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# Pipeline Monitoring Architecture based on observability and controllability Analysis

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**Abstract**— Recently many techniques with different applicability have been developed for damage detection in the pipeline. The pipeline system is designed as a distributed parameter system, where the state space of the distributed parameter system has infinite dimension.

This paper is dedicated to the problem of observability as well as controllability analysis in the pipeline systems. Some theorems are presented in order to test the observability and controllability of the system. Computing the rank of the controllability and observability matrix is carried out using Matlab.

**Keywords**— pipeline, Controllability, Observability.

## I. INTRODUCTION

Along with the development of industrial requirements, more and more long-distance pipelines have built and served. According to the experiences, pipelines, as a means of transport, are the safest but this does not mean they are risk-free. Therefore, assuring the reliability of the gas pipeline infrastructure has become a critical need for the energy sector. Pipeline monitoring systems are indispensable in the areas of power plants' petroleum and chemistry industries. When passing through the harsh environment, pipelines may be broken or blocked which would cause huge economic loss [1][2][3][4]. The main threat considered, when looking for means of providing the reliability of the pipeline network, is the occurrence of leaks.

When the pipeline has problem same as leaking, broken or blocked, will cause harm to nature, and the environment. Therefore, detecting the leakage in pipelines in order to avoid wastage and damage to nature is very important. Most studies are based on the design of the model leak detection methods and develop the investigation of friction coefficients [5]. In [5] the authors studied the methods for leak detection. Furthermore compared the capabilities of the techniques in order to identify the advantages and disadvantages of using each leak detection solution.

On the other hand, various researches have been done by introducing new algorithms in order to detect the leakage [6]. Bensacon et al. [7] used the observation technique in order to

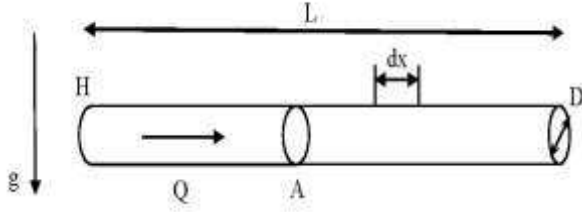
observe the leakage in the pipeline directly. Billman and Isermann in [7] introduced a new method for leak detection in pipelines. In recent years, there have been widespread studies based on frequency viewpoint analysis in the detection of deficiency in pipelines [8]. Mpesha et al. [9] proposed a frequency response model. Covas et al. [10] presented the standing wave difference method SWDM for leak detection. Verde et al. [1][2][11][12][13] have so many research in leak isolation and modeling observation in the detection of multi-leaking in the fluid pipeline. Zecchin et al. [14], [15] introduced the network admittance matrix method, for clarifying the frequency domain analysis of complex systems, and they combined admittance forms of the link equations in the frequency domain with graph-theoretic matrices.

The most current published researches are based on designing the observer, controller or fault diagnosis techniques. Nevertheless, investigating the observability as well as controllability of the pipeline system is exceptionally significant. The classical techniques for determining the observability as well as controllability is in direct relation with the fullness of the rank of the discrimination matrices. In this paper, an intuitive technique is introduced for analyzing the observability as well as the controllability of the pipeline system. The parameters such as pressure and flow are maintained constantly by implementing control valves depending on the different pressure and flow rate of the transmitting pipe.

The rest of the paper is structured as follows: Section II presents the pipeline mathematical modeling for constant friction coefficient. Section III describes the observability and Controllability of the model of the pipeline. Finally, Conclusions are given in Section IV.

## II. PIPELINE MODELING

In this paper, the convective change in velocity, as well as the compressibility in the line of length ( $L$ ), are neglected, and the liquid density ( $\rho$ ), the flow rate ( $Q$ ), and pressure ( $P$ ) at the inlet and outlet of the pipeline are measurable for evaluation. The cross-sectional area ( $A$ ) of the pipe is constant throughout the pipe. The designed pipeline is shown in Figure 1.



**Fig. 1.** Schematic of pipeline

The dynamics of fluid in a pipeline is defined on the basis of the mass and momentum, and the conservation equations in hydrodynamics and hydromechanics, respectively. In order to obtain the mass and momentum in hydrodynamics by implementing Newton's second law ( $F = ma$ ) to a control volume in the continuum and body force pipe ( $s = \frac{f}{2D}v^2$ ) the following relation is given,

$$\frac{\partial v}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{f}{2D} v^2 = 0 \quad (1)$$

By substituting  $v = \frac{Q}{A}$ , and  $p = \rho gH$  in (1) the following relation is extracted,

$$\frac{\partial Q}{\partial t} + Ag \frac{\partial H}{\partial x} + \frac{fQ^2}{2DA} = 0 \quad (2)$$

where  $H$  is the pressure head ( $m$ ),  $Q$  is the flow rate ( $m^3/s$ ),  $x$  is the length coordinate ( $m$ ),  $t$  is the time coordinate ( $s$ ),  $g$  is the gravity ( $m/s^2$ ),  $A$  is the section area ( $m^2$ ),  $D$  is the diameter ( $m$ ), and  $f$  is the friction coefficient.

