A Computer-Aided Tolerance Specification Method Based on Multiple Attributes Decision-Making

Qijian Zhao[1]*, Tukun Li[2], Yanlong Cao[3], Jiangxin Yang[3], Xiangqian Jiang[2]

[1] *Institute of Mechanical Manufacturing Technology, China Academy of Engineering Physics, Mianyang, 621900, China*
[2] *EPSRC Future Advanced Metrology Hub, Centre for Precision Technologies, School of Computing and Engineering, University of Huddersfield, Huddersfield, HD1 3DH, UK*
[3] *Key Laboratory of Advanced Manufacturing Technology of Zhejiang Province, College of Mechanical Engineering, Zhejiang University, Hangzhou, 310027, China*

* Corresponding author. Tel: +86-0816-2491346; Fax: +86-0816-2491346; E-mail: zqj@zju.edu.cn

Abstract:

In the mechanical product design, tolerance specification is a crucial process to select tolerance types to control the geometrical variation of the features on a workpiece. Tolerance specification is a kind of decision-making process which needs to consider many influential factors of the manufacturing cost and quality requirement of a workpiece. In this context, those factors are classified into two categories, that of static factors from the rules and standards, and that of dynamic factors based on the manufacturing site's information. This paper presents a novel method for computer-aided tolerance specification to evaluate both types of factors while the traditional methods only take the static factors into considerations. Those factors are assessed by the developed MADM (Multiple Attributes Decision-Making) algorithm, assisted with the rule-based algorithm and Axiomatic Design algorithm. A case study is undertaken, and its result shows that the effect of dynamic factors on the output of tolerance specification. The results adhere to the ISO standard.
Keywords: geometric tolerances; tolerance specification; Computer-Aided Tolerancing (CAT); Multiple Attributes Decision-Making (MADM); Axiomatic Design (AD).

1. Introduction

It is of importance that a mechanical engineer balances the quality requirement and the manufacturing cost of a workpiece in the design stage. To this end, the tolerance design is usually employed after the functional/structure design [1]. Tolerance design is to develop the permissible variation limit of the dimensions and geometric parameters of workpieces, which aims to address functional and assembly requirements under a limited cost. Tolerance design usually consists of three tasks: tolerance specification, tolerance allocation and tolerance analysis [2-6]. This paper mainly focuses on the tolerance specification, which is to determine Datum Reference Frame (DRF) and select the geometrical tolerances on a workpiece.

Some works have been done for the computer-aided tolerance specifications via different approaches to model information from various resources. Clemént [7, 8] introduced the technologically and topologically related surfaces (TTRS) model which calculates the extracted information of the surfaces from a geometric model, and it requires a designer's experience for further selection. The TTRS model has been further optimised [9, 10]. Variational geometric constraints network is defined to generate a network to model the relationship between features and specified tolerance types by Hu [11]. Zhang [12] proposed an algorithm for specification by introducing the polychromatic set theory. Qin [13, 14] suggested and developed an ontology-based model for tolerance specification, which the model has been further developed by Qie [15]. The assembly relationship of the parts is also considered. Anselmetti [16, 17] defined the positioning table method, which was implemented in Quick GPS [18]. Cao [19] applied graph theory to develop the positioning table method. Qin [20] introduced a top-down strategy to carry out tolerance specification for an arbitrary assembly designed in a Computer-Aided Design (CAD) system. The empirical knowledge can also be useful. Haghighi [21] have developed a set of rules for the generation of the tolerance specification. Armillotta [22] proposed a tolerance specification method by combining experimental factors and assembly relation. Some tolerance specification methods are developed based on previous cases. Cao [23] proposed a statistical learning-based method for datum selection of tolerance specification. Qin [24] introduced a case-based reasoning approach to developing the ontology-based tolerance specification method.

Table 1 lists a comparison of the factors considered in some typical tolerance specification methods. The factors can be classified into five groups, namely geometry, position, assembly, experience and cases. Those factors (refers to static factors in this paper) are developed based on the rules, standards, previous experience/cases. In the manufacturing site, an engineer must also consider the factors related to her/his environment (e.g. the ability of the measuring instrument, the up-limited of manufacturing cost, the level of the quality requirement and the precision of machine tools). Those factors are related to the conditions of the work floor (refers to dynamic factors in this context). However, the current computer-aided method for the tolerance specification did not consider dynamic factors in their evaluation.

Table 1 Comparison of the factors in some tolerance specification methods

The utilisation of the factors	Geometry	Position	Assembly	Empirical rules	Cases	Compatible with TTRS	Standard [25, 26]
Clemént [8]	✓	✓	✓			✓	ISO
Zhang [12]	✓	✓	✓			✓	ISO
Anselmetti [18]	✓	✓	✓			✓	ASME
Armillotta [22]	✓	✓	✓	✓		✓	ASME
Qin [24]	✓	✓	✓		✓	✓	ISO

There are several reasons to consider the impact of dynamic factors in the computer-aid of tolerance specification, as follows.

1. In current practice, dynamic factors play an essential role in the development of tolerance specification. According to a survey and interviews were undertaken in 2019 [47], engineers in the manufacturing site are often determined tolerance specification via both factors. Static factors must be followed due to that the rules are standardised and mathematically defined. It is often that the same static factors can produce several tolerance specification schemes. In this situation, an engineer will consider the dynamic factors to decide the final choice. For example, a factory with many developed gauges will prefer to use a tolerance scheme employed the dependent rule, and one with more digital instruments (e.g. CMM) would like to use the scheme developed by the independent rule.
2. In the future digital/smart manufacturing, dynamic factors will play an essential role in the tolerance design. Currently, the exchange of data/information is very limited in the life

cycle of a product. Therefore, a designer does not consider the dynamic factor when he/she design tolerance due to the lack of information. In the shift toward industry 4.0, the data and information from the machine tools and measurement instruments will be available, and knowledge will be accumulated with the aid of artificial intelligence. A designer will make a better decision in the development of tolerance with considering dynamical factors, together with the static factors.

To this end, this paper will design and develop a computer-aided algorithm to produce tolerance specification by assessing both dynamic factors and static factors. Dynamic factors are investigated and classified. Static factors produce a list of geometric tolerance to describe a feature on a workpiece. MADM algorithm is developed to assess the weight of dynamic factors on each tolerance scheme. AD algorithm is implemented to evaluate the outcome of MADM algorithm and to select geometric tolerance. MADM method is the first time to be introduced into this field, and the AD approach has been used in the assessment of one type of tolerance, namely flatness. Thus, some groundwork about MADM approach and AD approach is presented in Section 2. The developed algorithms are detailed in Section 3. Section 4 presents a case study to illustrate the use of the proposed method. The conclusion and future work are summarised in Section 5.

