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Multifactor consumption based asset pricing models using the US stock market as a reference: evidence from a panel of developed economies.

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Abstract

In this paper we extend the time series analysis to the panel framework to test the C-CAPM driven by wealth references for developed countries. Specifically, we focus on a linearised form of the Consumption-based CAPM in a pooled cross section panel model with two-way error components. The empirical findings of this two-factor model with various specifications all indicate that there is significant unobserved heterogeneity captured by cross-country fixed effects when consumption growth is treated as a common factor, of which the average risk aversion coefficient is 4.285. However, the cross-sectional impact of home consumption growth varies dramatically over the countries, where unobserved heterogeneity of risk aversion can also be addressed by random effects.

Keywords: Consumption-CAPM, Excess Returns, Generated Regressor, GMM, Habits, Panel, Wealth Reference

JEL Classification: C52, E44, G12

1 Introduction

Empirical studies have already shown that the conditional covariances between the Intertemporal Marginal Rate of Substitution (IMRS) in consumption and returns cannot satisfy the equilibrium restrictions imposed by the representative agent Consumption-based CAPM for different countries (C-CAPM) (Kocherlakota, 1996). This has led to a great research interest in C-CAPM of taking account of heterogeneity and idiosyncratic risk (Lund and Engsted, 1996). The issue of heterogeneous risk in asset pricing was first addressed by Miller (1977), and then revisited by other authors, i.e., Constantinides and Duffie (1996), Jacobs and Wang (2004). It has been concluded that heterogeneous risk has a better chance of explaining the data than standard representative-agent C-CAPM models (Jacobs and Wang, 2004). In this sense, Hunter and Wu (2009) also address the importance for the UK market of simultaneous heterogeneity that is proxied by the US wealth reference.

Panel data analysis is a conventional econometric technique of bias correction for heterogeneity. However, it has been less applied in asset pricing. Following Hunter and Wu (2009), this paper further investigates in a panel context the importance of wealth references proxied by the US wealth reference as an explanation of systematic risk in C-CAPM models for a further group of major developed countries. By taking into account the US excess return as a proxy of wealth reference, different specifications of a two-way error component panel model are used to study whether there is any measurable heterogeneity or idiosyncratic risk related to excess returns and consumption growth either across countries or over time.

The rest of this article is organised as follows. A brief literature review is given in section 2. Section 3 and 4 describe the data properties and the methodology applied in this paper, respectively. Section 5 reports the empirical results. Finally, section 6 contains some concluding remarks.

2 The Consumption CAPM Literature

In the last three decades, the poor performance of the standard Consumption-based CAPM(C-CAPM) has been well examined by time series analysis. Within the C-CAPM framework, research has improved the performance by applying different price kernels to incorporate different types of heterogeneity that has been neglected by the standard C-CAPM, i.e., preferences related to standard power utility function (Abel 1990, Constantinide 1990, Ferson and Constantinide 1991, Heaton 1993, Epstein and Zin 1991), complete asset markets (Constantinides and Duffie 1996; Heaton and Lucas 1997; Mankiw 1986; Storesletten, Telmer, and Yaron 1999), limited market participation (Mankiw and Zeldes, 1991 etc) and frictionless markets (Fisher 1994, He and Modest 1995, Margueerling and Verbeek 1999, Gregoriou and Ioannidis 2006).

Constantinides and Duffie (1996) provide a theoretical framework for assessing the effects of market incompleteness on financial market equilibrium, under which the conditional covariances between returns and the Intertemporal Marginal Rate of Substitution (IMRS) in consumption mean that it is not possible to attain equilibrium. They derive a pricing kernel for an economy where individuals with isoelastic preferences are subject to idiosyncratic income

shocks. The pricing kernel depends on aggregate consumption growth as well as the cross-sectional variance of per-capita log consumption growth. If this variance is negatively correlated with equity returns, the heterogeneous economy has a higher equity premium and lower risk-free rate than what the standard model predicts. Thus, there is the potential for this type of model to resolve asset-pricing puzzles.

Following the theoretical intuition of Constantinides and Duffie, several papers have investigated empirically the role of heterogeneity induced by market incompleteness. However, the results are mixed. For example, Jacobs (1999) uses the US Panel Study of Income Dynamics (PSID) data on food consumption to estimate individual Euler equations for the 1974-1987 period, and finds that joint tests on the risky and riskless asset strongly reject the model. Cogley (2002) generalises the pricing kernel of Constantinides and Duffie (1996) to test it on US Consumption Expenditure Survey (CEX) data for the 1980-1994 period. However, his findings indicate that measures of the cross-sectional dispersion of log consumption growth are only weakly correlated with stock returns, and that pricing kernels depending on these cross-sectional measures generate unrealistically small equity premia for preference specifications with low degrees of risk aversion. With the same CEX data set, Brav, Constantinides, and Geczy (2002) test a pricing kernel obtained from the aggregation of IMRS models. By permitting heterogeneity, this kernel can reconcile the problems that arise with consumption-based models and their models suggest coefficients of risk aversion between 3 and 4 that are more consistent with theory.

More recently, Jacobs and Wang (2004), Semenov (2005) and Balduzzi and Yao (2005) also investigate idiosyncratic consumption risk within the cross-sectional C-CAPM. Jacobs and Wang (2004) compare the traditional CAPM with a two-factor C-CAPM that is related to cross-sectional consumption variation that captures the possibility of idiosyncratic risk. They demonstrate that consumption risk described by cross-sectional consumption variation can contribute to the cross-sectional average returns of stocks, and the performance is similar to the consumption surplus ratio of the conditional C-CAPM of Campbell and Cochrane (2000). Also, Semenov (2005) develops an appropriate equilibrium factor model using the cross-moments of asset returns and the cross-sectional moments of individual consumption, aggregated by a dummy variable for risk signs. He finds that the model explains the observed equity premium with realistic values of risk aversion. Instead of using the cross-sectional variance of log consumption growth (Constantinides and Duffie, 1996), Balduzzi and Yao (2005) employ the growth of the cross-sectional variance of log consumption and develop a new heterogeneous-agent pricing kernel based upon the cross-sectional aggregation of marginal utilities. With reasonable coefficients of relative risk aversion, their model can explain the US risk premium by the consumption of asset holders.

