

Hilbert Space Quantum Mechanics is Contextual (Reply to R. B. Griffiths)

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Abstract

In a recent paper Griffiths [38] has argued, based on the consistent histories interpretation, that Hilbert space quantum mechanics (QM) is noncontextual. According to Griffiths the problem of contextuality disappears if the apparatus is “designed and operated by a competent experimentalist” and we accept the *Single Framework Rule* (SFR). We will argue from a representational realist stance that the conclusion is incorrect due to the misleading understanding provided by Griffiths to the meaning of quantum contextuality and its relation to physical reality and measurements. We will discuss how the quite general incomprehension of contextuality has its origin in the “objective-subjective omelette” created by Heisenberg and Bohr. We will argue that in order to unscramble the omelette we need to disentangle, firstly, representational realism from naive realism, secondly, ontology from epistemology, and thirdly, the different interpretational problems of QM. In this respect, we will analyze what should be considered as *Meaningful Physical Statements* (MPS) within a theory and will argue that *Counterfactual Reasoning* (CR)—considered by Griffiths as “tricky”—must be accepted as a *necessary condition* for any representational realist interpretation of QM. Finally we discuss what should be considered as a problem (and what not) in QM from a representational realist perspective.

Keywords: quantum contextuality, physical reality, basis problem.

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Introduction

Contextuality is one of the main features of Quantum Mechanics (QM), a feature which has been present since the early discussions of the founding fathers. Indeed the relation of the quantum formalism to definite experimental arrangements is one that has no analogue in classical theories. However, there is still today no consensus in the community of physicists and philosophers who discuss the foundational problems of QM, about the exact definition or the physical meaning of quantum contextuality. This controversy goes back to the discussions and debates between, for example, Einstein and Bohr. While Einstein used the contextual character of the theory in order to show the inconsistencies of the quantum formalism when related to *elements of physical reality* —as defined in his 1935 EPR paper, co-authored jointly with Podolsky and Rosen [33]—, Bohr used exactly this same feature in order to answer the EPR argument [12]. In a recent paper Griffiths has argued that Hilbert space QM is actually noncontextual. According to Griffiths the problem of contextuality disappears if the apparatus is “designed and operated by a competent experimentalist” and we accept the *Single Framework Rule* (SFR). In the present paper we attempt to argue, not only that contextuality is one of the main features of the quantum formalism, but also that there are several flaws in the arguments presented by Griffiths which are mainly due to the multiple shifts in the metaphysical standpoint of analysis. In order to make clear our own stance we will begin by clearly stating our representational realist stance from which we attempt to discuss and analyze Griffiths’ arguments against quantum contextuality. The paper is organized as follows. In section 1, we discuss what are the main features of a representational realist stance with respect to physics distinguishing it from “naive” realism and other philosophical positions which deny the theory-ladenness of physical experience. Section 2 provides an analysis of the meaning of contextuality and the physical constraints it involves in relation to any interpretation of QM that attempts to stay close to the orthodox Hilbert space formalism. In section 3, we discuss the “quantum omelette” created by Bohr and Heisenberg by cooking together objective and subjective perspectives of analysis. Section 4 attempts to describe the orthodox problems of QM, their specific questioning, their metaphysical presuppositions and limits of applicability. We shall focus on the distinction between the basis problem and the measurement problem. Section 5 argues in favor of the definition of Meaningful Physical Statements (MPS) within a theory and the need to consider Counterfactual Reasoning (CR) as a *necessary con-*

dition in order to provide a physical representation of reality. In section 6 we analyze and discuss Griffiths’ arguments against quantum contextuality in terms of four main points put forward in [38]. Finally, in section 7, we discuss what should be considered as a problem (and what not) in QM from a representational realist perspective.

1 The Representational Realist Stance

Physics has been always connected to different philosophical stances, but certainly, realism is one of the main viewpoints within the history of physics. The main presupposition of realism with respect to physics is that physical theories talk about reality. According to our representational realist stance [21, 22], which will be presupposed through the rest of the paper, the fundament of any physical theory is physical representation and not experimental data —the latter should be regarded by a representational realist only as part of the confirmation (or failure) of the empirical adequacy of a theory. Indeed, the representational realist takes as a standpoint the existence of Nature adding to it the idea that such existence can be represented through the interrelation of mathematical formalisms and conceptual networks allowing us to predict and understand specific phenomena. This realist stance, which relates to Heisenberg’s closed theory approach [13], goes against any type of “naive” realism that denies the theory ladenness of physical experience. Against the idea that one could distinguish between phenomena and raw observable data we have argued in [22] that even a ‘click’ in a detector or a ‘spot’ in a cloud chamber are only determined through the logical and ontological principles of existence, non-contradiction and identity. Such principles are not something that we find out in the world but rather metaphysical presuppositions that shape even our most basic experience. We have argued that even ‘clicks’ and ‘spots’ found in a laboratory are theory laden. Any stance going against representational realism must be capable of producing arguments that explain how physical experience can account for phenomena without the need of presupposing a physical representation.

Physical representation allow us to think about experience and predict phenomena without the need of actually performing any experiment. This is of course a completely different standpoint from those of many empiricist approaches who argue instead that the fundament of physics is ‘actual experimental data’. For example, as remarked by van Fraassen [50, pp. 202-203]: “To develop an empiricist account of science is to depict it as involving

a search for truth only about the empirical world, about what is actual and observable.” Even though there are different positions with respect to the consideration of what accounts to be an empirically adequate theory (see e.g. [11, p. 351]) we would like to draw a line distinguishing between those stances that accept the theory ladenness of physical experience and those which deny it. Our analysis is only concerned with the former.¹

Following van Fraassen [51, p. xviii], we have called the attention to the importance of making explicit the metaphysical stance that one takes in order to analyze a specific problem [21, 22]. As we shall argue in this paper, the mixing and entanglement of realist and empiricist standpoints and of ontological and epistemological perspectives within QM has produced a weird mix of problems which are too frequently turned into pseudoproblems. In this respect we would like to make clear right from the start what should be considered to be the kernel of a representational realist account of physics:

Representational Realism about Physical Theories (RRPT): *A representational realist account of a physical theory must be capable of providing a physical representation of reality in terms of a network of concepts which relates to the mathematical formalism of the theory and allows to make predictions of definite phenomena.*

Contrary to our definition of realism in physics which considers representation as a main construct of physical theories, naive realism claims instead that physical observation provides direct access to reality *as it is*. This idea was already implicit in the logical positivist distinction between *theoretical terms* and *empirical terms*. But even though in the philosophy of science community this distinction is characterized as “naive”, many of the problems discussed in the literature still presuppose implicitly such distinction. Indeed, as remarked by Curd and Cover [18, p. 1228]: “Logical positivism is dead and logical empiricism is no longer an avowed school of philosophical thought. But despite our historical and philosophical distance from logical positivism and empiricism, their influence can be felt. An important part of their legacy is observational-theoretical distinction itself, which continues to

¹We acknowledge however that the theory ladenness of physical experience is still today controversial within philosophy of science. As remarked by Bogen and Woodward [11, p. 304]: “The positivist picture of the structure of scientific theories is now widely rejected. But the underlying idea that scientific theories are primarily designed to predict and explain claims about what we observe remains enormously influential, even among the sharpest critics of positivism.”

play a central role in debates about scientific realism.” We have argued that the naive realist stance is not only philosophically untenable but maybe even more importantly, closes the door to a fundamental development of physics —since such stance assumes we already know what reality *is* in terms of (naive) observation.

From a realist perspective, physics attempts to describe a world in which we humans have no special preeminence with respect to existence. In this respect, the description or representation provided by classical physics was clearly specified since Newton’s mechanics in terms of systems constituted by definite valued properties; i.e., in general terms, what is called an *Actual State of Affairs* (ASA).² Also from a realist viewpoint, measurement and observation have been always considered as a way of exposing or discovering such preexistent ASA. But, as we know —contrary to classical physics— QM places serious difficulties for such a realist representation. An evidence of the deep crisis of physical representation within the theory of the quanta is the fact that more than one century after its creation the physics community has reached no consensus about what the theory is talking about. Indeed, for many a consistent interpretation that would match this strange formalism to a physical representation of reality seems to difficult to be found, for others it is enough for QM to account for the correct measurement outcomes.

