Dual Theories: 'Same But Different' or 'Different But Same'?

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Abstract

I argue that, under the glitz, dual theories are examples of theoretically equivalent descriptions of the same underlying physical content: I distinguish them from cases of genuine underdetermination on the grounds that there is no real incompatibility involved between the descriptions. The incompatibility is at the level of unphysical structure. I argue that dual pairs are in fact very strongly analogous to gauge-related solutions even for dual pairs that look the most radically distinct, such as AdS/CFT.

Key words: Quantum gravity, string theory, duality, underdetermination, theory equivalence

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1 Same but Different

A duality is an equivalence relation between a pair of theories: a pair of theories is said to be *dual* when they generate the *same physics.*[2] The existence of a duality signals some freedom in the representation of the relevant physics of the dual pair. However, according to the standard story, unlike ordinary symmetries (including gauge symmetries), the representations connected by dualities are often radically *different* in terms of their formulations and, if we

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[2] Where "same physics" is understood in terms of a matching of observables and global symmetries (but not gauge-dependent variables and gauge symmetries).

are literal in our reading of these formulations, also in terms of the ontological pictures they paint. It is this marked formal/ontological difference combined with qualitatively indistinguishable physics that make dualities special from the point of view of philosophy of physics, also linking them to ongoing debates in general philosophy of science. [3]

As Yeminima Ben-Menahem has pointed out, whenever one has incompatible descriptions of the world that are equally capable of describing, predicting, and explaining phenomena one has a possibility that "constitutes a challenge to the realist and comforts his opponent" (Ben-Menahem, 1990, p. 261). Structural realism is often viewed as an adequate response to this challenge: be realist about the structure that is (inevitably) shared by the descriptions at a level deeper than where the incompatibilities lie. However, it might well be possible that this same 'incompatible descriptions + observational equivalence' problem reemerges at the deeper 'structural' level in which case a primary motivation for the structuralist programme is undermined. Steven French makes no bones about it: "if the anti-realist can come up with examples of UTE where there are no common (structural) parts beyond the empirical level, then the structural realist will be scuppered" (French, 2011, p. 118). [4]

Dualities at first sight appear to fit the bill: fully empirically equivalent but underdetermined descriptions of the world that appear to diverge in their deeper structural aspects. [5] After all, one has dualities holding between descriptions with gravity (curved space) and without gravity (flat space); in four dimensions and in ten dimensions (and so topologically inequivalent); at large coupling strength and small coupling strength (a highly non-trivial difference for all cases but $g = 1$); with spacetime of large radius and spacetime of small radius (and so non-isometric as Riemannian manifolds); for electric (elementary states) and magnetic (solitonic states); with order (low temperature) and with disorder (high temperature), and so on. Surely these are sufficiently structurally orthogonal to scupper the structuralist ship? What is shared but the observable content in these cases?

But this is too quick:

[3] For example, according to Kevin Coffey's recent 'interpretation based' (non-formal) account of theoretical equivalence, dual theories would be clearly inequivalent (Coffey, 2014). I will briefly return to this account below.

[4] Structuralism is not one of my central concerns here: I'm concerned with how the dual pairs are to be treated: as physically the same or physically different. However, structuralist themes are never far from the surface given the nature of the subject matter.

[5] "[A]lternative hypothetical substructures that would surface in the same observable ways" in Quine's words (Quine, 1975, p. 313). To the best of my knowledge, all authors that have considered dualities in the philosophical literature follow this line: see, e.g. Gryb and Thébault (forthcoming).

