The Self-Organization of Meaning and the Reflexive Communication of Information

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Abstract

Following a suggestion of Warren Weaver, we extend the Shannon model of communication piecemeal into a complex systems model in which communication is differentiated both vertically and horizontally. This model enables us to bridge the divide between Niklas Luhmann's theory of the self-organization of meaning in communications and empirical research using information theory. First, we distinguish between communication relations and correlations between patterns of relations. The correlations span a vector space in which relations are positioned and thus provided with meaning. Second, positions provide reflexive perspectives. Whereas the different meanings are integrated locally, each instantiation opens horizons of meaning that can be codified along eigenvectors of the communication matrix. The next-order specification of codified meaning can generate redundancies (as feedback on the forward arrow of entropy production). The horizontal differentiation among the codes of communication enables us to quantify the creation of new options as mutual redundancy. Increases in redundancy can then be measured as local reduction of prevailing uncertainty (in bits). The generation of options can also be considered as a hallmark of the knowledge-based economy: new knowledge provides new options. Both the communication-theoretical and the operational (information-theoretical) perspectives can thus be further developed.

Keywords: redundancy; horizontal and vertical differentiation; codification; triple helix; reflection

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Introduction

In his contribution to Shannon & Weaver's (1949) *The Mathematical Theory of Communication*, Warren Weaver stated (at p. 27) that "[t]he concept of information developed in this theory at first seems disappointing and bizarre—disappointing because it has nothing to do with meaning …" However, the author added that Shannon's "analysis has so penetratingly cleared the air that one is now, perhaps for the first time, ready for a real theory of meaning." However, Luhmann ([1984] 1995) argued that meaning ("*Sinn*") self-organizes in terms of communications among human beings. From this perspective meaning is generated in interactions among communications. As Luhmann (1996, at p. 261) formulated provocatively: "My argument is: it is not human beings who can communicate, rather, only communication can communicate."

Hitherto, Luhmann's theory has remained far from operationalization and measurement. Using Bateson's (1972, at p. 315) alternative definition of information as "a difference which makes a difference" (cf. MacKay, 1969), Luhmann (1984, pp. 102 ff.; 1995, pp. 67f.) defined information as implying a selection: a difference can only make a difference for a system of reference that selects this difference from among other possible differences. Others have also defined information with reference to a receiving system (e.g., an observer) for whom a difference can make a difference. Varela (1979, p. 266) argued that the word "information" is derived from "in-formare" and thus the semantics call for the specification of a system of reference. "Information," however, is then considered a substantive concept that varies with the system of reference instead of a formal measure of the uncertainty prevailing in a distribution. Kauffman *et al*. (2008, at p. 28), for example, defined information as "natural selection assembling the very constraints on the release of energy that then constitutes work and the propagation of organization." In summary, using Bateson's alternative definition of information, the meaning of "information" becomes dependent on the context.

Using the same word ("information") for different concepts has led to considerable confusion. In an assessment of this confusion, Hayles (1990, pp. 59f.) compared the discussion with asking whether a glass is half empty or half full. However, the semantic confusion can be avoided by using the words "uncertainty" or "probabilistic entropy" when Shannon-type information is meant. In our opinion, the advantage of measuring uncertainty—and redundancies, as we shall argue—in bits of information cannot be underestimated, since the operationalization and the measurement provide avenues to hypothesis testing and thus control of the theorizing (Theil, 1972). Whereas Shannon-type information cannot be specified in terms of Bateson's definition, "a difference which makes a difference" can sometimes be operationalized and measured in terms of (potentially negative) bits of information.

Can Luhmann's theory of meaning and the self-organization of communication also be made compatible with Shannon's mathematical theory of communication? Is it possible to specify how information and meaning are related, and why the recursivity of this relation in terms of differences which make a difference can lead to the self-organization of meaning from communication? In this study, we aim to contribute to bridging this gap between a focus on meaning versus uncertainty by decomposing the problem using Herbert Simon's (1973a) model of complex systems that are both vertically and horizontally differentiated, as follows:

1. In the vertical dimension, we will follow Luhmann's (e.g., 1975; 2000) distinction between (i) interactions among communications providing variation, (ii) the organization of meanings in historical instantiations, and (iii) the self-organization of reflexive meaning generating a next level of "horizons of meaning" as global systems of reference (Husserl, 1929; cf. Luhmann, 1995b). We argue that the construction in terms of layers is bottom-up from the (probabilistic) informational level, but that the emerging systems levels can be expected to take over control in terms of codified intentionalities and expectations.

   For example, a scholarly communication (e.g., a manuscript) can be expected to contain a knowledge claim. Knowledge claims provide in this case the historical variation. When the manuscript is submitted, an editorial process is instantiated in which referee comments, editorial judgments, etc., are combined. The referees, however, are expected to judge the manuscript in terms according with the standards of the field. These codes of the communication can be expected to control the process. The instantiation requires a reflexive de- and reconstruction of the code.

   Unlike Simon's model (of complex and artificial systems), these mental constructs—codes of communication—can operate as a next-order systems level with frequencies much higher than the underlying levels. The mental constructs have two sides: the one of the constructing agency (in history), and the resulting constructs themselves which can only be entertained reflexively. Consequently, the sharing of meanings is volatile and cannot be observed directly; but the expectations about this operation can be specified mathematically. On the basis of this specification, one is able to identify and measure the “footprints” of the self-organizing dynamics in historical instantiations.

In other words, intentions and intentional systems cannot be found as observables in *res extensa*—or “matter”—but can only be hypothesized reflexively in *res cogitans*—”thought” (Husserl, 1929; Luhmann, 1990). The consequent possibility of an inversion in the order of control from the material conditions to systems of expectations enables reflexive agents—human beings—to operate infra-reflexively across (vertical) levels and among (horizontal) compartments by changing their respective perspectives on the complexity (Latour, 1988; cf. Pickering, 1992).

2. In the horizontal direction, we follow Parsons’ (1968) proposal to consider the functional differentiation and symbolic generalization of the codes of communication as drivers of the increasing complexity in cultural evolution. Recalling another intuition of Herbert Simon, one can expect an alphabet of these codes (Simon, 1973, at pp. 19 ff.); for example, power, love, truth, law, art, etc. Because of the various codes operating, the same communication can mean something quite different in terms of its affective value, its truth value, or how power is reproduced in communications (Parsons, 1963a and b; 1968; Künzler, 1987; Luhmann, 1974; 1997). The complexity is increased because the codes can be recombined in their instantiations (Hoffmeyer & Emmeche, 1991).

Using the symbolically generalized codes of communication in scholarly discourse, for example, one is specifically able to code possible meanings in terms of models that can serve to predict future states. The consequent availability of representations of future states enhances the anticipatory capacities of the social system of communications to such an extent that it can be considered "strongly anticipatory" (Dubois, 2003): anticipations can be used for the reconstruction of the system by the system itself. Based on decoding and recoding from and into communications, for example, different and highly codified meanings can be recombined into new technological options (Arthur, 2009; Cowan & Foray, 1997). A technological evolution can thus be generated as the retention mechanism of cultural evolution. Because of this coupling between the cultural and technological evolutions, Icarus's dream of flying can nowadays be realized by booking a flight on the internet.

