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Aspects of spectator fields in post-inflationary resonant particle production

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ACADEMIC DISSERTATION

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Into this wild Abyss
The womb of Nature, and perhaps her grave–
Of neither sea, nor shore, nor air, nor fire,
But all these in their pregnant causes mixed
Confusedly, and which thus must ever fight...

- John Milton, *Paradise Lost*

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Abstract

Grounded in the increasingly accurate astronomical observations of the past few decades, the study of cosmology has produced a comprehensive account of the history of the universe. This account is contained in the Hot Big Bang cosmological model which describes the expansion of a hot and dense state to become the universe as we observe it today.

While the Big Bang model has been extremely successful in being able to account for a wide array of cosmological data, it leaves unexplained the special initial conditions that are required in order to produce the universe we find ourselves in. Such initial conditions are, however, a natural consequence of a period of quasi-exponential expansion of the universe known as inflation. Such a period of expansion can be realized if the universe is dominated by a scalar field–the inflation–which is slowly rolling down the slope of its potential. Inflation also provides a natural mechanism for the production of primordial seeds of structure in the universe through the growth of the quantum fluctuations in the inflaton field to super-horizon scales.

Together, inflation and the subsequent Big Bang evolution form the back bone of modern cosmology. However, the transition between the inflationary epoch and the thermal state which characterizes the initial conditions of the Big Bang evolution is not well understood. This process–dubbed reheating–involves the decay of the inflaton field into the particles of the Standard Model of particle physics, and may be highly non-trivial, with non-perturbative resonant processes playing a major role. Spectator fields–light scalar fields which are subdominant during inflation–may also play an important role during this epoch.

The aim of this thesis is to showcase aspects of non-perturbative decay of scalar fields after inflation, focusing in particular on the role of spectator fields. This includes the modulation of the non-perturbative decay of the inflaton by a spectator field, the non-perturbative decay of a spectator into the Standard Model Higgs, as well as the non-perturbative decay of the Higgs field itself.

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List of Included Papers

This thesis is based on the following publications [1–3]:

I **Modulated reheating and isocurvature perturbations**
K. Enqvist and S. Rusak
JCAP **1303**, 017 (2013)

II **Reheating dynamics affects non-perturbative decay of spectator fields**
K. Enqvist, R. N. Lerner, and S. Rusak
JCAP **1311**, 034 (2013)

III **Non-Abelian dynamics in the resonant decay of the Higgs after inflation**
K. Enqvist, S. Nurmi and S. Rusak

In all of the papers the authors are listed alphabetically according to particle physics convention.

The author's contribution to the joint publications

I The present author performed all of the analytic and numerical calculations and produced the first draft of the paper. The process was guided throughout by K. Enqvist who also edited the draft.

II The present author performed all of the analytical and numerical calculations with the exception of the section 7 on perturbative decay. This section was contributed by R. Lerner who also wrote the first draft of the paper which was then jointly edited by all three authors.

III All of the analytic and numerical calculations were performed by the present author who also drafted the first version of the paper jointly with S. Nurmi. The draft was subsequently edited by all three authors.

Contents

Chapter 1

Introduction

Throughout history, questions about the origin and the nature of the universe have been at the forefront of the human intellectual endeavor. Gradually attempts at answering such questions transitioned from the realm of mythology and theology to that of metaphysical speculation and then on to formal scientific study. But only in the past few decades, with the advent of high-precision astronomical measurements, has the study of cosmology become a quantitative and a rigorous discipline. While we can only observe the one universe we live in, making the study of cosmology observational rather than experimental, the ever-increasing accuracy of cosmological observations has allowed for robust testing of cosmological models.

As a result, a comprehensive account of the history of the universe has emerged describing the expansion of a hot and dense state to become the universe as we observe it today. This account is contained in the Big Bang cosmological model according to which the early universe consisted of a hot soup of relativistic particles which either annihilated or formed bound structures in the course of the evolution of the universe as it cooled down as a consequence of expansion. Big Bang cosmology has been able to account for a large number of cosmological observations, most notably the existence of a Cosmic Microwave Background [4–7], the Hubble expansion [8–10], the abundance of light elements [11, 12] and the formation of structure out of primordial seeds [13].

Despite its successes, however, the Hot Big Bang model does not explain the initial conditions of homogeneity and flatness required for the subsequent evolution to produce our universe. It also does not explain the origin of the primordial perturbations that are needed to seed the formation of structure. Another piece of the puzzle was required and turned out to be a period of nearly exponential expansion of the very early universe, dubbed 'inflation' [14–18]. In most models, inflation is realized by a scalar field slowly rolling down the slope of its potential. The inflationary scenario has also been successful; it has helped explain the flatness and homogeneity of the universe and provided a mechanism for the origin of structure through amplification of quantum fluctuations in light fields. On the observational side, it has accounted for the anisotropies in the Cosmic Microwave Background and for the structure of the distribution of voids and galaxy clusters.

The Hot Big Bang evolution amended by a period of inflation now forms the standard model of cosmology. However, the transition between the inflationary era and the Hot Big Bang is not yet well understood. During inflation the field responsible for the quasi-exponential expansion – the inflaton – stays almost constant. After inflation is over the energy contained in this field must be converted into the particles of the heat bath that serve as the starting point for the Hot Big Bang evolution. This decay may be rather complicated with non-perturbative effects involving resonant production of particles playing a major role.

Also other fields than the inflaton may be important for cosmology. Fields which are light during inflation and which contribute a small fraction of the energy density of the universe compared to the inflaton are known as spectator fields. These fields may have important effects on the subsequent evolution of the universe such as being involved in the breaking of symmetries as in the case of the Higgs field [19–22], generate cosmological perturbations as in the curvaton [23–27] and the modulated reheating [28–30] scenarios, or be responsible for dark matter as in the case of the axion [31, 32]. It is therefore important to understand also the decay of these fields after inflation.

The aim of this thesis is to discuss the details of non-perturbative decay after inflation in the context of spectator fields. In particular the focus will be on the possible effects of spectators on the resonant decay of the inflaton as well as their own decay through non-perturbative effects. The discussion is based on the research papers **I-III**. The organization of this thesis is as follows. In the remainder of this chapter I review the standard cosmology confined to the assumption of a classical homogeneous universe. The discussion is extended to non-homogeneous and quantum mechanical effects in Chapters 2 and 3 where the basics of cosmological perturbation theory and quantum field theory respectively are presented. In Chapter 4 I review aspects of quantum fields in the early universe. Chapters 5 through 7 summarize the research papers **I-III** forming the core of the thesis. Finally, I conclude in Chapter 8 with the discussion of the results.

1.1 General Relativity

The theoretical foundation of modern cosmology is the General Theory of Relativity [33] which describes the dynamics of the metric $g_{\mu\nu}$ determining the spacetime interval $\mathrm{d}s^2 = g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu$. The equations of motion are obtained from the Einstein-Hilbert action

$$S = \int \mathrm{d}^4x \sqrt{-\det g_{\mu\nu}} \left(\frac{R}{16\pi G} + \mathcal{L}_{\mathrm{matter}} \right) \tag{1.1}$$

where R is the Ricci curvature scalar, G the Newton's gravitational constant and $\mathcal{L}_{\mathrm{matter}}$ is the Lagrangian density of the matter content. This action leads to the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \tag{1.2}$$

with $R_{\mu\nu}$ being the Ricci tensor and $T_{\mu\nu}$ the stress-energy tensor of matter. The cosmological constant Λ may be explicitly included in the action or it can be implicitly inside $\mathcal{L}_{\mathrm{matter}}$ as vacuum energy.

1.2 Friedman-Robertson-Walker universe

Assuming a homogeneous and an isotropic universe the Einstein equation has a solution in the form of the Friedmann–Lemaître–Robertson–Walker metric [34–38]

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-Kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right)\right] \tag{1.3}$$

where K is related to the spatial curvature on constant time slices, with $K = -1, 0$ or 1 corresponding respectively to an open, flat or a closed universe, and $a(t)$ is the scale factor which evolves according to the Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho \qquad \text{and} \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right). \tag{1.4}$$

The first of these corresponds to the time-time component of the Einstein equation while the second is a combination of the time-time component and the trace of the spatial part. ρ and p denote the energy density and the pressure of the matter content of the universe and these quantities will determine the expansion history. As a consequence of the Bianchi identities for the Einstein tensor [39], the matter content will obey the continuity equation $\nabla_\mu T^{\mu\nu} = 0$, which for a perfect fluid reduces to

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{1.5}$$

and, in fact, if the universe is filled with several different non-interacting fluids these will obey the above continuity equation independently of each other. For fluids which have $p = w\rho$, with w constant, the continuity equation gives the behavior of the energy density as the universe expands:

$$\rho_i \propto a^{-3(1+w_i)}. \tag{1.6}$$

There are three important classes of fluids that are especially relevant in cosmology. These are: dust, radiation and vacuum energy, with $w = 0, 1/3$ and -1 respectively.

1) Dust

Non-relativistic particles, whose energy density is dominated by their rest energy, are referred to as dust. The energy density of dust scales as $\rho \propto a^{-3}$, which can be interpreted simply as the dilution of the particle density by the expansion of the universe. A universe dominated by dust expands as $a \propto t^{2/3}$.

2) Radiation

Relativistic particles, whose momenta dominate over their rest masses, are referred to as radiation and their energy density scales as $\rho \propto a^{-4}$. This can be interpreted as the combined effect of the dilution of the particle number density and the redshifting of the momenta of the particles by the expansion of the universe. A radiation dominated universe expands according to $a \propto t^{1/2}$.

3) Vacuum energy/cosmological constant

Vacuum energy in the context of cosmology has an energy density that stays constant in time. A universe dominated by vacuum energy is described by a *de Sitter* spacetime and expands exponentially, $a \propto e^{Ht}$. The current expansion of the universe appears to be accelerating [40, 41], which could be caused by a vacuum energy component, dubbed 'dark energy'. However, the nature of dark energy is not currently understood. The vacuum energy that one would expect from quantum effects is much too large to be compatible with the current state of the universe [42] and understanding the nature of dark energy is one of the most important open problems in cosmology. The most popular scenarios involve either scalar fields [43] or modification of gravity [44, 45], the most simple of which is the cosmological constant. It has also been proposed that the accelerated expansion is only apparent, due to, for example, our solar system being located near the center of a large void [46], or a feature of structure formation backreacting on the average expansion rate [47–49].

1.2.1 Energy contents of the universe

The three components above are the major players determining the expansion history of the universe. The contribution of these components to the present energy budget of the universe as determined by cosmological data is $\Omega_m = 0.307 \pm 0.019, \Omega_r \simeq 5 \times 10^{-5}$ and $\Omega_{\rm vac} = 0.693 \pm 0.019$ [50] for matter, radiation and dark energy respectively, with $\Omega_i = \rho_i/\rho_c$ being the relative fraction of the critical energy density $\rho_c \equiv \frac{3H_0^2}{8\pi G}$.

A universe filled with matter and radiation would expand at an ever slowing rate as the gravity of these components would resist the expansion. In contrast, a universe filled with vacuum energy would expand at an accelerated rate due to the negative pressure of the vacuum energy component. Since the energy density of matter and radiation goes down as the universe expands while that of vacuum energy stays constant, a universe initially dominated by the former two will eventually become dominated by the latter and transition from decelerating to accelerating expansion. For our universe this transition happened about 9-10 billion years after the Big Bang which is quite close to the present time. The fact dark energy starts to be significant around the present time for us to be able to observe it is sometimes referred to as the coincidence problem. In addition there will also be a transition from radiation domination to matter domination because the energy density of radiation dilutes faster than that of dust.

1.2.2 Thermodynamics and the thermal history of the universe

Knowing the equation of state of the different components of the universe determines its expansion history but a more detailed description is needed in order to understand what went on during the course of its evolution. The early universe had high density allowing particles to interact efficiently and reach thermal equilibrium. Applying statistical physics and thermodynamics to the early universe allows for the description of the processes that were important at those early times.

The early universe consisted of a thermal bath of particles obeying the distribution function

$$n(k,T,\mu) = \frac{1}{e^{(\epsilon-\mu)/T} \pm 1} \tag{1.7}$$

with plus and minus corresponding to fermions and bosons respectively and where $\epsilon = \sqrt{k^2/a^2 + m^2}$ is the energy of the particles, μ the chemical potential and T the temperature. The energy density and the pressure of particles in the thermal bath are then

$$\rho = g \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \epsilon(k,m) n(k,T,\mu), \qquad p = g \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \frac{k^2}{3\epsilon(k,m)} n(k,T,\mu), \tag{1.8}$$

where g is the number of the internal degrees of freedom for the particle. For relativistic particles, this can be approximated by

$$\rho = 3p = \frac{\pi^2}{30} g_* T^4, \tag{1.9}$$

with $g_* = g$ for bosons and $g_* = \frac{7}{8} g$ for fermions. Since all of the relativistic particles behave essentially in the same way, one can define an effective number of degrees of freedom by counting all of the particles in the bath, $g_* = \sum_i \left(g_{i,BE} + \frac{7}{8} g_{i,FD} \right)$. When of all of the Standard Model particles are in thermal equilibrium $g_* = 106.75$. As the temperature of the universe goes down, heavy particle species will annihilate leaving less relativistic degrees of freedom.

The universe also underwent several important transitions, such as the QCD phase transition [51], electroweak symmetry breaking [52, 53], neutrino decoupling [54], Big Bang Nucleosynthesis [55] and recombination [56]. This thesis is mainly concerned with the transition from the inflationary era to the subsequent radiation dominated phase characterized by thermal equilibrium so these topics fall outside of its purview. I refer the interested reader to the references cited above.

1.3 Inflation

While the ΛCDM-model outlined above has been very successful in describing the evolution of the universe it requires initial conditions that are difficult to motivate. In particular, the universe must have started out as extremely flat and extremely homogeneous in order to exhibit these properties today to the observed accuracy. It also does not appear to contain exotic particles one would expect from high energy models, such as Grand Unification Theories (GUT). These are known as flatness, horizon and relic problems in cosmology. In order to solve these problems the theory of inflation was introduced which also provides a mechanism for generating the seeds of structure that are necessary for the formation of galaxies under the influence of gravity.

Flatness problem

The universe today is very close to being flat, with $|\Omega - 1| \lesssim 10^{-2}$ [50]. The deviation from critical density evolves as

$$|\Omega(t) - 1| = \frac{|K|}{\dot{a}^2}, \tag{1.10}$$

growing over time for a decelerating universe ($\ddot{a} < 0$) and decreasing for an accelerating universe ($\ddot{a} > 0$). Thus a universe dominated by matter would need to be more flat in the past. For a matter dominated universe, this means that the energy density at the time of nucleosynthesis must have deviated from the critical value by less than 10^{-16} and at the Planck epoch by less than 10^{-64} [57]. This constitutes a very specific initial condition which is difficult to motivate within Big Bang cosmology.

Horizon problem

Another problem relating to the initial conditions of the Big Bang is the horizon or the homogeneity problem. The universe appears to be homogeneous on much larger scales than the scale of the past causal horizon. This is most easily seen in the Cosmic Microwave Background which is uniform to the accuracy of 10^{-5} and yet the causal horizon at the time when the CMB was formed corresponds to approximately one degree in the sky. Thus, regions in the sky which are more than one degree apart have never been in causal contact and so it is puzzling why they would have the same temperature.

Relic problem

Yet another possible problem faced by the pure Big Bang cosmology is the problem of cosmic relics. High energy extensions of the Standard Model such as Grand Unified Theories and Supersymmetry predict the formation of various exotic particles and structures, such as magnetic monopoles, strings, gravitinos etc. at high temperatures. The predicted abundances of these artifacts in the pure Big Bang model are such that their effects should be detectable. The fact that they are not [58] creates another difficulty for the Big Bang scenario.

Inflation: basic idea

Inflation offers a mechanism by which these initial conditions are produced naturally as a result of fairly simple dynamics. The universe is said to inflate when the expansion is accelerating, though often the term is used exclusively to describe the accelerated expansion in the early universe and to exclude the current acceleration of the expansion. In order to achieve inflation one needs negative pressure since according to the Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{1.11}$$

Such negative pressure is easily achieved if the universe is dominated by a scalar whose potential energy exceeds its kinetic energy. For a homogeneous scalar field ϕ

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V, \qquad p_\phi = \frac{1}{2}\dot{\phi}^2 - V. \tag{1.12}$$

The energy continuity equation then gives the equation of motion for the field as

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0. \tag{1.13}$$

In order to have enough inflation the potential has to be sufficiently flat so that the change in the scalar field is sufficiently slow. This is achieved when the following slow-roll conditions are satisfied:

$$|\ddot{\phi}| \ll |H\dot{\phi}| \qquad \text{and} \qquad \dot{\phi}^2 \ll V \tag{1.14}$$

If these conditions are satisfied then the following slow-roll parameters

$$\epsilon \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \text{and} \qquad \eta \equiv M_{\rm pl}^2 \frac{V''}{V} \tag{1.15}$$

describing the flatness of the potential are small. Alternatively, one can define slow-roll parameters in terms of the spacetime geometry instead of the potential as

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} \qquad \text{and} \qquad \eta_H \equiv \frac{\dot{\epsilon}_H}{H\epsilon_H}. \tag{1.16}$$

When the slow-roll parameters are zero expansion is exponential and we are in *de Sitter* space. During inflation $\dot{a} \sim He^{Ht}$ grows so that deviations from critical density decay according to Equation (1.10) thus solving the flatness problem. The comoving causal horizon during inflation stays constant $d_p^c \sim (a_i H_i)^{-1}$ where the subscript 'i' corresponds to the beginning of inflation. In order to solve the horizon problem the scales corresponding to the observable universe $k = (a_0 H_0)$ must have been inside the causal patch so that

$$N \equiv \ln\frac{a_{\rm end}}{a_i} > 61 - \ln\frac{10^{16}\ \text{GeV}}{V_i^{1/4}} + \ln\frac{V_i^{1/4}}{V_{\rm end}^{1/4}} - \frac{1}{3}\ln\frac{V_{\rm end}^{1/4}}{\rho_{\rm reh}^{1/4}} \tag{1.17}$$

where subscripts 'reh' and 'end' refer respectively to the epochs of reheating and the end of inflation. For slow-roll inflation and efficient reheating the last two terms are small so that for a scale of 10^{16} GeV about 60 e-folds of inflation are needed to solve the horizon problem. Finally, inflation solves the relic problem by diluting the density of unwanted artifacts to be too small to be observable.

