A Novel Method for Computations of Ratios of Jet Cross Sections in Perturbative Quantum Chromodynamics

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Abstract

The strong interaction is the force responsible for binding quarks to form hadrons, such as protons and neutrons, and also for binding protons and neutrons to form the nuclei of atoms. The properties of the strong interaction can be studied in particle collisions from measurements of the production rates of collimated sprays of particles, called jets. In particular, the ratio of the number of collisions that produce three jets over the number of collisions that produce two jets is a direct measure of the strength of the strong interaction. This strength is quantified by the strong coupling constant, α_s. Determinations of α_s from particle collider data require theoretical calculations. In this paper, a new approach for the theoretical calculations is investigated that differs from the commonly used approach. Computations of the results are presented for different ratio measurements performed at the CERN Large Hadron Collider and the Fermilab Tevatron Collider. The results of the two different approaches are compared to each other and to the results of the experimental measurements. It is discussed in which kinematical regions the two approaches agree and where they differ.

1 Introduction

1.1 Particle Accelerators and Detectors

In high energy physics (HEP), particle accelerators are used to collide elementary particles together to investigate quantum effects of fundamental forces. Of interest in this paper are proton on proton collisions from the ATLAS detector at the CERN LHC and the DØ detector at the Fermilab Tevatron. In both accelerators, protons are accelerated in an underground

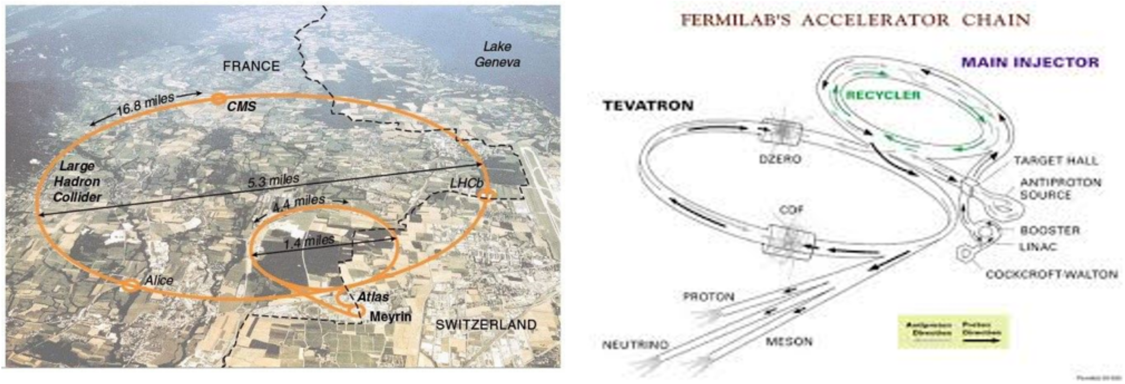

Figure 1: The Large Hadron Collider at CERN, left, and the Tevatron Collider at Fermilab right. Both accelerators collide protons on protons at 13 and 1 TeV energy, respectively. LHC image source: home.cern; Tevatron image source: fnal.gov

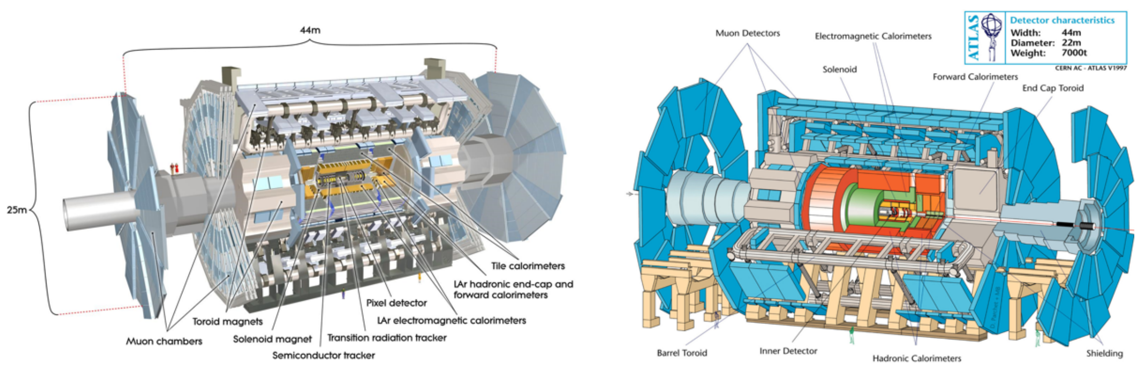

Figure 2: The ATLAS detector left and the DØ detector right. ATLAS image source: atlas.cern; DØ image source: Brad Abbott, OU HEP group

loop and then collided together in a particle detector. The above detectors are made out of a series of magnets that track the path of particles produced in a proton-on-proton collision as well as calorimeters to measure their energy.