The generation of meaning from (Shannon-type) information

How can the processing of meaning be conceptualized by elaborating on Shannon's theory given the author's explicit statement that the "semantic aspects of communication are irrelevant to the engineering problem" (Shannon, 1948, at p. 3)? As a first step in the specification of the relevance of Shannon's engineering model for developing a theory of meaning, Weaver (1949, at p. 26) proposed two "minor additions" to Shannon's well-known diagram (Figure 1), as follows:

> "One can imagine, as an addition to the diagram, another box labeled "Semantic Receiver" interposed between the engineering receiver (which changes signals to messages) and the destination. This semantic receiver subjects the message to a second decoding, the demand on this one being that it must match the statistical semantic characteristics of the message to the statistical semantic capacities of the totality of receivers, or of that subset of receivers which constitute the audience one wishes to affect.
>
> Similarly one can imagine another box in the diagram which, inserted between the information source and the transmitter, would be labeled "semantic noise," the box

previously labeled as simply “noise” now being labeled “engineering noise.” From this source is imposed into the signal the perturbations or distortions of meaning which are not intended by the source but which inescapably affect the destination. And the problem of semantic decoding must take this semantic noise into account.”

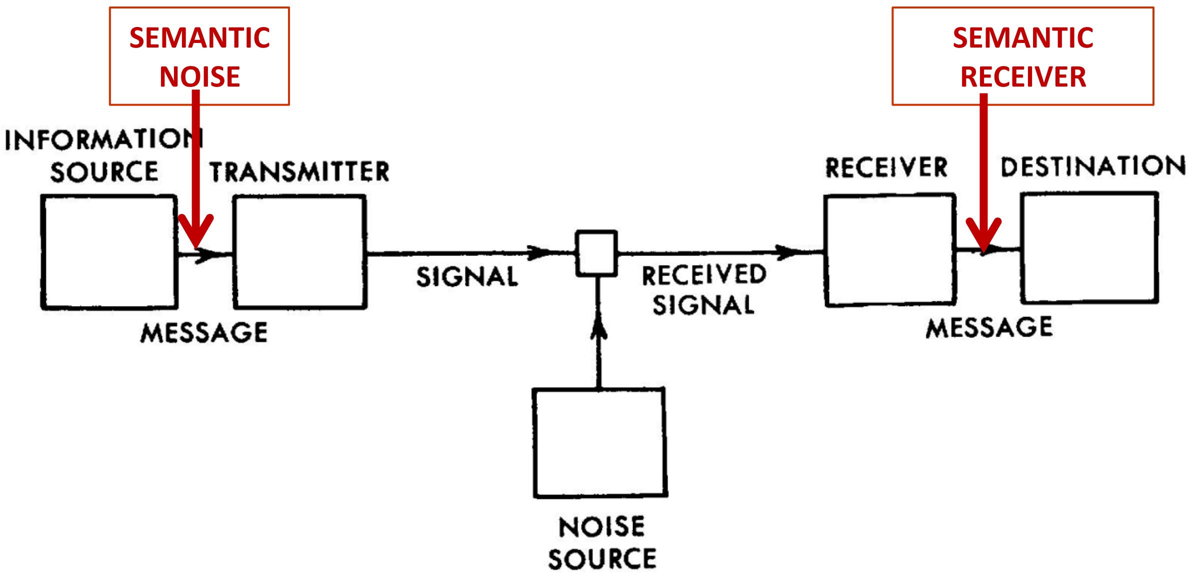

Figure 1: Weaver’s (1949) “minor” additions penciled into Shannon’s (1948) original diagram.

Since the “semantic receiver” recodes the information in the messages (received from the “engineering receiver” who only changes signals into messages) while having to assume the possibility of “semantic noise,” a semantic relationship between the two new boxes can be envisaged. Given Shannon’s framework, however, this relation cannot be another information transfer—since semantics are defined as *external* to Shannon’s engineering model. Meanings, however, can be shared without observable communications since semantics are based on patterns of relations or, in other words, correlations. The correlations span a vector space in a topology different from the network space—a graph—of relations.

Two synonyms, for example, can have the same position and meaning in the vector space, yet never co-occur in a single sentence as a relation. Similarly, two competing firms may have highly correlating patterns of relations with clients, but no relation with each other. In the case of a single relation, the relational distance is not different from the correlational one; but in the case of relations involving three (or more) agents, the distances in the vector space are different from the Euclidean distances in the network. Simmel (in 1902) noted that the transition from a group of two to three is qualitative. In a triplet {A, B, C}, the instantiation of AC or AB can make a difference with reference to the system of relations.

A system of relations can also be considered as a semantic domain. In other words, the sender and receiver are related in the graph of Figure 1, while they are correlated in terms of not (yet) necessarily instantiated relations in the background. The sender and receiver, for example, may share a (proto-)language. Note that this "language" is still naturalistic to the extent that one can also consider the gestures among monkeys as "languaging" (Maturana, 1978). The virtual structure of correlations provides a latent background that provides meaning to the information exchanges in relations. The correlations add up to a vector space that is a representation different from the network space, but no Shannon-information has to be added! In other words, meaning is not added to the information, but the same information is delineated differently and can be considered from a different perspective.

When one considers (Shannon-type) information as a set of differences in a probability distribution, "the differences make a difference" for a set of differences, or in other words, a system of reference. The systems perspective on the set generates meaning from the interactions among the relational communications. Note that such a "system of differences" is not delineated naturally, but remains an analytical construct. Depending on how one delineates in time and space, the meanings can be composed differently. Luhmann (1990) followed Von Foerster's (1979; cf. 1993) conjecture to use eigenvector analysis of the matrix of communications for its decomposition.[1] As against Shannon-type information which flows linearly along a vector—from the sender to the receiver—one can expect meanings to loop and by looping to develop next-order dimensionalities (Krippendorff, 2009a and b).

The third and fourth dimensions of the probabilistic entropy

As noted, a matrix of communications is shaped as soon as one adds a second communication to the single vector in Figure 1. The communication matrix can also be considered as a two-dimensional probability distribution—different from the one-dimensional probability distribution in a vector. Since the operation of a "difference which makes a difference" for a set of communications is recursive, further extensions to more than two dimensions are also possible.

Whether one's perspective is extended by taking more dimensions into account depends on one's research question: new types of analysis become possible with increasing dimensionality (Table 1), but by Occam's razor the representation should not be made more complex than necessary for the systems under study. Thus, there may be a trade-off in

[1] Luhmann indicates the latent dimensions with the word "eigenvalue" (Luhmann, 1990, p. 113, n. 59; cf. Von Glasersfeld, 2008, p. 64, n. 4). Technically, the eigenvalue of an eigenvector is the factor by which the eigenvector is scaled when multiplied by the matrix.

empirical research in terms of requisite variety (Ashby, 1958) and parsimony. However, we first pursue the analysis here mathematically,[2] in order to further specify the expected operation of a system of intentionality that cannot be observed directly (Luhmann, 1984, at p. 226; 1995, at p. 164). The historically observable instantiations are less complex than the evolutionary system which is instantiated; but the hypothesized systems and their respective selection mechanisms have the status of hypotheses. The observable instantiations can serve us in guiding our imagination of alternatives (Leydesdorff, 2015, in press).

