Are Newtonian Gravitation and Geometrized Newtonian Gravitation Theoretically Equivalent?

James Owen Weatherall

Department of Logic and Philosophy of Science
University of California, Irvine, CA 92697

Abstract

I argue that a criterion of theoretical equivalence due to Clark Glymour [*Noûs* **11**(3), 227–251 (1977)] does not capture an important sense in which two theories may be equivalent. I then motivate and state an alternative criterion that does capture the sense of equivalence I have in mind. The principal claim of the paper is that relative to this second criterion, the answer to the question posed in the title is "yes", at least on one natural understanding of Newtonian gravitation.

Keywords: Theoretical equivalence, Categorical equivalence, Gauge theory, Geometrized Newtonian gravitation

1. Introduction

Are Newtonian gravitation and geometrized Newtonian gravitation (Newton-Cartan Theory) equivalent theories? Clark Glymour (1970, 1977, 1980) has articulated a natural criterion of theoretical equivalence and argued that, by this criterion, the answer is "no".[1] I will argue here that the situation is more subtle than Glymour suggests, by characterizing a robust sense in which two theories may be equivalent that Glymour's criterion does not capture. This alternative sense of equivalence, which is in the same spirit as Glymour's, is best construed as a friendly amendment. Still, it will turn out that by this alternative criterion, Newtonian gravitation *is* equivalent to geometrized Newtonian gravitation—at

Email address: weatherj@uci.edu (James Owen Weatherall)

[1]Glymour's criterion has recently been a topic of debate on other grounds: see, for instance, Halvorson (2012, 2013), Glymour (2013), and Coffey (2014).

least on one way of construing Newtonian gravitation.[2] It follows that there exist realistic theories that are equivalent in a robust and precise sense, but which apparently disagree regarding certain basic features of the world, such as whether spacetime is flat or curved.

The paper will proceed as follows. I will begin by briefly reviewing the two versions of Newtonian gravitation. I will then describe Glymour's criterion for theoretical equivalence, according to which the two versions of Newtonian gravitation fail to be equivalent. Next, I will apply Glymour's criterion to two formulations of electromagnetism that, I will argue, should be (and typically are) taken to be equivalent. It will turn out that these theories fail to be equivalent by Glymour's criterion of equivalence. In the following sections, I will develop an alternative notion of equivalence between theories that I will argue does capture the sense in which these two formulations of electromagnetism are equivalent. I will then return to the question of principal interest in the present paper, arguing that there are two ways of construing standard (nongeometrized) Newtonian gravitation. I will state and prove a simple proposition to the effect that, by the alternative criterion, on one of the two ways of construing standard Newtonian gravitation (but not the other), it is theoretically equivalent to geometrized Newtonian gravitation. I will conclude by drawing some morals concerning the interpretation of physical theories. Proofs of selected propositions appear in an appendix.

2. Two formulations of Newtonian gravitation

The two theories with which I am principally concerned are Newtonian gravitation (NG) and a variant of Newtonian gravitation due to Élie Cartan (1923, 1924) and Kurt Friedrichs (1927), called "Newton-Cartan theory" or "geometrized Newtonian gravitation" (GNG).[3] In NG, gravitation is a force exerted by massive bodies on other massive bodies. It is mediated

[2] David Zaret (1980) has also replied to Glymour on this question. But his argument is markedly different than the one presented here, and Spirtes and Glymour (1982) offer what I take to be an effective reply.

[3] For background on geometrized Newtonian gravitation, see Malament (2012) or Trautman (1965).

by a gravitational potential, and in the presence of a (non-constant) gravitational potential, massive bodies will accelerate. In GNG, meanwhile, gravitation is "geometrized" in much the same way as in general relativity: the geometrical properties of spacetime depend on the distribution of matter, and conversely, gravitational effects are manifestations of this geometry. Despite these differences, however, there is a precise sense, which I will state below, in which the theories are empirically equivalent. The central question of the paper is whether they are also equivalent in some stronger sense.

On both theories, spacetime is represented by a four dimensional manifold, which I will assume throughout is contractible. This manifold is equipped with two (degenerate) metrics: a temporal metric t_{ab} that assigns temporal lengths to vectors, and a spatial metric h^{ab} that (indirectly) assigns spatial lengths to vectors.[4] These are required to satisfy $h^{ab}t_{bc} = \mathbf{0}$ everywhere. There always exists (at least locally) a covector field t_a such that $t_{ab} = t_a t_b$; in cases where the spacetime is "temporally orientable", this field can be defined globally. In what follows, I will limit attention to temporally orientable spacetimes. Finally, we assume spacetime is endowed with a derivative operator ∇ that is compatible with both metrics, in the sense that $\nabla_a t_{bc} = \mathbf{0}$ and $\nabla_a h^{bc} = \mathbf{0}$ everywhere. These four elements together define a *classical spacetime*, written (M, t_a, h^{ab}, ∇). Matter in both theories is represented by its mass density field, which is a smooth scalar field ρ. Massive point particles are represented by their worldlines—smooth curves whose tangent vector fields ξ^a satisfy $\xi^a t_a \neq 0$. Such curves are called *timelike*.

In this context, NG is the theory whose models are classical spacetimes with flat ($R^a{}_{bcd} = \mathbf{0}$) derivative operators, endowed with a gravitational potential, which is a scalar field φ satisfying Poisson's equation, $\nabla_a \nabla^a \varphi = 4\pi\rho$.[5] A massive point particle whose worldline has tangent field ξ^a will accelerate according to $\xi^n \nabla_n \xi^a = -\nabla^a \varphi$. In the geometrized

[4]Throughout the paper I use the abstract index notation, explained in Malament (2012, §1.4).
[5]Here $\nabla^a \varphi = h^{ab} \nabla_b \varphi$.

version of the theory, meanwhile, the derivative operator is permitted to be curved and the gravitational potential is omitted. The curvature field associated with the derivative operator satisfies a geometrized version of Poisson's equation, $R_{ab} = 4\pi\rho t_a t_b$, and in the absence of any external (i.e., non-gravitational) interactions, massive particles traverse timelike geodesics of this curved derivative operator. In both cases, we take the "empirical content" of the theory to consist in the allowed trajectories of massive bodies, given a particular mass density.