Therefore, it can be seen from the above discussion that although the debate over the specific pattern of heterogeneity in either consumption or returns has not reached a conclusion, the approach has a capacity to solve both of the puzzles. Hunter and Wu (2009) suggest a C-CAPM framework that includes US wealth reference as an alternative of home consumption habit to reconcile different volatilities between returns and consumption growth data for the UK. The introduction of this new risk factor can well mimic the cross-country heterogeneity in both returns and consumptions whenever they exist. They find

that for the UK model, the US stock market is the primary source of the low correlation between UK returns and consumption growth rates, since effects resulting from the external market are much stronger than even the UK consumption habit. Therefore, the integration of stock markets can at least alter the investors' expectations of risk returns and account for the disequilibrium of the conditional covariances between risk premia and consumption for the UK C-CAPM model. This would seem reasonable for a market that for more than a century has had a regard for the influence of global returns.

However, it can be argued whether imperfect asset diversification across international securities markets is also the primary source of low cross-country correlation of consumption growth rates and whether high cross-country correlation of excess returns is supported by evidence from other countries, or unique only for UK. Today, the international integration of financial markets is a central characteristic of the globalisation process and a potential force for driving changes in the institutions of corporate governance. For example, cross-border portfolio investment funds have expanded dramatically. Also, the number of foreign companies listed on the two major US stock markets has increased significantly, though there is still evidence for a home bias in investors' portfolio decisions (Opoku, 2007). These trends indicate a convergence in the institutions of corporate governance at national levels to the US system and the standards of US institutional investors. The ultimate impact will be the worldwide dominance of the US markets with respect to monetary policy and corporate governance.

In this paper we argue that it is natural to consider a panel as a more appropriate econometric approach to analyse the C-CAPM when non-US stock markets are converging to the US market, and it is also natural to use panel techniques to handle heterogeneity and measurement errors.¹ Panel data techniques provide a coherent methodology to flexibly deal with both the homogeneous and heterogeneous parameters of the country models, and after correcting for individual effects, any further heterogeneity at the level of the individual country would suggest that important predictive variables have been omitted from the models. On the other hand, measurement errors can lead to under-identification of an econometric model. However, the availability of multiple observations for a given individual or at a given time may allow a researcher to estimate different model specifications, and thus observe alternative parameters of these models. The problem of measurement error in variables can be particularly important and relevant for the two-step regressions estimated in this thesis. Although IV and GMM estimation have been deployed to minimise the extent to which it affects the time series analysis (Hunter and Wu, 2009), it is still worthwhile to investigate this problem further in a panel setting. A third issue is the effect of correlation and causation, both of which are of interest in statistical studies. In particular, correlation is very important for econometric studies. Panel data analysis permits us to extract from the data a range of different kinds of correlation: correlation at the level of individual country (autocorrelation), across individual countries (contemporaneous correlation) and across time.

A small number of panel data studies exist for C-CAPMs that consider country-specific effects while cross-country studies are more common in the

¹Panel data techniques provide a coherent methodology to flexibly deal with both the homogeneous and heterogeneous parameters of the country models. For more general issues, see the survey: Panel Data Analysis — Advantages and Challenges (Hsiao, 2006)

literature (i.e. Lund and Engsted, 1996), where C-CAPM models have been estimated separately without considering cross-country correlation. The cross-sectional approach can limit the potential robustness and efficiency of the findings with respects to country specific effects and time related effects. There are only a few studies with panel data on returns and portfolio allocation, for example, well known two dimensional panel - the 25 Fama-French portfolios, sorted with respect to five size and five book-to-market categories (Fama and French, 1996; Hodrick and Zhang, 2001; Campbell and Vuolteenaho, 2002).²

3 Descriptive Statistics

The historical quarterly panel data used cover nine typical developed stock markets, which are all collected from Datastream. Specifically, the nine countries are Australia (AU), Germany (BD), Canada (CN), Denmark (DK), Spain (ES), France (FR), Italy (IT), Switzerland (SW) and the United Kingdom (UK), as they are all important countries with regard stock market capitalisation except Denmark.³ The reason for including the Denmark market is that it retains its own currency and monetary policy, which is rare in Europe today, and thus can be expected to help prove whether heterogeneity exists. The consumption data (C_t) are aggregated, seasonally adjusted private consumption expenditures and measured at constant prices. Due to data availability short-term interest rates (r_t^f) of corresponding countries are chosen as either 3-month rates of either Treasury/Government bills or interbank rates. As far as the stock indices are concerned, recognised world indices are preferred due to their impact on the market, trading volumes and the corresponding derivatives centring on them. However, when we have too few observations, we use corresponding composite indices. Table 1 gives detailed information about all the selected variables.

The effect of the reunion of placeGermany at the end of 1990 should be noted as it might introduce a structural break by virtue of the economic change engendered and since the consumption data used are now aggregated across the former East and West Germany. However, this country cannot be excluded from the panel since it is so important,⁴ and thus we limited the sample to 1991:Q1-2004:Q4.

