Conventionalism, Structuralism and Neo-Kantianism in Poincaré's Philosophy of Science

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Abstract

Poincaré is well known for his conventionalism and structuralism. However, the relationship between these two theses and their place in Poincaré's epistemology of science remain puzzling. In this paper I show the scope of Poincaré's conventionalism and its position in Poincaré's hierarchical approach to scientific theories. I argue that for Poincaré scientific knowledge is relational and made possible by synthetic *a priori*, empirical and conventional elements, which, however, are not chosen arbitrarily. By examining his geometric conventionalism, his hierarchical account of science and defence of continuity in theory change, I argue that Poincaré defends a complex structuralist position based on synthetic *a priori* and conventional elements, the mind-dependence of which departs him from metaphysical realism.

Keywords: Conventionalism, Henri Poincaré, Neo-Kantianism, Structural Realism, Synthetic *A Priori*

1. Introduction

Two of the most prominent debates in the philosophy of science literature − namely the scientific realism debate and the conventionalism debate − originate in the work of Henri Poincaré[1]. However, Poincaré is often attributed seemingly conflicting positions when it comes to his stance towards scientific theories and geometry. While Poincaré's conventionalism is often generalised, equating his position to instrumentalism, he is also regarded as developing a form of selective scientific realism, namely structural realism.

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[1] Poincaré's most influential work is *Science and Hypothesis*, first published in 1902. One of the most discussed chapters in the book, 'The Theories of Modern Physics', where he develops the argument for the bankruptcy of science, was presented already in 1900 at the International Congress of Physics in Paris, under the title "The Relation Between Experimental Physics and Mathematical Physics". His second book *The Value of Science*, first published in 1905, further develops his ideas on the status of arithmetic, geometry and experimental science, and defends scientific knowledge against the generalised conventionalism with which he was associated after the publication of *Science and Hypothesis*. In particular, in the chapters 'Is Science Artificial' and 'Science and Reality', Poincaré argues against the conventionalist Le Roy, who had taken Poincaré's argument for the conventionality of geometry and mechanics to generalise to the whole of science. Poincaré's third book *Science and Method* was first published in 1908. A collection of essays that Poincaré had presented or previously published were published posthumously in 1913 under the title *Mathematics and Science: Last Essays.*

What is the scope of Poincaré's conventionalism and how does it relate to his structuralism and his general philosophy of science?

This article addresses the above question by analysing Poincaré's arguments for conventionalism, structuralism and his epistemology of science in order to show how these theses fit together. I examine the arguments offered by Poincaré that are usually taken to establish geometric conventionalism: (1) the argument for the underdetermination of geometry by experience; (2) the argument for the intertranslatability of different geometries; and (3) the argument for the constitutive but conventional status of geometry in physics. I show how these arguments relate and establish conventionalism as an epistemological thesis concerned with the status of geometry. Furthermore, I present Poincaré's argument for the conventionality of Newton's laws of motion and show how it differs from the arguments for the conventional status of geometry. I discuss Poincaré's arguments against instrumentalism and his defence of continuity and progress in science, taken to motivate structuralism. I present Poincaré's "layered" approach to scientific theories, according to which empirical science is made possible by synthetic *a priori*, conventional and empirical elements, and explore how it relates to his structuralism. I conclude that Poincaré's structuralism is deeply entrenched into his neo-Kantianism and conventionalism, and his views are much more complex than contemporary forms of selective realism often attributed to him.

2. Theory Change, Continuity and Structuralism

One of the central questions Poincaré addresses in his work regards the aim of science: do scientific theories function merely as predictive devices to be abandoned in light of more empirically adequate ones? Or do they aim at providing true descriptions of the unobservable world? This question took central stage at the turn of the 20th century and was widely debated amongst the scientific community. Poincaré played a central role in this debate by arguing that one can defend the continuity of science and scientific progress in light of the apparent discontinuities in theory change.

In *Science and Hypothesis* Poincaré articulates what the aim of science should be:

> The aim of science is not things in themselves, as the dogmatists in their simplicity imagine, but the relations between things; outside those relations there is no reality knowable. (Poincaré 2001, xxiv)

> The object of mathematical theories is not to reveal to us the real nature of things; that would be an unreasonable claim. Their only object is to coordinate the physical laws with which physical experiments make us acquainted, the enunciation of which, without the aid of mathematics, would be unable to effect. (Poincaré 2001, 117)

Despite appearing to defend instrumentalism, Poincaré strongly opposes the instrumentalism of his contemporaries Abel Rey and Édouard Le Roy because he believes that it cannot explain how theories manage to retain some of their content in theory change and also how theories get unified into a single framework.[2] In his chapter 'Is Science Artificial' from *The Value of Science*, Poincaré responds to Le Roy's instrumentalism, which takes scientific theories to be 'practical recipes' for prediction. He argues that if science has value because of its ability to make successful predictions, then this value must be due to the fact that theories are more than just practical recipes (Poincaré 2001, 320). Scientific theories, Poincaré argues, are "a classification, a manner of bringing together facts which appearances separate, though they were bound together by some natural and hidden kinship. Science, in other words, is a system of relations. Now we have just said, it is in the relations alone that objectivity must be sought" (ibid., 347).