Given a model of NG, it is always possible to produce a (unique) model of GNG that agrees on empirical content in this sense.

Proposition 2.1 (Trautman (1965)). *Let* $(M, t_a, h^{ab}, \overset{f}{\nabla})$ *be a flat classical spacetime, let* φ *and* ρ *be smooth scalar fields satisfying Poisson's equation with respect to* $\overset{f}{\nabla}$*, and let* $\overset{g}{\nabla} = (\overset{f}{\nabla}, C^a{}_{bc})$*, with* $C^a{}_{bc} = -t_b t_c \overset{f}{\nabla}{}^a \varphi$.[6] *Then* $(M, t_a, h^{ab}, \overset{g}{\nabla})$ *is a classical spacetime;* $\overset{g}{\nabla}$ *is the unique derivative operator on* M *such that given any timelike curve with tangent vector field* ξ^a, $\xi^n \overset{g}{\nabla}_n \xi^a = \mathbf{0}$ *iff* $\xi^n \overset{f}{\nabla}_n \xi^a = -\overset{f}{\nabla}{}^a \varphi$*; and the Riemann curvature tensor relative to* $\overset{g}{\nabla}$, $\overset{g}{R}{}^a{}_{bcd}$*, satisfies (1)* $\overset{g}{R}_{ab} = 4\pi\rho t_a t_b$*, (2)* $\overset{g}{R}{}^a{}_b{}^c{}_d = \overset{g}{R}{}^c{}_d{}^a{}_b$*, and (3)* $\overset{g}{R}{}^{ab}{}_{cd} = \mathbf{0}$.

It is also possible to go in the other direction, as follows.

Proposition 2.2 (Trautman (1965)). *Let* $(M, t_a, h^{ab}, \overset{g}{\nabla})$ *be a classical spacetime that satisfies conditions (1)-(3) in Prop. 2.1 for some smooth scalar field* ρ*. Then there exists a smooth scalar field* φ *and a flat derivative operator* $\overset{f}{\nabla}$ *such that* $(M, t_a, h^{ab}, \overset{f}{\nabla})$ *is a classical spacetime; given any timelike curve with tangent vector field* ξ^a, $\xi^n \overset{g}{\nabla}_n \xi^a = \mathbf{0}$ *iff* $\xi^n \overset{f}{\nabla}_n \xi^a = -\overset{f}{\nabla}{}^a \varphi$*; and* φ *and* ρ *together satisfy Poisson's equation relative to* $\overset{f}{\nabla}$.

It is important emphasize that the pair $(\overset{f}{\nabla}, \varphi)$ in Prop. 2.2 is not unique. A second pair $(\overset{f}{\nabla}{}', \varphi')$ will satisfy the same conditions provided that (1) $\nabla^a \nabla^b (\varphi' - \varphi) = \mathbf{0}$ and (2) $\overset{f}{\nabla}{}' = (\overset{f}{\nabla}, C^a{}_{bc})$, with $C^a{}_{bc} = t_b t_c \nabla^a (\varphi' - \varphi)$. Note, too, that Prop. 2.2 holds only if conditions (1)-(3) from Prop. 2.1 are satisfied. The geometrized Poisson equation, condition (1), has already been assumed to hold of models of GNG; for present purposes, I will limit attention to models of GNG that also satisfy conditions (2) and (3).

[6]The notation $\nabla' = (\nabla, C^a{}_{bc})$ is explained in Malament (2012, Prop. 1.7.3).

3. Glymour on theoretical equivalence

I will now turn to Glymour's account of theoretical equivalence. The underlying intuition is that two theories are theoretically equivalent if (1) the are empirically equivalent and (2) they are mutually inter-translatable.[7] In general, empirical equivalence is a slippery concept, but we will not discuss it further. For present purposes, it suffices to stipulate that the theories being compared *are* empirically equivalent, in the precise senses described.

Glymour makes the second condition, of mutual inter-translatability, precise via the notion of definitional equivalence in first order logic.[8] Suppose that L and L^+ are first-order signatures, with $L \subseteq L^+$. An explicit definition of a symbol in L^+ in terms of L is a sentence in L^+ that asserts the equivalence between that symbol (appropriately used) and some formula in L. Given a theory T in L, by appending explicit definitions of the symbols in L^+/L to T, we may extend T to a theory in L^+. The resulting theory is a *definitional extension of* T *in* L^+. Now suppose T_1 and T_2 are first-order theories in signatures L_1 and L_2, respectively, with $L_1 \cap L_2 = \emptyset$. Then T_1 and T_2 are *definitionally equivalent* if and only if there are first order theories T_1^+ and T_2^+ in $L_1 \cup L_2$ such that T_1^+ is a definitional extension of T_1, T_2^+ is a definitional extension of T_2, and T_1^+ and T_2^+ are logically equivalent. Definitional equivalence captures a sense of inter-translatability in that, given any pair of definitionally equivalent theories T_1 and T_2 and a formula ϱ in the language of T_1, it is always possible to translate ϱ into a formula in the language of T_2, and then back into a formula in the language of T_1 that is T_1-provably equivalent to ϱ.

Definitional equivalence is a natural notion of equivalence for first order theories. But it is difficult to apply to physical theories, since we rarely have first order formulations available.

[7]Glymour does not state that empirical equivalence is a necessary condition for theoretical equivalence, though he does appear to take theoretical equivalence to be strictly stronger than empirical equivalence, and, as Sklar (1982) emphasizes, empirical equivalence is a substantive interpretive constraint that goes beyond any formal relations between two theories.

[8]For details on explicit definability and definitional equivalence, see Hodges (1993, Ch. 2.6). See also the classic work by de Bouvere (1965b,a).

