CASE STUDY

Rearranging equations: (concepts – misconceptions) × peer discussion

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Abstract

Transposition of formulae (also known as rearranging equations and changing the subject) is a skill vital for professionals in many fields of science and engineering. It is however a topic with which many students, and particularly students of weaker algebraic competency, struggle and often do not master sufficiently. This paper proposes an intervention strategy for improved teaching and learning of transposition of formulae at third level. The intervention aims to address three key issues thought to inhibit students' understanding of the topic: (1) a lack of conceptual understanding of equations and equality, (2) prior misconceptions and (3) a fast paced learning environment that does not account for diversity in knowledge and aptitude. The strategy consists of three hour-long lesson plans that emphasise conceptual understanding while also dispelling the relevant misconceptions, using a peer discussion teaching model as a vehicle for consolidating and propagating the right concepts. To account for diversity in prior knowledge and aptitude an online tutorial on the topic of transposition has been developed using an online e-assessment platform that allows students to practice at their own pace and receive instant feedback as they progress.

Keywords: Transposition of formulae, rearranging equations, changing the subject, algebra.

1. Introduction

Transposition of formulae (also known as rearranging equations and changing the subject) is a fundamental skill for quantitative disciplines; not only does it form a key step in more advanced problem solving but it also facilitates understanding of the relationships between quantities of interest. From a learner's prospective, it is a rather demanding task as it represents a culmination of many algebra skills and concepts coming into play at once. As a result, it is a topic with which many students, and particularly students of weaker algebraic competency, struggle and often do not master sufficiently. Despite the topic being taught using a variety of styles and approaches both at school and often again at third level (higher education in the Republic of Ireland), students' difficulties persist (O'Brien & Ní Ríordáin, 2017; Stephens, et al., 2013; Pendergast M. & Treacy P., 2015; Chow, 2011). In our own experience, after giving a diagnostic test on transposition to over 350 first year students at Cork Institute of Technology, 75% of respondents were not able to correctly rearrange the following benchmark equation to isolate g:

$$T = \frac{2v}{g} + 5.$$

The problem is not limited to the topic of transposition itself and in fact perpetuates far beyond a class on rearranging equations. The difficulties that students encounter in a wide variety of further mathematical problems, e.g. separating variables in differential equations, arise from a lack of proficiency in rearranging equations. Moreover, mathematical proficiency is a major contributory factor to the success of a student in any science or engineering course. In many applied disciplines the topic of transposition is so important and integral to the understanding of applied concepts that students' struggles with transposition significantly impact on their progress in the applied subject. At Cork Institute of Technology, lecturers from applied disciplines, such as engineering, physics and business studies, reported that difficulties with the topic of transposition inhibited the teaching and assessment of applied concepts. Interestingly, even some online resources in Physics and Chemistry have dedicated sections to rearranging equations thus confirming the importance of the transposition skills in these disciplines but also highlighting students' difficulties with it in an applied context (Konig, 2015; Scott, 2012; Southall, 2016; Isaac-Physics, 2019). These resources also emphasise the importance of a conceptual learning of the method versus memorising some shortcuts like 'a formula triangle'.

The origins of students' difficulties with transposition, discussed in detail in (Lishchynska M. et al, 2019), are complex and multifaceted. Two major technical contributors to the issue, identified in (Lishchynska M. et al, 2019), are the lack of understanding the concept of balancing equations, as well as strong misconceptions of an algebraic and conceptual nature. It seems that the issue persisting does not so much depend on a particular style or method of teaching but more on the fact that mathematical fluency in the topic of transposition can only be acquired if one has formed a strong conceptual understanding of the principles while also managing not to form erroneous beliefs (misconceptions). The third issue identified in (Lishchynska M. et al, 2019) is the limited time available to teach the topic to a large number of students that vary in prior knowledge. Since the topic is taught at second level, transposition is viewed as a prerequisite in first year Mathematics modules and the three-hour lecturing time allocated is for the purpose of revision and consolidation. However, for many students the gaps in their knowledge are too large to overcome in this short time frame. In response to the technical findings and with the aim to rectify the problem, this paper concentrates on cultivating the key concepts and dispelling students' misconceptions pertinent to rearranging equations. We also seek to address the time constraints and variation in prior knowledge. The work aims to develop an approach to teaching transposition that will address the above deficiencies effectively and will ultimately help the students to master the skill.