2. Preliminary

2.1 MADM methods

Multiple Attributes Decision Making (MADM) methods are an approach to solve a selection problem with the aim of "making preference decisions over the available alternatives that are characterised by multiple attributes" [31]. Their historical origins can be traced back to Nicolas Bernoulli in 1738 when he proposed the concept of utility function to reflect human pursuit [31]. The applications of MADM methods are employed in many fields, such as computer science [32] and engineering applications [33]. In the field of tolerance specification, it is probably the first application of the MADM method.

The MADM methods integrate the attribute values of each candidate scheme into a matrix and then apply an algorithm to calculate the priority of each scheme based on the matrix [31]. Each value of the matrix could be number, interval or fuzzy set. In general, the form of MADM is expressed as:

$$optimize[u_1(x), u_2(x), \ldots, u_m(x)] \quad (1)$$
$$\textit{Subject To: } x \in X$$

where $X=\{x_1, x_2, \ldots, x_n\}(n\in N^+)$ represents the set of candidate schemes; x represents a scheme of. $\boldsymbol{U}=\{u_1, u_2, \ldots, u_m\}(m\in N^+)$ is the attributes set and $u_1, u_2, \ldots, u_m$ represent the attributes. The matrix is $\boldsymbol{C}=[c_{ij}]_{n\times m}$ a decision-making matrix, where $c_{ij}=u_j(x_i)$ $(i\in[1,n],\ j\in[1,m],\ i,\ j\in N^+)$. A MADM algorithm is to select an optimal scheme x based on $\boldsymbol{C}$.

For example, there is a reducer selection problem. The candidate reducers set is $\{x_1, x_2, x_3, x_4\}$. The price (y_1), efficiency (y_2) and weight (y_3) are the attributes. In this example, the preference is the lower price, lower weight and higher efficiency. The weights of the attributes are known, i.e. $\boldsymbol{w} = [0.2 \quad 0.5 \quad 0.3]^T$. The calculation for the optimal scheme is illustrated in Fig. 1. Firstly, the attribute values of the schemes are determined. Secondly, the decision-making matrix is normalised. Thirdly, the indexes are calculated and the optimal reducer selection scheme is determined. It is often that the weight of each attribute is unknown. The entropy algorithm [44, 45], which evaluates the unknown weight based on the design matrix, can be used. This method will be used in the Section 3.

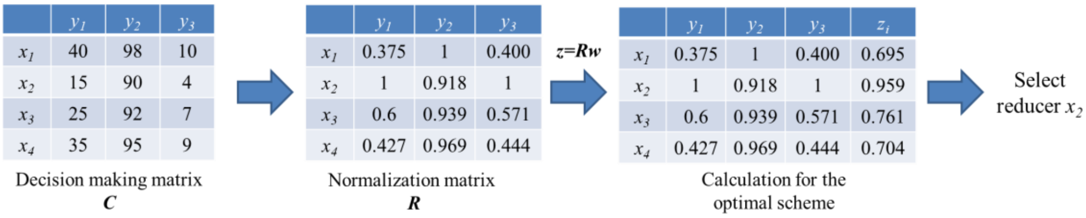

Fig. 1 The optimal reducer selection procedures by MADM approach

2.2 AD method

Axiomatic Design (AD) is a design methodology to address the transformation issues of customer needs into functional requirement and design parameter. It was proposed by Suh [27] in the 1990s and has been widely used to address the fundamental issues in Taguchi methods. In the field of computer-aided tolerance specification, an AD method for specification was firstly introduced in 2018 [30]. It shows that the tolerance specification can be interpreted to a design problem.

AD method uses two types of axioms, namely independent axiom and information axiom. The independent axiom (i.e. maintain the independence of the functional requirements) means that the functional requirements must comprise a minimum set of independent requirements that utterly satisfy the customer needs to provide a robust design. The information axiom (i.e. minimise the information content of the design) means that the design with the highest probability of success is the best. In a design problem, a set of Functional Requirements (FRs) are related to a set of Design Parameters (DPs) by a Design Matrix ***A***, which can be represented by

$$[FR] = [A_{m\times n}][DP], A_{ij} = \frac{\partial FR_i}{\partial DP_j}, i = 1, \dots, m; j = 1, \dots, n \quad (2)$$

$$Subject\ To: \{C_1\}, \{C_2\}, \dots, \{C_k\}$$

where $[FR]$ is the vector of functional requirements, $[A_{m\times n}]$ is the design matrix and $[DP]$ is the vector of design parameters. m and n are the numbers of FRs and DPs. $\{C_1\}$, $\{C_2\}$, …, $\{C_k\}$ represents the constraints of the design, such as the value range of the DPs. In this paper, the FR is the variation of a feature, and DP is applied to limit the variation, i.e. a geometric tolerance of the candidate tolerances.

3. The MADM method for computer-aided tolerance specification

3.1 The framework

Fig. 2 illustrates the framework of the implementation of the MADM method, and Table 2 lists the descriptions of the key terms used in this paper. The input data are features on the CAD model of a workpiece, together with factors to develop its tolerance specification. The classification of the factors is detailed in Section 3.1.1. The output data are the developed geometrical tolerances, together with the selected Datum Reference Frame (DRF), which is an orthogonal coordinate system for geometrical tolerance. The data is processed by a set of algorithms and operations: A Minimum Geometric Datum Elements based algorithm (MGDE-based algorithm) to generate DRF; a rule-based operation to create the candidate schemes set ***S***; MADM method to select optimal scheme (detailed in Section 3.2), and AD algorithm to determine geometrical tolerances (documented in Section 3.3). The MGDE-based algorithm and the rule-based operation are developed based on previous work [8, 11-14], and will be presented in a case study in Section 4.

All methods listed in Table 1 are compatible with TTRS, and they are adhering to the ISO standards or ASME standards correspondingly. For the proposed method, therefore, we select two criteria, that of to be compliable with TTRS, and that of to adhere to the ISO standards.