When the communication matrix, for example, is repeated over time, one obtains a three-dimensional array of information (Figure 2a). In such a three-dimensional array, the development of information can also be considered in terms of trajectories; the information is then organized historically. A four-dimensional array or hyper-cube of information is more difficult to imagine or represent graphically: unlike a (three-dimensional) trajectory, a four-dimensional array can contain a regime (Dosi, 1982).

For example, one can consider this next-order system as having one more degree of freedom that allows it to select among the possible trajectories in three dimensions as representations of its past (Figure 2b). This additional selection implies a reflection by the system. The reflection is performative in the present and can therefore be considered as the self-organization of an adaptive system. In the four-dimensional representational system, one representation of its history in three dimensions can be acknowledged (weighted) more than another.

[2] "Mathematically" means in this context without reference to material substance (Heidegger, 1962).

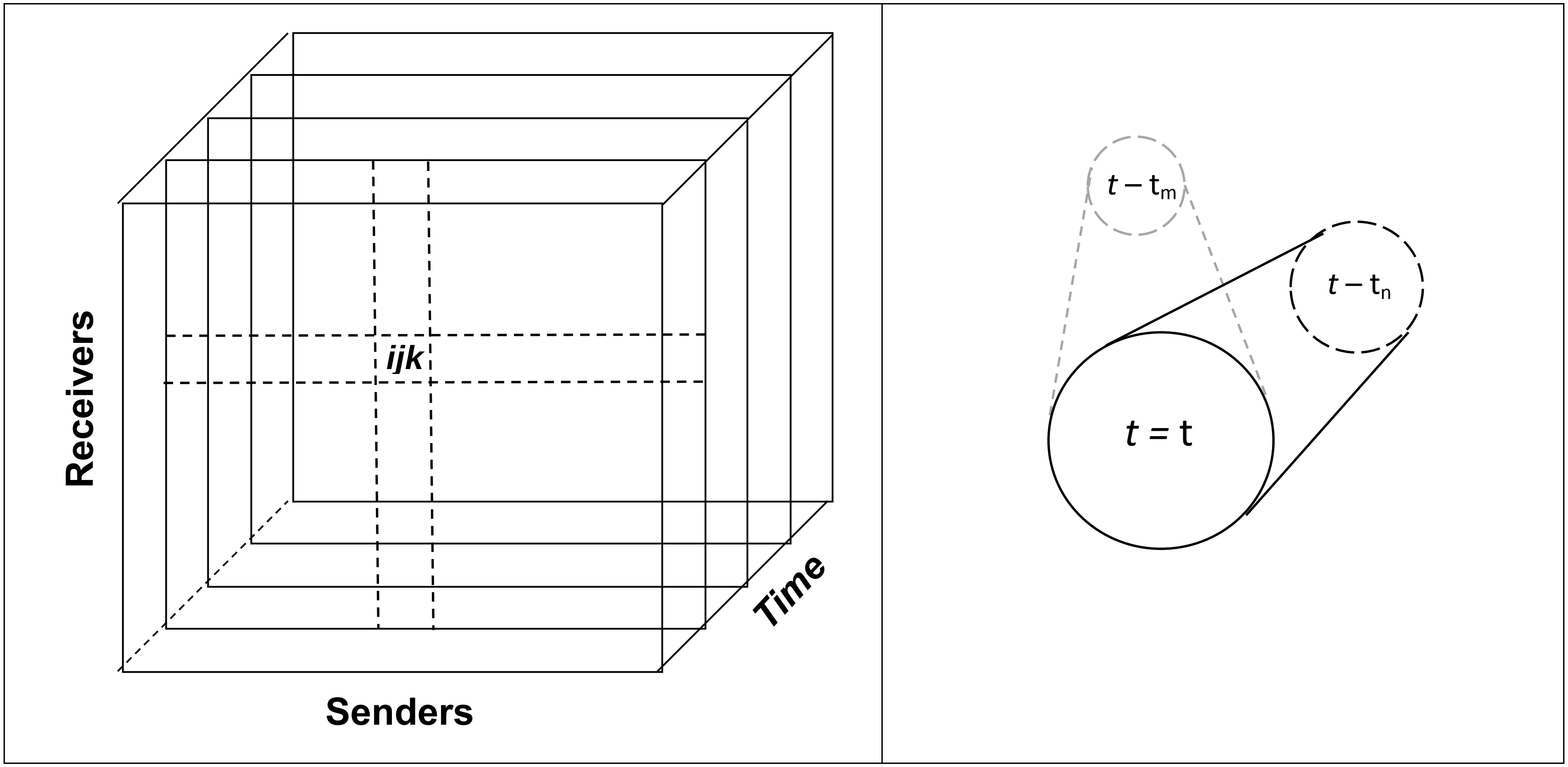

Figure 2a (left) and 2b (right): A three-dimensional array of information can contain a trajectory; a four-dimensional hypercube contains one more degree of freedom and thus a variety of possible trajectories. Source: Leydesdorff (1996), p. 289.

	first dimension	**second dimension**	**third dimension**	**fourth dimension**
Operation	variation	selection	stabilization, retention	globalization and self-organization
Nature	entropy; disturbance	extension; network	localized trajectory	identity or regime
Character of operation	probabilistic; uncertain	deterministic; structural	reflexive; reconstructive	globally organized; resilient
Appearance	instantaneous and volatile	spatial; multi-variate	historically contingent	emerging hyper-cycle
Unit of analysis	change in terms of relations	latent positions	stabilities during history	virtual expectations
Type of analysis	descriptive registration	multi-variate analysis	time-series analysis	non-linear dynamics

Table 1: Organization of concepts in relation to degrees of freedom in the probability distribution. Source: Leydesdorff (1994), at p. 223.

We ourselves as psychologies can be considered as self-organizing systems with the reflexive capacity to reconstruct the possible representations of our history. Luhmann's (e.g., 1986) suggestion is to model the social system of communications as a system without psychological consciousness, but with an equal level of complexity. However, a psychological system is centered on the *individuum* and will therefore tend to integration (Haken & Portugali, 2014), whereas the communication system is distributed (as a *dividuum*) and has the option of exploiting additional degrees of freedom for differentiation. When communication is centered, a high culture (e.g., the Holy Roman Empire) is shaped as a cosmological order with a king, an emperor, or a priest at the top of an integrating hierarchy. However, a modern society is based on prevailing differentiation among various codes of communication, so that a set of juxtaposed coordination mechanisms can be used. The different coordination mechanisms are partially integrated when the systems are instantiated in terms of historical organization and action.

Table 1 provides an overview of the semantics associated with different dimensions of the probabilistic entropy. Note that the terminology is fluid and not mutually exclusive because a two- or higher-dimensional array can also be considered as containing uncertainty (in the first dimension). Uncertainty provides the variation that is recursively selected: some selections are selected for stabilization (along the time axis), and some stabilizations can be selected for globalization of the expectations. The recursion of structures shaped by the interactions among the communications leads to interactions among structures providing second-order variations. For next-order or next-moment selections, however, the historical origin of the variation is irrelevant. Thus, the system may continuously loop into itself at different levels and from different perspectives. The horizontal and vertical differentiations lead historically ("phenotypically") to developments along diagonals. The systems under study may be far less decomposable than we envisage in the analytical model.

