The de Broglie Wave as Evidence of a Deeper Wave Structure

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August 9, 2015

Abstract

It is argued that the de Broglie wave is not the independent wave usually supposed, but the relativistically induced modulation of an underlying carrier wave that moves with the velocity of the particle. In the rest frame of the particle this underlying structure has the form of a standing wave. De Broglie also assumed the existence of this standing wave, but it would appear that he failed to notice its survival as a carrier wave in the Lorentz transformed wave structure. Identified as a modulation, the de Broglie wave acquires a physically reasonable ontology, evidencing a more natural unity between matter and radiation than might otherwise be contemplated, and avoiding the necessity of recovering the particle velocity from a superposition of such waves. Because the Schrödinger and other wave equations for massive particles were conceived as equations for the de Broglie wave, this interpretation of the wave is also relevant to such issues in quantum mechanics as the meaning of the wave function, the nature of wave-particle duality, and the possibility of well-defined particle trajectories

Keywords matter wave · wave packet · Schrödinger equation · wave-particle duality · wave function · particle trajectories

1 Introduction

De Broglie's intuition that solid matter might be wave-like revealed a unity in Nature that was to play a central role in the formulation of quantum mechanics (de Broglie [1] to [3]). It was this unity that allowed all particles, whether massive or massless, to be treated in like manner as evolving and interfering in accordance with associated wave characteristics.

However, it will be argued here, essentially from a reconsideration of de Broglie's thesis of 1924 [3], that the de Broglie or matter wave is not the independent wave conventionally supposed, but the relativistically induced modulation of an underlying carrier wave that moves with the velocity of the particle[1].

[1] The matters to be now considered were dealt with by de Broglie in two brief sections (I

In the rest frame of the particle, this underlying structure has the form of a standing wave[2].

So regarded, the de Broglie wave is not itself, strictly speaking, the matter wave of quantum mechanics, but evidences the existence of a deeper wave structure more deserving of that title. If, consistently with special relativity, this underlying structure were assumed to comprise influences evolving at the velocity c of light, its existence would imply a deeper and more natural unity between matter and radiation than could be contemplated if the only wave associated with a massive particle were its superluminal de Broglie wave.

This is not to question the significance of de Broglie's wave or the importance to physics of his famous thesis. Einstein remarked that de Broglie had "uncovered a corner of the great veil" [9], and it was from the thesis that Schrödinger was led to wave mechanics. The Schrödinger equation was itself conceived as a general equation for the de Broglie wave (see Bloch [10], and Bacciagaluppi and Valentini [11], esp. Chaps. 2 and 11).

Yet the de Broglie wave and the Schrödinger wave function have seemed curious affairs. The de Broglie wave is not only superluminal but strangely disassociated from the subluminal particle that it is forever overtaking but never outruns. And despite the utility of the wave function, there has been much debate as to what it actually means.

In his report to the Solvay Conference of 1927, Schrödinger observed that this "ψ-function" seems to describe, not a single trajectory, but a "snapshot .... with the camera shutter open" of all possible classical configurations (see Bacciagaluppi and Valentini [11], p. 411). In standard quantum mechanics, this superposition is intrinsically probabilistic, but that view has led to the measurement problem, as well illustrated by Schrödinger's *reductio ad absurdum* of the unobserved cat that is at once dead and alive (Schrödinger [12]).

However, it has been assumed in quantum mechanics, both in its standard form and in its many reinterpretations, as it was by de Broglie and Schrödinger, that the de Broglie wave is a wave in its own right. How the de Broglie wave might arise instead from the Lorentz transformation of an underlying standing wave may be explained shortly as follows. The crests of a standing wave rise and fall as one, each simultaneously with the next (as shown for a one-dimensional wave in Fig. 1(a)(i)). But according to special relativity, simultaneity is itself relative. In an inertial frame in which the rest frame of the standing wave is moving, the crests of the wave are observed to peak, not in unison, but in sequence. The standing wave suffers a dephasing - a failure of simultaneity - in the direction of travel (as suggested by Fig. 1(a)(ii)). This dephasing, moving through the underlying wave at superluminal velocity, is the de Broglie wave, not a wave in its own right, but a modulation, or as it might be termed, "a wave of simultaneity". To an observer for whom the particle is moving at the

and III) of the opening chapter of his thesis [3], numbering only a dozen or so pages, clearly written and rewarding to close scrutiny. For interesting discussions of the emergence of de Broglie's ideas, as reflected in his earlier papers, see Medicus [4] and Lochak [5].

[2] Several earlier such proposals were listed in Shanahan [6]; others include Mellen [7] and Horodecki [8].

velocity v, the standing wave will have the character of a carrier wave moving at that same velocity v, but subject to a modulation of velocity c^2/v, where c is the speed of light in vacuum. (The significance of Fig. 1(b) will be explained in Sect. 2.)

