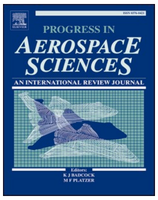
Progress in Aerospace Sciences 120 (2021) 100681

Contents lists available at ScienceDirect

Progress in Aerospace Sciences

journal homepage: http://www.elsevier.com/locate/paerosci



A review of mathematical modelling techniques for advanced rotorcraft configurations

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ARTICLE INFO

Keywords:
Advanced rotorcraft
Flight dynamics
Wake model
Aerodynamic interference
Engine/fuel control system
Manoeuvrability analysis

ABSTRACT

The paper will review the development and application of the mathematical modelling of the advanced rotorcraft configuration, including compound helicopter configurations and tilt-rotor vehicles. The mathematical model is the basis for the design of the flight control system and an essential tool to assess the flying and handling qualities for helicopters. As the helicopter is a multi-body system, the mathematical modelling of helicopter should consider the coupling effects among motion, inertia, structure, and aerodynamics, as well as the unsteady and nonlinear characteristics, to give the physical principles and mathematical expression of each part. Therefore, the mathematical modelling of a helicopter is a process of analysing and synthesizing different hypotheses and subsystem models. Moreover, the advanced helicopter configuration puts forward higher requirements for the helicopter mathematical modelling in terms of the aerodynamic interference, blade motion characteristics, and manoeuvre assessment. The critical issues of helicopter modelling, especially the modelling of the advanced rotorcraft configurations, will be illustrated in this paper. The emphasis is put on the modelling of rotor aerodynamics and aerodynamic interaction among the rotor, fuselage, and other parts. Integrated modelling methods and the manoeuvrability investigation are also the foci of the paper. Suggestions for future research on helicopter flight dynamics modelling are also provided.

1. Introduction – challenges and requirements for rotorcraft mathematical modelling

The aim of helicopter flight dynamics modelling is to construct a correlation among the helicopter motion, the external forces (moments), and the controllers based on the physical laws associated with aerodynamic theory and structural dynamics results. The helicopter flight dynamics model is not only the basis of its control system design but also the primary measurement to develop and analyse the handling qualities feature of the helicopter. The U.S. military handling qualities requirement for rotorcraft (ADS-33F-PRF) is explicitly stipulated that any new rotorcraft should examine the handling qualities with the flight dynamics model in each developing phase [1]. With recent advances in the helicopter industry, a range of advanced rotorcraft configurations have been developed, and the primary types are shown in Fig. 1, namely, the tilt-rotor aircraft, coaxial compound helicopter, and hybrid compound helicopter. These configurations have the capability to further improve the manoeuvrability and performance characteristics, such as the maximum flight speed, flight range, and flight duration. Also, the advanced rotorcraft configurations put forward a higher requirement of the helicopter flight dynamics modelling technique in order to meet the extension of the flight range and the manoeuvrability.

More consideration should be taken in modelling flight dynamics characteristics of helicopters. Considering the conventional single-rotor helicopter, the rotor system has to provide all the force and moments that the helicopter needs, except the yawing moment is provided by the tail rotor [2]. This implies that the lift, the control forces, and the propulsive force are coupled with each other. Furthermore, it should be mentioned that in most of the advanced rotorcraft configurations, such as the tiltrotor aircraft and the hybrid compound helicopter, the yawing moment is provided by a multi-rotor system. Meanwhile, the advanced rotorcraft is usually equipped with an auxiliary propulsion system or tilts the rotor disc to provide the propulsive force in high-speed flight, and the elevator and rudder may also be involved to compensate control power in high-speed flight. There are therefore strong interactions between the rotor system and other components of the helicopter.

Furthermore, the rotor is composed of flexibility blades with high aspect ratio, and it utilises the blade rotational motion to produce lift force [3]. As a result, there are two outstanding features of the rotor

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https://doi.org/10.1016/j.paerosci.2020.100681
Received 24 June 2020; Received in revised form 10 September 2020; Accepted 12 September 2020

Nomenclature

Symbol	Description
$A1C$	Longitudinal cyclic pitch (*Deg*)
$B1C$	Lateral cyclic pitch (*Deg*)
$\tilde{D},\tilde{K}$	Damping and stiffness matrix
K_{re}	Wake curve parameter
K_β	Equivalent flapping spring stiffness ($kg.m^2/(rad.s^2)$)
M_β	Static moment of the blade mass ($kg.m$)
I_β	Inertia moment of the blade ($kg.m^2$)
R	Rotor radius (m)
W_{tip}	Flapping amplitude of the original flapping motion (m)
$W'_{0.75}$	Flapping angle at 0.75R (rad)
V	Forward speed (m/s)
a_0, a_1, a_2	Coning angle, lateral and longitudinal flapping angle (rad)
e	Flapping hinge offset (m)
$\bar{e}$	Non-dimensional flapping hinge offset
$\tilde{f}$	External excitation vector
k	Downwash and side-wash factor
q	Pitching angular velocity (rad/s)
$\bar{q}_v$	Dynamic pressure ratio
t	Time (s)
v_h	Induced velocity at hover state (m/s)
v_i	Induced velocity (m/s)
Ω	Rotor rotation speed (rad/s)
β	Flapping angle (rad)
ζ	Downwash or side-wash effect parameter
$\bar{\omega}_n$	Non-dimensional frequency of the first-order flapping

aerodynamics phenomena. Firstly, a blade element may suffer from non-linear flow phenomena, such as separated flow and shock waves. The resulting stall condition at the advancing tip brings about great difficulty in performing useful aerodynamic analysis. Moreover, due to the development of advanced rotorcraft with increased flight speed there is an increased likelihood of the blade experiencing this stall condition. Secondly, the wake vortex at the blade trailing edge is rapidly rolled up near the rotor tips and formulates the rotor wake led by the tip vortex. In hover and low speed forward flight, the tip vortex is trapped near the rotor disc because of the low flow velocity, leading to severe geometric distortion in the wake. This induces a significant non-uniform component of the inflow on the rotor disc and consequently alters the aerodynamic load and motion of the blade, which further influences the trim characteristics, stability, and control features of the helicopter.

On the other hand, the change of the blade aerodynamic load distribution and motion would, in turn, affect the strength and geometry of the rotor wake vortex. The interaction among rotor wake, blade motion, and blade aerodynamic load forms a dynamic system with high-level coupling. Also, the overall motion of the helicopter and the Coriolis force on the blade change the features of the aerodynamic coupling phenomenon in manoeuvring flight, and this effect will be more significant for the advanced rotorcraft configurations that equipped with the multi-rotor system. In multi-rotor systems each rotor can influence the tip vortex motion and the non-linearity in the associated rotor disc inflow of the other rotors, and therefore alters the aerodynamic load distribution and the vehicle motion.

The aerodynamic interaction between the rotor system and other parts in the helicopter is another critical issue that should be considered in the helicopter flight dynamics modelling process, especially for advanced rotorcraft configurations. As well as the interaction between the rotor system and the horizontal tails, vertical fins, or fuselage, the unique interference for advanced rotorcraft should also be taken into consideration, including the aerodynamic interference between the rotor system and the auxiliary propeller and wing. This aerodynamic interaction has unique characteristics, determined by the rotor flow field and the helicopter configuration. Firstly, the rotor flow field contains various types of aerodynamic features, such as the non-stationary, non-linearity, and a three-dimension effect. These features include the tip vortex structure in the flow field, the vortex-blade interference, the dynamic separation, the periodic transonic motion, and the revolution of the trailing vortex and trapped vortex. Secondly, the rotor system creates a rotational flow, and the multi-rotor system even makes the rotor flow field further distorted. Effectively, aerodynamic interaction could change the rotor performance and the pertinent forces and moments provided by other parts of the helicopter to a large extent. Wind tunnel experiments [4] have demonstrated that the interaction between the rotor system and wing could occupy around 25% of the overall vertical force when the tiltrotor aircraft is in hover state, for example.

Rotor dynamics characteristics are significant aspects that should be carefully considered during the flight dynamics modelling process. The rotor dynamics feature largely influences the flapping, lagging, and torsion motions of the rotor blade, and these blade motions determine the direction of the rotor force and drag, which plays a significant role in determining the flight dynamics characteristics. In addition, advanced rotorcraft has unique rotor dynamics features. The tiltrotor aircraft usually equips with a gimbal rotor system [5], and the rigid rotor is widely utilised in the coaxial compound helicopter configurations. Furthermore, the rotor dynamics have a direct correlation with the rotor rotational speed, and a variable rotor speed strategy is usually involved in the advanced rotorcraft to reduce the compressibility effect at the advancing blade tip at higher flight speeds. Meanwhile, the variable rotor speed feature and the flight dynamics characteristics in large amplitude manoeuvres are coupled with the helicopter turboshaft engine. Thus, with the aim to enhance the accuracy of the flight dynamics modelling, the rotor dynamics and turboshaft engine characteristics should also be taken into account.

The flight dynamics mathematical modelling primarily includes the aerodynamic modelling of rotor, fuselage, horizontal tail, vertical tails, and the potential propulsion device, as well as the coupled rotor/engine/fuel control system dynamics model. The coupling dynamics and

(a) Tiltrotor Aircraft V22 Osprey

(b) Coaxial Compound Helicopter SB-1 DEFIANT

(c) Hybrid Compound Helicopter X^3 helicopter

Fig. 1. The primary advanced rotorcraft configurations.

kinematic model are also needed to capture detailed dynamics features in manoeuvring flight [2]. Considering the structure dynamics, aerodynamics, and the multidisciplinary nature of the interactions present, the mathematical modelling applied should be carefully selected and improved. Meanwhile, the flight dynamics model should be transformed and generalised based on the actual situation using a series of integrating methods.

This article discusses the core issues of the flight dynamics modelling of the advanced helicopter. The foci contain the rotor aerodynamic modelling, the aerodynamic interference modelling, the coupling dynamics modelling of rotor/engine/fuel control, and the helicopter flight dynamic modelling integration. Meanwhile, the manoeuvrability analysis methods for the advanced helicopter will also be discussed in this article. This article puts forward the current status and trends of the flight dynamics modelling development for the advanced helicopter industry.

2. Overview of helicopter flight dynamics modelling

The helicopter flight dynamics mathematical model has been developed from a simple 6 degree-of-freedom (DOFs) rigid-body model to multi-DOFs model. Also, research objectives have been extended from steady flight investigation to large amplitude manoeuvring flight analysis.