Poincaré's main concern is to account for the so-called 'bankruptcy of science'.[3] He argues that

> The ephemeral nature of scientific theories takes by surprise the man of the world. Their brief period of prosperity ended, he sees them abandoned one after another; he sees ruins piled upon ruins; he predicts that the theories in fashion to-day will in a short time succumb in their turn, and he concludes that they are absolutely in vain. This is what he calls the bankruptcy of science. (ibid., 122)

Poincaré is not convinced that this argument from theory change, if it is properly understood, shows that there is no progress in science. Poincaré continues his argument in defence of continuity in theory change by examining the famous transition from Fresnel's ether theory of light to Maxwell's electromagnetic theory of light. The point Poincaré makes is that Fresnel's equations are entailed by Maxwell's theory, which shows that Fresnel correctly identified some relations between optical phenomena, despite the fact that the nature of light is differently understood. Poincaré argues that while the referent of 'light' has changed from Fresnel to Maxwell, both theories have identified the same relations in the optical phenomena expressed in the equations of these theories. Since the equations survived the theory change, this gives confidence that Fresnel's theory had identified the correct relations despite the change in ontology between the two theories. Poincaré argues that:

> The true relations between these real objects are the only reality we can attain, and the sole condition is that the same relations shall exist between these objects as between the images we are forced to put in their place. If the

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2 Poincaré is often regarded as an instrumentalist or 'global conventionalist', a position called at his time 'nominalism'. Poincaré's contemporary Pierre Duhem is also often classified as an instrumentalist. However Duhem (1954) also argues against the instrumentalism of Rey and Le Roy and articulates two arguments in defence of a moderate position, arguing that instrumentalism cannot explain how scientific theories make novel predictions and get unified into a single 'natural classification'. See Worrall (1989) and Ivanova (2010).

3 Laudan (1981) develops this argument into a meta-induction on the history of science to argue that we cannot be confident in believing that our currently successful theories are true since such confidence in past successful theories was misguided. For Laudan, the history of science allows us to infer that theories that are currently regarded as approximately true and whose terms we regard as referring are actually false, and their theoretical terms fail to refer.

relations are known to us, what does it matter if we think it convenient to replace one image by another? (Poincaré 2001, 122-123)

Poincaré argues that science cannot teach us the nature of 'things' but it can teach us the true relations among them. The evidence for this claim comes from the history of science where one appreciates that despite the constant ontological revisions, the relations that scientific theories reveal remain stable throughout theory change; "there is in [theories] something which usually survives. If one of them has taught us a true relation, this relation is definitely acquired, and it will be found again under a new disguise in the other theories which will successively come to reign in place of the old" (ibid. 347-349).[4]

Worrall (1989) takes Poincaré's argument to be a middle ground between instrumentalism and scientific realism, which he calls structural realism.[5] By committing to the relational content of the theory, while remaining agnostic as to its ontology, Poincaré accommodates both the argument from theory change and the argument from the success of science. Poincaré maintains that there are no radical discontinuities at the level of mathematical structure. The discontinuities that occur in the history of science concern the 'nature' of theoretical entities, not the structure of the theory. This focus on 'structure' and not 'nature' allows the structural realist to argue that science is cumulative; in theory change there are elements of the old theory that are retained in the new one. In the transition from Fresnel's theory to Maxwell's theory of electromagnetism, the equations of the former theory are completely preserved in the latter theory. These equations carry different interpretations: in Fresnel's theory light is a disturbance in the ether, in Maxwell's theory the disturbance is due to the nature of the electromagnetic field. Nevertheless, Worrall argues, "Fresnel's theory had correctly identified certain relations between optical phenomena, the equations of Fresnel's theory are directly and fully entailed by Maxwell's theory" (ibid, 159).[6]

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[4] Note that despite opposing the acceptance of unobservable entities in his work, Poincaré changed his mind regarding the status of atomism after Perrin's experiments. In his (1913) paper, Poincaré argues for the acceptance of atoms due to there being decisive empirical evidence. Stump (1989) argues that Poincaré is flexible with regard to his realism and does not adopt an all-or-nothing instrumentalism. Krips (1986) argues that Poincaré's shift was not from instrumentalism to scientific realism, instead, the shift was in Poincaré regarding the atomic hypothesis from a purely metaphysical, or 'indifferent', to an empirical one. Ivanova (2013), further argues that, apart from acknowledging the change of status of the atomic hypothesis, Poincaré's aim in his paper was to raise the problem of whether science gives us reasons to believe in a fundamental level.

[5] Giedymin (1982) and Gower (2001) also classify Poincaré as a structural realist.

[6] It has been objected that the case of Fresnel-Maxwell is singular and that there are not many examples in the history of science of mathematical equations that survive completely in the subsequent theory (Redhead 2001). However, Worrall (2007) argues that structural realism can be defended by showing that in the transition from one theory to another, the equations of the old theory are limiting cases of the equations of the new theory, and are fully recovered by it. Worrall claims that this is the case with successful theories of the past that satisfy the general correspondence principle. According to Post (1971), the general correspondence principle states that the superseding theory can account for the success of the superseded theory by 'degenerating' into earlier theories in domains in which the earlier ones are well confirmed. For example, Newtonian mechanics and the Galilean transformations can be recovered from the special theory

The structuralist reading has convincingly shown Poincaré's desire to argue in favour of scientific progress and its continuity in light of the 'bankruptcy of science' without appealing to the referential continuity of theoretical terms. It shows that Poincaré defends some form of structuralism – he believes that the content of a scientific theory amounts to its structural claims – claims about the relations among 'things' and not information about the 'nature' of those things. Furthermore Poincaré claims that the relational content of the theory, captured by the mathematical equations, survives theory change. The question that remains open is how this structuralist argument fits with Poincaré's overall epistemology of science, his conventionalism and neo-Kantian epistemology. What is controversial is whether we can characterise Poincaré's position as realist in an externalist sense. In order to address this issue, we need to understand the conventionalist and neo-Kantian elements in his philosophy.

