. But the game rules are the game, and changing them will give us a new semantics. In effect, what Hintikka and Carlson have said is that P&C Semantics treats paycheckery because they can describe the failure of P&C Semantics to generate paycheck readings. Unfortunately, to describe one's failures does not convert them to successes.

This point may be made in a more general way. Given a sentence S, a world w, and a set of game rules, it is the case that either The Speaker will have a winning strategy for the play of S in w or he won't. This means that, according to the P&C Semantics, either S will be true in w or it will be false. This fact implies that any sentence will have exactly one truth value in a given world. And this fact in turn implies that the P&C Semantics gives no sentence an ambiguous reading.

At many points Hintikka et al write as though by reordering their rules they could generate all the readings behind ambiguous sentences, and something of the sort is true. But the implication is constantly made that the reordering is done within P&C Semantics, and this implication is false.

Let us assume, for example, that the rules are changed so that paycheck readings are generated. The Pronominalization Clause is probably deleted and (O.gen) is changed. Now we have a new set of rules which generate some paycheck readings, but the new set will never generate ambiguous readings, for its rules are ordered (albeit in a new way), and the reasoning above leads to the conclusion that it gives each sentence exactly one reading.

Once again Hintikka et al are presented with a dilemma. If the game rules are ordered then sentences are unambiguous. If they are not ordered, then there seems to be no way to play the game with them, and there will be no way to tell whether or not The Speaker has a winning strategy (and hence no semantics). In representing P&C Semantics thus far I have chosen the first horn of the dilemma, but Hintikka et al seem not to appreciate the fact that their remarks will impale them on the second.

The reader may wonder if this difficulty could be evaded by having a number of different semantics, each of which generates a certain type of reading. P&C Semantics might be one such semantics, and the Paycheck Semantics hinted at above, would be another. Then we might be able to correlate the standard reading of a sentence with its P&C Semantics reading, the paycheck reading of it with its Paycheck Semantics reading, etc. The difficulties here are twofold. First, such a maneouver would generate many inappropriate readings. For example, the following sentence gets an appropriate reading in P&C Semantics only because of the Pronominalization Clause.

38. Every team member does the high jump or she does the mile.

(A less awkward example would employ "unless" for "or", but we don't have (G.unless)).

In the Paycheck Semantics where the Pronominalization Clause is done away with , (O.every) requires that (G.or) apply to (38) first, yielding (39a) or (39b).

39. a. Every team member does the high jump.
    b. Every team member does the mile.

Both (39a) and (39b) are false in a world of the following description, where (38) itself is true:

The World.
Carol, Ellen, and Ruth are team members. Carol and Ellen do the high jump. Ruth does the mile.

Hence the Paycheck Semantics reading assigns inappropriate truth conditions to (38). (I take it no amount of straining yields a paycheck reading of the sort just described for (38)).

There are arguments of this sort for each of the ordering principles. Hintikka et al never reconcile their talk of generating ambiguous readings by reordering rules with their own arguments that the rules are ordered for very good reasons.

Another difficulty with such a proposal cannot be worked out in detail here without a more precise specification of the form the new semantics would take. The idea is this. There are going to be paycheck sentences in which we will want (G.gen) to apply as (O.gen) demands, after the paycheckery has been handled by doing away with (O.gen). The solution we are considering seems to result in these "mixed" readings not being generable.

Terence Parsons suggests a natural solution for these problems which may have potential for solving some of the other difficulties we have noted in the treatment of paycheckery. The solution stems from the observation that the game is defined on surface sentences rather than on trees representing derivational histories of surface sentences in the grammar. The lack of ambiguous readings would be an asset if the semantics were used to interpret unambiguous trees, (of course, it seems possible that there would then be a natural way for the predicate calculus to generate similiar readings if such a tack were taken.)