2. Building concepts

To gain a better insight into the students' difficulties, and more importantly the basis for those, an error analysis was performed on students' work (Lishchynska M. et al, 2019). One of the two major deficiencies identified was a lack of understanding of the equality sign and how equivalent equations are generated. This agrees with the findings in studies by Bush and Karp (2013), Stephens et al. (2013) and Byrd et al. (2015) where the lack of understanding of the equality sign and equivalent equations was at the core of students' problems with solving equations. The lack of conceptual understanding plays a major role in rearranging equations as without the deep understanding of what equations are, how they are formed and what the equality sign means, other algebraic skills become unproductive in the context of transposition. Conversely, when a student has developed a solid understanding of the main concepts and principles, they should be able to apply them and rearrange any formula or equation, not just a particular set of easy or typical ones.

To account for the above, teaching with a strong emphasis on conceptual understanding of balancing equations needs to precede the delivery of the method itself and practical work on rearranging equations. Moreover, to foster a greater understanding of the fundamental principles and concepts, students need to be confronted with a variety of concept questions relevant to transposition. A good concept question should satisfy the following criteria (Crouch, 2007): focus on a single important concept, ideally corresponding to a common student difficulty; require thought, not just plugging numbers into equations; provide plausible incorrect answers; be unambiguously worded; be neither too easy nor too difficult. In relation to transposition of formulae, the thought process behind forming concept questions applied in this work is schematically presented in Figure 1 and explained in more detail in (Lishchynska M. et al, 2019). With these criteria and approach in mind, a bank of concept questions on equations and transposition has been developed, with several examples given in the Appendix. Such questions can be used as a basis for peer discussion and teacher-students discussions in class as well as in diagnostic tests.

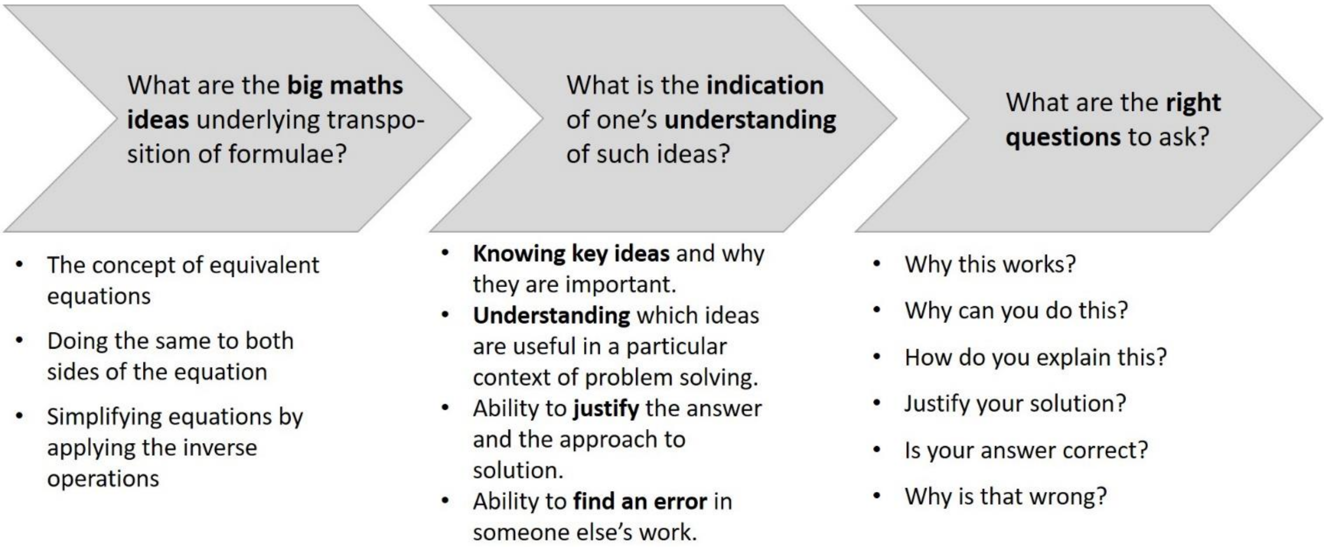

Figure 1. The approach to creating concept questions.