- Dualities are not quite cases of underdetermination, at least not in the sense of the thesis of underdetermination of theory by data (*logically incompatible* yet empirically equivalent theories). There is no *competition* between dual descriptions, and the incompatibility can be transformed away in a certain sense (not available in standard underdetermination scenarios) by application of the duality group, as we shall see.
- Nor are dual descriptions along the lines of Glymour's indistinguishable (yet incompatible) spacetimes (Glymour, 1977), in which observational equivalence is guaranteed by the fact that differences show up only behind causal horizons inaccessible to observers so that no body of experimental or observational evidence could decide between competing hypotheses about certain global features of such universes. [6] There are no structures hidden from our gaze by the world's causal structure in the case of dualities: the equivalence applies to *complete sets* ('totalities') of observables (whether or not they are measurable in practice). [7]

In this article I argue that dualities are examples of equivalent descriptions *simpliciter* (when supplemented with the duality mapping) and therefore we should not be at all surprised when they generate equivalent physical content. The apparent differences emerge from the freedom in assigning physical meanings to the formulations. We should withhold realist, literal readings when a duality exists since they are examples of 'unphysical structure' functioning more or less in the same way that arbitrary coordinates and other gauges do.

Whenever a duality exists, I argue (by analogy with gauge freedom) that a formulation's natural interpretation should be taken as a mere representation of a deeper underlying structure rather than a 'fundamental' representation of the world. I claim that this procedure of finding a deeper structure eliminating the representational freedom found in any dual pair is a general feature of dualities. [8] As with gauge freedom, one can look upon dualities as vital clues

[6] Holger Lyre has called these cases of "empirical limitations" (Lyre, 2009, p.237).
[7] In fact, inasmuch as there is anything 'hidden' in one side of a dual description (say because of the breakdown of a perturbative description at higher coupling), the weakly coupled dual description *uncovers* it.
[8] They are indeed, as Michael Atiyah has claimed, "two different points of view of looking at the same object" (Atiyah, 2007, p. 69)—in this case the same *physical content*. In general, the more perspectives one has, the more information one has. But, again, this is quite unlike standard cases of underdetermination where knowing two incompatible descriptions does *not* provide more information about some 'underlying structure.' For example, having Poincaré's 'discworld' theory (with its distorting forces) *and* an infinite curved space theory (without such forces) does not provide us with more information than either would alone. [9] The fact that the descriptions are incompatible (rather than, say, complementary) means that discoveries in one do not apply to the other. Hence, it is the absence of *incompatibility* that separates dualities from underdetermination (and, as seen here, convention-

in uncovering (invariant) physical content without which one would have great difficulty in understanding the underlying structure of whatever aspect of the world one is trying to represent. Indeed, I think this is the proper way to understand the existence of gauge symmetries and dualities: having expanded mathematical ('surplus') structure leads to improvements in one's knowledge of the physics by providing different perspectives. Without the multiplication in perspectives surplus structure brings, one is in a situation not unlike the proverbial blind men and the elephant.

2 Different but Same

As Quine put it, "Scientists invent hypotheses that talk of things beyond the reach of observation" (Quine, 1975, p. 313). That much is obvious. The thesis of underdetermination of theory by data, so beloved of structural realists (since it provides a key motivation through its natural resolution), builds on this feature of science: since most working theories (certainly in physics) go beyond actual (and indeed possible) observations, for any theory T we can 'always' find some competing theory T^* that matches T with respect to the observable or observed content but differs elsewhere. More precisely, given some body of observed empirical evidence E_{obs} we have it that $\forall E_{obs}$ both $T \vdash E_{obs}$ and $T^* \vdash E_{obs}$ despite the fact that, by assumption (or construction), $T \neq T^*$. Structural realists are able to slyly evade this problem since E_{obs} involves (empirical) relational structure so that T and T^* will always wind up being structural isomoprhs and so can be identified.

So goes the story. But the general thesis [10] has recently come under fire from various directions. For example, (Earman, 1993) complains about the missing 'genuine' examples in science. Norton complains that all such cases show is that $T = T^*$, in the sense of being descriptive counterparts, for all genuine that cases we do have, and so should not trouble even standard realists. More recently (Lyre, 2009) has argued that there are in fact *no* interesting puzzling cases of UTD in the mature sciences. He dismisses the cases discussed by (Glymour, 1977) and (Malament, 1977), mentioned above, in which, as Lyre nicely expresses it, "even idealized observers, observers who live forever, are unable to determine the global topology of space" (Lyre, 2009, p. 238) on the grounds that they are based on the empirical limitations mentioned above, set by scales of various kinds.

alist moves) and, I will argue, means that dualities are in fact fully theoretically equivalent.