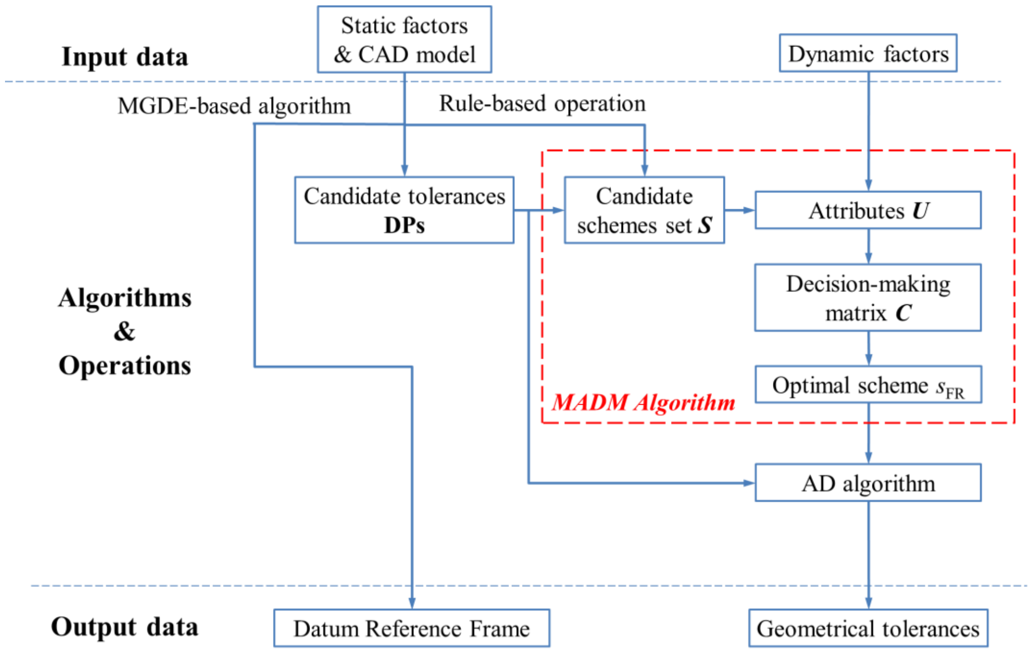

Fig. 2 The framework of the MADM method, assisted with the AD algorithm

Table 2 Key terms used in the MADM algorithm

Symbol	Name	Description	Example/Note
u	Dynamic Factor	A variable to describe factor from the manufacturing/measuring site.	Quality, precision, cost, …
DP	*Design parameter*	Candidate tolerances for FR (Functional requirement)	Flatness is a DP to specify form variation.
FR	*Functional Requirement*	Variations after *decomposition operations*	$\boldsymbol{FR}$ = {FR_1, FR_2, FR_3}, where FR_1, FR_2 and FR_3 represent the translation, rotation and form variation, respectively (see **Appendix 1**)
U	*Attribute*	A preference described as a set of dynamic factors	$\boldsymbol{U}=\{u_1, u_2, ..., u_m\}(m \in N^+)$.
S	*candidate schemes set*	A set ($\boldsymbol{S}=\{s_1, s_2, …, s_n\}(n \in N^+)$) of decomposition scheme, namely s_i, which is a selection of the decomposition of a feature.	see **Appendix 1**
C	*Decision-*	A matrix to integrate the values of the	An element of ***C*** is calculated as

	making matrix	attributes of the candidate schemes.	$c_{ij}=u_j(s_i)$ ($i\in[1,n], j\in[1,m], i, j \in N^+$).
s_{FR}	Optimal Scheme	A scheme selected from ***S***	

3.2 Classification of the factors

A dynamic factor is a quantified descriptor of a grouped of contributors for the tolerancing specification from the manufacturing/measuring site. A designer can change the weight of dynamic factors based on her/his requirement. In this paper, the dynamic factors were selected as the outcome of the literature review, a recent survey and interviews of experienced engineers. Some of them are listed in Fig. 3, and this paper will focus on five significant groups of factors: Quality, Precision, Applicative, Verification and Cost. They will be described as the corresponding attributes of a feature. Quality attribute estimates the quality requirements in manufacturing. Precision attribute quantifies precision levels. Applicative attribute accesses the agreement between the design criteria and functional requirement. Verification attribute estimates the complexity of the inspection. Cost attribute measures the cost based on tolerance requirements. Note that those factors are often conflicting. For example, we cannot make a selection to have the highest precision and the lowest cost.

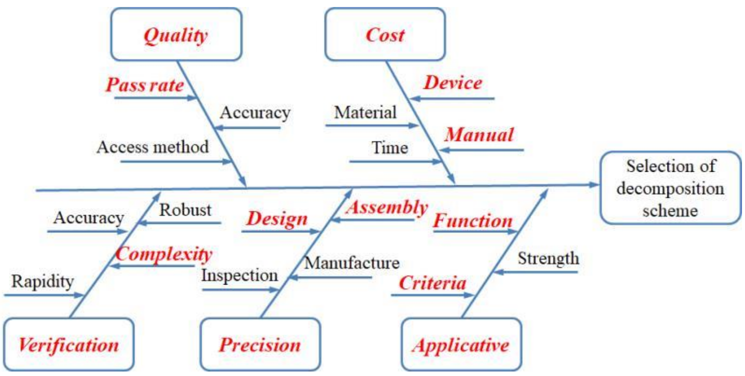

Fig. 3 The classification of dynamic factors

A static factor is an influential factor for the tolerancing specification, which is from the standard, rules, the geometrical relationship (see Table 1). Designers usually cannot change the weight of static factors. In this context, the static factors are geometrical type, position relationship with DRF and assembly relationship. The geometrical type could determine the candidate tolerances for a feature (see an example in Table 7); the position relationship with DRF could affect the selection of orientation and positioning tolerances; the assembly relationship determines the features that need to be specified.

3.3 The MADM method

As shown in Fig. 2, the input of the MADM method are both static factors (via some pre-processed) and dynamic factors, and output is the selected optimal scheme which will be used by AD algorithm. The MADM method is used to help the engineer to make decisions on the alternatives (i.e. different candidate schemes produced by static factors) according to their preferences in the form of dynamics factors. Fig. 4 illustrates the diagram of the MADM algorithm, which is consist of five steps.

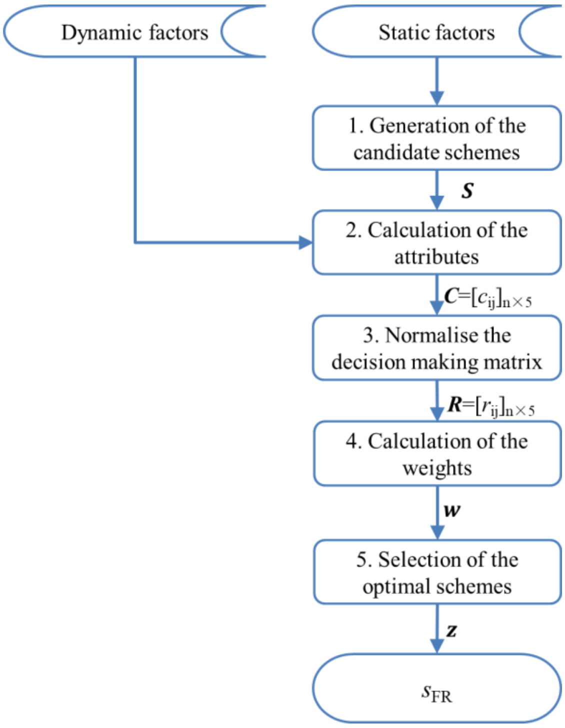

Fig. 4 The diagram of the MADM algorithm

Step 1: *Generation of the candidate schemes*

The first step is to generate the candidate schemes set ***S*** from the input, i.e. the static factors of a feature on a workpiece. A feature can be an integral feature or a derived feature. And the feature is described by the candidate schemes set ***S*** after this operation.