3. The Status of Geometry and Mechanics

This section presents and evaluates three arguments developed by Poincaré that are usually taken to motivate geometric conventionalism, as well as his argument for the conventional status of some physical principles. I start with the three arguments for the conventionality of geometry, followed by a presentation of the argument for the conventional nature of Newton's laws of motion. I evaluate the epistemological significance of these arguments, their relationship, and what kind of conventionalism they establish.

3.1 Geometric Conventionalism

Poincaré offers three arguments that are usually taken to motivate geometric conventionalism. These arguments establish the following conclusions: (1) geometry is underdetermined by experience; (2) different geometrical systems are intertranslatable; and (3) geometry has a conventional epistemological status. In this subsection I present these arguments and show how they are related.

3.1.1 Geometry is Underdetermined by Experience

In chapter 4 of *Science and Hypothesis*, Poincaré presents his famous argument for the empirical equivalence of Euclidean and non-Euclidean geometries. He urges us to imagine a world enclosed in a large sphere with a non-uniform heat field that affects the people and their measuring rods (Poincaré 2001, 55). The heat field affects everything in this sphere and when objects move from the centre to the circumference of the sphere, they all undergo contraction. So, when one performs measurements, one would shrink, together with one's measuring rods, as one moves away from the centre of the heat field. The appearances are compatible with two different alternatives. Either this is an infinite non-Euclidean world of constant negative curvature, or it is a finite Euclidean world with a heat force that affects all objects, including the measuring rods and the observers. Since experience is compatible with more than one geometry, there is a choice to be made as to

of relativity when the speed of light tends to infinity, also classical mechanics can be recovered from quantum mechanics when Plank's constant tends to zero.

which geometry to choose in order to describe the phenomena. For Poincaré this choice is not arbitrary and is limited only to three alternative geometries of constant curvature of which Euclidean geometry is preferred on the grounds of its simplicity.[7]

Poincaré's argument shows that one could cultivate an alternative, observationally indistinguishable theory by changing the geometry and modifying the physical laws (for example, by introducing distorting forces[8]). Here he elaborates further the argument for the empirical equivalence of different physical geometries:

> If Lobatschewsky's geometry is true, the parallax of a very distant star will be finite. If Riemann's is true, it will be negative. These are the results which seem within the reach of experiment, and it is hoped that astronomical observations may enable us to declare between the two geometries. But what we call a straight line in astronomy is simply the path of a ray of light. If, therefore, we were to discover negative parallaxes, or to prove that all parallaxes are higher than a certain limit, we have a choice between two conclusions: we could give up Euclidean geometry, or modify the laws of optics, and suppose that light is not rigorously propagated in a straight line. (ibid.)

It has been noted in the literature that the argument from the empirical underdetermination of geometry could be seen as a special case of Duhem's general argument for the underdetermination of theory by the data stemming from confirmational holism, suggesting that the argument does not apply in any special sense only to geometry.[9,10] However, this interpretation leads to the claim that the argument establishes global conventionalism, something Poincaré opposes. While this argument has often been taken to establish the conventional nature of geometry[11], what it shows is that our knowledge of geometry is not empirical since our experience is compatible with more than one alternative. Pure geometry in isolation makes no empirical predictions. Despite the available alternatives, Poincaré believes that we have good reasons to choose one of the alternative geometries and this choice is guided by considerations of simplicity and convenience:

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[7] Poincaré argues that our choice of geometrical space is limited to three alternative geometries of constant curvature: Euclidean, Lobachevskian and Riemannian. His argument relies on Lie's theory of groups, which assumes: (1) that each geometry corresponds to a possible group of transformations; and (2) free mobility.

[8] Note that Poincaré's argument relies on distorting, non-uniform, forces. Reichenbach (1958) later argues that non-uniform forces are in principle detectable and articulates the same argument using uniform (or universal) forces, that are in principle undetectable, in order to establish the empirical indistinguishability between Euclidean and non-Euclidean geometries.

[9] See Friedman (1999), Ben-Menahem (2001, 2006), Stump (1989) and Zahar (1997).

[10] Giedymin (1982) and Zahar (1997) argue that Poincaré's conventionalism about geometry is epistemic. Since he is not a verificationist, they argue, Poincaré is not led to the claim that there are no geometric facts and subscribes to a structural realist account of space. While Ben-Menahem (2006) agrees with Giedymin's and Zahar's reading that Poincaré is not a verificationist she disagrees with their claim that he is a realist in any sense.

[11] See Torretti (1978), Stump (1991), Giedymin (1982).

> Euclidean geometry is, and will remain, the most convenient: first, because it is the simplest, and it is not so only because of our mental habits or because of the kind of direct intuition that we have of Euclidean space; it is the simplest in itself, just as a polynomial of the first degree is simpler than polynomial of the second degree; second, because it sufficiently agrees with the properties of natural solids, those bodies which we can compare and measure by means of our senses. (Poincaré 2001, 45)

Poincaré argues that we choose between different geometries on grounds of simplicity and convenience, making Euclidean geometry always the most convenient option. It is important to note here that simplicity is not given epistemic but purely methodological status. While it guides our decision-making, Poincaré does not take simplicity to be a guide to the truth.

