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RASMUS AHVENNIEMI

STOCHASTIC DYNAMIC OPTIMIZATION  
MODELS IN THE BANKING SECTOR

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Helsinki, November 2014

Rasmus Ahvenniemi



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# Chapter 1

## Introduction

### 1.1 Background

Financial crises play an important part in explaining economic recessions. In order to be able to understand and explain economic developments there is therefore reason to account for the financial sector in economic thinking and models. The banking sector is a central part of the financial sector and its condition has a substantial influence on the other parts of the economy. This thesis considers the banking sector in times of economic distress, and intends to shed light on certain related issues with the help of stochastic dynamic models.

A bank's balance sheet is at the center of its operations and, in general, a bank obtains its profits from the difference between the interest rates on its assets and liabilities. There are several reasons as to why a bank has access to cheaper funding than the borrowers to which it lends money. These reasons relate to the financial intermediation services that a bank provides and include, among others, *maturity transformation*, i.e. the transformation of deposits that may be withdrawn on a short notice into loans with long maturities. Also, banks will carry the *credit risk* relating to loans, thus offering depositors a safer way of investing than direct investments into loans (see e.g. Freixas & Rochet 2008 for a more detailed description of a bank's functions). These issues, i.e. maturity transformation and credit risk, are considered in the essays of this thesis, in particular in the context of financial crises.

Banks are considered from two distinct viewpoints in this thesis. The first and the second essays of this thesis consider individual banks as utility-maximizing agents that try to optimize their behavior in a stochastic dynamic environment where financial crises may occur. The third essay takes a broader view and considers the whole banking sector as one entity that is a part of an economy. This change in

focus has substantial importance as the process of money creation in the banking system will then become a central issue. When the banking sector is considered as one aggregated entity, a causal link will appear from lending to funding since the creation of new loans will also increase the total amount of deposits in the banking system. When considering a single bank, this link would be of relatively small significance, in particular if the bank is of small size relative to the rest of the banking sector.

Financial crises relate to banks in two ways. On one hand, financial crises are a cause of potential problems in banks, as credit risks may be realized and the availability of funding may be reduced, leading to losses and a lack of liquidity. This is the focus of the first two essays of this thesis, which consider individual banks. On the other hand, banks may also have a role as amplifiers of crises and recessions. For example, when the economic outlook is worsening, banks, fearing credit risks or illiquidity, may reduce their lending, thereby causing the economic conditions to deteriorate further. The interdependence between the real economy and the banking sector is one aspect of the model presented in the third essay of this thesis.

Central banks are significant players in economies, their decisions affecting e.g. the money supply, inflation, GDP, interest rates, exchange rates etc. However, also commercial banks affect the money supply through lending, since lending creates new deposits, in particular as most money is held in the form of deposits rather than physical cash. Therefore, at least during times when lending is not constrained by regulatory restrictions, also the commercial banking sector is an important determiner of the money supply. This phenomenon is referred to as the *endogeneity of money*, which has been discussed e.g. by Lavoie (1984), Fontana & Venturino (2003), and Arestis & Sawyer (2006). The model presented in the third essay of this thesis considers an economy, in which money is endogenous during certain periods of time.

The first and the second essay of this thesis consider the dynamic optimization of balance sheets of individual banks in situations involving uncertainties regarding the development of the economic environment. In economic literature, microeconomic banking models often consider information asymmetries, as in the paper by Stiglitz & Weiss (1981), and bank runs, as in the paper by Diamond & Dybvig (1983). Microeconomic models considering the optimization of the balance sheet of a bank are rare, while some can be found within the Industrial Organization approach to banking (see e.g. Freixas & Rochet 2008). Klein (1971) presents one of the few models of this type. Within management science literature, on the other hand,



optimization models concerning the balance sheets of individual banks are relatively common, some examples being e.g. the models by Kusy & Ziemba (1986), Booth et al. (1989), and Oğuzsoy & Güven (1997). The model presented in the first essay of this thesis relates to this tradition of models while being more simple as its purpose is not that of solving real-world decision-making problems but rather to give insight into banking-related economic phenomena on a more general level. The second essay of this thesis applies a stochastic dynamic portfolio model to analyzing the behavior of a bank. Whereas the static model by Markowitz (1952) is the most well-known portfolio model, the papers by Samuelson (1969) and Merton (1969) introduced dynamic portfolio models. Models applying dynamic optimization to decision-making problems of banks include e.g. the early model by Daellenbach & Archer (1969) and e.g. the model by Mukuddem-Petersen & Petersen (2006). The model presented in the second essay of this thesis differs from existing models in that it includes maturity mismatch and the risk of a liquidity crisis, i.e. a temporary reduction in the availability of funding.

The third essay of this thesis considers an economy consisting of a banking sector, a central bank, and a real economy experiencing stochastic productivity shocks, which affect the amount of lending carried out by banks. The model could be categorized as a dynamic macroeconomic model, even though it is not an extension of any of the most common types of macroeconomic models. Modern macroeconomic models, i.e. the so called Dynamic Stochastic General Equilibrium (DSGE) models, include the Real Business Cycle (RBC) models, introduced by Kydland & Prescott (1982), and the New Keynesian DSGE models, presented e.g. in the textbook by Galí (2008). These models do not typically consider money explicitly (Galí 2008, p. 34), and most of the models ignore the banking sector altogether. Recently, however, there has been some growing interest in incorporating the banking sector into macroeconomic models, apparently as a reaction to the recent global recession and the financial crisis preceding it. There are now a few DSGE models including a banking sector, such as the models by Aslam & Santoro (2008), Aliaga-Díaz & Olivero (2010), Gerali et al. (2010), Gertler & Karadi (2011), and Benes & Kumhof (2012). Most of the macroeconomic models that involve the actions of a central bank derive the central bank's decisions from some relatively simple decision rule, typically some modification of the so called Taylor rule (see Taylor 1993), while models involving dynamic optimization of central bank policies are very rare. The model presented in the third essay of this thesis differs from modern macroeconomic models in that it includes a banking sector creating money endogenously as well as a central bank which makes decisions that are optimal in accordance with the

principle of dynamic programming.

## 1.2 Methodology

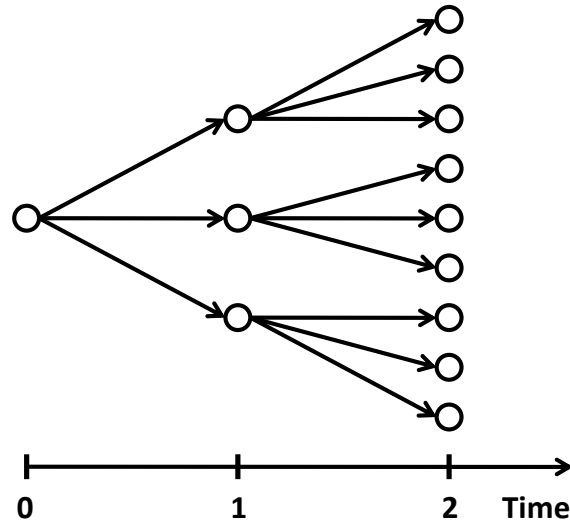
### 1.2.1 Scenario-based multistage stochastic programming

The first essay of this thesis applies a methodology called *scenario-based multistage stochastic programming* to solving the stochastic dynamic decision-making problem considered. It is common to use the shorthand *stochastic programming* when referring to this method and this convention is applied in what follows.

Stochastic programming is a method for finding optimal decisions under uncertainty, when multiple time periods are considered. Stochastic programming models have been proposed for solving problems of decision-making under uncertainty as early as the 1950s, by Dantzig (1955). The stochastic programming methodology is described e.g. by Dupačová (1995). Applications of the stochastic programming approach to financial decision-making include e.g. the banking model presented by Kusy & Ziemba (1986) and the model for fixed-income portfolio management by Zenios et al. (1998). Most of the existing research involving stochastic programming has been published in journals relating to management science and operations research. Economic and financial decision-making problems are typical application areas of the method.

In stochastic programming models the future is modeled with a *scenario tree* (sometimes also referred to as an *event tree*) representing alternative future developments of the decision-maker's environment, e.g. the economy and financial markets. The scenario tree consists of a number of nodes and the transitions between them. The planning horizon consists of a number of time periods. Each node of the scenario tree is followed by one or more child nodes in the subsequent period, representing alternative potential developments. Conditional probabilities are defined for the transitions from each node to its child nodes. An example of a simple scenario tree is depicted in Figure 1.1.

Each node of the scenario tree contains a number of parameters which characterize the scenario. In the case of models representing financial planning problems the set of parameters could include e.g. parameters describing macroeconomic developments (e.g. interest rates) or parameters relating to the assets or liabilities that are being modeled (e.g. return parameters). Each node includes decision variables representing actions that are to be carried out in that particular node. In financial planning problems such decisions may include e.g. changes in the composition of



**Figure 1.1:** An illustration of a simple scenario tree. The circles represent nodes while the arrows represent transitions from nodes to their child nodes. Each node contains a number of parameters and variables.

the portfolio. Stochastic programming models consist of a set of equations which e.g. determine the financial positions and cash flows in each node, and add other constraints to the optimization problem. The equations define an optimization problem, which is solved by a solver program. Stochastic programming problems may be formulated e.g. as linear programming models, and standard optimization software may therefore be applied to solving them efficiently.

### 1.2.2 Stochastic dynamic programming

The second and the third essay of this thesis apply the *stochastic dynamic programming* methodology to solving the stochastic dynamic decision-making problems considered. This approach allows exogenous stochastic shocks to be included in the model as well as the consideration of an infinite time horizon. The variant of the methodology applied is also known as *value function iteration* or *iteration on the value function* and is widely applied in macroeconomic models. It is commonly used e.g. in textbook examples of Real Business Cycle models, and has been described e.g. in the macroeconomics graduate level textbooks by Ljungqvist & Sargent (2004) and McCandless (2008). The state grids of the stochastic dynamic programming models developed in this thesis involve more than one continuous (though discretely modeled) state variables. As a result, the so called *curse of*

*dimensionality* becomes a serious challenge and computational issues need to be considered in order to keep the solution times feasible.

## 1.3 Summaries of the essays

### 1.3.1 A model for analyzing the dynamic behavior of a bank facing a financial crisis

The first essay presents a model for analyzing the optimal dynamic decision-making of a bank. The model considers the development of the bank's balance sheet in a situation involving the risk of a financial crisis which may or may not materialize, and the timing of which is uncertain. Depending on the parameter configurations, the crisis may involve defaulting of loans as well as a reduction in the availability of funding. The model considers the precautionary measures that the bank takes in preparation for the potential crisis as well as the bank's reactions to the crisis, in case it materializes. The bank may dynamically adjust the size and composition of its balance sheet, which includes cash, loans, deposits and equity. The model is formulated as a stochastic programming problem and solved by applying linear programming.

The model is shown to exhibit behavior that appears logical. Observations that can be made based on the outcomes of the model include e.g. the bank's tendency to deleverage its balance sheet, i.e. to reduce its funding and lending, in preparation for an anticipated financial crisis, as well as its tendency to accumulate cash reserves in order to maintain sufficient liquidity in case that a crisis should materialize. Running the model at different parameter configurations also indicates that the more severe the anticipated crisis is in terms of defaulting loans and reductions in the availability of funding, and the longer its duration, the more drastic are the precautionary measures taken in preparation for the crisis, i.e. the more the bank reduces its lending. These actions are taken before it is known to the bank, whether the crisis is going to materialize at all.

The *maturity mismatch*, i.e. the difference between the maturities of lending and funding, is an essential element of the model, and the maturing of loans and deposits taking place in each period is explicitly modeled. Thus, the process of maturity transformation is part of the model. Different maturities could cause problems if the bank's liquidity was suddenly reduced as a result of a financial crisis, since the slowly maturing loans might not provide a sufficient cash flow for repaying the more rapidly maturing deposits. The bank takes the difference in

maturities into account e.g. by holding cash reserves or by not applying too high leverage, when there is a risk of a financial crisis.

### **1.3.2 A stochastic dynamic portfolio model of the balance sheet of a bank**

The second essay presents a portfolio model for analyzing a bank making decisions over time in a stochastic environment. The balance sheet of the bank is assumed to consist of cash, loans, deposits and equity. The bank is assumed to make decisions regarding the amount of new loans given out in each period, thus affecting the allocation of its funds between liquid cash and nonliquid loans. The model involves different maturities of funding and lending, also known as *maturity mismatch*. The model also involves uncertainty regarding the availability of new funding, which is assumed to depend on the stochastically varying state of the economic environment. A reduction in the availability of new funding represents a *liquidity crisis*. The model is solved numerically using stochastic dynamic programming implemented as value function iteration on a state grid. The solution process produces a policy function, which is used as a decision rule in simulations of the model.

Simulations of the model show that some amount of the asset portfolio is allocated to cash even though no investment risk or credit risk is present in the model and even though cash provides zero returns. Simulations conducted with a number of different parameter configurations suggest that maturity mismatch and the risk of a liquidity crisis have a substantial impact on the holding of cash and thus on the asset allocation of the bank. The simulations indicate that in a model of this type, involving no credit risk or investment risk, holding of substantial amounts of cash appears to take place if and only if maturity mismatch and the risk of a liquidity crisis are both present. The outcomes of the model also show that when maturity mismatch is present, the turnover rate of funding is greater than that of lending, which in other words means that in a typical situation the bank is taking in new funding in order to pay back its older, maturing funding.

### **1.3.3 Optimal dynamic central bank policies under endogenous money**

The third essay presents a model of an economy consisting of a central bank, a commercial banking sector, and a real economy experiencing stochastic productivity shocks. A stochastic dynamic programming model is formulated for modeling the policy decisions of the central bank, which dynamically adjusts the size of the

monetary base. It is assumed that reserve requirements may or may not be binding at a given time. When the reserve requirements are binding, money creation is assumed to be determined by the required reserve ratio and the monetary base, in accordance with the money multiplier model. Otherwise, money creation is assumed to be *endogenous*, i.e. to be determined by lending decisions of commercial banks. These lending decisions are, in turn, assumed to be affected by the condition of the real economy and, to some extent, by central bank policies.

Central bank policies are assumed to affect the money supply in two ways. First, by determining the amount of money creation when reserve requirements are binding, and second, by affecting the amount of money creation through other monetary policy transmission channels when reserve requirements are not binding. Increases in the total amount of lending are assumed to affect the real economy positively, acting as a stimulant, whereas decreases are assumed to have the opposite effect. However, changes in lending are assumed to also determine the rate of inflation or deflation by affecting the money supply, since the money supply is assumed to be determined by the total amount of deposits, which in turn is assumed to be determined by the total amount of loans. The objective of the central bank is to keep inflation (or deflation) close to an inflation target over time by expanding or contracting the monetary base. The model is solved numerically using stochastic dynamic programming implemented as value function iteration on a state grid. The solution process produces a policy function, which is used as a decision rule in simulations of the model.

Simulations of the model are carried out in order to analyze model behavior during and after a recession. The focus in the simulations is particularly on how inflation (or deflation) develops over time, assuming that the central bank may carry out expansionary or contractionary policies. With certain parameter configurations the model produces substantial deflation during the recession and substantial inflation after the recession ends. However, parameter configurations resulting in developments approximately resembling those of the developments observed in the euro area during the recent global economic recession do not produce outcomes involving high inflation after the end of the simulated recession. The simulations indicate lower levels of inflation after the recession if the central bank's ability to conduct contractionary monetary policy is higher, and if the effectiveness of central bank policies is higher.

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# Chapter 2

## A model for analyzing the dynamic behavior of a bank facing a financial crisis

### 2.1 Introduction

One of the most central functions of a bank is that of maturity transformation, in which the bank transforms liabilities that have short maturities into assets, the maturities of which are longer. As a result, the turnover rate of the bank's liabilities is higher than that of its assets, i.e. the bank needs to renew its funding at a relatively high frequency. From the perspective of a bank, a financial crisis may involve on one hand the defaulting of some of its lending and on the other hand a difficulty of obtaining new funding as the market may have concerns regarding the solvency of the bank and since other financial institutions may want to hold on to their liquid assets in such a situation. Both of these issues reduce the bank's ability to pay back its maturing liabilities during a financial crisis. As a result, in order for a bank to remain liquid, i.e. being able to repay its maturing liabilities, it needs to adjust its balance sheet as a precautionary measure for the possibility of a financial crisis. This it needs to do even if it does not know if and when the anticipated crisis is going to materialize. In addition, it may need to take actions during the crisis.

This paper presents a model for analyzing the dynamic behavior of a bank and the development of the bank's balance sheet. The model is an optimization model, in which the bank's funding and lending, as well as their different maturities, are modeled explicitly, thereby exhibiting the process of maturity transformation. The model is applied to analyzing the bank's actions before and during a financial crisis in a situation where it is uncertain when the crisis is going to start and whether

it is going to materialize at all. These uncertainties have similarities with the ones concerning the subprime mortgage crisis of 2007–2008, as there were some early indicators of its possibility (e.g. the *Economist* warned about the housing bubble as early as May 2003) but it appeared to be uncertain if and when the potential crisis would materialize.

Much of the literature in economics relating to crises in banking considers bank runs, such as the famous paper by Diamond & Dybvig (1983) and contagious bank runs, i.e. bank panics, such as the papers e.g. by Bougheas (1999), Allen & Gale (2000), and Dasgupta (2004). There are also some simulation-based models, which concern financial crises from the viewpoint of systemic risk and financial contagion in the interbank markets. These include e.g. the models by Iori et al. (2006) and Ladley (2010).

Microeconomic models focusing on the optimization of a bank's balance sheet are rare. Some models of this kind may be found within the Industrial Organization approach to banking (see e.g. Freixas & Rochet 2008). One model along these lines is presented by Klein (1971). In contrast, in the field of management science there is a substantial amount of literature regarding the modeling of individual banks and their balance sheets. Many of these models are so called Asset and Liability Management (ALM) models based on the methodology of *scenario-based multistage stochastic programming* (see e.g. Dupačová 1995). Such models are presented e.g. by Kusy & Ziemba (1986), Booth et al. (1989), Oğuzsoy & Güven (1997), and Mulvey & Shetty (2004). This is also the approach applied in this essay.

The methodology as well as the dynamic approach to the management of the balance sheet of the modeled bank differentiate the model presented here from typical banking models in the field of economics. The methodology allows for a detailed analysis of the dynamics relating to the bank's optimal adjustments of its balance sheet. On the other hand, compared to existing management science literature involving stochastic programming ALM models, the main difference is that instead of presenting a model for the purpose of financial planning in a bank, the focus here is on developing a model for getting general-level, economic insight into the actions of a bank facing a financial crisis. Therefore, compared to similar models in the field of management science, the amount of parameters is relatively low and the model structure is relatively simple. The simplified model structure allows for increased transparency of the functioning of the model and makes it possible to draw economic conclusions from the outcomes. Another difference to typical stochastic programming models is the different structure of the scenario tree applied. In comparison to many stochastic programming models, the scenario

tree structure applied here is very simple, as it involves a very small number of points where the scenario tree diverges into more than one subsequent scenarios. This allows for a large number of decision-making periods whereas the number of alternative economic scenarios is modest. As a result of the larger number of periods, shorter period lengths may be applied, which in turn allows for more accurate modeling of decisions relating to lending and funding happening on a relatively short time-scale.