[10] In fact, I have in mind the weaker thesis in this paper, so that we needn't assume that such competing theories are *always* available, but only that some examples exist.

These claims are interesting and relevant, but my focus in this paper is on whatever special features dualities might bring to the table. My view is that in cases of dualities at least, Norton's view is vindicated: we have $T = T^*$ whenever T and T^* are dual. But then again, I don't think T and T^* are properly underdetermined whenever they are dual—T and T^* are dual but not *duelling* descriptions! So this might be somewhat distinct from Norton's view.

There are some distinctions to be made concerning underdetermination, depending on what content is underdetermined and by what. There are three broad classes:

- *Interpretive*[11] *underdetermination* occurs when there is a multiplicity of interpretations of one and the same formulation of a theory.
- *Formulational underdetermination* occurs when one and the same theory is presented through different formulations (which might then have ontologies of their own, which might or might not suffer from further interpretive underdetermination).
- *Theoretical underdetermination* occurs when there are multiple distinct theories compatible with the same body of evidence (usually assumed to be *complete* evidence, or all possible evidence that might observed in principle).

If dualities are to fit into any of these pre-existing categories, it would be most likely the 'formulational underdetermination' slot. Multiple formulations, in suggesting distinct ontologies, will raise a problem for the realist on any direct, literal reading of those formulations. However, as mentioned earlier, the difference with this case is that dual formulations are not incompatible: they are really the same. Why not use that same strategy whenever one meets formulational underdetermination? Because one doesn't have the kind of duality mapping linking formulations in standard cases. However, there is another kind of underdetermination that isn't quite covered by this taxonomy, namely that generated by gauge freedom. This is where dualities truly lie, though whether there is underdetermination depends on whether one chooses to treat the gauge freedom as physical, which runs strongly against the default.[12]

[11] Oliver Pooley (2006, p. 97) calls this "metaphysical underdetermination," which occurs when one formulation of a theory has many interpretations differing with respect to their ontologies (say, an interpretation with and without spacetime points). Coffey's approach to theoretical equivalence appears to simply line up with this kind of underdetermination: metaphysical underdetermination (when real) will be understood as pointing to distinct theories (and so a different level of underdetermination).

[12] The default is that gauge-related scenarios are "different but same": the difference is only at the level of unphysical structure.

It is important to bear in mind what is being claimed to be the same and what is different. In all dualities, it is the theories that are equivalent, not what the theories *refer to*. For example, in the case of T-dualities (and mirror dualities) the equivalence holds between a string theory compactified on one space (e.g. of a certain size and/or shape) and another (or the same) string theory on another space. However, even though it is the theories that are identified as equivalent, it clearly involves a certain kind of equivalence between the spaces too: it tells us that from the point of view of the observables, certain transformations of the space don't matter (leaving the observables [13] invariant and satisfying the same global symmetries). Thus, the theories are dual and the mapping between the different spaces is a symmetry of the theory (given the application of the other component of the duality switch, sending winding to momentum modes and *vice versa*). This same principle holds for other dualities. In AdS/CFT we claim that a pair of theories is equivalent (again *not* what the theories *refer* to in a natural interpretation) and this tells us something about the things that differ between the theories (in this case, the dimensionality of space, the gauge symmetries, and the degrees of freedom). [14] But once again, we can say that from the point of view of the observables and the physical symmetries, certain transformations 'don't matter.' The only difference between these transformations and standard gauge symmetries is that they seem to relate things that *look like they really should matter*! [15]

[13] To be clear, when I refer to 'observables' I am referring to genuine physical quantities in the sense of gauge-independent variables.

[14] This example is clearly far more complicated since it mixes 'spacetime dualities' (like those above) with non-spacetime dualities (namely 'entity dualities' relating the basic degrees of freedom and 'internal dualities' relating gauge symmetries).