²Dynamic Panel Data analysis (DPD) has to be discussed when panel data models are estimated by GMM. The general model that can be estimated with DPD is a single equation with individual and time specific effects, and an error term that should be serially uncorrelated to satisfy some set of common factor restrictions. However, DPD is not appropriate for the estimation of the current nine-country C-CAPM, since it is designed for cases where the cross section dimension is large and the time series dimension small. Moreover, DPD is indeed an autoregressive structure that includes AR terms as explanatory variables, which is not necessary for the C-CAPM definition when the residuals are not correlated.

³It should be noted that the second largest stock market - the Japanese stock market - has been neglected due to the illiquidity problem. Also, as documented in the literature, the Japanese stock market in the previous 15 years is not as relevant to the world market as those of other mature stock markets. Thus, it is not considered here in panel models.

⁴This can be seen by the central role of the Deutsch Market in the EMS and the decision to position the ECB in Frankfurt prior to the introduction of the Euro Zone.

Table 1 Data Information on 9 Selected Countries

Country	Market Capitalisation *	Source of interest rates	Market index
Australia	776.2	Interbank	ASX ALL ORDINARIES
Germany	1194.5	Interbank	DAX30
Canada	1177.5	Interbank	TORONTO SE COMPOSITE INDEX
Denmark	155.2	Interbank	COPENHAGEN OMXC20
Spain	940.7	Interbank	MADRID SE GENERAL
France	1435.7	Pibor	CAC40
Italy	789.6	G-bill	MILAN MEX
Switzerland	825.8	Interbank	SWISS MARKET PRICE INDEX
United Kingdom	2865.2	T-bill	FTSE100

*:Data source is from World Bank annual report 2005 (in billions of US dollars), where the US stock market is reported as \$16323.5.

It should be noted that \hat{r}_{US}^e and $\hat{r}_{US}^{e,2}$ are fitted values recalculated from the US return equation developed in Gregoriou, Hunter and Wu (2009) that are respectively with and without dummies.⁵ The reason that expectations on extreme observations have to be recalculated is because of the market timing which indicates that shocks are not predicted in advance. Apparently, it is not surprising as compared with the actual r_{US}^e , $\hat{r}_{US}^{e,1}$ is smoother while $\hat{r}_{US}^{e,2}$ is even more stable, and the correlation coefficients of nine major stock indices related to $\hat{r}_{US}^{e,2}$ are on average less than half of those related to r_{US}^e and $\hat{r}_{US}^{e,1}$. Unpredicted outliers may induce powerful correlations, suggesting their exclusion when the analysis requires rational asset pricing. However, some persistent correlations do exist across individual stock markets given by Table 2 and Table 3 for excess returns and consumption growth, respectively.

From Table 2, an obvious conclusion can be drawn that the Australian and Canadian stock markets are the two smallest markets that are still correlated with the US market even after correction for the effects of extreme observations associated with the sample period (0.17 and 0.19 respectively). An interesting example is the Canadian market. When the outliers are taken into account, we have so large correlation coefficients with the placeUS stock market, that it would appear that Canadian investors are not interested in their own market, rather, they prefer to participate in the US stock market. For Table 3, no further evidence can be found except that the growth in consumption across countries is not significantly correlated. The symbol \hat{r}_{US}^e will be used for simplicity as $\hat{r}_{US}^{e,2}$ for the remainder of this article.

Combining all the descriptive statistics associated with the panel data, it is apparent that the returns on different stock markets can be significantly different from each other, although their correlations are comparatively strong. However, consumption growth across countries is less inter-related, and thus might be as a result of the low level of variation relative to stock prices. All in all, it seems that the heterogeneity of national returns is worth investigating, particularly with the premise that consumer behaviour would appear to be heterogeneous.

⁵There are only three extreme observations, namely the Asian Crisis, "9.11" and the its anniversary that fall in the panel sample period.

Table 2 Correlation Coefficients for Excess Returns on 9 Developed Countries

	r_{AU}^e	r_{BD}^e	r_{CN}^e	r_{DK}^e	r_{ES}^e	r_{FR}^e	r_{IT}^e	r_{JP}^e	r_{UK}^e	r_{US}^e	$\hat{r}_{US}^{e,1}$	$\hat{r}_{US}^{e,2}$
r_{AU}^e	1											
r_{BD}^e	0.665	1										
r_{CN}^e	0.662	0.739	1									
r_{DK}^e	0.635	0.801	0.744	1								
r_{ES}^e	0.697	0.815	0.779	0.800	1							
r_{FR}^e	0.639	0.924	0.811	0.754	0.833	1						
r_{IT}^e	0.322	0.444	0.423	0.562	0.574	0.535	1					
r_{JP}^e	0.573	0.804	0.693	0.712	0.748	0.830	0.446	1				
r_{UK}^e	0.697	0.821	0.751	0.659	0.743	0.846	0.517	0.799	1			
r_{US}^e	0.687	0.803	0.810	0.660	0.782	0.834	0.430	0.738	0.867	1		
$\hat{r}_{US}^{e,1}$	0.378	0.668	0.604	0.589	0.578	0.667	0.414	0.710	0.619	0.587	1	
$\hat{r}_{US}^{e,2}$	0.166	0.301	0.189	0.236	0.288	0.301	0.249	0.313	0.308	0.317	0.564	1

Note: $\hat{r}_{US}^{e,1}$ and $\hat{r}_{US}^{e,2}$ are fitted values recalculated from the US returns equation with dummies and without dummies, respectively.