It is worthwhile acknowledging that in the Mathematics Education research there exists a decades long 'the chicken or the egg' type debate about what needs to come first, conceptual understanding or procedural fluency (Rittle-Johnson, et al., 2001; Haapasalo, 2003; Star, 2000). With regard to rearranging equations, we believe, a successful intervention must firstly focus on building up a solid understanding of the concepts of equality and equations. And this must come before the procedural fluency because *"to be able to apply the procedure for solving equations 'accurately, efficiently, and flexibly', students really need to know what they are doing to a far greater extent than when applying Pythagoras's theorem or multiplying fractions"* (Barton, 2018). In the case of rearranging equations, conceptual learning supports and generates procedural learning but procedural learning on its own does not support conceptual learning. Once students have a solid conceptual understanding of the principles of balancing equations, they can then be presented with a procedure, examples and practice questions.

However well intended and planned, focusing solely on cultivating a conceptual understanding may not be sufficient to affect students' skills in transposition. Students' erroneous beliefs about equations need to be dealt with as well.

3. Misconceptions cannot be ignored!

Another major contributor to the poor transposition skills pinpointed by the error analysis was a whole myriad of misconceptions that students hold about equations and manipulating them. In addition to this, there are also misconceptions related to the algebraic procedures involved in transposition of formulae. As it happens, students arrive at third level with some cognitive baggage and that likely includes erroneous beliefs and incomplete knowledge of some topics and concepts. Students' misconceptions are a known phenomenon with mathematics teachers and have been the focus of multiple studies. There is a general agreement in the literature that students' misconceptions form barriers to further learning and need to be addressed and dispelled in class (Lucariello, et al., 2014; Bush & Karp, 2013; Barbieri & Booth, 2016). Students can only benefit from developing good conceptual understanding while simultaneously challenging misconceptions (Stephens, et al., 2013). The research into the topic suggests that, while necessary, focusing an intervention solely on cognition or motivation may not lead to improvement in algebra skills whereas a combined intervention approach that includes error reflection may be beneficial (Barbieri & Booth, 2016).

Misconceptions most relevant to transposition may be roughly grouped in two classes: conceptual and algebraic. Conceptual misconceptions manifest in the lack of respect for the equality and hence mishandling the equations, incorrect use of inverse operations and, very commonly, illegal 'moving' entities across an equation in a random fashion. Algebraic misconceptions affecting transposition are mostly, but not limited to, misunderstanding and misuse of the distributive law, incorrect simplification of expressions (illegal 'cancelling') and the very common, intuitive but wrong assumption about distributivity of exponents (squaring, taking roots etc.). A review of algebraic misconceptions given in (Chow, 2011) highlights the latter group of misconceptions as over-generalisation of rules and/or operations. A schematic classification of misconceptions relevant to transposition and examples of typically encountered errors are given in Figure 2.

In our experience, the errors due to the above misconceptions are so common and persistent that they simply cannot be ignored and need to be resolved effectively. While there are plenty of resources of 'spot the mistake' style exercises, an active intervention needs to take place in class for a teaching strategy to be effective. The misconceptions of conceptual nature can be addressed through deliberate teaching with an emphasis on the right concepts of balancing equations discussed in the previous section. The issue of illegal 'moving' of entities across an equation is linked to that and can be addressed *after* the key concepts have been presented and practiced by the students. Algebraic misconceptions relate to rearranging somewhat harder, more convoluted formulae, and therefore can be addressed at a later stage but before considering discipline relevant equations/formulae. The idea here is to reinforce the necessary laws of algebra and principles of distributivity and simplification approaches, then challenge the students with concept questions testing these laws and discuss the common misconceptions.