Of course, when discussing about QM and its interpretation there are multiple standpoints and interpretative strategies that one can assume. For example, one can argue —as it has been done already by Fuchs and Peres [34, p. 70]— that “[...] quantum theory does not describe physical reality. What it does is provide an algorithm for computing probabilities for the macroscopic events (“detector clicks”) that are the consequences of experimental interventions. This strict definition of the scope of quantum theory is the only interpretation ever needed, whether by experimenters or theorists.” This instrumentalist perspective is satisfied with having an algorithmic recipe that allows us to calculate measurement outcomes from the formalism. But this is certainly not enough for a representational realist, for whom the formalism should be capable of relating coherently an interpretation which allows to provide a physical representation of reality according to the theory [22]. In contrast, within a representational realist approach to QM there are two main possibilities. The first one is to argue that QM makes reference to the same physical representation provided by classical physics; i.e. that it talks about an ASA. This is, for example, the main idea

²See for discussion and definition of this notion in the context of classical physics [26].

presupposed by the Hidden Variable Program (HVP) which, as noticed by Bacciagaluppi [7, p. 74], attempts to “restore a classical way of thinking about *what there is*.” The second possibility is to consider that QM might describe physical reality in a different, maybe even incommensurable, way to that of classical physics. This possibility seems to be endorsed by Griffiths [38, p. 174] who argues that many of the problems with the interpretation of QM come from “the view that the real world is classical, contrary to all we have learned from the development of quantum mechanics in the twentieth century.” In relation to this debate, one of the main aspects that divides the foundational community discussing the interpretation of QM is the issue of contextuality. Unfortunately, the meaning of quantum contextuality and its implication with respect to the interpretation of the theory remains within the literature not only unclear but also misleading due —we will argue— to an entangled analysis which mixes ontological and epistemological concerns, problems and solutions.

2 The Contextual Character of QM

We have argued extensively in [21] that quantum contextuality is directly related to the impossibility to provide a description in terms of an ASA. This is something completely new in physics. In classical physics, every physical system may be described exclusively by means of its *actual properties*, taking “actuality” as expressing the *preexistent* mode of being of the properties themselves, independently of observation —the “pre” referring to its existence previous to measurement.³ The evolution of the system may be described by the change of its actual properties. Mathematically, the state is represented by a point $(p; q)$ in the correspondent phase space Γ and, given the initial conditions, the equation of motion tells us how this point moves in Γ . Physical magnitudes are represented by real functions over Γ . These functions can be all interpreted as possessing definite values at any time, independently of physical observation. Thus, as mentioned above, each magnitude can be interpreted as being actually preexistent to any possible measurement without conflicting with the mathematical formulation of the theory. In this scheme, speaking about potential or possible properties usually refers to functions of the points in Γ to which the state of the system might arrive to in a future instant of time; these points, in turn

³Notice that even relativity theory allows for a description in terms of an ASA since space-time events are always considered as preexistent.

are also completely determined by the equations of motion and the initial conditions. It should be also remarked that just like the notions of possibility and potentiality, classical probability —considered as a physical notion⁴— also makes always reference (in terms of ignorance) to a preexistent ASA.

In QM, contrary to the classical scheme, physical magnitudes are represented by operators on \mathcal{H} that, in general, do not commute. This mathematical fact has extremely problematic interpretational consequences for it is then difficult to affirm that these quantum magnitudes are *simultaneously preexistent* (i.e., objective). In order to restrict the discourse to sets of commuting magnitudes, different Complete Sets of Commuting Operators (CSCO) have to be (subjectively) chosen. And here is where the mixing of the objective and the subjective takes place. Indeed, the way to solve this uncomfortable situation within the orthodox approach is to introduce a subjective choice —in between the many contexts— that reintroduces superficially the classical structure (see for discussion [26]). It is this *ad hoc* move, which mixes the subjective with the objective, what needs to be physically justified. But even what many consider the best candidate to account for this interpretational maneuver, namely the principle of decoherence, has failed to provide a convincing physical explanation of the quantum to classical limit [1, 35]. As remarked by Bacciagaluppi [8], some physicists and philosophers still believe “decoherence would provide a solution to the measurement problem of quantum mechanics. As pointed out by many authors, however (e.g. Adler 2003; Zeh 1995, pp. 1415), this claim is not tenable. [...] Unfortunately, naive claims of the kind that decoherence gives a complete answer to the measurement problem are still somewhat part of the ‘folklore’ of decoherence, and deservedly attract the wrath of physicists (e.g. Pearle 1997) and philosophers (e.g. Bub 1997, Chap. 8) alike” (see also [9]). As a matter of fact, still today there is no convincing physical explanation of the so called “quantum to classical limit”.

The idea that one needs to choose (subjectively) a context in order to determine which are the (objective) observables that have definite values is not only a *contradictio in adjecto* since it is the subjective choice which deter-

⁴As remarked in [21, chapter 7] probability should be distinguished as a physical notion from a mathematical one. The latter does not need to be considered necessarily in terms of a physical interpretation. However, this mathematical understanding of probability departs from a realist understanding of it. As Schrödinger [17, p. 115] makes the point: “[a probabilistic statement] has meaning only if one is indeed convinced that the something in question quite definitely *is* or *is not* the case. A probabilistic assertion presupposes the full reality of its subject.”

mines objective physical existence explicitly —the ASA is only determined after the choice of the experimenter— but also violates explicitly *counterfactual reasoning* which is maybe the most important feature of physical representation itself—a feature which allows us to go beyond the discourse about mere ‘measurement outcomes’ (we will come back to this point in section 5). Let us be now more specific about the formal meaning of contextuality through the analysis of the formalism of the theory (see also [28]).

In QM the frames under which a vector is represented mathematically are considered in terms of orthonormal bases. We say that a set $\{x_1, \dots, x_n\} \subseteq \mathcal{H}$ an n -dimensional Hilbert space is an *orthonormal basis* if $\langle x_i | x_j \rangle = 0$ for all $1 \leq i < j \leq n$ and $\langle x_i | x_i \rangle = 1$ for all $i = 1, \dots, n$. A physical quantity is represented by a self-adjoint operator on the Hilbert space \mathcal{H} . We say that \mathcal{A} is a *context* if \mathcal{A} is a commutative subalgebra generated by a set of self-adjoint bounded operators $\{A_1, \dots, A_s\}$ of \mathcal{H} . Quantum contextuality, which was most explicitly recognized through the Kochen-Specker (KS) theorem [46], asserts that a value ascribed to a physical quantity A cannot be part of a global assignment of values but must, instead, depend on some specific context from which A is to be considered. Let us see this with some more detail.

Physically, a global valuation allows us to define the preexistence of definite properties. Mathematically, a *valuation* over an algebra \mathcal{A} of self-adjoint operators on a Hilbert space, is a real function satisfying,

1. *Value-Rule (VR)*: For any $A \in \mathcal{A}$, the value $v(A)$ belongs to the spectrum of A , $v(A) \in \sigma(A)$.
2. *Functional Composition Principle (FUNC)*: For any $A \in \mathcal{A}$ and any real-valued function f , $v(f(A)) = f(v(A))$.

We say that the valuation is a *Global Valuation (GV)* if \mathcal{A} is the set of all bounded, self-adjoint operators. In case \mathcal{A} is a context, we say that the valuation is a *Local Valuation (LV)*. We call the mathematical property which allows us to paste consistently together multiple contexts of LVs into a single GV, *Value Invariance (VI)*. First assume that a GV v exists and consider a family of contexts $\{A_i\}_I$. Define the LV $v_i := v|_{A_i}$ over each A_i . Then it is easy to verify that the set $\{v_i\}_I$ satisfies the *Compatibility Condition (CC)*,

$$v_i|_{A_i \cap A_j} = v_j|_{A_i \cap A_j}, \quad \forall i, j \in I.$$

The CC is a necessary condition that must satisfy a family of LV s in order to determine a GV . We say that the algebra of self-adjoint operators is VI if for every family of contexts $\{A_i\}_I$ and LV s $v_i : A_i \rightarrow \mathbb{R}$ satisfying the CC , there exists a GV v such that $v|_{A_i} = v_i$.

If we have VI , and hence, a GV exists, this would allow us to give values to all magnitudes at the same time maintaining a CC in the sense that whenever two magnitudes share one or more projectors, the values assigned to those projectors are the same in every context. The KS theorem, in algebraic terms, rules out the existence of GV s when the dimension of the Hilbert space is greater than 2. The following theorem is an adaptation of the KS theorem —as stated in [31, Theorem 3.2]— to the case of contexts:

Theorem 2.1 (KS Theorem) *If \mathcal{H} is a Hilbert space of $\dim(\mathcal{H}) > 2$, then a global valuation is not possible.* \square

After having recalled the KS theorem we are now ready to add some physical discussion. Some remarks go in order:

- I. *KS type theorems preclude a physical representation of the formalism in terms of an ASA.*

If physical reality is conceived in terms of an ASA, then it is not possible to claim that Ψ describes an actual situation irrespectively of the choice of a context. Not all observables can be considered to be simultaneously actual (real). Notice that such consequence only applies when the metaphysical equality, ‘Actuality = Reality’, is assumed.

- II. *KS type theorems have nothing to do with probabilities.*

Going against a common phrase —continuously repeated within the literature— that says that “QM is a probabilistic theory” it should be clear that KS theorem does not talk about mean values of observables —as it is the case for Bell inequalities— but instead discusses about the definite values of quantum properties [21, chapter 5].