2. Subgamed Semantics.

So far we have not found a convincing argument that the game-theoretical semantics for quantification is preferable to a semantics employing the predicate calculus. In another, unpublished, paper, Hintikka and Carlson develop a different type of game-theoretical semantics and apply it to the sort of sentence which frequently concerns the discipline of a beast of burden. The paper is entitled "Conditionals, generic quantifiers, and other applications of subgames", and it deals with a treatment of the celebrated donkey-beating sentences such as

40. If Bill owns a donkey he beats it.

It is a fact that neither the P&C Semantics nor the Paycheck Semantics hinted at above will evaluate such sentences correctly, even if those systems are extended to include the following game rule:

(G.if') If S is of the form ⌜if p q⌝, the Verifier chooses ⌜neg p⌝ or q and play proceeds with respect to the sentence chosen.

Taken strictly, the Pronominalization Clause will block any move by The Speaker which involves choosing the second clause of (40) through (G.if'). If we allow (as Hintikka et al sometimes seem to) (G.a) to apply before (G.if) in such cases, then the wrong result follows. Consider the play of (40) in a world of this description:

> The World. Bill owns Pedro and does not beat him. Bill does not own Pete and beats him. Pete and Pedro are donkeys.

The first move to apply is (G.a). The Speaker should choose Pete, and the scoreboard reads:

IN PLAY	(41)	(If Bill owns Pete he beats it) and Pete is a donkey.

Nature does best to choose the first conjunct, and the board reads:

IN PLAY	(42)	If Bill owns Pete he beats it.

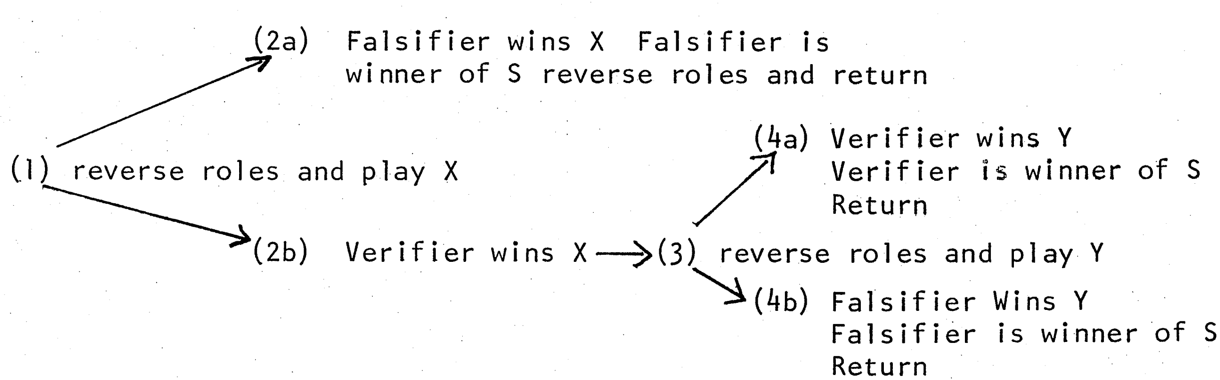
(G.if') will apply now, since "it" has a proper name as its antecedent. The Speaker may choose "neg Bill owns Pete" or "Bill beats Pete". Either choice wins, and the semantics will judge (40) true of the world (but it is false).

The Paycheck Semantics also does the wrong thing. In a Paycheck game the first move will be an application of (G.if) which yields "neg Bill owns a donkey" or "Bill beats a donkey". If The Speaker takes the second choice he will win, and the semantics will thus judge (40) true (but it is false).