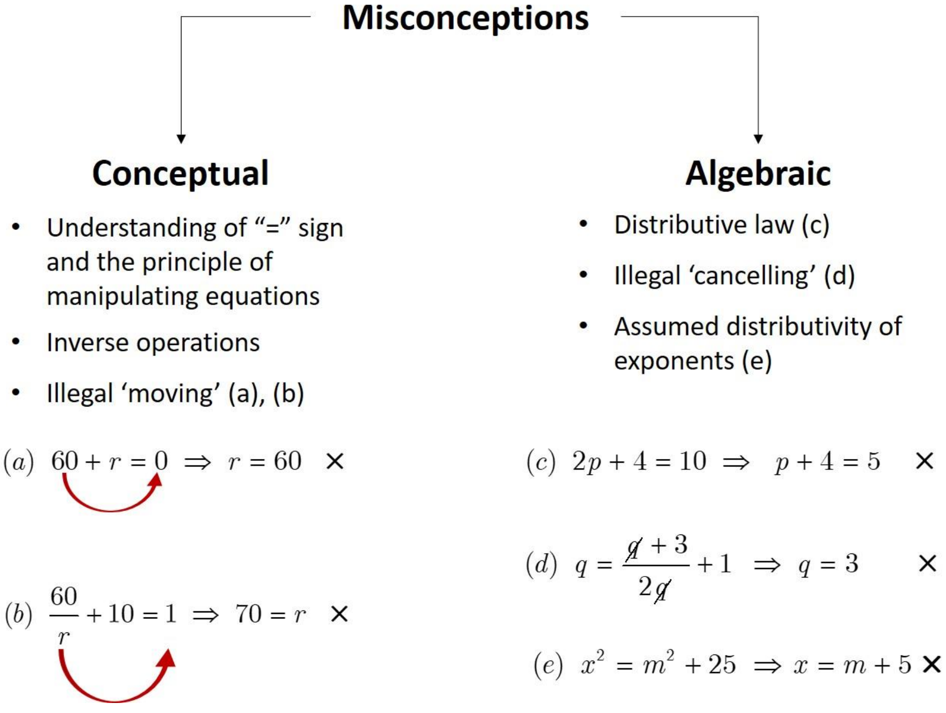

Figure 2. Summary of misconceptions and typical errors relevant to transposition.

Given the limited time lecturers at third level can dedicate to teaching the topic, e.g. a lecturer at Cork Institute of Technology has at most three hours of lectures and a tutorial, it is vital to plan the classes carefully and use this time effectively. Traditional teaching approaches, even with an increased emphasis on concept building and addressing misconceptions, may still be insufficient to induce a tangible change in students' learning and acquiring a solid skill of transposition. To be effective, an intervention needs to include a radical aspect in comparison with existing methods. *"Memory is the residue of thought"* and therefore *"students remember what they think about …"* (Barton, 2018). If one aims for the right ideas to be firmly planted in students' memories and discourage the wrong ones from forming, one needs to stimulate students to think, question and remember. Therefore, a peer discussion teaching model was adopted to support the taught element of the topic.

4. Peer discussion: a vehicle for propagating right concepts

Eric Mazur was teaching introductory physics to undergraduates and used a traditional method: the lecturer speaks, the students listen and take notes. One day Mazur decided to test the students' understanding of a particular concept not by doing traditional problems, but by asking them a set of basic *conceptual* questions, he discovered that the students had great difficulty with the conceptual questions. As a result, Mazur had to completely re-think his approach to teaching and proposed a peer instruction (also known as peer discussion) model to be used in conjunction with lectures (Mazur, 1997). Peer instruction is designed to engage students during class through activities that require each student to apply the core concepts being presented, and then to explain and discuss those concepts with their fellow students. After a particular topic is presented by a lecturer, a concept question (a question probing understanding of a particular concept) is posed. Students are given one or two minutes to formulate individual answers and vote, using clicker devices or mobile phones, for an answer from a choice displayed on a screen. Students then discuss their answers with others sitting around them; the instructor urges students to try to convince each other of the correctness of their own answer by explaining the underlying reasoning. After a few minutes, the instructor calls an end to the discussion, polls students for their answers again which may have changed based on the discussion.

