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ESTIMATING PERSISTENCE IN THE VOLATILITY OF ASSET RETURNS WITH SIGNAL PLUS NOISE MODELS

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Abstract

This paper examines the degree of persistence in the volatility of financial time series using a Long Memory Stochastic Volatility (LMSV) model. Specifically, it employs a Gaussian semiparametric (or local Whittle) estimator of the memory parameter, based on the frequency domain, proposed by Robinson (1995a), and shown by Arteche (2004) to be consistent and asymptotically normal in the context of signal plus noise models. Daily data on the NASDAQ index are analysed. The results suggest that volatility has a component of long-memory behaviour, the order of integration ranging between 0.3 and 0.5, the series being therefore stationary and mean-reverting.

Keywords: Fractional integration; Long memory; Stochastic volatility; Asset returns.

JEL Classification: C13 ; C22.

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1. Introduction

In recent years international stock markets have become increasingly volatile, and the persistence of volatility in asset returns has received a great deal of attention in the literature. A high degree of persistence and time dependence has often been found in their conditional variances. In particular, asset returns typically exhibit high persistence in the autocorrelation of some transforms such as squares or other powers of absolute values.

Two main approaches have been taken to model conditional heteroscedasticity. The first is the Autoregressive Conditional Heteroskedasticity (ARCH) model of Engle (1982), which models the conditional variance as an exact function of the squares of past observations. Thus, volatility is a stochastic process and both the mean and the volatility equations have separate and independent error terms. This paper uses the second approach, namely stochastic volatility (SV) models, extending them to the case of long memory behaviour.

Long range dependence (or long memory) processes have become very popular in recent years when modelling macroeconomic or financial time series (see, e.g., Diebold and Rudebusch, 1989; Baillie, 1996; Gil-Alana and Robinson, 1997; etc.). Moreover, the existence of long memory in powers of the absolute value of asset returns was studied by Ding et al. (1993). Later, Baillie et al. (1996), Bollerslev and Mikkelsen (1996) and Ding and Granger (1996) proposed the Fractionally Integrated ARCH (FIGARCH) model. Breidt et al. (1998), Harvey (1998) and, also, Deo and Hurvich (2001) and Arteche (2004) have developed parametric and semiparametric methods to estimate the memory parameter. In this paper, we use the approach proposed in Arteche (2004) to obtain a semiparametric estimate in the frequency domain based on the Whittle function, which is an approximation to the likelihood function, but does not require Gaussianity of the series, a feature that is rarely satisfied in financial time series. This estimate is shown to be consistent and asymptotically normal under very mild conditions in the context of signal plus noise models.

The outline of the paper is as follows: Section 2 briefly describes the model and the estimation procedure for the memory parameter. In Section 3 the procedure is applied to daily data on the NASDAQ, while Section 4 contains some concluding remarks.

2. The model and the estimation procedure

The Long Memory Stochastic Volatility (LMSV) model proposed in Breidt et al. (1998), Harvey (1998), Deo and Hurvich (2001) and Arteche (2004) is given by:

$$x_t = \sigma \sigma_t \varepsilon_t, \tag{1}$$

where x_t is the observed time series; σ is a positive constant; ε_t is i.i.d., with mean zero and variance 1, and σ_t is given by:

$$\sigma_t = \exp\left(\frac{v_t}{2}\right), \tag{2}$$

where v_t is stationary with long memory. That means that the covariance structure of v_t is the following:

$$\gamma_v(h) = Cov(v_t, v_{t+h}) \approx c_1 h^{2d-1}, \quad as \ h \to \infty, \qquad |c_1| < \infty, \tag{3}$$

and its counterpart in the frequency domain implies that the spectral density function of v_t satisfies:

$$f_v(\lambda) \approx c_2 \lambda^{-2d} \quad as \ \lambda \to 0^+, \quad 0 < c_2 < \infty, \tag{4}$$

where ≈ means that the ratio of the left-hand side and the right-hand side of (3) and (4) converges to 1 as $h \to \infty$ in (3) and as $\lambda \to 0^+$. Conditions (3) and (4) are not always equivalent, but Zygmund (1995, Cap.V, Sect. 2), and, more generally, Yong (1974) derive conditions under which both expressions are equivalent. A typical model satisfying the above two properties is the fractionally integrated I(d) model, namely