While inflation was first introduced in order to solve the initial condition problems of the Big Bang scenario discussed above, perhaps its biggest success has been to provide a natural mechanism for generating structure in the universe. It turns out that quantum fluctuations of scalar fields in a *de Sitter* background are stretched to macroscopic scales [59–62] to become classical perturbations in energy density which eventually lead to formation of galaxy clusters and other structures in the universe. In order to understand this process it is necessary to go beyond the assumption of a homogeneous and a classical universe and therefore I postpone the discussion of it until Chapter 4. In the next two chapters I shall review the basics of cosmological perturbation theory and quantum field theory.

Chapter 2

Cosmological perturbation theory

While the assumption of homogeneity and isotropy simplifies calculations considerably, the real universe is not completely homogeneous and isotropic and therefore more sophisticated methods are required in order to describe it. In fact, the universe today is extremely non-homogeneous with matter clumped into stars while galaxies are separated by huge expanses of empty space. Fortunately, the universe is still statistically homogeneous and especially the early universe was almost uniform. For this reason perturbation theory provides an appropriate tool to describe it at early times, and even today on very large scales [63, 64].

The metric may be separated into the Robertson-Walker part and a small perturbation around it, $g_{\mu\nu} = g_{\mu\nu}^{(\mathrm{RW})} + \delta g_{\mu\nu}$. As a symmetric rank-2 tensor the metric has 10 degrees of freedom which may be decomposed as

$$\mathrm{d}s^2 = -(1+2A)\mathrm{d}t^2 + 2aB_i\mathrm{d}t\mathrm{d}x^i + a^2[(1-2\psi)\delta_{ij} + 2E_{ij}]\mathrm{d}x^i\mathrm{d}x^j. \tag{2.1}$$

The background metric was taken to be spatially flat. These metric perturbations can be further decomposed according to their transformation properties under spatial rotations resulting in 4 scalar, 4 vector and 2 tensor degrees of freedom. The matter content of the universe can be similarly perturbed and the perturbations obey the Einstein equation

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}, \tag{2.2}$$

where $\delta G_{\mu\nu}$ is the perturbation in the Einstein tensor calculated from the metric (2.1) and $\delta T_{\mu\nu}$ the perturbation in the stress-energy tensor. Focusing on the scalar perturbations, the perturbed Einstein equations take the form [65, 66]

$$\begin{aligned}
3H(\dot\psi + HA) - \frac{\nabla^2}{a^2}[\psi + H(a^2\dot E - aB)] &= 4\pi G\delta T^0{}_0, &(2.3)\\
(\dot\psi + HA)_{,i} &= -4\pi G\delta T^0{}_i, &(2.4)\\
\left[(2\dot H + 3H^2)A + H(\dot A + 3\dot\psi) + \ddot\psi + \frac{1}{2}\nabla^2 D\right]\delta^i_j - \frac{1}{2}D_{,ij} &= 4\pi G\delta T^i{}_j, &(2.5)
\end{aligned}$$

where $D \equiv a^{-2}[(A-\psi) - H(a^2\dot{E} - aB) - \frac{\mathrm{d}}{\mathrm{d}t}(a^2\dot{E} - aB)]$.

Not all of these degrees of freedom are physical, however, as two of the scalar and two of the vector perturbations are gauge degrees of freedom corresponding to coordinate reparametrization. The tensor perturbations are gauge independent. Because of the gauge freedom quantities depend on the choice of coordinate system. There are two ways of dealing with this ambiguity. One can fix the gauge by imposing constraints on the perturbations. A variety of different gauges are employed in the literature as different choices prove useful in different circumstances. Some of the most frequently used include the Newtonian or the longitudinal gauge ($E = B = 0$), synchronous gauge ($A = B = 0$), the flat gauge ($\psi = 0$), and the uniform density gauge ($\delta\rho = 0$) [65, 67].

Another approach is to define combinations of the perturbations which are invariant under gauge transformations. These include the Bardeen potentials [68]:

$$\Phi = A - \frac{\mathrm{d}}{\mathrm{d}t}\Big(a^2\dot{E} - aB\Big), \tag{2.6}$$
$$\Psi = \psi + H(a^2\dot{E} - aB). \tag{2.7}$$

which correspond to gravitational potential and curvature perturbations in the Newtonian gauge as well as the perturbation

$$\zeta \equiv -\psi - \frac{H}{\dot{\rho}}\delta\rho \tag{2.8}$$

which can be identified with the curvature perturbation in the uniform density gauge. This quantity is especially useful because in the absence of non-adiabatic pressure perturbations it stays constant on super-horizon scales [69]. One can also define a corresponding quantity ζ_i for each non-interacting fluid separately, corresponding to the curvature perturbation on a hypersurface of uniform density for that fluid.

2.1 Isocurvature perturbations

So far we have been discussing perturbations which can be traced to the overall fluctuation in energy density. These are referred to as adiabatic perturbations. It is, however, also possible to have perturbations in the relative densities of different components which leave the overall density unperturbed. These are referred to as isocurvature or entropy perturbations. An isocurvature perturbation between two cosmic fluids, labeled by i and j can be defined as

$$\mathcal{S}_{ij} \equiv -3H\left(\frac{\delta\rho_i}{\dot{\rho}_i} - \frac{\delta\rho_j}{\dot{\rho}_j}\right) = -3(\zeta_i - \zeta_j). \tag{2.9}$$

The isocurvature perturbations are typically defined with respect to the density perturbation in the radiation fluid resulting in possible isocurvature perturbations for baryons, cold dark matter and neutrinos[1] [70, 71]. So far, no isocurvature perturbations have been detected and cosmological data

[1] There is also a possibility that the neutrino fluid has a different velocity from the photon fluid resulting in a neutrino velocity isocurvature perturbation $S^V_{\nu r} \equiv v_\nu - v_r$.

are consistent with purely adiabatic perturbations [72–74].

2.2 ΔN formalism

An extremely powerful tool for analyzing the evolution of perturbations is the ΔN-formalism also known as the separate universe approach [69, 75, 76]. The idea is to treat different regions of the universe on super-horizon scales as separate FRW-universes which are locally homogeneous and isotropic. The curvature perturbation is then defined as the difference between different patches which can be obtained by calculating the difference in integrated expansion. The spatial part of the metric may be expressed as

$$g_{ij} = \tilde{a}^2(\mathbf{x}, t)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j \tag{2.10}$$

where $\tilde{a}(\mathbf{x}, t) \equiv a(t)e^{-\psi(\mathbf{x},t)}$ is the local scale factor and $\gamma_{ij} = \left(\mathbf{1}e^h\right)_{ij}$ contains the tensor degrees of freedom. Then the local integrated expansion from some time t_* to a later time t is given by

$$N(t, t_*; \mathbf{x}) = \psi(\mathbf{x}, t_*) - \psi(\mathbf{x}, t) + \ln\frac{a(t)}{a(t_*)}. \tag{2.11}$$

Specifically, by choosing the flat slicing on the initial hypersurface ($\psi_* = 0$) and the final slicing to give the surface of uniform density we obtain the curvature perturbation

$$\zeta(\mathbf{x}, t_1) = -\psi(\mathbf{x}, t)|_{\rho=\rho(t)} = N(t, t_*; \mathbf{x}) - N(t) \equiv \Delta N \tag{2.12}$$

where $N(t)$ gives the integrated expansion between two flat hypersurfaces. Essentially the curvature perturbation is the difference in integrated expansion from a flat hypersurface to one of uniform density. The curvature perturbation may then be easily connected to inflation since the different number of e-foldings N of inflation result from superhorizon perturbation in the inflaton so that the curvature perturbation may be written simply as

$$\zeta = N'\delta\phi_* + \frac{1}{2}N''\delta\phi^2 + ... \tag{2.13}$$

The ΔN formula is extremely useful because it describes the curvature perturbation to all orders of perturbation theory on large scales and therefore lends itself to the calculation of non-Gaussianity where the required second-order cosmological perturbation theory is cumbersome. However, if the spatial gradients are large it may no longer be applicable, and in fact, ΔN formalism corresponds to the first order in gradient expansion [77, 78] so it is still a perturbative result in this sense.

2.3 Spectra of perturbations

Since the seeds of the cosmological perturbations come from random quantum fluctuations during inflation it is the statistics of the perturbations rather than their actual values that contain the information about the underlying physics. It is therefore necessary to be able to quantify the statistical behavior of the cosmological perturbations. For a Gaussian perturbation ξ the statistics are fully determined by the power spectrum $\mathcal{P}_\xi$ defined through

$$\langle \xi_{\mathbf{k}} \xi_{\mathbf{k}'} \rangle \equiv (2\pi)^3 P_\xi(k) \delta(\mathbf{k}+\mathbf{k}') \equiv \frac{2\pi^2}{k^3} \mathcal{P}_\xi(k) \delta(\mathbf{k}+\mathbf{k}') \tag{2.14}$$

where the $\xi_{\mathbf{k}}$ are the Fourier modes and $\langle ... \rangle$ denotes the ensamble average. In particular, we are interested in the power spectrum for the curvature perturbations ζ which can be expressed using the ΔN formula simply as

$$\mathcal{P}_\zeta(k) = \left(\frac{\partial N}{\partial \phi}\right)^2 \mathcal{P}_{\delta\phi} = \frac{\mathcal{P}_{\delta\phi}}{2\epsilon_H M_{\mathrm{pl}}^2} \tag{2.15}$$

where $\mathcal{P}_{\delta\phi}$ is the power spectrum of the inflaton perturbations which to lowest order in slow-roll is given by $\left(\frac{H}{2\pi}\right)^2$ evaluated at the moment when a particular scale exits the horizon during inflation as will be shown in Chapter 4. From the CMB measurements the power spectrum is determined to be $\mathcal{P}_\zeta \approx 2.4 \times 10^{-9}$ [72]. Inflation produces an almost scale invariant spectrum but the deviation from scale invariance is an important discriminator between various inflation models and can be quantified with the spectral index

$$n_s - 1 \equiv \frac{\mathrm{d} \ln \mathcal{P}_\zeta}{\mathrm{d} \ln k}. \tag{2.16}$$

The experimentally determined value for the spectral index is $n_s \approx 0.96030 \pm 0.0073$ [72].

2.4 Non-Gaussianity

The perturbations may also deviate from Gaussianity in which case one can define higher order spectra in a similar manner. The so called bispectrum can be defined in terms of the three-point functions as

$$\langle \xi_{\mathbf{k_1}} \xi_{\mathbf{k_1}} \xi_{\mathbf{k_3}} \rangle \equiv (2\pi)^3 \delta(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}) B_\xi(k_1,k_2,k_3) \tag{2.17}$$

This is a good quantity to look at because the three-point function, as well as all other odd functions, vanishes for Gaussian perturbations. The most common type of Non-Gaussianity studied in the literature is the local type where the curvature perturbation is given by $\zeta = g + bg^2$ with g being a Gaussian perturbation. The deviation from Gaussianity is conventionally parametrized by the non-linearity parameter

$$\frac{6}{5} f_{\mathrm{NL}} \equiv \frac{B_\zeta(k_1,k_2,k_3)}{P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)} \tag{2.18}$$

Using the ΔN-formalism non-Gaussianity from inflation is readily calculated as

$$\frac{6}{5} f_{\mathrm{NL}} = \frac{N''}{N'^2}. \tag{2.19}$$

So far cosmological data are consistent with Gaussian perturbations with deviation from Gaussianity being constrained to be $f_{\rm NL} = 2.7 \pm 5.8$ [79]. In simplest single-field inflation models, non-Gaussianity is proportional to the slow-roll parameters and expected to be negligible [80, 81] and so detecting significant non-Gaussianity would immediately rule out the simplest models. Multi-field models, especially those where the curvature perturbation is generated after inflation can produce significant levels of non-Gaussianity as will become apparent in subsequent chapters.

Chapter 3

Quantum Field Theory

The current understanding of the fundamental building blocks of matter in the universe is expressed in terms of quantum fields permeating the whole of spacetime. A quantum theory of fields is obtained by taking a classical field theory, characterized by an action $S = \int \mathrm{d}^4x \mathcal{L}(\phi_a, \partial_\mu \phi_a)$ which is a functional of fields ϕ_a and their derivatives, and demanding that the fields obey an equal time commutation relation $[\phi_a(\mathbf{x}), \pi_b(\mathbf{y})] = i\delta_{ab}\delta(\mathbf{x} - \mathbf{y})$ where π_a is a canonical variable conjugate to the field ϕ_a. Quantum excitations of the fields can then be interpreted as corresponding particles. A powerful alternative method of quantization exists which is known as the path integral formalism [82, 83]. This method employs functional integrals of e^{iS} over all possible field configurations. Nevertheless, its usefulness is limited in cosmology because of the time dependent background. Therefore I focus on the canonical quantization procedure in the following.

The Standard Model of particle physics contains 12 fields corresponding to leptons and quarks — the building blocks of matter. The theory also obeys a number of symmetries which require the existence of gauge fields corresponding to photons, massive gauge bosons $W^{\pm}$ and Z, as well as gluons for $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ symmetries respectively. Finally, the theory contains the Higgs field responsible for generating the masses of gauge bosons, leptons and quarks.

In all likelihood there are more fields beyond the Standard Model because many problems, such as the existence of dark matter, neutrino masses, inflation and the baryon and lepton asymmetries in the universe, cannot be addressed within the Standard model.

3.1 Scalar fields

The simplest case in quantum field theory is that of a real scalar field with zero spin. A non-interacting scalar field ϕ obeys the Klein-Gordon equation of motion

$$\ddot{\phi} + (-\nabla^2 + m^2)\phi = 0. \tag{3.1}$$

A field is quantized by promoting the field to an operator and imposing an equal time commutation relation between the field and its canonical momentum field so that $[\hat{\phi}(t, \mathbf{x}), \hat{\pi}(t, \mathbf{y})] = i\delta(\mathbf{x} - \mathbf{y})$

while other commutators vanish. The field may be expanded as

$$\phi(x) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}} \left[u_{\mathbf{k}}(t)\hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + u^*_{\mathbf{k}}(t)\hat{a}^\dagger_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}} \right], \tag{3.2}$$

where $u_{\mathbf{k}}(t)$ are the solutions of the classical equation of motion and $\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}}$ are the annihilation and creation operators with commutation relations $[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = \delta(\mathbf{k}-\mathbf{k}')$, $[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}] = [\hat{a}^\dagger_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = 0$. The commutation relation for the fields then implies a normalization condition $u\dot{u}^* - u^*\dot{u} = i$ for the mode functions. The general solution for the harmonic oscillator equation of motion is $u_k(t) = \alpha_k e^{i\omega_k t} + \beta_k e^{-i\omega_k t}$ which is compatible with the commutation relations if $|\alpha_k|^2 - |\beta_k|^2 = \hbar/2\omega_k$. The preferred mode function in Minkowski space is the positive frequency solution with $\beta = 0$ and $\alpha = 1$ as will become apparent in the next section.

The solutions can then be used to build a Fock space of quantum states with the vacuum state given by $a_{\mathbf{k}}|0\rangle = 0$ and excited states

$$|n_{\mathbf{k}}\rangle = \frac{(a^\dagger_{\mathbf{k}})^n}{\sqrt{n!}}|0\rangle. \tag{3.3}$$

Once the free theory has been constructed interactions can be taken into account by perturbative expansion around the free solutions as long as interaction terms are small compared to the free Lagrangian.

3.2 Bogolyubov transformation

An important caveat in the quantization of fields is the choice of the basis. For example, a scalar field may be quantized by introducing the expansion (3.2) where the mode functions $u_k(t)$ obey the equation of motion for the field. However, a linear combination of the mode functions $v_k = \alpha u_k + \beta u^*_k$ and its complex conjugate are also solutions to the equation of motion so one may equally well use the expansion

$$\phi(x) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}} \left[v_{\mathbf{k}}(t)\hat{b}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + v^*_{\mathbf{k}}(t)\hat{b}^\dagger_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}} \right], \tag{3.4}$$

where b and $b^\dagger$ are another pair of creation and annihilation operators which can be used to build a Fock space of quantum states. If the first set of mode functions is canonically normalized then the quantization condition imposes the relation $|\alpha|^2 - |\beta^2| = 1$. The relation between the two sets of operators is known as the Bogolyubov [84] transformation and is then

$$\left\{ \begin{array}{rcl} \hat{b}_{\mathbf{k}} & = & \alpha_{\mathbf{k}}\hat{a}_{\mathbf{k}} + \beta^*_{-\mathbf{k}}\hat{a}^\dagger_{-\mathbf{k}} \\ \hat{b}^\dagger_{\mathbf{k}} & = & \alpha^*_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{k}} + \beta_{-\mathbf{k}}\hat{a}_{-\mathbf{k}} \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{rcl} \hat{a}_{\mathbf{k}} & = & \alpha^*_{\mathbf{k}}\hat{b}_{\mathbf{k}} - \beta^*_{\mathbf{k}}\hat{b}^\dagger_{-\mathbf{k}} \\ \hat{a}^\dagger_{\mathbf{k}} & = & \alpha_{\mathbf{k}}\hat{b}^\dagger_{\mathbf{k}} - \beta_{\mathbf{k}}\hat{b}_{-\mathbf{k}} \end{array} \right. \tag{3.5}$$

As this transformation mixes creation and annihilation operators the vacuum state of one set of operators will not be the vacuum of the other and will instead contain particles with the occupation number $n_{\mathbf{k}} = |\beta_k|^2$. In the original basis the Hamiltonian is given by

$$
\begin{aligned}
\hat{H} &= \int \mathrm{d}^3\mathbf{x}\left[\frac{1}{2}\dot{\hat{\phi}}^2 + \frac{1}{2}(\nabla\hat{\phi})^2 + \frac{1}{2}\hat{\phi}^2\right] \\
&= \int \mathrm{d}^3\mathbf{k}\left[\frac{1}{2}(\dot{u}_k^2 + \omega_k^2 u_k^2)\hat{a}_{\mathbf{k}}\hat{a}_{-\mathbf{k}} + \frac{1}{2}(\dot{u}_k^{*2} + \omega_k^2 u_k^{*2})\hat{a}^\dagger_{\mathbf{k}}\hat{a}^\dagger_{-\mathbf{k}}\right. \\
&\quad \left. + \left(|\dot{u}_k|^2 + \omega_k^2|u_k|^2\right)\left(\hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{k}} + \frac{1}{2}\right)\right].
\end{aligned}
\tag{3.6}
$$

while in the new basis it is obtained by replacing u_k with v_k and $\hat{a}$ with $\hat{b}$. If the vacuum state $|0\rangle_a$ defined by operators $\hat{a}$ is to be an eigenstate of the Hamiltonian we must have $\dot{u}_k^{*2} + \omega_k^2 u_k^{*2} = 0$. Then a corresponding expression for the new mode functions is $\dot{v}_k^{*2} + \omega_k^2 v_k^{*2} = 2\alpha^*\beta^*(|\dot{u}_k|^2 + \omega_k^2|u_k|^2)$ and does not vanish for non-zero α and β. Therefore, the vacuum state $|0\rangle_b$ defined by operators $\hat{b}$ will not be an energy eigenstate.