The geometric tolerance (e.g. roundness) is represented as a *DP* (see Table 7 which is **DP**s for all geometric tolerances defined in ISO 1101:2017). A feature (e.g. a sphere) will be decomposed to the basic elements (i.e. **DP**s as shown in Table 8 which listed candidate tolerance for different features based on TTRS), together with the essential reference (i.e. datum). The purpose of decomposition is to find all possible **FR**s (namely the candidate schemes set ***S*** in this context) for a feature (i.e. ***S*** as shown in Table 9 are the possible sets for different features). This operation is detailed in Appendix 1.

Step 2: *Calculation of the attribute values*

If the output of step 1 provides more than one *DP*, it becomes a selection problem with consideration of dynamic factors. A dynamic factor is modelled as an attribute ***u*** of each candidate scheme (i.e. *DP*) for further assessment. The Quality attribute, for example, is estimated by the pass rates based on the standard [36]. Assuming that the verification of each geometrical tolerance is independent of the others, the qualification rate c_{i1} is

$$c_{i1} = u_1(s_i) = \prod_{k=1}^{i} \eta_k \quad (3)$$

where the success rate of a single tolerance type inspection be η_k, the number of tolerances is i $(i=1,\dots,n)$.

Then, the attribute value of each scheme is calculated by $c_{ij}=u_j(s_i)$., which could be represented as $\boldsymbol{C} = \boldsymbol{U(S)}$, where the attributes models $u_j()$ could be estimated (see Table 10). For example, the decision-making matrix of the feature f_2 in Fig. 6 is calculated as Eq. (4).

$$\boldsymbol{C} = \boldsymbol{U}(\boldsymbol{S}) = \begin{bmatrix} u_1(s_1) & \dots & u_5(s_1) \\ \vdots & \ddots & \vdots \\ u_1(s_5) & \dots & u_5(s_5) \end{bmatrix} \quad (4)$$

Step 3: *Normalisation of the decision-making matrix*

In this step, the attributes are normalised to [0, 1] via Eq. (5) and Eq. (6) as follows.

$$c_{i1}:\ r_{i1} = \frac{c_{i1}}{max\{c_{i1}, i=1,..,n\}} \tag{5}$$

$$c_{i2},\ c_{i3},\ c_{i4},\ \text{and}\ c_{i5}:\ r_{ij} = \frac{min\{c_{ij}, i=1,\ldots,n\}}{c_{ij}} \tag{6}$$

Then, attributes matrix $\boldsymbol{R}=[r_{ij}]_{n\times 5}$ is obtained.

Step 4: *Calculation of the weights*

The significance of each attribute is assessed by entropy algorithm adopted from [44]. The entropy $\boldsymbol{E} = [E_1, E_2, \ldots, E_5]^T$ of each attribute is calculated as:

$$E_j = -\frac{1}{logn}\sum_{i=1}^{n} \bar{r}_{ij} log\bar{r}_{ij}\ , j = 1,2, \ldots, 5 \tag{7}$$

$$\text{where}\ \bar{r}_{ij} = \frac{r_{ij}}{\sum_{i=1}^{n} r_{ij}}, i = 1,2, \ldots, n, j = 1,2, \ldots, 5 \tag{8}$$

if $\bar{r}_{ij} = 0$, then $\bar{r}_{ij} log\bar{r}_{ij} = 0$.

The weight $\boldsymbol{w} = [w_1, w_2, w_3, w_4, w_5]^T$ is calculated as:

$$w_j = \frac{1-E_j}{\sum_{k=1}^{5}(1-E_k)}, j = 1,2, \ldots, 5 \tag{9}$$

Step 5: *Selection of the optimised schemes*

The evaluation vector $\boldsymbol{z} = [z_1, z_2, \ldots, z_n]^T$ is calculated as:

$$\boldsymbol{z} = \boldsymbol{Rw} \tag{10}$$

Let $z_i = \max(z_1, z_2, \ldots, z_n)$, then $s_{FR} = s_i$. Thus, the optimal decomposition scheme s_{FR} is obtained.

3.4 The generation of geometrical tolerances by AD algorithm

As shown in Fig. 2, s_{FR} and *DP*s will be processed by the AD algorithm to generate the geometrical tolerances. More detail of the AD algorithm can be found in [30]. Fig. 5 shows the information flow of the AD algorithm. The selected scheme s_{FR} is decomposed (see Appendix A) into a set of FRs which represent the variations of form, orientation and translation. Then, s_{FR} and *DPs* is evaluated by the AD algorithm to select geometrical tolerance for the feature.

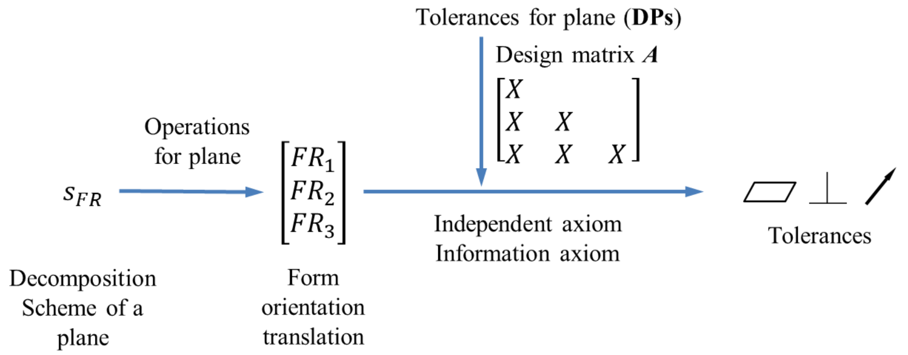

Fig. 5 The generation of the tolerances for a plane feature

In this algorithm, two axioms are used in operations: 1) the independent axiom requires each selected *DP* to satisfy each *FR* independently or decoupling, which means the selected *DP*s should make the design matrix *A* independent and decouple; 2) the information axiom requires the information of using *DP* is the least in candidate *DP*s. The information value of each *DP* is calculated. For example, DP_i in Table 7 is assessed as

$$I_i = I_d + I_{ci} + I_p + I_m = I_d + log_2 \frac{c_i}{c_{min}} + log_2 \frac{P_{sys}}{V} + log_2 \frac{3m_{sys}}{V}, i = 1, \dots, 15 \quad (11)$$

where I_d, I_{ci}, I_p, I_m are the information value of the datum, cost, process and inspection respectively; c_i is the relative cost and c_{min} is the least cost in the same class; V is the estimated value of FR_1, FR_2 or FR_3; P_{sys} is the process precision; m_{sys} is the measuring precision.