[15] It is important to keep in mind the fact that the equivalence is at the level of theories first and foremost: the symmetries hold, e.g., from the point of view of *strings* propagating within the spacetimes. In other words, the physics of the strings is insensitive to what we would ordinarily think of as physically significant transformations (switching manifolds of distinct shapes and sizes). However, the differences *would* be physically significant from the perspective of point particles. In this case, modal considerations alone would render distinct the two worlds-according-to-the-descriptions. We must therefore remain purely in the realm of theoretical descriptions to avoid such 'external' confounders. One might object to my claim that dual pairs are theoretically equivalent on the grounds that it seems silly to think that adding particles (to both theories) could render them inequivalent. However, I think that since these are distinct theories, this is exactly what happens: in the context of the string-only pair, there is equivalence. Indeed, if we consider the strings on orbifolds (in which certain discrete symmetries are eliminated through identifications), then it becomes inconsistent to add point particles: orbifolds are not (always) manifolds, which is fine for strings but not for particles. One can make precisely the same point about the 'line-point' duality in projective geometry. Given that line and point descriptions are isomorphic, switching 'line' and 'point' (e.g. in the 'bumper book of projective geometry theorems') will not lead to falsehoods. But clearly not

This is just the same principle as occurs in, say, general relativity with its diffeomorphism invariance. Because it is a topologically invariant theory, the observables don't care about certain transformations. if one replaces the spacetime manifold with a topologically equivalent (i.e. differentiably homeomorphic) manifold, then the physics 'stays the same.' Since homeomorphic manifolds do not alter the topological properties, the observable content of the theory is unaltered under the action of a diffeomorphism. We don't call this a duality precisely because we treat the diffeomorphs as part of the same theory. We can, if we like, give the points of the manifold (via coordinates) a 'physical interpretation' (i.e. treat them *as real spacetime points*), but we know this is to be taken with a pinch of salt: we choose our coordinate grids for utility. [16] We now know better than to treat the diffeomorphs as physically inequivalent even though we could.

In the case of mirror duality that I discussed above, we have a relation involving topologically inequivalent manifolds, but again the physics is genuinely identical: it is not a matter of limitation. There is no possible experiment that could be made to distinguish M from M^* using strings living in the space. We could, again, view the two scenarios (strings on M and strings on M^*) as physically distinct, or we could accept that it points to a redundancy just like gauge freedom grounded in general relativity's topological invariance. [17] This

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everything is preserved by this duality: we are restricted to a certain kind of formal discourse. As Robert Kirk argues (Kirk, 1986, p. 178), the theorems of projective geometry are preserved under the switch only if we assume that the syntactic structures are left *uninterpreted*. Clearly if we perform the line-point switch in a sentence like 'you are looking at a line,' then if it is true for one it will not be true for the other: the duality symmetry is broken in such a case. One might also view along these lines the possible breaking of the gauge interpretation of the vector potential in classical electromagnetism by introducing quantum charges.

[16] It has become customary to drop the term 'symmetry' in the case of gauge transformations in favour of *redundancy*: symmetry is seen to be a misnomer here since solutions are not mapped to *distinct* solutions but to the same solution. Redundancy simply means 'unphysical,' and this refers to the fact that one can transform the solution in a specific way *without altering anything physical*. Gauge transformations leave the physical content (the observables) invariant. But the terminology of 'redundancy' also is a misnomer in a certain sense. The transformations are not, of course, redundant in the sense of utterly superfluous: they have a purpose after all. While physically equivalent, different gauges can differ dramatically in how useful they are in terms of modelling the physical situation. Some can be better suited to some particular physical situation. Ditto dualities.

[17] Nick Huggett (this volume) treats the intermediate case of T-duality in which the manifolds are topologically equivalent but non-isometric. This is clearly much closer to the general relativity example and submits far more readily to the same kind of 'gauge freedomesque' treatment: spatial radius is a gauge degree of freedom in the theory, and the 'true' degrees of freedom depend on the orbifold $\mathbb{R}/Z_2$ in which the freedom is eliminated (essentially deleting a portion of the real line, from

is just an invariance at a different level, deeper than we are used to dealing with, which makes it look like a more radical move to identify them physically.