Table 3 Correlation Coefficients for Consumption Growth on 9 Developed Countries

	c_{SAU}^g	c_{SBD}^g	c_{SCN}^g	c_{SDK}^g	c_{SES}^g	c_{SFR}^g	c_{SIT}^g	c_{SJP}^g	c_{SUK}^g	c_{SUS}^g
c_{SAU}^g	1									
c_{SBD}^g	0.10	1								
c_{SCN}^g	0.19	-0.11	1							
c_{SDK}^g	0.21	0.32	0.03	1						
c_{SES}^g	0.45	-0.06	0.08	0.10	1					
c_{SFR}^g	0.32	0.26	-0.003	0.17	0.36	1				
c_{SIT}^g	0.21	0.09	0.29	0.26	0.47	0.17	1			
c_{SJP}^g	0.22	0.17	0.37	0.15	0.30	0.42	0.39	1		
c_{SUK}^g	-0.16	0.01	0.31	0.05	-0.05	0.14	0.23	0.29	1	
c_{SUS}^g	0.14	0.06	0.28	-0.01	0.22	0.12	0.29	0.27	-0.01	1

4 The Methodology

4.1 The Panel Model

We consider the heterogeneous panel linear regression model based upon Hunter and Wu (2009)⁶:

$$r_t^e = -\log \alpha - \log \beta + \gamma_1 cg_t + \gamma_2 \hat{r}_{US,t}^e + \mu_t. \quad (1)$$

In the above two-factor model, β is the time discount factor; r_t^e , cg_t and $\hat{r}_{US,t}^e$ denote the excess returns and consumption growth in one specific country, and the predicted excess returns on the country-regionplaceUS stock market. Eq (1) can be estimated by 12-lagged-instruments either IV or GMM, although the latter is often preferred in time series analysis.

Accordingly, the empirical two-way error component specification of a balanced panel data regression can be defined as:

$$\begin{aligned} r_{i,t}^e &= c_i + (\log \beta) + \gamma_{i,1} cg_{i,t} + \gamma_{i,2} \hat{r}_{US,t}^e + \mu_{it} & (2) \\ i &= 1 \dots I, t = 1 \dots T \\ \mu_{it} &= \alpha_i + \lambda_t + \epsilon_{it} \end{aligned}$$

As usual, β is the time discount factor, which is assumed to be the constant value of 0.99. $i=1 \dots 9$ is the country index representing the nine alphabetical country codes (used by Datastream) for the major stock markets of developed countries other than the US. That is: placeAustralia, placeGermany, placeCanada, placeDenmark, placeSpain, placeFrance, placeItaly, placeSwitzerland and United Kingdom, respectively. While $t=1 \dots 51$ is a quarterly time index starting at 1992:Q2. In this panel data framework, the common constant c_i is a scalar parameter and the parameters $\gamma_{i,1}$ and $\gamma_{i,2}$ are allowed to be heterogeneous. μ_{it} defines an error term that may contain any of following effects: country-specific α_i , period specific λ_t and an idiosyncratic disturbance, ϵ_{it} independently distributed over time and the cross-section with a mean of zero and heterogeneous variance, σ_i^2 .

A virtue of panel data analysis is that it permits a simple specification for heterogeneity either at the country specific level via α_i or the period specific level via λ_t . Investors are different to one another, then consumer behaviour is likely to be heterogeneous (i.e., Constantinides and Duffie, 1996). Also, investors/consumers may be sensitive to trends and fashions suggesting that behaviour may fluctuate over time, and thus, heterogeneity can exist over time. Departing from this idea, it appears that aggregated heterogeneity can outperform investors/consumers heterogeneity in that it is more persistent, yet reduces the measurement errors in regressors, i.e. population. Consequently, the persistence of heterogeneity can be found in panel data with positive correlations, that can be induced either through a dynamic pattern or unobserved variables.

Conditional on data availability fixed effect errors are suitable for a small number of factors while a large sample is required for random effect errors. Therefore, it is likely that α_i and λ_t should be treated as fixed effects and random effects respectively. α_i are all zeros if treated as random effects, while too many λ_t can easily lead to singularity problem in estimated residual correlation

⁶Hunter and Wu (2009) suggest that the degree of non-linearity is quite small.

matrices if treated as fixed effects. Consequently, α_i can capture cross section heterogeneity, which means that it can show the net effect of any unobserved variables on the dependent variable (individual market returns). On the other hand, any latent period heterogeneity that varies over time can be captured by λ_t .⁷

Econometric issues are particularly important in this two-stage regression because Hunter and Wu (2009) suggest that expected excess returns from the US stock market are likely to be measured with errors, leading to biased estimates of these coefficients. Firstly, Two Stage Least Square/Instrumental Variable Estimator (2SLS/IV) IV/2SGLS is required to eliminate the correlation between endogenous regressors and the disturbances. Secondly, Panel Corrected Standard Errors (PCSE), pioneered by Beck and Katz (1995, 1996) is employed to construct robust coefficient covariances for panel corrected residuals.⁸

Thirdly, although there is no obvious reason to believe that regressors are non-stationary such that a panel model has autocorrelated errors, it may suffer from heteroscedastic errors that can be either across individuals or over time periods, therefore, a Generalised Least Squares (GLS) approach is used to generate estimates of the robust coefficient covariances. Analogous to GLS in time series analysis, GLS applications for panel data analysis have four basic variance structures as weights that are conditional on the combinations of i and t : cross-sectional heteroscedasticity, period heteroscedasticity, contemporaneous covariances (cross-section SUR) and period heteroscedasticity and serial correlation (period SUR). For example, the covariance structure of the cross section SUR allows for conditional correlation between contemporaneous residuals for cross section i and j , but restricts residuals in different periods to be uncorrelated.⁹

Lastly, for robust covariance estimation for generated regression, the standard errors of disturbances in panel models should be corrected through cross-sectional residuals that are recalculated by the actual values of the US excess returns,¹⁰ although robust standard errors are still asymptotically consistent in each stage.¹¹ However, in a two step regression model, inferences is calculated using

$$\begin{aligned} & V\left(\hat{\beta}_{\text{F2SGLS}}\right) \\ &= \frac{N}{N-K} \left(\sum X^{*'}QP_{\bar{Z}_i}QX^*\right)^{-1} \left(\sum X^{*'}\hat{\Omega}_IX^*\right) \left(\sum X^{*'}QP_{\bar{Z}_i}QX^*\right)^{-1} \\ &\xrightarrow{a} \frac{N}{N-K} \left(\sum X'QP_{\bar{Z}_i}QX\right)^{-1} \left(\sum X'\hat{\Omega}_IX\right) \left(\sum X'QP_{\bar{Z}_i}QX\right)^{-1}. \quad (3) \end{aligned}$$

⁷Hausman specification tests imply under the null hypothesis that the random effects specifications are not significant.