In order to derive the continuity equation, by applying the law of conservation of mass and using the Reynolds transport theorem to a control volume and simplifying, the following relation is obtained,

$$\frac{\partial p}{\partial t} + \rho a^2 \frac{\partial v}{\partial x} = 0 \quad (3)$$

By substituting the head pressure ( $H$ ) and flow rate ( $Q$ ) in (3) the following is extracted,

$$\frac{\partial H}{\partial t} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} = 0 \quad (4)$$

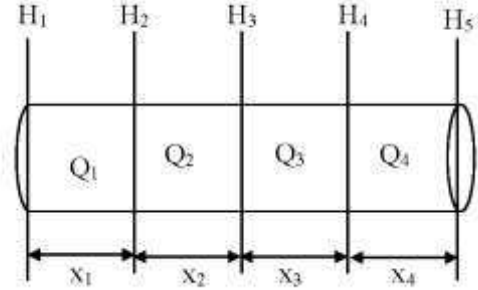
in which  $a$  is the velocity of the pressure wave ( $m/s$ ) in an elastic conduit filled with a slightly compressible fluid.

The pressure head ( $H$ ) and Flow rate ( $Q$ ) are functions of position and time as  $H(x, t)$  and  $Q(x, t)$ , where  $x \in [0, L]$ , also  $L$  is the length of pipe. The initial and boundary conditions that can be controlled and measured are the pressure heads in the beginning and end of the pipeline. The conditions are stated as

$$\begin{cases} H(0, t) = H_{in}(t) \\ H(L, t) = H_{out}(t) \end{cases} \quad (5)$$

For the system with small changes in the flow rate  $Q$ , the momentum equation from the nonlinear system can be linearized as below

$$\frac{\partial Q}{\partial t} + Ag \frac{\partial H}{\partial x} + \frac{fQ}{DA} = 0 \quad (6)$$



**Fig. 2.** The finite difference for the pipeline with four sections and actuation at the beginning and end of sections

The pipeline model is designed based on (4) and (6). Obtaining the solutions of these equations are complex. However, several methods are utilized in order to numerically integrate them. Some of the main methods such as characteristics and finite difference techniques are proposed by Chaudry [16] and Wyllie [17]. In this work, the finite difference method is applied as it is a simple way to get a more convenient model in order to observe and control the structure of the nonlinear system. The finite difference method is a discretization technique which divides the entire pipeline into  $N$  number of sections [18]. The flow dynamics are then described via the flow rate in each section and the pressures head at the end of each section.

$$\begin{aligned} \dot{H}_1 &= \frac{-a^2}{gA\Delta x}(Q_1 - Q_0) & \dot{Q}_1 &= \frac{-Ag}{\Delta x}(H_2 - H_1) - \frac{f}{DA}Q_1 \\ \dot{H}_2 &= \frac{-a^2}{gA\Delta x}(Q_2 - Q_1) & \dot{Q}_2 &= \frac{-Ag}{\Delta x}(H_3 - H_2) - \frac{f}{DA}Q_2 \\ \dot{H}_3 &= \frac{-a^2}{gA\Delta x}(Q_3 - Q_2) & \dot{Q}_3 &= \frac{-Ag}{\Delta x}(H_4 - H_3) - \frac{f}{DA}Q_3 \\ \dot{H}_4 &= \frac{-a^2}{gA\Delta x}(Q_4 - Q_3) & \dot{Q}_4 &= \frac{-Ag}{\Delta x}(H_5 - H_4) - \frac{f}{DA}Q_4 \\ \dot{H}_5 &= \frac{-a^2}{gA\Delta x}(Q_5 - Q_4) \end{aligned}$$

For example, with four sections that are depicted in Figure 2, the variables are  $H_1, H_2, H_3, H_4, H_5$  for Head pressure and  $Q_1, Q_2, Q_3, Q_4$  are flow rates. Control variables are the pressures at the beginning and end of the pipe ( $H_1$  and  $H_5$ ). Furthermore, the flow rates ( $Q_1$  and  $Q_4$ ) are directly measured.