For this reason, Glymour works with a model-theoretic variant of definitional equivalence. Suppose T_1 and T_2 are definitionally equivalent theories, and suppose that A_1 is a model of T_1. Then it is always possible to expand A_1 into a model A of T_1^+, the definitional extension of T_1. Since T_1^+ and T_2^+ (the extension of T_2) are logically equivalent, A is also a model of T_2^+. We may thus turn A into a model A_2 of T_2 by restricting A to symbols in the language of T_2. The whole process can then be reversed to recover A_1. In this sense, definitionally equivalent theories "have the same models" insofar as a model of one theory can be systematically transformed into a model of the other theory, and vice versa.[9]

Using this model-theoretic characterization of definitional equivalence as inspiration, Glymour proposes the following criterion of equivalence for physical theories expressed in terms of covariant objects on a manifold.[10]

Criterion 1. *Theories T_1 and T_2 are theoretically equivalent if for every model M_1 in T_1, there exists a unique model M_2 in T_2 that (1) has the same empirical content as M_1 and (2) is such that the geometrical objects associated with M_2 are uniquely and covariantly definable in terms of the elements of M_1 and the geometrical objects associated with M_1 are uniquely and covariantly definable in terms of M_2, and vice versa.*

GNG and NG fail to meet this criterion. The reason is that, although it is always possible given a model M_1 of NG to uniquely and covariantly define a model M_2 of GNG, it is *not* possible to go in the other direction: given M_2, there are many corresponding models of NG.

4. A problem case for Glymour?

I will presently argue that criterion 1 does not capture an important sense in which two theories may be equivalent. I will do so by displaying two "theories" (actually, formulations of a single theory) that usually are (I claim correctly) taken to be equivalent, but which fail

[9]It is essential that one can go from a model A_1 of T_1 to a model A_2 of T_2, and then back to the *same* model A_1 of T_1. See Andréka et al. (2005).

[10]Actually, all Glymour claims is that clause (2) of this criterion is a necessary condition for theoretical equivalence. I am extrapolating when I say that the two clauses together are also sufficient.

to meet Glymour's criterion. These theories correspond to two ways of presenting classical electromagnetism on Minkowski spacetime, (M, η_{ab}).[11]

On the first formulation of the theory, which I will call EM_1, the dynamical variable a smooth, antisymmetric tensor field F_{ab} on M. This field is called the Faraday tensor; it represents the electromagnetic field on spacetime. The Faraday tensor satisfies Maxwell's equations, which may be written as (1) $\nabla_{[a}F_{bc]} = \mathbf{0}$ and (2) $\nabla_a F^{ab} = J^b$, where J^b is a smooth vector field representing charge-current density. (Here ∇ is the Levi-Civita derivative operator compatible with η_{ab}.) Models on this formulation may be written (M, η_{ab}, F_{ab}). On the second formulation, which I will call EM_2, the dynamical variable is a smooth vector field A_a on M, called the $4-$vector potential. This field satisfies the differential equation $\nabla_a \nabla^a A^b - \nabla^b \nabla_a A^a = J^b$. Models may be written (M, η_{ab}, A_a).

These two formulations are systemically related. Given a vector potential A_a on M, one may define a Faraday tensor by $F_{ab} = \nabla_{[a}A_{b]}$. This tensor will satisfy Maxwell's equations for some J^a if A_a satisfies the differential equation above for the same J^a. Conversely, given a Faraday tensor F_{ab} satisfying Maxwell's equations (for some J^a), there always exists a vector potential A_a satisfying the required differential equation (for that J^a), such that $F_{ab} = \nabla_{[a}A_{b]}$. We stipulate that on both formulations, the empirical content of a model is exhausted by its associated Faraday tensor. In this sense, the theories are empirically equivalent, since for any model of EM_1, there is a corresponding model of EM_2 with the same empirical content (for some fixed J^a), and vice versa.

But are EM_1 and EM_2 equivalent by Glymour's criterion? No. Given any model (M, η_{ab}, A_a) of EM_2, I can uniquely and covariantly define a model (M, η_{ab}, F_{ab}) of EM_1 by taking $F_{ab} = \nabla_{[a}A_{b]}$. But given a model (M, η_{ab}, F_{ab}) of EM_1, there are generally many corresponding models of EM_2. In particular, if $F_{ab} = \nabla_{[a}A_{b]}$ for some 4-vector potential A_a,

[11]Minkowski spacetime is a (fixed) relativistic spacetime (M, η_{ab}) where M is $\mathbb{R}^4$, η_{ab} is a flat Lorentzian metric, and the spacetime is geodesically complete.

then $F_{ab} = \nabla_{[a}\tilde{A}_{b]}$ will also hold if (and only if) $\tilde{A}_a = A_a + G_a$, where G_a is a closed one form (i.e., $\nabla_{[a}G_{b]} = \mathbf{0}$). Thus unique definability fails in the EM_2 to EM_1 direction.

What should one make of this result? On the one hand, Glymour's criterion seems to capture something important: the failure of uniqueness suggests that EM_2 distinguishes physical situations that EM_1 cannot distinguish. On the other hand, EM_1 and EM_2 are usually taken to be different formulations of the same theory; they are intended to have precisely the same theoretical content. The tension concerns the relationship between the models of EM_2. The transformations between models of EM_2 associated with the same Faraday tensor are often called "gauge transformations". On their standard interpretation, models related by a gauge transformation are *physically equivalent*, in the sense that they have the capacity to represent precisely the same physical situations.[12] Thus EM_2 does *not* distinguish situations that EM_1 cannot. And indeed, it seems to me that there is a clear and robust sense in which two theories should be understood as equivalent if, on their standard interpretations, they differ only with regard to features that, by the lights of the theories themselves, have no physical content.

5. An alternative criterion

Thus far, I have introduced a criterion of theoretical equivalence and argued that it fails to capture the sense in which EM_1 and EM_2 are equivalent. In the present section, I will present a criterion of equivalence that does capture the sense in which EM_1 and EM_2 are equivalent. To motivate this new criterion, note first that there are actually two reasons that EM_1 and EM_2 fail to meet Glymour's criterion. The first concerns the failure of a model of EM_1 to correspond to a unique model of EM_2. The second problem concerns "covariantly definability". Though the Faraday tensor F_{ab} is always definable in terms of a vector potential, in general there is no way to define a vector potential in terms of a

[12]The status of the vector potential arguably changes in quantum mechanics. See Belot (1998).

covariant formula involving the Faraday tensor. Rather, one has general existence results guaranteeing that an associated vector potential must exist.