- III. *KS type theorems have nothing to do with measurements.*

There is no need of actual measurements for the KS theorem to stand. The theorem is not talking about measurement outcomes, but about the *preexistence* of properties, or in other words, about the constraints implied by the formalism to projection operators (interpreted in terms

of properties that pertain to a quantum system). Therefore, quantum contextuality cannot be tackled through an analysis in terms of measurements simply because there is no reference at all to any measurement outcome.

IV. *KS type theorems have nothing to do with evolution or dynamics of properties.*

There is no evolution or dynamics considered for the KS theorem to stand. The theorem makes reference to the possible values of projection operators (interpreted as properties of a system) at one single instant of time. There is no question regarding the evolution of such properties, it is only their simultaneous consistent values considered from different contexts which is at stake.

V. *KS type theorems in a nutshell.*

Put in a nutshell, quantum contextuality deals with the formal conditions that any realist interpretation which respects orthodox Hilbert space QM must consider in order to consistently provide a physical representation of reality.

Contextuality does not imply that QM is condemned to describing a subjectivist world in which each experimenter decides what is real and what is not, it only shows that there are severe constraints to any attempt to interpret the orthodox formalism of QM in terms of “classical physical reality”. But of course taking contextuality into account, there are different possibilities to be discussed and analyzed (section 7). However, we understand that to deny contextuality in QM is simply to deny the formalism of the theory, for this feature is a consequence of the fundamental mathematical structure of the theory, a formal scheme which has been allowing us for more than one century to produce the most outstanding predictions of phenomena. The feature of contextuality emerges from the formalism itself, it is not something external to it. Trying to neglect contextuality by adding *ad hoc* rules in order to recover a particular metaphysical scheme seems to us highly problematic for physics should be considered as a field in continuous development and not one that attempts to justify or match what we already know.

Unfortunately, instead of regarding quantum contextuality as a new interesting feature of a physical theory most attempts go against it in order to recover a physical representation in terms of an ASA. The first attempt

is the HVP which searches for a new formalism of QM in order to satisfy certain metaphysical desires. A second, less explicit approach, is assumed by several interpretations (e.g., CH interpretation) which introduce *ad hoc* rules in order to recover a classical discourse.⁵ The CH interpretation, just like the HVP, also denies the formalism, not by changing it explicitly — as in the case of the former— but by “blocking” its consequences through the introduction of the *Single Framework Rule* (SFR). We believe that to deny contextuality just because it obstructs an interpretation of the theory in terms of actuality would be tantamount to trying to deny the Lorentz transformations in special relativity simply because of its implications to the contraction of rigid rods. Indeed, this was the attempt of most conservative physicists until Einstein made the strong interpretational move of taking seriously the formalism of the theory and derived the physical consequences from it. If we accept the orthodox formalism, then contextuality is the crux of QM, it is that which needs to be physically interpreted instead of something that needs to be destroyed because of its non-classical consequences.

3 The ‘Objective-Subjective Omelette’ in QM

One of the main reasons for the general incomprehension of the deep consequences of contextuality within the literature is the mixing of ontological and epistemological perspectives and problems. As most clearly stated by Jaynes:

“[O]ur present [quantum mechanical] formalism is not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature —all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble. Yet we think that the unscrambling is a prerequisite for any further advance in basic physical theory. For, if we cannot separate the subjective and objective aspects of the formalism, we cannot know what we are talking about; it is just that simple.” [45, p. 381]

Indeed, we have argued extensively that Bohr is responsible for producing an epistemological interpretation of QM that does not only limit

⁵As we shall discuss in section 5 such interpretations block the set of meaningful physical statements that can be derived from the theory itself.

physical representation in terms of classical language and phenomena but also precludes the very possibility of introducing and developing new (non-classical) concepts [23, 24]. This was done by Bohr by means of two main desiderata. Unfortunately, since the mid 20th century the Orthodox Line of Research (OLR), presupposed these two Bohrian desiderata as almost necessary standpoints to think about the interpretation of QM. The first metaphysical presupposition is the idea that there must exist a “quantum to classical limit”, implying what Bokulich calls an “open theory approach” [13]. This idea was put forward by Bohr in terms of his *correspondence principle* [15].

1. **Quantum to Classical Limit:** The principle that one must find a “bridge” or “limit” between classical mechanics and QM.

The second metaphysical principle which has guided the OLR can be also traced back to Bohr’s claim that physical experience needs to be expressed exclusively in terms of classical physical language [16].

2. **Classical Representation of Physics:** The principle that one needs to presuppose the classical representation of physics in order to discuss about *phenomena* and interpret QM.

Bohr [53, p. 7] claimed that: “[...] the unambiguous interpretation of any measurement must be essentially framed in terms of classical physical theories, and we may say that in this sense the language of Newton and Maxwell will remain the language of physicists for all time.” In this respect, also according to Bohr [*Op. cit.*], “it would be a misconception to believe that the difficulties of the atomic theory may be evaded by eventually replacing the concepts of classical physics by new conceptual forms.”

Heisenberg, probably forced by his controversial political situation after the war, helped Bohr to support his epistemological approach even putting a name to it: “The Copenhagen Interpretation of QM” (see for discussion [44]). The book *Physics and Philosophy* [42] —where the term appeared for the first time— shows, on the one hand, Heisenberg’s fantastic historical and philosophical knowledge about physics, and on the other, his troubled —close to complete inconsistency— interpretation of QM. The so called Copenhagen interpretation invented by Heisenberg attempted to bring together not only Bohr’s epistemological approach but also his own Platonist realism about mathematical equations in physical theories [?, p. 91]. While Bohr’s anti-metaphysical commitment considered the language of classical

physics as the very fundament of phenomena, Heisenberg's closed theory approach insisted in the radical incommensurability of the concepts used in different physical theories [14]. A good example of the quantum omelette cooked in Heisenberg's book is the following passage:

“With regard to this situation Bohr has emphasized that it is more realistic to state that the division into the object and the rest of the world is not arbitrary. Our actual situation in research work in atomic physics is usually this: we wish to understand a certain phenomenon, we wish to recognize how this phenomenon follows from the general laws of nature. Therefore, that part of matter or radiation which takes part in the phenomenon is the natural ‘object’ in the theoretical treatment and should be separated in this respect from the tools used to study the phenomenon. *This again emphasizes a subjective element in the description of atomic events, since the measuring device has been constructed by the observer, and we have to remember that what we observe is not nature in itself but nature exposed to our method of questioning.* Our scientific work in physics consists in asking questions about nature in the language that we possess and trying to get an answer from experiment by the means that are at our disposal. In this way quantum theory reminds us, as Bohr has put it, of the old wisdom that when searching for harmony in life one must never forget that in the drama of existence we are ourselves both players and spectators. It is understandable that in our scientific relation to nature our own activity becomes very important when we have to deal with parts of nature into which we can penetrate only by using the most elaborate tools.” [42, p. 9] (emphasis added)

Of course, the fact we should acknowledge physical representation and experience has been created through concepts and tools specifically designed is completely different from claiming that the choice of a specific experimental arrangement determines explicitly physical reality itself. As we argued above, the KS theorem (formulated in 1967) showed that if one wishes to retain —as Bohr did— the classical representation of physics in terms of an ASA then the choice made by a competent experimentalist of a subset of observables —between the many possible *incompatible* subsets of observables— determines in a subjective manner what is to be considered as actual (real). This is because, due to the non-commutative structure of the theory, not all observables can be considered as real (actual) before the choice. This

situation regarding the definition of physical reality was clearly recognized by Einstein who remained always uncomfortable with the epistemological reasoning of Bohr. As recalled by Pauli:

“Einstein’s opposition to [quantum mechanics] is again reflected in his papers which he published, at first in collaboration with *Rosen* and *Podolsky*, and later alone, as a critique of the concept of reality in quantum mechanics. We often discussed these questions together, and I invariably profited very greatly even when I could not agree with *Einstein*’s view. ‘Physics is after all the description of reality’ he said to me, continuing, with a sarcastic glance in my direction ‘or should I perhaps say physics is the description of what one merely imagines?’ This question clearly shows *Einstein*’s concern that the objective character of physics might be lost through a theory of the type of quantum mechanics, in that as a consequence of a wider conception of the objectivity of an explanation of nature the difference between physical reality and dream or hallucination might become blurred.” [47, p. 122]

But quite independently of Einstein’s efforts to discuss the possible representation of quantum reality, the Bohrian approach has become a silent orthodoxy that limits the analysis of QM down to the almost exclusive set of problems—which capture Bohr’s main desiderata—that presuppose always the representation of reality that results from classical physics. Firstly, the presupposition that there must exist a continuous limit between QM and classical physics. Secondly, the idea that phenomena must be always represented and understood as classical space-time phenomena. Unfortunately, these problems have been also entangled with incompatible philosophical stances, producing a lot of confusion not only with respect to the metaphysical presuppositions and standpoints of the problems themselves but also with respect to their limits of inquiry.