The idea in this paper is to design an observer and controller for the systems mentioned in (4) and (6). A General linearized model is stated as

$$\frac{dx}{dt} = Ax + Bu \quad (7)$$

$$y = Cx \quad (8)$$

where  $x(t)$  is the state vector containing the  $n$  unknown flow perturbation quantities at each point. The vectors  $u(t)$  and  $y(t)$  contain the system forcing inputs and outputs respectively.

The suggested model, along with inputs and outputs are  $x = (Q_1 \ Q_2 \ Q_3 \ Q_4 \ H_2 \ H_3 \ H_4)^T$ ,  $u := [H_{in} \ H_{out}]^T = (H_1 \ H_5)^T$  and  $y = (Q_1 \ Q_4)^T$ , respectively.

$$\begin{aligned}
\dot{x}_1 &= \frac{-Ag}{\Delta x} (x_5 - u_1) - \frac{f}{DA} x_1 \\
\dot{x}_2 &= \frac{-Ag}{\Delta x} (x_6 - x_5) - \frac{f}{DA} x_2 \\
\dot{x}_3 &= \frac{-Ag}{\Delta x} (x_7 - x_6) - \frac{f}{DA} x_3 \\
\dot{x}_4 &= \frac{-Ag}{\Delta x} (u_2 - x_7) - \frac{f}{DA} x_4 \\
\dot{x}_5 &= \frac{-a^2}{gA\Delta x} (x_2 - x_1) \\
\dot{x}_6 &= \frac{-a^2}{gA\Delta x} (x_3 - x_2) \\
\dot{x}_7 &= \frac{-a^2}{gA\Delta x} (x_4 - x_3)
\end{aligned}$$

The values of  $A, B$ , and  $C$  in (7) and (8) can be written in the form of the block matrices as below

$$\begin{aligned}
A &= \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, C = [C_1 \quad C_2] \\
A_1 &= \begin{bmatrix} N & 0 & 0 & 0 \\ 0 & N & 0 & 0 \\ 0 & 0 & N & 0 \\ 0 & 0 & 0 & N \end{bmatrix}, A_2 = \begin{bmatrix} M & 0 & 0 & 0 \\ -M & M & 0 & 0 \\ 0 & -M & M & 0 \\ 0 & 0 & -M & M \end{bmatrix} \\
A_3 &= \begin{bmatrix} S & 0 & 0 & 0 \\ -S & S & 0 & 0 \\ 0 & -S & S & 0 \\ 0 & 0 & -S & S \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
B_1 &= \begin{bmatrix} M & 0 \\ -M & 0 \\ 0 & 0 \\ 0 & M \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\
C_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\end{aligned}$$

where  $N, M, S$  are defined as follows:

$$\begin{aligned}
S &= \frac{-a^2}{gA\Delta x} \\
N &= \frac{-f}{DA} \\
M &= \frac{-Ag}{\Delta x}
\end{aligned}$$

### III. OBSERVABILITY AND CONTROLLABILITY

In this section, the controllability and observability of the system are studied in Lemma1 and Lemma2, respectively. For more details, refer to [19] [20].

**Lemma. 1.** For a linear system defined in (6) and (7) with  $N_u$  as a control input, that is Head pressure and  $N_y$  as an output, that is measured flow rate, the controllability of matrix  $\mathcal{C}_i$  is given by [17],

$$\mathcal{C}_i(A, B) := [B \quad AB \quad A^2B \quad A^3B \quad \dots \quad A^{i-1}B] \quad (9)$$

By utilizing the Cayley-Hamilton Theorem [21], the rank of  $\mathcal{C}_i$  is determined by the first column of  $N \times N_u$ , where  $N$  is the state dimension and is equal to the dimension of matrix  $A$ .

A linear system (or, equivalently, the matrix pair  $(A, B)$ ) is named as the controllable if the controllability matrix has full row rank (i.e.  $\{Rank(\mathcal{C}) = i\}$ ), where  $i$  is termed as the state space dimension,  $\dim X$ .

**Lemma. 2.** For a linear system stated in (6) and (7), the observability of matrix  $O_i$  can be defined as [22],

$$O_i(C, A) := \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{i-1} \end{bmatrix} \quad (10)$$

Again by utilizing Cayley-Hamilton Theorem, the rank of  $O_i$  is determined by the first row of  $N \times N_u$ , where  $N$  is termed as the state dimension and is equal to the dimension of matrix  $A$ . In Other words,  $\ker(O_\infty) = \ker(O_i) = i \subset R^N$ .