⁸A key advantage of PCSE is that it takes into account the complexity of cross-sectional error processes while it does not require the data to be contemporaneously or serially uncorrelated, or panel homoscedastic. Thus, PCSE can be used when residuals are nonspherical. PCSE also has better small sample properties due to the block diagonal variance-covariance matrix.

⁹This weighting transformation is named cross section SUR since it only considers the contemporaneous correlations, that resemble the common SUR definition.

¹⁰The literature on whether it is still necessary in generated regression to correct standard error biases induced by generated variables is inconclusive (i.e. see Liang and Zeger, 1986; Hu and Lachin, 2001; Souleles, 2004).

¹¹See Hunter and Wu (2009). The same result is concluded using Lemma 12.1 in Woodridge (2002).

As such, in the second step they are consistent only with the equations regressed on the residuals of the actual variables, not those of generated ones. In other words, due to the incorrect residuals used for the variance matrix, standard errors are still biased compared with the results of models regressed directly on the actual variables. Therefore, the transformation matrix $\Psi = \sum X' \hat{\Omega}_I X$ take the cross-sectional PCSE as an example, needs to be recalculated by $\hat{\Omega}_I = \hat{\epsilon}' \hat{\epsilon} / T \otimes I_T$ rather than by $\hat{\Omega}_I = \tilde{\epsilon}' \tilde{\epsilon} / T \otimes I_T$, where $\hat{\epsilon}$ is a vector of stacked residual series of I cross-section specific regressions with the actual values of US excess returns, where \otimes is the Kronecker Product. The actual values of US excess returns $\hat{\epsilon}$ can be calculated using:

$$\hat{\epsilon}_i = r_i^e - \hat{c}_i - \begin{bmatrix} \hat{\gamma}_{i,1} & \hat{\gamma}_{i,2} \end{bmatrix} \times \begin{bmatrix} cg_{i,t} \\ r_{US,t}^e \end{bmatrix} - \hat{\alpha}_i - \hat{\lambda}_t. \quad (4)$$

By applying a procedure similar to the one used in Hunter and Wu (2009), with:

$$\tilde{\epsilon}_i = r_i^e - \hat{c}_i - \begin{bmatrix} \hat{\gamma}_{i,1} & \hat{\gamma}_{i,2} \end{bmatrix} \times \begin{bmatrix} cg_{i,t} \\ \hat{r}_{US,t}^e \end{bmatrix} - \hat{\alpha}_i - \hat{\lambda}_t. \quad (5)$$

It follows that:

$$\hat{\epsilon}_{i,t} = \tilde{\epsilon}_{i,t} + \hat{\gamma}_{i,2} (\hat{r}_{US,t}^e - r_{US,t}^e). \quad (6)$$

Applying the alternative recalculation of $\hat{\epsilon}$ to compute the contemporaneous equation covariances $\hat{\Omega}_I = \hat{\epsilon}' \hat{\epsilon} / T \otimes I_T$. This gives rise to a bias adjusted coefficient variance-covariance matrix:

$$\begin{aligned} & V_{BC} \left(\hat{\beta}_{F2SGLS} \right) \\ &= \frac{N}{N-K} (X' Q P_{\bar{z}_i} Q X)^{-1} \left(X' \hat{\epsilon}' \hat{\epsilon} / T \otimes I_T X \right) (X' Q P_{\bar{z}_i} Q X)^{-1}. \end{aligned} \quad (7)$$

Biase adjusted standard errors of coefficients can be obtained using:

$$SE_{BC} \left(\hat{\beta}_{F2SGLS} \right) = \text{diag} V_{BC} \left(\hat{\beta}_{F2SGLS} \right). \quad (8)$$

5 Empirical Results

In panel setting, we consider the linear form of the C-CAPM pooling the excess returns for nine individual countries. In the case of the extended two factor models we regress the pooled variable on country specific consumption growth and expected US excess returns. As far as the instrument set is concerned, consumption growth is implicitly explained by 4 lags of country-specific excess returns and rates of consumption growth; expected US excess returns up 2 lags are included as a two step regressor that instruments itself; 2 lagged actual US excess returns are also employed as additional instruments in order to capture potential heteroscedasticity in panel residuals. As the econometric methodology, we deploy Feasible Generalised Two stage Least Squares (FG2SLS).

Dynamic tests chosen here are for autocorrelation, heteroscedasticity and the validity of instruments. The autocorrelation test is the first-order Breusch and Godfrey LM test that operates across all vectors of cross-sectional residuals, while the heteroscedasticity test is an augmented Breusch-Pagan test proposed by Bickel that can take account of both within and between country

heteroscedasticity. The validity of the instrumental variables is tested using a Sargan’s test of overidentifying restrictions. This is a J-statistic that evaluates whether instruments and estimated residuals are orthogonal given the estimated parameters (Arellano and Bond, 1991):

$$J = \hat{\epsilon}' Z \left(\sum Z_i' \hat{\epsilon}_i \hat{\epsilon}_i' Z_i \right)^{-1} Z' \hat{\epsilon} \sim \chi_{p-k}^2. \quad (9)$$

The existence of either autocorrelation or heteroscedasticity in the residuals can lead in the dynamic panel context to biased and inconsistent estimates, and thus models that suffer from these problems are often mis-specified. The Sargan’s test is sensitive to any form of mis-specification, but for models that are otherwise correctly formulated a significant J-statistic suggests that the instruments are invalid.