4 Disentangling Orthodox Problems in QM

As analyzed in [23] the OLR deals with a specific set of problems which analyze QM from a classical perspective. This means that all problems assume as a standpoint a classical representation and only reflect on the formalism in “negative terms”, that is, in terms of the failure of QM to account for the classical representation of reality and its concepts: separability, space, time,

locality, individuality, identity, actuality, etc. The “negative” problems are thus: *non-separability*, *non-locality*, *non-individuality*, *non-identity*, etc.⁶ These problems start their analysis from the notions of classical physics assuming implicitly as a standpoint the strong metaphysical presupposition that QM should be able to represent physical reality according to such classical notions. But in between the many problems that can be found in the literature there are two unsolved main problems which show most explicitly the impossibilities of QM with respect to classical physics. The first problem relates directly to the issue of contextuality and is called the “basis problem”:

Basis Problem (BP): *Given the fact that Ψ can be expressed by multiple incompatible bases —given by the choice of a CSCO— and that due to the KS theorem the observables arising from such bases cannot be interpreted as simultaneously preexistent, the question is: how does Nature make a choice between the different bases? Which is the objective physical process that leads to a particular basis instead of a different one?*

Once again, the BP is a way of discussing quantum contextuality in “negative terms”. The problem already sets the solution through the specificity of its questioning. The problem presupposes that there is a path—in accordance to the quantum to classical limit imposed by Bohr—from the weird contextual quantum formalism to a classical noncontextual experimental set up in which classical discourse holds. If one could explain this path through an objective physical process then the choice of the experimenter could be regarded also as part of an objective process as well—and not one that determines reality. Unfortunately, still today the problem remains with no solution within the limits of the orthodox formalism. There is no physical representation of the process without the addition of strange *ad hoc* rules; rules which not only lack any physical justification but, more importantly, also limit the MPS of the theory (section 5).

A very different problem—sometimes also mixed and partly confused with the BP—is the so called “measurement problem” which deals explicitly with the superposition principle and takes as a standpoint a specific basis or context.

Measurement Problem (MP): *Given a specific basis (or CSCO) QM describes mathematically a state in terms of a superposition (of states), since*

⁶I am grateful to Bob Coecke for this linguistic insight.

the evolution described by QM allow us to predict that the quantum system will get entangled with the apparatus and thus its pointer positions will also become a superposition,⁷ the question is why do we observe a single outcome instead of a superposition of them?

The measurement problem is also a way of discussing the formalism in “negative terms” with respect to classical physics. In this case the problem concentrates in the justification of measurement outcomes —instead of trying to physically represent quantum superpositions in terms of new non-classical concepts (section 7). It should be remarked that the MP presupposes that the basis (or context) —directly related to a measurement set up— has been already determined. Thus it should be clear that there is no question regarding the contextual character of the theory within this specific problem. But once the experimental arrangement is settled —leaving aside the BP— a new problem appears: due to the superposition principle one can find, within a context, that the state is mathematically described in term of the so called “quantum superpositions” which are weird mathematical expressions composed by sets of states, $|\alpha_i\rangle$, each one of them giving rise to a set of (compatible) projectors, $|\alpha_i\rangle\langle\alpha_i|$, interpreted as *compatible observables* (or properties). As we have discussed in [19], such quantum superpositions can be, in general, composed by *contradictory properties*. Notice that given a Ψ we call a quantum superposition to each different representation of the Ψ in a specific basis. Thus the Ψ gives rise to different superpositions, each one of them determined within a CSCO. This goes against the orthodox assumption that every superposition arising from Ψ is “the same” superposition irrespectively of the basis, implying through this idea a reintroduction of contextuality within the MP. This interpretation confuses the whole problem and changes the question at stake. We have extensively discussed this interpretational maneuver and the meaning of quantum superpositions in [25].

In the MP the weirdness appears because a superposition can be composed simultaneously by a specific property (e.g., the system has the prop-

⁷Given a quantum system represented by a superposition $\sum c_i|\alpha_i\rangle$, when in contact with an apparatus ready to measure, $|R_0\rangle$, QM predicts that system and apparatus will become “entangled” in such a way that the final ‘system + apparatus’ will be described by $\sum c_i|\alpha_i\rangle|R_i\rangle$. Thus, as a consequence of the quantum evolution, the pointers have also become —like the original quantum system— a superposition of pointers $\sum c_i|R_i\rangle$. This is why the MP can be stated as a problem only in the case the original quantum state is described by a superposition of more than one term.

erty ‘spin up in the x direction’) and its contradictory property (e.g., the system has the property ‘spin down in the x direction’).⁸ On the one hand, due to the seeming violation of the Principle of Non-Contradiction (PNC), it makes no sense to interpret both properties as actual ones (see for discussion [19]). This was cleverly exposed by Schrödinger in his 1935 *gedankenexperiment* in which a cat, after interacting with an atom represented by a quantum superposition, possessed at the same time the property of being ‘alive’ and the property of being ‘dead’ [48]. On the other hand, all terms in the superposition must be considered in the evolution of the state and the predictions that can be made from it. One could analyze what such weird quantum superpositions represent in physical terms, instead the orthodox literature—following Bohr—has limited the analysis through the MP to the justification of actual measurement outcomes.

Now that we have clearly separated these two different problems some remarks go in order. Firstly, it should be clear that the mix of subjective and objective only appears within the BP. Due to the formalism, orthodoxy has argued extensively that in order to recover a “classical set up” and learn which properties are definite valued and which are not, one needs to *choose* a specific context.⁹ But if the “choice” is not physically justified in terms of an objective process, the definition of reality given by the subset of properties that are actual is obviously subjective—it depends explicitly on the choice of the experimenter. In other words, the context is not determined prior to the choice of the experimenter and thus cannot be considered as a preexistent ASA. Secondly, the MP has no relation to the mix of subjective and objective. The problem here is the justification of the path from a superposition (of, in the most general case, contradictory properties) to a single measurement outcome. Notice also that in the case the basis is such that we can write the superposition as one single term, $|\phi\rangle$, the MP cannot be stated.

Since there is no classical causal explanation which explains the path from the superposition to the actual outcome there seems to be an indeterministic aspect within Nature itself. The question here deals with determinism and the process of measurement within a definite experimental arrangement. But it should be clear that once the experimental arrangement is set, the experimenter has no relation whatsoever to the actual measure-

⁸There is an ongoing debate regarding the interpretation of such properties in terms of ‘contradiction’ and ‘contrariety’: [4, 5, 6, 23].

⁹As we have argued extensively in [26] this is not the case for QM, the choice of a context does not transform quantum properties into classical ones.

ment outcome that will be found in the recording apparatus. Any subjective intromission is completely out of the question at this stage. Once the context is chosen the measurement could be performed and the outcome recorded by a robot.

To summarize, quantum contextuality and the BP deal with the incompatibility of sets of contexts or bases which are in turn interpreted in direct relation to experimental set ups and definite properties. There is no measurement process involved here but the question of how an experimental set up and the properties it can expose is physically represented—in accordance to the formalism of QM. The MP deals with the process of measurement within a specific experimental set up, but in this stage the experimenter has no intromission whatsoever. Let us make these two points clear:

- I. *The BP introduces the question of the physical justification of the choice of a particular basis. The choice of the context determines which observables can be considered as actual and which cannot. This implicitly determines what is to be considered as real—the definite observables within the chosen context.*
- II. *The MP attempts to justify the path from “weird quantum superpositions (of more than one term)” to a single outcome within a particular context. Once the experimental arrangement is set the experimenter has no influence whatsoever on the sudden appearance of the measurement outcome. There is no relation to a subjective choice within the MP, the question only regards the physical representation of the path from a specific superposition (of more than one term) to an actual outcome.*

As we have remarked, the entanglement of these two different problems goes back to the way in which Bohr and Heisenberg argued in favor of the idea that the weirdness of QM could be dissolved by analyzing the measurement process. The seed of this idea is already present within the answer of Bohr to the EPR paper in 1935 which focuses its argumentation in the need of choosing a definite experimental set up in order to talk about definite observables [12]. Even though history has told us that somehow Bohr won the EPR battle, there was no clear physical explanation to the subtle question raised by Einstein regarding the meaning of physical reality according to QM. Heisenberg also helped Bohr to sweep the dirt under the carpet and tried to wipe the subjective dust away through a supposedly clear analysis of the measurement process (see for example [42, p. 89]). But

independently of such attempts, from a representational realist perspective —as Einstein remarked to Pauli many years ago—, a physical theory should be able to account coherently for what it talks about in terms of physical reality.