1.2 Particle Jet Production

Under the standard model, protons are made up of three elementary particles, two up quarks and one down quark. These quarks interact with each other and gluons, the force carrying particle of the strong nuclear force. When protons are collided together at high energies, the quarks and gluons that make up the individual protons interact to form new particles.

In the above figure, two quarks collide to produce a Quark\anti-quark pair. These quarks will break down into other particles inside the detector in a process called hadronization.

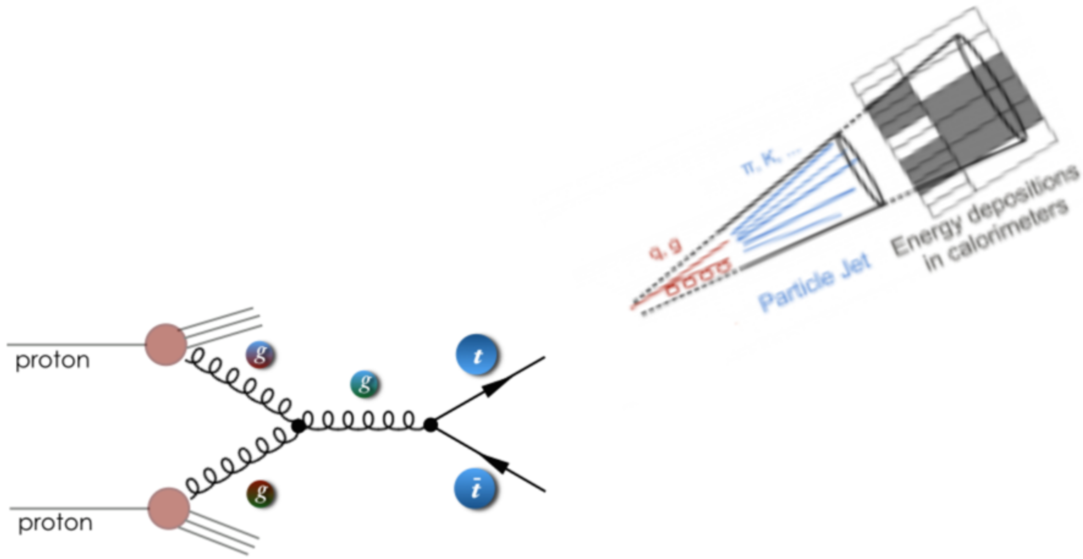

Figure 3: Quark\anti-quark production from proton\proton collision. Image source: Thomas G. Mccarthy and cms.cern

The data collected from these collisions are jets, and the rate at which a certain process occurs is used to determine the cross-section for that process experimentally.

1.3 Cross Sections

The strong coupling constant, α_s, paramaterizes the strength of the strong interaction, and acts at each vertex in the below diagrams.

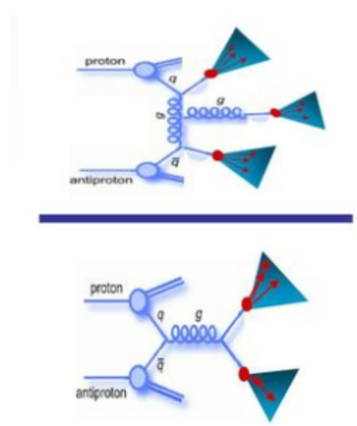

Figure 4: Diagrams for three and two jet processes. Image source: Markus Wobisch, Louisiana Tech HEP group

In the top diagram, three jets are produced, and in the bottom, two jets are produced. So, the three jet cross section would be experimentally found by adding the number of events that produced three jets and dividing by the total number of events. Often, ratios of cross sections are taken because uncertainties in measurements that are due to the detector; such as luminosity, energy calibrations, and resolution; are the same for each cross section, and cancel upon taking the ratio.