Fig. 6 shows an example which needs to specify feature f_1, a plane with two datums. f_1 is decomposed to FR_1, FR_2 and FR_3, and the decomposition scheme for feature f_1 is s_{p3}. According to Eq. (2) and the independent axiom, a form tolerance, an orientation tolerance and a positioning tolerance are applied to control the variation of feature f_1. It will obtain a design matrix A with decouple form as:

$$\boldsymbol{A} = \begin{bmatrix} X & & \\ X & X & \\ X & X & X \end{bmatrix}$$

Assume that the other conditions are the same to the case study, the comparison (see in Table 13) shows the DP_2(⏥), DP_6(⊥) and DP_{11}(↗) are applied to specify feature f_1. Therefore, the flatness, perpendicularity and circular runout are selected to specify feature f_1.

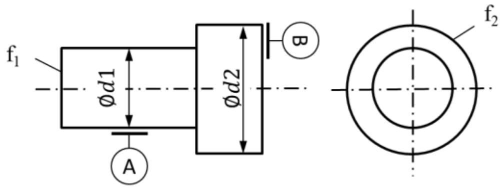

Fig. 6 An example of features to be specified, f_1 is a plane and f_2 is a cylinder surface.

4. Case study

4.1 Tolerance specification of a cycloidal

A typical part, a cycloidal gear of an RV reducer, is selected to demonstrate the use of the MADM method to tolerance specification. This part is consist of features defined in ISO 1101:2017. Fig. 7 illustrates the structure of the RV reducer, which includes two cycloidal gears. Fig. 8 is the drawing of the gear. In this case, the input data is listed in Table 3 and Table 12. Table 3 is generated according to the CAD model of the product. Table 12 is obtained based on an investigation in a factory. Tables are stored in an Excel file. Algorithms are developed in Matlab.

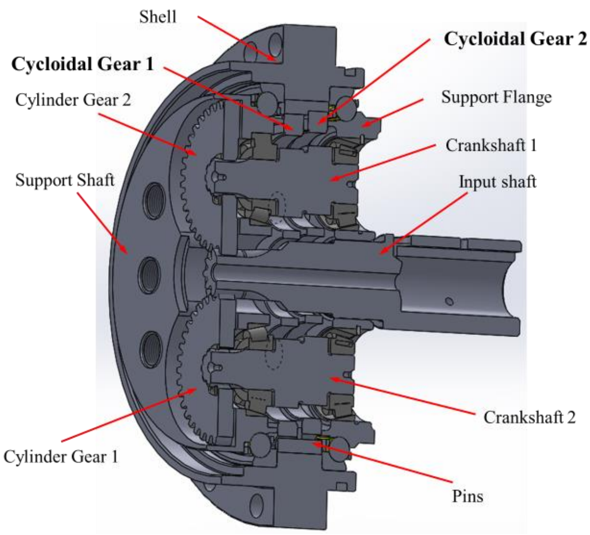

Fig. 7 Structure of the RV reducer

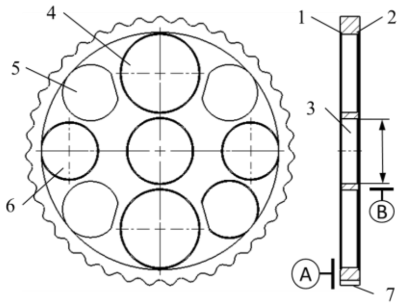

Fig. 8 Drawing of a cycloidal gear

Feature 2 on the gear is selected to shows the data flow of tolerance specification (see Fig. 9). Firstly, the DRF is established by the MGDE-based algorithm. In this case, the primary datum is Feature 1, and the secondary datum is Feature 3. Seven features on the gear are selected to develop specified tolerance. Fig. 8 illustrated the results of the DRF selection. Secondly, the static factors of each feature are analysed, and their candidate scheme sets DPs is generated. Table 3 shows the results. Thirdly, the MADM algorithm evaluates the dynamic factors (listed in Table 13) and DPs (see Table 9). It calculates the decision-making matrix $\boldsymbol{C}$ (see Eq. (4), Table 10 and Table 11), $\boldsymbol{R}$ (see Eq. (5~6)), $\boldsymbol{w}$ (see Eq. (7~9)) and z (see Eq. (10)). Thus, the optimal scheme s_{FR} is obtained according to z. Fourthly, AD algorithm assesses the s_{FR} and DPs (see Table 8) by independent axiom and Eq. (11) (parameters are illustrated in Table 13). Finally, tolerance is selected. For example, the algorithm selects position tolerance and flatness tolerance for feature 2. The optimised schemes for other features of the cycloidal gear are selected, and the results are listed in Table 4. The tolerance specification results for the cycloidal gear are illustrated in Table 5 and Fig. 10. They are adhered to the ISO standard [25], and satisfy the requirements of TTRS.

Table 3 The static factors of each feature

Feature	Geometrical type	Position relation with primary datum	Assembly relationship	Candidate Scheme Set
1	Plane	Primary datum	Supporting flange	$\boldsymbol{S_p} = [s_{p1}]$
2	Plane	Parallel	Shim	$\boldsymbol{S_p} = [s_{p1}, s_{p2}, s_{p3}]$

3	Revolving feature (Cylinder)	Perpendicular (Secondary datum)	Input shaft	$\boldsymbol{S_r} = [s_{r1}, s_{r2}]$
4	Pattern	Perpendicular	Bearing	$\boldsymbol{S_z} = [s_{z1}, s_{z2}, s_{z3}]$
5	Pattern	Perpendicular	Pin	$\boldsymbol{S_z} = [s_{z1}, s_{z2}, s_{z3}]$
6	Pattern	Perpendicular	Pin	$\boldsymbol{S_z} = [s_{z1}, s_{z2}, s_{z3}]$
7	Gear	Perpendicular	Gear	/