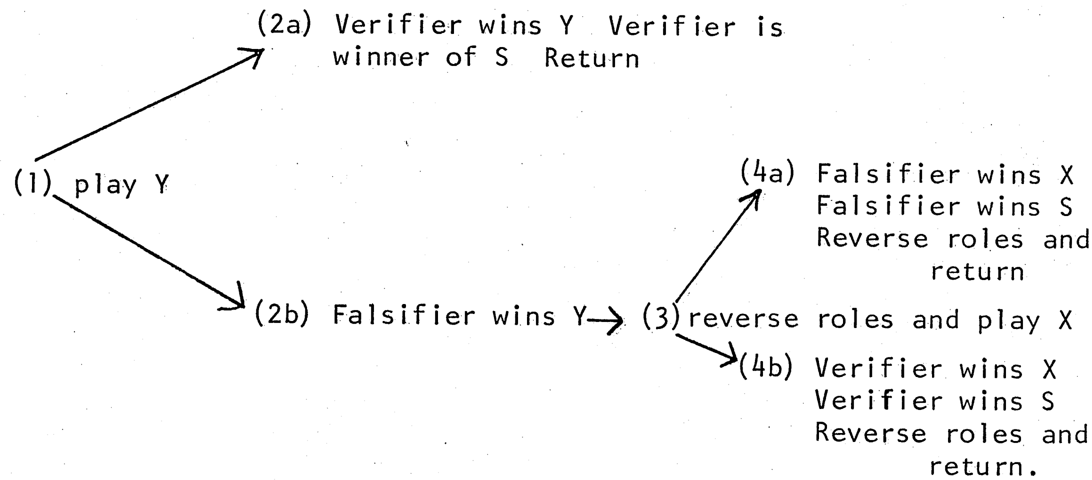
In response to this sort of problem, Hintikka and Carlson propose to "subgame" the play of (40). The idea is that some (perhaps all) of the picking and choosing is done away with. Instead, when a logical operator is being eliminated, all of the clauses involved are played through. That way a pronoun occurring later may be able to refer back to an individual introduced during the play of an earlier clause.

This idea is made more precise by the introduction of two rules, (G.cond1) and (G.cond2), which replace (G.if') and (G.if). The rules are written in flow chart form:

(G.Cond1). If S is of the form ⌜if X Y⌝,



(G.Cond2) If S is of the form ⌜Y if X⌝,



I have made two modifications in these rules. First, I have regarded the Skolem-function notation in which they are presented by Hintikka and Carlson as a dispensable device, for reasons elucidated earlier. Second, I have added a Return step to the flow charts. The idea is that, when a sentence of the form ⌜X if Y⌝ is being played as part of the play of a higher subgame, the Return notation tells the players to return to the higher subgame. The flow charts are constructed so that, at each return, The Speaker and Nature are playing the same roles they were playing when they entered the current subgame. It is to be understood that, if the move does not occur as a part of a higher-level subgame, the game ends at Return.

These flow charts are all we are given by Hintikka and Carlson as descriptions of their rules. In the course of their paper no sentences are actually played out as examples. Thus we are left to attempt the games for ourselves and, in the process, to attempt a formulation of the rules which makes their truth conditions turn out as Hintikka and Carlson say that they will.

Let us illustrate the workings of these rules with a play of (40). The first rule to apply is (G.Cond1). We are at step (1) in the (G.Cond1) flow chart. The players switch roles and the scoreboard reads:

IN PLAY (43) Bill owns a donkey.

Nature picks Pedro under (G.a) and the board reads:

IN PLAY (44) Bill owns Pedro and Pedro is a donkey.

Both conjuncts of (44) are true and Nature, as Verifier, must win (43). We are now at (2b) in the flow chart. We go to 3. The players reverse roles again. The scoreboard reads:

IN PLAY (45) Bill beats Pedro.

(45) is atomic and false, so Nature, as Falsifier, wins (45). We are now at (4b) in the flow chart. Nature is declared the winner of S and the game ends. At last we have a semantics which declares (40) false.

The move which resulted in (45) needs to be justified. The point of the subgaming is to have "it" pronominalize "Pedro", and in (45) that is what has happened. We are now in a position where we need a rule which the authors do not give, for things to work out in this way. Such a rule might be:

> Subgame Pronominalization Rule. When a sentence S is played, any pronoun pronominalizing a phrase P in some sentence S' outside S must be replaced by the reference P received in the play of the subgame on S'.

The Subgamed Semantics together with this rule generates appropriate readings for a number of simple donkey-beating sentences. In fact, the authors assert that

> "Our treatment of pronominalization in sentences like (40) is immediately extended to a large class of relative clauses. The following are cases in point.
>
> 46. Everyone who owns a donkey beats it.
> 47. Everyone who doesn't like a donkey beats it." (p 26)

The extension is not quite so simple as it would appear, however. The first move of a play of (46) will generate

48. John beats it, if (John is a one and John owns a donkey).

Sentences of this sort will be assigned inappropriate truth conditions, since "a donkey" occurs in a later clause than "it" and does not get a specific reference until too late in the play of (G.Cond2). Perhaps what is intended is a change in the order of the clauses in the output of (G.every), (G.some), (G.an) etc, and then a subgaming of those clauses. It does not seem that this alone will solve the problem, however. Consider the play of

49. If a man who owns it beats a donkey which kicks him, it will respect him.

Assuming that (G.an) is amended so that the first clause of the old output now comes last, the play of (49) will eventually result in the play of a sentence like

50. John is a man and John owns it and John beats a donkey which kicks him.

We are not told what to do now, since "it" pronominalizes a non-proper name occurring in a phrase to be played later.