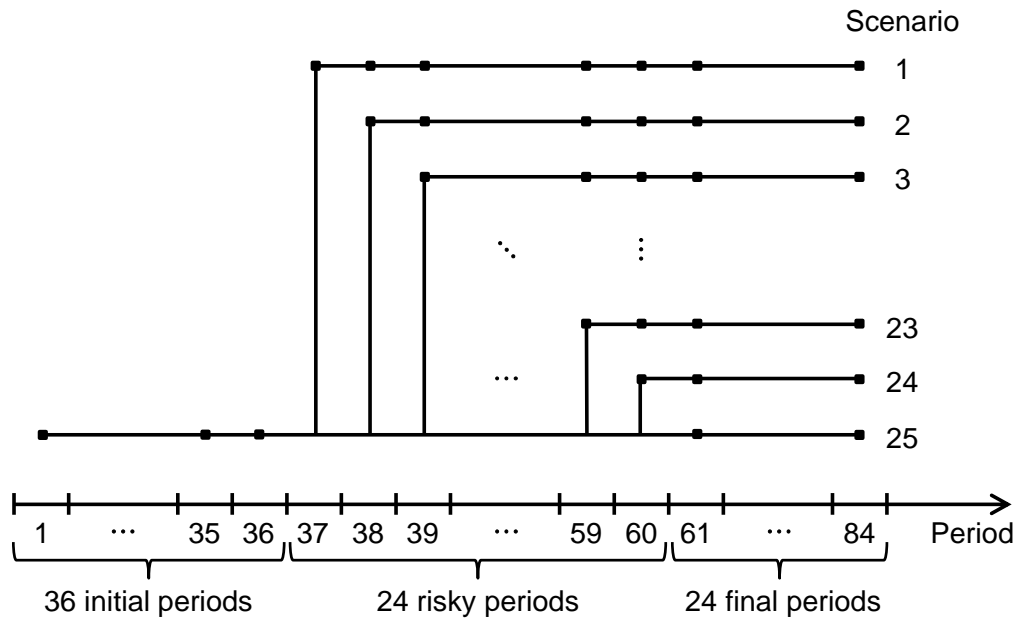
The purpose of this paper is to present a model for analyzing the dynamic behavior of a bank, particularly in a situation involving the risk of a financial crisis, and to confirm that the model behaves logically. The model has potential for being applied to the analysis of a wider range of crises, scenario structures and parameter configurations than the ones considered here. The model is presented in Section 2.2. Results based on the model are presented in Section 2.3. The outcomes of the model are analyzed in the case of financial crises of three different types in Section 2.3.1. First, a crisis involving defaulting loans is considered. Then, a liquidity crisis is considered, i.e. a crisis where the bank has difficulties obtaining new funding from the market. Third, these two basic types of crises are combined by considering a liquidity crisis involving defaulting loans. Section 2.3.2 considers the impact that different parameter configurations have on the bank's behavior before a financial crisis starts. This involves varying the type, severity and duration of the crisis. Conclusions are presented in Section 2.4.

## 2.2 Model

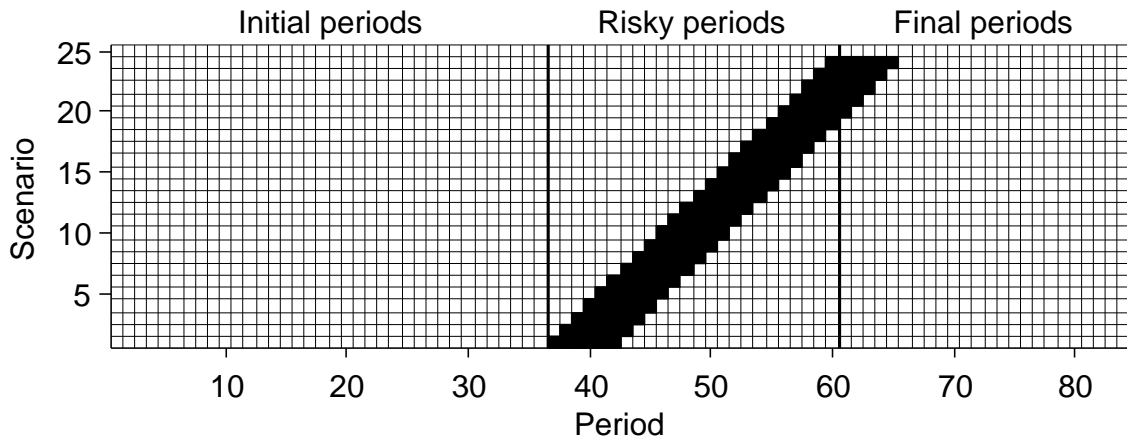
### 2.2.1 Assumptions

The methodology applied in the model is *scenario-based multistage stochastic programming* (see e.g. Dupačová 1995). The model is formulated as a linear programming model. The model differs from typical stochastic programming models in that the total number of scenarios in the scenario tree is very small relative to the number of periods. The planning horizon considered is divided into a large number of periods in order to allow for a high resolution in the modeling of the bank's dynamic behavior.

Figure 2.1 presents the scenario tree structure applied in the model. The total number of periods in the model is 84, and each period represents the time span of one month. The first 36 periods, i.e. the *initial periods* are spent in preparation for the crisis, and it is assumed, that there is no stochasticity during these periods.



**Figure 2.1:** The scenario tree structure of the model.



**Figure 2.2:** The scenarios and periods of the model. There are 84 time periods and 25 scenarios. The boxes painted in black represent those scenario-period combinations where there is a financial crisis going on. The financial crisis lasts for 6 consecutive periods. Scenario 25 represents the development in which the financial crisis does not materialize at all.

**Table 2.1:** The initial balance sheet of the bank.

Assets		Liabilities & Capital	
Cash	0	Deposits	90
Loans	100	Equity	10

The next 24 periods, i.e. periods 37–60, are *risky periods*, during which the crisis may or may not start. Once the crisis has started, it is assumed that the bank knows how the crisis is going to continue in that particular scenario. The 24 *final periods*, i.e. periods 61–84, constitute a phase during which the crisis will not start anymore, but may continue, if it has started in one of the last risky periods. These last 24 periods do not involve stochasticity.

There are two issues involving uncertainty in the model. First, it is uncertain if the crisis will materialize at all, and second, it is uncertain at what point in time the crisis starts if it materializes. There is a total of 25 scenarios in the model representing different alternative developments. Scenarios 1–24 represent the developments in which a crisis starts in periods 37–60, respectively. Scenario 25 represents the outcome where the crisis does not materialize at all. In case that the crisis materializes, it is assumed to last for 6 consecutive periods. Figure 2.2 depicts the scenario-period combinations of the model and indicates which of these combinations involve a financial crisis.

The assumptions regarding the scenario tree structure are of course simplifications. In reality, e.g. a bank in a financial crisis would not know for sure how the crisis would be going to continue. In the context of analyzing the problem at hand, such simplifications may, however, be considered appropriate, as the focus is on getting an overview of the precautionary measures taken by a bank in preparation for an uncertain crisis, as well as its reactions to a crisis of a specified kind. A simple scenario tree structure also improves the transparency of the model. Many of the simplifying assumptions of the model, such as those relating to the scenario tree structure, could be relaxed in future extensions of the model.

The asset side of the bank’s balance sheet is assumed to consist of liquid assets (“cash”), and nonliquid assets (“loans”). The liabilities of the bank consists of funding (“deposits”), which is assumed to include, in addition to actual deposits, also a number of other forms of funding, such as borrowing from other financial institutions. In addition, the balance sheet contains capital (“equity”). While being a simplification, the balance sheet applied in the model is sufficient for capturing some of the most relevant aspects of the functioning of a bank, and for analyzing some

of the most relevant decisions of the bank. These decisions include the allocation of the bank's funds between liquid assets and nonliquid assets, as well as decisions regarding the amount of funding, which adjusts the size of the bank's balance sheet. The maturities of the bank's loans are longer than those of its deposits, exhibiting the bank's function of maturity transformation, which is a central element of the model. The amounts of each balance sheet item in the initial balance sheet are presented in Table 2.1.

It is implicitly assumed that the bank is of relatively small size, since in the case of a bank with a large market share, the amount of lending would have some effect on the amount of deposits, as a result of money creation, and this effect is not included in the model considered here. It should also be noted, that whereas the bank's ability to make decisions regarding the amount of new "deposits" that it obtains may at first sight appear as a strange property, reasonable explanations may be given for it. First, as noted above, the balance sheet item "deposits" does not only consist of actual deposits but is assumed to also contain other forms of funding, such as borrowing from other financial institutions. Second, a bank may have some means of restricting the amount of deposits that it takes in, thus controlling its leverage. Third, a bank may attempt to attract new depositors from competitors and from other markets (e.g. other geographical areas). For these reasons, the bank has been given a relatively large amount of freedom in making decisions regarding the amount of funding it takes in, and thus, in determining its amount of leverage.

### 2.2.2 Equations

Tables 2.2 and 2.3 list the parameters and variables of the model, respectively. The indices  $i = 1, \dots, I$  and  $t = 1, \dots, T$  refer to the scenarios and periods of the model, respectively. The values of parameters  $d_L^{i,t}$  and  $m_D^{i,t}$  define the economic environment in given scenarios ( $i$ ) at given points in time ( $t$ ) by determining the amounts of defaulting loans and the availability of new funding. During a crisis involving defaulting loans, the parameter  $d_L^{i,t}$  obtains positive values. Similarly, during a liquidity crisis, the parameter  $m_D^{i,t}$  obtains values that are small or zero. Both of these aspects are present when a liquidity crisis involving defaulting loans is considered.

The bank is assumed to maximize its expected final-period utility. The bank's utility is assumed to equal its scenario-specific amount of equity and thus the bank is assumed to be risk-neutral. Therefore, the corresponding objective function, presented in Equation 2.1 is defined as the expected amount of equity in the final period.



**Table 2.2:** The parameters of the model. The values of parameters  $d_L^{i,t}$  and  $m_D^{i,t}$  presented in the table represent a situation, where there is no financial crisis going on, and they will obtain different values during financial crises. The length of a period is one month.

Parameter	Value	Description
$I$	25	Number of scenarios
$T$	84	Number of periods
$T_1$	36	Number of initial periods
$C^{i,0}$	0	Initial amount of cash
$L^{i,0}$	100	Initial amount of loans
$D^{i,0}$	90	Initial amount of deposits
$E^{i,0}$	10	Initial amount of equity
$r_L$	0.003274	Interest on lending per period (4 % per year)
$r_D$	0.001652	Interest on deposits per period (2 % per year)
$\tau_L$	24	Maturity of loans (periods)
$\tau_D$	12	Maturity of deposits (periods)
$d_L^{i,t}$	0	Ratio: defaulting maturing loans / all maturing loans
$m_D^{i,t}$	0.5	Ratio: maximum amount of new deposits / amount of equity
$p^1, \dots, p^{24}$	0.5/24	Probabilities of the scenarios involving a financial crisis
$p^{25}$	0.5	Probability of the scenario involving no financial crisis

**Table 2.3:** The variables of the model.

Variable	Description
$C^{i,t}$	Cash
$L^{i,t}$	Loans
$D^{i,t}$	Deposits
$E^{i,t}$	Equity
$\Delta_L^{i,t}$	New loans
$\Delta_D^{i,t}$	New deposits
$M_L^{i,t}$	Loans maturing
$M_D^{i,t}$	Deposits maturing

Equations 2.2 and 2.3 define how the amounts of loans and deposits develop. They are decreased by the amounts maturing and increased by the amounts of new loans or new deposits. Equation 2.4 defines the amount of equity based on equity in the previous period and the profits obtained in the current period. Profits depend on the interest income on loans, the interest costs relating to deposits, and the losses from defaulting loans. Equation 2.5 states that the balance sheet identity holds, i.e. that assets and their funding are equal. The balance sheet identity determines the amount of cash in the balance sheet as it is implied by the amounts of loans, deposits and equity.

It is assumed that the principals relating to loans and deposits are paid back at maturity. The maturities of loans and deposits are assumed to differ from each other, which reflects the bank's function of maturity transformation. It is implicitly assumed that the loans initially in the balance sheet are composed of equal amounts of securities of each maturity, thus determining the amounts maturing in periods  $t = 1, \dots, \tau_L$ . The amounts of deposits maturing in periods  $t = 1, \dots, \tau_D$  are determined similarly by the deposits initially in the balance sheet. Equations 2.6 and 2.8 reflect these assumptions. Equations 2.7 and 2.9 determine the amounts of loans and deposits maturing in periods in which all the securities held initially have already matured. Therefore, in these periods, the amounts of loans and deposits maturing are determined by the amounts of new loans given out and new deposits taken in  $\tau_L$  and  $\tau_D$  periods earlier, respectively.

The bank's ability to obtain new funding in the form of deposits is defined by Equation 2.10. It is assumed that the maximum amount of new deposits available depends linearly on the amount of equity that the bank has at the time. On one hand, this assumption represents, as a proxy of the size of the bank, its ability to attract depositors, and on the other hand it represents, in the form of a measure of solvency, its ability to obtain funding from other financial institutions.

There are also so called *nonanticipativity constraints* (see e.g. Dupačová 1995 or Mulvey & Shetty 2004), which define the structure of the scenario tree by equating the variables of those scenarios, which have a common past. Intuitively, these constraints can be interpreted as restricting the knowledge that the decision-maker has regarding the future developments of the economic environment by making it unknown, which transition in the scenario tree is going to take place in the following period. The nonanticipativity constraints fixing the scenario tree structure shown in Figure 2.1 are defined by Equation 2.11. The nonanticipativity constraints state that before the crisis begins, it is impossible to distinguish a given scenario from the scenario where the financial crisis does not materialize at all. This also implies

that scenarios, in which the crisis has not yet begun in a given period, cannot be distinguished from each other. Before the crisis starts in a given scenario, also the parameters defining the economic environment are equal to those in the scenario where the crisis does not materialize at all. The nonanticipativity constraints correspond to Figure 2.2, which shows the combinations of scenarios and periods where there is a financial crisis going on. It should be noted that in the case of some variables, the nonanticipativity constraints are redundant, since some of the variables of the model are determined by others.

Finally, the *nonnegativity constraints*, presented in Equation 2.12, determine which of the variables may only obtain nonnegative values. The only variable to which these restrictions are not applied is equity, as the equity of a bank could, in principle, become negative if the value of the bank's assets fell below the value of its liabilities. All other balance sheet items are assumed to be nonnegative. Also the amounts of new loans, new deposits, maturing loans, and maturing deposits are clearly nonnegative.

**The objective function:**

$$\max \sum_{i=1}^I p^i E^{i,T} \quad (2.1)$$

**The constraints defining the positions:**

$$L^{i,t} = L^{i,t-1} - M_L^{i,t} + \Delta_L^{i,t}, \quad i = 1, \dots, I, \quad t = 1, \dots, T \quad (2.2)$$

$$D^{i,t} = D^{i,t-1} - M_D^{i,t} + \Delta_D^{i,t}, \quad i = 1, \dots, I, \quad t = 1, \dots, T \quad (2.3)$$

$$E^{i,t} = E^{i,t-1} + r_L L^{i,t-1} - r_D D^{i,t-1} - d_L^{i,t} M_L^{i,t}, \quad i = 1, \dots, I, \quad t = 1, \dots, T \quad (2.4)$$

**The constraints defining the balance sheet identity:**

$$C^{i,t} + L^{i,t} = D^{i,t} + E^{i,t}, \quad i = 1, \dots, I, \quad t = 1, \dots, T \quad (2.5)$$

The constraints defining the amounts of maturing loans and deposits:

$$M_L^{i,t} = L^{i,0}/\tau_L, \quad i = 1, \dots, I, \quad t = 1, \dots, \tau_L \quad (2.6)$$

$$M_L^{i,t} = \Delta_L^{i,t-\tau_L}, \quad i = 1, \dots, I, \quad t = (\tau_L + 1), \dots, T \quad (2.7)$$

$$M_D^{i,t} = D^{i,0}/\tau_D, \quad i = 1, \dots, I, \quad t = 1, \dots, \tau_D \quad (2.8)$$

$$M_D^{i,t} = \Delta_D^{i,t-\tau_D}, \quad i = 1, \dots, I, \quad t = (\tau_D + 1), \dots, T \quad (2.9)$$

The constraints defining the maximum amount of new deposits:

$$\Delta_D^{i,t} \leq m_D^{i,t} E^{i,t}, \quad i = 1, \dots, I, \quad t = 1, \dots, T \quad (2.10)$$

The nonanticipativity constraints:

$$\begin{bmatrix} C^{i,t} \\ L^{i,t} \\ D^{i,t} \\ E^{i,t} \\ \Delta_L^{i,t} \\ \Delta_D^{i,t} \\ M_L^{i,t} \\ M_D^{i,t} \end{bmatrix} = \begin{bmatrix} C^{I,t} \\ L^{I,t} \\ D^{I,t} \\ E^{I,t} \\ \Delta_L^{I,t} \\ \Delta_D^{I,t} \\ M_L^{I,t} \\ M_D^{I,t} \end{bmatrix}, \quad i = 1, \dots, I-1, \quad t = 1, \dots, T_1 + i - 1 \quad (2.11)$$

The nonnegativity constraints:

$$\begin{aligned} C^{i,t} &\geq 0, & L^{i,t} &\geq 0, & D^{i,t} &\geq 0, \\ \Delta_L^{i,t} &\geq 0, & \Delta_D^{i,t} &\geq 0, & & \\ M_L^{i,t} &\geq 0, & M_D^{i,t} &\geq 0, & & \end{aligned} \quad i = 1, \dots, I, \quad t = 1, \dots, T \quad (2.12)$$

## 2.3 Results

### 2.3.1 Dynamic behavior of the bank

#### Case: Defaulting loans

The first type of financial crisis considered is one during which some of the bank's loans default at maturity, whereas it is assumed that there is no liquidity crisis going on, i.e. the bank's ability to obtain new funding is not reduced. It is assumed that during the crisis, 25% of the loans maturing in each period default, i.e.  $d_L^{i,t} = 0.25$ , while the other values of the parameters presented in Table 2.2 remain unchanged.

Figure 2.3(a) depicts how the amounts of each balance sheet item develop over the planning horizon in case of the middle scenario, i.e. Scenario 12, in which the financial crisis starts in period 48. Figure 2.3(b), on the other hand depicts the developments in the case that the financial crisis does not materialize at all. It can be seen, that the two scenarios are identical before period 48, i.e. before the crisis starts. Figures 2.4(a) and 2.4(b) depict how cash and loans, respectively, develop over the planning horizon in all the 25 scenarios of the model. The corresponding developments of deposits and equity are depicted in Figures 2.5(a) and 2.5(b), respectively. The following list presents the observations that can be made, and their intuitive explanations.

- The bank starts to reduce its lending before the potential crisis begins while at the same time obtaining less deposits. This deleveraging is carried out because it reduces the losses from the loans that would be maturing and defaulting during the crisis, if it materialized.
- At some point in time (approximately when the risky periods begin), the bank starts to accumulate cash. However, a few periods further, or in the event that the crisis begins, the bank transfers this accumulated cash into new loans. The explanation for this is that these new loans would be maturing only after the end of the potential crisis, and would thus not result in losses from defaults, but they are a source of interest income, and thus a profitable investment.
- When the crisis starts, the bank's loans and deposits are reduced in most scenarios, since the losses it takes from defaulting loans reduce its total assets and equity, and the reduction in equity also reduces its ability to obtain new funding. However, this does not happen if the crisis starts relatively late, since in that case the precautionary measures taken earlier will have reduced

the amount of loans that are going to mature during the crisis, thus reducing the potential for defaults.

### **Case: Liquidity crisis**

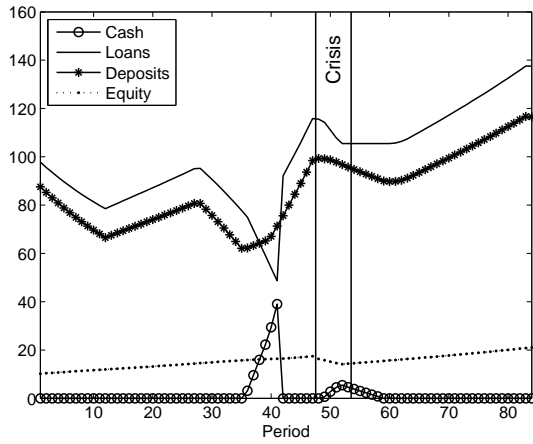
The second kind of crisis considered is a liquidity crisis, in which the bank's ability to obtain funding (deposits) is drastically reduced, whereas the crisis considered here does not involve defaulting loans. It is assumed that during the crisis  $m_D^{i,t} = 0.1$ , while the other values of the parameters presented in Table 2.2 remain unchanged.

Figures 2.3(c) and 2.3(d) depict the development of the bank's balance sheet in the middle scenario and in the scenario in which the financial crisis does not materialize at all. Figures 2.4(c) and 2.4(d) depict how cash and loans, respectively, develop over the planning horizon in all the 25 scenarios of the model. The corresponding developments of deposits and equity are depicted in Figures 2.5(c) and 2.5(d), respectively. The following list presents the observations that can be made, and their intuitive explanations.