Levels B and C in the Shannon diagram

Weaver (1949, p. 24) suggested taking Shannon's original diagram as a representation of "level A" which should be complemented with more levels (B and C) that represent how meaning is conveyed at level B, and how and why the received meaning can affect behavior (at level C)? In our opinion, meaning can only be conducive if it is codified and operates at the level of the social (that is, inter-personally); for example, as legislation. Thus, we have to look more carefully into the development and functions of codes in inter-human communication.

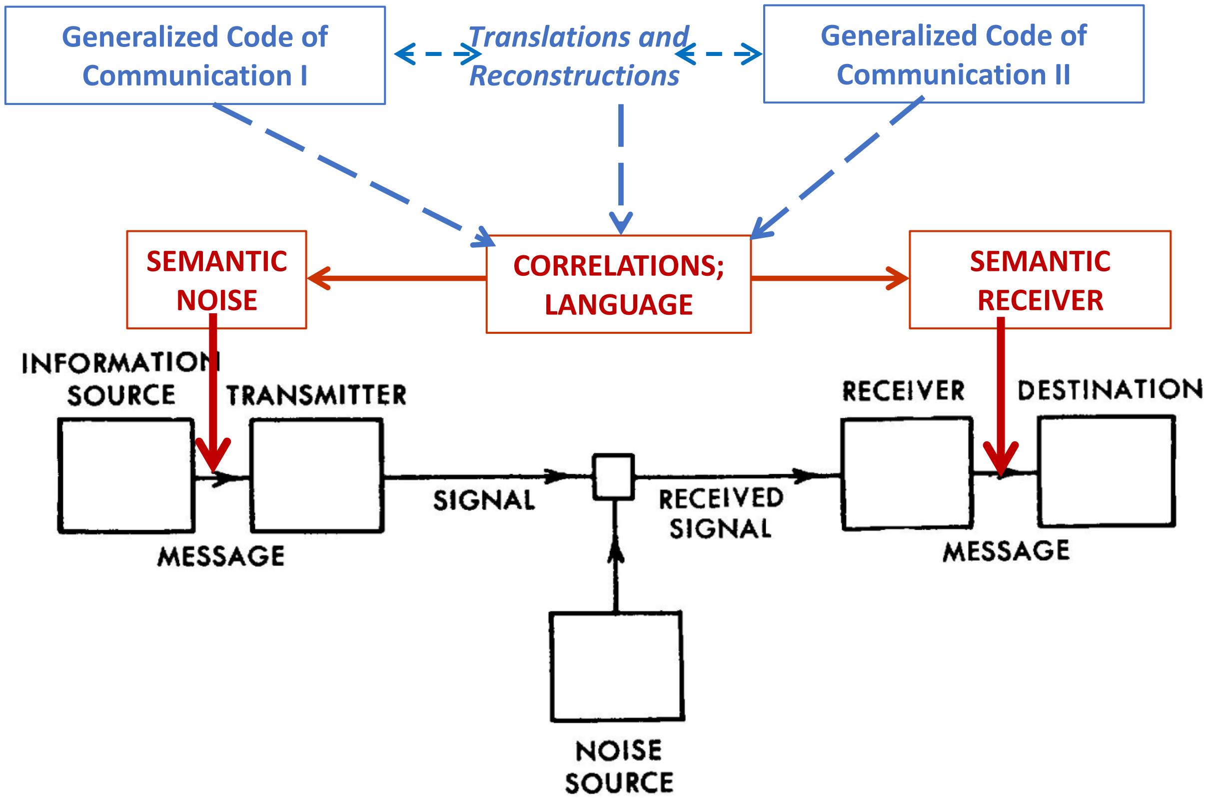

Figure 3: Levels B and C added to the Shannon diagram (in red-brown and dark-blue, respectively).

Figure 3 provides the scheme that we propose for levels B and C. We specified in the previous section that the semantic and potentially linguistic relation between the semantic receiver and semantic noise is based on correlations among sets of relations at level A. In the vector space (level B), meanings can be shared, but not communicated (because otherwise one operates at level A). The use of language facilitates, supports, and potentially reinforces the options for sharing meaning. Unlike the primitive forms of language discussed above, human language enables us to explicate the meaning of the communication in messages that can also be sent over telephone lines (as Shannon-type information at level A).

Natural languages can be considered as the as yet undifferentiated and therefore common medium of communication. But, as Talleyrand noted, "*La parole a été donnée à l'homme pour déguiser sa pensée.*"[6] In other words, codes of communication are also used at the symbolic level for regulating the use of language. The codes enable us to short cut the communication; for example, by paying the market price of a good instead of negotiating it using language. In our opinion, the codes of communication are thus candidates for Weaver's level C: the codes and their combinations also enable us to make the communications far more precise and efficient than is possible in natural languages.

The next-order loop of level C has been discussed in the social sciences as emerging from the so-called "double contingency" in inter-human communications. Building on American pragmatists (Cooley, 1902; Mead, 1934), the sociologist Talcott Parsons specified this double contingency as follows: in addition to relating to each other, one (*Ego*) expects the other (*Alter*) to entertain expectations about the other and about possible interactions just as one entertains such expectations oneself (Parsons, 1951 and 1968; Parsons & Shils, 1951). Thus, both the network space and the vector space are implicated in each inter-human interaction; and beyond being positioned, one can change the horizons of meaning emerging in one's communications by changing and refining (positional) perspectives reflexively.

Furthermore, Parsons (1968, p. 440) specified a sociological appreciation of level C as follows:

[6] "Speech was given to man to disguise his thoughts." Retrieved from http://forum.quoteland.com/eve/forums/a/tpc/f/99191541/m/251100355 (June 28, 2015); (cf. Voltaire, 1763).

> “At the cultural level [language] is clearly the fundamental matrix of the whole system of media. Large-scale social systems, however, contain more specialized media (if you will, specialized “languages”), such as money, power, and influence (see Parsons 1963a; 1963b). Such media, like language, *control* behavior in the processes of interaction.”

Parsons (1963a and b; 1968) described these media as symbolically generalized. Specific codes of communication can generate symbolically generalized media of communication (Distin, 2012). Using these same notions, Luhmann (1994, pp. 194 ff.) further argued that these generalized codes would have to be binary—like true and false in logics—in order to be binding. In our opinion, the already noted intuition is more fruitful given that eigenvectors of the communication matrix (or in higher-order communication arrays) span orthogonally and thus induce functional differentiation among the codes. The codes can also be more complex than one-dimensional (cf. Hoffmeyer & Emmeche, 1991).

For example, in the case of the science system, Luhmann (1990) argued for true/not-true as the specific code that provides a binary criterion for quality control in scholarly discourse. However, in the empirical sciences, the truth of statements is not unambiguous: some statements can be more true than others. As the sciences develop as discursive networks, uncertainty is always present. In a study of the debates about oxidative phosphorylation—which led to the Nobel Price of Chemistry for Peter Mitchell in 1978—Gilbert & Mulkay (1984) found that statements were relabeled using different repertoires when they were considered true or erroneous from the perspective of hindsight.