$$(1 - L)^d v_t = u_t, \tag{5}$$

where d can be any real number, and u_t is an I(0) process, defined, for the purpose of the present paper, as a covariance stationary process with spectral density function that is positive and finite at the zero frequency. Note that the polynomial on the left-hand side of (5) can be expressed in terms of its Binomial expansion, such that

$$(1 - L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - d L + \frac{d(d-1)}{2} L^2 - \ldots.$$

for all real d. Thus, higher the d is, the higher will be the level of association between the observations. This type of model was introduced by Granger (1980, 1981), Granger and Joyeux (1980) and Hosking (1981), and it was theoretically justified in terms of aggregation by Robinson (1978), Granger (1980), and more recently in terms of the duration of shocks by Parke (1999) and others.[1]

Taking logs of the squares of x_t in (1) and (2), we obtain:

$$y_t = \log x_t^2 = \mu + v_t + \xi_t, \tag{6}$$

where $\mu = \log\sigma^2 + E\log\varepsilon_t^2$ and $\xi_t = \log\varepsilon_t^2 - E\log\varepsilon_t^2$ is i.i.d. with zero mean and variance σ_ξ^2. For example, if $\varepsilon_t \sim N(0, 1)$ then ξ_t is a centred $\log\chi_1^2$ variable with E log ε_t^2 = 1.27 and $\sigma_\xi^2 = \pi^2/2$. Apart from the constant μ, y_t takes the form of a signal plus noise model, where the signal is a long memory process uncorrelated with the noise, which in this case is (non-Gaussian) i.i.d. (see Arteche, 2004).

The autocovariance function of y_t is then given by:

$$\gamma_y(h) = E\, y_t\, y_{t+h} = \gamma_v(h) + \sigma_\xi^2\, I(h=0), \tag{7}$$

implying that the corresponding spectral density function is:

[1] Cioczek-Georges and Mandelbrot (1995), Taqqu et al. (1997), Chambers (1998) and Lippi and Zafferoni (1999) also use aggregation to motivate long memory processes, while Diebold and Inoue (2001) propose another source for long memory based on regime-switching models.

$$f_y(\lambda) = f_v(\lambda) + \frac{\sigma_\xi^2}{2\pi}, \quad -\pi \leq \lambda < \pi. \tag{8}$$

In the context of SV models, maximum likelihood methods are hard to implement due to the existence of separate errors in the mean and log-volatility equations. Moreover, in the presence of long memory, the problem is even harder. Other techniques, such as the one based on the method of moments, were proposed by Taylor (1986) and Melino and Turnbull (1990), but these methods were shown to be inefficient in the context of AR disturbances with roots which are close to unity (see, e.g., Jacquier et al., 1994). Harvey et al. (1994) proposed a quasi-maximum likelihood method based on the Kalman filter in the context of short memory SV models. However, for long memory, this method requires a truncation in the AR expansion of the process, which may lead to a loss of relevant information.

Different estimators have been suggested for d in (5). Some of them are parametric, in the sense that the model is specified up to a finite number of parameters of which d is just one. Sowell (1992) analysed in the time domain the exact maximum likelihood estimates of the parameter of a fractional ARIMA (ARFIMA) model, using recursive procedures that allow a quick evaluation of the likelihood function. A limitation of this procedure is that the roots of the AR polynomial cannot be multiple and the theoretical mean parameter must be either zero or known. In the frequency domain, Fox and Taqqu (1986) assumed Gaussianity of the process, and, minimising the Whittle function, they showed that the estimate is consistent and asymptotically normal under appropriate conditions, which are satisfied by fractional models as in (5) with $0 < d < 0.5$. Dahlhaus (1989) also assumed Gaussianity but considered the exact likelihood function. He proved that this estimate and the one studied in Fox and Taqqu (1986) are both not only asymptotically normal but also asymptotically efficient in the sense of Fisher.

It is worth pointing out that all these parametric estimates have the same asymptotic properties of $T^{1/2}$-consistency and asymptotic normality, and, if the process is Gaussian, asymptotic efficiency. Giraitis and Surgailis (1990) relax the Gaussianity assumption and analyse the Whittle estimate for linear processes, showing that it is $T^{1/2}$-consistent and asymptotic normal, although it is no longer asymptotically efficient, while Hosoya (1997) extends the previous analysis to a multivariate framework.