Above anything else peer instruction makes each student *think* and think twice: once individually and then again when trying to explain their point of view to the fellow students. Mazur's results show a significant increase in the percentage of students answering the concept question correctly. Peer instruction has been shown to be effective across disciplines at second and third level (Cummings & Roberts, 2008; Smith, et al., 2011) and combining peer discussion with instructor explanation increases student learning from in-class concept questions (Smith, et al., 2009). The logic underlying the success of peer discussion based on concept questions is that it continuously engages students' minds and provides feedback to both students, and the lecturer, about the level of understanding. It also allows students to construct their own knowledge of the topic. Other benefits include the (usually positively received) social aspect of the peer discussion and the students' greater openness to accept information from their peers who know and are convincing in their explanations. There is also the use of a different, and perhaps simpler, language with explanations.

Importantly, as a basis for peer discussion, concept questions can be designed to uncover and highlight students' misconceptions in the material thus bringing the students' attention to the issues and allowing firstly the students themselves, and if necessary the lecturer, to address those. The authors in (Giuliodori, 2006) have found that 56.8% of the students with incorrect individual answers switched to a correct answer after peer discussion. Equally significant was the fact that only 6.5% of students changed their originally correct response to an incorrect one. Quoting (Mazur, 1996) it is *"easier to change the mind of someone who is wrong than it is to change the mind of someone who has selected the right answer for the right reason",* thus peer discussion tends to propagate the right ideas not the misconceptions.

Given the thought stimulating potential of the peer discussion teaching model, it may prove to be an effective part of an intervention strategy for teaching transposition. We want our students to think hard about the concepts and principles underpinning the process of rearranging equations and we want these thoughts to make a lasting imprint in their memories. Peer discussion may act as a vehicle for propagating and consolidating the right concepts.

5. Intervention strategy and lesson plan

This section brings together the ideas from the previous three sections in an intervention strategy. Considering the above, a successful intervention teaching model comprises three main components: (1) building of a solid conceptual basis, (2) resolving/dissolving relevant misconceptions and (3) a peer discussion acting as a vehicle for propagating and consolidating the right concepts (Figure 3). It is important to emphasise here that one needs to be cautious when dealing with misconceptions in terms of presenting the right ideas first, through explicit instruction, and only then moving onto clearly labelled incorrect examples once the students are familiar with the concepts.

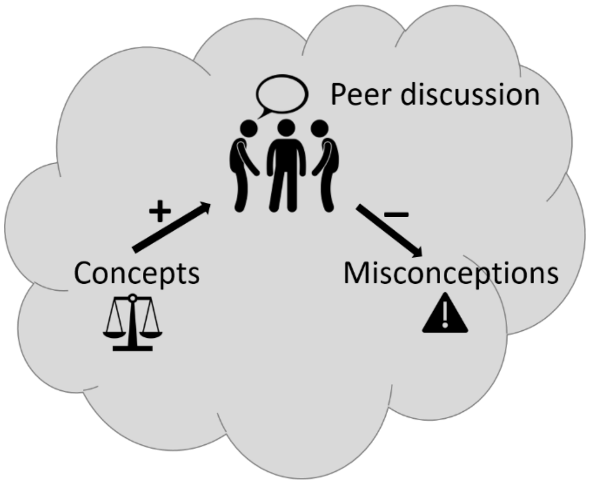

Figure 3. Schematic view of the intervention model.

Going back to the dilemma of the right order between procedural and conceptual knowledge acquisition, O'Connor and Norton (2016) investigated the mathematical difficulties encountered with quadratic equations by analysing student error patterns and the relationships between them. The study found that students' lack of procedural skills in prerequisite algebra hindered their conceptual understanding of quadratic equations. Similarly, our own error analysis found that many of our students did not have the pre-requisite algebra skills to apply to transposition problems. Star (2000) gives an interesting dissection of the 'which comes first' debate and concludes that *"understanding in mathematics is the synthesis of knowing and doing, not the accomplishment of one in the absence of the other"*. Indeed, conceptual errors and procedural errors are often intertwined. Rittle-Johnson et al. (2001) put forward the idea of an iterative model of the development of conceptual and procedural knowledge which we think may be especially relevant to the process of learning to rearrange equations. This factor is also taken into account in the intervention strategy.