1.4 Perturbative Expansion in α_s

Because the strong coupling constant is much less than one ($\alpha_s \approx .11$), the cross section for a process can be written in a perturbative expansion in α_s. The cross sections for the three and two jet processes, for example, can be written as:

$$\sigma_{3j} = c_1\alpha_s^3 + c_2\alpha_s^4 + c_3\alpha_s^5 + ...$$

and

$$\sigma_{2j} = c_1\alpha_s^2 + c_2\alpha_s^3 + c_3\alpha_s^4 + ... \tag{1}$$

where the constants in the sum are given by the S-matrix elements for the interaction. Currently, these values are only known only at next-to-leading order (NLO) so the NLO cross section is written as a leading order (LO) result plus a NLO correction:

$$\sigma_{3-jet} = \sigma_{3LO} + \sigma_{3NLO} + H.O.$$

and

$$\sigma_{2-jet} = \sigma_{2LO} + \sigma_{2NLO} + H.O. \tag{2}$$

For the 3-jet cross-section, the individual terms are given by

$$\sigma_{3LO} = \alpha_s^3 C_{3L}$$

and

$$\sigma_{3NLO} = \alpha_s^4 C_{3N} \tag{3}$$

and for the 2-jet cross-section

$$\sigma_{2LO} = \alpha_s^2 C_{2L}$$

and

$$\sigma_{2NLO} = \alpha_s^3 C_{2N} \tag{4}$$

2 Method

At NLO, the ratio of any two cross sections is typically found by simply dividing the two cross sections. For example: $R_{3/2} = \frac{\sigma_{3-jet}}{\sigma_{2-jet}}$.[2] Instead of taking the ratio of two perturbative expansions, a new method for computing ratios of jet cross sections can be found by taking the perturbative expansion of the ratio at NLO. At NLO, the old method is found by:

$$R_{3/2} = \frac{\alpha_s^3 C_{3L} + \alpha_s^4 C_{3N}}{\alpha_s^2 C_{2L} + \alpha_s^3 C_{2N}} \tag{5}$$

The new method is found from the old result by:

$$R_{3/2} = \frac{\alpha_s^3 C_{3L} + \alpha_s^4 C_{3N}}{\alpha_s^2 C_{2L}} \frac{1}{1 + \alpha_s \frac{C_{2N}}{C_{2L}}} \tag{6}$$

This factorization was done so that the last term can expanded in a Taylor series

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - ...$$

with

$$x = \alpha_s \frac{C_{2N}}{C_{2L}} \tag{7}$$

Which gives

$$R_{3/2} = \frac{\alpha_s^3 C_{3L} + \alpha_s^4 C_{3N}}{\alpha_s^2 C_{2L}} [1 - \alpha_s \frac{C_{2N}}{C_{2L}} + \alpha_s^2 (\frac{C_{2N}}{C_{2L}})^2 + ...] \tag{8}$$

$$= [\alpha_s \frac{C_{3L}}{C_{2L}} + \alpha_s^2 \frac{C_{3N}}{C_{2L}}][1 - \alpha_s \frac{C_{2N}}{C_{2L}} + \alpha_s^2 (\frac{C_{2N}}{C_{2L}})^2 + ...] \tag{9}$$

$$= \alpha_s [\frac{C_{3L}}{C_{2L}}] + \alpha_s^2 [\frac{C_{3N}}{C_{2L}} - \frac{C_{3L} C_{2N}}{C_{2L}^2}] + \alpha_s^3 [\frac{C_{3L} C_{2N}^2}{C_{2L}^3} - \frac{C_{3N} C_{2N}}{C_{2L}^2}] + ... \tag{10}$$

Because the first two terms are first and second order in α_s, they are the leading order result and next-to-leading correction for $R_{3/2}$

$$R_{3/2LO} = \alpha_s [\frac{C_{3L}}{C_{2L}}] \tag{11}$$

and

$$R_{3/2NLO} = \alpha_s^2 [\frac{C_{3N}}{C_{2L}} - \frac{C_{3L} C_{2N}}{C_{2L}^2}] \tag{12}$$