I will begin with the uniqueness problem. If we want a sense of theoretical equivalence that captures the sense in which EM_1 and EM_2 are equivalent to one another, we need to be able to accommodate the possibility that not all of the structure of models of EM_2 is salient. That is, we want a sense of unique recovery *up to physical equivalence*. One way to make this idea precise is to modify our definition of models of EM_2. Instead of characterizing a model as a triple (M, η_{ab}, A_a), we may take a model to be a triple $(M, \eta_{ab}, [A_a])$, where $[A_a]$ is the equivalence class of physically equivalent vector potentials, $[A_a] = \{\tilde{A}_a : \tilde{A}_a = A_a + G_a \text{ for closed } G_a\}$. This approach explicitly equivocates between physically equivalent vector potentials. Call the theory whose models are so characterized EM_2'.

Proposition 5.1. *For any model (M, η_{ab}, F_{ab}) of EM_1, there is a unique model $(M, \eta_{ab}, [A_a])$ of EM_2' such that $F_{ab} = d_a X_a$ for every $X_a \in [A_a]$.*

Thus we *do* have unique recovery of models of EM_2' from models of EM_1.

We still face the second problem, however, since it is still not clear that there is an expression by which we may covariantly define an equivalence class $[A_a]$ in terms of an antisymmetric tensor F_{ab}. To address this problem, we return to first order logic. In that context, a classic result known as Svenonius' theorem that states (roughly) that a relation is (explicitly) definable in a first order theory just in case the relation is invariant under the automorphisms of the models of that theory (see Hodges, 1993, §5.10). This result provides an alternative model-theoretic characterization of definitional equivalence. Of course, as in our discussion of Glymour's criterion, we cannot apply the new characterization directly. But it does suggest an alternative way of adapting definitional equivalence to the present context, by using the invariance properties of the models of our theories. Indeed, there is a sense in which Glymour's characterization in terms of *covariant* definability already incorporates an invariance requirement of precisely this sort, since covariance amounts to a condition on how objects behave under diffeomorphisms.

These considerations suggest the following idea. Instead of requiring explicit covariant definability, we require that the geometrical objects characterizing models of two theories be invariant under the same diffeomorphisms.

Proposition 5.2. *Let* (M, η_{ab}, F_{ab}) *and* (M, η_{ab}, F'_{ab}) *be models of* EM_1 *and let* $(M, \eta_{ab}, [A_a])$ *and* $(M, \eta_{ab}, [A_a]')$ *be the unique corresponding models of* EM'_2*. If* $\chi : M \rightarrow M$ *is an isometry such that* $\chi_*(F_{ab}) = F'_{ab}$*, then* $[\chi_*(A_a)] = [A_a]'$*.*[13]

This result provides a sense in which the models of EM'_2 might be said to be *implicitly* definable from the models of EM_1. Putting Props. 5.1 and 5.2 together, meanwhile, (almost) captures the sense in which EM_1 and EM'_2 are equivalent.

It would be desirable to abstract away from this specific case and articulate a more general criterion. To do so, observe that the relationship between EM_1 and EM'_2 that is captured by Props. 5.1 and 5.2 concerns not only the models, but also maps between the models—namely, the diffeomorphisms that preserve the structure of the models. This suggests that we might be able to extract a general criterion of equivalence by representing these theories not as *collections* of models, but as *categories* of models.[14] That is, we define a category $\mathbf{EM}_1$ whose objects are models of EM_1 and whose arrows are isometries of Minkowski spacetime that preserve the Faraday tensor, and a category $\mathbf{EM}'_2$ whose objects are models of EM'_2 and whose arrows are isometries of Minkowski spacetime that preserve the equivalence classes of vector potentials, as in Prop. 5.2.

Given these categories, we may then prove the following result.

Corollary 5.3. *There exists an isomorphism of categories between* $\mathbf{EM}_1$ *and* $\mathbf{EM}_2$ *that preserves empirical content.*

Prop. 5.3 captures the sense of equivalence given by the conjunction of Props. 5.1 and 5.2, but it is slightly stronger: it says not only that the corresponding models of EM_1 and EM_2

[13]Here χ_* is the pushforward along χ, defined for differential forms because χ is a diffeomorphism.

[14]For more on this proposal, see Halvorson (2012) and Halvorson and Weatherall (2015); for more on categories and functors, see Mac Lane (1998), Awodey (2006), or Borceux (2008), among many other excellent texts.

are invariant under the same diffeomorphisms, but also that those invariances are compatible with what might be thought of as the algebraic structure of the diffeomorphisms.

Prop. 5.3 suggests the following new criterion of equivalence.

Criterion 1′. *Two theories are theoretically equivalent just in case there exists an isomorphism between their categories of models that preserves empirical content.*

I call this criterion 1′ because it bears a very close relationship to Glymour's original criterion. In effect, all we have done is move from one version of the definability clause of criterion 1 to another—equivalent in first order logic—to generate a criterion that is a bit more convenient to use in the present context. This is not to say the resulting criteria are the same—as we have seen, criterion 1′ is strictly weaker than 1, since EM_1 and EM_2' are equivalent by criterion 1′ and not by criterion 1—but rather to say that both are motivated by the same basic intuition, implemented in different ways.

I claim that criterion 1′ and Cor. 5.3 capture the sense in which EM_1 and EM_2' are equivalent theories. But what about EM_2, the alternative formulation of electromagnetism we began with? After all, it was *this* theory that we originally wanted to claim was equivalent to EM_1. We may define a category of models of this theory, too: as a first pass, we take $\mathbf{EM}_2$ to be the category whose objects are models of EM_2 and whose arrows are isometries of Minkowski spacetime that preserve the vector potential. But this category is *not* isomorphic to $\mathbf{EM}_1$—and so, on this representation of EM_2, EM_1 and EM_2 are still not equivalent, even by criterion 1′. The problem is the same as with Glymour's criterion: roughly speaking, there is a failure of uniqueness.

We have already argued that this sort of non-uniqueness is spurious, at least on the standard interpretation of EM_2, because models related by a gauge transformation should be counted as physically equivalent. The category $\mathbf{EM}_2$ does not reflect this equivalence between models, because in general, two models that differ by a gauge transformation will not be isomorphic in this category. On the other hand, we also know that there is another class of mapping between models that *does* reflect this sort of physical equivalence—namely, the gauge transformations themselves. These maps do not appear as arrows in the category $\mathbf{EM}_2$, which suggests that if we want to represent EM_2 accurately, in the sense of representing it in a way that accords with what structure we take to be physically significant on the standard interpretation, we need a different category, one that includes information about the gauge transformations.