In summary, the proposed intervention strategy flows as follows (Figure 4):

- Develop concept of an equation and balancing equations (supported by peer discussion);
- Address misconceptions about handling equations (supported by peer discussion);
- Teach, through explicit instruction, manipulating equations and basic principles of transposition;
- Address relevant algebraic misconceptions (supported by peer discussion);

- Teach and practise transposition of harder (more convoluted) formulae.

Concept questions similar to the examples given in the Appendix can be used as a basis for peer discussions.

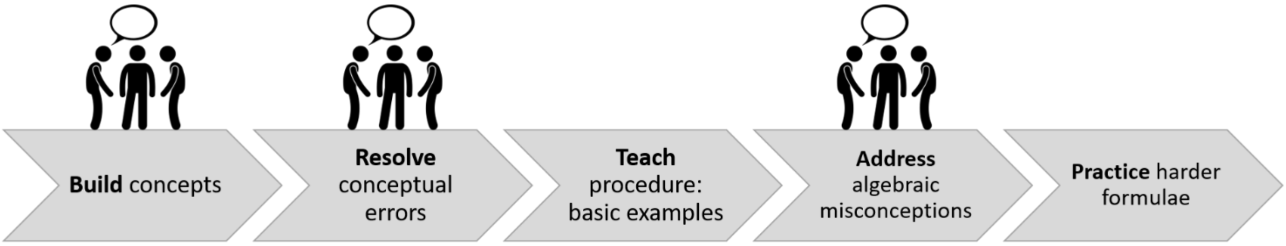

Figure 4. Teaching sequence in the proposed intervention strategy.

Specifically, the proposed intervention comprises a three-lecture plan. Lecture 1 focuses on the main concepts and fundamentals of balancing equations: motivation for transposing formulae; the concept of an equality and balancing scales; inverse operations; common conceptual misconceptions. The objectives of Lecture 2 are to review and reinforce algebra pre-requisites of transposition (simplifying expressions, distributive law, non-linearity of exponents), to discuss the relevant misconceptions and to present the general approach to rearranging equations. This class also addresses the difficulty of 'where to start' which is a common stumbling stone with weaker students. Lecture 3 is dedicated to dealing with harder (more convoluted) formulae involving the use of the distributive law, a subject appearing on both sides of the equation and fractions. Purposeful practice using students' discipline specific formulae here aims to help students master the skill.

6. Time constraints and variation in prior knowledge

One of the non-technical factors contributing to the students' difficulties with transposition identified in (Lishchynska M. et al, 2019) is the difference in prior knowledge. The students have been exposed to the topic at school and this prior exposure made the topic particularly challenging to teach; some students already know how to transpose equations and are bored in class distracting others whereas some think they know it (when they do not) and are not paying attention. There is also the issue of fast delivery pace at third level which does not suit some learners. All this does not create an optimum learning environment. It is hoped that the interactive nature of peer discussion will engage students who already understand the topic allowing them to consolidate their own knowledge through explanation to their peers. The quiz element of the peer discussion will also provide instant feedback to those students who think they know the material when in fact they do not.

To deal with the diversity of skill in class as well as encouraging all students to practise more, an online tutorial using the open source maths e-assessment platform NUMBAS (NUMBAS, 2015) has been developed at Cork Institute of Technology (NUMBAS-CIT-Transposition, 2019). This is an optional tool to be used as a supplement to lectures and standard tutorials. NUMBAS allows a lecturer to create a question framework from which an unlimited number of random variations of a question can be generated, allowing a student to practice a question of a certain type or difficulty as often as they feel necessary. As well as instantly informing the student whether their answer is correct, NUMBAS also provides options for 'show steps' and 'advice' where either partial solutions or full solutions can be displayed thus providing instant detailed feedback to a student when required. In line with the *"learning for mastery"* strategy (Bloom, 1968), where the emphasis is on students achieving a level of mastery in prerequisite knowledge before moving forward to learn subsequent information, the NUMBAS online tutorials are intended to help the students to spend as much time as individually necessary on tasks of various difficulty, and hence master the skill at an individual pace.