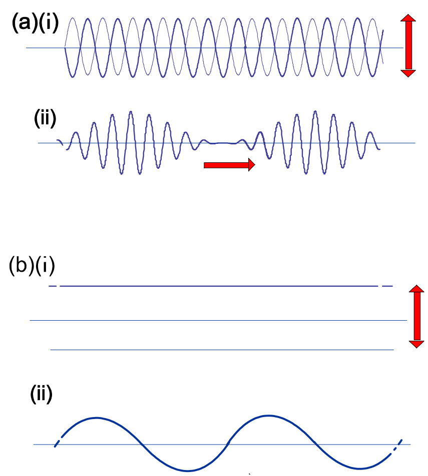

Fig. 1. The Lorentz transformation of two standing waves. Standing wave (a)(i) becomes the modulated wave (a)(ii) comprising a carrier wave of velocity v subject to a modulation of superluminal velocity c^2/v. The modulation is explained in the text as the de Broglie wave. De Broglie too assumed that the de Broglie wave emerges from a standing wave, but described a nodeless standing wave of form (b)(i) that has everywhere the same amplitude. When transformed, this wave has the form (b)(ii) in which the sinusoidal effect evolving at velocity c^2/v is again the de Broglie wave, not an independent wave, but the modulation of a nodeless carrier wave moving at velocity v.

That this is indeed the correct interpretation of the de Broglie wave is suggested by the curious manner in which its velocity varies with particle velocity. The velocity of the wave is not only superluminal but increases as the particle slows and becomes infinite as the particle comes to rest. This is not the behaviour of any independent wave, not that at least of any energy-carrying independent wave, but is precisely the behaviour of the modulation of an underlying carrier wave.

As we consider de Broglie's thesis, it will become apparent not only that he too assumed the existence of this antecedent standing wave, but that in two of his three demonstrations of the de Broglie wave, or as he then called it, the "phase wave"[3], this wave arises exactly as contended here. But in view of the tentative nature of his proposals, and as he explained in the conclusion to the thesis, he "left intentionally vague" (*intentionnellement laissé assez vagues*) the definitions of these wave forms. In the absence of a description of the antecedent standing wave, he seems to have assumed that it simply becomes the superluminal de Broglie wave when Lorentz transformed. It will be shown that this is not so.

Once the de Broglie wave is identified as a modulation, its velocity becomes consistent with special relativity, avoiding the necessity of assimilating the velocity of the particle to the group velocity of a superposition of such waves. With even greater significance for quantum mechanics, the Schrödinger and other equations for massive particles, including the Pauli, Klein-Gordon and Dirac equations, which were all contrived from the wave characteristics of the de Broglie wave (see, for example Dirac [13]), must then be in some sense equations for a modulation.

This might explain why the wave functions that emerge as solutions of these equations have seemed incapable of defining a trajectory for the particle in question. The existence of an underlying wave structure, moving at the velocity of the particle, might also shed light on such issues in quantum mechanics as the meaning of the wave function and the nature of wave-particle duality.

De Broglie's interpretation of his wave varied over time, but it was never the modulation proposed here. It was always a wave in its own right, or a superposition of such waves, as described in the thesis, and it is on the thesis that we now concentrate. It is also from the famous thesis, rather than from de Broglie's later writings, that it is possible to discern where and how the existence of the underlying carrier wave came to be overlooked.

2 The "periodic phenomenon"

De Broglie saw that if the Planck-Einstein relation,

$$E = h\nu, \tag{1}$$

for the photon were extended to matter and equated with Einstein's statement,

$$E = mc^2, \tag{2}$$

of the equivalence of mass and energy, a massive particle could be associated in its rest frame with a characteristic frequency ν_0 or, when expressed as an angular frequency,

$$\omega_0 = 2\pi\nu_0 = \frac{mc^2}{\hbar},$$

where m and $\hbar$ are respectively the rest mass of the particle and the reduced Planck constant (de Broglie [3], Chap. 1, Sect I).

[3]Not the usual sense in which "phase wave" is used, but suggested to de Broglie by the wave's role in the "harmony of phases" to be discussed in Sect. 4, below.

De Broglie rejected the possibility that this frequency could be the measure of some wholly internal oscillation. Observing that the energy of an electron "spreads throughout all space", he argued that a massive particle must be surrounded in its rest frame by what he referred to as a "periodic phenomenon".

He clearly regarded this phenomenon as some form of standing wave. This is apparent from his modelling of the phenomenon as an assemblage of springs oscillating in unison (see Sect. 5 below) and is explicit in his depiction of the wave in Minkowski spacetime (see Sect. 6 below). He expressly described the phenomenon as a stationary or standing wave in his report to the Solvay Conference of 1927 (see Bacciagaluppi and Valentini [11], p. 341), as also in other works ([14], and [15], Chap. 3) including his Nobel lecture of 1929 [16]. Consistently with that description, he suggested elsewhere that in the rest frame of the particle the de Broglie wave comprises a superposition of incoming and outgoing waves [17].