5.1 Pooled Panel Models with Common Coefficients

The panel model with common coefficients assumes cross section consumption growth has the same coefficient across the panel, and then such a model can be estimated by FG2SLS with cross section Panel Corrected Standard Errors (PCSE) as robust covariances. Further consideration of fixed and random effects yields four different specifications, results of which are reported in Table 4.

The second column of Table 4 depicts the outcome of a panel model without any error component correction. In this case, the model simply stacks all the data over the cross sections, and thus a single variable is regressed with $T \times I$ observations. Nevertheless, the estimates do not demonstrate that such a model can predict the returns of cross section stock markets based on the insignificant coefficient of consumption coefficient even at the 10% level. Further, one may argue that both the $Cov(\hat{r}_{US,t}^e, \mu_{it}) \neq 0$ and $Cov(cg_{i,t}, \mu_{it}) \neq 0$ in the presence of inter-country unobserved heterogeneity. Hence, the next three columns of Table 4 give the results of panel models with cross section fixed effects, period random effects and both kinds of effects, respectively.

Comparing the three coefficient sets, from a theoretical perspective we prefer the results associated with the model with cross section FE, since all three coefficients are significant at the 5% level after correcting for biases in the standard errors.¹² Further, the model also seems well specified on the basis of the dynamic tests for autocorrelation and heteroscedasticity that cannot reject the null, and the validity of the instrument set also cannot be rejected at the 5% level.

The introduction of FE dramatically increases the size of the risk aversion parameter to 4.285, which is consistent with economic theory.¹³ The fixed effect is a country specific intercept that can be interpreted as capturing fixed differences in country-level average excess returns over the sample period. These

¹²The bias correction for the coefficient variance-covariance matrix based upon recalculated residuals suggests the standard errors are adjusted downward in line with the results in Hunter and Wu (2009).

¹³Mehra and Prescott (1985) quote several micro-econometric estimates that bound risk aversion by 3, and they (1988) later clearly chose an upper bound as large as ten merely as a rhetorical flourish. Therefore, it would appear that the restriction that the risk aversion coefficient should be less than ten is more controversial (Kocherlakota, 1996), and an individual with a coefficient of relative risk aversion above ten would be willing to pay unrealistically large amounts to avoid bets (Mankiw and Zeldes, 1991).

fixed effects thus can help control for any time constant omitted variable bias that may influence consumers' decision on expenditure. In this sense, a panel model with country fixed effects can more clearly reflect the risk aversion of consumers across different countries than asset pricing models without the fixed effects, because it captures a component of country specific risk that then does not exonerate, intra-country differences in the pooled estimates of the rate of risk aversion.

The panel model with FE effect suggests that long run stock market behaviour across countries in a fixed way is different. Although the inclusion of a constant drift (-0.382) can also capture some long run effect, it cannot remove all of them when long run averages are heterogeneous as would appear to be the case here. For example, the UK and the Australian stock markets have the largest, negative long run average returns, which are -.018 and -.017, respectively. On the other hand, the average returns of Switzerland and Italy in the long run are respectively .017 and .012. The rest of countries are less variable with long run average returns in the range [-.007,.009].

Although the panel C-CAPM model with only common regressors is simple, it cannot give insight into any unobserved heterogeneity caused by simultaneous correlations, omitted variables, or measurement errors. Given the overall FE heterogeneity of country long run averages in the panel C-CAPM varies dramatically, it is natural to wonder whether country specific rates of risk aversion themselves are heterogeneous. Indeed, analysis of consumption heterogeneity is necessary and obvious since the effect of consumption on returns can be quite complicated, casting doubt on the static linear panel C-CAPM where consumption growth is a common regressor. The idea that consumption behaviour may differ in a non-constant and non-random way would suggest that it ought to be treated differently for each country in the panel.¹⁴

5.2 Pooled Panel Models with Country Specific Consumption Growth Effects

Table 5 presents a selection of panel models with country specific consumption growth coefficients estimated by 2SGLS. For comparison, we use the same sets of weighting and covariance matrices as those applied in Table 4. As far as the dynamic tests are concerned, there is no any first-order autocorrelation and Sargan's test of instrument validity/overidentifying restrictions can also be satisfied. However, rejection of homoscedasticity at the significant level of 10% in all the four specifications in Table 5 reveals that the residuals are heteroscedastic. Further inspection of the residuals by country suggests that there is not any heteroscedasticity up to four orders within each individual series, but cross-sectional heteroscedasticity across the panel.

Not surprisingly, as Beck and Katz (1995) suggest that panel data tends to suffer from non-spherical behaviour in the disturbances caused by the cross-sectional dimension of the problem.¹⁵ If the non-spherical behaviour of the

¹⁴Here, we feel the panel evidence is a useful adjunct to the UK study, suggesting that multifactor models driven by US excess returns are supported in the main. Country specific models are beyond the scope of this study and given the data limits associated with Germany the analysis would not be viable.

¹⁵Panel Heteroscedasticity means that the variance of the error term within a cluster is constant, but it varies across clusters.

disturbances arises purely by virtue of non-time invariant heteroscedasticity, then the usual heteroscedasticity-robust standard errors and test statistics from the pooled addressStreetLeast Square regression can be used (Wooldridge, 2002, pp.178); to this purpose, we use PCSE weights.