Thus, there is a preferred basis of mode functions—that which yields the eigenstate of the Hamiltonian. The situation changes when one moves into the realm of time dependent backgrounds which one encounters in cosmology. Then in general the vacuum state that is initially an eigenstate of the Hamiltonian will not remain so.

3.3 Fermions

Fermions are fields with half-integer spin and they constitute the matter degrees of freedom: quarks, leptons and neutrinos. The free fermionic action is

$$
S_F = -\int \mathrm{d}^4x \bar{\psi}^\dagger\gamma^0(i\gamma^\mu\partial_\mu + m)\psi \tag{3.7}
$$

and the corresponding equation of motion is the Dirac equation

$$
(i\gamma^\mu\partial_\mu + m)\psi = 0 \tag{3.8}
$$

where γ^μ are Dirac matrices obeying the anticommutation relation $\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$ and whose solutions are spinors. Fermionic fields are quantized similarly to scalar fields but instead of imposing a commutation relation the fields are required to satisfy the anticommutation relation $\{\psi_a, \psi_b^\dagger\} = \delta_{ab}\delta(x-y)$ where a and b refer to the spin degrees of freedom. After that the quantum field theory may be constructed analogously to the spin-0 case.

3.4 Vector bosons

Another important class of fields are the vector bosons which transform as four-vectors under Lorentz transformations. The vector fields within the Standard Model arise as a consequence of gauge symmetries. The action for a gauge boson A_μ is

$$
S_{\mathrm{gauge}} = -\int \mathrm{d}^4x\left[\frac{1}{2}\,\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \left(\mathcal{D}_\mu\Phi\right)^\dagger\left(\mathcal{D}^\mu\Phi\right) + \bar{\psi}\gamma^\mu\mathcal{D}_\mu\psi\right] \tag{3.9}
$$

where the field strength tensor is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig\left[A_\mu, A_\nu\right] \tag{3.10}$$

and Φ and ψ are respectively scalar and fermion fields charged under the gauge symmetry with the covariant derivatives being $\mathcal{D}_\mu = \partial_\mu - igA_\mu$. If the matter fields transforms as $\Phi \to \zeta\Phi$, $\psi \to \zeta\psi$ under a local gauge transformation then the vectors will transform according to

$$A_\mu \to A'_\mu = \zeta^{-1} A_\mu \zeta + \frac{i}{g}\zeta^{-1}\left(\partial_\mu \zeta\right). \tag{3.11}$$

The gauge groups present in the Standard Model are $U(1)$, corresponding to the electromagnetic interaction (in the broken electroweak phase) with the gauge field being the photon, $SU(2)$ describing weak interactions with $W^\pm$ and Z as the gauge bosons, and $SU(3)$ corresponding to the strong interactions with eight gluon fields being the gauge bosons. A free gauge field obeys the equation of motion

$$\partial_\mu F^{\mu\nu} + g\left[A_\mu, F^{\mu\nu}\right] = 0 \tag{3.12}$$

where the $\nu = 0$ component is in fact not a dynamical equation but a constraint known as *Gauss' law*. The field strength tensor also obeys the Bianchi identity which can be expressed in terms of the Jacobi commutator identity

$$[\mathcal{D}_\mu, [\mathcal{D}_\nu, \mathcal{D}_\lambda]] + [\mathcal{D}_\lambda, [\mathcal{D}_\mu, \mathcal{D}_\nu]] + [\mathcal{D}_\nu, [\mathcal{D}_\lambda, \mathcal{D}_\mu]] = 0. \tag{3.13}$$

Vector fields may be quantized analogously to scalar field quantization by imposing commutation relations on fields and their canonical conjugates. However, the difficulty in the procedure involves identifying the relevant degrees of freedom. Out of the four vector components only three are physical in the case of massive (Proca) fields and two in the case of massless fields. One may first note that the temporal component A_0 does not possess a canonical conjugate due to the antisymmetricity of the field strength tensor. For a massive vector field, however, A_0 may be expressed in terms of the other components so it is not an independent field. For a massless field gauge invariance entrails another unphysical degree of freedom which is typically removed by choosing a particular gauge through requiring some simplifying condition to hold for the fields. A further caveat arises because this gauge condition or the Gauss' law may be inconsistent with the canonical commutation relation though this can be remedied by requiring that these conditions hold for the physical states rather than operators [85–88].

3.5 Radiative corrections to classical dynamics

Interactions between fields will induce quantum corrections to the dynamics of the fields. For example the classical trajectory of a scalar field in a potential $V(\phi)$ is determined from the equation of motion

$$\phi'' - \nabla^2\phi + V' = 0. \tag{3.14}$$

To leading order, the quantum fluctuations $\delta\phi$ around the classical trajectory $\phi_{\rm cl}$ will obey the Klein-Gordon equation (3.1) with an effective mass $m_{\rm eff}^2 \equiv k^2 + V''(\phi_{\rm cl})$ and can be expanded using (3.2). Using the Hartree approximation the last term in (3.14) may be expanded as $V' = V'(\phi_{\rm cl}) + \frac{1}{2}V'''(\phi_{\rm cl})\langle\delta\phi^2\rangle$ where

$$\left\langle \delta\phi^2 \right\rangle = \int \frac{{\rm d}^3\mathbf{k}}{(2\pi)^3\omega_k}\left(\frac{1}{2} + n_{\mathbf{k}}\right). \tag{3.15}$$

The first term corresponds to the correction coming from the vacuum while the second is due to the presence of particles.

Vacuum correction

Let us first consider the vacuum contribution, i.e., the $n_{\mathbf{k}} = 0$ case in (3.15). The integral in question is in fact divergent but can be regularized by introducing a cutoff Λ for the momentum. This gives

$$\frac{1}{2}V'''(\phi_{\rm cl})\langle\delta\phi^2\rangle_{\rm vac} = \frac{1}{8\pi^2}\frac{{\rm d}m_{\rm eff}^2}{{\rm d}\phi_{\rm cl}}\int_0^\Lambda \frac{{\rm d}k\, k^2}{\sqrt{k^2+m_{\rm eff}^2}} = \frac{\rm d}{{\rm d}\phi_{\rm cl}}\left[\frac{1}{8\pi^2}\int_0^\Lambda {\rm d}k\, k^2\sqrt{k^2+m_{\rm eff}^2}\right]. \tag{3.16}$$

The expression in the square brackets is then the vacuum contribution to the effective potential. After calculating the integral this can be written as [89]

$$V_{\rm eff}^{\rm vac} = V_\infty + \frac{m_{\rm eff}^4}{64\pi^2}\ln\frac{m_{\rm eff}^2}{\mu^2}, \tag{3.17}$$

where μ is some arbitrarily chosen energy scale and V_∞ is the part containing terms which diverge as $\Lambda \to \infty$. This infinity can be removed through renormalization by a redefinition of the bare mass and the couplings [90].

Running of the couplings

The potential arising from vacuum corrections depends on an arbitrary renormalization scale μ (see equation (3.17)). For example for the self-coupling term $\frac{\lambda}{4}\phi^4$ the resulting effective potential is

$$V_{\rm eff} = \frac{1}{4}\left[\lambda + \frac{9\lambda^2}{16\pi^2}\ln\left(\frac{3\lambda\phi^2}{\mu^2}\right)\right]\phi^4. \tag{3.18}$$

Since the physics must be independent of this scale the coupling must have corresponding dependence given by

$$\beta_\lambda \equiv \frac{\partial\lambda}{\partial\ln\mu} = \frac{9\lambda^2}{8\pi^2} \tag{3.19}$$

known as the running of the coupling. Thus the coupling depends on the energy scale of the process under consideration. If its value is known at some energy scale, for example from experiment, it must be run from that scale to the scale of interest in order to obtain the correct effective potential. This can be continued at higher orders in perturbation theory to obtain higher loop corrections and all of couplings will run. This produces a set of coupled differential equations known as renormalization group equations leading to a renormalization group improved potential

$$V = \frac{\lambda(\phi)}{4}\phi^4. \tag{3.20}$$

Temperature correction

If the field is not in the vacuum state then the effective potential will also receive a contribution from the presence of particles. This may be estimated once the distribution of particles n_k is known. In particular, for bosons in a thermal bath it is the Bose-Einstein distribution $n_k = 1/(e^{\omega_k/T} - 1)$ and we have

$$\frac{1}{2}V'''(\phi_{\rm cl})\langle\delta\phi^2\rangle_T = \frac{1}{4\pi^2}\frac{\mathrm{d}m_{\rm eff}^2}{\mathrm{d}\phi_{\rm cl}}\int_0^\infty \frac{\mathrm{d}k\, k^2}{\sqrt{k^2+m_{\rm eff}^2}\left(e^{\sqrt{k^2+m_{\rm eff}^2}/T}-1\right)} \tag{3.21}$$

If $m_{\rm eff} \gg T$ the integral is negligible so that there is no thermal correction for non-relativistic particles. In the relativistic limit the integral gives $\pi^2/6$ so that the correction to the effective potential is

$$V_{\rm eff}^T = \frac{1}{24}\frac{\mathrm{d}^2 m_{\rm eff}^2}{\mathrm{d}\phi_{\rm cl}^2}T^2\phi^2 \tag{3.22}$$

which manifests as a thermal correction to the mass of the field. A more detailed analysis using finite-temperature field theory reveals also other effects such as dissipative effects of the thermal bath [91–93]; however, the above approximation will be sufficient for the purposes of this thesis.

3.6 The electroweak sector

The elements discussed above comprise the basic building blocks of quantum field theory and when applied to the known constituents of matter form the Standard Model of particle physics. As this thesis will in large part focus on the production and decay of the Higgs field and the weak gauge bosons after inflation, the electroweak sector of the Standard Model warrants a closer examination. The action of the EW sector is given by

$$S_{\rm EW} = -\int \mathrm{d}^4x\left[\frac{1}{2}\,\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \left(D_\mu\Phi\right)^\dagger D^\mu\Phi + \lambda\left(\Phi^\dagger\Phi - \frac{\nu^2}{2}\right)^2\right] \tag{3.23}$$

where $F_{\mu\nu}$ is given by (3.10) with $SU(2)$ fields A_μ, $G_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$ being the field strength tensor of a $U(1)$ gauge field B_μ and Φ being the Higgs field. The covariant derivative in this case is $D_\mu = \partial_\mu - igA_\mu - \frac{1}{2}ig'B_\mu$.

Higgs mechanism

The Higgs field is responsible for making the gauge fields massive through the Higgs mechanism [19–22] when it has a non-zero vacuum expectation value (VEV). Let us choose the unitary gauge by requiring that the Higgs can be expressed as $\Phi = \frac{1}{\sqrt{2}}\left(0 \;\; h+\delta h\right)^T$ where h is the VEV of the Higgs field. Then the kinetic term $(D_\mu\Phi)^\dagger D^\mu\Phi$ gives a quadratic interaction contribution for the gauge fields proportional to h^2. This term may be diagonalized to obtain mass and charge eigenstates if new gauge fields are defined as

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\Big(A_\mu^1 \pm iA_\mu^2\Big), \qquad Z_\mu \equiv \cos\theta_W A_\mu^3 - \sin\theta_W B_\mu, \qquad \mathcal{A}_\mu \equiv \sin\theta_W A_\mu^3 + \cos\theta_W B_\mu$$

with masses

$$m_W^2 = \frac{g^2h^2}{4}, \qquad m_Z^2 = \frac{m_W^2}{\cos^2\theta_W} \qquad \text{and} \qquad m_{\mathcal{A}}^2 = 0, \tag{3.24}$$

where A_μ^a are the components of A_μ in the basis of $SU(2)$ generators and the weak mixing angle is defined by $\tan\theta_W \equiv g'/g$. The fields $W^\pm$ and Z are the charged and the neutral weak gauge bosons respectively and $\mathcal{A}_\mu$ is the photon.

As can be seen from the action (3.23) at low energies the minimum of the Higgs potential is at $h = \nu$ which has been experimentally determined to be 246 GeV. However, temperature corrections change the potential so that for sufficiently high temperatures characteristic of the early universe, the vacuum expectation value is zero making the gauge fields massless and restoring the electroweak symmetry. If the Standard Model is valid all the way up to inflationary scales then the symmetry would have been broken also during inflation as the Higgs would have been a light field with a VEV comparable to the Hubble scale during inflation as we shall see in the next chapter.

Stability of the vacuum

As was discussed in section 3.5 the couplings run with energy because of the quantum vacuum corrections. The situation is especially interesting in the case of the self-coupling of the Higgs field. All of the fields the Higgs is coupled to contribute to the running and since bosons and fermions produce contributions of opposite signs the coupling becomes negative above the energy scale of $\mu_c \sim 10^{10}...10^{11}$ GeV when run from the values measured at the electroweak scale [94–97]. At energies higher than this scale the potential becomes unstable. However, the value of the instability scale is sensitive to the SM parameters, especially the mass of the top quark, so it may be pushed all the way to the Planck scale if the values of the parameters differ sufficiently from best-fit values from measurements. This is depicted in Figure 3.1.

This feature is important for inflation where energies might easily exceed the instability scale so that we couldn't find ourselves in the current electroweak vacuum. Thus, high inflationary energies suggest either that new physics intervene to stabilize the potential or that the true values of the SM parameters are somewhat different than the best-fit values obtained from measurement. A related

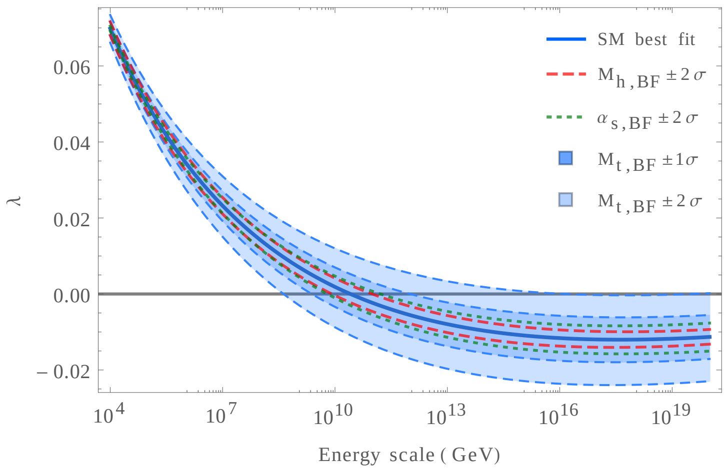

Figure 3.1: Running of the Higgs self-coupling. The thick blue line corresponds to the best-fit values [98, 99] of the SM parameter measurement. The blue regions correspond to 1σ and 2σ deviation for the top mass while the red dashed and the green dotted lines correspond to 2σ deviations in the Higgs mass and the strong coupling respectively.

issue is the possibility that the vacuum we occupy may not be the true vacuum state and that the universe might tunnel into the true vacuum. Our vacuum should nevertheless be at least metastable meaning that its lifetime should exceed the age of the universe [94].

Chapter 4

Quantum fields in the early universe

Having reviewed cosmological perturbation theory and the theory of quantum fields we are now in a position to describe the behavior of quantum fields in the early universe. The discussion in this chapter will be limited to scalar fields and will focus on the generation of perturbations from quantum fluctuation during inflation as well as post-inflationary decay processes.