However, in the case of parametric approaches, the correct choice of the model is crucial: if it is misspecified, the estimates of d are liable to be inconsistent. In fact, misspecification of the short run components can invalidate the estimation of the long run behaviour. Thus, there might be some advantages in estimating d on the basis of semiparametric approaches. These parameterise only the long run characteristic of the series. There is a price to be paid in terms of efficiency in not using a correct parametric model, but when the sample size is large the robustness of semiparametric procedures is important. Examples in this context are the log-periodogram regression estimator (LPE), initially proposed by Geweke and Porter-Hudak (1983) and later modified by Künsch (1986) and Robinson (1995b), the average periodogram estimator of Robinson (APE, 1994) and a local Whittle estimator (Robinson, 1995a). In the context of signal plus noise and SV models, Arteche (2004) showed that the latter procedure (Robinson, 1995a) is consistent and asymptotically normal.[2] The estimator is implicitly defined by:

$$\hat{d} = \arg\min_d \left(\log \overline{C(d)} - 2\,d\,\frac{1}{m}\sum_{j=1}^{m} \log \lambda_j \right), \tag{9}$$

$$\overline{C(d)} = \frac{1}{m}\sum_{j=1}^{m} I(\lambda_j)\,\lambda_j^{2d}, \qquad \lambda_j = \frac{2\pi j}{T}, \qquad \frac{m}{T} \to 0,$$

where $I(\lambda_j)$ is the periodogram of the raw time series, x_t, given by:

$$I(\lambda_j) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} x_t\, e^{i\lambda_j t}\right|^2 .$$

and d ∈ (-0.5, 0.5).[3] Under finiteness of the fourth moment and other mild conditions, Robinson (1995a) proved that:

$$\sqrt{m}\,(\hat{d} - d_o) \;\to_d\; N(0, 1/4) \quad as\ T \to \infty,$$

where d_o is the true value of d, and with the only additional requirement that m → ∞ slower than T.[4] Robinson (1995a) shows that m must be smaller than T/2 to avoid aliasing effects. A multivariate extension of this estimation procedure can be found in Lobato (1999).

The other methods also based on semiparametric models (such as the APE and the LPE) have been applied to economic time series (see, e.g. Gil-Alana, 2002). Here we use the Whittle approach, firstly because of its computational simplicity, as it does not require any additional user-chosen numbers in the estimation (as is the case with the LPE and the APE). Secondly, it is not necessary to assume Gaussianity in order to obtain an asymptotic normal distribution, Robinson's (1995a) method being more efficient than the LPE.[5] In addition, several Monte Carlo experiments carried out, for example, by Gil-Alana (2008) showed that, in finite samples, the Whittle approach has better statistical properties compared with the other procedures.[6]

Arteche (2004) shows that in the context of this procedure the spectral density of y_t ($f_y(\lambda)$) inherits the asymptotic behaviour of $f_v(\lambda)$ if the memory parameter d is positive. Further, under very mild regularity conditions, the estimate of d based on Robinson's (1995a) method preserves the same consistency and asymptotic normality properties as under normal circumstances.

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[2] In fact, he showed that it satisfies these two properties not only for the case of long memory at the zero frequency, but also when the spectrum is unbounded at any frequency in the interval (0, π] (see also Arteche and Robinson, 2000).

[3] Velasco (1999a, b) showed that the fractionally differencing parameter can also be consistently semiparametrically estimated in nonstationary contexts by means of tapering.

[4] The exact requirement is that $(1/m) + ((m^{1+2\alpha} (\log m)^2)/(T^{2\alpha}) \to 0$ as $T \to \infty$, where α is determined by the smoothness of the spectral density of the short run components. In the case of a stationary and invertible ARMA, α can be set equal to 2 and the condition is $(1/m) + (m^5 (\log m)^2)/(T^4) \to 0$ as $T \to \infty$.

[5] Velasco (2000) showed that Gaussianity is not necessary for the LPE either.

[6] Other recent approaches using the Whittle function are Phillips and Shimotsu (2004) and Shimotsu and Phillips (2005, 2006).