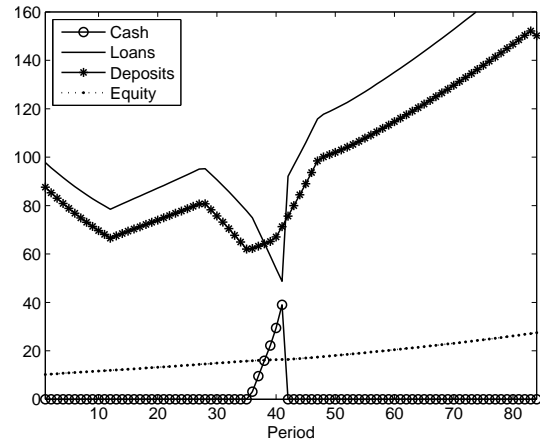
- Before the potential crisis begins, the bank reduces its lending, and instead starts to accumulate cash. It needs the extra cash in order to be able to repay the deposits that would be maturing during the potential crisis. Loans, having longer maturities than deposits, mature too slowly in order to provide a sufficient inflow of cash during a liquidity crisis, while at the same time the bank's ability to obtain new funding is reduced. By accumulating cash, the bank can remain sufficiently prepared for the potential liquidity crisis, and thus ensure that it can pay back its deposits when they mature.
- During the liquidity crisis, the bank reduces its lending even further. When new funding becomes difficult to obtain, the bank cannot anymore afford to reinvest the cash flow that it obtains from maturing loans, as it needs this cash flow for repaying funding that has been obtained in earlier periods and is maturing during the liquidity crisis.
- When there is not anymore a risk of the crisis materializing, the bank ceases to hold cash in its balance sheet, investing all in loans, thus obtaining profits from interest income.

### **Case: Liquidity crisis with defaulting loans**

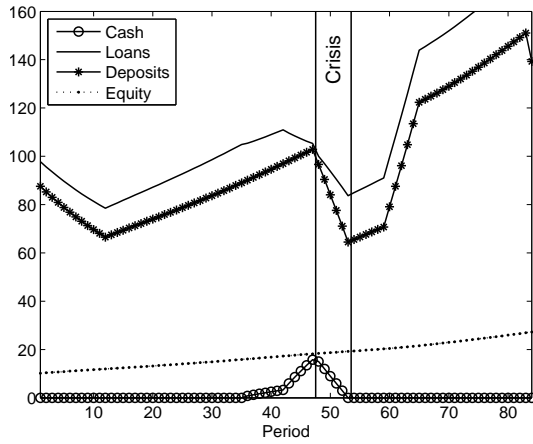
The third kind of crisis considered is a combination of the two types of crises considered above, i.e. a combination of a liquidity crisis and a crisis involving defaulting



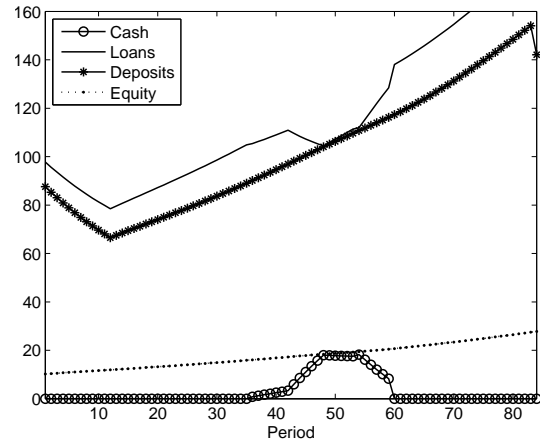
(a) Defaulting loans: Middle scenario



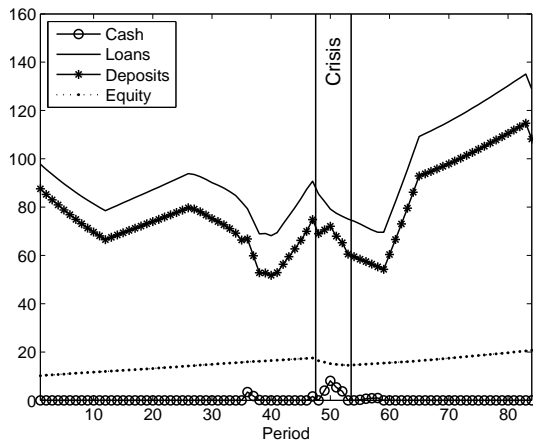
(b) Defaulting loans: No crisis materializes



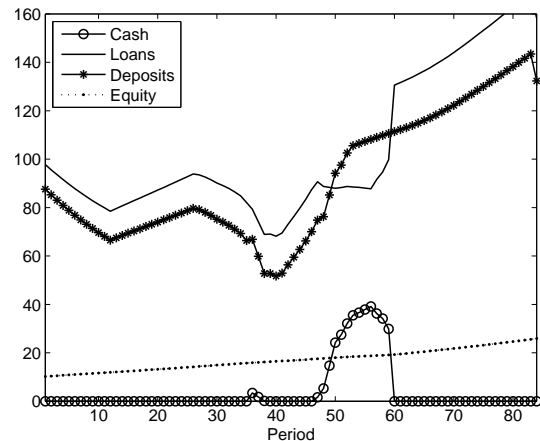
(c) Liquidity crisis: Middle scenario



(d) Liquidity crisis: No crisis materializes

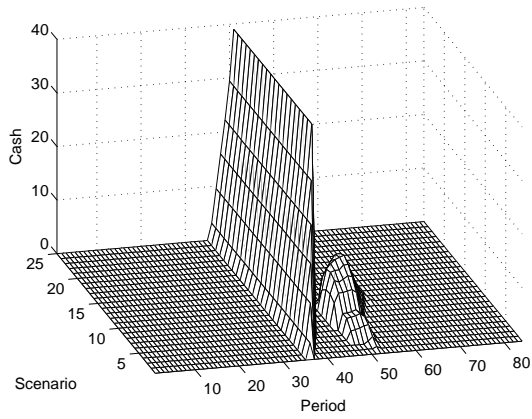


(e) Liquidity crisis with defaulting loans: Middle scenario

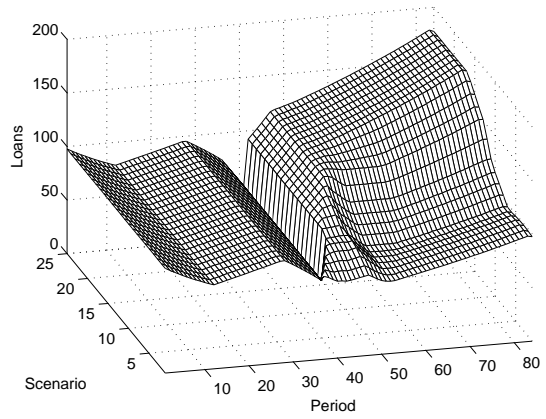


(f) Liquidity crisis with defaulting loans: No crisis materializes

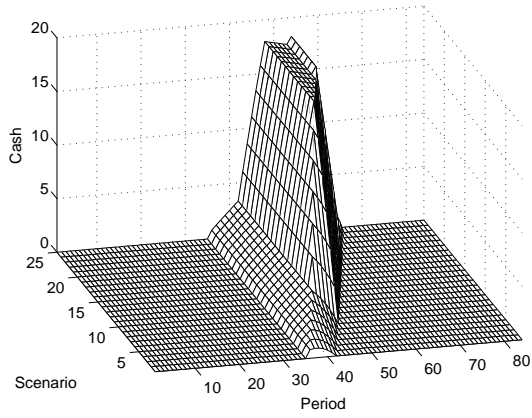
**Figure 2.3:** Development of the balance sheet in the three types of crises considered. Panels (a), (c), and (e) depict the middle scenario (Scenario 12), where the crisis begins in the middle of the risky phase (in period 48). Panels (b), (d), and (f) depict the scenario where the crisis does not materialize at all (Scenario 25).



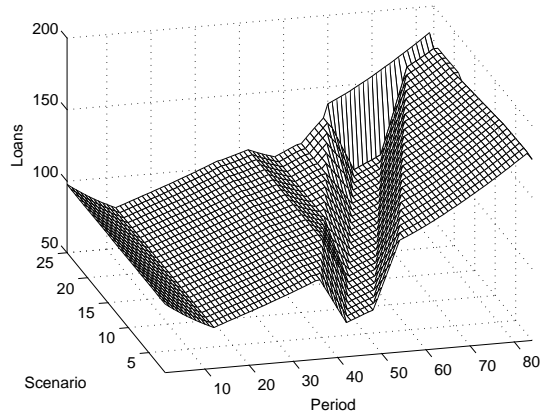
(a) Defaulting loans: Amount of cash



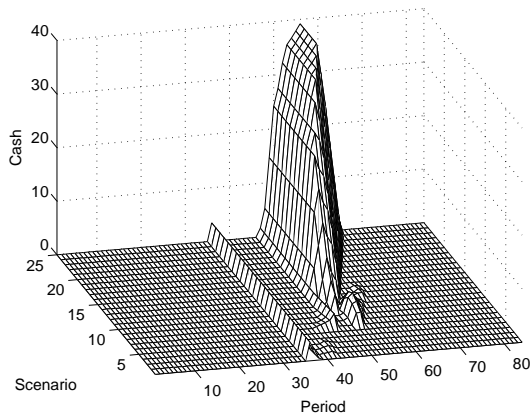
(b) Defaulting loans: Amount of loans



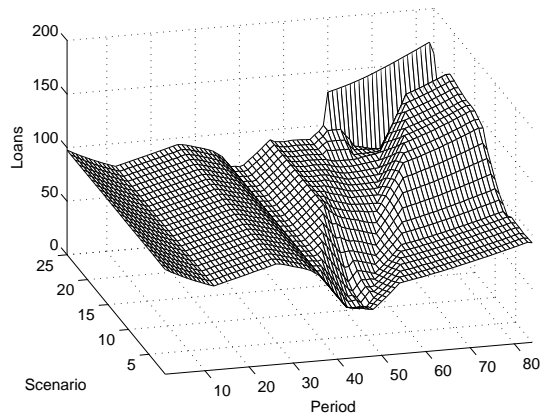
(c) Liquidity crisis: Amount of cash



(d) Liquidity crisis: Amount of loans



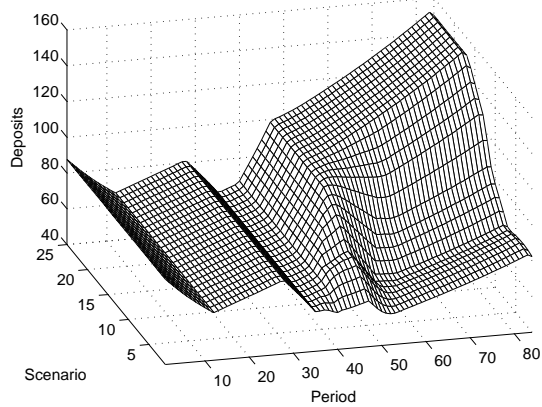
(e) Liquidity crisis with defaulting loans: Amount of cash



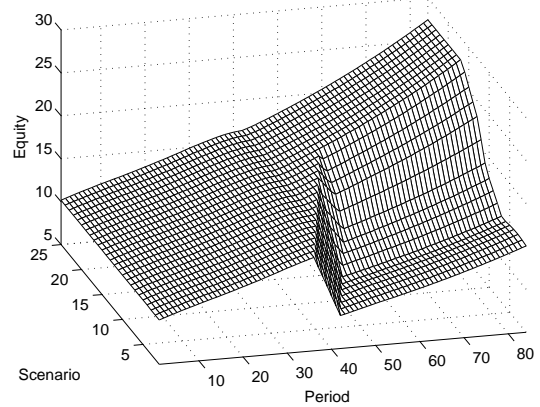
(f) Liquidity crisis with defaulting loans: Amount of loans

**Figure 2.4:** Development of the amounts of cash and loans in the three types of crises considered.

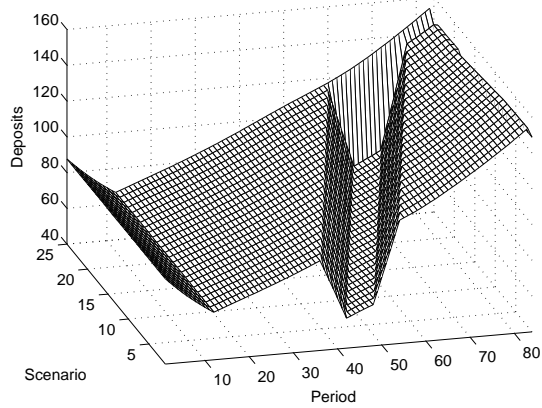




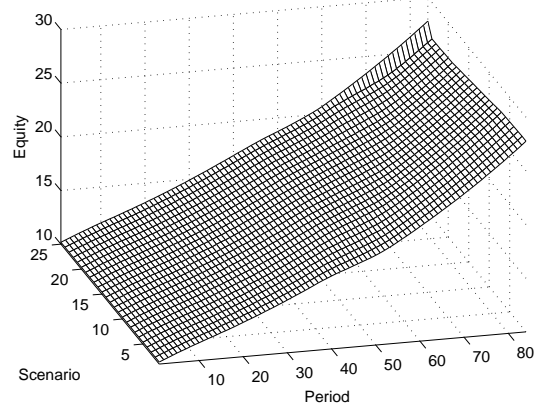
(a) Defaulting loans: Amount of deposits



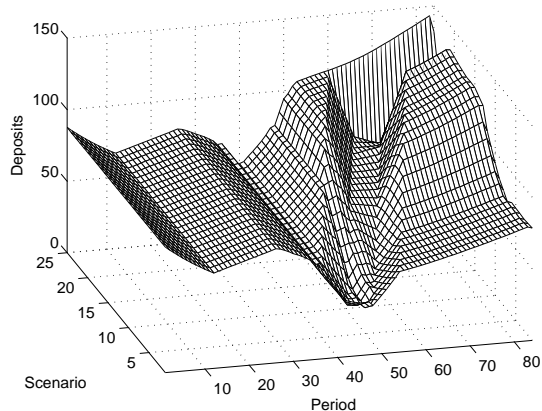
(b) Defaulting loans: Amount of equity



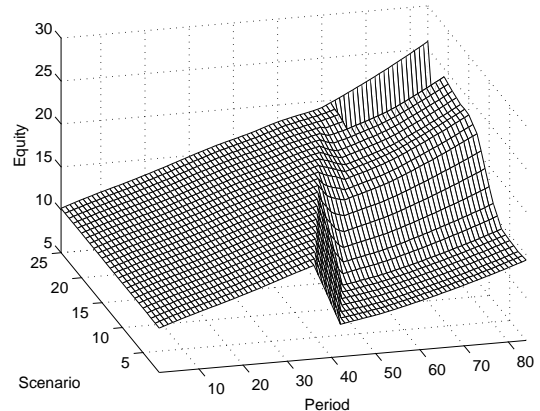
(c) Liquidity crisis: Amount of deposits



(d) Liquidity crisis: Amount of equity



(e) Liquidity crisis with defaulting loans: Amount of deposits



(f) Liquidity crisis with defaulting loans: Amount of equity

**Figure 2.5:** Development of the amounts of deposits and equity in the three types of crises considered.

loans. It is thus assumed, that during the crisis, 25% of the loans maturing default while at the same time the bank's ability to obtain new funding is reduced. Thus,  $d_L^{i,t} = 0.25$  and  $m_D^{i,t} = 0.1$  during the crisis.

Figures 2.3(e) and 2.3(f) depict the development of the bank's balance sheet in the middle scenario and in the scenario in which the financial crisis does not materialize at all. Figures 2.4(e) and 2.4(f) depict how cash and loans, respectively, develop over the planning horizon in all the 25 scenarios of the model. The corresponding developments of deposits and equity are depicted in Figures 2.5(e) and 2.5(f), respectively. The following list presents the observations that can be made, and their intuitive explanations.

- The bank's behavior in case of the crisis considered here appears, by visual inspection, to be similar to a combination of its behavior in the two basic types of crisis considered above. However, this observation holds only approximately, and can be made more clearly in the case of loans, deposits and equity than in the case of cash.
- As in the case of the crisis involving defaulting loans, the bank starts deleveraging before the potential crisis begins, in order to reduce the losses potentially resulting from defaults. After some time, loans and deposits are increased again, because the new loans would be maturing only after the end of the potential crisis.
- During the crisis, the bank's balance sheet shrinks, as it becomes difficult to obtain new funding. Also the losses due to defaulting of loans contribute to the shrinking of the balance sheet.

### 2.3.2 Varying the parameters defining the crisis

The bank's behavior before and during the financial crisis is affected by the type of crisis considered. In this section, the values of the parameters defining the type, severity and duration of the anticipated crisis are varied, and the implications on the bank's dynamic behavior are considered.

The surface plots in Figure 2.6 depict the development of the amount of loans in the bank's balance sheet in the scenario where the crisis does not materialize at all. The development is depicted at different values of the parameters determining the type, severity and duration of the crisis. While these plots consider only the case where no crisis materializes, they nonetheless provide insight into how the bank

prepares for the potential crisis in case of different parameter configurations. Based on Figure 2.6, the following observations can be made.

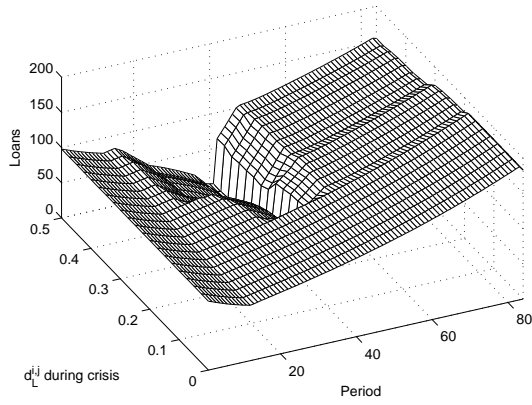
- The more defaulting of loans the anticipated crisis involves, the more the bank reduces its lending as a precautionary measure
- The more severe the reduction in funding during the anticipated crisis is, the more the bank reduces its lending as a precautionary measure
- The longer the anticipated crisis is expected to last, the more the bank reduces its lending as a precautionary measure
- Precautionary measures in the form of reduced lending are taken only if the severity of the anticipated crisis is above some threshold

## 2.4 Conclusions

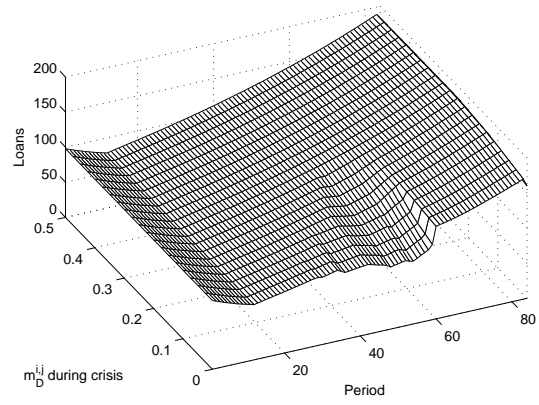
This paper presents a model, the purpose of which is to analyze the optimal dynamic behavior of a bank in an environment that involves uncertainties. The model sheds light on how a bank engaging in maturity transformation adjusts its balance sheet before and during a financial crisis. The scenario tree structure, as well as a number of parameters that are specified for each scenario and period, define the uncertainties involved. In this paper, there is assumed to be uncertainty as to when the crisis is going to start and also as to whether it is going to materialize at all. While the sources of uncertainty considered are relatively simple, the model could also be applied with more complex scenario tree structures.

Simplifications had to be made in order to maintain sufficient transparency of the model and to be able to draw conclusions regarding the impacts of different parameters. Despite the simplifications, the model includes features that are very central to the banking firm. These features include e.g. maturity transformation, the adjustment of the size of the bank's balance sheet, and the adjustment of the proportion of liquid assets in the balance sheet.