4.1 Inflaton perturbations

As I discussed in Chapter 1, a homogeneous field slowly rolling down a potential well will have negative pressure and therefore if it dominates the energy density of the universe this will result in a period of inflation. However, the field will also experience quantum fluctuations around the classical trajectory. The field may be divided into a homogeneous part and a perturbation $\delta\phi$ which in the flat slicing obeys the equation of motion [100]

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + \frac{k^2}{a^2}\delta\phi + \left[V'' - \frac{8\pi G}{a^3}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a^3}{H}\dot{\phi}^2\right)\right]\delta\phi = 0. \tag{4.1}$$

In conformal time, the equation of motion for the rescaled field $u = a\delta\phi$ is then

$$u'' + \left[k^2 - \mathcal{H}^2\left(2 + 5\epsilon_H - 3\eta + \epsilon_H\eta - 2\epsilon_H^2 + 2\frac{\dot{\epsilon}_H}{H}\right)\right]u = 0. \tag{4.2}$$

To leading order in slow-roll the solution can be expressed in terms of Hankel functions as $u = \alpha(-k\tau)^{1/2}H_\nu^{(1)}(-k\tau) + \beta(-k\tau)^{1/2}H_\nu^{(2)}(-k\tau)$, where $\nu = \frac{3}{2} + \frac{3}{2}\epsilon_H - \frac{1}{2}\eta$. Deep within the horizon ($k \ll \mathcal{H}$) the mode simply oscillates analogously to the Minkowskian case so it is natural to choose the initial condition at $\tau \to -\infty$ for the mode to be the Minkowski vacuum state $u \simeq \frac{1}{\sqrt{2k}}e^{-ik\tau}$. This choice is known as the Bunch-Davies vacuum [101]. With the correct normalization we get for the perturbations

$$u_k \simeq \sqrt{\frac{\pi}{4k}}e^{i\frac{\pi}{2}\left(\nu+\frac{1}{2}\right)}(-k\tau)^{1/2}H_\nu^{(1)}(-k\tau). \tag{4.3}$$

Outside the horizon ($k \gg \mathcal{H}$) the mode approaches

$$u_k \simeq \frac{1}{\sqrt{2\pi k}}\left(-\frac{k\tau}{2}\right)^{-\left(\nu-\frac{1}{2}\right)} e^{i\frac{\pi}{2}\left(\nu-\frac{1}{2}\right)}\Gamma(\nu) = \frac{e^{i\frac{\pi}{2}\left(\nu-\frac{1}{2}\right)}}{\sqrt{8k}}\left(-\frac{k\tau}{2}\right)^{-\left(\nu-\frac{1}{2}\right)}\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}. \tag{4.4}$$

The power spectrum for inflaton perturbations outside the horizon is then

$$\mathcal{P}_{\delta\phi} \simeq \left(\frac{H}{2\pi}\right)^2\left[\frac{\Gamma\left(\frac{3}{2}+\frac{3}{2}\epsilon_H-\frac{1}{2}\eta\right)}{\Gamma\left(\frac{3}{2}\right)}\right]^2\left[2(1-\epsilon_H)\left(\frac{k}{aH}\right)\right]^{\eta-3\epsilon_H}. \tag{4.5}$$

Since this changes very slowly we may evaluate it at the time of horizon crossing when $k = aH$. Thus, the perturbations become constant with the power spectrum to leading order in slow-roll $\mathcal{P} = \left(\frac{H_*}{2\pi}\right)^2$. This behavior is depicted in Figure 4.1. The above analysis in fact applies to any light field during inflation $(m^2 \ll H^2)$ so any such field will be left with a spectrum of superhorizon perturbations. The power spectrum for the resulting curvature perturbation is then given by equation (2.15) as discussed in Chapter 2.

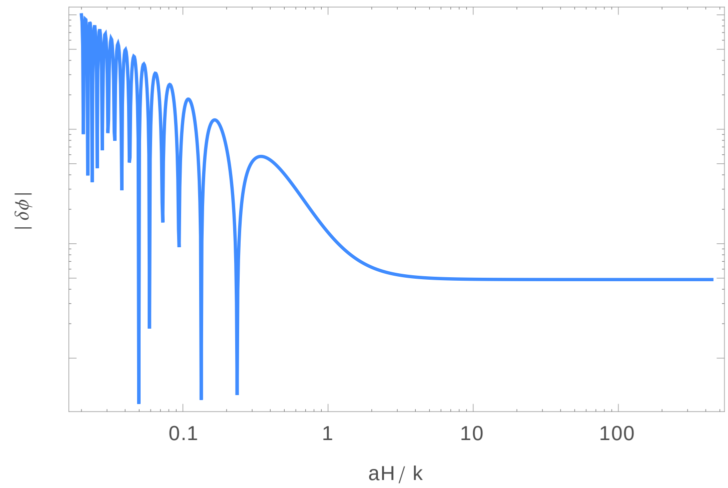

Figure 4.1: Freeze-in of inflaton perturbations.

Gravitational waves

The tensor modes of the metric obey the same equation of motion with $\eta = 0$, so they too will acquire a spectrum of superhorizon perturbations given by

$$\mathcal{P}_T = \frac{8}{M_{\rm pl}^2}\left(\frac{H_*}{2\pi}\right)^2. \tag{4.6}$$

Since the spectrum is given directly by the Hubble rate during inflation, a detection of gravitational waves would constitute a determination of the energy scale of inflation in a model-independent way. The ratio of the tensor power spectrum to that of scalar curvature perturbations has been constrained from CMB measurements by the Planck satellite to be $r < 0.11$ [72]; however, recently a detection corresponding to $r \sim 0.1..0.2$ has been claimed [102] though the result may be due to dust contamination [103, 104]. If the result is correct, however, it would correspond to an inflationary scale of $H \sim 10^{14}$ GeV.

4.2 Equilibrium state of light fields

Since super-horizon perturbations in light fields are constantly being generated from random quantum fluctuations during inflation, the field on superhorizon scales in essence experiences a type of Brownian motion where the generated perturbations stochastically jolt it from the background trajectory. This allows for a statistical description of light fields originally due to Starobinsky [105–109]. For simplicity let's consider a perfect *de Sitter* evolution with $\epsilon_H = 0$. The field may be decomposed into super- and subhorizon parts as follows

$$\phi(\mathbf{x}, t) = \bar{\phi}(\mathbf{x}, t) + \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}} \theta(k - aH) \left[\phi_k \hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + \phi_k^* \hat{a}_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}}\right] \tag{4.7}$$

where $\bar{\phi}$ is the field averaged on superhorizon scales and ϕ_k are the sub-horizon mode functions discussed in the previous section. From the slow-roll equations it then follows that the smoothed field $\bar{\phi}$ obeys the Langevin equation

$$\dot{\bar{\phi}}(\mathbf{x}, t) = -\frac{V'(\bar{\phi})}{3H} + f(\mathbf{x}, t) \tag{4.8}$$

with the stochastic noise term given by

$$f(\mathbf{x}, t) = \frac{aH^2}{(2\pi)^{3/2}} \int \mathrm{d}^3k \, \delta(k - aH) \left[\phi_k \hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + \phi_k^* \hat{a}_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}}\right] \tag{4.9}$$

having the two-point correlator $\langle f(\mathbf{x}, t) f(\mathbf{x}, t')\rangle = \frac{H^3}{4\pi^2}\delta(t - t')$. The quantities of interest are the averages of functions of $\bar{\phi}$ which may be calculated with the probability distribution function $P(\bar{\phi})$ which obeys the Focker-Planck equation

$$\frac{\partial P}{\partial t} = -\frac{V'(\bar{\phi})}{3H}\frac{\partial P}{\partial \bar{\phi}} + \frac{H^3}{8\pi^2}\frac{\partial^2 P}{\partial \bar{\phi}^2}. \tag{4.10}$$

If inflation lasts a sufficiently long time we expect this distribution to settle into the equilibrium state which is solved by

$$P(\bar{\phi}) \propto \exp\left[-\frac{8\pi^2}{3H^4} V(\bar{\phi})\right] \tag{4.11}$$

which should be normalized to one for each potential. This gives the probability distribution function for the light field after inflation.

4.3 Reheating

At the end of inflation the universe is left cold and void of any matter other than the condensate of the inflaton and other possible scalar fields which were light during inflation. Whatever matter might have been present before, will have been diluted away by nearly exponential expansion. In order to facilitate the beginning of the standard Hot Big Bang cosmology the energy stored in the inflaton field needs to be transferred to the Standard Model degrees of freedom and dark matter. This process is referred to as 'reheating' and it requires that the inflaton be coupled to the SM fields in some way.

The overall dynamics depend sensitively on the particle physics models and so the details of what went on during this phase of the universe's history are poorly understood. Nevertheless, many models share some basic features and it is the aim of this theses to outline the different processes that go into the decay of the scalar fields after inflation. After the end of inflation the inflaton starts to oscillate around the minimum of its potential and the equation of state parameter is [89, 110]

$$w \equiv \frac{p}{\rho} \simeq \frac{\langle V'\phi\rangle - \langle 2V\rangle}{\langle V'\phi\rangle + \langle 2V\rangle}, \tag{4.12}$$

where the brackets denote averaging over the oscillation. For a power law potential $V \propto \phi^n$ this gives $w = (n-2)/(n+2)$. Thus, the quadratic potential leads to matter-like expansion ($w = 0$) while a quartic potential results in a radiation-like evolution ($w = 1/3$). As the inflaton oscillates it decays producing particles it is coupled to. This may happen either through perturbative processes or non-perturbatively through parametric resonance.

Perturbative decay

The transfer of energy may proceed through three-point and four-point interactions. The three point interactions in the Lagrangian are of the form $-g\sigma\phi\chi$ and $-h\phi\bar{\psi}\psi$ for bosons and fermions respectively and they correspond to a decay of one inflaton particle into two bosons or fermions with the respective decay widths [111, 112]

$$\Gamma_\chi \equiv \Gamma(\phi \to \chi\chi) = \frac{g^2\sigma^2}{8\pi m} \qquad \text{and} \qquad \Gamma_\psi \equiv \Gamma(\phi \to \psi\psi) = \frac{h^2 m}{8\pi}. \tag{4.13}$$

Here m is the mass of the inflaton and σ is a mass scale which ensures the correct dimensions. The four-point interaction in the Lagrangian is of the form $-\frac{1}{2}g^2\phi^2\chi^2$ and it corresponds to two inflaton particles annihilating to produce two bosons as well as to the scattering of inflaton particles off of bosons χ.

The decay may be modeled phenomenologically with equations

$$\dot{\rho}_\phi + 3H(1+w)\rho_\phi = -\Gamma_{\rm eff}\rho_\phi \tag{4.14}$$

$$\dot{\rho}_{\rm SM} + 4H\rho_{\rm SM} = \Gamma_{\rm eff}\rho_\phi \tag{4.15}$$

where w is the equation of state parameter for the inflaton fluid and $\Gamma_{\rm eff}$ is the effective decay rate. The inflaton transfers its energy to the decay products when $H \sim \Gamma_{\rm eff}$. Until this point the density of the decay products evolves according to

$$\rho_{\rm SM} \simeq \frac{6M_{\rm pl}^2 H_*^2}{5-3w}\left(\frac{\Gamma_{\rm eff}}{H_*}\right)\left(\frac{a}{a_*}\right)^{-4}\left[\left(\frac{a}{a_*}\right)^{\frac{5-3w}{2}} e^{-\Gamma_{\rm eff}(t-t_*)} - 1\right] \tag{4.16}$$

If the decay products thermalize immediately the temperature can be obtained from equation (1.9). The temperature rises rapidly until it reaches the maximum[1] $T_{\rm max} \sim (H\Gamma_{\rm eff}M_{\rm pl}^2)^{1/4}$ after which it decays as $T \propto a^{-\frac{3(1+w)}{8}}$ until the inflaton has decayed at $H \sim \Gamma_{\rm eff}$ leaving the reheating temperature $T_R \sim \sqrt{\Gamma_{\rm eff} M_{\rm pl}}$.

4.4 Preheating

The perturbative decay analysis, which was developed shortly after the introduction of inflation and which has been outlined above, does not take into account the coherent nature of inflaton oscillations and the resulting time dependence of the particle masses. It was soon realised [117–122] that this leads to a non-perturbative effect, dubbed 'preheating', where particles may be produced exponentially in a much more efficient manner than the perturbative analysis would suggest (see [113, 123] for recent reviews). Consider the potential

$$V = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{1}{2}g^2\phi^2\chi^2 \ , \tag{4.17}$$

where ϕ is the inflaton and χ is some bosonic field coupled to it. Due to the large value of the inflaton, the field χ is heavy during inflation and its vacuum expectation value is zero. However, as the inflaton field starts to oscillate after the end of inflation the quantum fluctuations in the field χ are resonantly amplified in a non-perturbative effect known as parametric resonance. Of particular interest are the two limiting regimes $\lambda = 0$ and $m = 0$, henceforth referred to as the free and the massless case respectively.

The solution of the equation of motion for the oscillating inflaton is then of the form $\phi = \Phi(t)\Pi(t)$, where $\Phi(t)$ is the amplitude and $\Pi(t)$ is a periodic function. For the two cases of interest these are

$$\begin{cases} \Phi(t) = \Phi_* a^{-3/2}, & \Pi(t) = \sin(mt), \qquad \text{for } \lambda = 0 \\ \Phi(t) = \Phi_* a^{-1}, & \Pi(t) = {\rm cn}\left[\sqrt{\lambda}\frac{\phi_*}{H_*}\left(\sqrt{2H_* t} - 1\right), \frac{1}{\sqrt{2}}\right], \quad \text{for } m = 0 \end{cases}$$

where the subscript '$*$' refers to the onset of oscillations and ${\rm cn}[{\rm x},{\rm k}]$ is the Jacobi elliptic cosine.

[1] It should be noted that if the temperature is above the GUT scale of about 10^{16} GeV there could be a second phase of production of monopoles [113] putting an upper limit on the possible temperature. In the context of supergravity the limit can be even lower as gravitinos may be overproduced already at 10^9 GeV [114–116].

Production of χ-particles

The equations of motion for the rescaled fields $X \equiv a^{3/2}\chi$ can be rewritten as

$$\ddot{X} + \omega^2 X = 0 \qquad \text{with} \qquad \omega^2 = \left(\frac{k}{a}\right)^2 + g^2\phi^2 + \frac{3}{2}wH^2 \tag{4.18}$$

where w is the effective equation of state parameter of the universe and the term containing it is small after the onset of oscillation and vanishes identically in a matter-like background so it can be neglected. As the inflaton oscillates, the solutions grow exponentially within specific bands of momenta while those outside of these instability bands are oscillatory. The strength of amplification is characterized by the resonance parameter

$$q \equiv \frac{g^2\Phi^2}{4m_{\text{eff}}^2} \tag{4.19}$$

where m_{eff} is the effective mass of the inflaton. In particular, two important regimes can be identified as the narrow resonance, with $q \ll 1$, and the broad resonance, with $q \gg 1$. The nature of particle production is quite different in these two regimes: in the broad resonance regime solutions are amplified within broad bands of momenta $k \lesssim a m_{\text{eff}} q^{1/4}$ every time the inflaton crosses zero and the adiabaticity condition $\dot{\omega}/\omega^2 < 1$ is violated. Note that modes with momenta much larger than the inflaton mass can be amplified so that broad resonance can produce particles where perturbative decay would be kinematically blocked. In the narrow resonance regime, solutions are amplified within narrow bands of momenta centered around $k_{\text{phys}}^2 \sim m_{\text{eff}}^2(l^2 - 2q)$ of width $\Delta k \sim l^{-1} q^l m_{\text{eff}}$ where l is a positive integer. For the massless case the physical momentum k_{phys} should be replaced with the comoving momentum. Since the bands get progressively narrower the dominant contribution comes from the first band with $l = 1$.

For the massive non-interactive case, equation (4.18) corresponds to the time dependent Mathieu equation and for the massless self-interacting case to the Lamé equation [124]. The structure of resonance for the Mathieu equation is depicted in Figure 4.2; the Lamé equation shares a similar structure which can be seen from Figure 1 of reference [3] (note the slightly different parametrization). In an expanding universe the crucial difference between the two limiting cases is that the massless case is conformal to a corresponding Minkowski case so the momenta which are within resonance bands remain so indefinitely. In the massive case, both momenta and the resonance parameter q change with the expansion of the universe so that momenta which are initially within a resonance band will exit it eventually. As we shall see, this will produce a pattern of stochastic resonance.

Broad resonance

For $q \gg 1$, the evolution in the regions between zero-crossings is adiabatic ($\omega'/\omega^2 \ll 1$) and the solutions are well described by the WKB approximation:

$$X_\kappa(t) \simeq \frac{\alpha}{\sqrt{2\omega_\kappa}} e^{-i\int \omega_\kappa \mathrm{d}t} + \frac{\beta}{\sqrt{2\omega_\kappa}} e^{i\int \omega_\kappa \mathrm{d}t}. \tag{4.20}$$

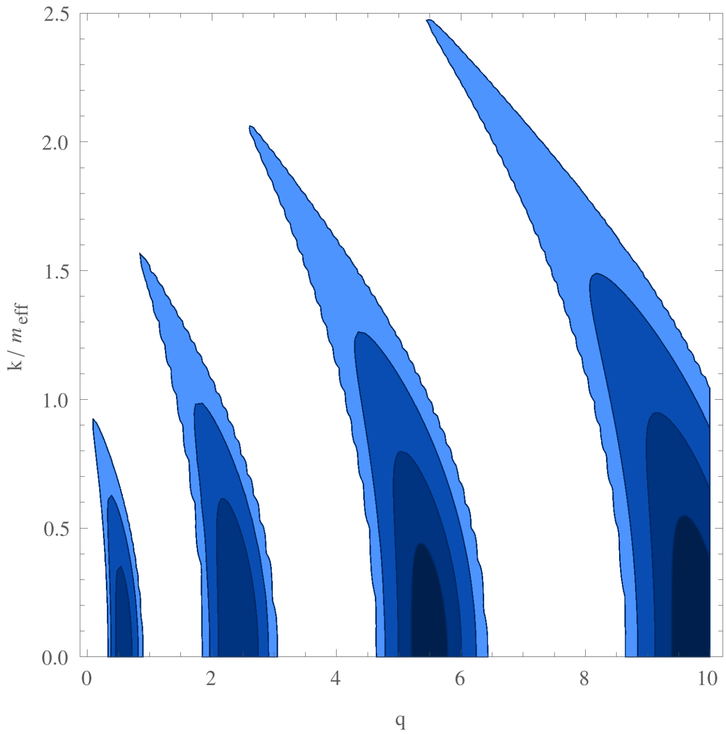

Figure 4.2: The structure of resonance for the Mathieu equation. The solutions are amplified exponentially within the blue bands with the shade indicating the strength of the amplification.