The conclusion of this subsection is that the argument for the underdetermination of geometry by experience does not in isolation establish the conventional status of geometry. The argument shows that geometry is not empirical. As I argue in section 3.1.3, this argument is a step in establishing the conventional status of geometry, which Poincaré develops in the later chapter of *Science and Hypothesis*. It is there that Poincaré introduces a new epistemic category in order to accommodate the existence of non-Euclidean geometries.

3.1.2 Intertranslatability of Different Geometries

In chapter 3 of *Science and Hypothesis*, Poincaré discusses non-Euclidean geometries and argues that the basic terms of Lobachevskian geometry can be translated into terms of Euclidean geometry by the construction of a 'dictionary'.

> [L]et us construct a kind of dictionary by making a double series of terms written in two columns …
> Space … … … the proportion of space situated above the fundamental plane
> Plane … … … sphere cutting orthogonally the fundamental plane
> Line … … … circle cutting orthogonally the fundamental plane
> Sphere … … … sphere
> Circle … … … circle
> Angle … … … angle
> Distance between two points … … … logarithm of the anharmonic ratio of these two points and of the intersection of the fundamental plane with the circle passing though these two points and cutting it orthogonally etc. (Poincaré 2001, 39 - 40)

Poincaré suggests that we can obtain the theorems of Lobachevskian geometry from Euclidean geometry by using such a dictionary. For example, the theorem that states that the sum of the angles of a triangle add to less than 180° will become "[i]f a curvilinear triangle has for its sides arcs of circles which if produced would cut orthogonally the fundamental plane, the sum of the angles of this curvilinear triangle will be less that two right angles" (ibid).[12]

It has been argued that the aim of this argument is to establish the isomorphism[13] between the two geometric systems (Black (1942), Nagel (1979)). Torretti (1978) finds this claim problematic because the models of Lobachevskian and Euclidean geometry are not isomorphic, and, furthermore, there is lack of textual evidence to support the claim that Poincaré wanted to show that these systems are isomorphic. Torretti argues that Poincaré's intertranslatability argument is based on the theory of group transformation which Poincaré adopted after Klein showed that Lobachevskian and Euclidean groups of motions are all subgroups of the projective group. Stump (1991) suggests that the best way to understand the argument from intertranslatability is to take Poincaré's topological view, according to which the two systems he considers are homeomorphic, that is, they share the same topological properties.

The goal of the intertranslatability argument is to establish the consistency of Lobachevskian geometry given the consistency of Euclidean geometry by showing how one could create a model for the former within the latter geometry. Can we read this argument as establishing geometric conventionalism and what would the conventionalist thesis amount to in this context? At first glance the argument seems to imply the problem of underdetermination. As Ben-Menahem (2001) argues, it might appear that the underdetermination argument Poincaré develops is redundant, since it follows logically from the intertranstalability argument. That is, geometric intertranslatability entails empirical equivalence. However, according to Ben-Menahem, the argument from underdetermination of geometry is not redundant because the intertranslatability argument applies only to formal systems and not physical theories. Only the latter are underdetermined by experience and thus the two arguments are logically distinct, even though the argument for empirical equivalence depends on the argument for intertranslatability. The intertranslatability argument is local and concerns only pure geometry and not applied geometry. The argument for empirical equivalence is concerned with theoretical systems that make factual claims. Geometry by itself makes no empirical predictions and therefore cannot be considered underdetermined by the data.

The conclusion of this subsection is that the argument for intertranslatability establishes the consistency of non-Euclidean geometries given the consistency of Euclidean geometries, but not geometry's conventional status. The intertranslatability argument concerns formal geometric systems and is thus distinct from the argument for the underdetermination of geometry by experience, since it does not regard physical geometry.

3.1.3 The Epistemic Status of Geometry

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12 Note that while the traditional notion of 'translation' preserves both the meaning and truth-value of two sets of sentences or theories, this notion of translation implies only preservation of truth-value and not meaning.

13 Two models $\langle D, R_1, \ldots R_n \rangle$ and $\langle D', R'_1, \ldots R'_{\underline{n}} \rangle$ are isomorphic iff there is a one-one mapping f of D onto D' such that for each i, R_i and R'_i have the same arity m (= number of argument places), and for every sequence of elements $\langle d_1, \ldots, d_m \rangle$ of D, R_i holds of $\langle d_1, \ldots, d_m \rangle$ iff R'_i holds of $\langle f(d_1), \ldots f(d_m) \rangle$.

Poincaré introduces the term 'convention' in a different context in which he examines the epistemic status of geometry. Poincaré works within the Kantian framework, according to which the axioms of Euclidean geometry are synthetic *a priori* truths (Kant (1781), (1783)). Within the Kantian framework, the axioms of geometry are both: (1) necessary truths; and (2) constitutive of our knowledge. Poincaré adopts the Kantian framework, but wants to account for the emergence of non-Euclidean geometries. He argues that if Euclidean geometry were to be synthetic *a priori*, two things would have to hold: (1) we would not be able to conceive the negation of its axioms; and (2) we would not be able to construct consistent systems using the negation of these axioms (Poincaré 2001, 45).[14] Poincaré argues that both claims are dismissed by the existence of non-Euclidean geometries. First, we have managed to conceive the negation of the Euclidean axioms, something which would be impossible were the axioms of geometry necessary truths. Second, we have managed to construct consistent geometries based on the negation of these axioms.[15]

It is clear that there is a tension with Kant's synthetic *a priori* and the only option Poincaré has, in order to accommodate non-Euclidean geometries into the existent framework, is to shift them to the following categories permitted within the Kantian framework: analytic *a priori* or synthetic *a posteriori*. Poincaré, however, argues against these options. Geometry is not analytic *a priori* because it would mean that it reduces to a tautology. Geometry, Poincaré argues, expands our knowledge. Geometry is not empirical either. We do not find the objects of geometry in our experience because they are idealised and we cannot perform empirical tests on these objects. Moreover, Poincaré argues, were it empirical, geometry would not be an exact science; it would be subject to constant revisions. Last, as the argument from underdetermination of geometry by experience shows, our experience is compatible with more than one geometrical system (Poincaré 2001, 45).