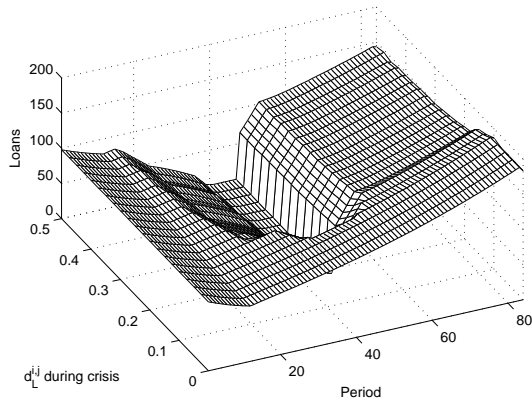
The outcomes of the model show that the bank takes actions in order to prepare for the anticipated financial crisis even though it is uncertain whether the crisis is going to materialize at all. These actions illustrate the bank's optimal dynamic behavior subject to a number of constraints that e.g. force the bank to keep its balance sheet in such a condition that it will be able to repay its maturing deposits under all the alternative developments considered. The analysis considers financial



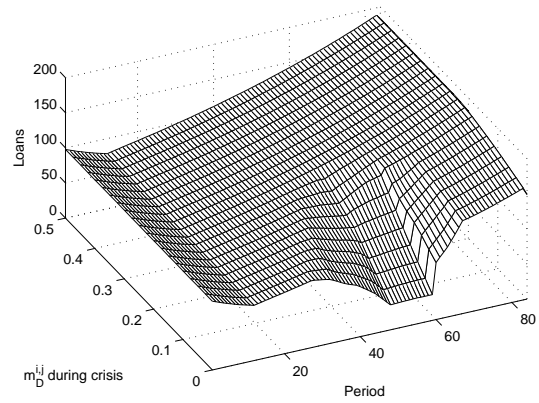
(a) Amount of defaults varied.  
Duration of crisis: 6 periods.



(b) Availability of funding varied.  
Duration of crisis: 6 periods.



(c) Amount of defaults varied.  
Duration of crisis: 12 periods.



(d) Availability of funding varied.  
Duration of crisis: 12 periods.

**Figure 2.6:** Development of loans in the scenario where the crisis does not materialize, considered at different values of the parameters determining the type, severity and duration of the crisis.

crises that involve defaulting loans, reduced availability of funding (i.e. a liquidity crisis), and combinations of these two basic types of crises.

Observations that can be made based on the outcomes of the model include e.g. the bank's tendency to deleverage its balance sheet, i.e. to reduce its funding and lending in preparation for an anticipated financial crisis, as well as its tendency to accumulate cash reserves in order to maintain sufficient liquidity in case that a crisis should materialize. The fact that the bank holds cash reserves even though they provide zero returns contradicts the assumption sometimes made in the theory of banking that banks would only hold the minimum amount of cash required by regulations (see e.g. Freixas & Rochet 2008, p. 71). Running the model at different parameter configurations also indicates that the more severe the anticipated crisis is in terms of defaulting loans and reductions in availability of funding, and the longer its duration, the more drastic are the precautionary measures taken in preparation for the crisis, i.e. the more the bank reduces its lending.

The actions taken by the bank in some of the outcomes of the model may seem unrealistically radical e.g. involving significant changes in the composition and size of the balance sheet. In reality, a bank's actions might be constrained e.g. by market illiquidity, transaction costs, policies, regulations etc. However, constraining the model as little as possible gives good insight into the incentives of a banking firm in the economic situations considered. On a general level, the phenomena observed by using a model such as the one considered here are likely to appear also in more constrained and realistic environments. Despite being perhaps somewhat radical, the bank's actions in the outcomes of the model appear to be logical and they are in line with what one might intuitively expect to observe. The model is sufficiently detailed in order to be able to capture relevant dynamics of a bank facing a financial crisis. The bank also appears to react logically to different values of the parameters determining the type, severity, and duration of the crisis.

The model presented here could be used for analyzing the dynamic behavior and incentives of a generic bank in different kinds of dynamic economic and financial environments involving uncertainties. Whereas the main purpose of this paper is to present the model and verify that it behaves logically, further research could be carried out applying more realistic settings, using the same model or a model closely resembling the one presented here. This could involve e.g. redefining the scenario tree structure and using different values for the parameters defining the economic environment in the scenarios and periods of the model.

In the model presented here, the future involves uncertainties from the view point of the bank only until the moment when the potential crisis starts. After this

moment, the future is completely known to the bank. A more realistic assumption would be that both the length of the crisis and the development of its severity would be unknown to the bank in advance. One approach to modeling macroeconomic developments could be to construct a Markov chain for representing the transitions from one state of the economy to another. These states could differ from each other at least for the part of the values of the parameters defining the percentage of loans defaulting and the availability of new funding. Also the interest rate parameters could be made dependent on the economic environment. Incorporating a stochastic process of this kind into the scenario tree would require a far larger number of scenarios. This might turn out to be computationally challenging and the lesser transparency of the outcomes would make it harder to interpret the results. It would however constitute a more realistic way of modeling the information that the bank has at a given moment. The alternative potential developments of financial crises could then also be more realistically modeled.

One possible modification to the equations defining the optimization model could be to allow the bank to default on its own deposits, while such defaults perhaps could be assumed to have negative impacts on the bank's utility. A benefit of this relaxation would be that the optimization model would then not turn out to be infeasible, i.e. to have no solutions satisfying all constraints, in situations where no precautionary measures could have saved the bank from defaulting. This modification would increase the flexibility of using the model, as it would allow a larger range of potential future economic developments to be considered, including crises of extreme severity. In addition, this would increase the realism of the model, as defaults do happen in the real world, and also since it is unlikely that banks have the ability or the incentives to take sufficient precautions against *every* scenario, some of which may have very low probabilities.

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# Chapter 3

## A stochastic dynamic portfolio model of the balance sheet of a bank

### 3.1 Introduction

A lack of liquid assets is a potential threat to a bank's ability to pay back the funding it has obtained from depositors and other lenders. This risk is aggravated in times when the availability of new funding is lower than usual. This paper presents a portfolio model for analyzing this issue. The model is applied to analyzing how a bank, represented by the portfolio, reacts to this risk.

In a typical banking institution, *maturity mismatch* is present, i.e. the bank's liabilities have shorter maturities than its assets, on average. This reflects one of the main functions of a bank, i.e. that of maturity transformation (see e.g. Freixas & Rochet 2008). A bank's maturing assets together with the interest income may not provide a sufficient cash flow for repaying its maturing liabilities and carrying out the interest payments relating to its liabilities. Therefore, the bank may need to obtain new funding in order to be able to repay its older, maturing funding. If, however, the availability of funding in the market is reduced, the bank may experience a shortage of liquidity, which may result in the bank defaulting on its funding. Such an outcome could materialize e.g. as a result of a financial crisis in which depositors and lenders may not have trust in the bank's ability to repay its debts. This may turn into a self-fulfilling prophecy as even a temporary lack of new funding may result in an outcome in which the bank is unable to pay back its older funding on time. This paper considers such situations as well as the precautionary measures taken by a rational bank in order to avoid defaults in these situations.

In order to analyze how a bank reacts to uncertainty, a dynamic stochastic approach is desirable, because different potential future development paths of the economic environment need to be considered, and because decisions taken in a given period have impact on the bank's balance sheet and expected utility in subsequent periods. The approach applied in this paper is that of stochastic dynamic programming. While this approach is widely applied e.g. in dynamic macroeconomic models (see e.g. the textbooks by Ljungqvist & Sargent 2004 or McCandless 2008), its applications to the analysis of a bank's balance sheet are relatively rare. In particular, stochastic dynamic programming models involving both maturity mismatch and a stochastically varying availability of funding do not appear to exist. The model developed in this essay includes these aspects.

Regarding portfolio models in general, the most well-known one is the static portfolio model by Markowitz (1952). Early stochastic dynamic portfolio models include the discrete-time dynamic programming model by Samuelson (1969) and the continuous-time optimal control problem by Merton (1969). These models consider the dynamic asset allocation of a portfolio over the investor's lifetime, and they can be solved analytically.

Eppen & Fama (1968) present a dynamic discrete-time stochastic cash balance model, in which the amounts of cash required varies stochastically from period to period. Their objective is to find the optimal amount of cash to be held when, on the one hand, cash shortages involve a penalty cost, and on the other hand, there are holding costs (representing opportunity costs) associated with the holding of cash. There are similarities between their model and the model presented in this essay, as the need for liquidity varies stochastically in both models, and both models involve opportunity costs associated with the holding of cash. An important difference is, however, that the model presented in this essay explicitly considers the whole balance sheet of a bank, and not just the level of cash balances. One implication of this is that the opportunity cost relating to the holding of cash emerges endogenously here, in contrast to the holding cost parameter applied by Eppen & Fama. Their model is solved numerically, by applying a formulation based on linear programming.

One of the first stochastic dynamic optimization models considering the balance sheet of a bank is presented by Daellenbach & Archer (1969). They consider stochastic, uncontrolled cash transactions, for which the bank needs to prepare by holding liquid assets in order to keep the risk of cash shortages sufficiently low. Their model considers the optimal cash balance but, unlike the model by Eppen & Fama, it also includes as state variables the holding of securities and the amount

of borrowing. In this respect, their way of modeling the balance sheet of a bank is similar to the one applied in this essay. A significant difference is, however, that in the model by Daellenbach & Archer, profits obtained in a given period do not affect the balance sheet in subsequent periods, whereas in this essay it is assumed that profits may be reinvested. Another difference is that Daellenbach & Archer allow securities to be sold and borrowing to be repaid at will, whereas in this essay, loans and deposits cannot be reduced based on decisions by the bank. Instead, here loans and deposits mature at given rates, which differ from each other, reflecting maturity mismatch. Daellenbach & Archer apply the principle of dynamic programming and their model is solved analytically.

Mukuddem-Petersen & Petersen (2006) present a continuous-time stochastic optimal control model of the balance sheet of a bank. Their model involves optimal security allocation and optimal bank capital inflow. A main difference to the model considered in this essay is that their model focuses on market risk and capital adequacy risk while liquidity risk is ignored, whereas in this essay liquidity is a central issue. Also, their model does not consider the maturities of loans and deposits, whereas they play a significant role in this essay.

In the field of management science, there are numerous models, which consider the balance sheet of a bank in a stochastic dynamic setting. These models include, in particular, so called multistage scenario-based stochastic programming models, such as the ones presented by Kusy & Ziemba (1986), Booth et al. (1989), Oğuzsoy & Güven (1997), and Mulvey & Shetty (2004), to name but a few. Also the first essay of this thesis applies this approach to the theoretical analysis of a bank facing a financial crisis, and considers maturity mismatch as well as the case of a liquidity crisis. However, while this approach may be a useful tool for decision-making in financial institutions, its applicability to theoretical, general-level analysis of banking-related economic phenomena is limited due to the method's reliance on a scenario tree for describing the potential development paths of the economic environment. Using a scenario tree for describing the future imposes limits on the number, and thus the variety, of different potential future development paths that can be considered in a model, thus reducing generality. Stochastic dynamic programming models, however, do not involve restrictions of this kind, and are therefore better suited for theoretical analysis. The approach applied in this paper, i.e. formulating the model as a stochastic dynamic programming problem, allows for a more general solution to be found, as economic developments may be defined as a stochastic process determining transitions between states of the economy. Also, the solution process produces a policy function which may be used in simulations.

Solving models of this type is, however, challenging, whether done analytically or numerically, which may be a reason for why applications of dynamic programming models to the dynamic analysis of balance sheets of banks are relatively rare.

This paper presents a portfolio model, which is applied to the dynamic management of a stylized balance sheet of a bank. The portfolio is assumed to include a liquid asset ("cash"), an illiquid asset ("loans"), external funding ("deposits"), and capital ("equity"). The model considers decisions regarding the allocation of funds between investments in the liquid and illiquid assets, whereas new funding is assumed to be taken in to the extent to which it is available. In order to capture the aspect of maturity transformation, central to the operations of a bank, different maturities are applied to loans and deposits, reflecting maturity mismatch. The availability of new funding is assumed to vary randomly. When the availability of new funding is low, the bank is experiencing a *liquidity crisis*. Such a situation may be a result e.g. of a bank run as discussed by Diamond & Dybvig (1983), or a "drying up" of the availability of funding from other financial institutions, as happened in the liquidity crisis of 2008.

While the model developed in this essay is assumed to represent a bank, the model could equally well be applied to any portfolio with similar characteristics. It could e.g. be used to model a leveraged asset portfolio if the assets are relatively illiquid or even an industrial company that borrows money and invests e.g. in physical capital or R&D, which bring returns only after a longer period.

The stochastic dynamic programming model presented in this essay is solved numerically using value function iteration on a state grid. The solution process gives a policy function, i.e. an optimal decision rule, as an outcome, which allows for the simulation of the development of the bank's balance sheet over time in a stochastic environment.

The structure of this essay is the following. The model is presented in Section 3.2, which also explains in detail how the model was solved. Section 3.3 presents simulations based on the policy function obtained as a solution to the model. Finally, Section 3.4 presents an analysis of how maturity mismatch affects the behavior of the modeled bank. This involves solving the model with a number of different assumptions regarding the maturities and the probability of a liquidity crisis, and simulating the solved model under each set of assumptions. Conclusions are presented in Section 3.5.

## 3.2 Model

### 3.2.1 Assumptions

The model considers a stylized balance sheet of a bank in a dynamic stochastic setting with an infinite time horizon. It is assumed that the economy is in one of two states: state 1, which represents a normal situation or state 2, which represents a situation in which a liquidity crisis is going on, i.e. the availability of new funding is significantly reduced. It is assumed that transitions between these states follow a Markov chain.

The balance sheet of the bank is assumed to consist of a liquid asset ("cash"), an illiquid asset ("loans"), external funding ("deposits"), and capital ("equity"). Cash, loans, and deposits each require their own state variables, whereas equity may be computed as the difference between total assets and deposits, and does not thus require a state variable of its own. In addition to these 3 continuous state variables, there is a fourth, binary state variable, which determines the state of the economy.

The bank is assumed to make decisions regarding the amount of new loans that it gives out in each period. It is assumed that the amount of new funding taken in depends on the state of the economy as well as the amount of equity in the balance sheet at a given time. This assumption implies that banks that are large or in good economic shape have easier access to new funding. Funding, represented by deposits, may be assumed to comprise actual bank deposits as well as different forms of borrowing from other institutions. It is implicitly assumed that the market share of the bank considered is small, since the amount of loans does not affect the amount of deposits (in the case of a large bank, loans would affect deposits, as a result of the money creation process).

The objective is to maximize expected discounted utility, assuming an infinite horizon and discrete time. Periodic utility is assumed to be linear in profits, i.e. to exhibit risk-neutrality with respect to profits. However, in case of defaults, i.e. if the bank is not able to repay its maturing funding, a significant reduction in utility is assumed. The lower the utility in case of defaults, the more incentives the bank has to avoid defaulting. It should be noted, that in contrast to many portfolio models, dividend payments have not been considered in this model. Therefore, the formulation applied here could be interpreted as representing the decision problem of the bank's top management rather than the problem of maximizing the utility of the shareholders. The preference for profits could then be explained e.g. by profit-based management bonus systems and the aversion to defaults could be explained by the

negative impact that the failure of the bank could have on the bank management's career prospects.

### 3.2.2 Equations

The model is formulated as a problem of maximizing expected discounted utility subject to a number of constraints. The objective function is presented in Equation 3.1, while Equations 3.2, 3.3, and 3.4 define the constraints determining the values of the continuous state variables in the next period. Equation 3.5 determines the amount of new funding obtained in a given period.

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t, D_t) \quad (3.1)$$

$$C_{t+1} = C_t + \left(r_L + \frac{1}{\tau_L}\right) L_t - \left(r_D + \frac{1}{\tau_D}\right) D_t + \Delta_t^D - \Delta_t^L \quad (3.2)$$

$$L_{t+1} = \left(1 - \frac{1}{\tau_L}\right) L_t + \Delta_t^L \quad (3.3)$$

$$D_{t+1} = \left(1 - \frac{1}{\tau_D}\right) D_t + \Delta_t^D \quad (3.4)$$

$$\Delta_t^D = \max \{0, m_t^D (C_t + L_t - D_t)\} \quad (3.5)$$

The state variables  $C_t$ ,  $L_t$ , and  $D_t$  represent the amounts of cash, loans, and deposits in the balance sheet, respectively, in period  $t$ . The amounts of new loans given out and new deposits obtained in period  $t$  are represented by  $\Delta_t^L$  and  $\Delta_t^D$ , respectively,  $\Delta_t^L$  being the control variable of the problem. Periodic utility is represented by the utility function  $u(\cdot)$ .

The parameter  $\beta$  is the discount factor, whereas the parameters  $r_L$  and  $r_D$  are the interest rates on loans and deposits, respectively. The parameters  $\tau_L$  and  $\tau_D$  represent the average maturities of loans and deposits, respectively. Therefore, the terms  $(1/\tau_L)L_t$  and  $(1/\tau_D)D_t$  represent approximations of the amounts of loans and deposits maturing in a given period  $t$ . This is an approximation, because to be exact, the amounts maturing in a given period should depend on the actual transaction history rather than the total amounts of loans or deposits in that period. This approximation is, however, necessary for computational feasibility, since maintaining a history of transactions taking place in several previous periods as state variables would make the problem size explode. The parameter  $m_t^D$  represents the

availability of new funding in a given period, and depends on the stochastic state  $s_t$  of the economy.

Equations 3.2, 3.3, and 3.4 define the total amounts of cash ( $C_{t+1}$ ), loans ( $L_{t+1}$ ), and deposits ( $D_{t+1}$ ) in the next period. The amount of cash in the next period depends on the amount of cash in the current period, as well as the amount of money obtained through interest income and maturing of loans, and the money paid out in the form of interest on deposits and maturing deposits. Also the amounts of new deposits obtained and new loans given out in the current period affect the amount of cash in the next period. The amount of loans in the next period depends on the amount of loans in the current period, the amount of loans maturing in the current period, and the amount of new loans given out in the current period. Similarly, the amount of deposits in the next period depends on the amount of deposits in the current period, the amount of deposits maturing in the current period, and the amount of new deposits obtained in the current period.

The amount of new deposits obtained in a given period depends on the stochastic parameter  $m_t^D$  and on the amount of Equity in the balance sheet (as explained in Section 3.2.1). The amount of equity in a given period is defined as a residual of the other balance sheet items, i.e. equity in period  $t$  equals  $C_t + L_t - D_t$ . The amount of new deposits is restricted to be non-negative, as is implied by the max operator in Equation 3.5.

The utility function applied in the problem is presented in Equation 3.6. The utility function involves utility that is linear in profits (i.e. interest income minus interest costs) in situations when cash is non-negative. A negative value of cash represents a default, i.e. a situation, in which the bank is unable to pay back all its maturing funding and the interest on its funding. If cash obtains a negative value in a given period, a constant utility  $F$  will be applied in that period. In periods after a default, utility is assumed to be zero. One interpretation of this formulation is that  $F$  is the present value (in period  $t$  involving a default) of the expected utility over time in all coming periods  $t, t + 1, t + 2, \dots$ . The reason for applying a formulation involving zero utility in periods after a default is that it simplifies the model and makes the model easier to solve as computation does not need to consider situations after a default. Lower values of  $F$  imply stronger incentives to avoid defaulting. In order to avoid defaulting in case of a liquidity crisis, the bank can take precautionary measures in the form of holding a sufficient amount of cash. Computation of the model is assumed to end in case of default, i.e. the equations formulating the model only apply if  $C_t \geq 0$ . Thus,  $C_t$  is not defined if  $C_{t'} < 0$  for some  $t' < t$ .

$$u(C_t, L_t, D_t) = \begin{cases} r_L L_t - r_D D_t & \text{if } C_t \geq 0 \\ F & \text{if } C_t < 0 \\ 0 & \text{if } C_{t'} < 0 \text{ for some } t' < t \end{cases} \quad (3.6)$$

Equation 3.7 defines the upper and lower limits to new loans. It is assumed that the maximum amount that can be lent out in a given period is the amount of cash remaining after all other transactions relating to that period have taken place. These transactions include the interest payments relating to loans and deposits, the maturing of loans and deposits, and the obtaining of new funding (new deposits). If this value turns out to be negative, there will not be any funds available to lend and the upper limit for the amount of new lending needs to be set to zero. This is done using the max operator in Equation 3.7.

$$0 \leq \Delta_t^L \leq \max \left\{ 0, C_t + \left( r_L + \frac{1}{\tau_L} \right) L_t - \left( r_D + \frac{1}{\tau_D} \right) D_t + \Delta_t^D \right\} \quad (3.7)$$

In this implementation of the model, the uncertain economic environment is modeled as a Markov chain of 2 states. State 1, i.e.  $s_t = 1$ , represents a "normal" situation, in which the availability of new funding is on a normal level, while state 2, i.e.  $s_t = 2$ , represents a liquidity crisis, in which the availability of new funding is reduced. The transitions between the states are determined stochastically, based on a  $2 \times 2$  matrix of state transition probabilities. The availability of funding, represented by the parameter  $m_t^D$  is a stochastic parameter, and its value is thus determined by whether the economic environment is in state 1 or 2.