We define such a category as follows: we take $\overline{\mathbf{EM}}_2$ to be the category whose objects are models of EM_2 and whose arrows are pairs of the form (χ, G_a), where G_a is closed and χ is an isometry that preserves the (gauge transformed) vector potential $A_a + G_a$.

Proposition 5.4. $\overline{\mathbf{EM}}_2$ *is a category.*

Note that $\overline{\mathbf{EM}}_2$ is naturally understood to include the arrows of $\mathbf{EM}_1$, which may be identified with pairs of the form $(\chi, 0)$, the gauge transformations, which are arrows of the form $(1_M, G_a)$, and compositions of these.

Intuitively speaking, $\overline{\mathbf{EM}}_2$ is the result of taking $\mathbf{EM}_2$ and "adding" arrows corresponding to the gauge transformations. Simply adding arrows in this way, however, does not yield a category that is (empirical-content-preservingly) isomorphic to $\mathbf{EM}_1$. The reason is that the extra arrows do not address the failure of unique recovery. But that does not mean this exercise was in vain. Although there is not an *isomorphism* between $\mathbf{EM}_1$ and $\overline{\mathbf{EM}}_2$ that preserves empirical content, there is an *equivalence* of categories that does so.

Proposition 5.5. *There is an equivalence of categories between* $\mathbf{EM}_1$ *and* $\overline{\mathbf{EM}}_2$ *that preserves empirical content.*

Equivalent categories may be thought of as categories that are isomorphic "up to object isomorphism"—which is precisely the notion of equivalence we argued we were looking for between EM_1 and EM_2 at the end of the last section.

The considerations in the last paragraph suggest a new, still weaker criterion.

Criterion 2. *Two theories are theoretically equivalence just in case there exists an equivalence between their categories of models that preserves empirical content.*

Prop. 5.5 establishes that EM_1 and EM_2 *are* theoretically equivalent by this new criterion—so long as we represent EM_2 by $\overline{\mathbf{EM}}_2$, rather than $\mathbf{EM}_2$. It is in this sense, I claim, that the two formulations of electromagnetism should be taken to be equivalent.

As a final remark, let me observe that criterion 2 also captures the sense in which EM_2 and EM_2' are equivalent. In particular, these theories are *not* equivalent by criterion $1'$, even though $\mathbf{EM}_2'$ and $\overline{\mathbf{EM}}_2$ may seem to be equally good ways of capturing "gauge equivalence" in a formal representation of EM_2. This fact reflects the more general fact that for most mathematical purposes, equivalence of categories is a more natural and fruitful notion of "sameness" of categories than isomorphism. It also suggests that criterion $1'$ is at best an awkward half-way point once we have begun thinking in the present terms.

6. Are NG and GNG theoretically equivalent?

With these new criteria in hand, we now return to the question at the heart of the paper. To apply either criterion to NG and GNG, however, one first needs to say what categories we will use to represent the theories. For GNG, there is a clear choice. We represent GNG by the category **GNG** whose objects are classical spacetimes (M, t_a, h^{ab}, ∇) satisfying the required curvature conditions from Prop. 2.1, and whose arrows are diffeomorphisms that preserve the classical metrics and the derivative operator.[15]

NG is more complicated, however. There is a natural option for the objects: they are classical spacetimes with gravitational potentials $(M, t_a, h^{ab}, \nabla, \varphi)$, where ∇ flat. But we face a choice concerning the arrows, corresponding to a choice about which models of NG are physically equivalent.

Option 1. One takes models of NG that differ with regard to the gravitational potential to be distinct.

[15] Given a diffeomorphism $\chi : M \to M'$ and derivative operators ∇ and ∇' on M and M' respectively, we say that χ preserves ∇ if for any tensor field $\lambda^{a_1\cdots a_r}_{b_1\cdots b_s}$ on M, $\chi_*(\nabla_n \lambda^{a_1\cdots a_r}_{b_1\cdots b_s}) = \nabla'_n \chi_*(\lambda^{a_1\cdots a_r}_{b_1\cdots b_s})$.

Option 2. One takes models of NG whose gravitational potential and derivative operators are related by the transformation $\varphi \mapsto \varphi' = \varphi + \psi$ and $\nabla \mapsto \nabla' = (\nabla, t_b t_c \nabla^a \psi)$, for any smooth ψ satisfying $\nabla^a \nabla^b \psi = \mathbf{0}$, to be equivalent.[16]

In the second case, one takes the gravitational potential to be a gauge quantity, much like the vector potential in electromagnetism. In the first case, one does not.

These two options suggest different categories. In particular, we define $\mathbf{NG}_1$ to be the category whose objects are as above, and whose arrows are diffeomorphisms that preserve the classical metrics, the derivative operator, and the gravitational potential, and we define $\mathbf{NG}_2$ to be the category with the same objects, but whose arrows are pairs (χ, ψ), where ψ is a smooth scalar field satisfying $\nabla^a \nabla^b \psi = \mathbf{0}$, and χ is a diffeomorphism that preserves the classical metrics and the (gauge transformed) derivative operator $\nabla' = (\nabla, t_b t_c \nabla^a \psi)$ and gravitational potential $\varphi + \psi$. The first category corresponds to option 1, while the second corresponds to option 2. Since these options correspond to different interpretations of the formalism, I will treat them as prima facie distinct theories, labeled as NG_1 and NG_2, respectively, in what follows.

What considerations might lead one to prefer one option over the other? The first option better reflects how physicists have traditionally thought of Newtonian gravitation. On the other hand, this option appears to distinguish between models that are not empirically distinguishable, even in principle. Moreover, there are systems in which option 1 leads to problems, such as cosmological models with homogeneous and isotropic matter distributions, where option 1 generates contradictions that option 2 avoids.[17] These latter arguments strike me as compelling, and I tend to agree with the conclusion that option 2 is preferable. But I will not argue further for this thesis, and for the purposes of the present paper, I will remain agnostic about which way of understanding NG is preferable.

[16]Note that, since *all* of the derivative operators considered in NG and GNG agree once one raises their index, one can characterize the gauge transformation with regard to any of them without ambiguity.