But in the thesis itself, de Broglie avoided saying much at all about this underlying structure. As mentioned above, this vagueness was intentional, the theory being, as he also said in concluding the thesis, "not entirely precise" (*n'est pas entièrement précisé*). De Broglie did state that the periodic phenomenon varies sinusoidally in time and that it "is distributed throughout an extended region of space" (de Broglie [3], Chap. 1, Sect I). But there is no description in the thesis of the manner in which the wave varies spatially, or any analysis in mathematical terms of how a standing wave changes under the Lorentz transformation. Nor is consideration given to the constraints imposed by special relativity on the form that such a wave might take.

By leaving the spatial variation of the phenomenon undefined, de Broglie allowed the possibility that it is a standing wave with no spatial variation - a node-free standing wave that has everywhere the same phase and amplitude (as in Fig. 1(b)(i)). A wave of that form would comprise counter-propagating waves of infinite wavelength and velocity contrary to the assumptions of special relativity. But it is nonetheless a featureless standing wave of this kind that de Broglie seems to contemplate in the thesis, and when Lorentz transformed, this featureless standing wave becomes (as will be seen in Sects. 5 and 6 below) a correspondingly featureless carrier wave subject to a sinusoidal modulation (as in Fig. 1(b)(ii)).

In the transformed wave structure, the only "wave" that is distinguished by a spatial variation is thus the modulation, which for a particle moving at velocity v, evolves through the carrier wave at the superluminal velocity c^2/v. It became possible for de Broglie to conclude then (or so we surmise) that when Lorentz transformed the standing wave simply becomes an independent superluminal wave of velocity c^2/v.

However, it will be shown in the next section that a standing wave, whatever its form or frequency, becomes to an observer for whom the rest frame of the wave is moving at velocity v, a carrier wave of that velocity, subject to a phase modulation having the wave characteristics and superluminal velocity c^2/v of the de Broglie wave.

3 The modulation

Consider a standing wave of form,

$$R(x, y, z)\, e^{i\omega t}, \tag{3}$$

where $R(x, y, z)$ describes the spatial variation of the wave, and $e^{i\omega t}$ is its evolution in time.

This wave thus has a well-defined sinusoidal frequency and might be expected to vary sinusoidally in space as well as time. But for now, we will follow de Broglie in leaving the spatial variation of the wave "intentionally vague".

We want to know the form that this "periodic phenomenon" would take when observed from a frame of reference in which the particle is moving with velocity v. Assuming a boost in the x direction, and applying the Lorentz transformation, $\Lambda_x(v)$:

$$x' = \gamma\,(x - vt)\,,$$
$$y' = y,$$
$$z' = z,$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right),$$

where γ is the Lorentz factor,

$$(1 - \frac{v^2}{c^2})^{-\frac{1}{2}},$$

standing wave (3) becomes the moving wave,

$$R(\gamma\,(x - vt)\,,\, y,\, z)\, e^{i\omega\gamma\left(t - vx/c^2\right)}. \tag{4}$$

which has two wave factors. The first,

$$R(\gamma\,(x - vt)\,,\, y,\, z), \tag{5}$$

is a carrier wave, which is moving at the velocity v and, as indicated by the inclusion of the Lorentz factor γ, has suffered the contraction of length predicted by special relativity.

The second wave factor,

$$e^{i\omega\gamma\left(t - vx/c^2\right)}, \tag{6}$$

is a transverse plane wave, which is moving through the carrier wave (5) at the superluminal velocity c^2/v. Identifying the frequency ω with the characteristic frequency ω_0 of a massive particle, wave factor (6) can be rewritten in terms of the Einstein frequency,

$$\omega_E = \gamma\omega_0, \tag{7}$$

and de Broglie wave number,

$$\kappa_{dB} = \gamma\omega_0 \frac{v}{c^2}, \tag{8}$$

as,

$$e^{i(\omega_E t - \kappa_{dB} x)}, \tag{9}$$

and is now more clearly identifiable as the de Broglie wave. However, it is not here the independent wave supposed by de Broglie, but as we have stressed, the modulation of the carrier wave (5), defining the dephasing of that wave (and the failure of simultaneity) in the direction of travel.

It is the modulated wave,

$$R(\gamma\,(x - vt)\,,\, y,\, z)\, e^{i(\omega_E t - \kappa_{dB} x)}, \tag{10}$$

rather than the de Broglie wave (9) that displays the full complement of changes in length, time and simultaneity contemplated by special relativity. It is suggested that it would be anomalous if any spatially extended phenomenon, wave or otherwise, could be Lorentz transformed into something that did not incorporate all these changes. Yet, as will be seen in Sects. 4 to 6, the carrier wave is effectively suppressed in de Broglie's derivations.