Having established that the axioms of geometry cannot be accommodated within the Kantian category of analytic *a priori*, synthetic *a priori* or synthetic *a posteriori*, Poincaré introduces the new epistemic category of 'convention'[16]: "The geometrical axioms are therefore neither synthetic *a priori* intuitions, nor experimental facts. They are conventions […] *the axioms of geometry* (I do not speak of those of arithmetic) *are only definitions in disguise*. What then are we to think about the question: Is Euclidean geometry true? It has no meaning" (ibid., 45, emphasis in original). His notion of convention does not equate with an arbitrary element in the theory, however. Conventions play a constitutive role in the theory, just like the Kantian synthetic *a priori*, despite the fact they are no longer regarded as unique; there is freedom as to which ones to employ. This choice, however, is not arbitrary; it is restricted to the three possible geometries of constant curvature – Euclidean, Lobachevskian and Riemannian – from which Euclidean geometry is the most

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14 Even though, according to Kant, we could construct logically consistent systems using the negation of some Euclidean axioms, these systems would be synthetic *a priori* false.

15 Lobachevskian geometry negates the fifth axiom of Euclidean geometry (the parallel lines postulate) and postulates that an infinite number of parallel lines to a given line can be drawn from a single point. Riemannian geometry negates the fifth axiom and omits the second axiom (which postulates that a line can be extended infinitely).

16 Ben-Menahem (2001) introduces the reading of 'conventions' in Poincaré as a new epistemic category.

convenient option due to its simplicity. Having examined the arguments for geometrical conventionalism, we can now analyse Poincaré's arguments for the conventional status of the principles of mechanics, and subsequently see how these arguments relate in Poincaré's hierarchical understanding of science.

3.2 The Conventionality of Mechanics

In chapter 6 of *Science and Hypothesis*, Poincaré develops the next stage of his argument: the conventional status of Newton's laws of motion. This argument has traditionally been taken to imply global conventionalism, according to which not only geometry but physical science itself is a conventional construction.[17] Poincaré wants to establish the status of the three laws of motion. The first argument concerns the empirical independence of the three laws of motion. The justification here is the same as in the case of geometry – these laws are neither directly testable by experiment nor *a priori*. Poincaré examines the law of inertia and argues that were this law *a priori*, it would have been imposed on us by reason and we would not have been able to conceive any alternatives to it. The history of science, however, shows that this is not the case. According to Aristotle's law of motion, for example, motion is possible only when there is a cause. The existence of alternative and contrary laws implies that they are not necessary truths. However, for Poincaré the law of inertia is not strictly speaking empirical either:

> An experimental law is always subject to revision; we may always expect to see it replaced by some other and more exact law. But no one seriously thinks that the law of which we speak [the law of inertia] will ever be abandoned. Why? Precisely because it will never be submitted to a decisive test […] this law, verified experimentally in some particular cases, may be extended fearlessly to the most general cases; for we know that in these general cases it can neither be confirmed nor contradicted by experiment. (Poincaré 2001, 77)

The laws of motion are not strictly empirical because they are concerned with idealised objects and processes. Poincaré argues that we cannot be led to the mathematical rigor of the laws of motion by experimental observation, we reach these laws by abstraction and idealisation and the product of this procedure is a model that does not directly represent the physical reality.

The second argument Poincaré offers for the conventionality of the three laws of motion is more significant as it concludes that the laws of motion serve as definitions.[18] Testing the second law requires prior knowledge of the meaning of 'force' and 'mass'. Mass is defined as the product of the volume and density of a body. Force, on the other hand, is understood as the cause of non-inertial motion; we have no independent way of measuring it. If we take force to be the product of the mass and the acceleration we end

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[17] The distinction between local and global conventionalism is drawn in Giedymin (1982).

[18] The literature on the status of Newton's laws of motion is large and I cannot do it justice here. A detail discussion can be found in Sklar (1974), Friedman (1983), DiSalle (2002) and Ryckman (2005).

up with circular definitions of the terms involved, without having an independent way to test this law. According to Poincaré: "For a definition to be of any use it must tell us how to measure force; and that is quite sufficient, for it is by no way necessary to tell what force is in itself, not whether it is the cause or the effect of motion" (ibid., 79). We can provide a definition for the equality of two forces – two forces are equal if they produce the same acceleration in bodies with equal mass, and Newton's third law provides us with another definition – two forces are equal when they are in equilibrium when acting in opposite directions. What is problematic about these definitions, according to Poincaré, is that a particular force which acts on one body cannot be independently tested by applying it to another body, so we cannot experimentally know whether this force will produce the same acceleration in another body. The point is that there is a circularity involved in testing Newton's laws – if we want to perform a test to check the equality of two forces (and so test the second law), we need to appeal to Newton's third law – the principle of equality of action and reaction. Poincaré concludes, "this principle can no longer be regarded as an experimental law but only as a definition" (ibid., 80). What Poincaré establishes with this argument is that the laws function as implicit definitions of the central terms of the theory; they define the concepts of force, mass, inertial frames of reference, and thus enable Newton's theory of gravitation to have empirical applicability.