3. Persistence in the volatility of the NASDAQ-100

Over the past few years the presence of long memory in the volatility of equity returns has dominated the literature on temporal dependencies in financial volatility. Recently, both Bollerslev and Jubinski (1999) and Ray and Tsay (2000) investigated the long memory behaviour in the volatility of the Aluminum Corporation of American (AA) daily stock returns. Bollerslev and Jubinski (1999) use Robinson's (1995b) bivariate version of the GPH estimator to estimate the long memory parameters of absolute returns and volume, whereas Ray and Tsay (2000) apply both the univariate GPH estimator and Breidt et al.'s (1998) QMLE to log-squared returns. Both studies find evidence of strong persistence in the volatility of the AA daily stock returns with a long memory parameter estimate of approximately 0.35. Jensen (2001) proposes a Bayesian estimator based on wavelets, and using the same dataset he concludes that the value of d is around 0.36.

In this section we analyse the persistence in the volatility of the NASDAQ-100 Index. It includes 100 of the largest non-financial domestic and international companies listed on the NASDAQ National Market tier of the NASDAQ Stock Market Inc. The index reflects NASDAQ's largest companies across major industry groups. The frequency of the series is daily and the sample covers the period from January 2, 2001 to February 20, 2004.

INSERT FIGURE 1 ABOUT HERE

Figure 1 plots the return series. These appear to be stationary, although the variance seems to exhibit a higher degree of volatility in the first half of the sample. Figure 2 contains the plot of the transformed series, i.e. y_t^*, which still appears to be stationary, but more persistent. Figure 3 plots the periodogram of the transformed series, which has a large peak at the smallest frequency, suggesting that the series has long memory behaviour at the long run or zero frequency.

Figure 2 displays the estimated values of d based on Robinson's (1995a) method, i.e., $\hat{d}$ is given by (9). The upper part of the figure reports the results for all values of m from 1 to T/2.[7] We also include in the figure the 95%-confidence interval corresponding to the hypothesis of d = 0. It can be seen that all the estimates are above the interval, implying the existence of a component of long memory behaviour.

INSERT FIGURE 2 ABOUT HERE

In the lower part of the table, we display the estimates for a grid of values of m from 25 to 100. In general, lower values are obtained for higher bandwidths, which may be explained by the fact that negative biases are produced by the added noise to the SVLM models. The estimated values range of d between 0.3 and 0.5, implying stationary long memory and mean-reverting behaviour. This result has some implications in terms of financial policy and planning inference: any shock affecting the volatility process will die out in the long run, though the adjustment process will be slow, according to a hyperbolic rate of decay. In this context, policy actions might be appropriate to eliminate the effects of a shock more quickly and accelerate mean-reversion.

INSERT TABLE 1 AND FIGURE 3 ABOUT HERE

[7] Some methods to calculate the optimal bandwidth numbers have been examined in Delgado and Robinson (1996) and Robinson and Henry (1996). However, in the case of the Whittle estimator employed here, the use of optimal values has not been theoretically justified. Other authors, such as Lobato and Savin (1998) use an interval of values for m but we have preferred to report the results for the whole range of values of m.

Figure 2 also shows that the parameter d is most stable when m is between 50 and 75. In such a case the estimate of d is around 0.41. Table 1 reports the first twenty impulse responses for a 1-unit shock in the context of an I(d) process with d = 0.41. It can be seen that, 10 periods after the initial shock, 11.7% of its effect is still present in the series, 7.84% being the corresponding figure after 20 periods. Figure 3 plots the responses over a 50-period horizon, and shows clearly the hyperbolic decay in the effect of the shocks.

4. Conclusions

This paper has examined the long memory property in the stochastic volatitlity models of Harvey (1998) and Breidt et al. (1998) by using a Gaussian semiparametric or local Whittle estimator of the fractional differencing parameter (Robinson, 1995a). In a recent paper, Arteche (2004) shows that this estimator is consistent and asymptotically normal in the context of signal plus noise models. Moreover, the conditions needed for consistency and asymptotic normality are less restrictive than those required in Deo and Hurvich (2001) for the estimator based on the LPE. Thus, for example, Gaussianity, which is a condition rarely satisfied in financial series, is not necessary.

Daily data on the NASDAQ-100 Index (x_t) for the time period January 2, 2001 – February 20, 2004 are used, and the analysis is carried out on the transformed series $y_t^* = \log x_t^2$. The results show that the volatility process has long memory, with an order of integration ranging between 0.3 and 0.5. The fact that the estimated values of d are strictly smaller than 1 implies mean reversion, with the effect of the shocks dying away in the long run.