The NLO correction can be rewritten as

$$R_{3/2NLO} = \alpha_s [\frac{C_{3N}}{C_{3L}}] \alpha_s [\frac{C_{3L}}{C_{2L}}] - \alpha_s [\frac{C_{2N}}{C_{2L}}] \alpha_s [\frac{C_{3L}}{C_{2L}}] \tag{13}$$

A k-factor is defined to be the ratio of the NLO and LO cross-sections for a given process.[3] The k-factors for the 2-jet and 3-jet processes are

$$k_2 = \frac{\alpha_s^2 C_{2L} + \alpha_s^3 C_{2N}}{\alpha_s^2 C_{2L}}$$

and

$$k_3 = \frac{\alpha_s^3 C_{3L} + \alpha_s^4 C_{3N}}{\alpha_s^3 C_{3L}} \tag{14}$$

For the old method, the NLO correction is given in terms of k-factors by:

$$R_{NLO} = \frac{k_3}{k_2} R_{LO} \tag{15}$$

The NLO correction for the new method is given by:

$$R_{3/2NLO} = (1 + k_3 - k_2) R_{3/2LO} \tag{16}$$

Both methods of computing $R_{3/2}$ are equal if the sum is taken to infinity. However, of interest is the difference in the two ratios, which is found by summing over all the terms of order 3 or higher. The term that contributes most to this sum is the term that is of order 3 in α_s, which appears in Eq. 10

$$\alpha_s^3 [\frac{C_{3L} C_{2N}^2}{C_{2L}^3} - \frac{C_{3N} C_{2N}}{C_{2L}^2}] \tag{17}$$

$$= \alpha_s \frac{C_{3L}}{C_{2L}} \alpha_s^2 \frac{C_{2N}^2}{C_{2L}^2} - \alpha_s^2 \frac{C_{3N}}{C_{2L}} \alpha_s \frac{C_{2N}}{C_{2L}} \tag{18}$$

$$= R_{3/2LO}(k_2 - 1)^2 - \alpha_s^2 \frac{C_{3N} C_{3L}}{C_{2L} C_{3L}} (k_2 - 1) \tag{19}$$

$$= R_{3/2LO}(k_2 - 1)(k_2 - k_3) \tag{20}$$

This term goes to zero as k_2 goes to 1 or when $k_2 = k_3$. When either of these conditions are met, the two approaches for computing $R_{3/2}$ agree at NLO.

3 Results

3.1 Definition of Ratios

The results for both methods will now be compared to data collected at ATLAS and DØ for the $R_{\Delta\phi}$, $R_{\Delta R}$, and $R_{3/2}$ measurements.

The quantity ϕ_{dijet} gives the angle in the azimuthal plane between the two most energetic jets. Due to conservation of momentum requirements, for a strictly dijet event, $\phi_{dijet} = \pi$. [4]However, smaller amounts of radiation may also be produced in the collision, which could lower ϕ_{dijet}, called azimuthal decorrelations. The ratio $R_{\Delta\phi}$ measures the fraction of dijet events that have an azimuthal decorrelation less than some chosen value, $\Delta\phi_{max}$ and is given by:

$$R_{\Delta\phi} = \frac{\sigma_{dijet}(\Delta\phi_{dijet} < \Delta\phi_{max})}{\sigma_{dijet}} \tag{21}$$

for a specified range of rapidity.[1]

The quantity $R_{\Delta R}$ is ensemble average over the number of neighbor jets withing a region of rapidty, ΔR, that a jet with transverse momentum P_T has,

And, $R_{3/2}$ is again defined by:

$$R_{3/2} = \frac{\sigma_{3-jet}}{\sigma_{2-jet}} \tag{22}$$

3.2 $R_{\Delta\phi}$

In all ratio plots, the solid red line is the old method of computing the ratio and the blue dotted line is the new method of computing the ratio. The markers for the below two plots indicate the value of $\Delta\phi_{max}$ for those data points. For rapidity $0 < y < .5$ and $.5 < y < 1$,