The 6 DOFs rigid body flight dynamics model for helicopter derives from the modelling method of the fixed-wing aircraft. This flight dynamics model is obtained by linearization of the 6 DOFs rigid body model. This method is convenient for researchers to get the essential control and stability characteristics of helicopters, and it can be used for the initial design of the helicopter control system [5–10]. The theory and practice process proves that the linear model of the flight dynamics is only suitable for the helicopter with lower manoeuvrability requirements as it fails to take the non-linear effect of the flight dynamics into consideration.

The helicopter flight dynamics characteristics have distinctive nonlinearity due to the aerodynamic coupling among rotor, fuselage, tailplane, and the potential auxiliary propulsion device. Also, the vehicle's motion, structural dynamics, and inertia should be considered in investigating the flight dynamics modelling process [11–15]. Many researchers have focused on the modelling of the non-linear characteristics of the helicopter, and a range of helicopter mathematical models have been developed for theoretical analysis [16–18], numerical simulation [19–23], and real-time simulation [24–28]. Other authors have also contributed to the body of work on helicopter mathematical models [29–32].

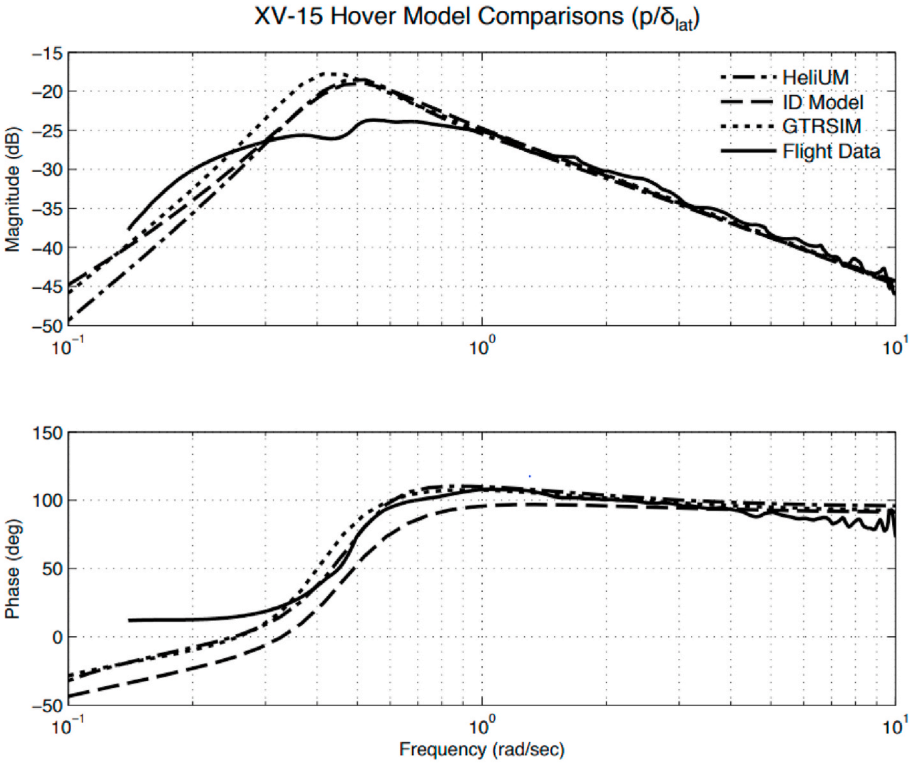
There are two different kinds of helicopter flight dynamics non-linear mathematical model. The first one is to describe the non-linearity using the differential equations of helicopter motion. The linearity and nonlinearity of the sub-system, such as the rotor model and the dynamics model of other parts, are implicitly expressed in this method. This modelling method provides a relatively accurate and reliable foundation for the helicopter flight control system design. The typical example of this model is the ARMCOP model [27,31]. This model is a low-order model, which adopts a static inflow model and utilises the blade element theory to calculate the forces and moments of the rotor system in a periodic average form. In order words, this method has the feature of time-efficiency and is appropriate for the initial design and analysis procedure [19–21].

The second type of modelling technique contains not only nonlinearity in the helicopter motion but also includes the non-linearity in every sub-system. The modelling method is widely used for helicopter flying simulation. The GENHEL simulation package [23] developed by Sikorsky helicopter company is one of the typical examples using this modelling method. It still assumes that the fuselage is a rigid body. However, except for the six-rigid-body DOFs of helicopter motion, this model also has the DOFs of rotor motion, including the flapping, lagging, and torsion motions, as well as the DOFs of rotor rotational motion. Meanwhile, this model utilises empirical equations to fit the torsion motion of the blade. Its rotor aerodynamic model applies the combination of static non-uniform inflow model and the blade element theory in order to calculate the blade aerodynamics. The rotor downwash on other parts of the helicopter and other aerodynamic interference effects are also included based on the experimental and theoretical analysis results. The accuracy of this modelling technique is well understood, and the method has been widely used for various studies, such as the ground numerical simulation [24], non-linear equation parallel processing investigation [25], and high-order linear model simplification [26]. It is worthy of mention that the GENHEL model also includes a representation of the engine/fuel control system so that a variable rotor speed strategy can be simulated with higher accuracy, which is vital for the flight dynamics analysis of more advanced rotorcraft configurations. According to the relevant flight test results [23,26], GENHEL model has relatively high precision in mid-speed forward flight range, but its accuracy in hover and high-speed ranges requires a further improvement due to the lack of the study on the rotor flow field and its aerodynamic interference on the sub-components at this flight range.

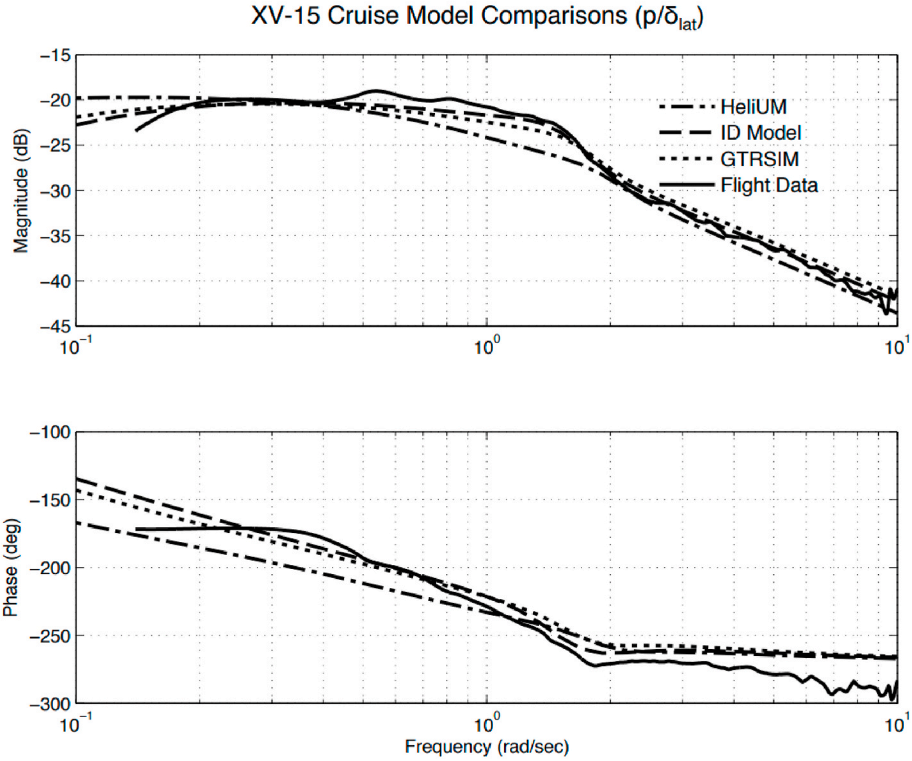
The University of Maryland developed an improved flight dynamics simulation code, HeliUM 2, to overcome the deficiencies in the GENHEL code [33]. It includes flexible rotors and free-vortex wake models to improve its accuracy across the flight range, and it has been expanded to include flexible wings and multi-rotor calculation capabilities. This model allows flight dynamics investigation to be executed for various rotorcraft configurations, from single rotor helicopter to compound helicopters and tilt-rotor aircraft. Fig. 2 indicates the roll rate response comparison in hover and cruise flight state of XV-15 tilt-rotor aircraft. where HeliUM curves represent the results obtained from HeliUM 2 code; the curve marked "ID Model" comes from a state-space model derived from "Flight Data" using system identification method; the "GTRSIM" represents the results from a state-space model derived from the GTRSIM code, which is constructed based on the wind tunnel experiments of the XV-15 tiltrotor aircraft [34]. The cruise state means that the tilt-rotor aircraft is flying in an aeroplane mode with the forward speed of 180 knots, and the hover state indicates the tilt-rotor aircraft is in hover state with the helicopter mode. According to Fig. 2, the HeliUM 2 curves follow the ID Model curves with good agreement, suggesting good accuracy of the HeliUM 2 simulation code for the flight dynamics modelling of tilt-rotor aircraft.

In order to validate the accuracy of the HeliUM 2 model in dealing with the coaxial compound helicopter, the trim results of X2TD helicopter (x2 technology demonstrator, a coaxial compound helicopter) in different forward speeds are shown in Fig. 3. The results derived from the GENHEL code are also provided in Fig. 3, where *A1C* and *B1C* represent the longitudinal and lateral cyclic pitches, respectively. It should be mentioned that due to differences in the propeller modelling method, HeliUM 2 data are not presented in the propeller collective results. Based on the results, the HeliUM 2 software has better accuracy in the longitudinal control comparison, indicating the increased precision in the aerodynamic interference calculation between the rotor wake and other parts. However, the accuracy of the collective pitch is lower than the GENHEL model. This is because the aerodynamic interference inside the coaxial rotor system is significant so that the rotor wake calculation method used in HeliUM 2 (free wake model) may lead to additional errors.