Let us now suppose that instead of a spatially extended wave, we have the oscillating point or point particle,

$$\delta[x_0, y_0, z_0]\, e^{i\omega_0 t}, \tag{11}$$

where $\delta[x, y, z]$ is the Dirac delta function and the particle is located in its rest frame at the point (x_0, y_0, z_0). Under a boost in the x direction, oscillating point (11) becomes,

$$\delta[\gamma(x_0 - vt),\, y_0, z_0]\, e^{i(\omega_E t - \kappa_{dB} x)}, \tag{12}$$

which describes not a wave, but a moving and oscillating point.

Like a child on a pogo stick, this moving point might describe the form of a wave, but it is not itself a wave. It is not possible for a point to become, by Lorentz transformation, an extended wave. Under a Lorentz transformation, a point remains a point, and a wave, although changed in form, remains a wave. While the second factor in Eqn. (12) does have the functional form (9) of the de Broglie wave, it is not in this case a wave, and it is not therefore the de Broglie wave.

That this is so may be intuitively obvious, but will be of some importance in understanding de Broglie's analyses. De Broglie was correct in insisting that if the moving particle is to be associated with a spatially extended waveform, some such waveform must also exist in the rest frame of the particle. But it will become apparent in the next section (Sect. 4) that de Broglie disregarded this extended waveform when applying what he referred to as the theorem of the harmony of phases. It will be seen that by confining his consideration of phase to the phase of the particle at the position of the particle, he derived, not the de Broglie wave, but what was in effect the variation in phase of a moving and oscillating point.

A difficulty of a different kind will be encountered in Sects. 5 and 6 below when we consider de Broglie's two further demonstrations of his wave. It is not every wave of the form (3) to which the Lorentz transformation can be validly applied. It was not stipulated when defining wave (3) that it should comprise underlying influences propagating at the velocity c of light. Yet it is that velocity (together with relative velocity v) that determines the velocity c^2/v of the modulation. It is the Lorentz transformation itself that imposes the velocity c, and it does so because it is assumed in special relativity that all underlying influences develop ultimately at that velocity (see, for example, Ref. [6], Sect. 6).

Or to put this another way, the de Broglie wave is not merely evidence of the wave-like nature of matter, but provides confirmation through its velocity c^2/v that the wave-like influences underlying matter and its interactions evolve ultimately at the free space velocity c of light.

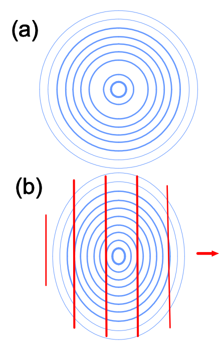

Fig. 2. A model particle wave is represented: (a) as a spherical standing wave; and (b) as a relativistically contracted carrier wave of velocity v, subject to a modulation (a de Broglie wave) of superluminal velocity c^2/v. The ellipses in (b) represent the ellipsoidal maxima of the carrier wave, while the vertical lines are intended to suggest the transverse planar wave fronts of the de Broglie modulation.

The Lorentz transformation can be appropriately applied to a standing wave formed from waves of a velocity other than c, but only if the velocity of underlying influences is nonetheless c, as is so for instance in the case of counter-propagating sound or water waves, where changes in underlying electromagnetic fields propagate at that velocity, or counter propagating light waves of velocity c/n in a medium of refractive index n. But the Lorentz transformation cannot be validly applied to a standing wave of the form (3) if the spatial variation $R(x, y, z)$ is unphysical, as in the case of the standing wave that is entirely without spatial variation referred to in the previous section (Sect. 2). Such a wave would be composed from influences of infinite wavelength and thus infinite velocity, contrary to special relativity.

But a wave of that kind might nonetheless be simulated, at least in principle, by an array of identical oscillators. Such a simulation will be encountered in the second of de Broglie's three demonstrations (to be discussed in Sect. 4), while the unphysical wave simulated will itself be seen in the third (Sect. 5).

The relativistic transformation of such an unphysical wave was illustrated in Fig. 1(b). The corresponding transformation of a physically reasonable standing wave comprising underlying influences of velocity c and varying sinusoidally in space and time was as shown in Fig. 1(a). The Lorentz transformation of a simple ansatz or model of the wave of Fig. 1(a) in three dimensions is shown in Fig. 2 (see, also Ref. [6]).

Finally, it is instructive to apply the reverse Lorentz transformation $\Lambda_x(-v)$ to the de Broglie wave,

$$e^{i(\omega_E t - \kappa_{dB} x)}, \tag{13}$$

whereupon with the aid of Eqns. (7) and (8) it becomes the oscillating point,

$$e^{i\omega_0 t},$$

which, as we have stressed, cannot become by Lorentz transformation, a wave. When Lorentz transformed, it describes the track of a moving and oscillating point.

It has thus been shown that the de Broglie wave is not an independent wave, but the modulation of an underlying carrier wave. We now consider in the next three sections (Sects. 4 to 6) how de Broglie came to conclude otherwise.