Table 3.1 presents the parameters of the model. The length of a period is assumed to be one quarter of a year and the interest rates have been adjusted accordingly. The initial amounts of cash, loans and deposits have been given values producing model behavior which initially approximates that of a system in a steady state, i.e. one involving monotonic growth at a constant rate as long as the economic environment remains in state 1. The model is solved by applying dynamic programming, which involves the Bellman equation presented in Equation 3.8.

$$V(C_t, L_t, D_t, s_t) = \max_{\Delta_t^L} [u(C_t, L_t, D_t) + \beta E_t V(C_{t+1}, L_{t+1}, D_{t+1}, s_{t+1})] \quad (3.8)$$

The Bellman equation may be rewritten in the form of Equation 3.9 by applying the utility function assumed in Equation 3.6 and, in particular, its property of assigning zero utility to all periods after a default.



**Table 3.1:** The parameters of the model, and their assumed values. Some of the parameters obtain other values in the analyses carried out in Section 3.4. Period length is one quarter of a year.

Parameter	Value	Description
$\beta$	0.97	Discount factor (quarterly)
$r_L$	0.0098534	Interest on loans (4 % annually)
$r_D$	0.0049629	Interest on deposits (2 % annually)
$\tau_L$	10	Average maturity of loans
$\tau_D$	4	Average maturity of deposits
$m_t^D$	0.8 or 0.2	Availability of new funding (varies stochastically)
$F$	0	Utility in the period when the bank defaults
$p_{11}$	0.9	Probability of remaining in state 1
$p_{12}$	0.1	Probability of transition from state 1 to state 2
$p_{21}$	0.2	Probability of transition from state 2 to state 1
$p_{22}$	0.8	Probability of remaining in state 2
$C_0$	7	Initial amount of cash
$L_0$	60	Initial amount of loans
$D_0$	50	Initial amount of deposits

$$\begin{aligned}
 V(C_t, L_t, D_t, s_t) = & \\
 & \begin{cases} \max_{\Delta_t^t} [r_L L_t - r_D D_t + \beta E_t V(C_{t+1}, L_{t+1}, D_{t+1}, s_{t+1})] & \text{if } C_t \geq 0 \\ F & \text{if } C_t < 0 \end{cases} \quad (3.9)
 \end{aligned}$$

### 3.2.3 Solving the model

The model was solved numerically using stochastic dynamic programming implemented as value function iteration on a state grid. This involved assuming a state grid of the 4 state variables  $C_t$ ,  $L_t$ ,  $D_t$ , and  $s_t$  for each grid point. The problem was solved by proceeding backwards in time according to the Bellman equation defined in Equation 3.9. The process consisted of a number of iterations, each of them involving the optimization of the control variable and the evaluation of the value function at every grid point. The iterations were continued until the value function had converged, i.e. until further iterations did not change it significantly. At this point, the value function was assumed to represent the solution to the infinite-horizon problem.

Since there were as many as 4 state variables (3 continuous and 1 binary), the computational effort was substantial. The state space was discretized into a state grid consisting of 100 states for cash, 200 states for loans, 200 states for

deposits, and 2 states for the binary state of the economy, thus resulting in a total of  $100 \times 200 \times 200 \times 2 = 8$  million states. The values of the continuous variables were discretized at even intervals of 1 unit, starting from zero. Thus, the grid points of cash represented the values 0, 1, 2, ..., 99 whereas those of loans and deposits represented the values 0, 1, 2, ..., 199. Multilinear (trilinear) interpolation was applied in the evaluation of the value function at locations between these grid points. Outside of the grid, the value function was assumed to obtain the same value as in the geometrically closest point within the state grid.

A grid-based approach was applied also to the optimization of the state-specific decisions, i.e. the values of the control variable  $\Delta_t^L$ . Thus, a number of different values of the control variable  $\Delta_t^L$  were tried in each state of the state grid. A discretization interval of 1 unit was applied in the case of the control variable, the range of feasible values being determined by the state as defined by Equation 3.7. Whereas this approach of applying a grid also to finding the optimal value of the control variable made it reasonably likely that a global optimum would be found, the optimum was likely to be somewhat imprecise as a result of the discretization, and thus only an approximation of the true optimum.

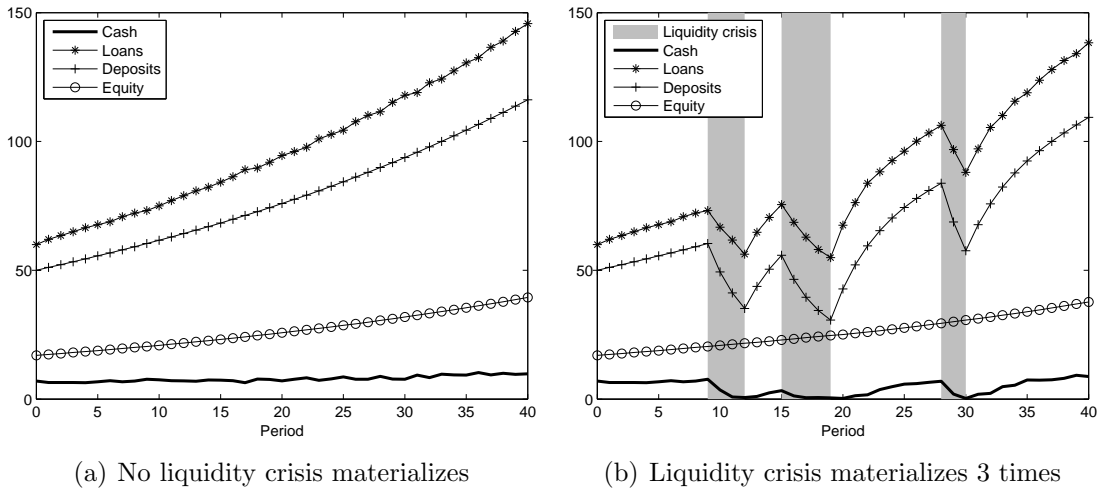
The large size of the state grid combined with the "brute force" approach to optimization described above, caused the problem to be computationally challenging. In addition, the problem had to be solved separately for each parameter configuration considered. In order for the problem to be solvable in feasible time, an implementation in the *C++* programming language was used for carrying out the value function iteration process. The efficiency of computation was also enhanced by carrying out the optimization of the policy function only on every 7th round of value function iterations. The total number of value function iterations required for convergence was typically approximately 150–200 rounds of iterations. Convergence was assumed to have occurred when the maximum change of the value function between two iterations was less than 0.01 for any state. When the solution process was implemented as described above, the problem took approximately 30–40 minutes to solve for each parameter configuration on a PC with a 3.16GHz Intel Core 2 Duo CPU. After the optimization process had finished, the solution, i.e. the policy function, was imported to *Matlab*, where simulations were conducted based on it.

### 3.3 Simulations

The model was solved assuming the parameter configuration of Table 3.1. The policy function obtained as a result of the solution process was applied as a decision

rule in simulations of the model. Figure 3.1 depicts a simulation of the balance sheet of the bank in two cases: one in which no liquidity crisis materializes and another, which involves 3 distinct liquidity crises. The decisions regarding new lending in the simulations are determined by the policy function, and these decisions, in turn, determine the allocation of funds between cash and loans. Figure 3.2 presents the development of the balance sheet items in 100 simulations, in which the variable  $s_t$ , determining the state of the economic environment, varies stochastically between state 1 (normal situation) and state 2 (liquidity crisis) according to a Markov chain.

It can be observed in the figures that some amount of cash appears to be held in the balance sheet as a precautionary measure as this reduces the risk of defaulting if a liquidity crisis should materialize. When a liquidity crisis starts, the amount of cash drops as it is used for repaying maturing old funding in a situation in which new funding is difficult to obtain. In addition, the amount of deposits drops during the crisis, as there are more old deposits maturing than new deposits obtained. A drop in the amount of loans can also be observed during the crisis, which may be explained with two reasons. First, there is less funding available during the liquidity crisis and thus the size of the whole balance sheet will shrink. Second, liquid assets (cash) may still be needed in the following periods for repaying maturing old funding, as it is not known for how long the liquidity crisis is going to continue, and as a result, less is invested in loans.



**Figure 3.1:** Simulations of the model

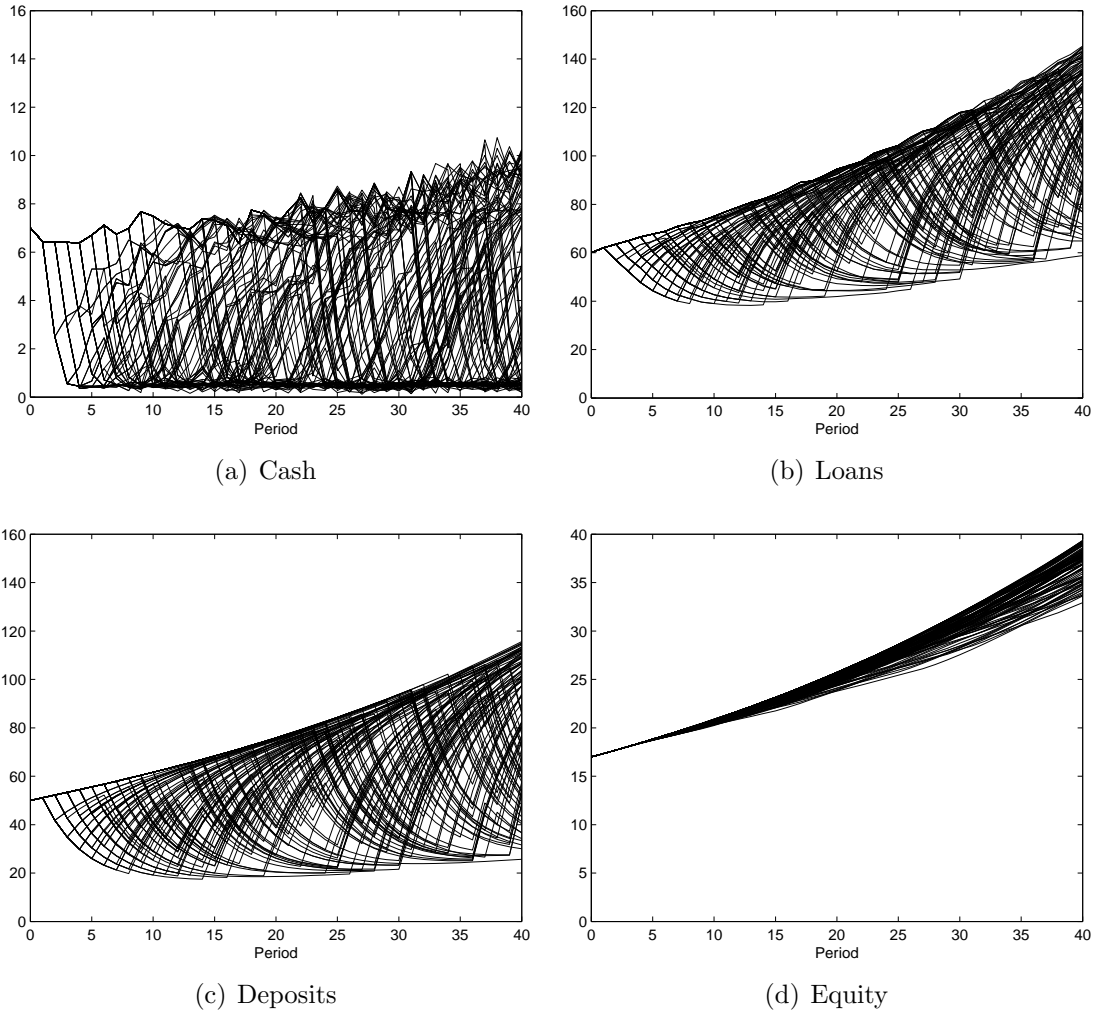


Figure 3.2: 100 simulations of the model

## 3.4 Analysis of the impact of maturity mismatch

### 3.4.1 Parameter adjustments

The model was solved and simulated with a number of different assumptions regarding the maturities. In the normal case, the maturity of lending would be assumed to be longer than that of funding. However, in order to analyze what impacts maturity mismatch has, cases of more similar maturities and even equal maturities were also considered. Therefore, in these analyses the maturity of deposits was increased towards that of loans. This was done by increasing the parameter  $\tau_D$ , determining the maturity of deposits, towards the value of  $\tau_L$ , determining the maturity of loans.

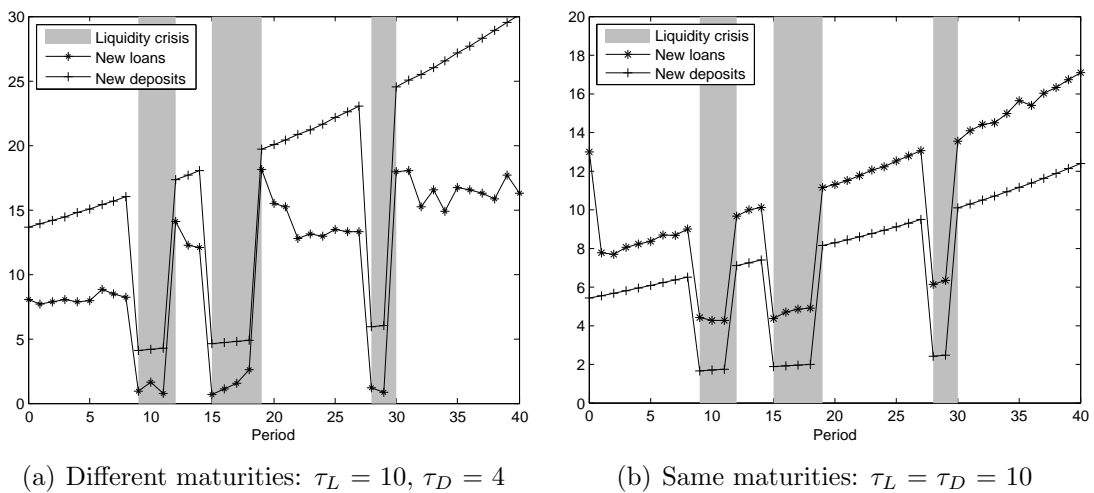
However, longer maturities of deposits would also imply that if the parameter  $m_D$  determining the availability of funding remained the same, the total amount of deposits in the balance sheet would grow at a substantially higher rate. In order to

keep the outcomes resulting from different parameter settings comparable with each other, and to prevent the balance sheet from growing at a high rate, the change in the maturity of deposits needs to be compensated for. This was done by reducing the parameter  $m_t^D$  determining the availability of funding, accordingly. The effect of changing the maturity of deposits is (approximately) compensated for if the product  $\tau_D m_t^D$  is kept constant in all the different parameter configurations applied in the analysis. In Table 3.1, these parameters are given the values  $\tau_D = 4$  and  $m_t^D = 0.8$  or  $0.2$ , depending on the state of the economic environment. Thus, in periods not involving a liquidity crisis the product obtains the value  $\tau_D m_t^D = 4 \times 0.8 = 3.2$ , while in periods of crisis it obtains the value  $\tau_D m_t^D = 4 \times 0.2 = 0.8$ . If the product  $\tau_D m_t^D$  is to be held constant under different values of  $\tau_D$ , the product needs to obtain the value  $3.2$  in normal times and  $0.8$  during a crisis. This will be the case if the value of  $m_t^D$  is determined according to Equation 3.10. These adjustments will be applied in the analyses of Sections 3.4.2 and 3.4.3 as these analyses involve the varying of the value of  $\tau_D$ .

$$m_t^D = \begin{cases} \frac{3.2}{\tau_D} & \text{if in normal situation in period } t \\ \frac{0.8}{\tau_D} & \text{if in liquidity crisis in period } t \end{cases} \quad (3.10)$$

### 3.4.2 New loans and new deposits

Figure 3.3 presents simulations of the development of the amounts of new lending and new funding over a time span of 40 periods. The figure presents the cases of equal and different maturities of lending and funding.



**Figure 3.3:** New lending and new funding at different and equal maturities

Figure 3.3(a) depicts the case where the maturity of lending is longer than

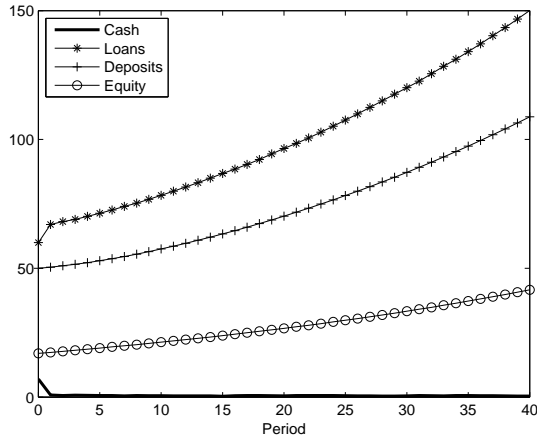
that of funding. In this case, the amount of new loans given out in each period is substantially smaller than the amount of new funding obtained. The intuitive explanation for this difference is that part of the new funding is used for paying back funding that has been obtained in earlier periods and is maturing in the current period, as the amounts of loans maturing (and the interest income) do not provide a sufficient cash flow for repaying the bank's maturing funding (and the interest cost). Thus only some part of the new deposits is invested in loans or held as cash.

Figure 3.3(b) depicts the case where lending and funding have equal maturities. It can be seen that in contrast to the case of different maturities, new loans is now clearly above new deposits. In this case the intuitive explanation is that the amounts of loans maturing, together with the interest income, provide a sufficient cash flow for paying back the older, maturing funding, together with interest costs, and therefore all the funding obtained in a given period may be invested in loans or held as cash. The amount of new loans is, however, above the amount of new deposits because the bank's profits are invested in loans.

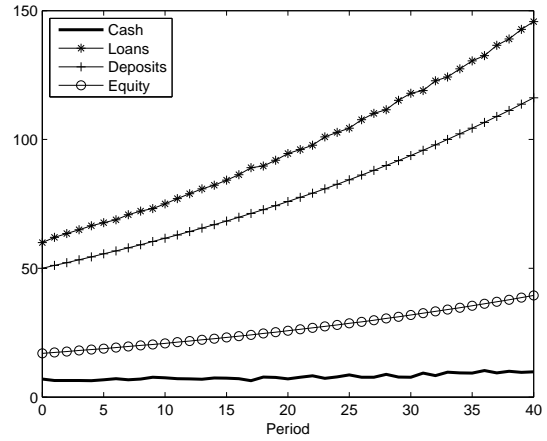
### 3.4.3 Conditions for holding cash

Figure 3.4 shows the outcomes in 4 different different cases, representing 4 configurations of model parameters. The configurations involve settings with and without the risk of a liquidity crisis. The configurations also involve settings where the maturities of funding and lending are equal as well as settings where they are different, the maturity of lending being longer, as is typically the case. It appears that holding of substantial amounts cash takes place only in the case of Figure 3.4(b), which involves both differing maturities and the risk of a liquidity crisis.