In a number of studies, Herbert Simon (e.g., 1973b) argued in favor of truth-finding and puzzle-solving as combined "logics" in scientific discovery. In our opinion, also this operationalization has also remained chiefly a philosophical appreciation of the evolutionary process (cf. Popper, 1959 [1935]) more than a proposal for empirical operationalization (cf. Newell & Simon, 1972). Empirically, the sciences evolve as systems of expectations rationalized by arguments in discourses; after a further development, the criteria may also have changed (Fujigaki, 1997; Kuhn, 1962). Using the conceptualization of the codes as evolving eigenvectors of the communication matrix, one can appreciate the dynamic and uncertain character of these codes of communications.

Codes can also be nested. For example, different specialties may operate with different codes while sharing some general criteria for the quality control of scientific communications. Similarly, in economic transactions, a variety of payment methods can be distinguished under the umbrella of an economic logic that differs from a scholarly or normative one (Boudon, 1979; Bourdieu, 1976). In his last book, Bourdieu (2004, at p. 83) precisely formulated a reflection on the empirical study of the sciences, as follows:

> Each field (discipline) is the site of a specific legality (a nomos), a product of history, which is embodied in the objective regularities of the functioning of the field and, more precisely, in the mechanisms governing the circulation of information, in the logic of the allocation of rewards, etc., and in the scientific habitus produced by the field, which are the condition of the functioning of the field. […]
>
> What are called epistemic criteria are the formalization of the 'rules of the game' that have to be observed in the field, that is, of the sociological rules of interactions within

the field, in particular, rules of argumentation or norms of communication. Argumentation is a collective process performed before an audience and subject to rules.

Bourdieu (at p. 78) calls this a "Kantian"—that is, transcendental—transition from "objectivity" to "intersubjectivity" as the carrying ground of scientific inferences. However, the philosopher most associated with this transition is Edmund Husserl, who criticized the increasingly empiristic self-understanding of the modern (European) sciences (Husserl, [1935/36] 1962).

In his *Cartesian Meditations*, Husserl (1929, at pp. 182f.) argues that a radical continuation of Descartes' (1637) reasoning is needed for developing a philosophy of science: in addition to the *cogitantes*—that is, the reflexive agents who remain in doubt—one can entertain *cogitata*—that is, things about which we remain in doubt, such as intentions and expectations.[7] Both the *res cogitans* and the *res extensa* belong to our reality. According to Husserl, the possibility to communicate expectations intersubjectively grounds the empirical sciences "in a concrete *theory of science"* (p. 155).

In our opinion, indeed one tests expectations entertained at the supra-individual level (in discourses) against observations, and the observations can update these expectations since they can function as arguments. We shall take from Husserl that the self-organizing codes of the communication at level C are not material, but belong to our reality as structures of expectations or *res cogitans.* Popper (1972) denoted this domain as World 3, but neither he nor Husserl

[7] Husserl uses the singular form "*cogitatum*" for this intersubjective intentionality. Note that for Descartes, the *cogitatum* was God, whom the contingent *Cogito* knows as the transcendent Other.

specified the evolutionary dynamics of expectations in terms of communications (Luhmann, 1986b).

We submit that *res cogitans* can be expected to develop in terms of redundancies instead of probabilistic entropy, unlike the material world (*res extensa*) where the Second Law prevails. Language and the symbolic media of communication enable us to multiply meanings as options at a speed much faster than can historically be realized. The codes of communications provide us with horizons of other possible meanings. The different codes can be recombined and reconstructed. We shall argue that at level B, meanings are instantiated in specific combinations of codes, while at level C the codes themselves are further developed in response to the temporal integrations in the instantiations.

The development of redundancies

As is well-known, Shannon's (1948) probabilistic entropy (H) is coupled to Gibbs' formula for thermodynamic entropy $S = k_B * H$. In Gibbs' equation, k_B is the Boltzmann constant that provides the dimensionality Joule/Kelvin to S, while H is dimensionless and can be measured in bits of information (or any other base of the logarithm) given a probability distribution (containing uncertainty). The Second Law of thermodynamics states that entropy increases with each operation, and Shannon-type information is accordingly always positive.

The redundancy R is defined in information theory as the fraction of the capacity of a communication channel that is not used. In formula format:

$$R = 1 - \frac{H}{H_{\max}} = \frac{H_{\max} - H}{H_{\max}} \tag{1}$$

Brooks & Wiley (1986) noted that in the case of an evolving (e.g., biological) system, the number of options N may increase, and therefore both the H of the system under study and the maximum entropy (H_{max}) can increase ($H_{max} = \log(N)$). In Figure 4a, we added green to the redundancy as part of the entropy: these are the options that were not realized by the system, but could have been realized. Kauffman (2000), for example, calls these possible realizations "adjacent," but his argument remained in the biological domain. Above this area, however, Brooks & Wiley (1986, at p. 43) added the label "impossible" as a legend (Figure 4a).

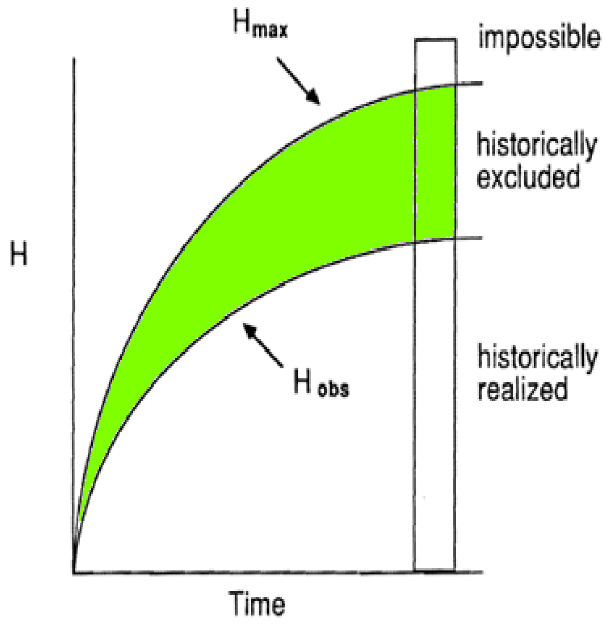

Figure 4a: The development of entropy (H_{obs}), maximum entropy (H_{max}), and redundancy ($H_{max} – H_{obs}$). Source: Brooks & Wiley (1986, at p. 43).

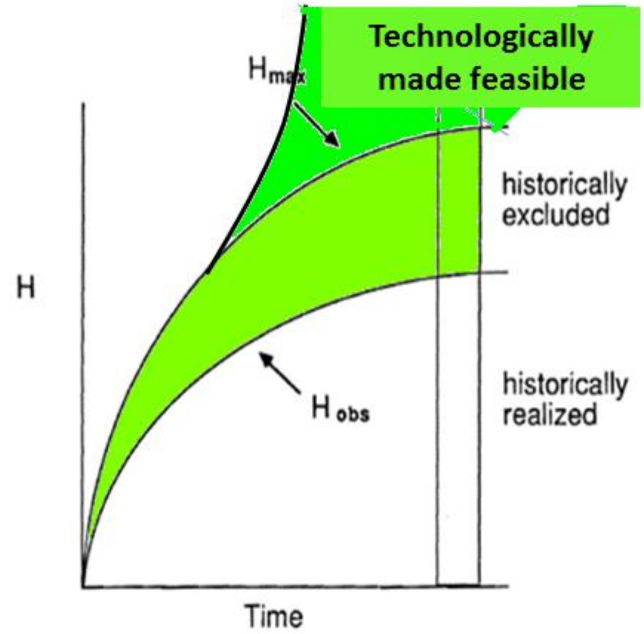

Figure 4b: Hitherto impossible options are made possible because of cultural and technological evolution.