Coordinates in general relativity do not have quite the same meaning as in pre-GR theories. They can serve to label points, but not in a way that latch onto 'real' features of spacetime points: there is a freedom (albeit with constraints) in how we label a spacetime's points. *Gauge* is, of course, just a more general way of speaking of coordinates in this sense. One can arbitrarily label things other than bits of space, and in the same way these labels are often devices set up with convenience in mind (and with some increase in overall information about the system underlying the labelling). Sometimes a problem will naturally 'choose' a specific labelling, thus *fixing* (selecting) a particular gauge. Terence Tao [18] has a useful way of making sense of gauge fixing (or gauge symmetry *breaking*) in terms of 'spending' a symmetry. Presumably one could also 'invest' in a gauge symmetry by *adding* mathematical structure to a theory to make it more transparent or make tractable in some way. Here is where I take the duality-gauge link to be particularly strong. One can think of the 'theories' as *frames* of reference (like coordinates), appropriate for particular situations. [19]

Richard Healey adopts the stance that gauge symmetries constitute examples of multiple realizability generated by surplus theoretical structure. As he says, "[u]nderstood realistically, the theory is epistemologically defective, because it postulates a theoretical structure that is not measurable even if the theory is true" (Healey, 2006, p. 158). I argue that dual pairs have this same flavour: in any situation in which one could be said to be measuring some observable quantity of one of a dual pair, with equal justification one could apply the

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the point of view of physical structure: string radius is not a physical observable). The two theories are really representations (with unphysical structure) of a deeper invariant structure. Indeed, it is well recognised that self-T-dual pairs are gauge symmetries in the usual sense: I propose that this viewpoint should be generalised to *all* dualities. However, as Huggett points out, one could view the duality as leading to indeterminacy about the radius (no fact of the matter), which I take to be analogous to the indeterministic bullet-biting response to the hole argument of general relativity. One could. But the point I wish to make in this paper is that the more complex duality cases are not in fact qualitatively different from the treatment of gauge freedom in general relativity (and hole argument considerations): one has the same set of interpretive options (as Huggett also mentions, this includes the lesser-held 'one true gauge (radius)' option).

[18] See "What is a gauge?": `http://terrytao.wordpress.com/2008/09/27/what-is-a-gauge/`.

[19] Michael Redhead describes the process of 'stretching' (into reality) some surplus structure by giving it a realistic interpretation (Redhead, 2003, pp. 128–9). I think that the discussions of most dualities in physics thus far are guilty of unwarranted stretching in just this sense.

duality mapping and speak of measuring some other observable quantity. One can measure neither: given the freedom, these would *ipso facto* be unobservables. However, to speak of this as an "epistemological defect' misses the great practical feature that both cases have: one can choose a gauge or dual picture according to what is most useful (or practical, simple, etc.). [20]

As Healey points out, the natural course of action is to treat the multiple realizations (that is, the various descriptions linked by gauge symmetry) as representations of one and the same physical situation. For example, one could develop an electromagnetic potential's value in many ways off an initial hypersurface, but without empirical difference, thus providing multiple realisations of the physical content. One cannot say the same for the loop integral of the potential of course, and so it is often suggested that these invariant objects provide the 'true physical content'. Now, in the case of AdS/CFT duality [21] , we know that in addition to the apparently significant differences that there is important agreement. For example, the symmetries of the Yang-Mills theory are identical with the isometries of the $AdS_5 \times S_5$ geometry, namely $SO(4,2) \times SO(6)$. Hence my proposal that the differences be viewed as unphysical in exactly the sense of gauge freedom: the duality mapping leaves invariant all of the physically meaningful quantities and symmetries.