There are some other sentences which seem to get inappropriate readings in such a semantics. Consider the play of (51) in a world of the following description:

51. Bill beats a donkey if he owns it.

The World.
Bill owns Pete and doesn't beat Pete. Fred owns Pedro. Pete and Pedro are donkeys. Bill doesn't beat Pedro.

On the natural reading of (51) it is false in the world, since Bill owns Pete and does not beat Pete. In a play of the game on (51), (G.Cond2) applies first. The scoreboard reads:

IN PLAY	(52) Bill beats a donkey.

The Speaker chooses Pedro and loses. Play advances to step (3) of the flow chart for (G.Cond2). Roles reverse and (53) is played:

53. Bill owns Pedro.

The Speaker wins (53) and thereby wins (51). Hence (51) is judged true in the world. But this seems wrong. Hintikka and Carlson discuss (51) only to give it as an example of a sentence which is syntactically ill-formed. This judgement seems questionable. Hintikka and Carlson do not note that, if (51) is well-formed, it will counterexample the Subgamed Semantics.

A final example is a simplified version of one suggested by Hintikka and Carlson themselves. Consider

54. If every child gets a present, some child will open it.

Hintikka and Carlson say of (54) that

> "Here a winning strategy of [The Speaker's] for the antecedent assigns to every child a present. Hence, when "some child" in the consequent invites us to pick out one, he or she comes already with an associated present, recoverable by the pronoun "it" in the consequent of (54)."

This is a misrepresentation of the game as we have formalized it here. Perhaps there is a different subgame pronominalization rule which Hintikka and Carlson have in mind. If so, they will clear up many misunderstandings when they give it. What is certain is that (54) will not be assigned the correct reading by the Subgamed Semantics given here, for consider its play in a world of the following description, where it is false.

The World. Bill gets P1 and doesn't open it. Carol gets P2 and she will open it. P1 and P2 are presents and Bill and Carol are children.

(G.Cond1) applies first. The players reverse roles and the board reads:

IN PLAY (55) Every child gets a present.

The Speaker chooses Bill, yielding

IN PLAY (56) Bill gets a present, if Bill is a child.

Now (G.Cond2) applies. The board reads:

IN PLAY (57) Bill gets a present.

Nature must choose P1 or she will lose immediately. Hence she chooses P1 and wins (57), going on to win (56). They return to the play of (54) at stage (2b) in the (G.Cond1) flow chart. They reverse roles and the board reads:

IN PLAY (58) Some child will open P1.

(Note that "it" is assigned the reference given "a present" in the previous subgame.) The Speaker chooses Carol, and the board reads:

IN PLAY (59) Carol will open P1 and Carol is a child.

No matter which choice Nature makes now, she will lose the play of (59). We are at stage (4a) in the flow chart. The Speaker is declared winner of (54), and the Subgamed Semantics judges (54) true in the world, (but it is false). What has gone wrong ?

The problem seems to lie in the fact that playing the first subgame does not assign to each child a present <u>recoverable</u> in the second subgame. Through a combination of cunning and foresight, The Speaker chooses a child in the first subgame with a present which will win the second subgame. The Subgamed Semantics given here does not bring it about that <u>Carol's</u> present is recovered in the second subgame. Instead it inappropriately recovers Bill's present.