5 Meaningful Statements and Counterfactual Reasoning in Physics

In order to discuss and analyze physical interpretations of a theory one should first agree with respect to what should be considered as *Meaningful Physical Statements* (MPS) within that theory. Furthermore, from a representational realist perspective, the theory should be capable of representing physically the MPS it talks about.¹⁰

Definition 5.1 Meaningful Physical Statements (MPS): *If given a specific situation a theory is capable of predicting in terms of definite physical statements the outcomes of possible measurements, then such physical statements are meaningful relative to the theory and must be constitutive parts of the particular representation of physical reality that the theory provides. Measurement outcomes must be considered only as an exposure of the empirical adequacy (or not) of the theory.*

It must be remarked that a MPS pertains to the physical representation provided by the particular theory, this means we leave open —going against Bohr’s second desiderata— the possibility that experience is not reduced to actual measurement outcomes. It should be also noticed that this definition of MPS should be only followed by those who attempt to provide an objective description of physical reality. Because MPS pertain to physical representation, from a representational realist account of physics MPS are necessarily related to counterfactual reasoning.

Definition 5.2 Counterfactual Reasoning (CR): *The possibility to make MPS in terms of a physical representation allows in general for counterfactual statements in physical discourse. If the theory is empirically adequate then such MPS are presupposed to be that of which the theory talks about.*

¹⁰This is of course not the case for the sort of empiricists who takes actual observation as its fundament. The empiricist position is however in deep disagreement with the fact that physical experience is always theory laden (see for discussion [22]).

MPS are not necessarily statements about future events, they can be also statements about past and present events. CR about MPS comprise all actual and non-actual physical experience.

If we accept the RRPT stance (section 1), physical representation must take into account the MPS produced by the theory. This also implies that CR is a *necessary condition* that a theory must uphold for without it there is no possibility of physical representation nor physical discourse. Without CR in physical discourse one cannot imagine experience beyond actuality. For a representational realist, the power of physics is CR itself, it is the capability that allows us to predict that “if I perform this or that experiment” then —if it is a MPS— the physical theory will tell me that “the outcome will be x or y ”, and I do not need to actually perform the experiment! I know what the result will be independently of actually performing the experiment or not.¹¹ That is the whole point of being a realist about physics, that I trust the theory to be talking about a physical representation of reality. CR in physical discourse has nothing to do with time, evolution nor dynamics, it has to do with the possibility of representing experience. A physical theory allows me to make claims about the future, the present and the past, just in the same way physical invariance in classical mechanics connects the multiple frames of reference without me being in any particular one. CR is the discursive invariance with respect to physical phenomena. We do not need to be in a specific frame to know what will happen in that specific frame or a different one. Notice, once again, that CR is a necessary condition only for representational realist approaches, not for those philosophical positions which denying the theory-ladenness of physical experience are grounded on raw experimental data.

Remark: *CR is a necessary condition for a coherent discourse about MPS that pertain to a particular physical representation —consequently, also for supporting RRPT.*

After providing these definitions we are now ready to discuss which statements can be considered as MPS within quantum theory. The quantum wave function Ψ gives rise to clear definite physical statements regarding observ-

¹¹The fact that what we know is given in statistical terms does not imply that we need to explain such predictions in classical terms. Or that knowledge boils down only to ‘knowledge that accounts for certainty about actuality’. In the case of QM we have ‘certain knowledge in statistical terms’.

ables through the Born rule. Are the statements that one can make using the Born rule meaningful? Of course they are! This is what the theory is all about. All statements of the type, “the average value of observable A is a (given by the Born Rule)”, are MPS for QM. Each one of the measurement outcomes of QM has been tested to be in accordance with such type of MPS.¹²

Definition 5.3 MPS in QM: *Given a vector in Hilbert space, Ψ , the Born rule allows us to predict the average value of (any) observable O .*

$$\langle \Psi | O | \Psi \rangle = \langle O \rangle$$

This prediction is independent of the choice of any particular basis.

This definition of MPS in QM implies that according to the formalism all observables —independently of the context— must be considered as part of physical (quantum) reality simultaneously and independently of the choice of the context. This is of course —due to contextuality— not possible if we consider physical reality only in terms of an ASA. This shows that, either the formalism should be changed in order to recover a classical representation of reality or, that we should shift to a non-classical representation of physical reality.

6 Revisiting Griffiths’ Arguments Against Quantum Contextuality

Griffiths argues already in the abstract of his paper that:

“[...] quantum mechanics is noncontextual if quantum properties are represented by subspaces of the quantum Hilbert space (as proposed by von Neumann) rather than by hidden variables. In particular, a measurement using an appropriately constructed apparatus can be shown to reveal the value of an observable A possessed by the measured system before the measurement took place, whatever other compatible ($[B, A] = 0$) observable B may be measured at the same time.” [38, p. 174]

¹²Notice that the idea that only actual properties are real has led quantum logic and also modal interpretations to restrict the MPS in QM only to a specific subset of them; i.e. those which imply a certain knowledge of an outcome. Those observables which have probability equal to 1.

According to Griffiths [*Op. cit.*, p. 174]: “a substantial literature has accumulated which would throw doubt on this or suggest the opposite on the basis of various arguments related to the Kochen-Specker theorem.” Griffiths builds up his argumentation against contextuality from the analysis and discussion of four main points or ideas:

- i. The idea that physical reality according to QM is not classical reality.*
- ii. The idea that in order to understand QM one needs to use the SFR and abandon CR.*
- iii. The idea that one can “get rid” of Schrödinger’s cat through the SFR.*
- iv. The idea that contextuality can be discussed through the analysis of quantum measurements.*

In this section we attempt to show that the arguments which support these ideas contain several flaws mainly due to multiple shifts in the metaphysical stance of analysis. Even though Griffiths claims to provide a realist account of QM, his own justification and analysis is in many respects close from instrumentalism. But as we have discussed above, a realist interpretation of QM cannot be justified using a set of instrumentalist arguments. The criticisms provided in this section against the CH interpretation can be easily extended to several (supposedly) realist interpretations of QM.

6.1 “Quantum Reality” Is Not “Classical Reality”

Griffiths argues that the wrong assumption, that QM talks about classical reality, has led to the idea that QM is contextual:

“How have so many come to [the] conclusion [that QM is contextual]? By adopting, we shall argue, the view that the real world is classical, contrary to all we have learned from the development of quantum mechanics in the twentieth century. In particular, discussions couched in terms of hidden variables typically assume that they are classical rather than the sort of thing one might expect in a quantum mechanical world.” [*Op. cit.*, p. 174]

As we have discussed above (section 2), KS theorem makes exactly the same point against classical representations of QM: because of contextuality QM cannot be described in terms of a classical ASA kind of representation. KS

should be understood as an *ad absurdum* proof of the failure of the notion of (classical) *actual preexistence* to account for what QM is talking about [26]. The conclusion that must be drawn is the following: in case we want to stay close to orthodox Hilbert space QM, then projection operators cannot be interpreted in terms of actually preexistent properties¹³ —which is what, as we shall see, Griffiths ends up trying to do.

According to Griffiths:

“The way our approach avoids any conflict with the Kochen-Specker theorem is by denying *Re* [Realism about properties]. The claim that every observable possesses a value at every time is, indeed, inconsistent with a representation of quantum properties by subspaces of a Hilbert space. Consider, for example, a spin-half particle. There are distinct rays in the two-dimensional Hilbert space corresponding to $S_x = +\frac{1}{2}$ (in units of \hbar) and to $S_x = -\frac{1}{2}$; also rays corresponding to $S_z = +\frac{1}{2}$ and $S_z = -\frac{1}{2}$, but there is no ray that can represent a simultaneous value of S_x and S_z . *Students are told, correctly, that S_x and S_z cannot be measured simultaneously, and they ought to be told that the reason for this is that there is nothing there to be measured.* The projectors corresponding to S_z do not commute with the projectors corresponding to S_x , and once one has accepted the connection between quantum properties and Hilbert subspaces proposed by von Neumann it makes no sense to speak of a spin half system in which, for example, $S_x = +\frac{1}{2}$ at the same time that $S_z = +\frac{1}{2}$.” [*Op. cit.*, p. 179] (emphasis added)

As we have discussed above, the condition *Re* is perfectly well founded in the physical representation provided by classical Newtonian mechanics in terms of an ASA. Now, the fact that two observables do not commute does not only imply that the observables cannot be measured together but more importantly, that one cannot assign a *GV* to them, and this in turn precludes the possibility of representing what is going on in terms of an ASA (see also for discussion [28]). But of course there are things to be measured! As a matter of fact every observable when related to a Ψ determines a MPS in QM in terms of the Born rule (section 5). If I analyze spin in the x -direction then I will find out that the average value of ‘spin up’ is a and the average value of ‘spin down’ is b , and the same happens with the spin in the z -direction or in any other direction I would like to measure. QM provides

¹³Even worse, as we have proven in [28], the fact there is no value invariance of dynamical observables makes untenable the very idea of discussing about a physical system.

the correct predictions to all such MPS. For these reasons, the author of this paper does not understand at all what is Griffiths trying to say when he claims that “there is nothing to be measured”.