7. Conclusion and future work

Students' lack of proficiency in rearranging equations is an issue perpetuating far beyond a maths class and often hinders progress in a science or engineering course. This paper proposes an intervention strategy for improved teaching and learning of transposition of formulae. The intervention is based on an increased emphasis on conceptual understanding while also dispelling the relevant misconceptions. This is to be supported by a peer discussion teaching model as a vehicle for consolidating and propagating the right concepts. Differences in prior learning and a fast-paced learning environment are countered through the use of interactive online NUMBAS tutorials.

The intervention is currently being implemented at Cork Institute of Technology. A full evaluation study is underway. Future work will focus on quantitative analysis of the outcomes of the implemented intervention. The 'before' and 'after' diagnostic tests will aim to quantify the impact of the intervention on student attainment in the topic of transposition in a first-year service mathematics course. A qualitative study intended to gauge the effect on students' experience and attitudes when learning transposition will also be undertaken.

8. Acknowledgements

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Appendix

Examples of concept questions on equations and transposition.

Q1. Which of the following are equations? Choose all the options that apply.

□ x^2

□ $x^2 = 2x$

□ $x + 5$

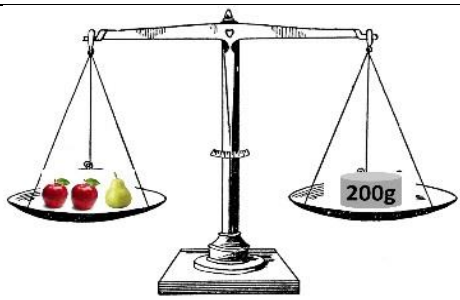
□ $x^2 - 5x + 6 = 0$

□ $x = 2b$

□ $T = 2\pi\sqrt{\frac{l}{g}}$

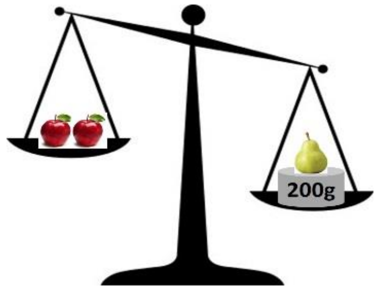
Q2.

$$2a + p = 200$$



What happened here?!!

$$2a = 200 + p$$



Can you explain why the scales are no longer balanced?

Q3. If $3m = 5$, which of the following are true? Explain why. Choose all the options that apply.

☐ $6m = 10$

☐ $9m^2 = 25$

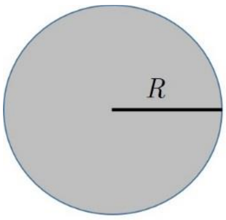
☐ $3m - 2 = 3$

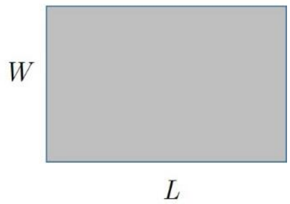
☐ $\sqrt{3m} = \sqrt{5}$

☐ $3m^2 = 25$

☐ $m = \frac{5}{3}$

Q4. Two plots of land, of circular and rectangular shapes, have equal areas.





Is the following statement true or false? Justify your answer.

$$L = \frac{\pi R^2}{W}$$

Q5. Apples and pears are used to balance the scales shown below.

If a is the weight of an apple and p is the weight of a pear, which of the following statements are correct? Explain why.

(a) $10a = 2a + 6p$ (b) $3p = 4a$ (c) $5a = 3p$ (d) $a + 3p = 5a$

Q6. In an exam, students had to re-arrange (transpose) the following formula to express r which represents the thickness of an engineering part:

$$p = \frac{r^2 + q^2}{L}$$

Below are two solutions by two students. One of these solutions is incorrect.

a) Identify the incorrect solution.
b) Circle the part of the incorrect solution where the mistake is made.
c) Use the box below to explain briefly why it is wrong.

Solution 1	Solution 2	
$p = \frac{r^2 + q^2}{L}$ $pL = r^2 + q^2$ $pL - q^2 = r^2$ $r = \sqrt{pL - q^2}$	$p = \frac{r^2 + q^2}{L}$ $pL = r^2 + q^2$ $\sqrt{pL} = r + q$ $r = \sqrt{pL} - q$	

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