It is possible to tune the flight dynamics model using empirical factors to improve the accuracy in steady and small amplitude manoeuvring flight. However, the aerodynamic characteristics are significant in large amplitude manoeuvres, especially for the advanced rotorcraft due to its multi-rotor system and potential auxiliary propulsion device. Using empirical factors can be difficult to establish an accurate model in such scenarios. Ferguson [36,37] and Yuan [38–40] have investigated the manoeuvrability characteristics of the compound helicopter, but the accuracy of the simulation results is still questionable due to the lack of



(a) Hover state

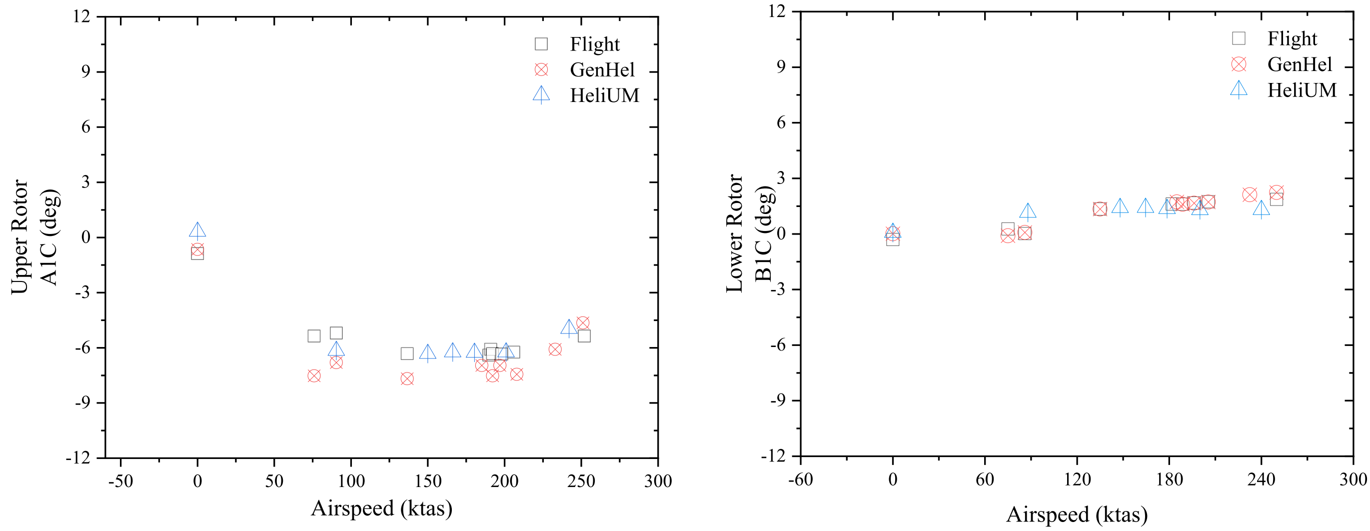


(b) Cruise state

Fig. 2. XV-15 cruise pitch rate response comparison [33].

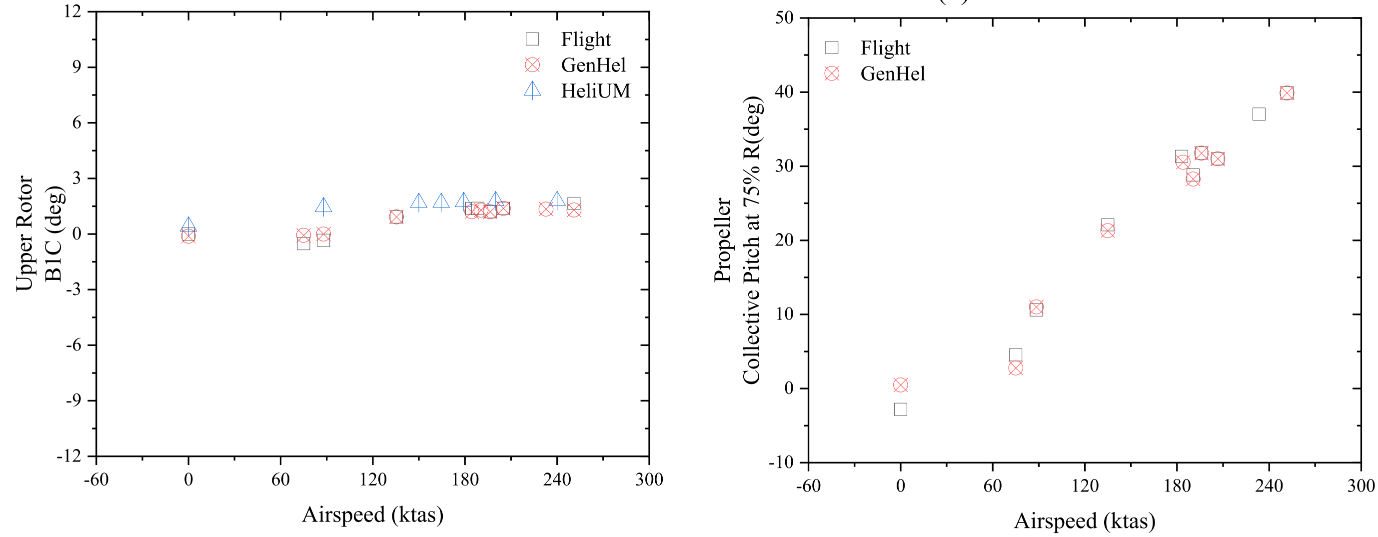
the relevant flight test data. The verified simulation research on the large amplitude manoeuvre is mainly related to the conventional helicopter. The off-axis response simulation results of UH-60 helicopter from different researchers are shown in Fig. 4. In these figures, t is the time, and q represents the pitching angular velocity.

According to these results, a significant error can be found in different modelling methods, indicating that the modelling technique for the large-amplitude manoeuvre still does not replicate the real



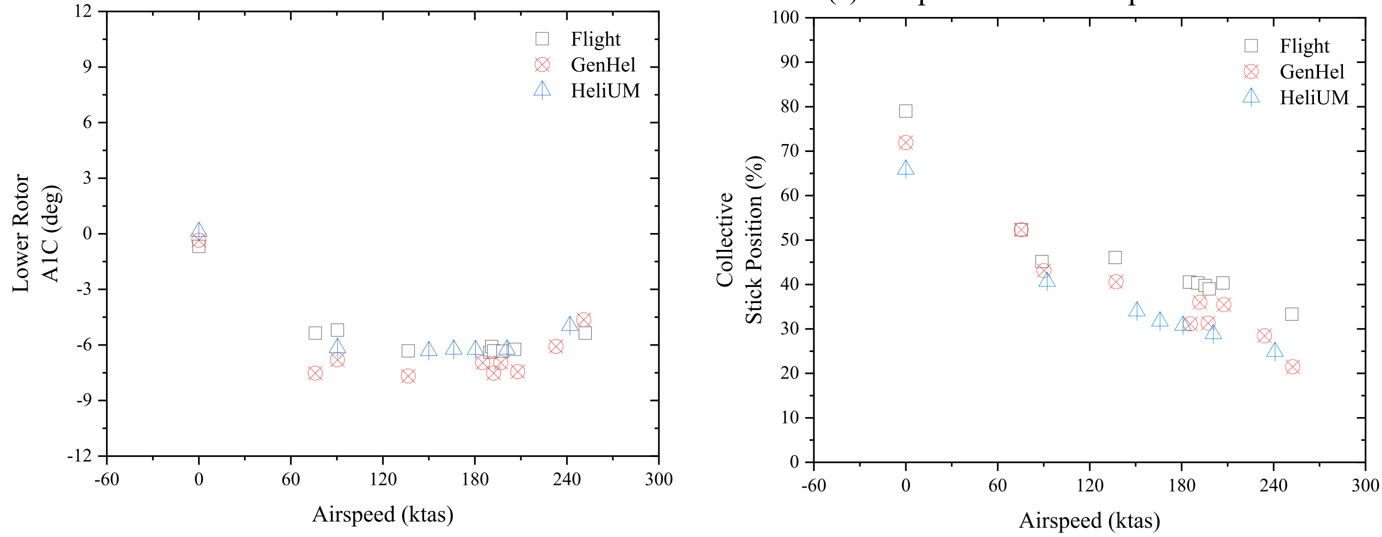
(a) Upper rotor *A1C*

(d) Lower rotor *B1C*



(b) Upper rotor *B1C*

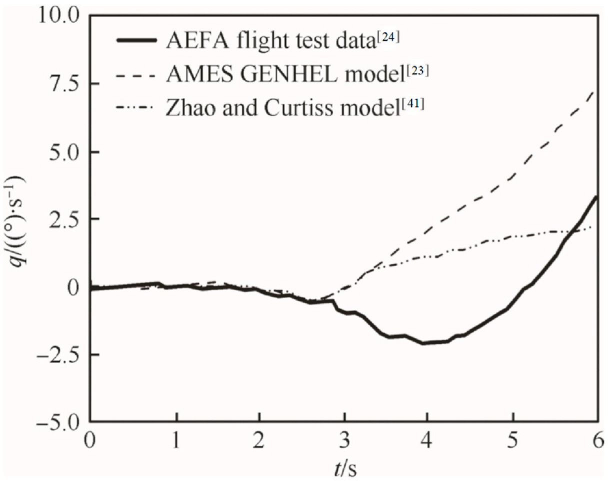
(e) Propeller collective pitch at 75% *R*



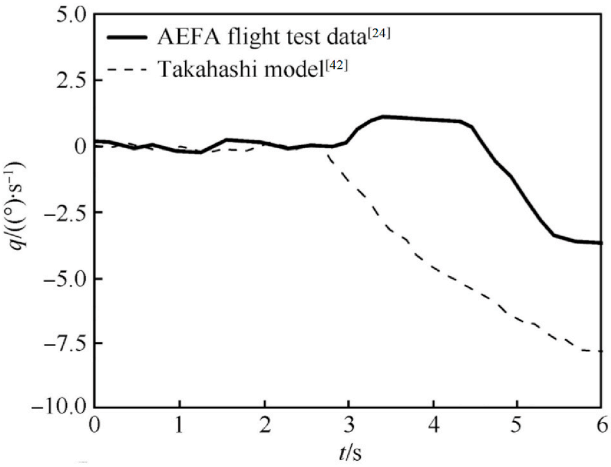
(c) Lower rotor *A1C*

(f) Collective stick position

Fig. 3. Trim comparison of X2TD coaxial compound helicopter [35].



(a) Comparison with Zhao and Curtiss's model, GENHEL's result with AEFRA flight test data



(b) Comparison of Takahashi model with AEFA flight test data

Fig. 4. Large amplitude manoeuvre comparison with different researchers [23,24,41,42].

vehicle. This inaccuracy exists in the different types of helicopter flight dynamics models. The comparison results for BO-105 and AH-64 helicopters also show similar phenomena in large amplitude manoeuvring flight [43,44].

In order to improve the precision of the helicopter flight dynamics model, researchers have investigated this coupling effect with different aspects, including the rotor aerodynamic characteristics, the blade dynamics, the unsteady rotor wake feature, and the 2-D airfoil unsteady aerodynamics characteristics [45–55]. In their rotor blade aerodynamic models, the influence of airfoil aerodynamic unsteady and dynamic stall characteristics are both involved. The blade dynamics model can include elastic blade representations based on the finite element method (FEM), and an improved rotor free wake model can also be used in the helicopter flight dynamic modelling. The relevant results demonstrate that a significant improvement can be seen when these methods are combined [48,54].