Poincaré argues for the conventionality of principles in two different contexts – mechanics and geometry. Both arguments are based on the observation that geometry and mechanics study idealised objects and also function as definitions of certain concepts. But are the arguments analogous and do they establish the same kind of conventionalism? The next section positions these two different understandings of conventionalism into Poincaré's 'hierarchical' account of scientific theories. This account shows that while the laws of motion are regarded as constitutive conventions, they are nevertheless idealised from our experience and 'elevated' into constitutive principles. On the other hand, geometry is constructed by the concept of a group, which pre-exists in the mind, but is guided by our experience. The next section presents how Poincaré intergrates these two understandings of constitutive but conventional principles in his overall epistemology of science.

4. Poincaré's Series of Sciences

According to Poincaré, not all statements in a scientific theory have the same epistemic status. Contrary to epistemological holism, endorsed by his contemporary Pierre Duhem (1906), Poincaré argues that some statements are more epistemically fundamental and play an asymmetric role in sciences insofar as they make them physically possible. Poincaré's idea is based on what he calls 'the series of sciences' in which empirically meaningful theories are constructed and made possible by a series of conventional and synthetic *a priori* elements. Friedman (1999) gives the most detailed reconstruction of Poincaré's hierarchical approach to scientific theories and explains how each layer of the hierarchy is constituted by the more fundamental one.

Let us describe, following Friedman, how Poincaré understands the status and function of different elements in this hierarchy. Poincaré starts his description of the 'series of sciences' in chapter 2 of *Science and Hypothesis*, where he examines the epistemological status of arithmetic. According to him, arithmetical knowledge is based on our reasoning by recurrence, or 'mathematical induction', which allows us to show that a given theorem does not hold only for a finite amount of cases, but can be taken to hold of all, infinite, cases. As he suggests, "reasoning by recurrence […] is the only instrument which enables us to pass from the finite to the infinite" (Poincaré 2001, 16). For Poincaré arithmetic is synthetic *a priori* because whereas it enriches our knowledge, since it takes us from the finite to the infinite (and is thus non-tautological), it is nevertheless based on our intuition. Arithmetic is based on our intuitive capacity to represent infinite instances of the very same operation.

Having argued for the synthetic *a priori* status of arithmetic, Poincaré turns to the study of mathematical magnitude. He argues that we create the mathematical continuum by applying the law of non-contradiction using as a raw material the idea of the physical continuum which we build from our sensory experience (ibid., 24). Because the law of non-contradiction plays an essential part in creating the notion of mathematical continuum, Poincaré concludes, as in the case of the synthetic *a priori* status of arithmetic, that "this notion has been created entirely by the mind, but it is experience that has provided the opportunity" (ibid.). Poincaré's next step is to introduce a way to measure magnitudes on the continuum and he does this by introducing the addition operation. On the next level of Poincaré's hierarchy of science he places geometry. He takes the one-dimensional mathematical continuum, applies it to many dimensions and adds a metric.

> Space is another framework which we impose on the world. Whence are the first principles of geometry derived? Are they imposed on us by logic? Lobatschewsky, by inventing non-Euclidean geometries, has shown that this is not the case. Is space revealed to us by our senses? No, for the space revealed to us by our senses is absolutely different from the space of geometry. Is geometry derived from experience? Careful discussion will give an answer − no! We therefore conclude that the principles of geometry are only conventions; but these conventions are not arbitrary. (ibid., 6)

The laws of Newtonian mechanics are found on the next level of Poincaré's hierarchy bearing the same epistemological status as geometry − that of a convention. The three laws of motion, according to Poincaré are not empirical but conventional. While originating in our experience, these laws have been elevated to the level of conventional principles that define force, mass and the inertial reference frames, without which the law of gravitation would not have empirical meaning. As Friedman explains,

> The laws of mechanics do not therefore describe empirical facts governing independently given concepts. On the contrary, without these laws we would simply have no such concepts: no mechanical concepts, that is, of time, motion, mass and force. In this sense the laws of mechanics are also free creations of our mind, which we must first inject, as it were, into nature. (Friedman 1995, 76)

The conventional status of the laws of motion, however, does not imply conventionalism about all physical laws. Having defended the conventional status of the laws of mechanics, the next level of Poincaré's hierarchy contains empirical claims that are subjected to test and revision. As Poincaré explains,

> We now come to the physical sciences, properly so called, and here the scene changes. We meet with hypotheses of another kind, and we fully grasp how fruitful they are. No doubt at the outset theories seem unsound, and the history of science shows us how ephemeral they are; but they do not entirely perish, and of each of them some traces still remain. (Poincaré 2001, 6)

With this hierarchical approach to scientific theories Poincaré shows that some principles are more fundamental insofar as they make the empirical claims possible. This view strictly opposes the Duhem-Quine holism, according to which all sentences in a scientific theory have the same (empirical) status.[19]

One important point to note here is the difference in the epistemological status of geometry and mechanics. While they are both regarded as constitutive conventions they have different epistemological origin. The laws of mechanics, according to Poincaré, originate in experience and become generalisations of 'great fertility'. The axioms of geometry are not related to experience in the same way; they are not generalisations of our experience. On the contrary, geometry is constructed from the concept of a group, which Poincaré argues pre-exists in our mind. As Poincaré argues,