Griffiths continues arguing that:

“It is somewhat odd that this particular principle [*Re*] should be identified with realism, since at the present time all available experimental evidence is in accord with Hilbert space quantum mechanics, and not with classical physics when the two disagree. If a Hilbert space provides the appropriate mathematics to describe everything from the quarks to the quasars, where in the real world, the one we live in, is there any part that satisfies the condition of “realism” given by *Re*? It would be much less confusing if whenever “realism” were used in this way the adjective “classical” were prepended. The hidden variables of typical hidden variable theories are classical hidden variables, and it is for this reason that the attempt to use them for interpreting quantum theory has given rise to numerous conflicts with the latter.” [*Op. cit.*, p. 179]

We completely agree with Griffiths that the realism discussed is in fact “classical realism”, however we disagree that one can, from a realist stance, argue that it is enough to predict the correct measurement outcomes. What we need, if one attempts to be a realist, is to either develop a new formalism that describes things in terms of an ASA (i.e., in terms of “classical reality”) —as the HVP attempts to do— or to find a new non-classical representation of physical reality which takes into account all MPS in QM. It is simply incorrect to argue in favor of realist interpretation of QM using an instrumentalist argument, that is, that the formalism provides the correct “experimental evidence”. What matters for a realist is the physical representation of reality provided by the theory, and still today we do not know what QM is talking about —what we know is that due to contextuality the orthodox formalism is not taking about an ASA.

6.2 Applying the SFR (Instead of CR) in QM

In several papers, Griffiths —who is certainly not alone in supporting this idea— has argued against CR in physical discourse [36, 37, 38, 39, 40, 41]. According to him [38, p. 178]: “Analyzing counterfactual questions is a bit tricky even in situations not entangled with quantum mysteries.” Indeed, this idea of CR being something “tricky” or “weird” is quite widespread

within the literature. In order to escape the consequences of CR Griffiths proposes to “block” this discursive condition in QM through the introduction of what he calls the *Single Framework Rule* (SFR).¹⁴

“The single framework rule asserts that any sort of discussion of the quantum system must be carried out in some framework of the sort just discussed, which is typically chosen because it has some events which are interesting for some reason or another. The physicist is free to choose any framework he pleases for describing the world; what the single framework rule prohibits is combining frameworks. [...] The well-known Kochen-Specker paradox is constructed precisely by forming a bridge, or one might better say bridges, between incompatible frameworks in such a way that one eventually ends up with a contradiction. The histories approach disposes of the paradox by declaring the bridging invalid.” [*Op. cit.*, p. 180]

As we have argued above, one thing is a representation of reality according to a theory and a very different one is a specific observation of one particular aspect of such physical representation. Indeed, for a representation to be coherent one needs a consistent relation between such multiple observations and perspectives. Invariance of Galilei transformations in classical mechanics or Lorentz transformation in relativity theory are good examples of such consistency between different frames of reference. CR is the discursive invariance which allows one to claim that, if the physical representation is adequate, then the phenomenon A will have the result a while the phenomenon B will have the result b , and so forth. CR presupposes—according to the realist stance—that there is a physical representation of reality according to the theory. The fact that we can talk about such physical representation without the need to actually perform experiments is the very condition of coherent discourse in physics.

Continuing his argumentation Griffiths makes the point that:

“It is important to note that the single-framework rule does not state that there is only one framework which can be used for a valid quantum

¹⁴An equivalent formulation or use of SFR is provided, just to mention a few interpretations in Dieks original modal interpretation which restricts the context to the one in which the Born rule gives probability 1 (certainty) to an observable [29, 30], by Dieks and Vermaas in their generalization of the modal interpretation to density operators [52], by Bene and Dieks within their perspectival version of the modal interpretation [10] and by Svozil in [49].

description of a situation. The quantum physicist is free to choose any framework, consistent with the Hilbert space structure of quantum mechanics (and, in the case of histories, satisfying consistency conditions if the extension of the Born rule is to be used to assign probabilities), in order to describe a quantum system. In some frameworks a particular observable A may possess some value, while in other frameworks it may not. The existence of the latter does not preclude the possibility or the validity of a framework which include the former. The single framework rule is not a restriction on the use of frameworks; it is instead a prohibition against combining incompatible frameworks.” [Op. cit., p. 180]

With respect to this argumentation by Griffiths some remarks go in order. Firstly, physical representation involves the need to be able to discuss of all contexts simultaneously; properties of different contexts cannot be brought into reality by virtue of a choice if we want to claim at the same time that our representation of reality is not defined subjectively. There should be no subjective choice in order to claim that a given property is actually existent or not (i.e., has a value or not). Secondly, the *ontological incompatibility* of quantum properties should not be confused with the *epistemological incompatibility* of distinct measurements.¹⁵ As a matter of fact, even in classical physics we can certainly have a situation in which there is an epistemological incompatibility between different measurements of the same system. Due to the specific questioning I might not be able to perform two measurements simultaneously. In general, not all properties of a classical physical object can be measured at the same time. However, and this is what really matters, in classical physics all properties in a given situation are consistent with possessing a definite value, making it possible to understand any classical situation in terms of an ASA —all classical properties of a system are ontologically compatible. In other words, it is not the epistemological incompatibility of measurements which is at stake through quantum contextuality but the ontological incompatibility of the values of properties which allow for a GV —and thus a description in terms of an ASA.

But the strangest aspect of the argumentation of this second idea by Griffiths (section 6.2) is its contradiction with his first own argument by which QM is not to be represented through “classical reality” (section 6.1).

¹⁵See in this respect the analysis provided by Aerts regarding classical and quantum experiments and properties in operational quantum logic [2, 3].

Griffiths now argues—in this second point—that the SFR should be used in order to talk about “properties with preexistent values”. But this is exactly the “classical” physical representation that Griffiths had criticized in the first place (section 6.1). The two arguments (from sections 6.1 and 6.2) are flagrantly in contradiction, both cannot stand together within the same approach.

Contrary to what is claimed by Griffiths, according to the author of this paper, the SFR is an *ad hoc* rule which explicitly shows that Hilbert space QM is contextual. It is an external addition that blocks an essential aspect of the formalism, an aspect which is key to the predictions of QM and the MPS it provides. In this sense the HVP is not so different from the CH interpretation which ends up doing exactly the same, namely, changing or restricting the formalism in order to recover a description in terms of “classical reality”.

6.3 “Getting Rid” of Quantum Superpositions

Griffiths¹⁶ [*Op. cit.*, p. 177] considers the question: “What will occur if the experimenter prepares an initial state $|\psi\rangle = \frac{1}{\sqrt{2}}(|a_1\rangle + |a_2\rangle)$ which is a superposition corresponding to two distinct eigenvalues of A and then carries out a measurement?” But as discussed in detail in [20, 25], there is a subtlety involved because the state $|\psi\rangle$ is not identical to the state $\frac{1}{\sqrt{2}}(|a_1\rangle + |a_2\rangle)$. Of course it is the same vector, but one can also argue that $|\psi\rangle$ makes special reference to an experimental arrangement such that only the observable $|\psi\rangle\langle\psi|$ is measured—obtaining with certainty its eigenvalue—and $\frac{1}{\sqrt{2}}(|a_1\rangle + |a_2\rangle)$ makes reference to an experimental arrangement in which A is measured—obtaining each result with probability 0.5; the result related to $|a_1\rangle$ and the result related to $|a_2\rangle$. If we call V , P_1 and P_2 the projection operators related to $|\psi\rangle$, $|a_1\rangle$ and $|a_2\rangle$, respectively, then, according to Griffiths, because the projections V , P_1 and P_2 do not commute one can apply the SFR [*Op. cit.*, p. 177].

Some remarks go in order. Firstly, a superposition is a particular mathematical representation of a vector in Hilbert space Ψ . Secondly, the SFR cannot help in dissolving or doing away with quantum superpositions because quantum superpositions are defined—as we discussed above—within a single context, they are not constituted by *incompatible* observables. Let

¹⁶The denial of the existence of quantum superpositions by Griffiths has been analyzed in detail in [25]. Here we only attempt to discuss Griffiths argumentation against quantum superpositions using the SFR.

us remark this important point: there is no quantum superposition composed by states which give rise to *incompatible observable*. Of course, given a typical Stern-Gerlach (SG) type experiment, one can have a particular quantum superposition such as $c_{1x}|\uparrow_x\rangle + c_{2x}|\downarrow_x\rangle$, and one can also find different superpositions through a change of basis. For example, if we produce a rotation of the SG to the y -direction we will obtain the superposition $c_{1y}|\uparrow_y\rangle + c_{2y}|\downarrow_y\rangle$. As we have argued extensively, these are to be considered —contrary to what is claimed by the orthodox interpretation— as two different superpositions which give rise to different sets of MPS [25]. While the former superposition provides information of what will happen if the SG is placed in the x -direction (MPS about the observable S_x), the latter superposition provides information in case the SG is placed in the y -direction (MPS about the observable S_y). Thirdly, the problem with quantum superpositions, as we have extensively argued in [19], is that the *same observable* (e.g., S_x) can give rise to *contradictory properties* within the same superposition. But due to the PNC —which defines both ontologically and logically the realm of actuality (see for discussion [27])— both properties cannot be simultaneously considered as actually preexistent. Even when interpreted in terms of possibilities (rather than actualities), as it is the case of quantum logic and several versions of the modal interpretation, such possibilities must be necessarily considered as *quantum possibilities* (see for discussion [26]).