With the development of the advanced rotorcraft industry, the importance of developing a universal manoeuvrability assessment method is growing [56,57], particularly for use in assessing vehicle subjective handling qualities features and to improve its flight dynamics characteristics. Researchers have done a great deal of work, trying to apply different types of flight dynamics models for manoeuvrability investigation. In theoretical research, initially energy methods were the primary approach to study the helicopter manoeuvring. This method is based on the theory of energy conservation to calculate the trajectory of helicopter manoeuvring flight. When considering real conditions and specific circumstances, the accuracy of the energy method is relatively low [58–62]. In the late 1980s, the inverse simulation method was developed to analyse the manoeuvrability of the helicopter [63–65]. The inverse simulation method utilises the mathematical description to give the flight trajectory and then obtains the control input during the manoeuvre by inversely calculating the flight dynamics equations. Although this method is successful in the research of helicopter manoeuvring flight and can acquire meaningful results, the inverse simulation method still needs improvement in practical application due to the increasing complexity of the helicopter flight dynamics model. Also, the non-linear optimization method and other manoeuvrability analysis methods have their inefficiency in practice. Therefore, how to investigate the helicopter manoeuvring flight characteristics is still a key issue in the research of the helicopter flight dynamics.

3. Development in the helicopter flight dynamics modelling

Helicopter flight dynamics mathematical modelling involves the rotor aerodynamic modelling, helicopter aerodynamic interference modelling, the engine/fuel control system modelling, and the manoeuvring flight analysis method. This section would illustrate the status and development of the above modelling and analysis methods.

3.1. Rotor aerodynamic modelling

The rotor provides most of the lift, the control power, and the propulsion that the helicopter needs all of which are dependent on aerodynamic characteristics. Meanwhile, the advanced rotorcraft usually adopts the auxiliary propeller to provide the propulsion in the high-speed flight, and its aerodynamic modelling process is similar to the rotor modelling methods.

In modelling the helicopter flight dynamics, the critical aspects of rotor aerodynamic modelling can be concluded in three aspects, which are the airfoil aerodynamic model, the rotor wake model, and the blade dynamics model. These three aspects have mutual effects on each other.

3.1.1. Airfoil aerodynamic model

The airfoil aerodynamic model is the basis of the rotor aerodynamics model, determining the lift and drag of each blade element (airfoil) in different flight ranges. The most simplified airfoil aerodynamic modelling method is to use the 2D lift-curve slope theory to calculate the lift coefficients at the corresponding angle of attack, in conjunction with an empirical equation to calculate the airfoil drag coefficients. This method has been widely utilised to the initial flight dynamics estimation and performance calculation. Its weakness is that it fails to consider unique features of the helicopter airfoil aerodynamics, such as reverse flow and dynamic stall characteristics [66–73]. Therefore, the accuracy of this method is relatively low and cannot be utilised to investigate the flight dynamics characteristics of the advanced helicopter comprehensively.

The airfoil aerodynamic characteristics of the helicopter are quite different from the fixed-wing aircraft due to phenomena of dynamic stall, compressibility effects, reverse flow, and the radial flow effect. These features are summarised in Fig. 5. The main challenge to improve the accuracy of the airfoil aerodynamic model is to determine the appropriate modelling method to simulate these features.

Firstly, the dynamic stall phenomenon is due to the leading-edge vortex, which provides additional suction over the upper airfoil

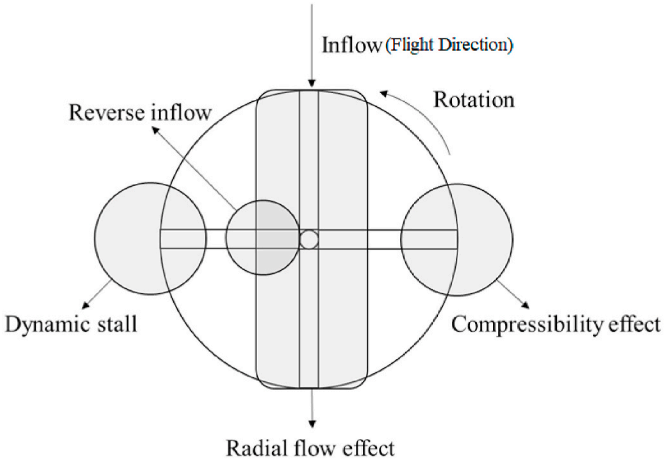


Fig. 5. The features of the airfoil aerodynamic characteristics.

surface as it converts downstream. This increased suction leads to performance gains in lift and stall delay, but it quickly becomes unstable and detaches from the airfoil, leading the lift to decrease rapidly [74–76]. The helicopter rotor usually occurs this phenomenon at the blade tip of the retreating side, and it is more significant in the advanced rotorcraft as the increase of the forward speed exaggerates the flapping motion. It is hard to calculate the dynamic stall effect on a blade section even with a detailed CFD method [77–79], and therefore, a series of empirical or semi-empirical methods were developed based on experimental results, including an unsteady/dynamic stall aerodynamic model constructed by Leishman and Beddoes [80–82]. The calculation process is composed of three parts, the bound flow aerodynamics calculation, the trailing separation estimation, and the dynamic stall (advancing separation) aerodynamics calculation. In the bound flow aerodynamics calculation, an exponential response method is adopted to calculate the airfoil aerodynamics. The pressure on the leading edge is used to estimate airflow separation occurrence based on the correlation between the vertical force coefficient and the critical vertical force coefficient of the airflow separation. The critical coefficient is a function of Mach number, which can be empirically determined according to steady airfoil aerodynamic test data. Also, The French Aerospace Lab (ONERA) developed a 2-D dynamic stall model for airfoil aerodynamics model based on the Hopf bifurcation [83–85]. The lift and moment coefficients are calculated on the basis of non-linear ordinary differential equations, and the relevant parameters in those equations are also based on wind tunnel experiments.

The compressibility effect and reverse flow also influence the airfoil aerodynamics to a large extent. As forward speed increases, the local Mach number at the blade tip of the advancing side is close to the local speed of sound, reducing the aerodynamic efficiency and sharply increasing the drag. Meanwhile, the area of the reverse flow spreads with forward speed. The angle of attack in the reverse flow area is much higher than the stalling incidence, which means that any simple approach to calculating the lift and drag characteristics of the blade section would be inaccurate. The maximum forward speed of the advanced helicopter is much greater than the conventional helicopter, and consequently, the effect of the compressibility and reverse flow have a more significant influence on its flight dynamics characteristics. Therefore, NASA provided the aerodynamic characteristics tables of various airfoil types according to a range of wind tunnel experiments, which demonstrates the aerodynamic lift and drag with different angle of attack (-180°–180°) and Mach number (0.0–1.0) [23]. Thus, the results obtained can be utilised to determine the airfoil aerodynamics in the reverse flow area and the influence of the compressibility effect. Other researchers have combined the airfoil aerodynamics model with a CFD technique to determine these effects and calculate the aerodynamic properties of the airfoil in different flight ranges [86–89], however, the additional time-cost of incorporating a CFD approach would deteriorate the computing efficiency of the flight dynamics model.

Radial flow can influence the airfoil aerodynamics characteristics, especially in the high-speed flight range, suggesting that it should be considered in the advanced high-speed rotorcraft. According to the relevant experiments [90], the radial flow can induce an additional normal force on inboard blade sections due to centrifugal and Coriolis forces and therefore alter the airfoil aerodynamics and flight dynamics characteristics. Thus, the radial flow could delay the stall of the rotor disc to some extent. A series of correction methods have been put forward to simulate the radial flow [90,91], but the parameters in the correction equations have to be determined by the relevant experiments or appropriate CFD technique. Breton [91] utilised the lifting-line-prescribed wake vortex scheme to calculate this effect and used a wind tunnel experiment to verify this method. The calculation results share a similar trend with the experimental results, indicating that it can be adopted in flight dynamics models to improve their accuracy.

In conclusion, different methods have been developed to enhance the precision of the airfoil aerodynamics model. Nevertheless, most of these methods have to rely on the relevant experiments or CFD calculations.

3.1.2. Rotor wake model

The induced velocity is determined by the rotor wake model, which alters the rotor aerodynamics and the flight dynamics characteristics. In addition, advanced rotorcraft usually utilises a multi-rotor system, and therefore, the aerodynamic interaction between rotor discs plays a significant effect on the induced velocity on each disc and consequently changes the flight dynamics features of the advanced rotorcraft. Therefore, with the aim to accurately simulate the flight dynamics characteristics of the advanced rotorcraft, the interaction in the multi-rotor system should also be considered in the rotor wake model.

There are two important requirements for the rotor wake model: accuracy and computing efficiency. The accuracy of the rotor wake model strongly influences the validity of the flight dynamics model, and the computing efficiency defines the capability of the flight dynamics model for analysis in practical circumstances. High time cost limits the ability of the flight dynamics model to investigate the control response and handling qualities.

Different types of wake models have been developed [92], such as the most straightforward rotor disc uniform induced velocity model [93–97], the finite-state inflow [98–101], the fixed wake method [102–108], the free wake model [109–113], and higher-resolution wake models developed [114–121] recently.

The uniform induced velocity model is a simplified rotor wake model with high computing efficiency, which is based on the relationship between the induced velocity and the aerodynamic loading on the rotor disc [92,95]. This model can be regarded as a particular wake model derived from the momentum theory. However, the feasibility of this method is limited. When the helicopter is in forward flight, the wake will tilt backwards, and the induced velocity distribution becomes non-uniform, reducing the accuracy of the uniform induced velocity model. Thus, Coleman at [93] built a linear induced velocity model with the wake correction, enabling the method to simulate the induced velocity distribution in the mid to high speed forward flight. However, the accuracy of this model is reduced in low-speed forward flight due to the wake distortion. In addition, these induced velocity models could only be utilised for the single rotor system, and cannot consider the aerodynamic interaction in the multi-rotor system, which means it cannot be directly used for the most of the advanced helicopter configurations.