[17]For more on this, see the debate between John Norton (1992, 1995) and David Malament (1995).

	Condition $1'$	Condition 2
NG_1 and NG_2	Inequivalent	Inequivalent
NG_1 and GNG	Inequivalent	Inequivalent
NG_2 and GNG	Inequivalent	**Equivalent**

Table 1: A summary of the equivalences and inequivalences of NG and GNG, by the standards set by conditions $1'$ and 2.

We may now ask: are any of these theories pairwise equivalent by either criterion? None of these theories are equivalent by criterion $1'$, effectively for the reason that NG and GNG fail to be equivalent by Glymour's original criterion.[18] Moreover, NG_1 is not equivalent to either GNG or NG_2 by criterion 2. But GNG and NG_2 *are* equivalent by criterion 2.

Proposition 6.1. *There is an equivalence of categories between* $\mathbf{NG}_2$ *and* **GNG** *that preserves empirical content.*

The situation is summarized by table 1.

7. Interpreting physical theories: some morals

I have now made the principal arguments of the paper. In short, criterion 1 does not capture the sense in which EM_1 and EM_2 are equivalent. However, there is a natural alternative criterion that does capture the sense in which EM_1 and EM_2 are equivalent. And by this criterion, GNG and NG are equivalent too, if one adopts option 2 above. Moreover, criterion 2 highlights an important distinction between two ways of understanding NG.

There are a few places where one might object. One might say that *no* formal criterion captures what it would mean for two theories to be equivalent.[19] One might also reject the significance of the particular criteria discussed here. I do not agree with these objections, but I will not consider them further. For the remainder of this paper, I will suppose that

[18] Note, however, that one could construct an alternative presentation of NG_2 analogous to EM_2', in such a way that this *would* be equivalent to GNG by criterion $1'$. Moreover, if one restricts attention to the collections of models of NG and GNG in which (1) the matter distribution is supported on a spatially compact region and (2) the gravitational field (for models of NG) vanishes at spatial infinity, then $\mathbf{NG}_1$, $\mathbf{NG}_2$, and **GNG** are all equivalent by *both* criteria.

[19] For versions of this worry, see Sklar (1982) and Coffey (2014).

criterion 2 does capture an interesting and robust sense in which these theories may be equivalent. If this is right, there are several observations to make.

First of all, the arguments here support one of Glymour's principal claims, which is that there exist empirically equivalent, theoretically inequivalent theories. This is because even if NG_2 and GNG are theoretically equivalent, NG_1 and GNG are still inequivalent, even by condition 2. Glymour's further claim that GNG is better supported by the empirical evidence, on his account of confirmation, is only slightly affected, in that one needs to specify that GNG is only better supported than NG_1. This makes sense: the reason, on Glymour's account, that GNG is better supported than NG is supposed to be that NG makes additional, unsupported ontological claims regarding the existence of a gravitational potential. But one can understand the difference between NG_1 and NG_2 in this way as well, since NG_2 explicitly equivocates between models that differ with regard to their gravitational potentials.

There is another purpose to which Glymour puts these arguments, however. There is a view, originally due to Poincaré (1905) and Reichenbach (1958), that the geometrical properties of spacetime are a matter of convention because there always exist empirically equivalent theories that differ with regard to (for instance) whether spacetime is curved or flat.[20] Glymour argues against conventionalism by pointing out that the empirical equivalence of two theories does not imply that they are equally well confirmed, since the theories may be theoretically inequivalent. But the present discussion suggests that there is another possibility that is not often considered: theories that attribute apparently distinct geometrical properties to the world may be *more* than just empirically equivalent.

As I have just noted, one way of understanding NG_2 is as a theory on which the gravitational potential is not a real feature of the world, because the gravitational potential is not

[20]For a clear and detailed description of the positions that have been defended on the epistemology of geometry in the past, see Sklar (1977); see also Weatherall and Manchak (2014).

preserved by mappings that reflect physical equivalence. GNG, meanwhile, does not make any reference to a gravitational potential. In this sense GNG and NG_2 appear to have the same ontological implications, at least with regard to gravitational potentials. Still, GNG and NG_2 do differ in one important way. In particular, in all models of NG_2, *spacetime is flat*. In generic models of GNG, conversely, spacetime may be curved. Thus, at least in this context, there is a sense in which classical spacetime admits equally good, *theoretically* equivalent descriptions as either curved or flat.[21]

Let me emphasize that this view is not a recapitulation of traditional conventionalism about geometry. For one, it is not a general claim about spacetime geometry; the view here depends on the details of the geometry of classical spacetime physics. Indeed, there is good (though perhaps not dispositive) reason to think that general relativity, for instance, is *not* equivalent to a theory on which spacetime is flat, by any of the criteria discussed here.[22] More generally, I do not believe that it is a matter of convention whether we choose one empirically equivalent theory over another. There are often very good reasons to think one theory is better supported than or otherwise preferable to an empirically equivalent alternative. Rather, the point is that in some cases, apparently different descriptions of the world—such as a description on which spacetime is flat and one on which it is curved—amount to the same thing, insofar as they have exactly the same capacities to represent physical situations. In a sense, they say the same things about the world.

The suggestion developed in the last few paragraphs will worry some readers. Indeed, one might be inclined to reject criterion 2 on the grounds that one has antecedent or even *a priori* reason for thinking that there is, in all cases, an important distinction—perhaps

[21]There is a caveat worth mentioning: although spacetime is flat in all models of NG_2, and thus in all models, parallel transport of vectors is path independent, the result of parallel transporting any particular vector along a given (fixed) curve will generally vary even between equivalent models, because the derivative operator varies with gauge transformations. Thus one might think that GNG provides a more perspicuous representation of spacetime geometry, since the geometrical facts are obscured by the gauge transformations on NG_2. (I am grateful to Oliver Pooley for pressing this point; Knox (2014) makes a closely related point.)

[22]See Knox (2011) and Weatherall and Manchak (2014) for evidence supporting this claim.

a *metaphysical* distinction—between a theory that says spacetime is flat and one that says spacetime is curved. Two theories that disagree in this regard could not both be true, because at most one could accurately reflect the facts about the curvature of spacetime, and thus, two such theories could not be equivalent. I think this position is probably tenable. But it seems to me to get things backwards. At the very least, let me simply say that there is another way of looking at matters, whereby one allows that the distinctions that one can sensibly draw may depend on the structure of the world. And the our best guide to understanding what those distinctions are will be to study the properties of and relationships between our best physical theories.