A drawback of analysis carried out here might be the use of a semiparametric method not taking into account the short-run dynamics in the series. However, as mentioned in Section 2, the use of parametric procedures is difficult to implement due to the existence of separate errors in the main and log-volatility equations. Theoretical work in this direction is now under way.

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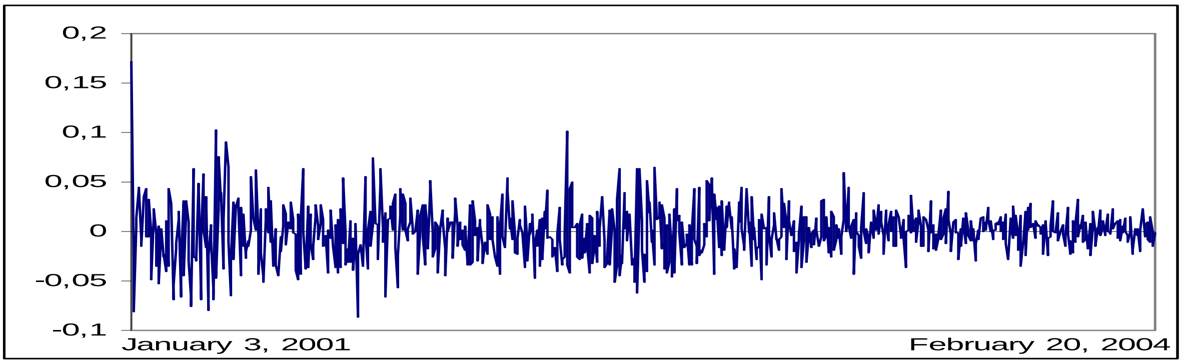
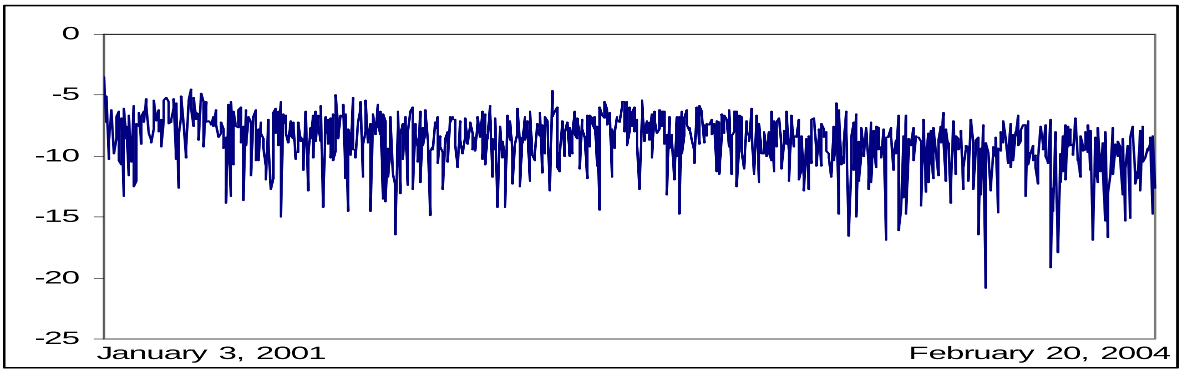
FIGURE 1

NASDAQ-100 Index returns (y_t)