One problem that arises from the application of corrected standard errors subject to the generated regressor problem relates to the application of Hausman test to compare the parameters of different fixed and random effects models. As the corrected standard errors may be larger or smaller than the conventional ones, their differential can be negative. When the differential is positive and the variance estimate consistent, then we consider a sequence of Hausman tests coefficient by coefficient. If such differences are deemed significant at the 1% level for more than half the parameters we give preference to the model that has both random and fixed effects. With the exception of cross sectional heteroscedasticity, the final specification is expected to satisfy all the tests of specification: no serial correlation, valid instruments and no correlation in squared country specific auto-correlations. It follows from the results in Table 5 and the results of Hausman Test for the period random effect comparisons in Table 6 that our preferred specification is the model with both country specific fixed effects and period random effects. As can be observed from the Hausman tests a sizeable number of the coefficients differ from the model without random effects.

The model with both country specific fixed effects and period random effects has the only one negative cross sectional consumption growth coefficient, which is as a result not significant. Also, Italy has the largest, positive consumption coefficient of 12.71, which is obviously different from those of other countries. I would appear that the lowest correlation (.43) between the Italian and the US stock markets can explain this highest risk aversion rate. The low correlation may be due to Italian own monetary policies for macroeconomic adjustments, and particularly in the 1990s, the personal consumption expenditure of Italy had more probability of reflecting national economic health, and the Italian stock market. In fact, except Spain and Italy, the model with fixed and random effects has a common excess return response for all countries and different consumption effects ranging between [1.53,5.32], which is again well in line with economic theories.

Both Table 4 and Table 5 reveal the significant and consistent influence (.869 and 1.043, respectively) of the US stock market over the stock markets of other countries, which is not surprising due to the dependency of non-US markets on the US stock market in some forms, i.e. stock prices, returns and/or volatilities. In practice, as interest rates can be used for evaluating riskless assets, financial practitioners may also look for some criteria for assessing performance of their risky investments, and due to the largest capitalisation of the US stock market, it can be treated as the performance benchmark for several reasons. First, more and more international companies are now traded in New York, accelerating the fusion of financial markets, and thus making the US market an efficient mechanism for portfolio and risk diversification. Also, there are more companies, particularly European companies that are running large businesses in North America, so the US consumer's sentiments will inevitably influence their revenues. Third, different industries are gathered to and may play equivalent roles in US, while only some national stock markets are highly sensitive to specific industries, i.e. car manufacturing in Germany. In the era of a globalised economy, shocks affecting these industries may further accelerate stock market

integration. All in all, at least when modelling risk using consumption, this research indicates a key role for the US stock market.

6 Conclusion

This article focuses on a linearised form of the Consumption-based CAPM in a pooled cross section panel model with two-way error components. Specifically, we assert that each country may have its own fixed effects across countries and random effects appear over time. The panel model is designed to extend the time series framework of Hunter and Wu (2009) that explains UK excess returns by UK consumption growth on US predicted excess returns. The panel model covers nine major developed stock markets with quarterly data over the period 1991:1-2004:4. The empirical findings of the panel models based on a range of specifications that capture fixed and random effects all indicate that there is significant heterogeneity and heteroscedasticity but no apparent autocorrelation across the nine countries. In particular, unobserved heterogeneity described by fixed effects offsets the effect of the US stock market. Although the average risk aversion coefficient is 4.285 across the sample, the cross-sectional impact of home consumption growth varies dramatically over the countries observed here, unobserved heterogeneity of which can also be addressed by random effects given the Hausman test statistic and other dynamic test results.

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Table 4 Pooled Panel Models with Common Coefficients

Parameters	Pooled-2SLS	Pooled-2SLS with cross-section FE	Pooled-2SLS with period RE	Pooled-2SLS with cross-section FE and period RE
	Coefficient (P-value)	Coefficient (P-value)	Coefficient (P-value)	Coefficient (P-value)
Constant	-.018(.48)	-.038(.02)**	-.014(.81)	-.024(.43)
\hat{r}_{US}^e	.994(0)***	.869(0)***	1.250(.02)**	1.181(.03)**
Cg_i	.166(.92)	4.285(.04)**	-1.305(.41)	0.573(.81)
Cross-sectional Fixed Effects				
Australia (AU)	-	-0.017	-	-0.003
Germany (BD)	-	0.009	-	0.0005
Canada (CN)	-	-0.007	-	-0.0003
Denmark (DK)	-	0.007	-	0.003
Spain (ES)	-	-0.001	-	0.005
France (FR)	-	-0.002	-	-0.005
Italy (IT)	-	0.012	-	0.002
Switzerland (SW)	-	0.017	-	0.007
United Kingdom(UK)	-	-0.018	-	-0.008
Period Random Effects	S.D(RHO)	S.D(RHO)	S.D(RHO)	S.D(RHO)
Period Random	-	-	.079(.65)	.079(.66)
Idiosyncratic Random	-	-	.057(.35)	.056(.34)
Hausman Test	-	-	$\chi^2(1) = .28(.59)$	$\chi^2(1) = .92(.34)$
Dynastic Tests				
Autocorrelation LM(t-test)	-820(.37)	-1.494(.22)	-827(.36)	-1.069(.30)
Heteroscedasticity (F-test)	2.04(.13)	1.27(.28)	2.921(.05)*	1.46(.23)
Sargan's test (J-statistic)	$\chi^2(11) = 37.5(0)***$	$\chi^2(19) = 29.14(06)^*$	$\chi^2(11) = 16.2(.13)$	$\chi^2(19) = 16.73(.61)$