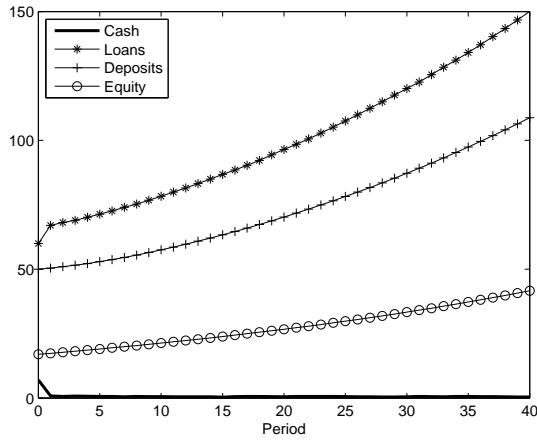
Based on the outcome, it may be concluded that there appears to be two conditions that need to be satisfied in order for funds to be allocated also into liquid assets: First, the maturity of lending needs to be longer than that of funding. Second, there needs to be a risk of a liquidity crisis. These conclusions, suggested by Figure 3.4, are confirmed for a wider range of parameters by Figure 3.5, which shows that significant holding of cash does indeed require both maturity mismatch ( $\tau_L > \tau_D$ ) and a positive probability of a liquidity crisis ( $p_{12} > 0$ ). The figure depicts how maturity mismatch and the risk of a liquidity crisis affect the amount of cash that is held in the balance sheet of the bank. The figure shows the average amount of cash in simulations over 40 time periods, under different parameter configurations. No liquidity crisis materializes in these simulations, but this, of course, is assumed not to be known in advance by the decision-maker. Since no crisis materializes, the bank holds cash only as a precautionary measure, and therefore, the



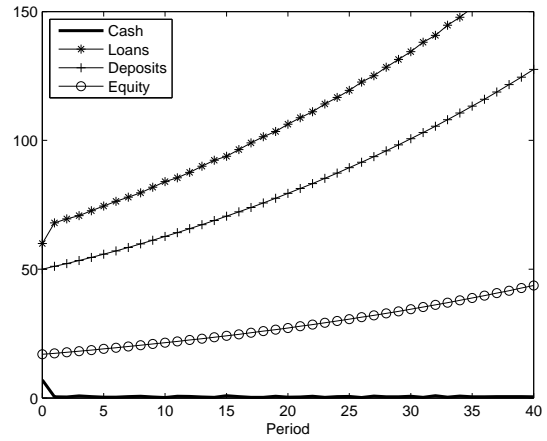
(a) Risk of crisis, same maturities:  
 $p_{12} = 0.1, \tau_L = \tau_D = 10$



(b) Risk of crisis, different maturities:  
 $p_{12} = 0.1, \tau_L = 10, \tau_D = 4$



(c) No risk of crisis, same maturities:  
 $p_{12} = 0, \tau_L = \tau_D = 10$

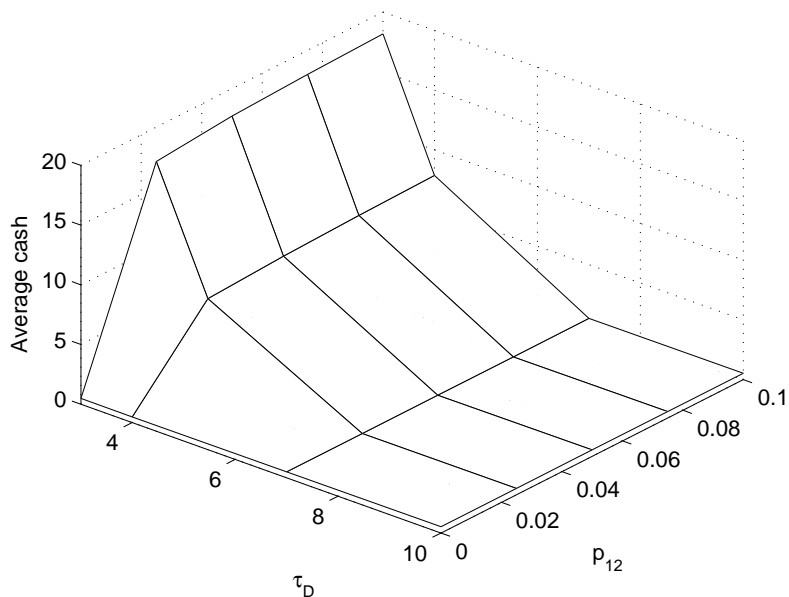


(d) No risk of crisis, different maturities:  
 $p_{12} = 0, \tau_L = 10, \tau_D = 4$

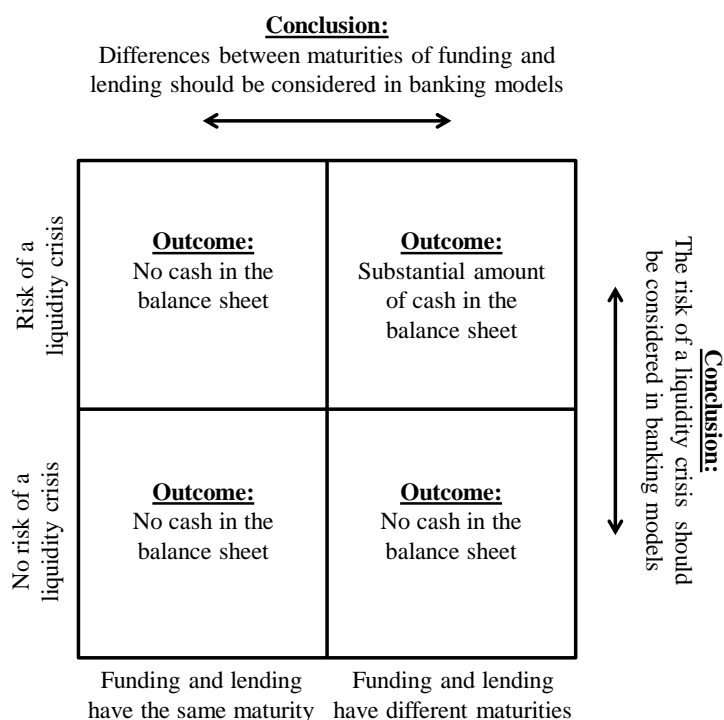
**Figure 3.4:** Simulations of the development of the balance sheet under 4 different parameter configurations. No liquidity crisis materializes in these simulations.

average amount of cash represents how the bank prepares itself for the risk of a liquidity crisis.

Including maturity mismatch and the risk of a liquidity crisis in the model has a substantial impact on the amount of cash held in the balance sheet. Therefore, it would be advisable, within computational possibilities, to pay attention to these two aspects when constructing bank portfolio models. The outcomes under different assumptions, as well as the conclusions drawn, are summarized in Figure 3.6.



**Figure 3.5:** The average amount of cash in the balance sheet depending on whether maturity mismatch and the risk of a liquidity crisis are present. The averages were computed over simulations of periods 1 to 40 with no liquidity crisis materializing.



**Figure 3.6:** A matrix of 4 sets of assumptions, the resulting outcomes, and the conclusions drawn



## 3.5 Conclusions

The main contribution of this essay is to present a portfolio model that can be applied to analyzing the development of the balance sheet of a bank in a dynamic stochastic setting. The model is formulated as a dynamic programming problem, which allows a general solution to be found that may be used as an optimal decision rule in simulations. The main difference to earlier banking models implemented as dynamic programming problems is that both maturity mismatch and the risk of a liquidity crisis are considered here. The solution process applied relies on stochastic dynamic programming implemented numerically as value function iteration on a state grid, and is described in the paper.

A shortage of liquidity in the market (i.e. a reduction in available funding) can be the cause of the defaulting of banks and other institutions, even if there is no investment risk or credit risk present. In particular, this is the case when maturity mismatch is present. The analysis shows that a rationally acting bank will take precautionary measures, i.e. hold some amount of cash in order to avoid such defaults, assuming that there is a cost, in terms of utility, associated with defaulting. Traditional portfolio models such as the static model by Markowitz (1952) and the dynamic models by Samuelson (1969) and Merton (1969) would not produce outcomes involving holding of cash, if there were other riskless assets available that earn positive returns. If funding is assumed to have a cost, investing in cash with leverage would, in fact, produce losses. It is presumably for these reasons, that it is assumed in many banking models that the amount of reserves held by a bank equals the minimum level required by regulations. This assumption is made e.g. in one of the banking models presented in the textbook by Freixas & Rochet (2008, p. 71). In a similar manner, reserves are assumed to be just a constant fraction of total deposits in the stochastic dynamic bank balance sheet model by Mukuddem-Petersen & Petersen (2006). On the other hand, the results arrived at in this essay are in line with those of the stochastic dynamic cash balance models of Eppen & Fama (1968) and Daellenbach & Archer (1969), as their optimal solutions involve the holding of positive amounts of cash. However, in contrast to the model considered in this essay, neither of these two models considers the maturities of assets and liabilities.

The model presented was applied to analyzing conditions that are necessary in order for the holding of cash to take place when the portfolio does not face investment risk. It was shown that this requires two conditions to be satisfied. First, there needs to be a risk of a liquidity crisis in which the availability of new funding

is reduced. Second, there needs to be a maturity mismatch, i.e. the maturity of lending needs to be longer than that of funding. If these conditions are satisfied, cash is being held even if the liquidity crisis never actually materializes. If, however, either of the two conditions is not satisfied, cash is not held in the balance sheet.

Obviously also regulations may cause a bank to hold liquid assets, and some amount of liquidity is needed in order to be able to respond to normal daily fluctuations e.g. in the amounts withdrawn. However, the outcomes of the model considered here indicate, that in addition to these reasons, rational banks will hold substantial amounts of liquid assets also as a precaution, in order to be able to survive a potential liquidity crisis. Thus, it may be advisable to consider the maturity mismatch and the risk of liquidity crises in models and analyses relating to banks, as they may have a significant effect on a bank's asset allocation.

The results of the simulations confirmed that when maturity mismatch is present, the portfolio needs to take in new funding in order to be able to repay its maturing, older funding, since the positive cash flow provided by the maturing of its loans together with interest income is not sufficient for this. When there is no maturity mismatch present, this phenomenon is not observed. It may thus be concluded that the turnover rate of deposits is higher than that of loans as a result of maturity mismatch.

The model was kept as simple as possible in this implementation for reasons of transparency of model behavior and reasons of computability. Relaxing or changing some assumptions might increase the model's applicability to the analysis of real-world cases. First, there could be a larger number of different states of the economic environment as opposed to the two states in the model presented here. Second, the interest rate parameters could be stochastic and dependent on the state of the economic environment. Third, the possibility of some part of the loans defaulting could be introduced in order to incorporate credit risk or investment risk in the model. Fourth, other utility function forms could be applied. Fifth, the model parameters could be calibrated to represent real-world situations as well as possible.

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# Chapter 4

## Optimal dynamic central bank policies under endogenous money

### 4.1 Introduction

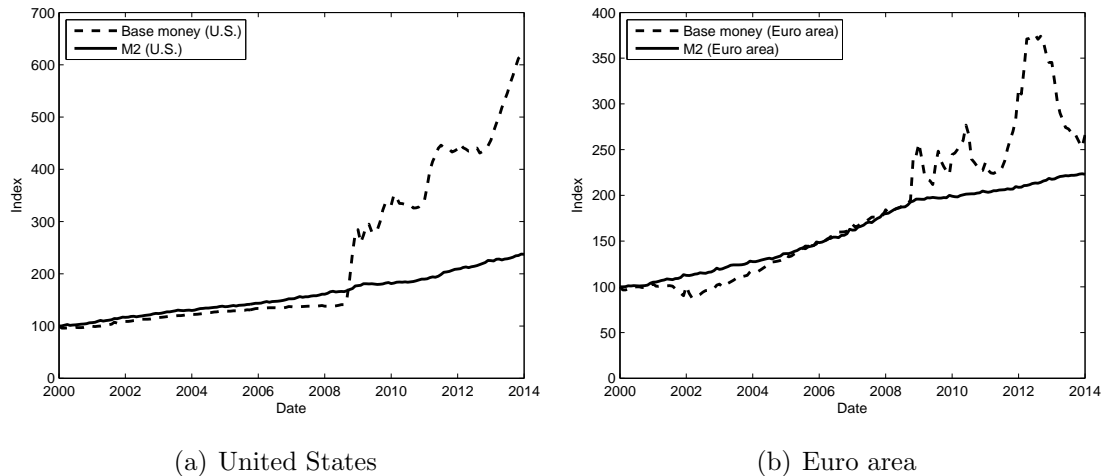
#### 4.1.1 Endogenous money

During the recent global economic recession there have been substantial increases in the monetary base in the U.S. and, to a somewhat lesser extent, in the euro area. As Figure 4.1 shows, the amount of base money quadrupled in the U.S. between the summer of 2008 and the end of 2014, whereas in the euro area it doubled between the summer of 2008 and the summer of 2012 when it reached its peak level. These unusually large increases in base money did, however, not result in high growth rates of the monetary aggregate M2. In the euro area the growth rate of M2 even appears to have declined since the summer of 2008. In addition, these economies have not been experiencing much inflation during this period despite commonly heard concerns regarding the "printing of money".

The fact that the substantial growth of the monetary base has not been accompanied by a corresponding growth of M2 implies that the money multiplier<sup>1</sup> cannot be assumed to be a constant value. Sheard (2013) explains, that many commentators describe this phenomenon as a "collapse" of the money multiplier. These commentators imply that the collapse might be only temporary, and therefore the banks' large excess reserves could lead to severe inflation in the future.

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<sup>1</sup>The money multiplier reflects the ratio of the money supply to the monetary base. If banks were assumed to hold only the minimum amount of reserves required by a reserve requirement and if physical cash held by the public was ignored, the money multiplier would simplify to the inverse of the required reserve ratio. Money multiplier models are explained e.g. in the textbooks by Burda & Wyplosz (2001, pp. 203-207) and Mishkin (2003, pp. 413-416).



**Figure 4.1:** Development of monetary aggregates. Sources of data: Federal Reserve Economic Data, European Central Bank.

The situation described above appears to imply that the reserves of commercial banks substantially exceed the level needed for fulfilling the reserve requirements, and thus the reserve requirements appear not to be binding. The observation that reserve requirements are not binding has also been made e.g. by Bennett & Peristiani (2002) and Martin et al. (2013). In addition, there appears to have been times earlier as well when the requirements have not been binding, such as the Great Depression in the U.S. as described by Feinman (1993).

An explanation to the large "excess" reserves held by commercial banks appears to be the so called *endogeneity* of money, which means that the decisions of commercial banks and borrowers determine the amount of new loans being created and, for a large part, the extent of money creation. E.g. in times of economic distress, banks may choose to restrict their lending because of the default risks involved, while at the same time there may also be less firms and households willing to borrow. This, combined with expansionary monetary policy, may produce a situation in which reserves are substantially above the reserve requirements, which are thus not binding. Thus, a larger monetary base, represented by larger reserves held by commercial banks, may not necessarily result in an expansion of the money supply on the level of monetary aggregates such as M2. Endogenous money has been discussed e.g. by Lavoie (1984), Fontana & Venturino (2003), and Arestis & Sawyer (2006). A bank's decisions regarding new loans is also part of e.g. the paper by Kopecky & VanHoose (2004), in which a bank's optimal quantities of loans and other balance sheet components are derived in a static setting. Also Stiglitz & Weiss (1988) describe how banks make lending decisions based on the condition of the economy and how these lending decisions, in turn, affect the economy. The arguments of

Carpenter & Demiralp (2012) also provide support for the endogeneity of money as they criticize the simple money multiplier models taught in textbooks.

In addition to actual reserve requirements, there may be other forms of regulations that effectively impose more or less implicit reserve requirements on banks. For example, Cabral (2013) describes the Basel III liquidity requirements as "essentially a more generic form of reserve requirements". Furthermore, according to Cabral, liquidity requirements as well as capital requirements are both intended to restrict bank leverage. Thus, also capital requirements may have effects that are somewhat similar to those of reserve requirements.

In the model presented in this essay, money is assumed to be partly endogenous so that during times when reserve requirements are not binding, the lending decisions of banks are not restricted and money is being created endogenously. Otherwise, however, the regulatory restrictions, represented by a reserve requirement, determine the amount of lending and money creation.

#### **4.1.2 Central bank policies**

Dynamic models considering central bank policies have typically been based on policy rules rather than dynamic optimization. The background for this lies in the famous paper by Kydland & Prescott (1977) who argue that in economic planning, policy rules are superior to discretion, since in the case of discretion, the selection of a certain policy will change the expectations that economic agents have regarding future policies and this will affect their decisions. According to Kydland & Prescott, better results would be obtained by commitment to a good policy rule and therefore discretion e.g. in the form of optimization approaches such as optimal control theory, is not an appropriate tool for economic planning. The most famous policy rule is the so called Taylor rule (see Taylor 1993), in which the nominal interest rate is set by the central bank, based on the deviations of inflation and GDP from their target levels.

There are also proponents of discretion and dynamic optimization of monetary policy. E.g. Blinder (1999) stresses the importance of applying the dynamic programming philosophy to monetary policy, and argues that making rational decisions today requires planning ahead for future developments instead of just considering the present situation. Furthermore, he argues that reputations of central bankers do not change very fast as a result of policy decisions, which is one reason why the problems relating to discretion described by Kydland & Prescott are not so serious.

It should be noted that even if it were optimal for central banks to follow policy rules, this does not in any way imply that this is what they would in reality be

doing, especially in times of financial or economic crisis. Instead, they might very well be making decisions based on discretion rather than rules. Thus, the dynamic programming philosophy may be a proper approach for describing the dynamic actions of a central bank regardless of its optimality as a tool for determining optimal policies. Optimal central bank policy rules have been discussed e.g. by Barro & Gordon (1983), Giannoni & Woodford (2004), and Singh & Nikolaou (2013). However, models considering dynamic optimization of central bank policies are hard to find.

One question regarding central bank policies is whether expansionary monetary policy carried out by a central bank is effective in times when the interest rates are close to zero and reserve requirements are not binding. Expansionary monetary policy has indeed been carried out during the recent recession despite the excess reserves that banks have and despite the very low interest rates. These policies, often referred to as *quantitative easing*, may positively affect the amount of borrowing in the economy, thereby stimulating economic activity, by lowering long-term interest rates even if short-term interest rates are close to zero. As explained by Sheard (2013), quantitative easing changes the asset portfolios held by the private sector, replacing e.g. government bonds with reserves and bank deposits. This, according to Sheard, leads to *portfolio rebalance effects*, which e.g. cause banks that have excess reserves to readjust their asset portfolios by increasing the amount of lending. In addition, as McLeay et al. (2014) explain, if the central bank purchases bonds held by non-bank financial institutions, these institutions will, as a result, rebalance their portfolios by acquiring new, similar assets, thereby lowering the cost of funding and stimulating economic activity. Also Joyce et al. (2011) emphasize the importance of the portfolio rebalance effect as a transmission channel of quantitative easing, focusing on the case of the United Kingdom. According to Carpenter & Demiralp (2012), empirical data since 1990 does not support the view that monetary policy would work through the standard money multiplier model where reserves determine the amount of loans. Channels of monetary policy transmission are discussed e.g. by Bernanke & Blinder (1992), Mishkin (1995), Meyer (2001), Arestis & Sawyer (2002), Diamond & Rajan (2006), and in the textbook by Freixas & Rochet (2008, pp. 196-203).

### **4.1.3 The model presented in this essay**

The banking sector has typically been ignored in modern macroeconomic models, such as the DSGE models. However, more recently, the interest in considering the banking sector as part of economic models has been growing. E.g. Aslam &



Santoro (2008), Aliaga-Díaz & Olivero (2010), Gerali et al. (2010), and Gertler & Karadi (2011) have incorporated the banking sector into DSGE models. A banking system is also embedded in the DSGE model presented in the IMF working paper by Benes & Kumhof (2012), which has obtained some publicity as a result of their controversial and radical proposal for reshaping the banking sector.

Two important features differentiate the model presented in this essay from modern macroeconomic models. First, it includes a banking sector creating money endogenously, as described in Section 4.1.1. Second, it includes a central bank, making decisions which are optimal in accordance with the principle of dynamic programming, as discussed in Section 4.1.2. Earlier macroeconomic models including both of these features do not appear to exist. The economy is modeled using a stochastic dynamic programming model, in which the decision-maker is the central bank controlling the monetary base, and the real economy experiences stochastic productivity shocks. It is assumed that there are restrictions on the magnitude of the expansionary and the contractionary actions that the central bank may carry out during one time period. The central bank therefore needs to plan several periods ahead so that its current policy is optimal with respect to a range of different potential economic developments that may occur, while at the same time considering the actions that it would be able to take if such developments were to materialize. For example, when deciding on monetary policy during an economic recession, the central bank would have to consider the trade-off between the beneficial effects on growth resulting from expansionary monetary policy and the risk of future inflation, which may materialize when the recession ends. When the recession ends, the money supply may start to expand rapidly in an endogenous manner, as the creation of new loans is made possible by the substantial reserves that were produced by the expansionary central bank policy during the recession.