$y_t^* = \log(y_t)^2$



Periodogram of y_t^2

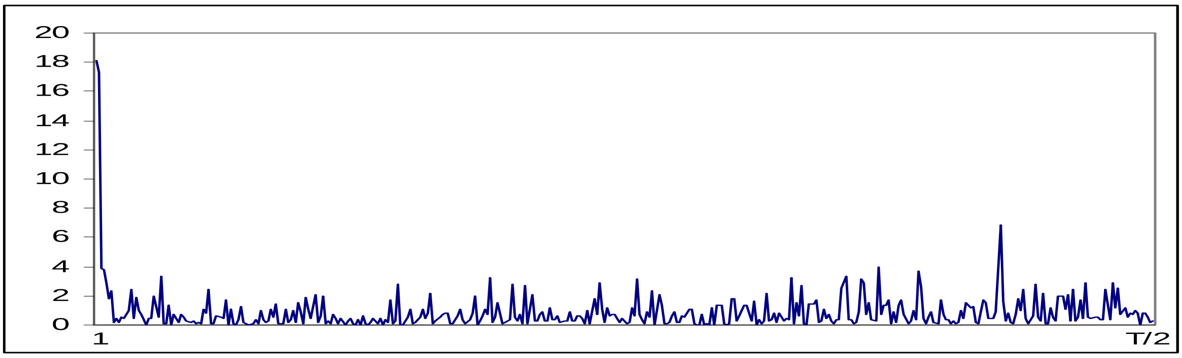
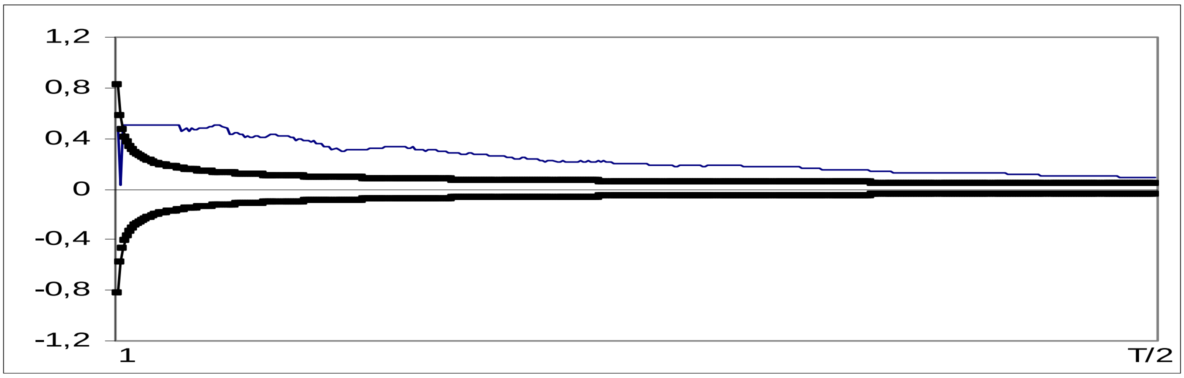
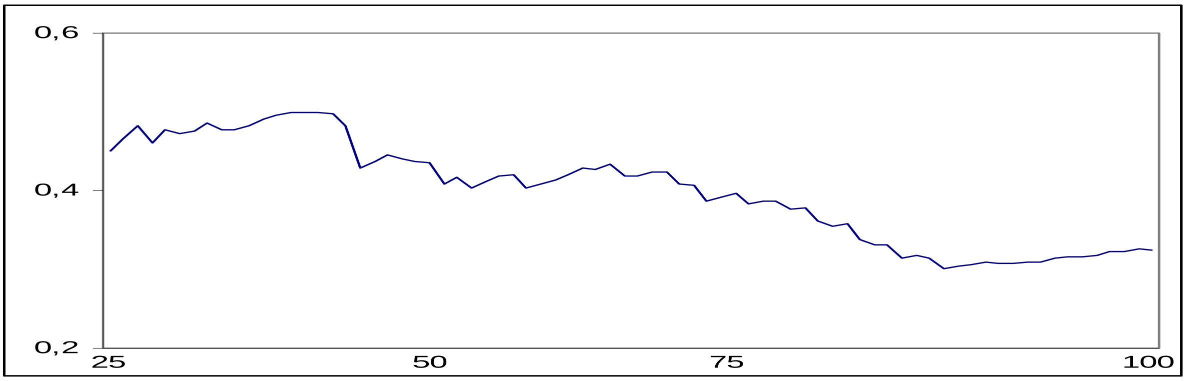


FIGURE 2

Estimates of d based on the Whittle method of Robinson (1994a) for the transformed returns



Estimates of d for a short grid of values of m



The horizontal axis refers to the bandwidth parameter m, while the vertical one corresponds to the estimated value of d.

TABLE 1			
Impulse responses for a value of d = 0.41			
Period	Value	Period	Value
1	0.4100	11	0.1111
2	0.2890	12	0.1056
3	0.2322	13	0.1008
4	0.1979	14	0.0966
5	0.1745	15	0.0928
6	0.1574	16	0.0893
7	0.1441	17	0.0862
8	0.1335	18	0.0834
9	0.1247	19	0.0808
10	0.1174	20	0.0784

FIGURE 3

Impulse responses for an I(d) process with d = 0.41