Near the jth zero-crossings the inflaton may be linearized $\phi \simeq \dot{\phi}_j(t-t_j)$ so that the equation of motion becomes

$$X'' + \left(\kappa_j^2 + x^2\right) X = 0 \tag{4.21}$$

where $x \equiv \sqrt{g|\dot{\phi}_j|}(t-t_j)$ and $\kappa_j^2 \equiv k^2/ga_j^2|\dot{\phi}_j|$ and is analogous to the Schrödinger equation with a parabolic potential. The problem can then be treated as a scattering between two WKB modes with some coefficients α^j and β^j before the zero-crossing and α^{j+1} and β^{j+1} after. The general solution to the above equation can be obtained in terms of parabolic cylinder functions which leads to the following relation between the coefficients before and after the zero-crossing [121]

$$\begin{pmatrix} \alpha_\kappa^{j+1} \\ \beta_\kappa^{j+1} \end{pmatrix} = \begin{pmatrix} \sqrt{1+W_\kappa^j}e^{i\varphi_\kappa} & i\sqrt{W_\kappa^j}e^{2i\theta_\kappa^j} \\ -i\sqrt{W_\kappa^j}e^{-2i\theta_\kappa^j} & \sqrt{1+W_\kappa^j}e^{-i\varphi_\kappa} \end{pmatrix} \begin{pmatrix} \alpha_\kappa^j \\ \beta_\kappa^j \end{pmatrix} \tag{4.22}$$

where $W_\kappa^j \equiv e^{-\pi\kappa_j^2}$, $\varphi_\kappa = \arg\Gamma\left(\frac{1+i\kappa_j^2}{2}\right) + \frac{\kappa_j^2}{2}\left(1+\ln\frac{2}{\kappa_j^2}\right)$ and $\theta^j \equiv \int_{t_j}^{t_{j+1}} \omega_k \mathrm{d}t$ is the phase accrued between zero-crossings. After j zero crossings the solution can be interpreted as a Bogolyubov transformation from an initial positive frequency solution ($\alpha^0 = 1$, $\beta^0 = 0$), and so the number of particles in the new vacuum is simply $n_\kappa^j = |\beta_\kappa^j|^2$ and particles are produced each time the inflaton crosses zero according to

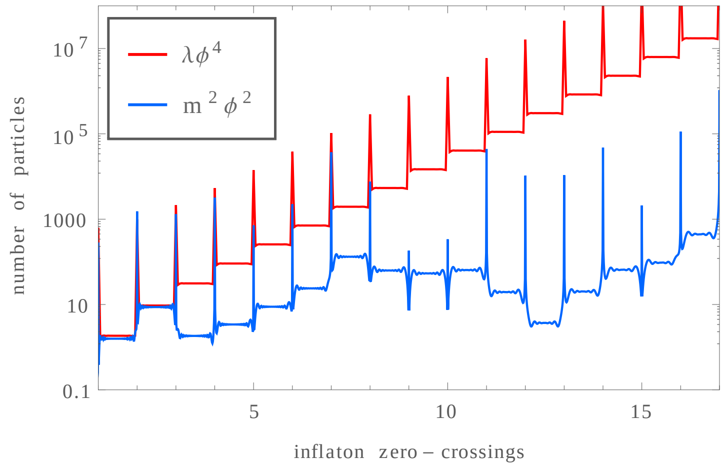

Figure 4.3: Number of created particles for mode $k = 0.01 m_{\text{eff}}$ for quartic (red) and quadratic (blue) inflaton potential. In the latter case the pattern of stochastic resonance can be seen where particles are sometimes destroyed due to destructive interference caused by the expansion of the universe. In the quartic case particles continue to be produced as it is conformal to resonance in Minkowski space.

$$n_\kappa^{j+1} = W_\kappa^j + \left(1 + 2W_\kappa^j\right) n_\kappa^j - 2\sin\theta_{\text{tot}^j}\sqrt{W_\kappa^j(1+W_\kappa^j)}\sqrt{n_\kappa^j(1+n_\kappa^j)}. \tag{4.23}$$

In the limit $n_\kappa \gg 1$ this reduces to $n_\kappa^{j+1} \simeq e^{2\pi\mu_\kappa^j} n_\kappa^j$ with the Floquet index

$$\mu_\kappa^j \equiv \frac{1}{2\pi}\ln\left[1 + 2W_\kappa^j - 2\sin\theta_{\text{tot}^j}\sqrt{W_\kappa^j(1+W_\kappa^j)}\right]. \tag{4.24}$$

The Floquet index can take values in the range $\ln(3-2\sqrt{2}) \leq 2\pi\mu \leq \ln(3+2\sqrt{2})$ depending on the value of θ_{tot} and thus production of particles will only occur within certain resonance bands of momenta where $\mu > 0$. In the massless case the parameters do not depend on time and so the modes which are initially within these bands will be produced with each zero-crossing of the inflaton. In contrast, in the free case the parameters evolve in time and modes which are inside a resonance band at one zero-crossing may be outside of it at the next. In fact, in the broad resonance regime the phase accrued between two consecutive zero-crossings is very large so that $\theta_{\text{tot}} \gg \pi$. Thus, this phase behaves essentially as a random variable leading to the pattern of stochastic resonance where particles may be either created or destroyed as the modes go through resonance bands. This is depicted in Figure 4.3.

Note that particles will be produced efficiently only for $\pi\kappa_j^2 < 1$. Since $\dot{\phi} \sim m_{\text{eff}}\Phi$ this leads to a cutoff scale $k_{\text{cut}}^2 \sim 2\pi^{-1}a_j^2\sqrt{q_j}m_{\text{eff}}^2$. The total number of χ-particles can be obtained by counting all of zero crossings and integrating over all momenta. With the use of the saddle-point approximation this can be estimated to be

$$n_\chi = \frac{1}{(2\pi a)^3}\int \mathrm{d}^3\, k n_k \simeq \frac{(g\Phi_0 m_0^{\text{eff}})^{3/2}}{16\pi^3\sqrt{2\mu m_{\text{eff}} t}} e^{2\mu m_{\text{eff}} t} \tag{4.25}$$

where μ is the maximum value of the average of Floquet index $\mu_k^{\text{av}} \equiv \pi(m_{\text{eff}}t)^{-1}\sum_j \mu_k^j$.

Backreaction

As more and more χ-particles are produced they will eventually reach high enough densities to affect the dynamics of the oscillating inflaton field. This effect can be estimated in Hartree approximation by adding a term

$$g^2\langle\chi^2\rangle\phi^2 = \frac{g^2\phi^2}{(2\pi a)^3}\int \mathrm{d}^3k\ |X|^2 \sim \frac{gn_\chi}{\Phi}\phi^2 \tag{4.26}$$

to the inflaton equation of motion. This is essentially an induced mass and backreaction becomes important when it becomes of the same order as the initial effective mass of the inflaton, $g^2\langle\chi^2\rangle \sim m_{\text{eff}}^2$. From equation (4.25) this happens at

$$m_{\text{eff}}t \simeq \frac{1}{4\mu}\ln\left[5\times 10^5\frac{m_{\text{eff}}^2 t\mu}{g^5\Phi}\right]. \tag{4.27}$$

This time can be considered as the end of preheating as it terminates shortly after backreaction becomes important [121].

4.5 Generating the curvature perturbation after inflation

In the beginning of this chapter we saw how curvature perturbations may arise from quantum fluctuations in the inflaton field during inflation. However, there are mechanisms which can generate the curvature perturbation after inflation from isocurvature perturbations in spectator fields. These are fields that are light during inflation and therefore also acquire a spectrum of perturbations but these fields give a negligible contribution to the energy density of the universe during inflation. The two most prominent examples of such mechanisms are the modulated reheating [28–30] and the curvaton scenarios [23–27].

Modulated reheating scenario

The idea behind the modulated reheating scenario is that the decay rate of the inflaton Γ depends on some spectator field σ. As was discussed earlier in this chapter the inflaton decays into radiation when $H = \Gamma$. Since Γ now depends on the spectator field which has slightly different values in different parts of the universe due to perturbations from inflation this transition will happen inhomogeneously in space. If the inflaton is assumed to oscillate in a harmonic potential after inflation then the evolution is locally matter-like prior to $H = \Gamma(\sigma)$ and radiation-like subsequently to it. Due to the inhomogeneity of the transition some parts of the universe will spend more time in the matter-like stage than others thus losing less of their energy, which results in density, or correspondingly, curvature perturbations.

Since $H \propto a^{-3(1+w)/2}$ for constant equation of state parameter w the number of e-folds from the end of inflation to some time after reheating is

$$N \equiv \ln\left(\frac{a}{a_{\rm end}}\right) = -\frac{2}{3}\ln\left(\frac{\Gamma}{H_{\rm end}}\right) - \frac{1}{2}\ln\left(\frac{H}{\Gamma}\right). \tag{4.28}$$

so that with the use of ΔN-formalism the curvature perturbation on uniform energy hypersurfaces is

$$\zeta_{\rm MR} = -\frac{1}{6}\frac{\Gamma'}{\Gamma}\delta\sigma_* = -\frac{g'}{3g}\delta\sigma_*, \tag{4.29}$$

where $\delta\sigma_*$ refers to field perturbations at the time of horizon crossing, and the last equality comes from assuming $\Gamma \propto g^2$ with $g = g(\sigma)$ a coupling constant. The power spectrum and non-Gaussianity in this model are

$$\mathcal{P}_\zeta^{\rm MR} = \left(\frac{\Gamma'}{6\Gamma}\right)^2\left(\frac{H_*}{2\pi}\right)^2, \qquad f_{\rm NL}^{\rm MR} = 5\left(1 - \frac{\Gamma''\Gamma}{\Gamma'^2}\right). \tag{4.30}$$

Curvaton scenario

In the curvaton scenario the curvature perturbation also originates from the quantum fluctuations of a spectator – the curvaton – and is generated after inflation. The central point of this mechanism is that a light spectator σ starts to oscillate in a harmonic potential at some time $t_{\rm osc}$ during radiation domination. Since the energy density of the field decreases slower than that of radiation its relative contribution to the overall density grows until it decays at some later time $t_{\rm dec}$. As its contribution to the energy density increases so does its contribution to the curvature perturbation to the point where it can account for the entire primordial perturbation if it decays after having become the dominant component. The number of e-folds from the onset of oscillation to the time of the decay of the curvaton is

$$N = \ln\left(\frac{\rho_{\sigma,\rm dec}}{\rho_{\sigma,\rm osc}}\right)^{1/3} = \ln\left(\frac{\rho_{r,\rm dec}}{\rho_{r,\rm osc}}\right)^{1/4} \tag{4.31}$$

Since $\rho_r = \rho_{\rm tot} - \rho_\sigma$ and we are interested in hypersurfaces of uniform total density the derivatives at $t_{\rm osc}$ and $t_{\rm dec}$ can be related by

$$\frac{\partial\rho_{\sigma,\rm dec}}{\partial\sigma_*} \simeq (1 - r_{\rm dec})\left(\frac{\rho_{r,\rm dec}}{\rho_{r,\rm osc}}\right)^{3/4}\frac{\partial\rho_{\sigma,\rm osc}}{\partial\sigma_*} \tag{4.32}$$

where $r_{\rm dec} \equiv \left.\frac{3\bar\rho_\sigma}{3\bar\rho_\sigma + 4\bar\rho_r}\right|_{\rm dec}$. During slow-roll evolution the field value changes very little so that $\rho_{\sigma,\rm osc} \simeq \frac{1}{2}m_\sigma^2\sigma_*^2$ leading to

$$\mathcal{P}_\zeta^{\rm curvaton} = \frac{4r_{\rm dec}}{9\sigma_*^2}\left(\frac{H_*}{2\pi}\right)^2, \qquad f_{\rm NL}^{\rm curvaton} = \frac{5}{3} + \frac{5}{6}r_{\rm dec} - \frac{5}{4r_{\rm dec}} \tag{4.33}$$

As we have seen quantum fields can play an important role in the very early universe and many different aspects of their dynamics can be relevant in the post-inflationary epoch, including the generation of super-horizon perturbations from light fields as well as perturbative and non-perturbative particle production. So far we have looked at these effects in isolation but in more realistic models such effects are likely to be present concurrently and the interactions of these effects may change the dynamics or produce wholly new effects. In the following chapters I shall discuss how non-perturbative decay of scalar fields is affected when modulation of couplings, perturbative decay and radiative corrections to the potential are taken into account.

Chapter 5

Cosmological perturbations from modulation of resonant decay

As was discussed in the previous chapter, the modulated reheating scenario [28–30] provides a mechanism for generating the curvature perturbation after inflation from isocurvature perturbations in a spectator field which modulates the decay rate of the inflaton. This setup assumes perturbative decay of the inflaton, but it is natural to extend this scenario to non-perturbative decay through parametric resonance. Analogously to the modulated reheating case the coupling of the inflaton to its decay products may be modulated by another field σ which was light during inflation and therefore acquired a spectrum of super-horizon perturbations [1, 125–127]. This would lead to slightly different initial conditions for the resonant production of particles in different parts of the universe leading to the generation of curvature perturbations in analogy with the modulated reheating scenario. Let the potential be

$$V = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2 \tag{5.1}$$

where ϕ is the inflation, χ is the preheat field, and their coupling depends on the spectator field σ, $g = g(\sigma)$. As we saw in the previous chapter, this model gives rise to preheating through resonant production of χ-particles. However, preheating will last longer in some parts of the universe than in others because of the modulation of the coupling by super-horizon perturbations in the spectator field σ. During preheating the evolution of the universe is matter-like and after it is over the effective pressure rises quickly to a value w_f somewhat below that of radiation [128–131]. Then the number of e-folds after inflation is given by

$$N = \frac{2}{3}\frac{w_f}{1+w_f}\ln\frac{t_1}{t_0} + \frac{2}{3}\frac{1}{1+w_f}\ln\frac{t}{t_0} \tag{5.2}$$

where t_0 is the end of inflation and t_1 the end of preheating, and with the use of the ΔN-formalism the resulting curvature perturbation from modulation of the duration of preheating is

$$\zeta_{\mathsf{MP}} = \frac{2}{3}\frac{w_f}{1+w_f}\frac{\partial \ln t_1}{\partial \ln g}\frac{g'}{g}\delta\sigma_*. \tag{5.3}$$

If preheating terminates before backreaction becomes important, $t_1 \propto g$ and so the curvature perturbation is of the same magnitude but opposite sign as in the modulated reheating scenario, $\zeta_{\mathsf{MP}} \simeq \frac{2}{3}\frac{w_f}{1+w_f}\frac{g'}{g}\delta\sigma_*$. The reason for the difference in sign is that, unlike perturbative decay, the duration of preheating is directly proportional to the strength of the coupling. Without backreaction, preheating terminates because the resonance parameter $q \propto g^2$ decreases as the universe expands, eventually becoming too small for the decay to be efficient. For larger couplings the initial value of the resonance parameter is greater, allowing for a longer period of preheating. In contrast, the time until perturbative decay is inversely proportional to the strength of coupling, $t \propto \Gamma^{-1} \propto g^{-2}$.

5.1 Termination by backreaction

A more interesting case to consider is if the parametric resonance is terminated by the decay products backreacting on the inflaton dynamics. In this case preheating ends when the effective inflaton mass induced by the produced particles becomes comparable to its bare mass, $g^2\langle\chi^2\rangle \sim m^2$ which occurs when the number density of produced particles reaches $n_\chi \sim g^{-1}m^2\Phi$. From the equation (4.27) this gives a curvature perturbation

$$\zeta_{\mathsf{MP}} = -\frac{2}{3}\frac{w_f}{1+w_f}\frac{4\mu m t_1 - 1}{4\mu m t_1 - 3}\left[\frac{5}{4\mu m t_1 - 1} + \frac{\partial \ln \mu}{\partial \ln g}\right]\frac{g'}{g}\delta\sigma_*, \tag{5.4}$$

where μ is the effective Floquet exponent characterizing the efficiency of production. Typically preheating lasts for $mt_1 \sim 10^2$ and $\mu \sim \mathcal{O}(0.1)$. Numerical analysis shows that the variation of the Floquet exponent with respect to the coupling can be as high as $\partial \ln \mu / \partial \ln g \sim \mathcal{O}(10)$ [1], resulting in much higher curvature perturbations than what perturbative modulated reheating can produce, $\mathcal{P}_\zeta^{\mathsf{MP}} > 10^2 \mathcal{P}_\zeta^{\mathsf{MR}}$.

However, μ is not a monotonic function of g. As the resonance parameter q is changed, the location of the resonance band also changes and at some point there will be a transition from one band to the next (see Figure 4.2). The situation is further complicated by the stochastic nature of the resonance. The location of the resonance bands is different for each consecutive zero-crossing of the inflaton in a manner which depends very sensitively on the initial conditions. As a result the produced curvature perturbation also is very sensitive to the initial conditions.

5.2 Contribution from the inflaton

Modulated preheating can produce curvature perturbations on super-horizon scales but whether the observed primordial curvature perturbation can be due to this process depends on the magnitude of the produced perturbations compared to those generated by the inflaton which are always present. The contribution from the inflaton can be obtained in the usual manner by considering the amount e-folds of inflation so that

$$\mathcal{P}_\zeta^{\rm inf} = \frac{1}{2\epsilon_H M_{\rm pl}^2}\left(\frac{H_*}{2\pi}\right)^2 \tag{5.5}$$

where ϵ_H is the slow-roll parameter describing deviation from *de Sitter* dynamics, evaluated at the time of horizon crossing. The combined spectrum of curvature perturbations is then $\mathcal{P}_\zeta = (1+\xi\lambda^2)\mathcal{P}_{\rm inf}$ where

$$\lambda \equiv \frac{1}{2}N_\sigma\sigma_* = \frac{w_f}{1+w_f}\frac{g'\sigma_*}{3g}\frac{\partial \ln t_1}{\partial \ln g}, \qquad \xi \equiv 8\epsilon_*\left(\frac{M_{\rm pl}}{\sigma_*}\right)^2 . \tag{5.6}$$

Here λ parametrizes the dynamics of preheating while ξ characterizes the initial conditions at horizon crossing. Modulated preheating produces the dominant contribution when $\xi\lambda^2 \gg 1$. As discussed above, t_1 changes non-monotonically with g and is sensitive to the initial conditions, but according to numerical estimates in [1], $\lambda < \mathcal{O}(1)$, so in order for modulated preheating to give the dominant contribution to the primordial curvature perturbation requires the amplitude of the spectator field to be much smaller than Planck mass ($\xi \gg 1$).