3. Conclusion.

We have given an exposition of the two sorts of game-theoretical semantics currently being employed by Hintikka <u>et al</u> and we have evaluated the claims made for those semantics. With respect to the question of second-order treatments the verdict is "not proven", with the added observation that P&C Semantics will be inadequate if Hintikka <u>et al</u> are correct about their test sentences. With respect to paycheckery we have seen that some revision is needed in order for the semantics to live up to its billing (and we have hypothesized that one possible revision -- the consideration of derivational trees rather than surface sentences -- may allow a predicate calculus treatment as well). With respect to donkey-beating sentences, the system is not worked out well enough to evaluate in detail. On the construction of it given here, the Subgamed Semantics fails in a number of crucial cases.

Appendix.

[Note: These proofs are based on a suggestion made by Terence Parsons.]

I. Proof of the semantic equivalence of the propositional game-theoretical semantics ("P") and the propositional calculus ("PC").

First let us define a translation function F as follows:

If S is atomic, F(S) is S.
If S is of the form ⌜p and q⌝, F(S) is ⌜p & q⌝.
If S is of the form ⌜p or q⌝, F(S) is ⌜p v q⌝.
If S is of the form ⌜p but q⌝, F(S) is ⌜p & q⌝.
If S is of the form ⌜neg p⌝, F(S) is ⌜~ p⌝.
If S is of the form ⌜p if q⌝, F(S) is ⌜p v~ q⌝.

Now we specify a mechanical procedure for turning any sentence S of P into T(S), its PC translation:

1. If S is atomic, T(S) is S. The translation is finished.
2. If S is non-atomic, locate the next logical operator to be eliminated from S by the P&C game rules. T(S) is F(S), where F(S) eliminates that operator. Continue to rewrite each of the clauses of F(S) by working in left-right order, applying these rules to them until their translations are atomic.

Now we sketch a proof by induction that for any sentence S, S is true in P iff T(S) is true in PC. The induction is on n, the number of logical operators in S.

Base Case: Assume n is 0. Then, by definition, S is atomic. The truth value of S in any world is determined by checking S against that world. T(S) is S when S is atomic. We assume that PC also assigns truth values to atomic sentences by checking them against the world. Therefore, trivially, S is true in P iff T(S) is true in PC.

Induction Step: Assume that for all sentences Q in P with fewer than n logical operators, Q is true in P iff T(Q) is true in PC. To prove: For all sentences S with n logical operators, S is true in P iff T(S) is true in PC.

Let "W(S)" abbreviate "The Verifier has a winning strategy for the play of S". Then this section of the proof has three parts.

1. Left to right. Assume that S is true in P. Then there are these cases to consider:

a. S is atomic. Then the result has been proved in the base case.
b. S is not atomic. Then there are these possibilities:

S is of the form ⌜p and q⌝, where "and" is the first logical operator to be eliminated from S by the P game rules. Then W(S), by the assumption that S is true and the P definition of truth. But W(S) only if W(p) and W(q). Otherwise, the Falsifier could choose the losing conjunct and win the game on S. But W(p) and W(q) only if p is true in P and q is true in P. p is true in P only if T(p) is true in PC, by the induction hypothesis and the observation that p has to have at least one fewer logical operators than S. Similarly for q. Hence T(p) is true in PC and T(q) is true in PC. But this is the case only if p & q is true in PC, by the definition of "&". But p & q is T(S). Therefore T(S) is true in PC.

Similarly for the other logical operators.

2. Right to left. We consider a sentence T(S) in PC which is the translation of some sentence S in P. Assume that T(S) is true in PC. Then there are these cases to consider:

a. T(S) is atomic. Then the result has been proved in the base case.
b. T(S) is not atomic. Then there are these possibilities:

T(S) is of the form T(p) & T(q) , where "&" is the major connective in T(S) and p and q are sentences of P. This is true only if T(p) is true in PC and T(q) is true in PC, by the definition of "&". By the induction hypothesis, T(p) is true in PC iff p is true in P. Similarly for T(q). Hence p and q are true in P. This implies that W(p) and W(q), by the P definition of truth. This implies W(p and q) and W(p but q), since the Falsifier will not be able to choose a falsifiable conjunct from either of these sentences. This implies that p and q is true in P and that p but q is true in P, by the P definition of truth. But an inspection of the T function shows that S must be either p and q, or p but q. Hence S is true in P.

Similarly for the other logical operators.