> The object of geometry is the study of a particular "group"; but the general concept of group pre-exists in our minds, at least potentially. It is imposed on us not as a form of our sensitiveness, but as a form of our understanding; only, from among all possible groups, we must choose one that will be the *standard*, so to speak, to which we shall refer natural phenomena. Experiment guides us in this choice, which it does impose on us. It tells us not what is the truest, but what is the most convenient geometry. (Poincaré 2001, 59)

On the other hand, when explaining the conventional nature of the laws of mechanics, Poincaré argues that:

> Principles are conventions and definitions in disguise. They are, however, deduced from experimental laws, and these laws have, so to speak, been erected into principles to which the mind attributes an absolute value. (Poincaré 2001, 105)

The principles of mechanics are regarded as conventions to which 'the mind attributes absolute value', however, their origin is empirical and before they are elevated into constitutive conventions, they are regarded only as approximate. However, once they are elevated into the status of conventions, they are regarded as absolute truths because new experiments are not going to be found in conflict with them (Poincaré 1913, 124). Just as

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19 According to Quine (1951), all sentences in a web of belief are likely to undergo revision in light of new evidence. The likelihood of revision is established not on epistemological grounds but on purely intentional grounds – by how likely it is that the scientific community will want to revise a given statement.

in the case of geometry, the decision as to which principles will be taken as conventions is not arbitrary but guided by experience.

One standard objection raised against Poincaré's view is that the rise of relativity theory undermines his diagnosis that geometry is conventional and Euclidean geometry will always be preferred on the grounds of simplicity and convenience. Since the general theory of relativity employs a pseudo-Riemannian geometry and the metric is linked to the distribution of mass and energy, which makes it empirically determined, Poincaré's view is untenable. However, as Friedman's (2001) defense of the relativized *a priori* illustrates, part of Poincaré's idea about the hierarchy of sciences can still be defended, even if his geometric conventionalism cannot. According to Friedman, the empirical content of physical theories is determined by the constitutive *a priori* principles. These principles, however, are not fixed as Poincaré thought. They are dynamical, relativised, and evolve throughout scientific progress. As Friedman explains:

> Relativity theory involves a priori constitutive principles as necessary presuppositions of its properly empirical claims, just as much as did Newtonian physics, but these principles have essentially changed in the transition from the latter theory to the former: whereas Euclidean geometry is indeed constitutive a priori in the context of Newtonian physics, for example, only infinitesimally Euclidean geometry − consistent with all possible values of the curvature − is constitutively a priori in the context of general relativity. What we end up with, in this transition, is thus a relativized and dynamical conception of a priori mathematical-physical principles, which change and develop along with the development of the mathematical and physical sciences themselves. (Friedman 2001, 31)

What is presupposed as a constitutive framework in Newtonian mechanics is the Euclidean geometry and Galilean boosts, which provide the Newtonian spacetime structure required to formulate the empirically testable law of gravitation. In the special theory of relativity the presupposed framework is formed by Minkowski spacetime structure on which the empirically testable laws are formulated (e.g. Maxwell's laws of electromagnetism). In the general theory of relativity we presuppose the infinitesimally Lorentzian manifold spacetime structure that admits any semi-Riemannian metric structure, which is locally Minkowskian. The laws that are empirically tested are Einstein's field equations.

On the picture drawn by Friedman, Quine's notion of entrenchment simply does not explain the relationship between the different elements of a theory. Friedman (2001, 35) argues that Quine's epistemological holism correctly captures the fact that a single empirical problem can promote the revision of any part of a theoretical framework. The essential difference between the Quinean picture and that of Friedman is that according to the former account the mathematical formalism would be simply another "element in a larger conjunction" while on the latter it is a "necessary presupposition without which the rest of the putative conjunction has no meaning or truth-value at all" (ibid., 36). The constitutive principles make the very empirical content of a theory possible and are thus essential presuppositions for knowledge.

Just as Friedman argues against Quine, Poincaré does not accept the epistemic implications of the holistic view advocated by his contemporary Duhem. Poincaré attributes different epistemic status to different conventions and their role in empirical science. The experimentally testable laws, according to Poincaré, are constructed by employing synthetic *a priori* as well as conventional elements. The conventions themselves have different origins, while geometry is constructed by the concept of a group, which is synthetic *a priori*, the laws of mechanics have empirical origin but are idealised and elevated into constitutive principles. The next section shows how this conception of scientific theories relates to Poincaré's structuralism.

5. Poincaré's neo-Kantian Structuralism

An aspect of Poincaré's philosophy that has remained unexplored is how to understand his structuralism given his conventionalism. The worry is especially pressing for those who take conventionalism to be a stance towards scientific theories, a thesis that, like instrumentalism, holds that scientific theories do not aim to discover truths about the underlying ontology of the world but simply at empirical adequacy. Many philosophers who take seriously Poincaré's argument for the underdetermination of geometry by experience believe that Poincaré was led to believe that theories are just conventions, since they are always underdetermined by experience and this underdetermination is resolved by employing pragmatic values. However, this line of argument does not capture Poincaré's own defence of conventionalism, which is restricted to geometry and some physical constitutive principles only, and not to empirical science as a whole. But even this clarification does not present a complete picture of Poincaré's view on the epistemology of science. The question still remains: does Poincaré believe that our theories can successfully reveal the structure of a mind-independent world, given his conventionalism? The above discussion shows that Poincaré's conventionalism is localised; it does not equate with taking physical theories as empirically adequate descriptions of the observable phenomena. But how are his neo-Kantianism and conventionalism related to his structuralism?