4 A "harmony of phases"

De Broglie's theorem of the harmony of phases (de Broglie [3], Chap. 1, Sect I) is essentially the requirement that relatively moving observers should agree on the phase that a wave has at each point of space and time. Phase is a scalar invariant and must have the same value in all inertial frames.

De Broglie concluded on the basis of this theorem that a moving particle is accompanied by a superluminal wave of velocity c^2/v. But in reaching that conclusion he confined his harmonizing of phase to the phase of the particle at the position of the particle. Thus what he was considering was not a wave but the sinusoidal trace of an oscillating point as defined by Eqn. (12).

De Broglie's argument proceeded as follows: From the standpoint of a "fixed observer" (for whom the particle is moving at velocity v) the particle has in its rest frame the reduced frequency,

$$\omega_{red} = \frac{\omega_0}{\gamma}, \tag{14}$$

yet in the frame of that same observer, the moving particle has an increased energy and correspondingly increased frequency,

$$\omega_{inc} = \omega_0 \gamma. \tag{15}$$

But a wave can have only one phase at any point of space and time, and all observers must agree on that phase. As de Broglie put it[4],

> The periodic phenomenon connected to a moving body whose frequency is for the fixed observer equal to $[\omega_0/\gamma]$ appears to him to be constantly in phase with a wave of frequency $[\omega_0\gamma]$ emitted in the same direction as the moving body, and with the velocity $V = [c^2/v]$.

So far so good. De Broglie refers in this passage to the periodic phenomenon and appears to be contemplating the transformation of the entire spatially extended phenomenon. But when he then proceeds to derive the velocity $V = c^2/v$ of the resulting "wave", he confines his consideration to a single point within the phenomenon, that is to say, the position of the particle, which is stationary in one frame and moving in the other, but is nonetheless a single point developing along a single world line.

De Broglie states (correctly) that as the particle travels the distance x in the time t (in the frame of the fixed observer), it follows from Eqn. (14) that it is considered to experience in its own frame the change of phase,

$$\omega_{red}\, t = \frac{\omega_0}{\gamma}\frac{x}{v}, \tag{16}$$

whereas in the fixed observer's frame, it follows from Eqn. (15) that the change of phase observed by that observer is,

$$\omega_{inc}\,(t - \frac{x}{V}) = \omega_0\gamma\,(\frac{x}{v} - \frac{x}{V}). \tag{17}$$

These changes of phase must be equal. Thus, equating (16) and (17),

$$\frac{\omega_0}{\gamma}\frac{x}{v} = \omega_0\gamma(\frac{x}{v} - \frac{x}{V}),$$

from which,

$$V = \frac{c^2}{v}.$$

De Broglie's analysis thus delivers the velocity of the de Broglie wave. But the change described by Eqn. (17) is simply the change in phase that occurs in a point particle as it moves the distance x in the time t. In terms of the Einstein frequency ω_E and de Broglie wave number κ_{dB}, the evolution of phase $\Delta\phi$ described by Eqn. (17) can be rewritten using Eqns. (7) and (8) as,

$$\Delta\phi = \omega_E\, t - \kappa_{dB}\, x, \tag{18}$$

from which it can be seen (compare, for instance, Eqn. (9)) that the oscillating point, moving at the subluminal velocity v, is maintaining consistency of phase

[4] In the quoted passage, we have simplified de Broglie's expressions for the frequencies and avoided here as elsewhere in this paper the practice of describing the velocity of the de Broglie wave as c/β.

with a superluminal wave having the characteristics of the de Broglie wave, which explains why de Broglie's analysis was able to produce the velocity of the de Broglie wave.

But by seizing upon the single point, and ignoring the disposition of phase across the extended wave, de Broglie suppressed the carrier wave of velocity v, which as shown in Sect. 3, must result from the Lorentz transformation of a standing wave. If a wave with the characteristics of the de Broglie wave were the only wave associated with the moving particle, harmony of phase could be guaranteed only at the position of the particle. It is the full modulated wave that harmonizes phase at all points for all observers.

De Broglie's two other demonstrations did involve the transformation of an extended wave. But from his theorem of the harmony of phases, he had already concluded that the de Broglie wave is a wave in its own right.

5 The mechanical model

Faced with the potentially embarrassing superluminality of his wave, de Broglie invoked a simple toy model to illustrate how a velocity greater than c might yet be consistent with special relativity provided the actual velocity of energy transport were less than c (de Broglie [3], Chap. 1, Sect I).