A linear system (or, equivalently, the matrix pair  $(A, C)$ ) is called observable if the observability matrix has full columns rank (i.e.  $\{Rank(O_i) = i\}$ ), where  $i$  is the number of independent columns in the observability matrix.

### IV. SIMULATION RESULTS

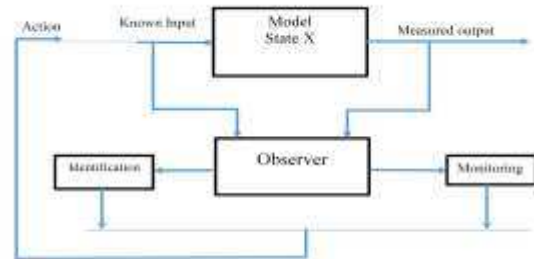
The simulations in this section are carried out by Matlab. An illustration of the model is shown in Figure 1. In this Figure the length of the pipeline is  $L = 1.7 \times 10^4 m$ , the dimension is  $D = 0.56 m$ , the cross section is  $A = 0.97 m^2$ , density is  $\rho = 1000 kg/m^3$ , gravity is  $g = 9.81 m/s^2$  and the wave speed coefficient is  $b = 1250 m/s$  that are summarized in Table 1.

**Table. 1.** The proposed characteristics of the pipeline

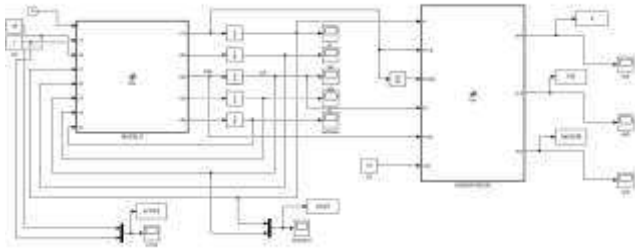
L	$1.7 \times 10^4 m$
D	$0.56 m$
A	$0.97 m^2$ ,
F	0.001
b	$1250 m/s$
g	$9.81 m/s^2$
$\rho$	$1000 kg/m^3$

Control variables are the pressures at the beginning and end of the pipe ( $H_1$  and  $H_5$ ). Furthermore, the flow rates ( $Q_1$  and  $Q_4$ ) are directly measured. According to Lemma 1 and 2, the rank of discriminant matrices is  $rank \mathcal{C} = rank O = 4$ , thus, the system is completely observable and controllable.

The model, the observer, and the state feedback structure are displayed in Figure 3.



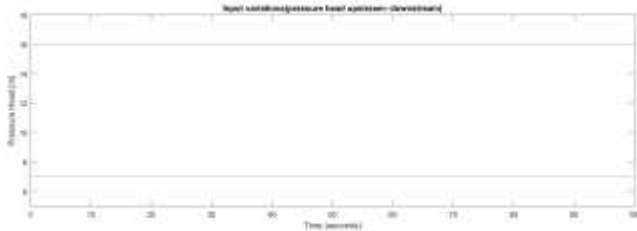
**Fig 3.** structure of model , observer and state feedback



**Fig 4.** simulation in Matlab

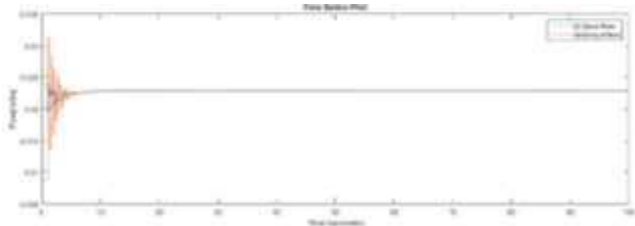
The simulation has the same structure as the designed system demonstrated in (7) and (8). (Figure 4)

Figure 5 displays the simulated pressure head at the inlet ( $H(in) = H_1 = 16\text{ m}$ ) and outlet ( $H(out) = H_5 = 7\text{ m}$ ) of the pipe.



**Fig 5.** Input variation

Figure 6 shows the evolution of the inflow and outflow,  $Q_1$  and  $Q_4$ , respectively. It can be seen that after a few second the  $Q$  from the initial amount (0) is approaching the real amount. Therefore, the pipeline system is completely observable and controllable.



**Fig 6.** output and input flow

## V. CONCLUSION

The objective of this paper is analyzing the observability and controllability of the pipeline system. The system is stated by hyperbolic partial differential equations. In this work, the finite difference method is applied as it is a simple way to get a more convenient model in order to observe and control the structure of the nonlinear system. This method divides the entire pipeline into  $N$  number of sections. Some theorems are presented in order to test the observability and controllability of the system. Future work is to study the stability of the pipeline system.

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