The contractionary or expansionary measures taken by the central bank represent a combination of the central bank's policy tools for adjusting the monetary base and affecting growth and stability. These include standard open market operations, quantitative easing and setting the discount rate. Changing the required reserve ratio, while also being a tool of monetary policy, is excluded from this analysis. Since there is a number of policy tools, monetary policy cannot be comprehensively described by just one interest rate, such as the Federal funds rate. Since, however, all the policy tools considered affect the monetary base, it has been assumed in this essay that the central bank conducts monetary policy by adjusting the monetary base. As a result, the level of interest rates is not considered explicitly in the model, but it is assumed to be one of the several determinants of the demand for loans.

The model applied in this essay considers the effect of monetary policy through 2 main channels: First, when reserve requirements are binding, the monetary base determines the amount of lending according to the standard money multiplier model. Second, however, when the reserve requirements are not binding, the combined effect of all other monetary policy transmission channels is represented by one single generalized channel, which may include e.g. the impacts of changes in short and long market interest rates, changes in the discount rate, as well as portfolio rebalance effects. The high level of abstraction applied here to the modeling of transmission channels of central bank policies is justifiable, as it greatly reduces model complexity, while still allowing for the analysis of essential issues.

#### **4.1.4 Issues considered**

An important question regarding monetary issues is whether the expansion of the monetary base that has been taking place during the recent years will at some point in the future result in high inflation, in particular when the economy starts to recover, and banks and borrowers alike will be more inclined to create loans, thereby expanding the money supply. The banks' large reserves in comparison to their reserve requirements would potentially allow money creation of significant extent to take place, which would increase the money supply e.g. on the level of the M2 aggregate, and might thereby cause a surge in inflation. An associated question is, therefore, whether the central banks will be able to contract the monetary base rapidly enough in order to prevent strong inflationary tendencies, if the above-mentioned developments start to materialize.

This essay attempts to give insight into these issues by developing a model for analyzing how central bank policies, money creation, the real economy and inflation interact and develop over time if money creation is partly endogenous, so that reserve requirements may or may not be binding at a given time. Balance sheets of banks are considered on a relatively abstract level in this essay, and therefore the reserve requirements considered here could also be interpreted as the effective result of reserve, liquidity or capital requirements, whichever is the most restrictive.

#### **4.1.5 The structure of this essay**

The structure of this essay is the following. Sections 4.2.1 and 4.2.2 present the assumptions and equations of the model, respectively, whereas Section 4.2.3 explains how the model was solved. The outcome of the solution process is a policy function describing the optimal actions of the central bank in different states. The

policy function is used as a decision rule in simulations, the outcomes of which are presented in Section 4.3. Conclusions are presented in Section 4.4.

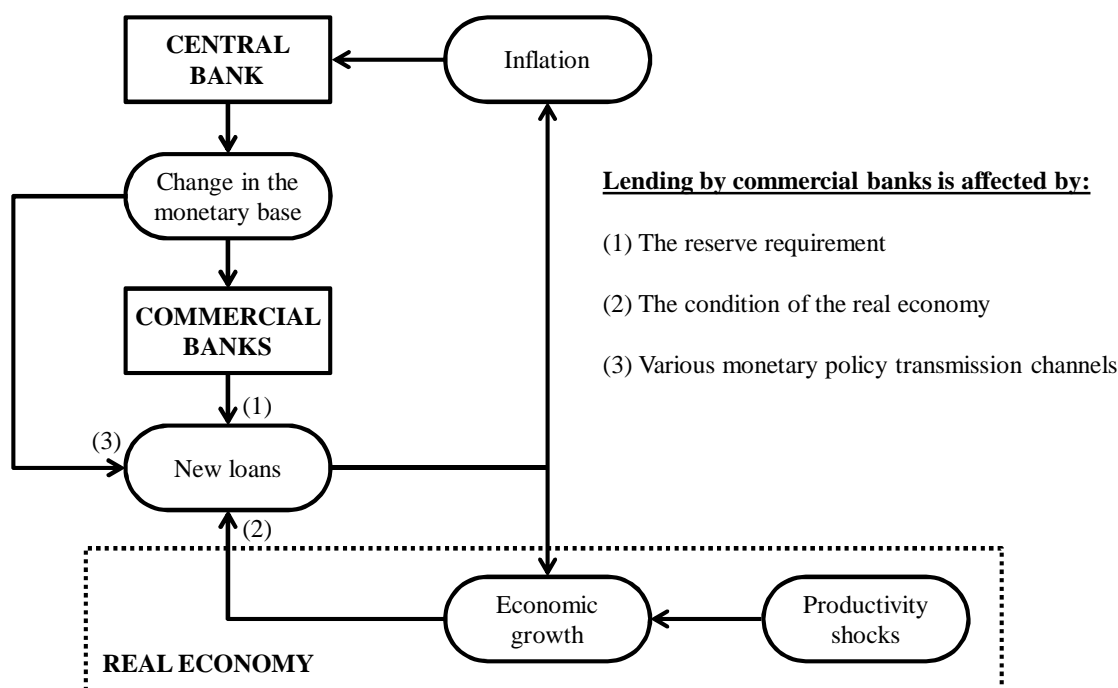
## 4.2 Model

### 4.2.1 Assumptions

The structure of the model considered is illustrated in Figure 4.2. In each period, the central bank adjusts the size of the monetary base, which is assumed to consist entirely of reserves held by commercial banks. The amount of physical cash held by the public is typically negligible in comparison to the reserves held by banks and it is therefore ignored for reasons of simplicity. The model considers all commercial banks as one entity. The commercial banks decide the amount of new loans that they give out in each period. The banks' lending decisions are affected by the general condition of the economy, as the banks avoid giving loans that would result in defaults. When a new loan is created, it is assumed that at the same time a new deposit is created on the other side of the balance sheet of the banking sector. Thus, the money supply (again, ignoring cash held by the public) is determined by the amount of loans given by banks. The reserve requirement sets an upper limit to the amount of lending but the requirement need not be binding, i.e. banks may hold substantially more reserves than is required.

Monetary policy affects lending through two main channels. First, in situations when the reserve requirement is binding, the money multiplier model is applicable and then the amount of base money determines the amount of lending. Second, when the reserve requirement is not binding, the impact of monetary policy takes place through a number of different channels described in Section 4.1.4, which are in this treatment of the problem combined into one general channel affecting lending by commercial banks. It is assumed that commercial banks decide only the amount of new lending in a given period, while loans mature over the time of several future periods. Interest rates are considered only implicitly as one of the several monetary policy transmission channels. The reason for not including the interest rates in the model explicitly is that it would require to lower the level of abstraction of these transmission channels, which would severely increase model complexity.

It is assumed that economic growth is affected on one hand by exogenous, stochastic productivity shocks, and on the other hand by the amount of new loans given by commercial banks. One interpretation of the latter one of these effects is that the ability of firms to obtain financing for new investment projects, and for



**Figure 4.2:** An illustration of the structure of the model.

paying for ongoing operations, affects overall economic activity. Also the ability of households to obtain loans for consumption, or for investments e.g. in new housing, affect the total amount of production in the economy.

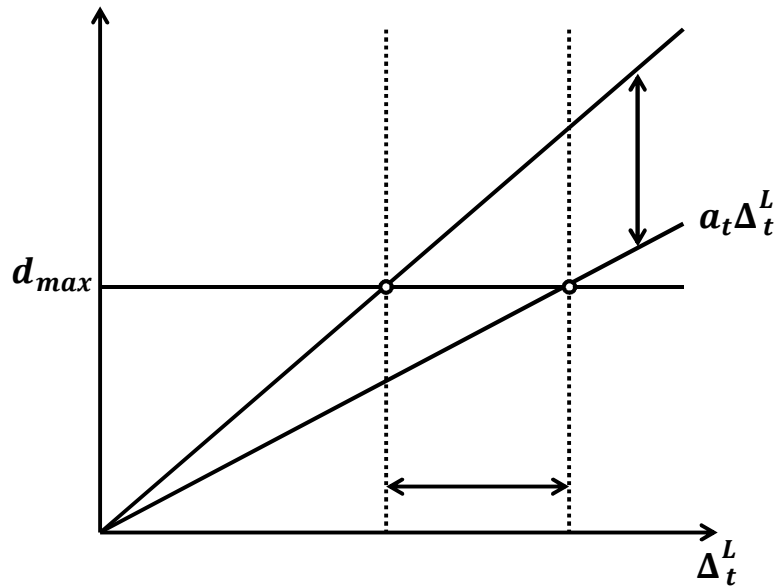
As noted above, the money supply is determined by the total amount of loans in the balance sheets of the banking sector. It is assumed that inflation is determined by changes in the money supply. The central bank adjusts the size of the monetary base, with the objective of keeping inflation close to its target over time. The rate of economic growth could, in principle also be included in the objective but has been omitted in order to simplify the analysis. Since an inflation target also implies an avoidance of deflation, and since periods of low growth are inclined to involve deflation, economic growth is, in fact, to some extent implicitly accounted for in the objectives even in the present formulation of the model.

The banking sector is modeled with a representative bank. While the banking sector could, in principle, also be modeled by considering  $N$  identical banks (and this would be a suitable approximation from the view point of this analysis) the following reasons justify the assumption of a representative bank. First, the balance sheets of a number of different banks can be aggregated into one large sectoral balance sheet as transactions between deposit accounts in different banks do not change the composition of the balance sheet of the aggregated banking sector. Second,

symmetry between individual banks would imply that during a given time period transactions between the deposit accounts in different banks should cancel each other out and, in addition, all banks should give similar amounts of new loans. Thus, the balance sheets of the  $N$  banks would continue to be identical to each other and the balance sheet of the aggregated banking sector would be a multiple of the balance sheet of any individual bank. Third, if individual banks are considered as having monopolies in their respective markets of new loans, then competition between the banks need not be considered and the aggregated lending decisions of individual banks can be modeled as a lending decision of the aggregated banking sector. While this last assumption is a strong one, it is justifiable as the aim is to have the banks' decisions express an avoidance of risky loans, whereas the consideration of a market equilibrium level of interest rates is of less importance here.

It is assumed that commercial banks make decisions regarding new loans. Furthermore, it is assumed that some loans are riskier than others, i.e. have higher default probabilities, and that loans with low default probabilities are preferred over ones with high default probabilities. Thus, the larger the amount of new loans is, the higher is the marginal default risk. When the reserve requirement is not binding, it is assumed that in each period, a bank gives new loans until the marginal default risk grows "too high". One interpretation of this maximum level of acceptable default probability could be that of the margin that the bank gets from the difference between the interest rates on its lending and funding. This margin could be assumed to be approximately constant either as a result of competition between banks (which was previously excluded for reasons of simplicity, though) or by assuming that interest rates on both sides of the balance sheet behave similarly. In any case this way of modeling the lending decision produces inner-point solutions, which is desirable. The marginal default probability is here assumed to grow linearly. Somewhat similar formulations relating to the lending decisions of banks have been applied also by Kopecky & VanHoose (2004), whereas their reasoning behind the formulation is somewhat different, being based on quadratically increasing costs, implying linearly increasing marginal costs. An approach similar to the one applied here is also presented in the model by Bossone (2001), in which default risk increases with the amount of loans given, and lending is increased until the default probability reaches the marginal return on loans.

The unconstrained lending decision of the representative bank is depicted in Figure 4.3. The riskiness of loans is assumed to depend on the condition of the economy. When the economic outlook is better, the slope of the linear marginal



**Figure 4.3:** The unconstrained lending decision of the representative bank is determined by the marginal default probability  $a_t \Delta_t^L$  and the maximum acceptable default probability  $d_{max}$ . The condition of the economy affects the general level of risk relating to loans and thus determines the slope of the line representing the marginal default risk. As a result, the amount of new loans  $\Delta_t^L$ , given in a period, varies over time.

default curve is lower, and more new loans can be given before the marginal default probability grows too high. It should be noted, that in addition to representing the changes over time in the riskiness of loans, the formulation applied here can also be interpreted as involving changes in the general demand for loans, since during times of economic distress the general demand for loans would be lower. Therefore, at each level of loan-specific credit risk, i.e. at each level of default probability, the public's demand for loans would be lower, which would explain how the slope of the marginal default curve becomes steeper.

The formulation of a bank's decision-making applied here implicitly assumes that the bank is unable to charge different interest rates based on the riskiness of the loans. This is, again, a strong assumption, but may be to some extent justified e.g. by information asymmetries. E.g. increasing the amount of lending is likely to involve the expanding of the customer base to include previously unknown clients. The risks associated with new clients may be harder to assess whereas their willingness to pay interest may not be above average. Regulatory issues may also to some extent limit the possibility of charging very high interest rates. In addition, on a general level it seems reasonable that banks cannot perfectly price discriminate between their clients, while at the same time it also appears evident

that aggressive lending by banks may result in high losses. For these reasons, the strong assumption of neglecting the consideration of interest rates and assuming that banks make lending decisions only based on the default probability is justifiable for the purposes of this analysis.

## 4.2.2 Equations

The model has two continuous state variables,  $B_t$  and  $L_t$ , which represent the monetary base and total amount of loans, respectively, in period  $t$ . In addition, there is a binary state variable  $s_t$  representing the state of the macroeconomy. There is one control variable,  $\Delta_t^B$ , representing the increase or decrease in the monetary base in period  $t$ . The amount of new loans given by commercial banks in period  $t$  is represented by  $\Delta_t^L$ . Equation 4.1 determines how the control variable adjusts the monetary base from period to period. Equation 4.2 determines how the amount of loans in the next period is determined by the amount of loans in the current period and the amount of new loans given in the current period. The parameter  $\tau$  is the average maturity of the loans and thus  $(1/\tau)L_t$  represents (as an approximation) the amount of loans maturing in period  $t$ . Equation 4.3 imposes restrictions on the magnitude of expansionary and contractionary measures that the central bank is able to carry out during one period.

$$B_{t+1} = B_t + \Delta_t^B \quad (4.1)$$

$$L_{t+1} = \left(1 - \frac{1}{\tau}\right) L_t + \Delta_t^L \quad (4.2)$$

$$\Delta_{min}^B \leq \Delta_t^B \leq \Delta_{max}^B \quad (4.3)$$

As illustrated in Figure 4.3, the marginal default probability depends linearly on the amount of new loans, the slope parameter  $a_t$  being the coefficient determining the general risk relating to new loans. The parameter  $a_t$  is assumed to depend on the condition of the economy, i.e. when there is a crisis going on,  $a_t$  obtains large values, implying high default probabilities and, thus, low amounts of new lending. When the reserve requirement is not binding, the amount of new loans given by the banking sector during one period is the amount at which the marginal probability of defaulting, i.e.  $a_t \Delta_t^L$ , equals the maximum acceptable level  $d_{max}$ . This is the case when  $\Delta_t^L = d_{max}/a_t$ . In addition, it is assumed that when the reserve requirement is not binding, monetary policy affects lending through a number of

different transmission channels, such as the portfolio rebalance effect, as discussed earlier. The impact of these effects is accounted for by adding to the amount of new lending the term  $\lambda\Delta_t^B$ , where the coefficient  $\lambda$  represents the general effectiveness of central bank policies in times when the reserve requirements are not binding. Thus, more expansionary monetary policy, represented by a higher value of  $\Delta_t^B$  results in more lending taking place. Equation 4.4 presents the resulting unconstrained amount of new lending. The max operator ensures that this amount cannot become negative.

$$\Delta_t^{L,free} = \max \left\{ 0, \frac{d_{max}}{a_t} + \lambda\Delta_t^B \right\} \quad (4.4)$$

It is assumed that there is a reserve requirement which may or may not be binding. It is not binding if reserves are above the level required at a given time. The decisions regarding lending in period  $t$  need to be such that the reserve requirement will be fulfilled in period  $t + 1$ . Equation 4.5 presents, for period  $t + 1$ , the reserve requirement, where total reserves are represented by the monetary base  $B_{t+1}$ , total deposits are represented by  $D_{t+1}$  and  $\alpha$  is the required reserve ratio.

$$\frac{B_{t+1}}{D_{t+1}} \geq \alpha \quad (4.5)$$

All loans are assumed to be held as deposits in the banking system and all deposits are assumed to have originated as loans at some point. This implies that the amount of deposits equals the amount of loans at any given time. Substitution of  $L_{t+1}$  from Equation 4.2 for  $D_{t+1}$  and  $B_{t+1}$  from Equation 4.1 renders the reserve requirement for period  $t + 1$  into the form presented in Equation 4.6. This requirement needs to be accounted for by the banking sector when making decisions regarding new lending in period  $t$ .

$$\frac{B_t + \Delta_t^B}{\left(1 - \frac{1}{\tau}\right) L_t + \Delta_t^L} \geq \alpha \quad (4.6)$$

This may be rewritten as follows.

$$\Delta_t^L \leq \frac{B_t + \Delta_t^B}{\alpha} - \left(1 - \frac{1}{\tau}\right) L_t \quad (4.7)$$

Since new lending cannot be negative, the following formulation of the upper limit to new lending is implied.

$$\Delta_t^{L,max} = \max \left\{ 0, \frac{B_t + \Delta_t^B}{\alpha} - \left(1 - \frac{1}{\tau}\right) L_t \right\} \quad (4.8)$$



If the unconstrained amount of lending presented in Equation 4.4 is above the maximum level defined by Equation 4.8, the amount of lending will be equal to the reserve requirement. Equation 4.9 presents the amount of new loans when the reserve requirement is accounted for. It should be noted that  $\Delta_t^{L,max}$  and  $\Delta_t^{L,free}$  are always non-negative as a result of their definitions and thus also  $\Delta_t^L$  will be non-negative.

$$\Delta_t^L = \begin{cases} \Delta_t^{L,free} & \text{if } \Delta_t^{L,free} < \Delta_t^{L,max} \\ \Delta_t^{L,max} & \text{otherwise} \end{cases} \quad (4.9)$$

Inflation is assumed to be determined by changes in the money supply and thus, a form of the quantity theory of money is applied. However, the impacts of changes in the velocity of money as well as economic growth are not accounted for. This simplification is sufficient for this analysis. Since cash held by the public is not considered, the money supply is assumed to equal total bank deposits. Furthermore, total bank deposits are assumed to equal the total amount of loans. Therefore, inflation may be defined based on the relative change in the total amount of loans, from period to period, as presented in Equation 4.10.

$$\pi_t = \frac{L_{t+1}}{L_t} - 1 \quad (4.10)$$

The condition of the economy is described with the variable  $g_t$  representing economic growth in period  $t$ . The value of the coefficient  $a_t$ , determining the general default risk relating to new loans, is assumed to depend on economic growth as described by Equation 4.11. There is thus an inverse relationship between the variables, scaled by the parameter  $\theta$ . Higher economic growth results in lower values of the variable  $a_t$  and thus, as expressed by Equation 4.4, in a higher amount of new loans being given, assuming that the reserve requirement is not binding.

$$a_t = \frac{\theta}{g_t} \quad (4.11)$$

Economic growth  $g_t$  is assumed to depend on the condition of the economy, represented by the state variable  $s_t$ . It is assumed that this state variable varies according to a Markov chain of two states, state 1, i.e.  $s_t = 1$ , representing a good condition of the economy ( $g_t = g^H$ ) and state 2, i.e.,  $s_t = 2$ , representing an economic crisis ( $g_t = g^L$ ). The transitions between these states are assumed to represent stochastic productivity shocks. However, as explained earlier, it is assumed that the amount of lending in a given period affects economic growth. This effect is incorporated into the model by assuming that the state transition probabilities

are affected by the amount of new lending. It should be noted that the formulation of Equation 4.11 only allows for positive growth rates  $g_t$ . This restriction becomes less problematic if periods of low economic growth, represented by state 2, are interpreted as representing approximations of relatively long periods of economic downturn, during which average growth rates may yet be slightly positive.