In Figure 4b, we have added the label "technologically made feasible" to this latter area in order to indicate how the generation of new options (and hence redundancy) can be enhanced by cultural evolution which includes the levels B and C. An intentional system operates by adding redundancy without necessarily adding information (as we argued above). The addition of information is likely—because of the coupling to historical time—but this newly added uncertainty is not a condition for the generation of redundancy. Symbolic generalization of the codes regulates the generation of redundancies from above, whereas Shannon entropy is continuously generated in the historical process from below.

New options can be generated as mutual redundancy when two (or more) codes of communication are instantiated, as in the case of introducing a new technology in a market or when writing an evidence-based report for a government agency. In this case, one needs text that can be read using the various codes involved, and thus one generates redundancies deliberately (Fujigaki & Leydesdorff, 2000). The redundancy observable in the green surfaces of Figure 4b is generated by the recombination of redundancy flows in *res cogitans*. Both the carrying agents (*cogitantes*) and the models entertained (*cogitata*) can provide options—under conditions to be specified—from which one selects in the instantiations. Let us now try to specify this process more precisely.

The measurement of redundancy

a. Redundancy in two dimensions

We propose to specify mutual redundancy between systems in analogy to the concept of mutual information as specified in Shannon's theory. In Figure 5, the overlap between two uncertainties in different variables is depicted as sets. The mutual information can then be formulated as follows:

$$T_{12} = H_1 + H_2 - H_{12} \qquad (2)$$

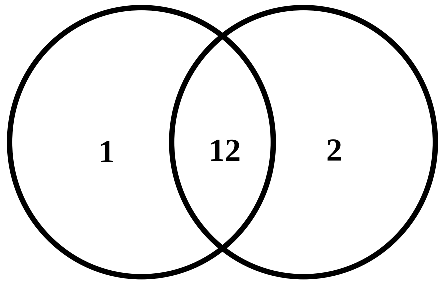

Figure 5: Overlapping uncertainties in two variables x_1 and x_2.

Note that the addition of entropies accords with the rules of set theory. But this changes when we consider the overlap as a redundancy to be appreciated twice. In addition to H_1 and H_2, the overlap contains redundancy as a surplus of information. We can thus define an "excess" information value Y_{12} —equivalent to H_{12} but with the plus sign, since we do not correct for the duplication in the case of redundancies—as follows:

$$Y_{12} = H_1 + H_2 + T_{12} = H_{12} + 2T_{12} \tag{3}$$

The mutual redundancy R_{12} can now be found by using Y_{12} instead of H_{12} in Eq. 2, as follows:

$$\begin{aligned} R_{12} &= H_1 + H_2 - Y_{12} \\ &= H_1 + H_2 - (H_{12} + 2T_{12}) \\ &= H_1 + H_2 - ([H_1 + H_2 - T_{12}] + 2T_{12}) \\ &= -T_{12} \end{aligned} \tag{4}$$

Since T_{12} is necessarily positive (Theil, 1972: 59 ff.), it follows from Eq. 4 that R_{12} is negative and *therefore* cannot be anything other than the consequence of an increased redundancy. This reduction of the uncertainty is measured in bits of information, but the sign is negative. Therefore, this is not a Shannon-type information, since the latter information is necessarily positive (Krippendorff, 2009a).

b. Redundancy in three and four dimensions

For the three-dimensional case and using Figure 6, one can define, in addition to the two-dimensional values of Y (in Eq. 3), a three-dimensional value including the redundancy as follows:

$$Y_{123} = H_1 + H_2 + H_3 + T_{12} + T_{13} + T_{23} + T_{123} \quad (5)$$

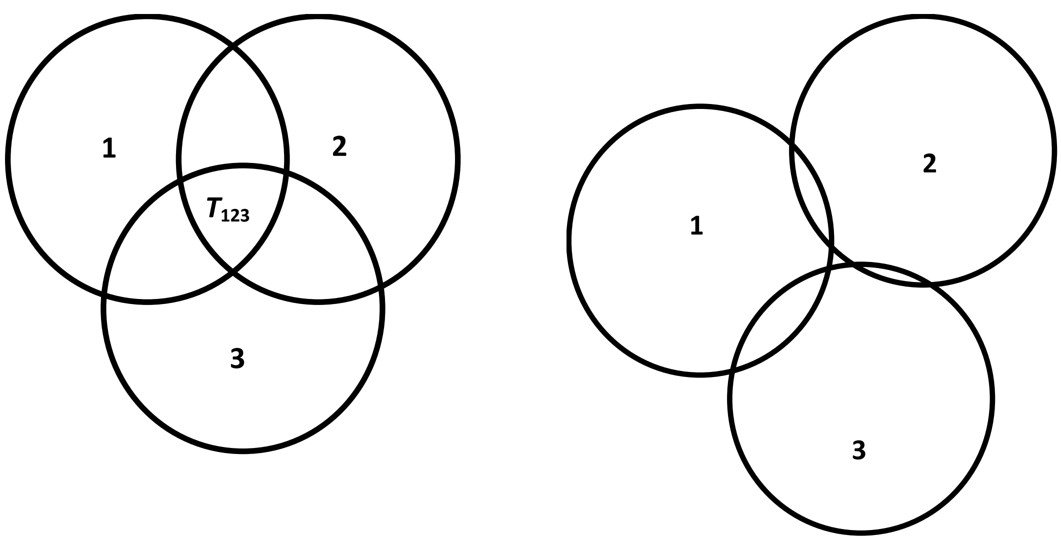

Figure 6: Overlapping uncertainties in three variables x_1, x_2, and x_3: two configurations with opposite sign of T_{123}.

Information-theoretically, however, one derives the following corrections for the overlaps by subtracting in Figure 6:

$$H_{123} = H_1 + H_2 + H_3 - T_{12} - T_{13} - T_{23} + T_{123} \tag{6}$$

It follows that the difference between Eqs. 5 and 6 (after subtraction) is:

$$Y_{123} - H_{123} = +2T_{12} + 2T_{13} + 2T_{23}$$
$$Y_{123} = H_{123} + 2T_{12} + 2T_{13} + 2T_{23} \tag{7}$$

Furthermore, the mutual information in three dimensions can be derived (e.g., Abramson, 1963, at p. 129; cf. McGill, 1954; Yeung, 2008) as:

$$T_{123} = H_1 + H_2 + H_3 - H_{12} - H_{13} - H_{23} + H_{123} \tag{8}$$

Using *Y*-values instead of *H*-values for the joint entropies in Eq. 8, one obtains the mutual redundancy as follows:

$$\begin{aligned} R_{123} &= H_1 + H_2 + H_3 - (H_{12} + 2T_{12}) - (H_{13} + 2T_{13}) - (H_{23} + 2T_{23}) \\ &\quad + (H_{123} + 2T_{12} + 2T_{13} + 2T_{23}) \\ &= T_{123} \end{aligned} \tag{9}$$