5.3 Constraints from non-Gaussianity

As the dependence of the resonance on the coupling is highly non-linear modulated preheating also has the potential to produce large non-Gaussianity with the $f_{\rm NL}$ parameter given by

$$\frac{6}{5}f_{\rm NL}^{\rm MP} \simeq \frac{1}{2}\frac{\xi^2\lambda^4}{(1+\xi\lambda^2)^2}\left[\left(\frac{g''\sigma_*}{g'} - \frac{g'\sigma_*}{g}\right)\frac{1}{\lambda} - \frac{1}{2}\left(\frac{g'\sigma_*}{g}\right)\frac{1}{\lambda^2}\frac{\partial^2 \ln\mu}{\partial(\ln g)^2}\right]. \tag{5.7}$$

Numerical analysis shows that non-Gaussianity is typically much larger than that produced in the modulated reheating scenario [1]. Since the bounds on non-Gaussianity are moderately tight, this implies constraints on the modulated preheating scenario. While it is possible for non-Gaussianity to be small in this framework because of the non-monotonic nature of $\mu(g)$ this would require tuning of the parameters.

On the other hand, since many models exhibit the feature of parametric resonance the fact that modulated preheating very easily produces too large non-Gaussianities can also constrain the framework of modulated reheating. Taking non-perturbative decay into account in considering modulated reheating models can result in much more stringent constraints from non-Gaussianity.

5.4 Constraints from gravitational waves

Additional constraints on the model may be obtained from the background of gravitational waves generated by inflation. As was discussed in the previous chapter, tensor modes generated from quantum fluctuations in the metric by the quasi-*de Sitter* expansion have the same spectrum as a massless scalar field. The tensor-to-scalar ratio at horizon crossing is then $r_* = 16\epsilon_*$. If curvature perturbations are generated after inflation from isocurvature perturbations in a spectator field, the scalar contribution grows while that from the tensors stays constant so that the final tensor to scalar ratio is suppressed:

$$r = \frac{16\epsilon}{1+\xi\lambda^2}. \tag{5.8}$$

Thus, if a sizable tensor-to-scalar ratio is detected then the contribution from the modulated reheating cannot dominate the curvature perturbation ($\xi\lambda^2 \ll 1$).

5.5 Constraints from residual isocurvature

It is also possible that the field σ will not decay into radiation but instead forms dark matter. In this case a residual isocurvature perturbation between radiation and the dark matter fluids will remain which could be used to constrain the model further. Let us consider a simple quadratic potential $V_\sigma = \frac{1}{2}m_\sigma^2\sigma^2$. As discussed in the previous chapter, while the field is in slow-roll the ratio $\delta\sigma_*/\sigma_*$ remains constant leading to isocurvature perturbation

$$\mathcal{S}_\sigma \equiv 3(\zeta_\sigma - \zeta_r) = \frac{\delta\rho_\sigma}{\rho_\sigma} = 2\frac{\delta\sigma}{\sigma}. \tag{5.9}$$

Eventually the field starts to oscillate and forms dark matter leading to CDM isocurvature perturbations with the isocurvature fraction and correlation

$$\alpha \equiv \frac{\mathcal{P}_\mathcal{S}}{\mathcal{P}_\zeta + \mathcal{P}_\mathcal{S}} = \frac{\xi}{1+\xi\,(1+\lambda^2)}, \quad \cos^2\Delta \equiv \frac{\mathcal{C}_{\zeta\mathcal{S}}^2}{\mathcal{P}_\zeta\mathcal{P}_\mathcal{S}} = \frac{\xi\lambda^2}{1+\xi\lambda^2}. \tag{5.10}$$

Since cosmological observations are consistent with adiabatic perturbations, isocurvature constraints provide further limits for this model. The combined constraints from non-Gaussianity and isocurvature for a particular realization are shown in Figures 4 and 5 of [1]. In particular in the regime where modulated preheating dominates the curvature perturbation the isocurvature perturbations are fully correlated or anticorrelated with the curvature perturbation and the isocurvature fraction $\alpha < 1/(1+\lambda^2)$. In this case tight isocurvature constraints require λ to be larger than what numerical estimates suggest. Thus we conclude that the curvature perturbation coming entirely from modulated preheating is ruled out. The opposite limit, where the contribution of the modulated preheating is small, produces uncorrelated isocurvature with isocurvature fraction $\alpha = \xi/(1+\xi)$ leading to constraint from $\sigma_* \gtrsim \sqrt{\epsilon}M_{\rm pl}$.

As we have seen, spectator fields may have very non-trivial effects on the resonant decay of the inflaton by means of modulation of the coupling strength resulting from perturbations in the spectator field. This may result in the generation of curvature perturbations from inhomogeneous end of preheating in analogy with the modulated reheating scenario. The duration of preheating when it is terminated by backreaction depends non-trivially on the coupling resulting in sensitivity to initial conditions and production of large non-Gaussianities. Further constraints may be obtained from gravitational waves, and in the case of the spectator producing dark matter, isocurvature perturbations. While small non-Gaussianity is achievable in this model it requires fine-tuning so in light of tight limits on non-Gaussianity from the data the model should be viewed as a constraining process when considering preheating rather than a viable model. While spectators may affect the non-perturbative decay of the inflaton they themselves may experience parametric resonance as they start to oscillate after the end of slow-roll. The next two chapters will cover such processes.

Chapter 6

Resonant decay of spectators during slow reheating

We have seen that spectator fields can play a significant role in the non-perturbative decay of the inflaton. However, spectator fields themselves can decay non-perturbatively through the same process of parametric resonance. It is then prudent to ask how the decay of the inflaton can affect the non-perturbative decay of spectators. In this chapter we focus on this issue, studying non-perturbative decay of a spectator σ in the thermal bath produced by the continuous perturbative decay of the inflaton. Let the spectator be coupled to the Standard Model Higgs Φ with the potential

$$V = \frac{1}{2}m_\sigma^2\sigma^2 + g^2\sigma^2\Phi^\dagger\Phi. \tag{6.1}$$

As was discussed in Chapter 3, radiative corrections both from the vacuum and from the interaction with the thermal bath will induce corrections to the effective potential for the spectator. The thermal corrections can be considered as thermal contributions to the effective mass $m_{\text{eff}}^2 = m^2 + \tilde{g}_T^2 T^2$. The correction to the Higgs mass comes from the loop contributions of all of the SM degrees of freedom and is known to be $g_T^2 = 0.1$ [132]. The correction to the spectator mass comes from the Higgs loops and is $g_T \sim g$. The vacuum radiative contribution to the potential in the limit $g^2\sigma^2 \gg m^2 + g^2T^2$ where it is non-negligible is

$$V_{\text{vac}} \simeq \frac{g^4\sigma^4}{64\pi^2}\left[\ln\left(\frac{g^2\sigma^2}{m^2}\right) - \frac{3}{2}\right]. \tag{6.2}$$

There are thus three different regimes where the dynamics of the spectator are dominated by: 1) the thermal mass gT, 2) the bare mass m_σ and 3) the vacuum contribution V_{vac}. The regions of the parameter space corresponding to these regimes are depicted in Figure 1 of Reference [2].

6.1 Inflaton decay

The decay of the inflaton into SM particles proceeds perturbatively with some effective decay rate Γ. Efficient reheating occurs when $H \sim \Gamma$. Before this point we assume that the evolution of the universe is matter-like corresponding to inflaton oscillating in a harmonic potential. We also assume that the decay products thermalize instantly so that according to the results of Section 4.3 the temperature shortly after the end of inflation and before reheating evolves as

$$T = \left(\frac{36 M_{\rm pl}^2 H \Gamma}{\pi^2}\right)^{1/4} a^{-3/8} \equiv T_* a^{-3/8}, \tag{6.3}$$

where H_* denotes the Hubble rate at the end of inflation and $g_* = 106.75$ is the number of SM degrees of freedom. The temperature decreases rather slowly because of the competing effects of the expansion of the universe and the production of particles by inflaton decay.

6.2 Spectator evolution

The evolution of the spectator field condensate is determined by the equation of motion

$$\ddot{\sigma} + 3H\dot{\sigma} + m_{\rm eff}^2(\sigma)\sigma = 0 \tag{6.4}$$

where $m_{\rm eff}^2(\sigma) = m_\sigma^2 + g^2T^2 + \lambda\sigma^2$ and we take $\lambda \simeq \frac{g^4}{16\pi^2}$ since the logarithm term in the vacuum contribution changes slowly. Once the Hubble rate becomes smaller than the effective mass the spectator starts to oscillate about the minimum of the potential. The frequency and the amplitude of the oscillation depend on which of the three contributions to the effective mass dominates.

Thermal mass dominant

When the Higgs particles are relativistic the spectator obtains a thermal mass gT. The effective mass of the Higgs is given by $m_h^2 \simeq g_T^2 T^2 + g^2\sigma^2$ so the spectator receives a thermal mass for $g\sigma \ll T$. If this contribution dominates over the bare mass m_σ the evolution of the spectator can be solved to be

$$\sigma(t) = \frac{2^{2/3}\Gamma(5/3)\sigma_*}{A^{2/3}a^{3/4}} J_{\frac{2}{3}}\left(Aa^{9/8}\right) \tag{6.5}$$

where $A \equiv \left(\frac{4^7 g^4 M_{\rm pl}^2 \Gamma}{3^6 g_* \pi^2 H_*^2}\right)^{1/4}$ and $J_\nu(z)$ is the Bessel function of the fist kind.

Bare mass dominant

If the bare mass of the spectator m_σ gives the dominant contribution the situation is analogous to the massive preheating case discussed in Chapter 4. The solution is then given by

$$\sigma \approx \frac{3H_*\sigma_*}{2m_\sigma\, a^{3/2}} \sin\left(m_\sigma t + \frac{2m_\sigma}{3H_*}\right) \tag{6.6}$$

and the field crosses zero at $t_j \simeq j\pi/m_\sigma$ with $j = 1, 2, 3, \ldots$. This regime is relevant when $m_\sigma^2 \gg g^2T^2, g^4\sigma^2/(16\pi^2)$.

Vacuum correction dominant

When the vacuum correction dominates the spectator is effectively in the quartic potential. The vacuum contribution is important if the Higgs is non-relativistic and the bare mass of the spectator is small compared to $g\sigma$. The situation is similar to massless preheating discussed in Chapter 4; however, now the background evolution is matter-like rather than radiation-like. This manifests as a delay of oscillation for the spectator because the term $-a''/a$ in the equation of motion keeps the field in slow-roll. This term decays, however, and once it becomes small the oscillations proceed as in the massless preheating case. The spectator field evolution in this case is

$$\sigma \simeq \begin{cases} \sigma_* & \text{for} \quad a < a_{\text{osc}} \\ \sigma_* \left(\frac{a}{a_{\text{osc}}}\right)^{-1} \text{cn}\left[11(\sqrt{\frac{a}{a_{\text{osc}}}} - 1), \frac{1}{\sqrt{2}}\right] & \text{for} \quad a > a_{\text{osc}} \end{cases} \tag{6.7}$$

where $\text{cn}(z, \frac{1}{\sqrt{2}})$ is the Jacobi elliptic cosine and the subscript 'osc' denotes the onset of oscillations.

6.3 Resonant decay in the presence of a thermal bath

In all three cases the spectator oscillates around zero and so produces Higgs particles through parametric resonance. The equation of motion for the normalized Higgs degrees of freedom $X = a^{3/2}\phi$ is

$$\ddot{X} + \left[\frac{k^2}{a^2} + g_T^2 T^2 + g^2\sigma^2\right] X = 0. \tag{6.8}$$

The resonance parameter characterizing the strength of the resonance is now $q \equiv \frac{g^2\Sigma^2}{4m_{\text{eff}}^2}$ where Σ is the amplitude of the spectator and m_{eff} its effective mass. Note that if the thermal mass of the spectator dominates then $q \gg g^{-2}$ so the resonance is always broad. Likewise, in the vacuum dominated case $m_{\text{eff}}^2 \simeq \frac{g^4\sigma^2}{16\pi^2}$ so that $q = 4\pi^2 g^{-2}$ and there is no narrow resonance. In contrast, both broad and narrow resonance can occur when the spectator evolution is dominated by the bare mass.

Broad resonance

The situation is analogous to preheating with the substitution $k^2 \to k^2 + a^2 g_T^2 T^2$. The condition for being inside the resonance band at the jth zero-crossing now becomes

$$k^2 < \frac{a_j^2 g|\dot{\sigma}_j|}{\pi}\left(1 - \frac{\pi g_T^2 T_j^2}{g|\dot{\sigma}_j|}\right). \tag{6.9}$$

If the second term in the parentheses is larger than one then no modes are inside the resonance band and no particles are produced. Thus the thermal mass of the Higgs can block the resonance completely if the temperature is sufficiently high. If the spectator starts to oscillate after the inflaton has completely decayed then the resonance may be blocked from the beginning although it may eventually become unblocked as the temperature falls down [133].

If the onset of oscillations happens before inflaton has decayed, however, then the resonance which is initially unblocked may become blocked as the thermal bath is generated by the decay of the inflaton. Since $\dot{\sigma} \sim m_{\text{eff}}\sigma$ and $T \propto a^{-3/8}$ the thermal blocking term in (6.9) grows as $\sim a^{3/8}$, $\sim a^{9/8}$ and $\sim a^{13/8}$ for the cases where the thermal, bare and vacuum contributions to the effective spectator mass dominate respectively.

The question then arises whether the spectator field can decay by parametric resonance before it becomes blocked. This is an important issue for example in the curvaton scenario where the field has to be sufficiently long lived in order to come to dominate the energy density of the universe and to generate the curvature perturbation. Thermal blocking of the non-perturbative decay can facilitate this but only if the resonance is blocked sufficiently early for the field to survive into radiation domination era. The estimate for the time when the field will have decayed is when the energy of produced particles reaches the energy in the spectator field, $\rho_\sigma \sim \rho_h$. Broad resonance can be terminated either by becoming thermally blocked as discussed above or by transitioning into the narrow resonance regime as the resonance parameter decreases. Therefore the field will have decayed by broad resonance if at the end of it the energy in decay products is greater than that of the spectator field.

Thermal mass dominant

When the thermal mass dominates the resonance is blocked after $j_{\text{block}}^{\text{th}}$ zero-crossings where

$$j_{\text{block}}^{\text{th}} = \left[\frac{g_* \Gamma(5/3)^3}{\pi^5 g_T^6 2^{3/2}} \right]^{\frac{2}{5}} g^2 \left(\frac{\sigma_*}{M_{\text{pl}}} \right)^{\frac{6}{5}} \left(\frac{M_{\text{pl}}}{\Gamma} \right)^{\frac{2}{5}} . \tag{6.10}$$

We have found that this is always smaller than one in the region of the parameter space where the thermal mass gives the dominant contribution [2]. Thus, in this case the resonance is always blocked and the field cannot decay before radiation domination.

Bare mass dominant

When the dynamics of the spectator are determined by the bare mass contribution the resonance is blocked after

$$j_{\text{block}}^{\text{bare}} = \frac{g_*}{24\pi g_T^4} g^2 \left(\frac{\sigma_*}{M_{\text{pl}}} \right)^2 \left(\frac{m_\sigma}{\Gamma} \right) \tag{6.11}$$

oscillations. In this case the resonance is initially unblocked in part of the parameter space, specifically for large mass and coupling and for slow inflaton decay rate [2]. In this region the resonance is sufficiently strong to efficiently transfer the energy from the spectator to the decay products or the buildup of the thermal bath due to inflaton decay is sufficiently slow to allow for efficient decay of the spectator.

Vacuum correction dominant

In the case of the vacuum radiative correction dominating the spectator evolution the resonance is blocked after

$$j_{\text{block}}^{\text{vac}} = \frac{11}{K\tau_{\text{osc}}}\left(\frac{\sqrt{\bar{q}}H_*^2}{\pi g_T^2 T_*^2}\right)^{2/5} - \frac{1}{2}\left(\frac{11}{K} - 1\right) \tag{6.12}$$

oscillations. Also in this case the resonance is unblocked in part of the parameter space and the field can decay efficiently before the onset of radiation domination for strong couplings and slow reheating. The region of efficient decay in this case is depicted in Figure 4 of [2].

Narrow resonance

So far we have been discussing the regime of broad resonance with $g^2\sigma_*^2/4m_{\text{eff}}^2 \gg 1$. However, narrow resonance can also occur for certain values of the parameters though only in the case where the bare mass of the spectator is dominant. The system can either be initially in the narrow resonance or it can transition into it from the broad resonance regime. Thus, even if the spectator field doesn't efficiently decay during broad resonance it can continue to produce particles through narrow resonance and eventually decay.

Again the situation is analogous to the preheating case, discussed in Chapter 4, with the substitution $k^2 \to k^2 + g_T^2 T^2$. Now the condition for being inside the resonance band is

$$k^2 \sim a^2 m_\sigma^2 \left[1 + 2q \pm q - \frac{g_T^2 T^2}{m_{\sigma^2}}\right] \tag{6.13}$$

and no modes are within the band for $g_T^2 T^2 > (1+3q)m_\sigma^2$. Thus, the heat bath also blocks the narrow resonance when the temperature is sufficiently high. In contrast to the broad resonance, the thermal blocking decreases whenever temperature does and so if the resonance is blocked initially it will inevitably become unblocked as the temperature goes down. Thus narrow resonance begins either by becoming thermally unblocked or by transitioning from broad resonance if the thermal unblocking condition is already satisfied during it. We find that the spectator field can then efficiently decay by narrow resonance in the regions where broad resonance is inefficient or blocked though regions of efficient decay are still confined to the strong coupling and weak inflaton decay region of the parameter space [2].