We have examined plenty of quotations that illustrate Poincaré's commitment to structural preservation in theory change. He claims that the aim of science is to discover how the phenomena are related and that the real relations we discover do not undergo revisions in theory change, their usefulness remains even after the ontology of the theory is modified. Do these arguments establish realism about scientific theories, even if this realism is to be selective, restricted only to the structural claims of a theory? Such a realist reading of Poincaré's position seems to clash with his claim that we do not have access to a mind-independent reality. Recall his claim that "a reality completely independent of the mind which conceives it, sees or feels it, is an impossibility. A world as exterior as that, even if it existed, would for us be forever inaccessible" (Poincaré 2001, 14). Given that Poincaré's thesis is based on the neo-Kantian premise that we cannot discover facts about the world that are independent of our cognitive apparatus, it departs from the view of contemporary structural realists who claim that our best theories can successfully track the structure a mind-independent reality.[20]

Friedman's (1999) reconstruction of Poincaré's hierarchical approach to science shows that empirical claims are made possible only once we have adopted a framework of

[20] Of course, not every structural relist is committed to this view. Michela Massimi (2011), for example, develops a neo-Kantian form of structural realism.

conventional and synthetic *a priori* elements. As a consequence, our knowledge of relations is conditioned on our employment of such elements. Poincaré notes on several occasions that there is no mind-independent access to the true relations.

> Does the harmony the human intelligence thinks it discovers in nature exist outside of this intelligence? No, beyond all doubt, a reality completely independent of the mind which conceives it, see it or feels it, is an impossibility. A world as exterior as that, even if it existed, would for us be forever inaccessible. But what we call objective reality is, in the last analysis, what is common to many thinking beings, and could be common to all; this common part, we shall see, can only be the harmony expressed by mathematical laws. It is this harmony then which is the sole objective reality, the only truth we can attain. (Poincaré 2001, 14)

In another place in *The Value of Science*, Poincaré argues that:

> It will be said that science is only a classification and that a classification cannot be true, but convenient. But it is true that it is convenient, it is true that it is so not only for me, but for all men; it is true that it will remain convenient for our descendants; it is true finally that this cannot be by chance. In sum, the sole objective reality consists in the relations of thing whence results the universal harmony. Doubtless these relations cannot be conceived outside of a mind which conceives them. But they are nevertheless objective because they are, will become, or will remain, common to all thinking beings. (Poincaré 2001, 350)

Poincaré takes the aim of science to be the development of a unified theory that uncovers 'hidden relations' or 'hidden kinships' between the phenomena. For Poincaré it is in the grasp of the harmony between the phenomena that one achieves understanding. He argues that it is not coincidental that we search for harmony in nature "[w]e take elements which at the first glance are unconnected; these arrange themselves in an unexpected order, and form a harmonious whole. We cannot believe that this unexpected harmony is a mere result of chance" (ibid. 100). The harmony our theories reveal cannot be understood either as an objective feature of the world outside our mental capacities nor simply as a subjective emotional response projected upon nature by us. Poincaré argues that this harmony is part and parcel of our intellectual capacities and an ideal we follow in our enquiries. He claims that "[t]his harmony is at once a satisfaction of our aesthetic requirements, and an assistance to the mind which it supports and guides" (ibid., 396-397). For Poincaré unification is the ultimate goal of science. It is in revealing 'hidden kinships' and 'real relations' in the phenomena that Poincaré finds the aim of science accomplished and our understanding of nature fulfilled.

The above passages illustrate that despite Poincaré's defence of structural continuity in theory change, his position cannot be equated with an externalist realist position. Poincaré argues that the relations revealed by scientific theories are conditioned on mind-dependent and conventional elements that give the theory its empirical content. Furthermore, the search for elegant and unified mathematical theories that reveal relations between distinct sets of phenomena is a regulative ideal that gives coherence to scientific knowledge and direction to our enquiry. It is us that impose this unity in the phenomena and search for relations among them. With these elements of Poincaré's thoughts in consideration, it becomes difficult to associate his relationalism with contemporary structural realist accounts that endorse an externalist stance. Poincaré's own structuralism is deeply entwined with his neo-Kantian, internalist, epistemology, his defence of conventional principles and his desire to defend scientific progress and rationality in science despite the apparent discontinuities in theoretical transitions.

6. Conclusion

While conventionalism, structural realism and neo-Kantianism can easily be seen as incompatible, Poincaré believes they can be consistently defended. He shows that successful reference does not need to have a central stage in the defence of scientific knowledge, that focusing on the empirical success and preserved mathematical relations can tackle arguments targeting the defence of scientific progress and its continuity. Poincaré's conventionalism does not entail arbitrariness about geometry and its scope does not go beyond the status of geometry and principles of mechanics. He shows that despite the need to introduce a new epistemic category of 'convention', important elements of Kantian epistemology are still to be taken seriously. Poincaré's hierarchical approach to scientific theories holds that scientific knowledge is conditioned on elements of empirical, synthetic *a priori* and conventional status. Furthermore, Poincaré argues that the discovery of relations, which is the ultimate aim of science, is a regulative ideal that guides our enquiries. Despite the fact that Poincaré appeals to structural preservation in theory change and believes that science can only discover relations, his structuralism is internalist and deeply entrenched into his neo-Kantianism and conventionalism.

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