This general approach to dualities allows us to dissolve a potential problem [22] to the view that neither description is fundamental (since they are equivalent), namely: the existence of *applied* dualities, in which, e.g. the gauge-gravity duality is applied to a real-world manipulable system, such as condensed matter systems. In such cases we simply *know* that we are not dealing with microscopic black holes in higher dimensions, so it makes sense to think that the description more closely corresponding to our knowledge of the system is, in this case, the more fundamental. But 'more fundamental' here is read as more applicable given the problem at hand. In other words, one might simply view this as 'spending the duality,' so that the symmetry is broken.

Let's finish off by linking the preceding discussion to more general issues of theoretical equivalence. Prima facie, dual descriptions appear to satisfy Quine's 'inter-translatability' identity criterion of theories:

[20] As Polchinski points out in his contribution to this issue, in the case of a strong-weak duality, as coupling is increased what might have been a good approximation for the first few terms in a perturbation series breaks down and can fail to reveal qualitatively new physics that exists at some energy (confinement in QCD for example). Investing in duality, like gauge symmetry, increases one's grip on 'the physics.' Duals form an entangled pair in a very specific way. Having duals increases the grasp one has of the 'underlying physical content'—as Polchinski puts it, "having an understanding of the two limits gives a powerful global picture of the physics".
[21] See the contributions of Polchinski and de Haro for the technical details.
[22] My thanks to Radin Dardashti for drawing my attention to this problem.

> I propose to individuate theories thus: two formulations express the same theory if they are empirically equivalent and there is a construal of predicates that transforms one theory into a logical equivalent of the other. (Quine, 1975, p. 320)

Quine's criterion is intended to reduce the amount of underdetermination—essentially by employing a Ramsey sentence framework. The price is a position in which the theoretical ontology is left *indeterminate*. While I think Quine's criterion provides a useful analogy, there is something rather different in the case of dual theories: it is not a case of the ontology being indeterminate precisely because there is no incompatibility involved: there is convergence on the same ontology though that is left unspecified to a certain extent—this is the content of the phrase "dual but not duelling" above.

With dualities we have syntactic isomorphism plus observational equivalence, yet the dual descriptions are nonetheless usually treated as distinct theories since the syntactic structures, though isomorphic and generating equivalent observable content, receive distinct *physical* interpretations. This is the claim at least: one does not view the interpretation as merely a useful means (a set of labels) for dealing with the syntax but as potentially describing reality: as physical degrees of freedom. That is what I think we should deny. Doing so has the potential to resolve various puzzles having to do with dualities. However, I don't view the situation as trivial as Norton's expression: "the same theory dressed in different clothes" suggests. ([2008], p. 33). What the dualities reveal is that there are deep lessons to be learned about the identification of unphysical degrees of freedom, in order to isolate the theory underlying the dual pair.

Let's return to Coffey's interpetational view of theoretical equivalence to help spell out this last remark. Coffey argues that certain 'physical' differences are strong enough to stop the conclusion that what are commonly seen to be equivalent formulations (e.g. Hamiltonian and Lagrangian formulations of Newtonian mechanics [23]) are simple 'notational variants.' His position is stated thus:

> Two theoretical formulations are theoretically equivalent exactly if they say the same thing about what the physical world is like, where that content goes well beyond their observable or empirical claims. Theoretical equivalence is a function of interpretation. It's a relation between completely interpreted formulations. (Coffey, 2014, pp. 834-5)

[23] Nic Teh (this volume) considers this case, and puts pressure on the arguments that suggest an inequivalence between such formulations (on account of the non-existence of an isomorphism between the two descriptions): he argues that finding an isomorphism is a work in progress rather than a settled matter.

He unpacks this in the following terms, which closely relate to our presentation of the issue facing dual pairs:

> Insofar as we can understand the physical pictures provided by different interpreted formalisms, and insofar as we're capable of comparing those pictures, we can straightforwardly determine whether two interpreted formulations are theoretically equivalent, whether they say the same thing about what the physical world is like. (ibid.)