In the three-dimensional case, the mutual redundancy is thus identical to the mutual information in three dimensions. Leydesdorff & Ivanova (2014, at p. 392) show that in the case of four dimensions $R_{1234} = - T_{1234}$. The sign of the mutual redundancy alternates with the number of dimensions. This corrects for the otherwise inexplicable sign changes in the mutual information with increasing dimensionality. In other words, mutual redundancy in three or more dimensions is consistent, while mutual information itself is not, because of the sign changes with the dimensionality.[8]

c. Generalization

Eq. 8 can also be written as follows:

$$T_{123} = [T_{12} + T_{13} + T_{23}] + [H_{123} - H_1 - H_2 - H_3] \tag{10}$$

since one can rewrite, as follows:

$$T_{123} = [(H_1 + H_2 - H_{12}) + (H_1 + H_3 - H_{13}) + (H_2 + H_3 - H_{23})] + [H_{123} - H_1 - H_2 - H_3]$$

$$T_{123} = H_1 + H_2 + H_3 - H_{12} - H_{13} - H_{23} + H_{123} \tag{8}$$

[8] Krippendorff (2009b, at p. 670) provided a general *notation* for this alteration with changing dimensionality—but with the opposite sign (which further complicate the issue; cf. Leydesdorff, 2010a:68)—as follows:

$$Q(\Gamma) = \sum_{X \subseteq \Gamma} (-1)^{1+|\Gamma|-|X|} H(X) \tag{9}$$

In this equation, Γ is the set of variables of which X is a subset, and $H(X)$ is the uncertainty of the distribution; |Γ| is the cardinality of Γ, and $|X|$ the cardinality of X.

Due to the subadditivity property (because of overlaps as in Figure 6): $H(x_1, \dots, x_n) \leq \sum_1^n H(x_i)$, which holds for any dimension *n*≥ *2*. The second bracket in Eq. 10 makes a negative contribution, whereas the terms in the first bracket of Eq. 10 are strictly positive.

It follows (inductively) that for any given dimension *n*, one can formulate combinations of mutual informations corresponding to $\sum_1^n H(x_i) - H(x_1, \dots, x_n)$ that are by definition positive (or zero in the null case of complete independence). For example (up to four dimensions) as follows:

$$
\begin{aligned}
&0 \leq \sum_{i=1}^{n=2} H(x_i) - H(x_1, x_2) = T_{12} \\
&0 \leq \sum_{i=1}^{n=3} H(x_i) - H(x_1, x_2, x_3) = \sum_{ij}^{3} T_{ij} - T_{123} \qquad (11) \\
&0 \leq \sum_{i=1}^{n=4} H(x_i) - H(x_1, x_2, x_3, x_4) = \sum_{ij}^{6} T_{ij} - \sum_{ijk}^{4} T_{ijk} + T_{1234}
\end{aligned}
$$

where the sums on the right-hand side are over the $\binom{n}{k}$ permutations of the indices. This relation can be extended for general *n* as,

$$
\begin{aligned}
0 &\leq \sum_{i=1}^{n} H(x_i) - H(x_1, \dots, x_n) \\
&= \sum_{ij}^{\binom{n}{2}} T_{ij} - \sum_{ijk}^{\binom{n}{3}} T_{ijk} + \sum_{ijkl}^{\binom{n}{4}} T_{ijkl} - \cdots + (-1)^{1+n} \sum_{ijkl\dots(n-1)}^{\binom{n}{n-1}} T_{ijkl\dots(n-1)} + \\
&\quad (-1)^n \sum_{ijkl\dots(n)}^{\binom{n}{n}} T_{ijkl\dots(n)} \qquad (12)
\end{aligned}
$$

where the last term on the right-hand side is equal to $(-1)^n T_{1234\dots n}$. Returning to the relation between R_{12} and T_{12}, it now follows instructively that:

$$R_{12} = -\,T_{12}$$
$$= H(x_1, x_2) - \sum_1^2 H(x_i) \leq 0 \tag{11}$$

and the analogous relations for R_{123} and R_{1234} follow in the same way from Eq. (12). More generally, in the case of more than two dimension, *n > 2*:

$$R_n = (-1)^{1+n} T_{1234\ldots n} = [H(x_1, \ldots, x_n) - \sum_1^n H(x_i)]$$
$$+ [\sum_{ij}^{\binom{n}{2}} T_{ij} - \sum_{ijk}^{\binom{n}{3}} T_{ijk} + \sum_{ijkl}^{\binom{n}{4}} T_{ijkl} - \cdots + (-1)^{1+n} \sum_{ijkl\ldots(n-1)}^{\binom{n}{n-1}} T_{ijkl\ldots(n-1)}] \tag{13}$$

The left-bracketed term of Eq. 13 is necessarily negative entropy, while the configuration of the remaining mutual information relations contribute a second term on the right which is positive (see the set of Equations 11 above). In other words, we model here the generation of redundancy on the one side versus the historical process of relating on the other, as an empirical balance. When the resulting *R* is negative, self-organization prevails over organization in the configuration under study, whereas a positive *R* indicates conversely a predominance of organization over self-organization as two different subdynamics.

Note that the resulting configuration of relations—as expressed in the right-hand term of Eq. 13—no longer refers to level A of the Shannon model because this formulation implies a systems view, whereas the Shannon model focuses on local relations. (As noted, a negative value of R_{12} cannot be appreciated in the Shannon model.) In other words, the right-side term of Eq. 13 is composed of both interactional and correlational information, and the resulting sign and value of

R thus include also the feed-forward and feedback loops between levels A and B. The one sign of *R* can also be associated with clockwise and the other with anti-clockwise rotation of the resulting vectors, whereas the values of the two terms in Eq. 13 measure the relative weights of the two rotations.

Clockwise and anti-clockwise rotations

As soon as the relation between two subdynamics (or agents) is extended with a third, the third may feedback or feed-forward on the communication relation between the two, and thus a system is shaped (Sun & Negishi, 2010). This principle is known in social network analysis as "triadic closure" (Bianconi *et al*., 2014; De Nooy & Leydesdorff, 2015). Triadic closure can be considered as the basic mechanism of systems formation. When three selection environments operate on variation in the interactions among them, the communication can proliferate auto-catalytically using each third mechanism as a feedback on or feed-forward to bi-lateral relations. At a next moment, the cycling may take control as a vortex.

Ulanowicz (2009 at p. 1888) depicted this possibility of auto-catalysis as follows:

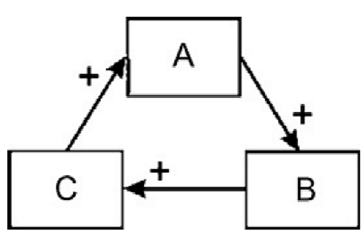

Figure 7: Schematic of a hypothetical three-component autocatalytic cycle.
(Source: Ulanowicz, 2009, at p. 1888, Figure 3.)

A second cycle with the reverse order of the operations is equally possible; the two cycles can be modeled as two vectors, and this system can then be simulated in terms of rotations of the vectors. One vector can be understood as corresponding to the tendency of historical realization and the other to self-organization. Using simulations, Ivanova & Leydesdorff (2014a and b) showed that the operation of these two three-dimensional vectors upon each other generates a value of *R* depending on the configuration. As we showed above, *R* can also be measured. This measure indicates the size of the footprint of the self-organizing fluxes of communication on the historical organization in the instantiation.