The criticism provided by Griffiths to the Many Worlds (MW) interpretation of QM also makes clear the fact there is a mix between the MP and the BP in his argumentation against Schrödinger’s cat.

“[...] the histories approach is distinctly different from the Everett or many-worlds interpretation with its insistence that “the wave function”, in this instance the state onto which V projects, represents fundamental physical reality. From the histories point of view the difficulties which many-worlds advocates have in explaining how ordinary macroscopic physics can be consistent with their perspective is not unrelated to the fact that they are seeking to assign simultaneous reality to properties which in the quantum Hilbert space are represented by incompatible projectors.” [*Op. cit.*, p. 177]

Contrary to this claim, MW focuses in solving the MP, and deals in no way with incompatible projectors [26, 32]. MW attempts to provide physical meaning to quantum superpositions by multiplying classical reality as many times as terms are found in a superposition. The fundamental solution

provided by MW to the MP does not deal in any way with contextuality nor the BP.

The discussion provided by Griffiths continues calling the attention to the need of considering the measurement process. Griffiths then argues [*Op. cit.*, p. 177] that: “[...] getting rid of the ghost of Schrödinger’s cat, we still need to show that the measurement apparatus actually carries out a measurement; i.e., the outcome pointer position is properly correlated with a previous property of the measured system. For this purpose we need an extension of Born’s rule that allows probabilities to be assigned to a closed quantum system at three or more times, and this in turn requires the use of consistent (or decoherent) families of histories.” Once again, the problem is shifted to the justification of the measurement process.

6.4 Measurement Process and Contextuality

According to Griffiths: “The most direct approach to determining whether quantum theory is or is not contextual is to analyze the process that goes on in a quantum measurement.” [*Op. cit.*, p. 176] We have argued extensively why this cannot be the case. Quantum measurements have nothing to say about the contextual character of QM simply because contextuality says nothing about measurements. Contextuality deals with the orthodox formalism and the limits of any consistent interpretation that stays close to it, while measurement outcomes should only be considered as particular expressions of what the theory tells us physical reality is about. As we have discussed above, from a realist perspective, measurements cannot be considered as more fundamental than the physical representation provided by the theory. For a representational realist about physics measurement outcomes are to be interpreted in terms of the physical representation and not the other way around. This is the reason why Einstein—a realist which clearly understood the importance of representation within the definition of phenomena [43]—said to Heisenberg a long time ago: “It is only the theory which can tell you what can be observed.”

The discussion about the measurement process in QM has been a way to hide all we do not understand about the theory. Bohr, and also Heisenberg, insisted on the idea that through an analysis of the measurement process the problems of the quantum disappeared. However, they were never truly able to resolve the MP nor the BP—which are in themselves attempts to justify the classical physical description of reality. In the present the situation hasn’t changed a single bit. As discussed above, it has become clear that

best candidate of orthodoxy to try to solve the quantum to classical limit, namely, decoherence, has also failed to provide a physical explanation and representation of the measurement process.

One of the main statements of Griffiths [*Op. cit.*, p. 174] goes against “the view that the real world is classical, contrary to all we have learned from the development of quantum mechanics in the twentieth century.” Griffiths remarks that: “at the present time all available experimental evidence is in accord with Hilbert space quantum mechanics, and not with classical physics when the two disagree.” However, empirical adequacy cannot be taken as an argument about the reality of the measurement process. The fact that we can do things in the lab does not mean that we can represent them or understand what is going on. This is according to our representational realist perspective, the main difference between physics and technology in contradistinction to instrumentalism which denies the very need of explaining physical phenomena. To say it differently: there can be no realist solution “For All Practical Proposes” (FAPP) as the decoherent orthodoxy claims and also Griffiths implicitly seems to imply. FAPP is just a gentle way of saying: “We do not know how it works, but it doesn’t matter, it (somehow) works!” A realist cannot remain satisfied with this answer.

7 What is the Problem (for a Representational Realist) in QM?

The fundament of a representational realist approach is that a mathematical formalism when coherently related to a network of physical concepts is capable of producing a physical representation of reality. The particular empirical exposures are just that, findings in experience of what is already expressed by the theory through its physical representation. This is the whole point of being a representational realist about physics. From this standpoint there are no epistemological concerns whatsoever. Epistemological questions are part of a different set of problems where we humans enter the picture in a fundamental manner. Thus, the aim of an ontological realistic approach to QM should be, in the very first place, to account for a representation of physical reality that coherently relates to the formalism. It is not enough to say that QM does not relate to “classical reality”. This is not doing the job. The formalism of QM has proven already to be empirically adequate. As we have argued elsewhere [23], instead of changing the formalism or adding *ad hoc* rules in order to go back a few squares

and recover a classical representation of physical reality, there is a different strategy that could be considered. According to this new strategy —which we have called the Constructive Metaphysical Line of Research (CMLR)— what needs to be done is to construct a new net of (non-classical) concepts capable of interpreting the formalism as it is.

Starting from the formalism, then adding *ad hoc* rules —such as the SFR— in order to make the formalism “more classical” (noncontextual) and then trying to get to a measurement outcome —despite the fact there is up to the present no satisfactory solutions given to the MP nor the BP— does not solve the problem at all. This seems to be instead some sort of hidden instrumentalist approach which remains silent about what really matters: the physical representation of reality according to QM.

An example of this instrumentalist move is witnessed by the fact that many approaches are obsessed —as Griffiths put it— with “getting rid of the ghost of Schrödinger’s cat”. This negative stance towards quantum superpositions shows that such approaches are not willing to take seriously the problem of finding a physical representation that matches the mathematical formalism. According to the author of this paper, contextuality should be neither regarded as a “ghost” which we need to fight or destroy, it should be taken as one of the key elements that must help us in understanding QM and the type of physical reality it implies. Technological and experimental developments are going much faster than the present discussion regarding foundational issues about QM, discussions which are still stuck by the limits imposed by Bohr to the OLR. This is one of the main reasons we believe that the field is in need of a strong criticism. The BP and MP which attempt to build a bridge between QM and our classical understanding of reality, could be replaced in the literature by new problems which assume the possibility of a new representation of physical reality according to QM. In [21, 23], we put forward two new problems which could help to think, from a different perspective, the problem of interpretation in QM. The first, which is the one that interests us here and should replace the BP, was called the Contextuality Problem (CP):

Contextuality Problem (CP): *Given the fact that Hilbert space QM is a contextual theory, the question is which are the concepts that would allow us to coherently interpret the formalism and provide a representation of physical reality that accounts for this feature of the theory?*

The CP —as well as the superposition problem discussed in [25]— opens

the possibility to truly discuss a physical representation of reality which goes beyond the classical representation of physics in terms of an ASA. We are convinced that without a replacement of the problems addressed in the literature there is no true possibility of discussing an interpretation of QM which provides an objective non-classical physical representation of reality. We know of no reasons to believe that this is not doable.

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References

- [1] Adler, S. L., 2003, “Why Decoherence has not Solved the Measurement Problem: A Response to P. W. Anderson”, *Studies in History and Philosophy of Modern Physics*, **34**, 135-142.
- [2] Aerts, D., 1982, “Description of many physical entities without the paradoxes encountered in quantum mechanics”, *Foundations of Physics*, **12**, 1131-1170.
- [3] Aerts, D. 1983, “The description of one and many physical systems”, In *Foundations of Quantum Mechanics*, C. Gruber (Ed.), pp. 63-148, Lausanne.
- [4] Arenhart, J. R. and Krause, D., 2014, “Oppositions in Quantum Mechanics”, in *New dimensions of the square of opposition*, J.-Y. Béziau and K. Gan-Krzywoszynska (Eds.), pp. 337-356, Philosophia Verlag, Munich.
- [5] Arenhart, J. R. and Krause, D., 2014, “Contradiction, Quantum Mechanics, and the Square of Opposition”, *Logique et Analyse*, accepted.