Notes: 1 The subjective discount factor is restricted to assume the value of $\hat{a} = 0.99$. 2 P-values are given in parenthesis and ***, ** and *: Significant levels of the 1%, 5% and 10%, respectively. 3. Pooled-2SLS and Pooled-2SLS with cross-section FE is estimated by 2SGLS with cross section weights and corrected for PCSE weights; Pooled-2SLS with period RE and Pooled-2SLS with cross-section FE and period RE are estimated by 2SGLS corrected for PCSE weights. 4.The instruments chosen are expected US excess returns, current and lags one and two, 2 lags in actual US excess returns, 4 lags of cross-country excess returns and lagged cross-country real consumption growth rates that can be either common or cross-country specific. 5 Common instrument sets are country-specific excess returns, the expected country-regionplaceUS returns (estimated without dummies) and the individual rates of consumption growth, up to 4 lags. 6. Autocorrelation test is based on first-order Breusch and Godfrey LM test across all vectors of cross-sectional residuals, that is, $LM = \sqrt{NT^2/T-1} \times r \sim \chi^2(1)$ where $r = \sum_i \sum_{t=2} \hat{\epsilon}_{it} \hat{\epsilon}_{i,t-1} / \sum_i \sum_{t=2} \hat{\epsilon}_{it}^2$. 7. The heteroscedasticity test is Bickel's version of the Breusch-Pagan test for the joint-significance of the two power predictions in the pooled equation: $\hat{\epsilon}_{it}^2 = \gamma_0 + \gamma_1 \hat{y}_{it} + \gamma_2 \hat{y}_{it}^2 + \nu_{it}$ 8. Instrument validity is tested using equation (9). 9 The variance-covariance matrix of the estimated coefficients is corrected using equaions (7) and (8).

Table 5 Pooled Panel Models with Consumption Growth as a Cross Section Specific Parameter

Parameters	Pooled-2SLS	Pooled-2SLS with cross-section FE	Pooled-2SLS with period RE	Pooled-2SLS with cross-section FE and period RE
	Coefficient (P-value)	Coefficient (P-value)	Coefficient (P-value)	Coefficient (P-value)
Constant	-.018(.48)	-.038(.02)**	-.014(.81)	-.024(.43)
\hat{r}_{US}^e	.994(0)***	.869(0)***	1.250(.02)**	1.181(.03)**
cg_i	.166(.92)	4.285(.04)**	-1.305(.41)	0.573(.81)
Cross-sectional Fixed Effects				
Australia (AU)	-	-0.017	-	-0.003
Germany (BD)	-	0.009	-	0.0005
Canada (CN)	-	-0.007	-	-0.0003
Denmark (DK)	-	0.007	-	0.003
Spain (ES)	-	-0.001	-	0.005
France (FR)	-	-0.002	-	-0.005
Italy (IT)	-	0.012	-	0.002
Switzerland (SW)	-	0.017	-	0.007
United Kingdom(UK)	-	-0.018	-	-0.008
Period Random Effects				
Period Random	-	-	.079(.65)	.079(.66)
Idiosyncratic Random	-	-	.057(.35)	.056(.34)
Hausman Test	-	-	$\chi^2(1) = .28(.59)$	$\chi^2(1) = .92(.34)$
Dynastic Tests				
Autocorrelation LM(t-test)	-.820(.37)	-1.494(.22)	-.827(.36)	-1.069(.30)
Heteroscedasticity (F-test)	2.04(.13)	1.27(.28)	2.921(.05)*	1.46(.23)
Sargan's test (J-statistic)	$\chi^2(11) = 37.5(0)***$	$\chi^2(19) = 29.14(.06)*$	$\chi^2(11) = 16.2(.13)$	$\chi^2(19) = 16.73(.61)$

Note: see Table 4. The variance-covariance matrix of the estimated coefficients is corrected using equations (7) and (8). Due to robust standard errors that are also corrected for the generated regressor, problem, the Hausman test reports some negative variance differences (var(FE)-var(RE)) and thus cannot correctly calculate Chisq statistic. Therefore, we use a t statistic to test a single consumption growth parameter each time, ignoring the other parameters.(see Table 6)

Table 6 Period Random Effect Comparisons - Hausman Test

Parameters	Pooled-2SLS	Pooled-2SLS with cross-section FE	Pooled-2SLS with period RE	Pooled-2SLS with cross-section FE and period RE
	Coefficient (P-value)	Coefficient (P-value)	Coefficient (P-value)	Coefficient (P-value)
Constant	-.018(.48)	-.038(.02)**	-.014(.81)	-.024(.43)
\hat{r}_{US}^e	.994(0)***	.869(0)***	1.250(.02)**	1.181(.03)**
CG_i	.166(.92)	4.285(.04)**	-1.305(.41)	0.573(.81)
Cross-sectional Fixed Effects				
Australia (AU)	-	-0.017	-	-0.003
Germany (BD)	-	0.009	-	0.0005
Canada (CN)	-	-0.007	-	-0.0003
Denmark (DK)	-	0.007	-	0.003
Spain (ES)	-	-0.001	-	0.005
France (FR)	-	-0.002	-	-0.005
Italy (IT)	-	0.012	-	0.002
Switzerland (SW)	-	0.017	-	0.007
United Kingdom(UK)	-	-0.018	-	-0.008
Period Random Effects				
Period Random	-	-	.079(.65)	.079(.66)
Idiosyncratic Random	-	-	.057(.35)	.056(.34)
Hausman Test	-	-	$\chi^2(1) = .28(.59)$	$\chi^2(1) = .92(.34)$
Dynastic Tests				
Autocorrelation LM(t-test)	-.820(.37)	-1.494(.22)	-.827(.36)	-1.069(.30)
Heteroscedasticity (F-test)	2.04(.13)	1.27(.28)	2.921(.05)*	1.46(.23)
Sargan's test (J-statistic)	$\chi^2(11) = 37.5(0)***$	$\chi^2(19) = 29.14(06)^*$	$\chi^2(11) = 16.2(13)$	$\chi^2(19) = 16.73(61)$