The cycles can be vicious or virtuous in the sense of providing opportunities for next-order systems formation. The horizontal circulation provides a subdynamics analytically different from the vertical dynamics, but the two dynamics influence each other as they provide one another with variation. For example, the system is constructed bottom-up in terms of triadic closures.[9] In these triads, the order of the operations matters because the circulation across the communication field can become increasingly an independent factor in the further dynamics. The emergence of a communication field in the circulation from the interactions leads to new non-linear interactions. In other words, the probabilistic entropy is extended with one more degree of freedom by the shaping of this field or, in other words, a new code becomes possible. The number of

[9] In the case of communication between two components, the symmetry is commutative or Abelian in terms of gauge theory. Addition of the third component changes the symmetry in the group to non-Abelian (non-commutative). Transformations in a non-Abelian group lead to the appearance of further interactions because the field generated by the interactions can itself become another source of interactions (Yang & Mills, 1954).

combinatorial options among codes is based on the multiplication of the numbers of categories in the respective code sets. Note that the operation of triadic closure is recursive.

The "Method of Reflections" between codes of communication

A perspective for further research is provided by applying Hidalgo & Hausmann's (2009) "Method of Reflections" between codes to the horizons of meaning implicated at level C. Each horizon of meaning can be considered as a code set or alphabet; two code sets can co-evolve. In their study of economic complexity, these authors proposed the following recursive equations for a model of how code sets reflect each other when co-evolving:

$$\begin{aligned} k_{p,n} &= \frac{1}{k_{p,0}} \sum_{c=1}^{N_c} M_{c,p} k_{c,n-1} \\ k_{c,n} &= \frac{1}{k_{c,0}} \sum_{p=1}^{N_p} M_{c,p} k_{p,n-1} \end{aligned} \tag{14}$$

In Eq. 14, p_j represents the set of product groups and c_j the set of countries; $M_{c,p}$ is the asymmetrical matrix of countries versus product groups; $k_{p,n}$ and $k_{c,n}$ denote row and column vectors of this matrix at the n^{th} iteration.

In a number of analytical steps, one can derive from these equations an economic complexity index (ECI) of countries in terms of their product portfolios, and *vice versa* an index of product complexity (PCI) in comparison to other countries. Using this "Method of Reflections," the one codification is reflected asymmetrically in the other (Kemp-Benedict, 2014). In principle, the reflections can be extended to more than two sets; for example, three code sets can be reflected into each other asymmetrically with six different translations among codes as a result (Ivanova *et al*., 2015).

The co-evolution of code sets modeled in Eqs. 14 can be expected to converge when one uses averages in the equations (Ourens, 2013). In this case, the one code set can be precisely translated into the other. In the case of communications, however, one expects highly skewed distributions (Ijiri & Simon, 1977; Seglen, 1992) and thus the translations among code sets—operationalized here as reflections—will remain uncertain. The more complex each code (that is, as a character in an alphabet of categories),[10] the more uncertain its meaning.

Importantly, it can be shown that a co-evolution between code sets transforms them to the extent that the observable input $k_{p,0} = \sum_{c=1}^{N_c} M_{c,p}$ stands orthogonally to the output vector $\vec{k}$ after sufficient iterations (Kemp-Benedict, 2014, Appendix I, at pp. 12f.). In other words, the reflections among the self-organizing codes can completely transform the observable configurations. Code sets ("alphabets") are self-generated as communication channels by the transformations. As noted, the addition of a new code set multiplies the number of options and thus increases the sustainability of the more diversified system (Leydesdorff, 1997, pp. 335f.).

Summary and conclusions

We have extended Shannon's model of communication (at level A) with two levels (B and C) that change the linear model into an evolutionary one because feedback and feed-forward loops

[10] The codes can also be considered as indicating a hierarchy such as the different branches of the Medical Subject Headings (MeSH) in MEDLINE/PubMed (Petersen *et al*., under submission).

are possible among the levels. At level A, information is transmitted; at level B, information is organized and thus made meaningful in a vector-space. Reflexivity reveals that this vector space is constructed and therefore a potential subject of reconstruction: the possibility of reconstruction opens horizons of meaning (level C). These horizons can be expected to originate along the eigenvectors of the communication matrix. Whereas the common language at level B tends to integration (into organization), the eigenvectors can be expected to span a vector space in different directions. Thus, this layer generates horizontal differentiation as a top-down pressure.

Codes of communication are no longer actor-attributes, but operate on the communications once they emerged in a self-organizing mode, that is, insofar as constraints on the communication are removed. The system has to find these resonances by varying historically. That can be a long and tedious process such as, for example, the invention of modernity since the late Middle Ages. New coordination mechanisms with specific codes are "invented" and added one by one. For example, the power of the Pope at the Vatican eventually assumed a sanctified meaning in the Middle Ages ("*Roma dixit*"), but this hierarchy became contested during the Struggle for the Investiture (11^{th} and 12^{th} centuries), the Reformation (16^{th} century), etc. The invention of the printing press (15^{th} century) made it possible to disseminate ideas, wishes, fantasies, etc., at an increasing speed. Sufficient volumes of variation may be needed to find resonances with next-order potentials.

The specification of the selection mechanisms is crucial in the cultural domain. "What is evolving?" becomes a relevant question when selection is no longer given by nature (Andersen, 1994). Boulding (1978, p. 33) answered this question at the time by formulating: "What evolves is something very much like knowledge." From our perspective, we consider "knowledge" as codified meaning. The codes of communication are evolving in a self-organization process of variously codified meanings. Note that meaning can also be codified in other directions, such as the meaning of communications in political discourse, etc.

In addition to vertical differentiation, the assumption of horizontal differentiation is needed for understanding the evolving complexity. By being coded, some meanings can assume performative roles more than others; for example in the case of legislation. The differentiation and the ensuing pluriformity of the competing coordination mechanisms, however, break the structural formations so that the systems can be expected to grow by being interrupted. The generation pf redundancy can thus enter the historical instantiations and under the condition of self-reinforcing loops tip the balance towards the prevalence of evolutionary self-organization over historical organization. We have shown how this trade-off can be followed by the measurement of mutual redundancies.

We argue that redundancy is a more crucial subject of study in a knowledge-based economy than information. Redundancies are not generated on the side of the variation, but by the selection mechanisms operating upon one another. When three or more selection mechanisms operate, auto-catalysis is an option, and options can then be generated at an increasing pace. Thus, horizontal differentiation is a necessary component of self-organization in the vertical dimension. The warp and the woof of meaning generation and self-organization are not harmoniously integrated as in textiles, but differentiated and disturbing one another. These dynamics lead to a fractal manifold in different directions. Through breakage new options are generated.

The generation of redundancy proceeds in a domain that reflexively can be assumed to exist. Derivates of the Latin *frangere* such as fragments, fragile, fractals, etc., may help to indicate that—as against *res extensa*— *res cogitans* does not "exist" in the sense of Latin *esse* (Heidegger, 1962). By turning away from an objectivistic self-understanding of the sciences, we find room for a general theory of meaning and knowledge-generation that we have depicted as an extension of Shannon's theory (Figure 3), but which can also be understood as this theory's proper intension. As we argued, meaning is not "added" to the information as uncertainty, but provided by constructing new and richer perspectives.

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