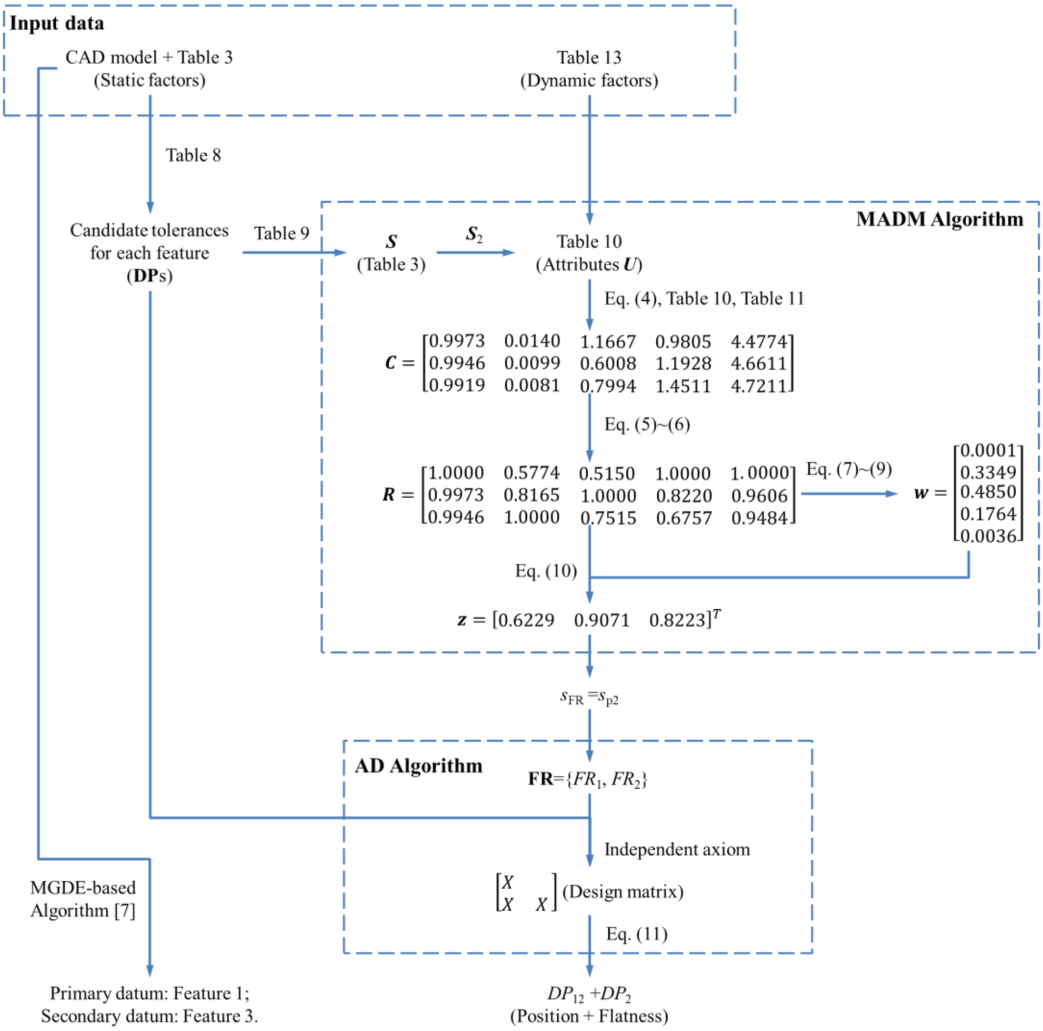

Fig. 9 Data flow of tolerance specification for feature 2 of the cycloidal gear

Table 4 The optimised decomposition scheme of each feature

Feature	1	2	3	4	5	6
d/mm	112	112	33	75	85	85
Types	Plane	Plane	Cylinder	Pattern	Pattern	Pattern
schemes	s_{p1}	s_{p2}	s_{r2}	s_{z2}	s_{z1}	s_{z1}

Table 5 The tolerance specification results of the cycloidal gear

No.	Feature Type	Tolerancing results
cg-1	Plane	DP_2(▱)
cg-2	Plane	DP_{12}(⌖)+DP_2(▱)

cg-3	Cylinder	DP_6(⊥)+DP_4(⌭)
cg-4	Pattern	DP_{10}(⌖)+DP_4(⌭)
cg-5	Pattern	DP_{10}(⌖)
cg-6	Pattern	DP_{10}(⌖)
cg-7	Gear	DP_{11}(↗)

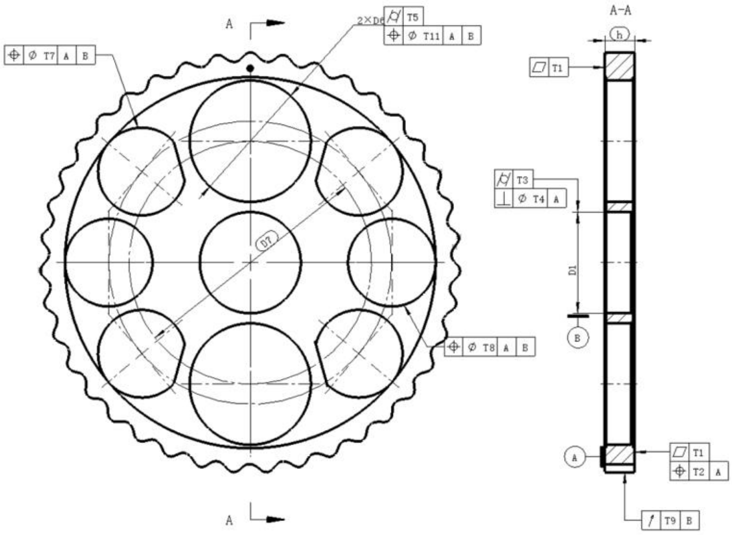

Fig. 10 The tolerance specification results of the cycloidal gear on the precision requirement of IT5

4.2 Discussion

This case study shows the dynamic factors can be used to select the tolerance specification scheme. Traditionally, the selection is based on the experience of the engineer. Dynamic factors model the preference as a set of attributes in the selection process. Five dynamic factors were defined and used to describe the preference. It provides a solution to store the expert's knowledge in the selection of the tolerance scheme.

The factors, usually conflicting, are quantified by their weight. The outcome could be varied for different preference. For example, a designer decides to upgrade the precision requirement (from the IT5 to IT6) on the gear shown in Fig. 8. The developed MADM algorithm undertook a re-assessment. Table 6 lists the optimised schemes and Fig. 11 illustrates the tolerance specification. It shows that more geometrical tolerances are selected to control feature 2 and feature 4. Thus, the developed MADM algorithm made a different selection for different preference. Furthermore, it also illustrates the effect of the dynamic factors for the tolerance specification.

Table 6 The optimised decomposition scheme of each feature with different precision requirements

Feature	1	2	3	4	5	6
d/mm	112	112	33	75	85	85
Types	Plane	Plane	Cylinder	Pattern	Pattern	Pattern
Schemes (IT 6)	s_{p1}	s_{p2}	s_{r2}	s_{z2}	s_{z1}	s_{z1}
Schemes (IT 5)	s_{p1}	s_{p3}	s_{r2}	s_{z3}	s_{z1}	s_{z1}