Appendix A. Proofs of propositions

Proof of Prop. 5.1.

Suppose there were vector potentials A_a and $\tilde{A}_a$ such that $[A_a] \neq [\tilde{A}_a]$, but for every $X_a \in [A_a]$, $\nabla_{[a}X_{b]} = \nabla_{[a}\tilde{A}_{b]} = F_{ab}$. Then $\nabla_{[a}(X_{b]} - \tilde{A}_{b]}) = \mathbf{0}$ for every $X_a \in [A_a]$, and thus $X_a - \tilde{A}_a$ is closed for every $X_a \in [A_a]$. Thus $[A_a] \subseteq [\tilde{A}_a]$. A similar argument establishes that $[\tilde{A}_a] \subseteq [A_a]$. $\square$

Proof of Prop. 5.2.

Suppose we have an isometry χ as described. Then for every $X_a \in [A_a]$, we have $\chi_*(\nabla_{[a}X_{b]}) = \chi_*(F_{ab}) = F'_{ab}$. But exterior derivatives commute with pushforwards along diffeomorphisms, and so $\chi_*(\nabla_{[a}X_{b]}) = \nabla_{[a}\chi_*(X_{b]}) = F'_{ab}$. Thus by Prop. 5.1, $[\chi_*(A_a)] = [A'_a]$. $\square$

Proof of Prop. 5.4.

$\mathbf{EM}_2$ includes identity arrows, which are pairs of the form $(1_M, 0)$; (2) it contains all compositions of arrows, since given any two arrows (χ, G_a) and (χ', G'_a) with appropriate domain and codomain, $(\chi', G'_a)\circ(\chi, G_a) = (\chi'\circ\chi, \chi^*(G'_a)+G_a)$ is also an arrow; and (3) composition of arrows is associative, since given three pairs (χ, G_a), (χ', G'_a), and (χ'', G''_a) with appropriate domain and codomain, $(\chi'', G''_a)\circ((\chi', G'_a)\circ(\chi, G_a)) = (\chi'', G''_a)\circ(\chi'\circ\chi, \chi^*(G'_a)+G_a) =$

$(\chi'' \circ (\chi' \circ \chi), \chi^* \circ \chi'^*(G''_a) + \chi^*(G_a)' + G_a) = ((\chi'' \circ \chi') \circ \chi, \chi^*(\chi'^*(G''_a) + G'_a) + G_a) = (\chi'' \circ \chi', \chi'^*(G''_a) + G'_a) \circ (\chi, G_a) = ((\chi'', G''_a) \circ (\chi', G'_a)) \circ (\chi, G_a)$. □

Proof of Prop. 5.5.

It suffices to show that there is a functor from $\overline{\mathbf{EM}}_2$ to $\mathbf{EM}_1$ that is full, faithful, and essentially surjective, and which preserves F_{ab}. Consider the functor $E : \overline{\mathbf{EM}}_2 \to \mathbf{EM}_1$ defined as follows: E acts on objects as $(M, \eta_{ab}, A_a) \mapsto (M, \eta_{ab}, \nabla_{[a}A_{b]})$ and on arrows as $(\chi, G_a) \mapsto \chi$. This functor clearly preserves F_{ab}. It is also essentially surjective, since given any F_{ab}, there always exists some A_a such that $\nabla_{[a}A_{b]} = F_{ab}$. Finally, to show that it is full and faithful, we need to show that for any two objects (M, η_{ab}, A_a) and (M, η_{ab}, A'_a), the induced map on arrows between these models is bijective. First, suppose there exist two distinct arrows $(\chi, G_a), (\chi', G'_a) : (M, \eta_{ab}, A_a) \to (M, \eta_{ab}, A'_a)$. If $\chi \neq \chi'$ we are finished, so suppose for contradiction that $\chi = \chi'$. Since by hypothesis these are distinct arrows, it must be that $G_a \neq G'_a$. But then $A_a + G_a \neq A_a + G'_a$, and so $\chi_*(A_a + G_a) \neq \chi_*(A_a + G'_a)$. So we have a contradiction, and $\chi \neq \chi'$. Thus the induced map on arrows is injective. Now consider an arrow $\chi : E((M, \eta_{ab}, A_a)) \to E((M, \eta_{ab}, A'_a))$. This is an isometry such that $\chi_*(\nabla_{[a}A_{b]}) = \nabla_{[a}A'_{b]}$. It follows that $\chi_*(\nabla_{[a}A_{b]} - \nabla_{[a}\chi^*(A'_{b]})) = \mathbf{0}$, and thus that $\nabla_{[a}A_{b]} - \nabla_{[a}\chi^*(A'_{b]})$ is closed. So there is an arrow $(\chi, \chi^*(A'_a) - A_a) : (M, \eta_{ab}, A_a) \to (M, \eta_{ab}, A'_a)$ such that $E((\chi, \chi^*(A'_a) - A_a)) = \chi$, and the induced map on arrows is surjective. □

Proof of Prop. 6.1.

This argument follows the proof of Prop. 5.5 closely. Consider the functor $E : \mathbf{NG}_2 \to \mathbf{NG}_1$ defined as follows: E takes objects to their geometrizations, as in Prop. 2.1, and it acts on arrows as $(\chi, \psi) \mapsto \chi$. This functor preserves empirical content because the geometrization lemma does; meanwhile, Prop. 2.2 ensures that the functor is essentially surjective. We now show it is full and faithful. Consider any two objects $A = (M, t_a, h^{ab}, \nabla, \varphi)$ and $A' = (M', t'_a, h'^{ab}, \nabla', \varphi)$. Suppose there exist distinct arrows $(\chi, \psi), (\chi', \psi') : A \to A'$, and suppose (for contradiction) that $\chi = \chi'$. Then $\psi \neq \psi'$, since the arrows were assumed to be distinct.