3. From 1 and 2, the result follows immediately.

II. Proof of the semantic equivalence of P&C Semantics("P&C") with first-order predicate calculus ("FPC").

First we define a translation function F as follows:

Let F(S) be as in Part I, when S is atomic or when the first logical operator to be eliminated from S by the P&C rules is a propositional operator. Otherwise (where "i" gets a value during the translation procedure to be described below):

If S is of the form ⌜X some Y who Z⌝, F(S) is ⌜$(\exists x_i)(Xx_iW$ & x_i is a Y & $x_iZ)$⌝. [Similarly for "an".]

If S is of the form ⌜X every Y who Z W⌝, F(S) is ⌜$(x_i)(Xx_iW \rightarrow$ (x_i is a Y and x_iZ))⌝. [Similarly for "any".]

If S is of the form ⌜X the Y who Z W⌝, then F(S) is ⌜$(\exists x_i)(Xx_iW$ & x_i is a Y & x_i Z and $(y_i)(x_i \neq y_i \rightarrow$ y_i is not a Y who Z))⌝.

If S is of the form ⌜X Y's Z W⌝, then F(S) is ⌜$(\exists x_i)(Xx_iW$ & x_i is the Z of Y)⌝, where Y is replaced by its antecedent if it pronominalized a proper name.

Now we specify a mechanical procedure for turning any sentence S of P into T(S), its FPC translation:

1. If S is atomic, the translation is finished, except for step 3.
2. If S is non-atomic, locate the next logical operator to be eliminated from S by the P&C game rules. Rewrite S as F(S). If F(S) involves a variable of the form "x_i" or "y_i", let i be the number of steps taken so far in the rewriting process. Continue to rewrite each of the clauses of F(S) by working in left-right order, applying these rules to them until their translations are atomic.
3. When S has been completely rewritten by following 1 and 2, replace each pronoun in T(S) by the phrase it pronominalizes.
4. The result of rewriting a P&C sentence S in this way is an FPC formula T(S).

Now we sketch a proof by induction that for any sentence S, S is true in P&C iff T(S) is true in FPC. The induction is on n, the number of logical operators in S.

Base Case: Same as in Part I.

Induction Step: Same induction hypothesis as in Part I.

1. Left to right. Assume that S is true in P&C. Then there are these cases to consider:

a. S is of a form dealt with in Part I. Then reproduce that proof here.

b. The first logical operator to be eliminated from S by the

P&C game rules is a quantificational operator. Then there are these possibilities:

S is of the form ⌜X the Y who Z W⌝. Then W(S), from the assumption and the P&C definition of truth. (G.Sing.the) guarantees that W(S) only if there is some individual c in the domain such that (i) is true:

i. X c W and c is a Y and c Z. Furthermore, (G.Sing.the) guarantees that, no matter what individual b the Falsifier chooses from the domain,

ii. if b is distinct from c, then b is not a Y who Z. The truth of (ii) guarantees the truth of (iii) in FPC:

iii. $b \neq c \rightarrow$ b is not a Y who Z. Since b has been introduced as the result of a choice of any individual from the domain, (iii) guarantees the truth of (iv) in FPC, by the FPC conditions on quantifiers:

iv. $(y_i)(y_i \neq c \rightarrow y_i$ is not a Y who Z). Since (iv) and (i) are both true in FPC, we may conjoin them as

v. X c W and c is a Y and c Z & $(y_i)(y_i \neq c \rightarrow y_i$ is not a Y who Z). We know that c is some individual in the domain. Hence (v) implies (vi) in FPC:

vi. $(\exists x_i)(Xx_iW$ & x_i is a Y & x_iZ & $(y_i)(y_i \neq x_i \rightarrow y_i$ is not a Y who Z)).

But (vi) is T(S). Hence T(S) is true in FPC.

Similarly for the other logical operators.

2. Right to left. The strategy is fairly obvious, although the operators must be considered in the correct order.

3. The result follows immediately from 1 and 2.

Footnotes.

This paper has benefited greatly from the insights and criticisms of Terence Parsons, Muffy Siegel and Barbara Partee, who nonetheless may not endorse each of its conclusions.

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