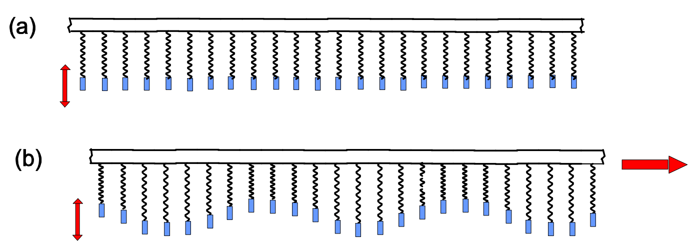

Fig. 3. De Broglie's mechanical model shown stationary in (a), and moving at the relativistic velocity v in (b). The sinusoidal effect of superluminal velocity c^2/v is a consequence of the failure of simultaneity in the direction of travel. This sinusoidal "wave" is the de Broglie wave, not an independent wave as assumed by de Broglie, but a dephasing (a modulation) of the standing wave modelled by the assemblage of springs.

Thus de Broglie was not seeking to derive his wave from this model, merely to justify its superluminal velocity. Even so, the de Broglie wave emerges from the model, not as an independent wave, but as the modulation of an underlying carrier wave.

As described by de Broglie, and as we have imagined this model in Fig. 3, it comprises a horizontal disk of large diameter, from which are suspended many identical spring weights oscillating in phase and at the same amplitude, but with the number of springs per unit area diminishing with distance from the centre of the disc in "very rough analogy", as de Broglie explained, to the distribution of energy around a particle. (But no attempt has been made to depict this diminishing intensity in Fig. 3).

An observer in the inertial frame of the disk observes these weights to be oscillating in unison (as in Fig. 3(a), and see also again, Fig. 1(b)(i)). But a second observer, for whom the disk is moving at velocity v (Figs. 3(b) and 1(b)(ii)), observes (along with other relativistic effects) what de Broglie described as the "dephasing of the movements of the various weights", that is to say, the failure of simultaneity in the direction of travel. To the first observer, the weights define a horizontal plane moving up and down. But to the observer for whom the disc is moving, this surface is not planar but sinusoidal, with the crests of this sinusoidal surface moving in the same direction as the disk, but at the superluminal velocity c^2/v of the de Broglie wave.

De Broglie did not provide a separate derivation of the velocity c^2/v of this sinusoidal effect. He merely said of the moving sinusoidal surface that:

> It corresponds .... to our phase wave. According to the general theorem, the surface has a speed [c^2/v] parallel to that of the disk ... With this example we see clearly (and this is our excuse for such protracted insistence on it) how the phase wave corresponds to the transport of the phase and not at all to that of the energy.

In other words, the sinusoidal surface defined by the moving springs is an instance of his phase wave (the de Broglie wave), while the "general theorem" de Broglie relies upon is his theorem of the harmony of phases.

De Broglie seems not to have noticed that the standing wave (the array of oscillating weights) has not simply become the de Broglie wave. It has become a structure moving at velocity v (a moving array of oscillating weights) subject to a modulation moving at velocity c^2/v. In treating the modulation as an independent wave, de Broglie has ignored the structure that it modulates.

In its rest frame, this toy model is not strictly speaking of course a standing wave. It is merely the *simulation* of a standing wave; in fact, the simulation of a physically unreasonable nodeless standing wave that has everywhere the same phase and amplitude. While an assemblage of synchronized oscillators would not itself be physically unreasonable, the wave thus modelled would comprise underlying influences of infinite velocity contrary to special velocity. But even ignoring that difficulty, it was shown in Sect. 3 that the application of the Lorentz transformation to a standing wave of any form results formally, not in an independent wave of velocity c^2/v, but in a carrier wave of velocity v subject to a modulation of velocity c^2/v.

De Broglie's model becomes under a Lorentz transformation, not the simulation of an independent de Broglie wave, but the simulation of a modulated wave of velocity v in which the de Broglie wave is the modulation.

If de Broglie had supposed an assemblage of springs that varies sinusoidally in space as well as time (and thus of the general form depicted in Fig. 1(a)), and in which underlying influences move at velocity c, he would have had in that model, a de Broglie wave that is physically reasonable and consistent with special relativity. But when he sought to recover the classical velocity of the particle from his superluminal wave, the analogy he drew was with the group velocity of a wave packet formed from the superposition of de Broglie waves of nearly equal frequency (see de Broglie [3], Chap 1, Sect II).

It was from that very different physical effect that the difficult concept of a particle wave packet was carried into quantum mechanics.

6 In Minkowski spacetime

By representing a standing wave in a spacetime diagram, de Broglie was able to derive the velocity of the de Broglie wave, while demonstrating in an intuitive manner the dephasing and consequent failure of simultaneity defined by this wave.

Referring to Fig. 4, the unprimed (x, ct) coordinates are those of the fixed observer, while the primed ($x^{'}$, $ct^{'}$) coordinates define the frame of the moving particle. The $ct^{'}$ and $x^{'}$ axes are inclined at $\alpha_1 = \arctan c/v$ and $\alpha_2 = \arctan v/c$, respectively, to the x axis, while the line OD defines one edge of the light cone. The particle itself is moving to the right at the velocity v, and its world line thus follows the primed $ct^{'}$ axis.