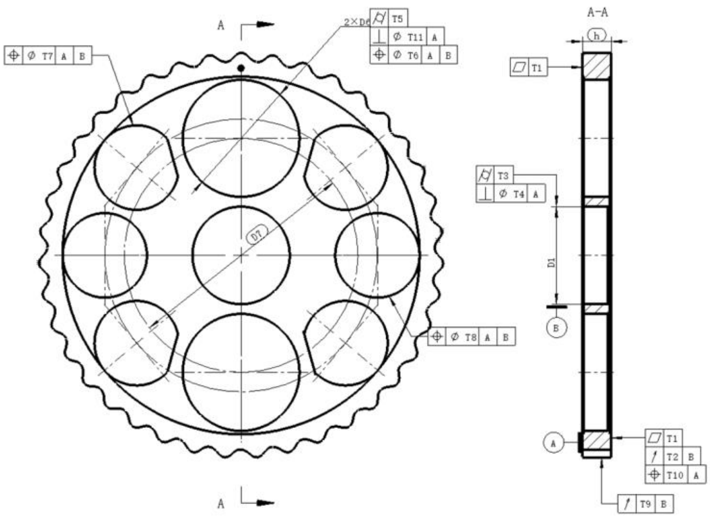

Fig. 11 The tolerance specification results of the cycloidal gear on the precision requirement of IT6

The developed MADM algorithm is employed to assess five dynamics factors only, and it should be able to deal with more factors/sub-factors according to the use of MADM method in other fields [31]. AD method was introduced in the assessment of the flatness [30]. However, it cannot be used directly to develop tolerance with DRF. Now, the AD method can select tolerance with DRF when using it with the MADM algorithm. Thus, the MADM method extends the usage of the AD method in the Computer-Aided Tolerance design.

5. Conclusion and future work

In this paper, the MADM method is introduced and applied on computer-aided tolerance specification. This new method is not only considering the factors used in the traditional methods but also including the influential factors varied according to the work floor situation. The dynamic factors are analysed and grouped to describe the preference of engineers. Those factors were handled by the developed MADM algorithm, together with the rule-based algorithm and AD algorithm. A case study was undertaken. The results show that the developed algorithms generate the specification which is adhered to ISO standard and compatible with the requirements of TTRS.

This paper has made two contributions to this field, that of the assessment of the dynamic factors via computer algorithms, and that of the introduce of MADM method. This method is relatively complicated, which reflect the situation in the manufacturing site. Further development will be undertaken in future, that of a study and verification work of the estimation of the attributes, and that of a reliable verification method to access tolerance specification.

Acknowledgements

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Appendix 1: Decomposition operation for a feature

The variation of a feature is considered as a combination of one/several types of variations in tolerance specification. In this context, decomposition refers to an operation to separate those variations into the variations of translation, rotation and form. In this paper, the decomposition is developed based on the kinematics [35] and the geometrical characteristics of the feature. An example of this operation is illustrated in Fig. 12. The tolerances can control variations decoupling by this decomposition method. According to the definition of each geometric tolerance, a positioning tolerance controls three variations; an orientation tolerance controls rotation variation and form variation [34]; a form tolerance only controls form variation (see Fig. 12). In this case, a plane could be decomposed to one, two or three variations and noted as s_{p1}, s_{p2} and s_{p3} respectively.

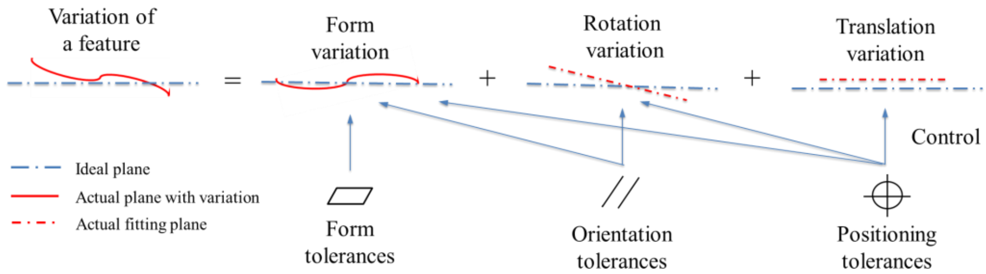


Fig. 12 Decomposition operation on a plane, viewed from the direction paralleled to the plane.

Some features are consist of a derive feature with an integral feature, which the decomposition has more than one results. ***Rule 1*** and ***Rule 2*** is supplied in order to provide unambiguous results.

Rule 1: *If a feature has derived feature(s), then (1) the rotation and translation variation of a feature are controlled through its derived feature first, and (2) the form variation of a feature is controlled through its integral feature.*

For example, the variations of a cylinder are decomposed to the positioning variation of its axis, the rotation variation of its axis, the form variation of its axis, the axial variation of integral feature and the radial variation of an integral feature. It means that the number of variations of a cylinder could be 1, 2, 3, 4 or 5.

Rule 2: *If a feature is a primary datum, its rotation and translation variation are not considered; if a feature is a secondary datum, its positioning variation is not considered.*

A *decomposition scheme* is a result of the decomposition, e.g. "*decompose to kinematic and form variation*". A *candidate schemes set* ***S*** is a group of the decomposition schemes of a feature. The candidate tolerances, geometric type of feature and position relationship with DRF are applied to generate the ***S***.

Appendix 2: The tables of DPs, candidate tolerances, candidate schemes set and estimation of attributes

Table 7 DPs for all geometric tolerances (adopt from [25])

DPs[1]	Geometric tolerance	Symbol	The requirement of datum	Tolerance type
DP₁	Straightness	—	No	Form

DP_2	Flatness	⏥	No	Form
DP_3	Roundness	○	No	Form
DP_4	Cylindricity	⌭	No	Form
DP_5	Parallelism	∥	Yes	Orientation
DP_6	Perpendicularity	⊥	Yes	Orientation
DP_7	Angularity	∠	Yes	Orientation
DP_8	Coaxiality	◎	Yes	Positioning
DP_9	Symmetry	⌯	Yes	Positioning
DP_{10}	Position	⌖	Yes	Positioning
DP_{11}	Circular Runout[1]	↗	Yes	Positioning
DP_{12}	Total Runout[1]	⌰	Yes	Positioning
DP_{13}	Concentricity	◎	Yes	Positioning
DP_{14}	Line Profile[2]	⌒	Yes/No	/
DP_{15}	Surface Profile[2]	⌓	Yes/No	/

[1] The circular and total runout determines the position, so they are classified as position tolerance.

[2] Line and surface profile can be used as form, orientation or position tolerance, so the datum requirement is dynamic.

Table 8 The candidate tolerances for different features

Geometrical Type	Candidate geometrical tolerances
Plane[1]	DP_1(—),DP_2(⏥),DP_5(∥),DP_6(⊥),DP_7(∠),DP_9(⌯), DP_{10}(⌖),DP_{11}(↗) or DP_{12}(⌰)
Revolving surface[1]	DP_1(—),DP_3(○),DP_4(⌭),DP_5(∥),DP_6(⊥),DP_7(∠),DP_8(◎), DP_9(⌯),DP_{10}(⌖),DP_{11}(↗) or DP_{12}(⌰)
Sphere[1]	DP_3(○) and DP_{13}(◎); or DP_{10}(⌖)
Pattern[1]	DP_1(—)/DP_4(⌭)[3],DP_5(∥),DP_6(⊥),DP_7(∠),DP_{10}(⌖)
Key Slot/Multiple keys [2]	Axial: DP_9(⌖) Radial: DP_{11}(↗)
Gear teeth[2]	DP_{11}(↗)
Free curve [2]	DP_{14}(⌒)
Freeform surface [2]	DP_{15}(⌓)

Note: [1] The candidate geometrical tolerances is bigger than one, and the selection will be continued. For example, the geometrical type of plane, there are nine possible geometrical tolerances (see Table 8).