Now, this looks like a sensible suggestion. We might easily transfer (as Doreen Fraser seems to do, in fact, in this volume) the idea to dual pairs: they clearly make very different physical claims *on the surface*. However, and this really forms the backbone of my position, one must be careful with one one considers to be *physical* structure in a representation (i.e. rather than surplus), as we have seen. Gauge theories also look like they make physically distinct claims: they can differ in which spacetime point does what, or in which vector potential is realised, and so on. But the point is, we can choose not to admit these differences as physically significant, and it is often a good idea not to do so. So one always has an initial choice to make before Coffey's analysis proceeds: let's work out just what is physical and unphysical, and then we can look and see whether they say physically different things about the world. In the case of dualities, however, there is no reason to assume that they are not just as in the gauge theory case. [24]

The claim that dual pairs are theoretically equivalent does not, then, amount to the simple positivistic claim that empirical equivalence is all that matters. There is a deeper argument going on here relaying on the duality mapping holding dual pairs together in a way that standard underdetermination (and conventionalist) scenarios do not. In the case of AdS/CFT one still has agreement on the genuine symmetries (i.e. those that change the physical state in a way that leaves the dynamics invariant). [25] As Polchinski puts it, the dualities

[24] Curiously, Coffey mentions that Norton says something similar to the point I'm making here, but he doesn't follow up on it (*cf.* Coffey (2014, pp. 840)). The basic idea of using dual pairs to identify unphysical structure (and physical structure!) fits rather closely with James Weatherall's approach to theoretical equivalence in which one must look to an equivalence class of theories to decide what is physical in any member (unpublished manuscript). The difference in my duality-specific claim, however, is that the existence of such an equivalence class points to the existence of a deeper theory characterised by the shared structure. It would be an interesting project to explicitly apply Weatherall's approach to dual pairs.

[25] One must bear in mind that there are several highly unrealistic elements in play in this example: crucially, being a conformal theory we find that the physical quantities (masses and couplings) don't 'run with the energy scale' used to measure them. We should not be overly confident of our intuitions about what is physical and what is unphysical here—note, of course, that 'unphysical' here does not mean 'unrealistic.'

are "blind to gauge invariances" but not to global symmetries since the latter "act non-trivially on physical states". The shared global symmetry group in this case is $PSU(2,2|4)$ which means that physical quantities are left invariant under this symmetry *on both sides.* This breaks down into a shared (bosonic) subgroup of $SO(4,2) \times SO(6)$, which corresponds precisely to the isometry group of the string theory background (the product $AdS_5 \times S^5$) and the global symmetries of the gauge theory (the product of the conformal group and the symmetry group of six scalars). Hence, the group structure of the dual pair is identical.

In their book on symmetries, Fuchs and Schweigert make the following insightful remark about the relationship between dual theories and gauge symmetries:

> In practice, there is frequently a natural notion of what one should consider as a theory, so that any further analysis of the difference between these types of symmetries [gauge versus duality] would be a rather academic exercise. On the other hand, it can be a major breakthrough in science to discover a symmetry between *a priori* different theories [a duality] and then promote it to a symmetry of the first type [a gauge symmetry]. (Fuchs, 1997, p. 14)

I hold that this can be generalized to *all* cases in which one has a duality symmetry: they can always be promoted to gauge-type symmetries because they just are gauge-type symmetries.

To sum up what I take to be the take home message of this paper: when we have (genuine) dualities apparently holding between a pair of theories, we ought not to take either picture as fundamental or (entirely) physical. The mere existence of a duality points to unphysical structure in the dual theories. Thus, any interpretation we give is 'provisional,' and points towards some common core. This common core might be fully understood (as a deeper theoretical structure encompassing both dual theories) or might be known only via the limited information provided by the dual pair. [26] I take this to be in agreement with gauge freedom and the relationship of such freedom to the gauge invariant structure it reveals: all dual pairs correspond to representations of a deeper structure. This is the lesson of taking the analogy with gauge freedom seriously.

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[26] One will automatically know certain things about the deeper theory, such as the (effective) physical observables and symmetries.

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