- [6] Arenhart, J. R. and Krause, D., 2014, “Potentiality and Contradiction in Quantum Mechanics”, in *The Road to Universal Logic (volume II)*, A. Koslow and A. Buchsbaum (Eds.), Springer, in press.
- [7] Bacciagaluppi, G., 1996, *Topics in the Modal Interpretation of Quantum Mechanics*, PhD dissertation, University of Cambridge, Cambridge.
- [8] Bacciagaluppi, G., 2012, “The Role of Decoherence in Quantum Mechanics”, *The Stanford Encyclopedia of Philosophy*, E. N. Zalta (Ed.), <http://plato.stanford.edu/archives/win2012/entries/qm-decoherence/>.
- [9] Bacciagaluppi, G., 2013, “Measurement and Classical Regime in Quantum Mechanics”, *Oxford Handbook of Philosophy of Physics*, R. Batterman (Ed.), pp. 416-459, Oxford University Press, Oxford.
- [10] Bene, G. and Dieks, D., 2002, “A Perspectival Version of the Modal Interpretation of Quantum Mechanics and the Origin of Macroscopic Behavior”, *Foundations of Physics*, **32**, 645-671.
- [11] Bogen, J. and Woodward, J., 1988, “Saving the Phenomena”, *The Philosophical Review*, **XCVII**, 303-352.
- [12] Bohr, N., 1935, “Can Quantum Mechanical Description of Physical Reality be Considered Complete?”, *Physical Review*, **48**, 696-702.
- [13] Bokulich, A., 2004, “Open or Closed? Dirac, Heisenberg, and the Relation between Classical and Quantum Mechanics”, *Studies in History and Philosophy of Modern Physics*, **35**, 377-396.
- [14] Bokulich, A., 2006, “Heisenberg Meets Kuhn: Closed Theories and Paradigms”, *Philosophy of Science*, **73**, 90-107.
- [15] Bokulich, A., 2014, “Bohr’s Correspondence Principle”, *The Stanford Encyclopedia of Philosophy (Spring 2014 Edition)*, E. Zalta (Ed.), URL: <http://plato.stanford.edu/archives/spr2014/entries/bohr-correspondence/>.
- [16] Bokulich, P. and Bokulich, A., 2005, “Niels Bohr’s Generalization of Classical Mechanics”, *Foundations of Physics*, **35**, 347-371.
- [17] Bub, J., 1997, *Interpreting the Quantum World*, Cambridge University Press, Cambridge.

- [18] Curd, M. and Cover, J. A., 1998, *Philosophy of Science. The central issues*, Norton and Company (Eds.), Cambridge University Press, Cambridge.
- [19] da Costa, N. and de Ronde, C., 2013, “The Paraconsistent Logic of Quantum Superpositions”, *Foundations of Physics*, **43**, 845-858.
- [20] da Costa, N. and de Ronde, C., 2015, “Logical Identity \neq Metaphysical Identity”, in preparation.
- [21] de Ronde, C., 2011, *The Contextual and Modal Character of Quantum Mechanics: A Formal and Philosophical Analysis in the Foundations of Physics*, PhD dissertation, Utrecht University, The Netherlands.
- [22] de Ronde, C., 2014, “The Problem of Representation and Experience in Quantum Mechanics”, in *Probing the Meaning of Quantum Mechanics: Physical, Philosophical and Logical Perspectives*, 91-111, D. Aerts, S. Aerts and C. de Ronde (Eds.), World Scientific, Singapore.
- [23] de Ronde, C., 2015, “Modality, Potentiality and Contradiction in Quantum Mechanics”, *New Directions in Paraconsistent Logic*, J.-Y. Béziau, M. Chakraborty and S. Dutta (Eds), Springer, in press.
- [24] de Ronde, C., 2015, “Epistemological and Ontological Paraconsistency in Quantum Mechanics: For and Against Bohrian Philosophy”, *The Road to Universal Logic (Volume II)*, A. Koslow and A. Buchsbaum (Eds.), Springer, in press.
- [25] de Ronde, C., 2015, “Quantum Superpositions Do Exist! But ‘Quantum Physical Reality \neq Actuality’ (Reply to Dieks and Griffiths)”, *Foundations of Physics*, sent.
- [26] de Ronde, C., Freytes, H. and Domenech, G., 2014, “Interpreting the Modal Kochen-Specker Theorem: Possibility and Many Worlds in Quantum Mechanics”, *Studies in History and Philosophy of Modern Physics*, **45**, 11-18.
- [27] de Ronde, C., Freytes, H. and Domenech, G., 2014, “Quantum Mechanics and the Interpretation of the Orthomodular Square of Opposition”, in *New dimensions of the square of opposition*, J.-Y. Béziau and K. Gan-Krzywoszynska (Ed.), 223-242, Philosophia Verlag, Munich.

- [28] de Ronde, C. and Massri, C., 2014, “Revisiting the First Postulate of Quantum Mechanics: Invariance and Physical Reality”, preprint *Los Alamos Archive*, quant-ph/arXiv:1412.2701.
- [29] Dieks, D., 1988, “Quantum Mechanics and Realism”, *Conceptus* **57**, 31-47.
- [30] Dieks, D., 1989, “Quantum Mechanics Without the Projection Postulate and Its Realistic Interpretation”, *Foundations of Physics*, **19**, 1397-1423.
- [31] Domenech, G. and Freytes, H., 2005, “Contextual logic for quantum systems”, *Journal of Mathematical Physics*, **46**, 012102-1-012102-9.
- [32] Domenech, G., Freytes, H. and de Ronde, C., 2009, “Many worlds and modality in the interpretation of quantum mechanics: an algebraic approach”, *Journal of Mathematical Physics*, **50**, 072108.
- [33] Einstein, A., Podolsky, B. and Rosen, N., 1935, “Can Quantum-Mechanical Description be Considered Complete?”, *Physical Review*, **47**, 777-780.
- [34] Fuchs, C. and Peres, A., 2000, “Quantum theory needs no ‘interpretation’ ”, *Physics Today*, **53**, 70.
- [35] Giulini, D., Joos, E., Kiefer, C., Kupsch, J., Stamatescu, I.-O., and Zeh, H. D., 1996, *Decoherence and the Appearance of a Classical World in Quantum Theory*, Springer, Berlin.
- [36] Griffiths, R. B., 2002, *Consistent quantum theory*, Cambridge University Press, Cambridge.
- [37] Griffiths, R. B., 2012, “Quantum counterfactuals and locality”, *Foundations of Physics*, **42**, 674-684.
- [38] Griffiths, R. B., 2013, “Hilbert space quantum mechanics is non contextual”, *Studies in History and Philosophy of Modern Physics*, **44**, 174-181.
- [39] Griffiths, R. B., 2013, “A consistent quantum ontology”, *Studies in History and Philosophy of Modern Physics*, **44**, 93-114.

- [40] Griffiths, R. B., 2014, “The Consistent Histories Approach to Quantum Mechanics”, *The Stanford Encyclopedia of Philosophy*, E. N. Zalta (ed.). URL: <http://plato.stanford.edu/archives/fall2014/entries/qm-consistent-histories/>.
- [41] Griffiths, R. B., 2015, “Consistent quantum measurements”, preprint.
- [42] Heisenberg, W., 1958, *Physics and Philosophy*, World perspectives, George Allen and Unwin Ltd., London.
- [43] Howard, D., 1993, “Was Einstein Really a Realist?”, *Perspectives on Science*, **1**, 204-251.
- [44] Howard, D., 2004, “Who Invented the ‘Copenhagen Interpretation?’ A Study in Mythology”, *Philosophy of Science*, **71**, 669-682.
- [45] Jaynes, E. T., 1990, *Complexity, Entropy, and the Physics of Information*, W. H. Zurek (Ed.), Addison-Wesley.
- [46] Kochen, S. and Specker, E., 1967, “On the problem of Hidden Variables in Quantum Mechanics”, *Journal of Mathematics and Mechanics*, **17**, 59-87. Reprinted in Hooker, 1975, 293-328.
- [47] Pauli, W., 1994, *Writings on Physics and Philosophy*, C. Enz and K. von Meyenn (Eds.), Springer-Verlag, Berlin.
- [48] Schrödinger, E., 1935, “The Present Situation in Quantum Mechanics”, *Naturwiss*, **23**, 807. Translated to english in *Quantum Theory and Measurement*, J. A. Wheeler and W. H. Zurek (Eds.), 1983, Princeton University Press, Princeton.
- [49] Svozil, K., 2014, “Unscrambling the Quantum Omelette”, *International Journal of Theoretical Physics*, **53**, 3648-3657.
- [50] Van Fraassen, B. C., 1980, *The Scientific Image*, Clarendon, Oxford.
- [51] Van Fraassen, B. C., 2002, *The Empirical Stance*, Yale University Press, New Haven.
- [52] Vermaas, P. E. and Dieks, D., 1995, “The Modal Interpretation of Quantum Mechanics and Its Generalization to Density Operators”, *Foundations of Physics*, **25**, 145-158.

- [53] Wheeler, J. A. and Zurek, W. H. (Eds.) 1983, *Theory and Measurement*, Princeton University Press, Princeton.