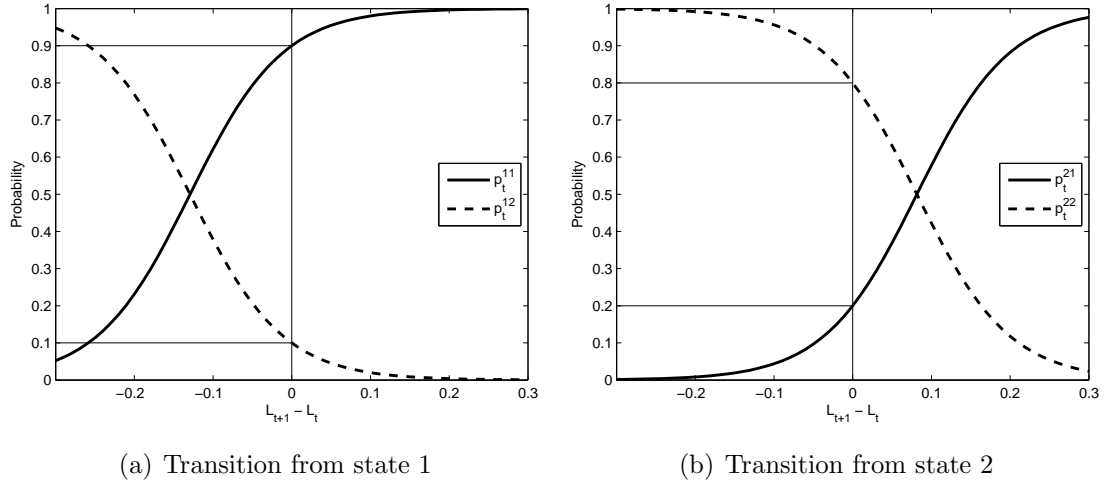
The probability of a transition from state  $i$  to state  $j$  is represented by the constant  $p^{ij}$  when the impact of lending on economic growth is not considered or is zero, whereas  $p_t^{ij}$  represents the state transition probability (between periods  $t$  and  $t + 1$ ) after this effect has been accounted for. The transition probabilities are modified according to Equation 4.12, in which a variant of the logistic function is applied to determining the modified probabilities.

$$\begin{cases} p_t^{i1} = 1 / \left[ 1 + \frac{1-p^{i1}}{p^{i1}} e^{-\gamma(L_{t+1}-L_t)} \right] \\ p_t^{i2} = 1 - p_t^{i1} \end{cases}, \quad i \in \{1, 2\} \quad (4.12)$$

The modified probability of a transition from either of the states to state 1 is represented as a function of the change in the total amount of loans  $L_{t+1} - L_t$ . The function has 4 essential properties. First, the function increases with changes in the total amount of loans  $L_{t+1} - L_t$ . Thus, increasing total lending results in a higher probability that the next state transition leads to state 1, where the economy is in a good condition. Second, if  $L_{t+1} = L_t$ , then  $p_t^{i1} = p^{i1}$ , i.e. if total lending does not change, the transition probabilities are not affected. Third, the function is defined on the whole real line and its limits at infinity and negative infinity are 1 and 0, respectively. Thus, very large increases or decreases in the amount of lending set the probability of a transition to state 1 to values close to 1 or close to 0, respectively, while always being between these two values. Fourth, the value of the parameter  $\gamma$  scales the function horizontally, and therefore  $\gamma$  may be used to describe how changes in the amount of loans affect the real economy. Larger values of  $\gamma$  imply a stronger effect.

Figure 4.4 illustrates the transition probabilities as functions of the change in the total amount of loans. The figure is based on those values of the parameter  $\gamma$  and of the unmodified transition probabilities  $p_{ij}$ , which are applied in the analyses carried out in this paper, and listed in Table 4.1. The horizontal lines in the figure illustrate how the transition probabilities are determined in the case that the total amount of loans does not change (i.e.  $L_{t+1} - L_t = 0$ ), thus representing the unmodified transition probabilities.

The objective is assumed to be the minimization (over time) of the expected squared difference between inflation  $\pi_t$  and its target level  $\pi^*$ . Objectives of similar



**Figure 4.4:** Transition probabilities as functions of the change in the total amount of loans

type, i.e. containing inflation in squared form, have been applied e.g. by Barro & Gordon (1983), Giannoni & Woodford (2004), and Woodford (2004). Typically also other targets, such as e.g. GDP growth or unemployment are included in the objective function of the central bank. Here, however, the objective is restricted to minimizing deviations from the inflation target. Equation 4.13 presents the central bank's objective to minimize, over time, the expected discounted deviations of inflation from its target level. Equations 4.1, 4.2, 4.3, 4.4, 4.8, 4.9, 4.10, 4.11, and 4.12 constitute the constraints of the stochastic dynamic programming problem.

$$\min E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t - \pi^*)^2 \quad (4.13)$$

Equation 4.14 presents the Bellman equation of the problem.

$$V(B_t, L_t, s_t) = \min_{\Delta_t^B} [(\pi_t - \pi^*)^2 + \beta E_t V(B_{t+1}, L_{t+1}, s_{t+1})] \quad (4.14)$$

### 4.2.3 Solving the model

The parameters of the model and their values are listed in Table 4.1. The parameters  $\Delta_{min}^B$  and  $\lambda$  will also be given some other values in the optimizations and simulations carried out in Section 4.3, whereas the other parameters' values will not be changed. It is assumed that one period in the model represents one quarter of a year.

The model was solved numerically using dynamic programming implemented as value function iteration on a state grid. This method is explained e.g. in the textbooks by Ljungqvist & Sargent (2004) or McCandless (2008). The solution

**Table 4.1:** The parameters of the model. Period length is one quarter of a year.

Parameter	Value	Description
$\alpha$	0.1	Required reserve ratio
$\beta$	0.98	Discount factor
$\gamma$	17	Parameter defining how changes in lending affect the probabilities of state transitions
$\theta$	0.00003	Parameter determining the dependence between economic growth and the general default risk of loans
$\lambda$	1.5	Effectiveness of monetary policy in affecting lending
$\pi^*$	0.005	Inflation target ( $\sim 2\%$ annually)
$\tau$	10	Average maturity of loans
$\Delta_{max}^B$	0.17	Maximum value of the control variable
$\Delta_{min}^B$	-0.12	Minimum value of the control variable
$d_{max}$	0.005	Maximum acceptable default probability
$p_{11}$	0.9	Transition probability (unmodified): state 1 to state 1
$p_{12}$	0.1	Transition probability (unmodified): state 1 to state 2
$p_{21}$	0.2	Transition probability (unmodified): state 2 to state 1
$p_{22}$	0.8	Transition probability (unmodified): state 2 to state 2
$g^H$	0.0075	Economic growth in state 1 ( $\sim 3\%$ annually)
$g^L$	0.0025	Economic growth in state 2 ( $\sim 1\%$ annually)
$B_0$	0.9	Initial monetary base
$L_0$	9	Initial amount of loans

process involved a state grid of the 3 state variables  $B_t$ ,  $L_t$  and  $s_t$ . The state grid consisted of 200 states for  $B_t$ , 200 states for  $L_t$ , and 2 states for  $s_t$  representing the binary state of the economy. Therefore, the total number of states was  $200 \times 200 \times 2 = 80\,000$  states. The grid points were spaced at even intervals, the ranges being  $[0, 4]$  for  $B_t$  and  $[1, 20]$  for  $L_t$ . Multilinear (bilinear) interpolation was applied in the evaluation of the value function at locations between the grid points. In order to prevent the state variables from obtaining values outside of the state grid, restrictions were applied to the decision variables  $\Delta_t^B$  and  $\Delta_t^L$  in the proximity of the outer border of the grid. Finding the optimal value of the control variable  $\Delta_t^B$  in each state was carried out by trying 1000 evenly spaced values of  $\Delta_t^B$  in the range defined by Equation 4.3. Approximately 200 rounds of iterations were typically required for convergence of the value function iteration process. Convergence was assumed to have occurred when the maximum change of the value function between two iterations was less than 0.00001 for any state.

### 4.3 Simulation results

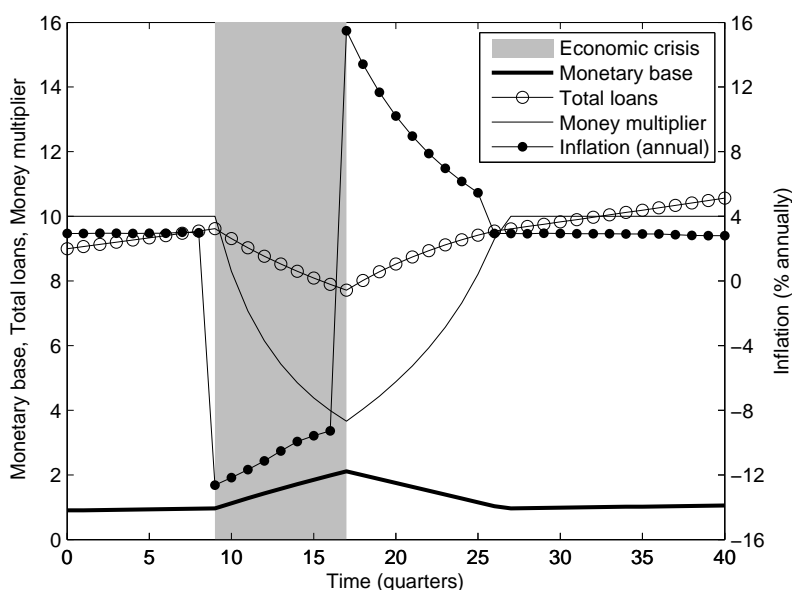
The model was optimized under 3 different parameter configurations, and simulations were carried out for each of the resulting 3 different policy functions. Figures

4.5, 4.6, and 4.7 present the simulation outcomes relating to these configurations. Whereas the parameters applied in the simulation presented in Figure 4.5 are those defined in Table 4.1, the simulations presented in Figures 4.6 and 4.7 involve modifications of some of those parameters.

In the 3 simulations presented here, the real economy is initially in a good condition, but then experiences a crisis, which lasts for a number of periods, after which the economy recovers. The graphs show how the monetary base, the total amount of loans, the money multiplier, and inflation develop over time before, during, and after the crisis. The length of one period is assumed to be a quarter of a year and the simulations, covering 40 periods, thus represent a time span of 10 years.

The parameter configuration applied in Figure 4.5 represents a *base case* to which the other outcomes are compared. In the base case, the total amount of loans drops during the crisis as a result of banks not wanting to lend because of the higher default risk. This, together with the expansionary monetary policy, results in the money multiplier "collapsing" during the crisis, and money becoming endogenous as the reserve requirement ceases to be binding. Substantial deflation is observed during the crisis. Once the crisis ends, also lending (and thus the money supply) starts to increase, which causes inflation, even though the central bank starts to conduct contractionary monetary policy in order to combat inflation. The money multiplier also starts to grow after the crisis has ended, and at some point reaches the level of 10 (i.e. the inverse of the required reserve ratio of 10%) at which the reserve requirement is binding again, and money ceases to be endogenous. With this parameter configuration inflation reaches annual rates higher than 10% during the first year after the crisis. It should, however, be noted that since developments of the real economy are modeled with only 2 states, the end of the crisis may be unrealistically sharp in the simulation. A smoother transition to normal economic conditions might result in lesser rates of inflation after the end of the crisis.

Figure 4.6 illustrates the outcome of the model if the central bank's ability to contract the monetary base is higher than in the base case presented in Figure 4.5. Here the upper limit to contractionary measures, represented by the parameter  $\Delta_{min}^B$ , has been doubled from -0.12 to -0.24, while the maximal expansionary measures remain at the value of 0.17 applied in the base case. As might be expected, the rate of inflation following the crisis is now lower than in the base case, since the central bank is now able to contract base money at a greater speed when attempting to combat inflation. Annual inflation reaches levels in the neighborhood of 5% after the crisis, which is substantially less than in the base case. In addition, after the

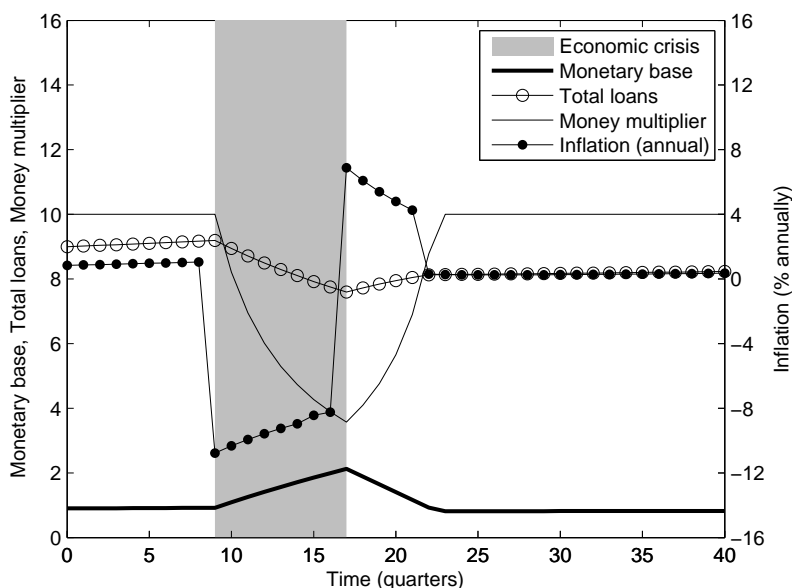


**Figure 4.5:** Base case

crisis, the money multiplier rises more quickly back to the level of 10 where money ceases to be endogenous.

Figure 4.7 depicts the outcome in a case in which the effectiveness of central bank policy is higher than in the base case. The effectiveness of central bank policy, represented by the parameter  $\lambda$  has been increased from 1.5 to 2.5, whereas the maximal expansionary and contractionary measures are the same as in the base case. This means that e.g. a given amount of quantitative easing has a stronger effect here than in the base case, and also that contractionary measures are more efficient. Therefore, the central bank is able to affect the money supply more effectively and, thus, keep inflation as well as deflation at moderate levels. As can be observed in the outcome, the drop in the amount of loans during the crisis is almost negligible and, thus, significantly smaller than in the previous two cases. As a result, significantly lesser rates of deflation are observed during the crisis. Also the rates of inflation observed after the end of the crisis are now lower.

In the model considered in this essay, the total amount of deposits is assumed to equal the total amount of loans, and thus, the total amount of loans may be assumed to approximately represent the monetary aggregate M2. If compared to the developments of M2 illustrated in Figure 4.1, the third case, presented in Figure 4.7 appears to most closely correspond to the outcomes actually observed during the recent recession, particularly in the euro area. In the euro area, the growth rate of the monetary aggregate M2 has dropped slightly during the recent recession, whereas in the United States the growth of M2 appears to have continued without



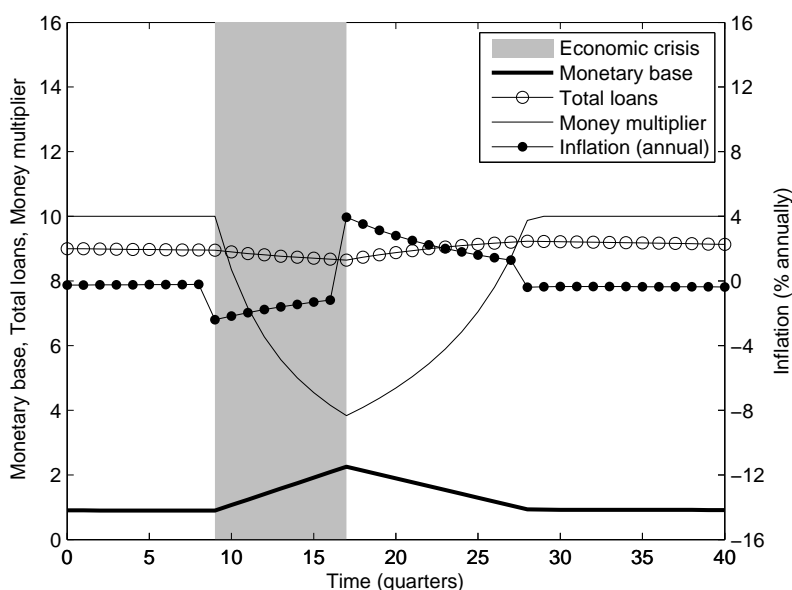
**Figure 4.6:** Fast contraction of the monetary base ( $\Delta_{min}^B = -0.24$ )

interruption. In the third one of the simulations considered above, the total amount of loans actually dropped slightly during the crisis, but the outcome is nonetheless relatively close to the developments observed in the euro area. Also the rates of deflation during the recession are only moderate in the third simulation, and thus at least to some extent in line with actually observed developments. It should be noted, that the model presented in this essay does not include a long-term positive trend of growth of the real economy. Thus, the results could be interpreted as approximately representing developments from which a long-term trend has been subtracted. From this point of view, the outcomes appear to be even closer to the ones observed during the recent recession.

## 4.4 Conclusions

The question whether the expansionary policies carried out by central banks during the recent recession will lead to inflation when the recession ends has obtained some interest in the recent years. In this essay, a model is developed in order to provide a way of analyzing situations such as the recent recession on a general, theoretical level. The focus in the essay is on the developments of monetary aggregates and inflation, whereas a simplified real sector of the economy has also been included in the model.

The model presented includes a central bank, a commercial banking sector and a real economy. The central bank was given the role of the decision-maker who



**Figure 4.7:** Efficient central bank policy ( $\lambda = 2.5$ )

controls the monetary base, thereby affecting the money supply and inflation. The central bank's objective is to keep inflation close to a target level over time. Since the reserve requirements were assumed to not necessarily be binding all the time, also the commercial banking sector was modeled as a decision-maker by allowing it to make decisions regarding the amount of new loans given out in each period. Since giving a new loan is assumed to result in a new deposit being created, the ability of commercial banks to make lending decisions enables them to affect the money supply. Therefore money can become endogenous, which means that the money supply (represented e.g. by a monetary aggregate such as M2) cannot at all times be determined by the central bank.

The model was formulated as a stochastic dynamic programming model, the solution of which is a policy function defining the expansionary or contractionary measures that the central bank takes under different circumstances. The model was solved under different parameter configurations, and the resulting policy functions were then applied in simulations that were used to analyze the outcomes under these parameter configurations. The analysis of the results in the form of three figures in Section 4.3 indicates that the rate of inflation after a crisis may depend, in particular, on two issues: First, the maximum speed of monetary contraction that the central bank can conduct after the crisis, and second, the effectiveness of central bank policy. Effectiveness here refers to how effectively the central bank's policy affects lending by commercial banks (and thus the money supply) in situations when reserve requirements are not binding. A high ability of the central bank to conduct



contractionary policy after the recession as well as high effectiveness of monetary policy have the effect of lowering the rate of inflation that may be expected after a crisis.

Of the three simulation outcomes considered in Section 4.3 the third one, which involved a higher effectiveness of monetary policy, was the one that appeared to most closely resemble the developments actually observed during the recent recession, in particular in the euro area. The parameter configurations resulting in developments resembling the recent recession do not produce radical inflationary outcomes after the end of the simulated recession. Thus, the prediction implied by the model would be that significant inflation is not likely to be observed after the end of this recession, at least not in the euro area. It should, however, be noted that the outcomes of the model depend on the particular values applied to a large number of parameters as well as on the particular model structure applied.

During the Great Depression in the 1930s the situation was somewhat similar to the current one. According to Feinman (1993), the amount of central bank money was then high and it was feared that this would result in high inflation at some point. As a result, the central bank in the U.S. then raised the reserve requirements. The Great Depression did, however, not involve inflation and the developments were perhaps more closely related to the phenomenon of debt deflation described by Fisher (1933). The currently ongoing period of low economic growth might also involve similar deflationary elements. Fears of inflation should be reduced by the fact that central banks have the power to increase the reserve requirements in the case that the contractionary measures are not otherwise sufficient. In addition, as is well-known, the phrase "pushing on a string" is often associated with monetary policy, referring to the asymmetry of the effectiveness of monetary policy, implying that expansionary monetary policy often has less effect than contractionary policy. The model applied here assumes equal effectiveness of measures taken in both directions. If the model was modified in such a way that contractionary measures would be more effective than expansionary ones, the non-inflationary predictions might become even stronger.

Further developments of the model could involve a more careful calibration of the model parameters. One particular improvement in the realism of the model would be to make the effectiveness of monetary policy asymmetric with respect to expansionary vs. contractionary policy. Currently, the same parameter  $\lambda$  is used to describe the effectiveness of monetary policy in both cases. However, the maximal amounts of monetary expansion or contraction (in terms of changes in the monetary base) may differ even in the current model. Increasing the number of different states

of the real economy would also increase realism. Additional states could e.g. include smaller and larger recessions as well as periods of transition between recessions and normal economic conditions. Furthermore, one might also consider modeling the macroeconomy in more detail, perhaps moving into the direction of DSGE models.

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