Carpenter and Friedovich [94] expanded the momentum theory to the dynamic inflow model, in which the additional inertia effect produced by the disturbance on the rotor disc is considered. This dynamic inflow model has the capability to simulate the dynamic change of the induced velocity, and this induced velocity is given in the form of first-order ordinary differential equations. Pitt, Peters, and He constructed the Pitt-Peters dynamic inflow model [95] and its generalised form (Pitt-He finite-state (high order) inflow model) [96,97] according to the acceleration potential theory. This wake model has been widely used for the flight dynamics analysis and control response calculation. Nevertheless, the prerequisite of these models is that the inflow velocity should be much more than the induced velocity on the rotor disc. Thus, the dynamic inflow model only can be used for rotors with low loading or when the helicopter is in mid to high speed forward flight. In addition, Ferguson [37] utilised a revised dynamic inflow model to investigate the flight dynamics characteristics of the coaxial compound helicopter. However, an empirical correction has to be applied to modify the parameters in the dynamic inflow model, hindering its further utilisation.

Peters further developed the finite state inflow method based on the Galerkin treatment of the potential flow equations, allowing this method to compute induced flow everywhere in the flow field [98–101], suggesting that it could be used to determine the aerodynamic interaction in the multi-rotor system. According to the comparison against flight tests, the obtained result gives acceptable precision for flight dynamics analysis so that this rotor wake model has the potential to be adopted into the flight dynamics modelling of the advanced rotorcraft, such as the coaxial compound helicopter and the tilt-rotor aircraft. However, this method is only verified in the hover state of the coaxial helicopter configuration [99], and the numerical convergence of this method is also a significant impediment for its further development.

In order to develop a generalised rotor wake model to accurately simulate the non-linearity of the induced velocity on the rotor disc throughout the flight range, as well as calculate the aerodynamic interaction in the multi-rotor system, Barocela [102], Krothapalli [103], Zhao [104], Rosen and Isser [105,106], and Keller [107,108] put forward different wake distortion models using the pre-scheduled curvature method. They defined a parameter K_{re} for the wake curve to reflect the proportional relationship between the rotor induced velocity gradient and the wake curvature. However, the parameter of K_{re} must change along with the forward speed to ensure its accuracy at different flight ranges. Bhagwat [109] constructed the correlation between K_{re} and the forward speed, rotor angular acceleration, and rotor thrust, enhancing the feasibility of the method. However, the pre-scheduled wake distortion method still fails to fully reflect the distortion of the rotor wake geometry and cannot take the aerodynamic interaction of the multi-rotor system into account. Thus, it cannot be used to precisely simulate the flight dynamics characteristics of the advanced rotorcraft.

The free-wake model is another approach to calculate the rotor wake of the helicopter, which was developed based on the rotor vortex theory [110], and this model solves for the rotor wake geometry directly, and in principle do not require experimental data for formulation purposes. In this method, the wake system was usually decomposed into two main parts. Firstly, a near wake of trailed and shed vorticity behind each blade and second, a far wake comprising the rolled-up tip vortices from the blade. Then, the numerical solution to the free-wake problem can be described by the integration of a system of ordinary differential equations. These are obtained after the spatial discretisation of a series of partial differential equations that govern the positions of the tip vortices. A set of collocation points are specified on the trailed vortex filaments, and these points are numerically converted through the flow field at the local velocity. The curved tip vortices generated by the blades are usually divided into a number of smaller straight-line segments. The local velocities at each collocation point on the vortex filament are then calculated by the application of the Biot-Savart law. Thus, this method allows the vortex element to move with the local airflow velocity and can automatically simulate the self-induction and distortion of the wake. The induced velocity vector at anywhere in the flow field can be obtained using this method. The free-wake method can be used for advanced rotorcraft modelling as it can not only calculate the aerodynamic interaction in the multi-rotor system but also be able of capturing the effect of wake distortion during flight. In its early development, the explicit Euler time marching method was widely utilised, however, its numerical stability is relatively weak. To solve this numerical instability, two numerical methods have been put forward. The first method is to introduce a constraint of the periodic condition during the renew process in each time step, referred to as the classical relaxation free wake method. The second method is to combine the forecast-correction method with the high order time marching format to reduce the numerical oscillation, referred to as the time-accuracy free wake method.

The line vortex discrete embedded free wake model is a relatively mature method for the flight dynamics modelling of helicopter. This method could not only guarantee the precision but also reduce the overall time cost. Meanwhile, this method shows a significant efficiency advantage in computing the aerodynamic interference between rotors, which is essential for the advanced rotorcraft modelling process. However, the line vortex discrete embedded free wake model is based on the potential flow theory, excluding the viscous effects. In order to improve the accuracy of this method, researchers applied the empirical coefficients into the vortex core model, and the position of the tip vortex distortion are used to include the effect of viscosity. However, these empirical coefficients impede its applicability for the manoeuvrability and control response analysis. Lee and Na [111,112] constructed a new rotor wake method with vortex blob method, successfully solving the problem of the numerical convergence. However, its low calculation efficiency in tackling the self-induced velocity limits its further development and utilisation for the flight dynamics modelling of the advanced rotorcraft.

Yuan, Chen, and Li developed a multi-transmutable-vortex-ring (MTVR) wake model based on the rotor disc assumption and fixed wake theory [113]. The induced velocity everywhere in the flow field is attainable with this rotor wake model, and its computing efficiency is much enhanced, making it able to achieve the real-time requirement. The validation results indicate that this model can accurately simulate the induced velocity distribution of the coaxial compound helicopter in different forward speeds. However, the wake variables of this method are extensive and that leads to difficulty in the convergence during the calculation.

In recent years, with the development of the fast multipole method (FMM) [114], many researchers tried to combine this method with high-resolution general vortex method to construct the rotor wake model. Brown [115,116] firstly built the vortex transport method (VTM) method for the high accuracy rotor wake calculation according to the finite volume method. He and Zhao [117,118] constructed the viscous vortex particle method (VVPM) for high precision rotor wake estimation. These methods not only inherit the advantage of non-viscous free wake method but also consider the effect of the viscous dissipation and the wake geometry alteration on the rotor aerodynamic characteristics. However, these methods usually utilise the lift-line or lift-surface model due to the convergence requirement, reducing the accuracy in calculating the airflow characteristics around the blade. The rotor CFD method is widely utilised to accurately simulate the rotor wake influence, and the effects of the airflow separation, dynamic stalling, and the shock wave on the rotor wake are all considered in this method [119]. This method usually suffers from numerical dissipation problems, leading to over fast attenuation on the vorticity. For this purpose, some researchers combined the wake method with a CFD method to develop a high precision wake calculation approach [120,121]. A CFD method is utilised to capture the flow field detail characteristics, and the rotor wake model is used to calculate the wake viscous dissipation and topological structural change. Therefore, this method can obtain the induced velocity in the flow field and be used to accurately construct the flight dynamics model for advanced rotorcraft, especially the simulation of the aerodynamic interaction in multi-rotor systems. Nevertheless, the time cost of this method is extremely high, decreasing the computing efficiency of the flight dynamics model.

3.1.3. Blade dynamics model

The blade motion consists of three parts: flapping motion, lagging motion, and torsion motion (pitching motion). The flapping motion is the most important component to helicopter flight dynamics modelling as it decides the control power and propulsion of the rotor system. The blade motion of the advanced rotorcraft is usually different from the conventional helicopter in order to improve its performance. Coaxial compound helicopters, such as X2TD and SB-1 helicopter, utilise what is referred to as a rigid rotor to delay the dynamic stall phenomenon in high-speed flight range [122–126]. The tilt-rotor aircraft adopts gimbal rotors to deal with the aeroelastic instability problem in high-speed aeroplane mode [127]. These features significantly change the blade motion characteristics and consequently alters the flight dynamics of the advanced helicopter.

In steady flight, the periodic characteristics of the blades' aerodynamics are similar as their motion trajectories are same, and the tip planes of different rotor blades maintain in the same shape. Thus, the coning angle, longitudinal flapping angle, and the lateral flapping angle can be used to fully describe the flapping motion of the blades. This description and modelling method are regarded as rotor plane method. The orientation of the aerodynamic forces and the effect on the vehicle motion can be easily determined using this method. However, the aerodynamic and inertia force on each blade will be different in large amplitude manoeuvring flight. Meanwhile, the turbulent environment could also make the trajectories of rotor blades located in different planes [50]· Consequently, the trajectories of different rotor blades are no longer kept in the same plane so that the accuracy of the rotor plane method is reduced. In order to improve the accuracy of the blade motion calculation for manoeuvring flight, two different approaches have been developed.

The first method is to assume that the rotor blades still have the same dynamic trajectory, and it can be described as [17]:

$$\begin{bmatrix} \ddot{a}_0 \\ \ddot{a}_1 \\ \ddot{a}_2 \end{bmatrix} + \tilde{D} \begin{bmatrix} \dot{a}_0 \\ \dot{a}_1 \\ \dot{a}_2 \end{bmatrix} + \tilde{K} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \tilde{f} \tag{1}$$

where: a_0, a_1, and a_2 represent the coning angle, lateral flapping angle, and longitudinal flapping angle; $\tilde{D}$, $\tilde{K}$, and $\tilde{f}$ denote the damping matrix, stiffness matrix, and external excitation vector, respectively. This approach only takes the dynamic change of the rotor disc into account, indicating that it is only suitable for the small-to-moderate amplitude manoeuvring flight.

The second method abandons the hypothesis that each blade should be kept in the same plane and separately investigates the flapping motion of each blade in rotational coordinates [23]. Compared with the first method, this approach can sufficiently capture the Coriolis force derived from the vehicle angular motion and the inertia force from manoeuvring flight. It should be mentioned that this method is also appropriate for the blade motion modelling in the steady flight. The path of each blade is similar to the others, and the trajectory would be therefore back to the same rotor plane.

The blade motion has a direct relationship with the design of the rotor hub. In terms of a rotor modelled by a centrally located flapping hinge, the first order flapping frequency of the blade is the same as the rotor rotational speed, which makes it easy to model. The design of the rotor hub becomes unique in some helicopter configurations, such as the gimbal and high-rigidity rotor hub design. These types of rotor hub alter the flapping frequency and blade motion characteristics and consequently change the control and stability characteristics of the helicopter. With the aim of taking these effects into consideration and maintaining the computing efficiency, the blade motion can be simplified by modelling the flapping dynamics by the simplification shown in Fig. 6, in which e is the flapping hinge offset, K_β is the stiffness of equivalent flapping spring, and β is the flapping angle.