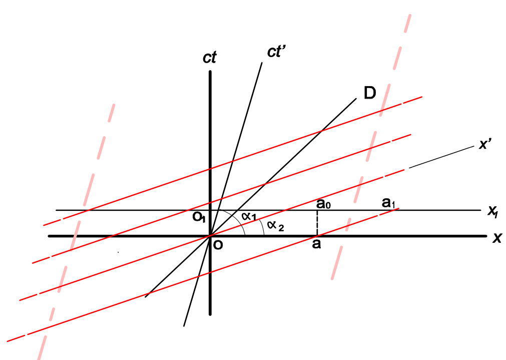

Fig. 4. The world line of a particle follows the $ct^{'}$ axis. Surrounding the particle in its rest frame is a standing wave represented by four parallel "equiphase planes". A world tube enclosing the world line of the particle has been defined by dashed lines parallel to the $ct^{'}$.axis. The equiphase planes are inclined at α_2=arctan v/c to the x axis, and define in the unprimed frame, a dephasing of velocity c^2/v. This dephasing is the de Broglie wave, considered by de Broglie an independent wave, but revealed by the drawing to be a modulation of the transformed standing wave. (Adapted from de Broglie [3], Chap. 1, Sect III, Fig. 1).

The standing wave is represented in the diagram by what de Broglie referred to as "equiphase spaces" (*espaces équiphases*), these being the four equally spaced lines drawn parallel to the x' axis. De Broglie also referred to these as "planes" (*plans équiphases*), presumably hyperplanes of three dimensions in the four dimensions of spacetime. Each such plane defines a sub-space in which the wave has reached a particular phase. In each inertial frame, these planes thus repeat after a time equal to the period of oscillation in that frame.

The four equiphase planes are parallel to the x' axis, leaving no doubt that it was assumed by de Broglie that in the rest frame of the particle the periodic phenomenon comprises some form of standing wave. What we want to know is what these planes mean in the unprimed frame of the fixed observer.

However, as they appear in the thesis (see de Broglie [3], Chap. 1, Sect. 3, Fig. 1), these planes are depicted in a way that could be a source of confusion. The lines representing the planes are not centred, as would be natural, on the world line of the particle. Indeed they are so positioned that they could suggest a wave propagating to the left of the diagram. The four planes have thus been repositioned in Fig. 4 so that they retain the same inclination to the x axis as in de Broglie's drawing but are centred on the world line of the particle.

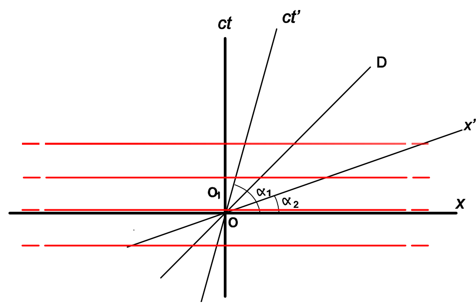

Fig. 5. As Fig. 4, but with the roles of primed and unprimed frames interchanged. The formerly fixed observer is moving to the right with velocity v, while the unprimed frame is now the inertial frame of the particle. The four equiphase planes representing the standing wave are thus parallel to the x axis.

To show how these planes would be represented from the standpoint of an observer who is in the frame of the particle itself, the roles of primed and unprimed frames have been interchanged in a further diagram (Fig. 5). In this, it is the formerly fixed observer, rather than the particle, that is moving to the right. Here again the planes are centred on the world line of the particle.

However, we concentrate now on Fig. 4. In the unprimed frame, the equiphase planes are inclined to the x axis and thus display the asymmetry in phase and failure of simultaneity in the direction of travel predicted by special relativity. From this asymmetry, de Broglie calculated the velocity of the de

Broglie wave. He first explains that as the line O_1x_1 represents the frame of the fixed observer at $t = 1$, the distance aa_0 is exactly c. He then says that at $t = 1$,

> The phase which at time $t = 0$ was at a is now at a_1. For the fixed observer it is then displaced in space by an amount a_0a_1 in the x direction, during one unit of time. One can then say that the speed is:
>
> $$V = a_0a_1 = aa_0 \cot \alpha_2 = \frac{c^2}{v}.$$

This is again the velocity of the de Broglie wave, but when de Broglie refers in this passage to the speed at which phase has become "displaced in space .... in the x direction", he is describing the velocity of a dephasing, that is to say, of a modulation.

To see that the standing wave has not simply become transformed into an independent de Broglie wave as supposed by de Broglie, it is only necessary to notice that the standing wave and the de Broglie wave do not have the identical representation in the diagram. In a spacetime diagram, such as those included here, the Lorentz transformation is a *passive* transformation under which an event or world line retains its location within the diagram[5]. Instead of the event or world line changing position, its coordinates differ according to the frame to which it is referred. Thus, if the de Broglie wave and the transformed standing wave were the same wave they would share the same world line in the drawings.

