Segmentation of ARX models through SDP-relaxation techniques

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Abstract

Segmentation of ARX models can be formulated as a combinatorial minimization problem in terms of the ℓ_0-norm of the parameter variations and the ℓ_2-loss of the prediction error. A typical approach to compute an approximate solution to such a problem is based on ℓ_1-relaxation. Unfortunately, evaluation of the level of accuracy of the ℓ_1-relaxation in approximating the optimal solution of the original combinatorial problem is not easy to accomplish. In this poster, an alternative approach is proposed which provides an attractive solution for the ℓ_0-norm minimization problem associated with segmentation of ARX models.

Problem description

Data-generating system:

$$y(k) = \phi^\top(k)\theta_o(k) + e(k)$$

Parameter estimation (with regularization):

$$\hat{\theta}_N = \arg\min_{\theta} \sum_{k=1}^{N} \left(y(k) - \phi^\top(k)\theta(k)\right)^2 + \gamma\|\Delta\theta\|_0$$

$$\Delta\theta_k = \|\theta(k+1) - \theta(k)\|_\infty$$

ℓ_1-approximation [1]:

$$\hat{\theta}_N = \arg\min_{\theta} \sum_{k=1}^{N} \left(y(k) - \phi^\top(k)\theta(k)\right)^2 + \gamma\|\Delta\theta\|_1$$

Polynomial formulation

Reformulation of ℓ_0-norm minimization in terms of polynomial optimization:

$$\min_{\theta,w} \sum_{k=1}^{N} \left(y(k) - \phi(k)^\top\theta(k)\right)^2 + \gamma \sum_{k=1}^{N-1} w_k$$

s.t.

$$(1-w_k)(\theta_i(k+1) - \theta_i(k)) = 0, \qquad i = 1,\ldots,n$$

$$w_k(w_k - 1) = 0, \qquad k = 1,\ldots,N-1$$

SDP-relaxation

Based on theory-of-moments relaxation [2], construct a sequence of SDP-relaxed problems to approximate the solution of the formulated polynomial optimization problem.

Advantages:

- Possibility to check if the global optimum of the original ℓ_0-norm minimization is attained by the constructed sequence of SDP-relaxed problems.
- Monotone convergence to the global optimum of the original ℓ_0-norm minimization (**no structural approximation**).
- The peculiar structure of the formulated optimization problem can be used to reduce the computation complexity in solving the SDP-relaxed problems (**computationally feasible**).

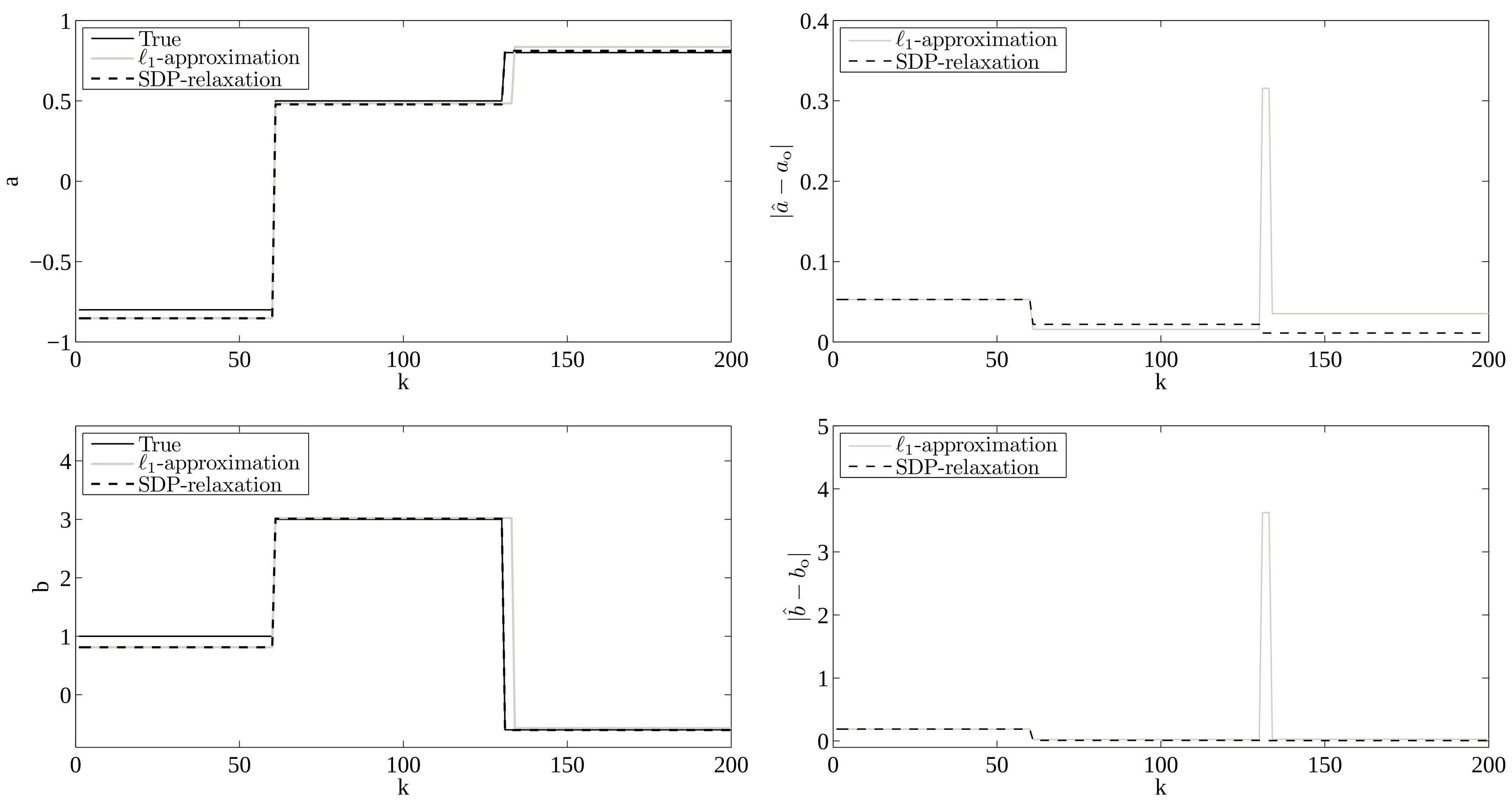
Simulation results

Data-generating system:

$$y(k) + a_o(k)y(k-1) = b_o(k)u(k) + e(k)$$

$$e(k) \text{ i.i.d.}, \quad e(k) \sim \mathcal{N}(0, 0.6^2)$$

Results



References

[1] H. Ohlsson, L. Ljung, S. Boyd, *Segmentation of ARX-models using sum-of-norms regularization*, Automatica, 46(6), 2010.

[2] J. Lasserre, *Global optimization with polynomials and the problem of moments*, SIAM Journal on Optimization, 11, 2001.

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