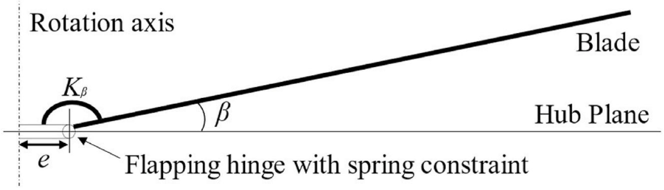


Fig. 6. Equivalent articulated or hingeless rotor [17].

In this simplified method, the equivalent flapping spring is used to adjust the flapping frequency in order to ensure it matches that of the real rotor. This simplified method is appropriate for rotors with the flapping frequency less than 1.1 Ω, where Ω is the rotor rotational speed.

The gimbal rotor hub of the tilt-rotor aircraft can be simulated using this method [2], however, there are some other advanced rotorcraft configurations equipped with highly rigid rotor blades where the flapping frequency can be more than 1.4 Ω. In order to simulate the blade motion of the rigid rotor, there are two requirements that should be met to guarantee accuracy: the first one is to ensure the flapping frequency remains the same before and after the equivalent system is defined; secondly, the flapping mode after the equivalence should be as similar as possible compared with the original flapping mode. The equivalent model of the rigid blade flapping motion is shown in Fig. 7, which can satisfy the conditions discussed above.

The non-dimensional equivalent flapping offset is calculated using the equation shown below to guarantee the similar flapping mode

$$\bar{e} = 1 - \frac{W_{tip}}{R \cdot W'_{0.75}} \tag{2}$$

where: W_{tip} is the flapping amplitude of the original flapping motion; R is the rotor radius; $W'_{0.75}$ denotes the flapping angle at 0.75 R.

To keep the flapping frequency, the additional flapping constraint spring is needed, and its stiffness can be expressed as [3].

$$K_\beta = \left(\bar{\omega}_n^2 - 1 - \frac{\bar{e} R M_\beta}{I_\beta} \right) I_\beta \Omega^2 \tag{3}$$

where: $\bar{\omega}_n$ is the non-dimensional flapping frequency of the first-order flapping; M_β is the static moment of the blade mass; I_β is the blade inertia moment.

This method assumes the blade is rigid in the flapping motion, which is the mainstream approach in the helicopter flight dynamics modelling as it would simplify the calculation process and improve the computing efficiency. With the development of the advanced rotorcraft, there are higher requirements for rotor motion modelling. The combination of an elastic blade model with a helicopter flight dynamics model has drawn growing attention due to the improvement in precision it provides [129–133]. The Finite-Element-Method (FEM) embedded elastic model can represent the elastic deformation of the coupling between blade flapping, lagging, and torsion motion, which further improves the precision in the blade model, especially for advanced rotorcraft equipped with the rigid rotors. Duval, He, and Turnour and Celi [[128,129, 129–131]] constructed different flight dynamics models with this elastic

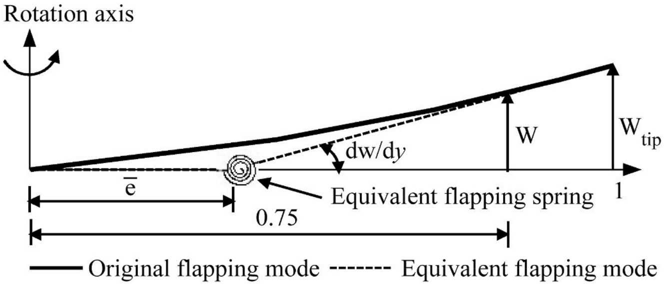


Fig. 7. Equivalent method for rigid blade flapping [128].

blade motion approach, and they pointed out that the elastic blade motion model could efficiently improve the precision of the helicopter in off-axis control response. References [132,133] combined the FEM method with advanced wake model and airfoil unsteady/dynamic stall model, suggesting that the accuracy is further improved when calculating the rotor loading in different flight ranges. The FEM incorporated an elastic blade model is a powerful approach for the advanced helicopter flight dynamics modelling to enhance the calculation accuracy in the high speed and manoeuvring flight. However, the additional time cost brought by the FEM method and other elastic blade motion models reduces the time efficiency of the flight dynamics model.

3.2. Model of aerodynamic interference among components

The aerodynamic interference among helicopter components is the most challenging feature to capture in a flight dynamics model. The interference among the rotor system, fuselage, horizontal and vertical tails alters the flow field and pressure distribution on each component, influencing the resultant force and moment. In addition, the aerodynamic interference becomes more extensive for advanced rotorcraft. As well as the additional force and moment due to the interference, advanced rotorcraft configurations usually experience other resulting problems. For example, the rotor wake changes the aerodynamic characteristics of the horizontal and vertical tails, and consequently control power of the elevator and rudder are altered. Further, for tilt-rotor aircraft aerodynamic interference influences the lift-to-drag ratio of the wing and affects the performance characteristics. Coaxial compound helicopters have different wake features due to the interference between the coaxial rotor system, which affects the aerodynamic characteristics of other parts of the helicopter significantly. The hybrid compound helicopter has two auxiliary propellers situated at each side of the wing, and their wakes will couple with the rotor wake, which leads to a significant wake effect on the vertical and horizontal tail. Also, poor propeller inefficiency may occur in the hybrid compound helicopter when the forward speed equals to the induced velocity of the propeller. Therefore, it is worth exploring aerodynamic interference among the helicopter components in more detail and detailing modelling methods to capture their effects.

Fig. 8 is a schematic diagram of the aerodynamic interference between rotor and horizontal tail when the helicopter is flying from hover to forward flight [134]. The rotor wake sweeps backwards and impacts the horizontal tails to produce the nose-up moment as the forward speed increases. Then, as the forward speed further increases, the rotor wake sweeps upward missing the horizontal tails area, which then returns to providing a nose-down moment. This phenomenon alters the trim characteristics of the helicopter and may induce the helicopter dynamic instability [27]. Also, the wake interference also changes the inter-axis coupling effect and consequently damages the handling qualities of the helicopter.

This interference of the rotor wake on the vehicle not only degrades the handling qualities of the helicopter but could also damage the helicopter during flight. During the development of the AH-64 helicopter, there were a number of horizontal tail redesigns to avoid rotor wake effect on the horizontal tail, and this was the reason for an accident during the flight test programme. Finally, manufacturers have had to change the location of the horizontal tail and adopt an all-moving horizontal tail to meet the handing qualities requirement [135], as shown in Fig. 9. Interference induced vibration occurred in the YUH-61 helicopter owing to the lower distance between the rotor and fuselage [136]. This led the Boeing Company to start a programme called UTTAS for seven years to research the internal mechanism between the aerodynamic interference. The programme included a large number of experiments [58,137], which produced a large volume of test data. This programme has pushed forward the research on the aerodynamic interference for helicopter development to a large extent.

The downwash and side wash effects of the rotor on other parts of the vehicle mainly influences the dynamic pressure, angle of attack and sideslip angle at the helicopter sub-components. Issues caused by rotor wake instability and time-varying characteristics can be very difficult to describe analytically within a model, and so data from wind tunnel experiments are widely used to predict this influence. Fig. 10 shows wind tunnel experimental results for dynamic pressure on the tailplane of YUH-61A helicopter [8]; where $\bar{q}_v$is the dynamic pressure ratio (the difference between the local pressure and the free flow pressure divided by the free-flowing pressure), and v_h is the induced velocity in hover state. As shown in Fig. 10, the horizontal tail is affected by the rotor wake, increasing its dynamic pressure above the pressure in the free-flow.

The downwash and side-wash caused by the rotor wake can be expressed as follow

$$\zeta = k\left(\frac{v_i}{V}\right) \tag{4}$$

where v_i represents the induced velocity; k denotes downwash or side-wash factor caused by rotor wake, which is determined by the rotor wake skewing angle and its relative position from the rotor hub. In practice, the factor k is obtained by the relevant wind tunnel experiments or numerical calculation techniques, such as CFD.

The correlation between the downwash factor k and the rotor wake skewing angle is illustrated in Fig. 11, demonstrating that the downwash factor increases with the rotor wake skewing angle. It also indicates that the effect of the rotor wake on the horizontal tail diminishes as forward speed increases. On the other hand, the downwash factor is close to 2.0 when the rotor wake skewing angle is 90°. In other words, the downwash velocity is twice that on the rotor disc in hover state, which is in line with the result derived from momentum theory.

Based on the experimental results mentioned above, the rotor wake effect is dependent on the flight status, the configuration, and the rotor

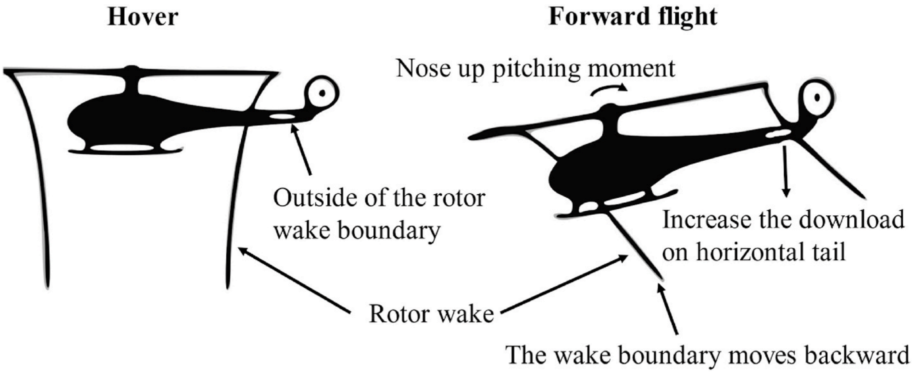


Fig. 8. Aerodynamic interference of rotor wake with horizontal tail.

Fig. 9. The horizontal tail before and after the redesign in AH-64 helicopter [12].

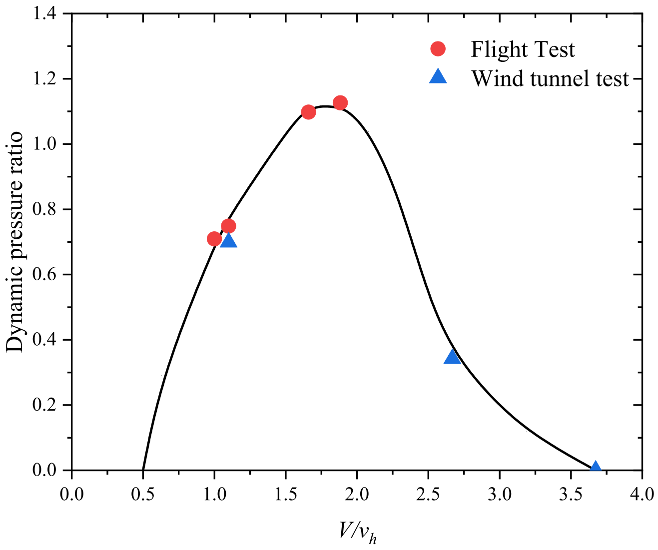


Fig. 10. Dynamic pressure ratio at the horizontal tail [8].