6.4 Perturbative decay due to the thermal bath

The presence of the thermal bath also means that additional perturbative decay processes for the spectator can be relevant, such as scatterings with the thermal background. For cases where either thermal or bare mass dominate the spectator potential the total decay rate is [2]

$$\Gamma_{\text{th}} = \frac{1}{576\pi}\frac{g^4 T^2}{m_\sigma(T)}, \tag{6.14}$$

which is non-negligible when $m_\sigma(T) \ll T$ and $m_h \simeq g\Sigma(t) \ll T$. When the vacuum correction dominates, the decay rate may be estimated as

$$\Gamma_{\rm th} = 6.6 \times 10^{-8} \frac{g^8 T^2}{m_\sigma(T)}, \tag{6.15}$$

which is also only valid for $m_\sigma(T) \ll T$. The spectator will decay by these processes when $H \sim \Gamma_{\rm th}$. We find that this can happen before the inflaton has decayed for sufficiently large couplings and small masses of the spectator (see Figure 5 in [2]).

As we have seen, the decay of the inflaton has significant consequences for the decay of spectators following inflation. The slow buildup of a thermal bath generated by the inflaton decay may allow for efficient decay through non-perturbative resonant particle production but whether this will be possible will depend on how fast thermal effects block the resonance. This depends in part on the evolution of the spectator which is governed by its effective mass which in turn may be due to the bare, thermal or radiative contribution. Also perturbative decay in the presence of the thermal bath may be relevant for certain parameter values.

The issue of when and how the spectators decay are especially important for the curvaton scenario which requires the spectator to survive well into the radiation domination era but may also be relevant for other spectators which should decay early in order to preserve the predictions of established physics such as Big Bang Nucleosynthesis. The effects studied in this chapter also do not exhaust all of the physics relevant for resonant particle production. We have studied the decay of a spectator into the Standard Model Higgs but the Higgs itself is also a light field during inflation and is expected to have a non-zero vacuum expectation value and to start oscillating some time after inflation. Parametric resonance is therefore expected to occur also for the Higgs field. We have also neglected the effects of non-linear terms of the decay products. In the next chapter, I shall discuss the non-perturbative decay of the Higgs condensate into weak gauge bosons and the effect of their non-Abelian terms on the resonance.

Chapter 7

Non-Abelian corrections in the resonant decay of the Higgs

In the previous chapter the decay of a spectator into the Standard Model Higgs was considered. However, if the Standard Model is valid all the way up to the inflationary scale then also the Higgs is a light spectator during inflation. As the Higgs starts to oscillate after inflation it should also experience parametric resonance and would non-perturbatively produce all of the particle species it is coupled to. This is perhaps the most concrete and realistic model of non-perturbative decay as all of the couplings of the Higgs and their runnings are known. The only unknown quantity is the initial amplitude of the Higgs field which determines the energy scale to which the couplings need to be run to. If inflation lasts a sufficiently long time this amplitude can be expected to be of the order of the Hubble parameter during inflation, as discussed in section 4.2. Thus, if the energy scale of inflation can be fixed, for example by a detection of primordial gravitational waves, then this model would in principle be fully determined.

The dominant channel is the production of weak gauge bosons $W^{\pm}$ and Z [134]. However, previous studies of resonant production of gauge bosons from the Higgs condensate have neglected the effects of the non-Abelian interactions between them. The aim of this chapter is to focus on the effects of the non-Abelian corrections on the resonance and the time scales when these become important.

7.1 Higgs dynamics

At high energies the dominant part of the Higgs potential is the quartic term

$$V \simeq \frac{\lambda(h)}{4} h^4, \tag{7.1}$$

where λ is the Higgs self-coupling at energy scale h. As was discussed in Section 4.2, the probability distribution for the field will reach an equilibrium value $P(h) \propto \exp\left(-\frac{2\pi^2 \lambda h^4}{3H^4}\right)$. With the correct normalization this gives for the variance of the field

$$\langle h^2 \rangle = \sqrt{\frac{3H_*^4}{2\pi^2\lambda_*}} \frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{4}\right)}. \tag{7.2}$$

Thus, the field value after inflation in our observable patch is expected to be of the order of inflationary scale, $h \sim H_*$. This is also the scale the couplings need to be run to in order to give the correct dynamics.

As was discussed in Section 3.6, the self-coupling of the Higgs actually becomes negative for energy scales higher than $E \sim 10^{11}$ GeV making the electroweak vacuum unstable. In order to consider inflationary scales higher than this, either new physics has to intervene to stabilize the potential or the values of the Standard Model parameters must be somewhat different than the best fit to the experiments. In particular lowering the top mass has the effect of shifting the instability scale to higher energies. Increasing the Higgs pole mass and the strong coupling will also improve the stability but less drastically than changing the top mass which has greatest uncertainty.

After inflation the Higgs condensate starts to oscillate once the Hubble rate becomes smaller than its effective mass $m_0^2 \simeq \lambda h^2$. The situation is the same as that considered in the previous chapter in the context of the radiative correction to the potential which was approximated by a quartic term. If the inflaton oscillates in a harmonic potential then the effective matter domination will delay the onset of resonance but once it has started the solution can be obtained in terms of the Jacobi elliptic cosine:

$$\chi = \chi_{\rm osc}\,{\rm cn}\left[\sqrt{\lambda\chi_{\rm osc}^2}(\tau - \tau_{\rm osc}), \frac{1}{\sqrt{2}}\right], \tag{7.3}$$

where $\chi \equiv a^{-1}h$, τ is the conformal time and the subscript 'osc' refers to the onset of oscillation.

7.2 Gauge field evolution

Because of conformal invariance of the kinetic term for the vector fields the weak gauge bosons only experience the expansion of the universe during inflation through the time dependence of their masses which are due to the Higgs field. Thus, after inflation their expectation values are zero. Once the Higgs field starts to oscillate, vacuum fluctuations in these fields can be resonantly amplified in the same way as for scalar fields discussed in the previous chapters. However, previously we had not considered the interactions between the decay products. Weak gauge bosons, on the other hand, interact with each other in a manner dictated by the structure of the $SU(2)\times U(1)$ symmetry group.

Linear stage of the resonance

At the start of the resonance the non-linear terms due to interactions are still much smaller than the linear terms and this stage of the resonance can be studied as before. The only difference in this respect is the vector nature of the fields. If the vectors are decomposed into longitudinal and transversal components then it turns out that only the transversal degrees of freedom are amplified by resonance. As the modes are amplified by parametric resonance the non-linear terms grow until they start to affect the dynamics of the resonance. We would like to find out when this will occur.

Non-linear stage of the resonance

With the use of the Hartree approximation the non-Abelian interaction terms coming into the equations of motion for the gauge field component A_i^a is obtained to be

$$-g^2\eta^{\mu\nu}\sum_{b\neq a}\left[\left\langle A_\mu^b A_\nu^b\right\rangle A_i^a - \left\langle A_i^b A_\mu^b\right\rangle A_\nu^a\right] \tag{7.4}$$

which in turn induces effective masses for the transversal modes of the gauge fields which are given by

$$m_W^2 = \frac{2g^2\lambda\chi_{\rm osc}^2}{3\pi^2}\int_0^\infty {\rm d}\kappa\kappa^2\Big(|X_W|^2+\cos\theta_W^2|X_Z|^2\Big), \tag{7.5}$$

$$m_Z^2 = \frac{4g^2\lambda\chi_{\rm osc}^2\cos\theta_W}{3\pi^2}\int_0^\infty {\rm d}\kappa\kappa^2|X_W|^2. \tag{7.6}$$

Here $X \equiv (\lambda\chi_{\rm osc}^2)^{1/4}A$ and the integrals depend only on $q_{W,Z}$, that is, where resonance band lies for a given renormalization scale.

7.3 Backreaction on the Higgs

The produced gauge bosons will also alter the dynamics of the Higgs field. The Higgs will likewise acquire an effective mass induced by amplified gauge fields which is

$$m_\chi^2 = \frac{g^2}{4}\Big[2\langle W_\mu^+W^{\mu-}\rangle + (\cos\theta_W)^{-2}\langle Z_\mu Z^\mu\rangle\Big]. \tag{7.7}$$

Thus, the mass induced for the gauge fields is $m_{W,Z}^2 \sim qm_\chi^2$. Therefore in the broad resonance regime ($q \gg 1$) the produced particles will alter the dynamics of the Higgs condensate before the non-Abelian terms become comparable to the linear ones. For SM couplings the resonance is always broad as a consequence of rather strong coupling of the Higgs to the gauge bosons. However, unless the energy is very close to the instability scale the resonance is not extremely broad ($q \sim 1..100$) so those two time scales are close to each other.

In the narrow resonance regime ($q \ll 1$) non-Abelian terms become important before the onset of backreaction so that the Abelian approximation breaks down already in the linear regime and lattice simulations are needed in order to investigate the details of the resonance. While narrow resonance does not occur for the pure Standard Model it may be relevant if new physics change the runnings of the couplings. It is also relevant for a generic model of a spectator other than the Higgs decaying into gauge bosons. In that case the couplings are free parameters and narrow resonance is a possibility.

We conclude that non-Abelian interactions may play an important role in the resonant production of gauge fields in the decay of the Higgs condensate. While in the broad resonance regime, typical of the Standard Model couplings, the non-Abelian corrections do not grow to the size of the linear terms before the produced particles backreact on the Higgs condensate, these two timescales are rather close for generic SM couplings suggesting that more detailed analysis using lattice simulations is warranted. In the narrow resonance regime especially the non-linear terms become important before the onset of backreaction calling for investigation on the lattice.

Chapter 8

Conclusions

While both the ΛCDM cosmological model and inflation have been extremely successful in accounting for the cosmological data, the transition between inflation and the thermal state necessary for the initial condition of the Big Bang cosmology remains poorly understood. The inflaton field and possible spectator fields must have transferred their energy to the particles of the Standard Model as well as dark matter in the reheating era but the underlying physics are highly model dependent. In particular, non-perturbative decay through parametric resonance is a feature of many models.

In this thesis I have discussed non-perturbative resonant decay of scalar fields after inflation in the context of spectator fields. Even though spectator fields are not important during inflation, they can have crucial consequences after inflation is over. Important examples of such effects are the modulated reheating and the curvaton scenarios where the curvature perturbation is generated from perturbations in the spectator fields. The manner in which scalar fields decay is of crucial significance in such circumstances and it is important to understand how the interplay between various effects will affect the dynamics.

In Chapter 5 the modulation of the resonant decay of the inflaton by a spectator field was discussed, where it was assumed that the coupling of the inflaton to the decay products depends on the value of the spectator field. As spectator fields acquire a spectrum of super-horizon perturbations during inflation, the strength of the coupling exhibits spatial variation resulting in modulation of the duration of the resonance. As a result, curvature perturbations are generated. The model is prone to producing large non-Gaussianity suggesting that considerable fine tuning is needed to reconcile it with the non-Gaussianity constraints from Planck. However, the situation may be viewed from the opposite angle in that a generic dependence of couplings on a spectator field easily produces large non-Gaussianity possibly ruling out models that otherwise seem viable.

An opposite scenario was considered in Chapter 6 where the inflaton was assumed to decay perturbatively while a spectator experienced parametric resonance producing Standard Model Higgs particles in the thermal background arising from the decay products of the inflaton. Also radiative corrections from the vacuum were considered. While it had previously been shown that interactions with the thermal bath can block the resonance [133], instantaneous reheating had been assumed. We have shown that prolonged reheating gives rise to a regime where the resonance may become blocked gradually as the thermal bath builds up allowing for efficient decay for sufficiently slow reheating. It was shown that the timescales of the thermal blocking was determined by whether the spectator effective potential was dominated by bare mass, thermal mass or radiative corrections. Specifically, the resonance was found to always be blocked for the regime where thermal mass dominates. The other two regimes allow for resonance to occur and for the spectator to decay efficiently for slow reheating and sufficiently large couplings. Also perturbative decay due to the thermal bath was found to allow for efficient decay of spectator.

In Chapter 7 the resonant decay of the Higgs field iteself was considered in the context of non-perturbative production of weak gauge bosons. The focus was specifically on the role of non-Abelian terms and the time scales of when these become important, as previous studies had neglected them [134–136]. It was found that the non-Abelian terms induce effective masses for the gauge fields which grow exponentially as particles are being produced. For the pure Standard Model the broad resonance is the appropriate regime and in this case the non-Abelian interactions do not grow to be very large before the onset of backreaction. Nevertheless, for generic SM couplings these events are not far apart so that the effect of non-Abelian corrections may still be relevant. For the narrow resonance regime the non-Abelian terms grow to be large already before the onset of backreaction and shut down the resonance. This analysis motivates a detailed investigation using lattice simulations to reveal the nuances of the resonance.

Bibliography

[1] Enqvist K. and Rusak S. *Modulated preheating and isocurvature perturbations*. JCAP. 1303, 017, 2013. 1210.2192

[2] Enqvist K., Lerner R. N., and Rusak S. *Reheating dynamics affects non-perturbative decay of spectator fields*. JCAP. 1311, 034, 2013. 1308.3321

[3] Enqvist K., Nurmi S., and Rusak S. *Non-Abelian dynamics in the resonant decay of the Higgs after inflation*. 2014. 1404.3631

[4] Dicke R., Peebles P., Roll P., and Wilkinson D. *Cosmic Black-Body Radiation*. Astrophys.J. 142, 414–419, 1965.

[5] Penzias A. A. and Wilson R. W. *A Measurement of excess antenna temperature at 4080-Mc/s*. Astrophys.J. 142, 419–421, 1965.

[6] Smoot G. F., Bennett C., Kogut A., Wright E., Aymon J., et al. *Structure in the COBE differential microwave radiometer first year maps*. Astrophys.J. 396, L1–L5, 1992.

[7] Mather J. C., Cheng E., Cottingham D., Eplee R., Fixsen D., et al. *Measurement of the Cosmic Microwave Background spectrum by the COBE FIRAS instrument*. Astrophys.J. 420, 439–444, 1994.

[8] Lemaître G. *Un univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques*. Annales de la Societe Scietifique de Bruxelles. 47, 49–59, 1927.

[9] Lemaître A. G. *A homogeneous universe of constant mass and increasing radius accounting for the radial velocity of extra-galactic nebulae*. Monthly Notices of the Royal Astronomical Society. 91(5), 483–490, 1931. http://mnras.oxfordjournals.org/content/91/5/483.full.pdf+html

[10] Hubble E. *A relation between distance and radial velocity among extra-galactic nebulae*. Proceedings of the National Academy of Sciences. 15(3), 168–173, 1929. http://www.pnas.org/content/15/3/168.full.pdf+html

[11] Alpher R., Bethe H., and Gamow G. *The origin of chemical elements*. Phys.Rev. 73, 803–804, 1948.

[12] Steigman G. *Primordial Nucleosynthesis: The Predicted and Observed Abundances and Their Consequences*. PoS. NICXI, 001, 2010. 1008.4765

[13] Bertschinger E. *SIMULATIONS OF STRUCTURE FORMATION IN THE UNI-*

VERSE. Annual Review of Astronomy and Astrophysics. 36(1), 599–654, 1998. http://dx.doi.org/10.1146/annurev.astro.36.1.599

[14] Guth A. H. *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*. Phys.Rev. D23, 347–356, 1981.

[15] Linde A. D. *CHAOTIC INFLATING UNIVERSE*. JETP Lett. 38, 176–179, 1983.

[16] Linde A. D. *Chaotic Inflation*. Phys.Lett. B129, 177–181, 1983.

[17] Starobinsky A. A. *Spectrum of relict gravitational radiation and the early state of the universe*. JETP Lett. 30, 682–685, 1979.

[18] Starobinsky A. A. *A New Type of Isotropic Cosmological Models Without Singularity*. Phys.Lett. B91, 99–102, 1980.

[19] Englert F. and Brout R. *Broken Symmetry and the Mass of Gauge Vector Mesons*. Phys.Rev.Lett. 13, 321–323, 1964.

[20] Higgs P. W. *Broken symmetries, massless particles and gauge fields*. Phys.Lett. 12, 132–133, 1964.

[21] Higgs P. W. *Broken Symmetries and the Masses of Gauge Bosons*. Phys.Rev.Lett. 13, 508–509, 1964.

[22] Guralnik G., Hagen C., and Kibble T. *Global Conservation Laws and Massless Particles*. Phys.Rev.Lett. 13, 585–587, 1964.

[23] Mollerach S. *ISOCURVATURE BARYON PERTURBATIONS AND INFLATION*. Phys.Rev. D42, 313–325, 1990.

[24] Linde A. D. and Mukhanov V. F. *Nongaussian isocurvature perturbations from inflation*. Phys.Rev. D56, 535–539, 1997. astro-ph/9610219

[25] Enqvist K. and Sloth M. S. *Adiabatic CMB perturbations in pre - big bang string cosmology*. Nucl.Phys. B626, 395–409, 2002. hep-ph/0109214

[26] Lyth D. H. and Wands D. *Generating the curvature perturbation without an inflaton*. Phys.Lett. B524, 5–14, 2002. hep-ph/0110002

[27] Moroi T. and Takahashi T. *Effects of cosmological moduli fields on cosmic microwave background*. Phys.Lett. B522, 215–221, 2001. hep-ph/0110096

[28] Dvali G., Gruzinov A., and Zaldarriaga M. *A new mechanism for generating density perturbations from inflation*. Phys.Rev. D69, 023505, 2004. astro-ph/0303591

[29] Dvali G., Gruzinov A., and Zaldarriaga M. *Cosmological perturbations from inhomogeneous reheating, freezeout, and mass domination*. Phys.Rev. D69, 083505, 2004. astro-ph/0305548

[30] Zaldarriaga M. *Non-Gaussianities in models with a varying inflaton decay rate*. Phys.Rev. D69, 043508, 2004. astro-ph/0306006

[31] Kim J. E. *Light Pseudoscalars, Particle Physics and Cosmology*. Phys.Rept. 150, 1–177, 1987.