But then $\varphi+\psi \neq \varphi+\psi'$, and so $(\varphi+\psi)\circ\chi \neq (\varphi+\psi')\circ\chi$. Thus $\chi \neq \chi'$ and E is faithful. Now consider any arrow $\chi : E(A) \to E(A')$; we need to show that there is an arrow from A to A' that E maps to χ. I claim that the pair $(\chi, \varphi'\circ\chi-\varphi) : A \to A'$ is such an arrow. Clearly if this arrow exists in $\mathbf{NG}_2$, E maps it to χ, so it only remains to show that this arrow exists. First, observe that since χ is an arrow from $E(A)$ to $E(A')$, $\chi : M \to M'$ is a diffeomorphism such that $\chi_*(t_a) = t'_a$ and $\chi_*(h^{ab}) = h'^{ab}$. Moreover, $\chi_*(\varphi + (\varphi'\circ\chi-\varphi)) = \chi_*(\varphi'\circ\chi) = \varphi'\circ(\chi\circ\chi^{-1}) = \varphi'$, so χ maps the gauge transformed potential associated with A to the potential associated with A'. Now consider the action of χ on the derivative operator ∇. We need to show that for any tensor field $\lambda^{a_1\cdots a_r}_{b_1\cdots b_s}$, $\chi_*(\tilde{\nabla}_n\lambda^{a_1\cdots a_r}_{b_1\cdots b_s}) = \nabla'_n\chi_*(\lambda^{a_1\cdots a_r}_{b_1\cdots b_s})$, where $\tilde{\nabla} = (\nabla, t_bt_c\nabla^a(\varphi'\circ\chi-\varphi))$ is the gauge transformed derivative operator associated with A. We will do this for an arbitrary vector field; the argument for general tensor fields proceeds identically. Consider some vector field ξ^a. Then $\chi_*(\tilde{\nabla}_n\xi^a) = \chi_*(\nabla_n\xi^a - t_nt_m\xi^m\nabla^a(\varphi'\circ\chi - \varphi)) = \chi_*(\overset{g}{\nabla}_n\xi^a - t_nt_m\xi^m\nabla^a\varphi - t_nt_m\xi^m\nabla^a(\varphi'\circ\chi-\varphi)) = \chi_*(\overset{g}{\nabla}_n\xi^a) - \chi_*(t_nt_m\xi^m\nabla^a(\varphi'\circ\chi))$, where $\overset{g}{\nabla} = (\nabla, t_bt_c\nabla^a\varphi)$ is the derivative operator associated with $E(A)$. Now, we know that $\chi : E(A) \to E(A')$ is an arrow of $\mathbf{GNG}$, so $\chi_*(\overset{g}{\nabla}_n\xi^a) = \overset{g}{\nabla}_n\chi_*(\xi^a)$. Moreover, note that the definitions of the relevant $C^a{}_{bc}$ fields guarantee that $\nabla^a\lambda^{a_1\cdots a_r}_{b_1\cdots b_s} = \overset{g}{\nabla}{}^a(\lambda^{a_1\cdots a_r}_{b_1\cdots b_s})$ and similarly for ∇' and $\overset{g}{\nabla}{}'$. Thus we have $\chi_*(\overset{g}{\nabla}_n\xi^a) - \chi_*(t_nt_m\xi^m\nabla^a(\varphi'\circ\chi)) = \overset{g}{\nabla}{}'_n\chi_*(\xi^a) - t'_nt'_m\chi_*(\xi^m)\nabla'^a(\varphi'\circ(\chi\circ\chi^{-1})) = \overset{g}{\nabla}{}'_n\chi_*(\xi^a) - t'_nt'_m\chi_*(\xi^m)\nabla'^a\varphi' = \nabla'_n\chi_*(\xi^a)$, where $\overset{g}{\nabla}{}' = (\nabla', -t_bt_c\nabla^a\varphi')$ is the derivative operator associated with $E(A')$. So χ does preserve the gauge transformed derivative operator. The final step is to confirm that $\nabla^a\nabla^b(\varphi'\circ\chi-\varphi) = \mathbf{0}$. To do this, again consider an arbitrary vector field ξ^a on M. We have just shown that $\nabla'_a\chi_*(\xi^b) - \chi_*(\nabla_a\xi^b) = -\chi_*(t_at_m\xi^m\nabla^b(\varphi'\circ\chi-\varphi)$. Now consider acting on both sides of this equation with ∇'^a. Beginning with the left hand side (and recalling that ∇ and ∇' are both flat), we find: $\nabla'^n\nabla'_a\chi_*(\xi^b) - \nabla'^n\chi_*(\nabla_a\xi^b) = \nabla'_a\overset{g}{\nabla}{}'^n\chi_*(\xi^b) - \chi_*(\nabla_a\overset{g}{\nabla}{}^n\xi^b) = \overset{g}{\nabla}{}'_a\overset{g}{\nabla}{}'^n\chi_*(\xi^a) - t'_at'_m(\nabla'^b\varphi')\overset{g}{\nabla}{}'^n\chi_*(\xi^m) - \chi_*(\overset{g}{\nabla}_a\overset{g}{\nabla}{}^n\xi^b) + \chi_*(t_at_m(\nabla^b\varphi)\overset{g}{\nabla}{}^n\xi^m) = \chi_*(t_at_m(\nabla^b(\varphi - \varphi'\circ\chi))\overset{g}{\nabla}{}^n\xi^m)$. The right hand side, meanwhile, yields $-\nabla'^n(\chi_*(t_at_m\xi^m\nabla^b(\varphi'\circ\chi-\varphi)) =$

$\chi_*(t_a t_m(\nabla^n \xi^m)\nabla^b(\varphi - \varphi' \circ \chi)) + \chi_*(t_a t_m \xi^m \nabla^n \nabla^b(\varphi - \varphi' \circ \chi))$. Comparing these, we conclude that $\chi_*(t_a t_m \xi^m \nabla^n \nabla^b(\varphi - \varphi' \circ \chi)) = \mathbf{0}$, and thus $t_a t_m \xi^m \nabla^n \nabla^b(\varphi - \varphi' \circ \chi) = \mathbf{0}$. But since t_a is non-zero and this must hold for *any* vector field ξ^a, it follows that $\nabla^a \nabla^b(\varphi' \circ \chi - \varphi) = \mathbf{0}$. Thus E is full. $\square$

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