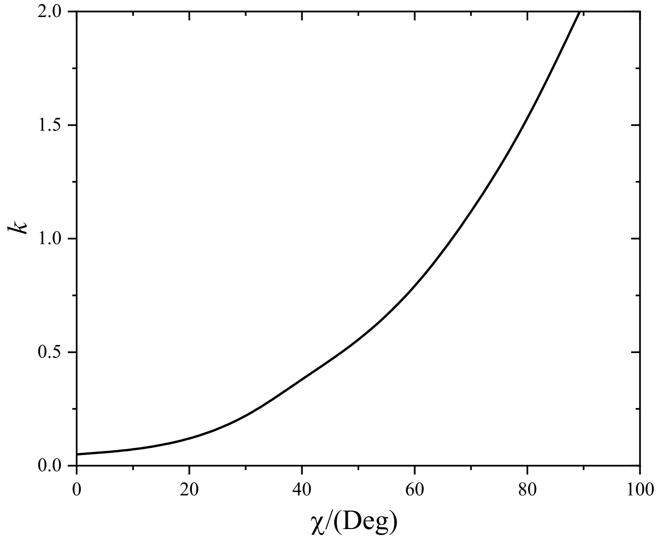


Fig. 11. Relationship between rotor downwash factor and wake angle [9].

design. During the modelling process, it is a significant challenge to determine which parts of the helicopter are affected by the rotor wake and what additional influence there is due to this interference. Moreover, the dynamic motion of the rotor wake also occurs during manoeuvring flight, enlarging the effects of aerodynamic interference. Both features should be considered during the modelling process.

In the 1980s–1990s, researchers started to implement theoretical studies to simulate aerodynamic interference with the developmental free wake research [138–145]. A theoretical prediction model of the rotor/fuselage aerodynamics was developed to determine the unsteady aerodynamic loading on the rotor and fuselage, including the effects of aerodynamic interference.

In order to validate the accuracy of this model, Georgia Institute of Technology [146–149] and the University of Maryland [150,151] separately built rotor/fuselage combined experimental model from 1989 to 1991. In these experiments, the flow field and the pressure distribution around the fuselage in different conditions were measured. These results have become the validation baseline for subsequent theoretical analysis methods [54,104].

With the breakthrough in the numerical stability of unsteady rotor free wake methods [109,152], the theoretical prediction model is relatively straightforward to use in helicopter flight dynamics modelling. Horn [46] and Ribera [48] combined the time-precision free wake analysis method with an existing flight dynamics model. Wachspress [153] introduced the time-precision free wake model into the rotorcraft simulation process, and validations illustrate that the free wake model could improve the accuracy of the predictions of helicopter aerodynamic characteristics, especially for the aerodynamic interference derived from the rotor wake. D′ Andrea [49] adopted the time-precision free wake model with unstructured surface element grid method and developed the ADPANEL method for the aerodynamic interference analysis.

Recently, with the development of the advanced rotorcraft, a set of research has been put forward towards the aerodynamic interference of the tiltrotor aircraft and compound helicopter.

The main focus in the aerodynamic interference of tiltrotor aircraft is the interference between the rotor system and the wing. This interference determines the flight dynamics characteristics and performance characteristics of the aircraft, and could be changed with the nacelle incidence angle, flight speed, and other flight states. Yeo [154] calculated the aerodynamic interference effect on the performance characteristics of the tiltrotor aircraft with CFD/CSD coupled method on the CAMRAD II platform. Based on the analysis results, the aerodynamic interference effect improves the aircraft lift-to-drag ratio, and the interference velocities reduce the total induced velocity along the wingspan and, thus, reduce wing induced power. Jung [155] investigated the aerodynamic interference between the rotor system the wing with different sideslip angles and nacelle incidence angles based on a CFD flow solver. The results indicated that aerodynamic interference magnifies the fluctuating amplitudes of the yaw and roll moments with the increase of the sideslip angle, and the aerodynamic interference is more significant when the nacelle is tilted forward. These results provide a deeper insight into the aerodynamic effect of the tiltrotor aircraft; however, the time cost of these calculation process is extremely high. Thus, the empirical factors are widely used in the current flight dynamics model [156–158], especially the model that needs to achieve the real-time requirement, to deal with the aerodynamic interference in the tiltrotor aircraft.

There are different types of aerodynamic interference inside the compound helicopter that play a major effect on its flight dynamics characteristics. Apart from the rotor-tail surface interference, the rotor-wing interaction and the rotor wake effect on the propeller also contribute to the flight dynamics features and performance characteristics. Yeo [159] investigated the performance characteristics of different compound helicopter based on CAMARD II. The results indicated that rotor/wing interference effects are examined for a compound helicopter at a high-speed cruise flight condition where the rotor carries about 7% of the gross weight and the wing carries about 93% of the gross weight. The interference velocity on the rotor is relatively large due to the large wing lift compared to small rotor induced velocity. However, interference power is very small in cruise. Stokkermans [160] utilised the unsteady CFD simulation technique to investigate the installation effects of the latera rotors for a hybrid compound helicopter featured a box-wing design. The results suggested that the main interaction in cruise was between the wing and lateral rotors, resulting in a propulsive efficiency increase up to 10.6% due to wingtip vortex energy recovery. In hover the main rotor slipstream resulted in a near perpendicular inflow to the lateral rotors, with a disturbance from the wings due to the deflection of the main rotor slipstream. Although the high-accuracy results can be obtained from these CFD solvers, their time cost is still significant and consequently cannot be directly adopted into the flight dynamics modelling. On the other hand, Ye applied the MTVR wake model into an aerodynamic interference calculation and utilised the flight test data to verify its accuracy [113]. The comparison indicates that this method could capture the primary influence of the rotor wake on the other parts of the helicopter, and the trim calculations follow the flight test results with good accordance. Additionally, the computing efficiency of the MTVR model is better than the free wake model and can be used to the control response simulation and manoeuvrability analysis.

3.3. Engine/fuel control system modelling

The engine has a significant coupling effect on the flight dynamics characteristics of the helicopter [161,162]. During the steady flight, the flight state of the helicopter is relatively constant so that the power requirement and the power output of the engine roughly remain constant. However, when the helicopter is in manoeuvring flight or experiencing atmospheric disturbances, the pilot needs to keep changing the control input of the helicopter and the airflow around the helicopter is therefore influenced. Thus, helicopter flight dynamics characteristics and the associated engine power output also vary to a large extent. Therefore, the dynamic characteristics of the engine must be taken into consideration as additional lag or overshoot effect may occur due to the engine characteristics, and consequently influence the flight dynamics and handling qualities of the helicopter [162]. Moreover, advanced rotorcraft configurations usually employ multi-rotor systems or rotor-propeller combination systems, further complicating the formation of the power output and engine/fuel control system modelling process.

There is significant research into engine dynamics characteristics independent from helicopter flight dynamics studies. Research relating to the rotorcraft engine dynamics usually assumes that the power output of the engine and the rotorspeed are invariable and fails to consider the coupling effect between the rotor dynamics and engine. Ballin built a real-time simulation model of the T-700 turboshaft engine taking into account the aerodynamic and thermodynamic characteristics of the engine [163]. Then, Ahmet utilised an identification method to construct a simplified linear model of the T-700 turboshaft engine on the basis of the Ballin model [164]. As mentioned above, the dynamics characteristics of the engine are excluded in many helicopter flight dynamics studies, and it is often assumed that the power required can be met instantly by the engine. For example, the ARMCOP model, widely used for rotor system design and analysis, ignores the dynamic influence of the engine [165].

Many researchers have tried to improve vehicle modelling accuracy by incorporating the dynamic effect of the turboshaft engine. Talbot put forward a simplified engine/fuel control system model for helicopter flight dynamics investigation [166]. This model utilised a second-order transfer function to represent the engine dynamics effect, including the compressor, throttle control, power turbine, and fuel control. This method is simple in structure and easily adopted in flight dynamics models. However, the detailed response of the engine is neglected in the simplified transfer function. The GENHEL helicopter simulation package included an engine model of the T-700 turboshaft engine and its fuel control system. This helicopter rotor/engine integrated model enhanced the accuracy of the flight dynamics analysis in manoeuvring flight. The engine model in the GENHEL package ignores the inlet pressure of the engine turbine, which reduces its precision during the large amplitude manoeuvre [167,168].

A specific schematic modelling method was developed for the T-53 turboshaft engine used in the XV-15 tilt-rotor aircraft [169]. The model is composed of equations to calculate engine horsepower during transient and steady-state based on the operating characteristics of the combined engine-fuel control system. The model includes the dynamic effect of the engine with more detail and ensures the real-time requirement for the associated flight dynamics can be met. However, the parameters in this model are determined by many experiments and can only be suitable for a specific combination of engine and helicopter type. In addition, Cranfield University developed a gas turbine performance simulation code called TURBOMATCH [170–172]. It is a long-standing and validated tool of the engine suitable for both steady and transient conditions. Researchers have used it for flight dynamics modelling and analysis of the advanced coaxial rotor configuration, such as the X2TD helicopter [172].

3.4. The integration and calculation of the non-linear systems

Helicopters experience a wide range of nonlinear effects which produces unique dynamic characteristics. The calculation and integration of this non-linear system is a great challenge in the flight dynamics modelling process. Also, the additional components present in advanced rotorcraft put forward the higher requirement for this integration and calculation procedure.

First, the vehicle body motion of the helicopter features low-frequency characteristics and strong coupling, and therefore, the governing differential equations describing the helicopter motion are non-linear. Furthermore, aerodynamic interference plays a significant effect on these non-linear characteristics. The downwash or side wash of the rotor wake causes the aerodynamics on the fuselage, horizontal tail, and vertical tail to be discontinuous, bringing about further problems in the calculation process.

Second, the flight dynamics model needs to incorporate a rotor wake model to capture the aerodynamic interference. However, current rotor wake models still suffer from the problems of numerical instability and calculation inefficiency. The combination of the flight dynamics model and the discrete rotor wake model exaggerates this effect. In order to improve the numerical stability, Theodore [173] and Ribera [48] adopt the simplified free wake model of Bagai [174] and Bhagwat [109] into the FLEXUM model. Spoldi [45] and Horn [47] utilised a similar simplification when combining the CHARM free wake model of the CDI company with the GENHEL model. A loose coupling method is needed for these applications in order to reduce the numerical instability and improve calculation efficiency. According to the published research mentioned above, the integrated method between wake model and flight dynamics model indicates the potential of the rotor wake model in enhancing the accuracy and computing efficiency of the flight dynamics model.