[32] Sikivie P. *Axion Cosmology*. Lect.Notes Phys. 741, 19–50, 2008. astro-ph/0610440

[33] Einstein A. *The Foundation of the General Theory of Relativity*. Annalen Phys. 49, 769–822, 1916.

[34] Friedman A. *Über die krümmung des raumes*. Zeitschrift für Physik. 10(1), 377–386, 1922.

[35] Lemaître G. *L'univers en expansion*. Annales de la Societe Scietifique de Bruxelles. 53, 51, 1933.

[36] Lemaître A. *The expanding universe*. General Relativity and Gravitation. 29(5), 641–680, 1997.

[37] Robertson H. P. *Relativistic cosmology*. Rev. Mod. Phys. 5, 62–90, Jan 1933.

[38] Walker A. G. *On milne's theory of world-structure*. Proceedings of the London Mathematical Society. s2-42(1), 90–127, 1937. http://plms.oxfordjournals.org/content/s2-42/1/90.full.pdf+html

[39] Carroll S. M. *Spacetime and geometry: An introduction to general relativity*. 2004.

[40] Riess A. G. et al. *Observational evidence from supernovae for an accelerating universe and a cosmological constant*. Astron.J. 116, 1009–1038, 1998. astro-ph/9805201

[41] Perlmutter S. et al. *Measurements of Omega and Lambda from 42 high redshift supernovae*. Astrophys.J. 517, 565–586, 1999. astro-ph/9812133

[42] Weinberg S. *The Cosmological Constant Problem*. Rev.Mod.Phys. 61, 1–23, 1989.

[43] Copeland E. J., Sami M., and Tsujikawa S. *Dynamics of dark energy*. Int.J.Mod.Phys. D15, 1753–1936, 2006. hep-th/0603057

[44] Ellis G., Nicolai H., Durrer R., and Maartens R. *Editorial on the GRG special issue on dark energy*. Gen.Rel.Grav. 40, 219–220, 2008.

[45] Durrer R. and Maartens R. *Dark Energy and Modified Gravity*. pages 48 – 91, 2008. 0811.4132

[46] Enqvist K. *Lemaitre-Tolman-Bondi model and accelerating expansion*. Gen.Rel.Grav. 40, 451–466, 2008. 0709.2044

[47] Buchert T. *Dark Energy from Structure: A Status Report*. Gen.Rel.Grav. 40, 467–527, 2008. 0707.2153

[48] Rasanen S. *The effect of structure formation on the expansion of the universe*. Int.J.Mod.Phys. D17, 2543–2548, 2009. 0805.2670

[49] Buchert T. and Räsänen S. *Backreaction in late-time cosmology*. Ann.Rev.Nucl.Part.Sci. 62, 57–79, 2012. 1112.5335

[50] Ade P. et al. *Planck 2013 results. XVI. Cosmological parameters*. 2013. 1303.5076

[51] Schwarz D. J. *The first second of the universe*. Annalen Phys. 12, 220–270, 2003. astro-ph/0303574

[52] Kajantie K., Laine M., Rummukainen K., and Shaposhnikov M. E. *The Electroweak phase transition: A Nonperturbative analysis*. Nucl.Phys. B466, 189–258, 1996. hep-lat/9510020

[53] Kajantie K., Laine M., Rummukainen K., and Shaposhnikov M. E. *Is there a hot electroweak*

phase transition at m(H) larger or equal to m(W)? Phys.Rev.Lett. 77, 2887–2890, 1996. hep-ph/9605288

[54] Hannestad S. *Primordial neutrinos*. Ann.Rev.Nucl.Part.Sci. 56, 137–161, 2006. hep-ph/0602058

[55] Olive K. A., Steigman G., and Walker T. P. *Primordial nucleosynthesis: Theory and observations*. Phys.Rept. 333, 389–407, 2000. astro-ph/9905320

[56] Seager S., Sasselov D. D., and Scott D. *How exactly did the universe become neutral?* Astrophys.J.Suppl. 128, 407–430, 2000. astro-ph/9912182

[57] Liddle A. and Lyth D. *Cosmological Inflation and Large-Scale Structure*. Cambridge University Press, 2000.

[58] Ade P. et al. *Planck 2013 results. XXV. Searches for cosmic strings and other topological defects*. 2013. 1303.5085

[59] Mukhanov V. F. and Chibisov G. V. *Quantum Fluctuation and Nonsingular Universe. (In Russian)*. JETP Lett. 33, 532–535, 1981.

[60] Vilenkin A. and Ford L. *Gravitational Effects upon Cosmological Phase Transitions*. Phys.Rev. D26, 1231, 1982.

[61] Linde A. D. *Scalar Field Fluctuations in Expanding Universe and the New Inflationary Universe Scenario*. Phys.Lett. B116, 335, 1982.

[62] Starobinsky A. A. *Dynamics of Phase Transition in the New Inflationary Universe Scenario and Generation of Perturbations*. Phys.Lett. B117, 175–178, 1982.

[63] Cole S. et al. *The 2dF Galaxy Redshift Survey: Power-spectrum analysis of the final dataset and cosmological implications*. Mon.Not.Roy.Astron.Soc. 362, 505–534, 2005. astro-ph/0501174

[64] Tegmark M. et al. *Cosmological Constraints from the SDSS Luminous Red Galaxies*. Phys.Rev. D74, 123507, 2006. astro-ph/0608632

[65] Mukhanov V. F., Feldman H., and Brandenberger R. H. *Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions*. Phys.Rept. 215, 203–333, 1992.

[66] Kurki-Suonio H. *Cosmological Perturbation Theory (lecture notes)*. 2010.

[67] Ma C.-P. and Bertschinger E. *Cosmological perturbation theory in the synchronous and conformal Newtonian gauges*. Astrophys.J. 455, 7–25, 1995. astro-ph/9506072

[68] Bardeen J. M. *Gauge Invariant Cosmological Perturbations*. Phys.Rev. D22, 1882–1905, 1980.

[69] Wands D., Malik K. A., Lyth D. H., and Liddle A. R. *A New approach to the evolution of cosmological perturbations on large scales*. Phys.Rev. D62, 043527, 2000. astro-ph/0003278

[70] Bucher M., Moodley K., and Turok N. *The General primordial cosmic perturbation*. Phys.Rev. D62, 083508, 2000. astro-ph/9904231

[71] Bucher M., Moodley K., and Turok N. *Characterizing the primordial cosmic perturbations*

using map and Planck. Phys.Rev. D66, 023528, 2002. astro-ph/0007360

[72] Ade P. et al. *Planck 2013 results. XXII. Constraints on inflation*. 2013. 1303.5082

[73] Savelainen M., Valiviita J., Walia P., Rusak S., and Kurki-Suonio H. *Constraints on neutrino density and velocity isocurvature modes from WMAP-9 data*. Phys.Rev. D88, 063010, 2013. 1307.4398

[74] Valiviita J., Savelainen M., Talvitie M., Kurki-Suonio H., and Rusak S. *Constraints on scalar and tensor perturbations in phenomenological and two-field inflation models: Bayesian evidences for primordial isocurvature and tensor modes*. Astrophys.J. 753, 151, 2012. 1202.2852

[75] Lyth D. H., Malik K. A., and Sasaki M. *A General proof of the conservation of the curvature perturbation*. JCAP. 0505, 004, 2005. astro-ph/0411220

[76] Lyth D. H. and Rodriguez Y. *The Inflationary prediction for primordial non-Gaussianity*. Phys.Rev.Lett. 95, 121302, 2005. astro-ph/0504045

[77] Salopek D. and Bond J. *Nonlinear evolution of long wavelength metric fluctuations in inflationary models*. Phys.Rev. D42, 3936–3962, 1990.

[78] Sasaki M. and Stewart E. D. *A General analytic formula for the spectral index of the density perturbations produced during inflation*. Prog.Theor.Phys. 95, 71–78, 1996. astro-ph/9507001

[79] Ade P. et al. *Planck 2013 Results. XXIV. Constraints on primordial non-Gaussianity*. 2013. 1303.5084

[80] Creminelli P. and Zaldarriaga M. *Single field consistency relation for the 3-point function*. JCAP. 0410, 006, 2004. astro-ph/0407059

[81] Cheung C., Fitzpatrick A. L., Kaplan J., and Senatore L. *On the consistency relation of the 3-point function in single field inflation*. JCAP. 0802, 021, 2008. 0709.0295

[82] Feynman R. and Hibbs A. *Quantum mechanics and path integrals*. International series in pure and applied physics. McGraw-Hill, 1965.

[83] Peskin M. and Schroeder D. *An Introduction to Quantum Field Theory*. Advanced book classics. Addison-Wesley Publishing Company, 1995.

[84] Bogoliubov N. N. *On the Theory of Superfluidity*. Jour.Phys. 11, (1), 1947.

[85] Gupta S. N. *Theory of longitudinal photons in quantum electrodynamics*. Proc.Phys.Soc. A63, 681–691, 1950.

[86] Bleuler K. *A New method of treatment of the longitudinal and scalar photons*. Helv.Phys.Acta. 23, 567–586, 1950.

[87] Bjorken J. *ELEMENTS OF QUANTUM CHROMODYNAMICS*. page 219, 1979.

[88] Jackiw R. *Introduction to the Yang-Mills Quantum Theory*. Rev.Mod.Phys. 52, 661–673, 1980.

[89] Mukhanov V. *Physical foundations of cosmology*. Cambridge University Press, 2005.

[90] Coleman S. R. and Weinberg E. J. *Radiative Corrections as the Origin of Spontaneous Sym-*

metry Breaking. Phys.Rev. D7, 1888–1910, 1973.

[91] Yokoyama J. *Can oscillating scalar fields decay into particles with a large thermal mass?* Phys.Lett. B635, 66–71, 2006. hep-ph/0510091

[92] Drewes M. *On the Role of Quasiparticles and thermal Masses in Nonequilibrium Processes in a Plasma*. 2010. 1012.5380

[93] Mukaida K. and Nakayama K. *Dissipative Effects on Reheating after Inflation*. JCAP. 1303, 002, 2013. 1212.4985

[94] Degrassi G., Di Vita S., Elias-Miro J., Espinosa J. R., Giudice G. F., et al. *Higgs mass and vacuum stability in the Standard Model at NNLO*. JHEP. 1208, 098, 2012. 1205.6497

[95] Chetyrkin K. and Zoller M. *Three-loop β-functions for top-Yukawa and the Higgs self-interaction in the Standard Model*. JHEP. 1206, 033, 2012. 1205.2892

[96] Bezrukov F., Kalmykov M. Y., Kniehl B. A., and Shaposhnikov M. *Higgs Boson Mass and New Physics*. JHEP. 1210, 140, 2012. 1205.2893

[97] Buttazzo D., Degrassi G., Giardino P. P., Giudice G. F., Sala F., et al. *Investigating the near-criticality of the Higgs boson*. JHEP. 1312, 089, 2013. 1307.3536

[98] Aad G. et al. *Combined search for the Standard Model Higgs boson using up to 4.9 fb^{-1} of pp collision data at $\sqrt{s} = 7$ TeV with the ATLAS detector at the LHC*. Phys.Lett. B710, 49–66, 2012. 1202.1408

[99] Chatrchyan S. et al. *Combined results of searches for the standard model Higgs boson in pp collisions at $\sqrt{s} = 7$ TeV*. Phys.Lett. B710, 26–48, 2012. 1202.1488

[100] Taruya A. and Nambu Y. *Cosmological perturbation with two scalar fields in reheating after inflation*. Phys.Lett. B428, 37–43, 1998. gr-qc/9709035

[101] Bunch T. and Davies P. *Quantum Field Theory in de Sitter Space: Renormalization by Point Splitting*. Proc.Roy.Soc.Lond. A360, 117–134, 1978.

[102] Ade P. et al. *Detection of B-Mode Polarization at Degree Angular Scales by BICEP2*. Phys.Rev.Lett. 112, 241101, 2014. 1403.3985

[103] Flauger R., Hill J. C., and Spergel D. N. *Toward an Understanding of Foreground Emission in the BICEP2 Region*. JCAP. 1408, 039, 2014. 1405.7351

[104] Adam R. et al. *Planck intermediate results. XXX. The angular power spectrum of polarized dust emission at intermediate and high Galactic latitudes*. 2014. 1409.5738

[105] Starobinsky A. *Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations*. Physics Letters B. 117(3–4), 175 – 178, 1982.

[106] Starobinsky A. A. in *Fundamental interactions*, page 55. MGPI Press, Moscow, 1984.

[107] Starobinsky A. A. in *Field Theory, Quantum Gravity and Strings*, volume 246, pages 107–126. Springer-Verlag, Berlin, 1986.

[108] Sasaki M., Nambu Y., and Nakao K.-i. *Classical Behavior of a Scalar Field in the Inflationary*

Universe. Nucl.Phys. B308, 868, 1988.

[109] Starobinsky A. A. and Yokoyama J. *Equilibrium state of a selfinteracting scalar field in the De Sitter background*. Phys.Rev. D50, 6357–6368, 1994. astro-ph/9407016

[110] Turner M. S. *Coherent scalar-field oscillations in an expanding universe*. Phys. Rev. D. 28, 1243–1247, Sep 1983.

[111] Dolgov A. and Linde A. D. *Baryon Asymmetry in Inflationary Universe*. Phys.Lett. B116, 329, 1982.

[112] Linde A. *Particle Physics: Inflationary Cosmology*. Harwood, 1990.

[113] Bassett B. A., Tsujikawa S., and Wands D. *Inflation dynamics and reheating*. Rev.Mod.Phys. 78, 537–589, 2006. astro-ph/0507632

[114] Ellis J. R., Hagelin J., Nanopoulos D. V., Olive K. A., and Srednicki M. *Supersymmetric Relics from the Big Bang*. Nucl.Phys. B238, 453–476, 1984.

[115] Kawasaki M. and Moroi T. *Gravitino production in the inflationary universe and the effects on big bang nucleosynthesis*. Prog.Theor.Phys. 93, 879–900, 1995. hep-ph/9403364

[116] Ellis J., Nanopoulos D. V., Olive K. A., and Rey S.-J. *On the thermal regeneration rate for light gravitinos in the early universe*. Astropart.Phys. 4, 371–386, 1996. hep-ph/9505438

[117] Dolgov A. and Kirilova D. *ON PARTICLE CREATION BY A TIME DEPENDENT SCALAR FIELD*. Sov.J.Nucl.Phys. 51, 172–177, 1990.

[118] Traschen J. H. and Brandenberger R. H. *PARTICLE PRODUCTION DURING OUT-OF-EQUILIBRIUM PHASE TRANSITIONS*. Phys.Rev. D42, 2491–2504, 1990.

[119] Kofman L., Linde A. D., and Starobinsky A. A. *Reheating after inflation*. Phys. Rev. Lett. 73, 3195–3198, 1994. hep-th/9405187

[120] Kofman L., Linde A. D., and Starobinsky A. A. *Nonthermal phase transitions after inflation*. Phys.Rev.Lett. 76, 1011–1014, 1996. hep-th/9510119

[121] Kofman L., Linde A. D., and Starobinsky A. A. *Towards the theory of reheating after inflation*. Phys.Rev. D56, 3258–3295, 1997. hep-ph/9704452

[122] Greene P. B., Kofman L., Linde A. D., and Starobinsky A. A. *Structure of resonance in preheating after inflation*. Phys.Rev. D56, 6175–6192, 1997. hep-ph/9705347

[123] Allahverdi R., Brandenberger R., Cyr-Racine F.-Y., and Mazumdar A. *Reheating in Inflationary Cosmology: Theory and Applications*. Ann.Rev.Nucl.Part.Sci. 60, 27–51, 2010. 1001.2600

[124] Arscott F. *Periodic differential equations: an introduction to Mathieu, Lamé, and allied functions*. International series of monographs in pure and applied mathematics. Macmillan, 1964.

[125] Kofman L. *Probing string theory with modulated cosmological fluctuations*. 2003. astro-ph/0303614

[126] Kohri K., Lyth D. H., and Valenzuela-Toledo C. A. *Preheating and the non-gaussianity of the*

curvature perturbation. JCAP. 1002, 023, 2010. 0904.0793

[127] Erratum-ibid. 1009, E01, 2011

[128] Podolsky D. I., Felder G. N., Kofman L., and Peloso M. *Equation of state and beginning of thermalization after preheating*. Phys. Rev. D73, 023501, 2006. hep-ph/0507096

[129] Frolov A. V. *DEFROST: A New Code for Simulating Preheating after Inflation*. JCAP. 0811, 009, 2008. 0809.4904

[130] Sainio J. *CUDAEASY - a GPU Accelerated Cosmological Lattice Program*. Comput.Phys.Commun. 181, 906–912, 2010. 0911.5692

[131] Easther R., Finkel H., and Roth N. *PSpectRe: A Pseudo-Spectral Code for (P)reheating*. JCAP. 1010, 025, 2010. 1005.1921

[132] Anderson G. W. and Hall L. J. *The Electroweak phase transition and baryogenesis*. Phys.Rev. D45, 2685–2698, 1992.

[133] Enqvist K., Figueroa D. G., and Lerner R. N. *Curvaton Decay by Resonant Production of the Standard Model Higgs*. JCAP. 1301, 040, 2013. 1211.5028

[134] Enqvist K., Meriniemi T., and Nurmi S. *Generation of the Higgs Condensate and Its Decay after Inflation*. JCAP. 1310, 057, 2013. 1306.4511

[135] Bezrukov F., Gorbunov D., and Shaposhnikov M. *On initial conditions for the Hot Big Bang*. JCAP. 0906, 029, 2009. 0812.3622

[136] Garcia-Bellido J., Figueroa D. G., and Rubio J. *Preheating in the Standard Model with the Higgs-Inflaton coupled to gravity*. Phys.Rev. D79, 063531, 2009. 0812.4624