But that is not so. Consider the standing wave. In its rest frame, a standing wave is, by definition, stationary. The world line of any point in that wave thus follows the time axis of its rest frame, which in this case is the ct' axis of the primed frame of Fig. 4.

Or we could consider instead some extended region within the wave, let us say a spherical region enclosing the particle. Instead of a world line, the passage through spacetime of such a region defines a "world tube" (see, for example, Misner, Thorne and Wheeler [18], p. 473), which in this case, and as shown in Fig. 4, would enclose the world line of the particle which is following the primed ct' axis.

On the other hand, the superluminal velocity c^2/v of the de Broglie wave implies a world line (or tube, but it will suffice now to speak of a line) lying beyond the light cone and parallel to the x' axis. Thus the standing wave does not simply become the de Broglie wave, as was assumed by de Broglie.

One further feature of Fig. 4 should be noticed. The fact that the world line of the de Broglie wave is parallel to the x' axis (or, in Fig. 5, the x axis) means that in the rest frame of the particle, this wave is of infinite velocity.

[5] To show the Lorentz transformation as *active*, the diagram would include two depictions of the equiphase planes, that shown in Fig. 4 and that shown in Fig. 5.

That velocity would be anomalous in an independent wave, but acquires a natural explanation once the de Broglie wave is understood as a modulation. At rest, the crests of the underlying wave are no longer peaking in sequence, but in unison, simultaneity has been restored, alignment of phase has become instantaneous, and the velocity of the modulation describing the progress of that alignment has thus become infinite.

In effect, modulation and carrier have merged in the standing wave, and the de Broglie wave has disappeared.

7 Discussion

There is in de Broglie's thesis an evocative and much-cited passage that captures the essence of his notion of wave-particle duality:

> The particle glides on its wave, so that the internal vibration of the particle remains in phase with the vibration of the wave at the point where it finds itself (de Broglie [3], Chap. 1, Sect. 1).

And this in a sense is true. As discussed in Sect. 4 above, the particle, considered as a subluminal moving point, changes phase (see Eqn. (18)) as if it were maintaining consistency of phase with a superluminal wave having the characteristics of an independent de Broglie wave. But in referring to an "internal vibration" de Broglie seems to deny the spatially extended standing wave from which, as we saw in Sect 2, he commenced his analysis.

This and other references in de Broglie's writings to the oscillation of frequency ω_0 being "intrinsic" or "internal" or even "fictitious" (see, for instance, de Broglie [19]) may be explained by de Broglie's insistence that wave and particle are ontologically distinct entities. But it has been shown here that if the frequency ω_0 is taken to be that of a standing wave, as de Broglie himself proposed, the de Broglie wave arises as an immediate consequence of the failure of simultaneity described by the Lorentz transformation.

In none of de Broglie's demonstrations in the thesis is the de Broglie wave the independent superluminal wave that de Broglie contemplated. In the first, harmonizing of phases occurs only for an oscillating point within each wave and not for the wave as a whole. The result of Lorentz transforming that oscillating point is not a spatially extended wave, but the path described by a moving oscillating point. In the second (the mechanical model) and the third (the Minkowski diagram), the de Broglie wave emerges from the antecedent standing wave, not as a wave in its own right, but as the modulation of an underlying standing wave.

The reinterpretation of the de Broglie wave as a modulation would not affect the formal core of quantum mechanics. The de Broglie wave or wave function would still evolve in accordance with the wave characteristics determined by the relevant wave equation. But much that has seemed anomalous would be explained. The superluminal velocity of the wave would not then be that of energy (or information) transport, consistency with special relativity would be achieved and there would be no need to equate the velocity of a massive particle with the group velocity of a superposition of such waves.

Nor would it be at all mysterious that the superluminal wave does not outrun the subluminal particle or fly off at a tangent from a particle orbit. A modulation must remain forever coextensive with the wave it modulates. As the particle changed direction, there would simply be a corresponding change in the direction of dephasing and a rearrangement of phase throughout the modulated wave. This rearrangement of phase would unfold not at the superluminal velocity of the de Broglie, but at the velocity of the constituent influences of the underlying wave, which for consistency with special relativity we assume to be the velocity c of light.

But it is the suggestion of a deeper wave structure underlying the de Broglie wave that could have the larger significance for quantum theory. In the mystery of wave-particle duality, the role of wave has been played solely by the de Broglie wave. If the de Broglie wave is the modulation of a underlying standing wave structure, it becomes possible to speculate that this underlying wave, moving at the velocity of the particle and following a well-defined trajectory, might explain both the wave and the particulate properties of the particle.

And composed of influences evolving at the speed of light, the existence of this underlying structure would imply a deeper and more natural unity in Nature than could be guessed at if the only wave associated with a massive particle were of superluminal velocity and unknown ontology.

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