Third, the unsteady and dynamic stall characteristics of the airfoil aerodynamics influence the air loading of the rotor blade. The Leishman-Beddoes model [80], ONERA model [175,176], or the Johnson model [3] can be used in blade loading calculation, to consider those effects. The unsteady blade aerodynamic characteristics mainly focus on the effect of shed vortex in the wake. When the rotor wake model and the airfoil unsteady aerodynamics model are both utilised to model the helicopter flight dynamics characteristics, care should be taken to avoid the repeated inclusion of the unsteady effect from the shed vortex.

Fourth, the governing equations of the helicopter flight dynamics model need to be expressed with a specific format due to the coupling effect of the blade dynamics feature and the interaction between the rotor and fuselage, impeding the utilisation of traditional solving methods. Tornour and Celi [131] utilised analytical solutions to separate the inertia coupling related to the vehicle acceleration from the rotor/fuselage coupling dynamic functions, and this part was rescheduled to a first-order ordinary differential equation. This method has been widely used for flight dynamics analysis and simulation.

Finally, the objective to introduce the engine/fuel control system is to make the helicopter flight dynamics model suitable for manoeuvrability investigation. Compared with the steady flight, the nonlinearity significantly increases in manoeuvring flight, and this nonlinearity will couple with the engine/fuel system, which will put forward higher requirements in the system integration and solution procedures.

Predictably, with the increasing complexity of the rotor aerodynamics model and aerodynamic interference model, a growing effort will be needed to develop its integration and calculation methods. The critical challenge for the high-confidence helicopter flight dynamics modelling method is to strike a balance of the modelling accuracy and computing time cost to ensure both the accuracy and efficiency of the flight dynamics model can be satisfied at all flight conditions.

3.5. *Progress in manoeuvrability analysis*

Flight test and pilot-in-the-loop simulation have been widely used for manoeuvrability analysis for the helicopter. Many the flight tests have focussed on different Mission-Task-Elements (MTEs) using various helicopter configurations [177–181], providing valuable material for the helicopter flight dynamics assessment. Flight testing is expensive, potentially dangerous and can only take place once a prototype aircraft is available, and not in the early design phase where potential benefits could be identified.

Pilot-in-the-loop simulation is widely used due to its economic efficiency and capability to be adopted in the early design process. There are a number of helicopter simulators being used across the world to assess the manoeuvrability of different helicopter configurations [182–185]. Nevertheless, the method puts forward a higher requirement for the flight dynamics modelling technique. The flight dynamics model has to meet the real-time requirement when adopted into the flight simulator. In other words, the application of the high-precision method, including the rotor free wake method and FEM rotor dynamics model, narrows the feasibility of this method. Effectively, the handling qualities assessment from the pilot-in-the-loop simulation still has a significant error compared with the flight test results, and it cannot be used to replace the flight test entirely at present.

On the other hand, with the increase of the flight dynamics modelling accuracy, a range of novel methods for manoeuvrability analysis have been put forward, including the nonlinear optimal control (NOC) method and the inverse simulation method.

The NOC method is based on the collocation and the numerical optimization method and has been adopted into the manoeuvrability investigation of the conventional helicopter [186–189] and tilt-rotor aircraft [190,191]. This method utilises a human operator model to take the pilot biometric lag into the simulation process and improve its accuracy. However, this method has internal numerical instability and can suffer from convergence problems once the discrete modelling technique, such as the rotor free wake model, is adopted. Therefore, the NOC approach is only suitable for the task profile (e.g. flight range and flight duration calculation) investigation and the small-to-moderate amplitude manoeuvrability analysis.

The inverse simulation has been created and steadily developed in recent years and has been employed to investigate the manoeuvrability and subjective handling qualities of various helicopter configurations [64,65,192–201]. Plenty of progress has been made for the inverse simulation method to improve its accuracy and efficiency. Rutherford [64,65] has adopted the individual-blade-motion into the inverse simulation approach, making it possible to consider the blade motion effect in more detail. Cameron introduced a pilot model into the inverse simulation method [197]. Thus, the pilot-induced oscillation is included in the calculation results. Hess [198,199] has tried to use a pilot model to conduct the inverse simulation of aggressive mission tasks, demonstrating that this analysis allows well-established compensatory models of human pilot behaviour to produce realistic pilot responses in discrete manoeuvres. Lee [200] utilised the pilot model to conduct the inverse simulation of helicopter shipboard operations. Results show that the unsteadiness of the ship airwake has a significant impact on pilot workload when the helicopter is operating near the deck and superstructure of the ship. Meanwhile, to enhance the efficiency and precision of the results, Ye [201] has adopted the Automatic Differentiation (AD) method into the flight dynamics model, accelerating the computing speed during the inverse simulation process. The AD method is based on the chain rule of the differentiation process. The calculation results indicate that the AD method embedded inverse simulation method could satisfy the real-time requirement. In other words, the proper control inputs can be calculated in advance of the real-time period using this inverse simulation method, which widens the application of the inverse simulation approach in the area of automatic control, control system design, and flight simulator assessment.

However, the inverse simulation method is still under developing. Firstly, it utilises a pre-determined trajectory, a mathematical description of the manoeuvring task needs to be given in advance. However, some manoeuvres, including some of the mission-task-elements (MTEs) defined in helicopter handling qualities requirements (ADS-33F-PRF) [1] do not have a fixed trajectory. In other words, it is hard to describe these MTEs mathematically. Secondly, inverse simulation obtains the control input by inversely solving the helicopter flight dynamics equations, which may have more than one solution in some flight states. Thus, additional constraint conditions are needed in order to obtain an optimized solution, and these constraints may not be entirely realistic in the context of actual vehicle piloting strategies. Although the inverse simulation approach can obtain the cockpit input that satisfies the performance requirement according to the handling qualities, the obtaining control strategy is one of the many control methods that could meet the requirement for given MTEs. Moreover, the advanced rotorcraft usually has redundant control inputs, including the rudder and the elevator, which also influences the inverse simulation results. In some research, additional boundary conditions need to be added to investigate its manoeuvrability [36,38]. In short, there are still many challenges for the further development of the inverse simulation method.

The main focus in manoeuvre studies recently has been the assessment of handling qualities ratings based on the control input results obtained from NOC or inverse simulation method. Wavelet analysis methods have been widely utilised due to its excellent time-frequency resolution [202–205]. In wavelet analysis, the finite-length bandpass filter is introduced to illustrate the signal energy in the frequency spectrum and time histories. With the wavelet analysis method, the main frequency components during the control input can be identified. According to the research of Thitschler and O' Conner [205], the pilot workload and pertinent handling qualities rating are dependent on the main frequency components. Additionally, the numerical correlation between the main frequency range and handling qualities ratings can be constructed. Thus, the pertinent handling qualities rating can be calculated in a straightforward manner once the control input is obtained.

4. Conclusion

Significant advances in helicopter mathematical modelling techniques are detailed in this paper. However, there is still a need for further improvements to investigate flight performance, flight dynamics, and handling qualities characteristics especially for advanced rotorcraft configurations. The specific research topics that will require future attention are as follows:

1) Improvement in the accuracy of the rotor aerodynamics model is still the most critical aspect of the helicopter flight dynamic modelling. Rotor flow field models have advanced from initial slipstream theory to the high-resolution rotor wake model able to calculate the aerodynamic characteristics found in advanced rotorcraft configurations. However, the high-resolution rotor wake model usually uses the lift-line or lift-surface models and excludes the detailed flow features around the blade, which limits its overall precision. The accuracy of conventional aerodynamic modelling methods is reduced to a large extent in high-speed flight due to dynamic stall and reverse area flow effects, making them of limited use in the simulation of advanced high-speed configurations. With multi-rotor systems used in the advanced helicopters, the distortion of the rotor wake put forward higher requirements for the rotor aerodynamics calculation process, and advances in this area are certainly a priority for improved predictions.
2) Although much progress has been achieved in the aerodynamic interference calculation, most methods are still dependant on the use of wind tunnel experiments or CFD simulation, and so are only valid for the vehicle tested (or class of vehicle at best). This lack of generality is an aspect of rotorcraft modelling that needs further attention. Methods such as the free wake model can numerically calculate the wake induced velocity in the flow field, and obtain aerodynamic interference, but their extreme time cost hinders their utilisation for the flight dynamics modelling. This problem is amplified for vehicles with multiple rotors or auxiliary propulsion devices where the time cost of free wake and other vortex-based methods can incur unmanageable computational overheads for flight dynamics research.
3) Engine/fuel system modelling has a significant influence on the flight dynamics characteristics predicted, especially for large-amplitude manoeuvring flight. The change of the required power leads to additional lag or overshooting effect produced by the engine/fuel control system. However, this effect is usually neglected in the flight dynamics modelling. At the moment, simplified transfer functions of the engine can be utilised in flight dynamics analyses of the helicopter. Nevertheless, relevant experiments are still required in these modelling methods in order to construct the engine/fuel system models, and in particular, a generalised modelling method to simulate the engine effect for various rotorcraft configurations is in urgent demand.
4) Rotorcraft dynamics can be considered as a set of highly coupled subsystems. The vehicle motion, the rotational motion of the rotor, and the wake motion are coupled with the unsteady aerodynamic and inertia loading to ensure the precision. This structure puts forward a higher requirement for the approach taken to solve the flight dynamics model. The associated approach should not only consider the coupling effect among the helicopter's systems, but also have the capability to allow each sub-component to exchange data efficiently and with a relatively low time cost. Only with these considerations, the flight dynamics model could strike a balance between precision and time efficiency.
5) The manoeuvrability of the advanced helicopter has drawn a range of research interest in recent years, and the NOC method and inverse simulation approach could be utilised to investigate the manoeuvrability of various rotorcraft configurations with relatively low time and financial cost. Also, the wavelet method can be used to obtain the handling qualities rating from the calculated control inputs. However, due to the deficiency of the NOC and inverse simulation methods, further improvements on the accuracy of the manoeuvrability analysis method and the resultant handling qualities rating method are still needed.

Funding sources

The financial support of the EPSRC project MENtOR: Methods and Experiments for NOvel Rotorcraft EP/S013814/1, is gratefully acknowledged.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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