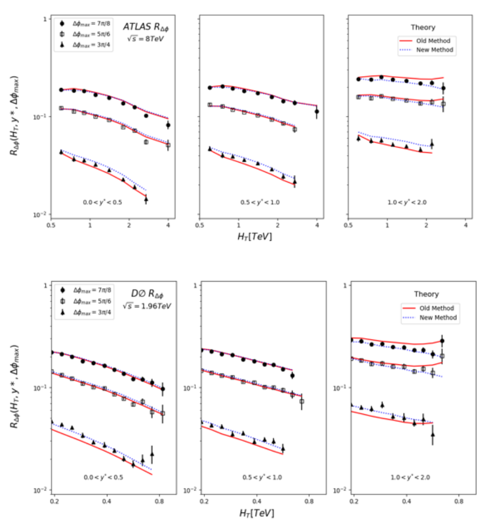

Figure 5: Measurement of $R_{\Delta\phi}(H_T, y^*, \Delta\phi_{max})$ as a function of H_T in three regions of y^* and for three choices of $\Delta\phi_{max}$ compared to theory predictions. Top plot: arXiv 1805.04691 [hep-ex], Bottom plot: arXiv 1212.1842 [hep-ex]

both methods have close agreement, and in general, fit the data. In the $1 < y < 2$ region, in the D∅, the last points have much uncertainty, and the old method trails up to follow it while the new method more accurately fits the overall trend of the data. This data points from this rapidity region were excluded from the data set used to make the determination of α_s because there was not good agreement between the data and theory. However, if the new method had been used as well, the D collaboration may have decided to included these points.

3.3 $R_{\Delta R}$

In the $R_{\Delta R}$ measurement, we see the closest agreement with the two methods. Both describe the data well by visual inspection. The difference between the two methods can be taken as

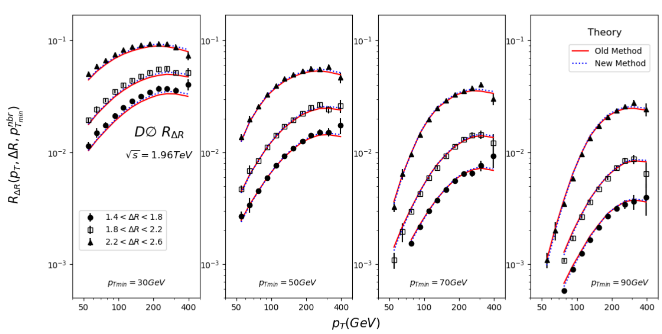

Figure 6: Measurement of $R_{\Delta R}$ as a function of P_T in three regions of ΔR and for four choices of P_{Tmin} compared to theory predictions. arXiv:1207.4957 [hep-ex]

a rough estimate of the amount of theory uncertainty there is for a particular measurement. Because the two methods are not very different from each other in the above plot, this suggests that there is little theory uncertainty for this ratio.

3.4 $R_{3/2}$

The new method for computing ratios performed the worst for the $R_{3/2}$ measurement, particularly for low choices of P_{Tmin}, indicating substantial theory uncertainty.

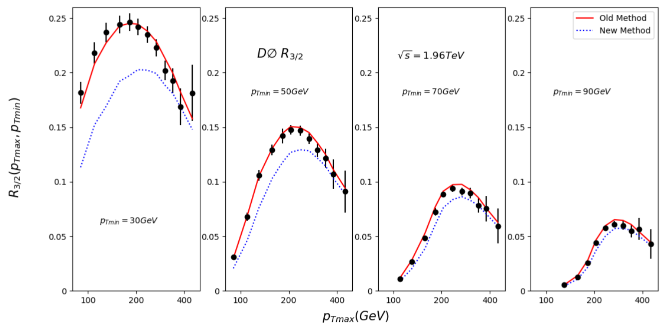

Figure 7: Measurement of $R_{3/2}$ as a function of P_{Tmax} and P_{Tmin} for four choices of P_{Tmin} and compared to theory predictions. arXiv:1209.1140 [hep-ex]

4 Summary

A new method was presented for computing ratios of jet cross sections that consists of taking the perturbative expansion in α_s of a ratio instead of taking the ratio of two perturbative expansions. When both methods are taken to all orders, there are equivalent, but as finite series, their difference can be used as an additional estimate of theory uncertainty for a certain measurement.

5 Future Work

Another determination of α_s, this time with data points that both methods describe, would be one possible interesting continuation of this idea. Another would be to investigate in more detail what is causing the difference between the two methods to be so large for the $R_{3/2}$ data.

References

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