[2] The candidate geometrical tolerance is equal to one, and the selection is made as there is only one result (i.e. no selection needed).

[3] The use of DP_1(—) or DP_4(⌭) depends on the geometry type of each unit. If the unit is a cylinder, the DP_4(⌭) is applied; otherwise, the DP_1(—) is employed.

Table 9 The candidate schemes set for each type of feature

Feature	Candidate schemes set	Notes
Plane	Primary datum: $\boldsymbol{S_p} = [s_{p1}]$;*	s_{p1}: No decomposition;
	Secondary datum: $\boldsymbol{S_p} = [s_{p1}, s_{p2}]$;	s_{p2}: To decompose to kinematic and form variation;
	Other cases: $\boldsymbol{S_p} = [s_{p1}, s_{p2}, s_{p3}]$.	s_{p3}: To decompose to translation, rotation and form

		variation.
Revolving feature	Primary datum: $\boldsymbol{S_r} = [s_{r1}]$;* Secondary datum: $\boldsymbol{S_r} = [s_{r1}, s_{r2}]$; Other cases: $\boldsymbol{S_r} = [s_{r1}, s_{r2}, s_{r3}, s_{r4}, s_{r5}]$.	s_{r1}: No decomposition; s_{r2}: To decompose to kinematic and form variation of the derived feature; s_{r3}: To decompose to translation, rotation and form variation of the derived feature; s_{r4}: To decompose to translation, rotation, central-line variation and axial variation of integral feature. s_{r5}: To decompose to translation, rotation, central-line variation, axial variation of integral feature and radial variation of integral feature.
Sphere	Primary datum: $\boldsymbol{S_s} = [s_{s1}]$;* Secondary datum: $\boldsymbol{S_s} = [s_{s1}, s_{s2}]$; Other cases: $\boldsymbol{S_s} = [s_{s1}, s_{s2}]$.	s_{s1}: No decomposition; s_{s2}: To decompose to translation and form variation.
Pattern	Primary datum: $\boldsymbol{S_z} = [s_{z1}]$;* Secondary datum: $\boldsymbol{S_z} = [s_{z1}, s_{z2}]$; Other cases: $\boldsymbol{S_z} = [s_{z1}, s_{z2}, s_{z3}]$.	s_{z1}: No decomposition; s_{z2}: To decompose to kinematic and form variation of the derived feature; s_{z3}: To decompose to translation, rotation and form variation of the derived feature.

*: It is unnecessary for further selection.

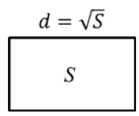
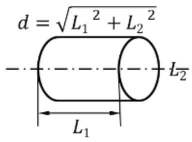
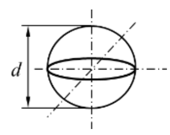
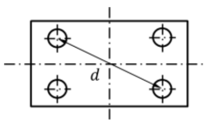
Table 10 The description and estimation of the attributes

Attributes	Symbol/Estimation	Description	Note
Quality	$c_{i1}=u_1(s_i)=\prod_{k=1}^{i}\eta_k$ *(i=1,2,…,n)**	The pass rate assesses it. A large c_{i1} shows the scheme has a high pass rate in the inspection.	η_k is the pass rate of a single tolerance type.
Precision	$c_{i2}=u_2(s_i)=\sqrt{\frac{\lambda_T}{id}}$ *(i=1,2,…,n)**	It is an estimation of the design/assembly precision of a feature. A small c_{i2} indicates the scheme has high precision.	λ_T is the tolerance value of the corresponding d** (Table 11) and IT (design tolerance grade) [38]
Applicative	$c_{i3}=u_3(s_i)=\sqrt{\lvert\lambda_1 \ln(\lambda_{IT})+\lambda_2-i\rvert}$ *(i=1,2,…,n)**	It is modelled to balance the precision and design function of a feature. A small c_{i3} indicates that the scheme is appropriate.	λ_{IT} is the value of the corresponding tolerance grades [38];
Verification	$c_{i4}=u_4(s_i)=\delta_1 e^{\delta_2 i}$ *(i=1,2,…,n)**	It is an estimation of verification complexity. A large c_{i4} shows that the verification of the scheme is complicated.	the constant coefficients λ_1 and λ_2 are determined by inspection conditions
Cost	$c_{i5}=u_5(s_i)=\alpha_1 e^{-\alpha_1 c_{i2}}+\alpha_2 e^{-\frac{\alpha_3}{c_{i2}}}$ *(i=1,2,…,n)**	It is an estimation of the cost of the device and manual. A small c_{i5} indicates that the scheme costs low. [41, 42]	α_0, α_1, α_2 and α_3 are determined by wages of a worker, craft and inspection method.

: "(i=1,2,…,n)*" means the order of the scheme in an $\boldsymbol{S}$. For example, to $\boldsymbol{S_z}=[s_{z1},s_{z2},s_{z3}]$, i of s_{z1},s_{z2} and s_{z3} is 1, 2 and 3 respectively.

**: The definition of d is illustrated in Table 11 for the estimation of the precision, which is referred to the literature [22].

Table 11 Determination of the length parameter of a feature

Features	Calculation methods	Illustration
Plane	The square root of the area of an integral feature.	
Revolving feature	The least-square norm of the axial and the radial length of an integral feature.	
Sphere	The diameter of a sphere.	
Pattern	The largest length between two units pattern.	

Appendix 3: The parameters in the case study

Table 12 The parameters for estimation of attributes [46]

Parameters	Plane	Revolving/Sphere	Pattern
η_k	0.9973	0.9973	0.9973
IT	6	6	6
λ_1	-1.762	-1.762	-1.762
λ_2	5.518	5.518	5.518
δ_1	0.806	0.806	0.806
δ_2	0.196	0.196	0.196
α_0	5.075	17.315	8.237
α_1	25.207	47.540	35.805
α_2	1.686	1.792	1.307
α_3	0.0086	0.033	0.0083

Table 13 The information value of each DP [30, 46]

DPs	information value
DP_1(—)	0
DP_2(⏥)	0
DP_3(○)	0
DP_4(⌭)	0
DP_5(//)	2.322
DP_6(⊥)	2.322
DP_7(∠)	2.322
DP_8(◎)	4.322
DP_9(⌯)	+∞
DP_{10}(⌖)	4.322
DP_{11}(↗)	0.585
DP_{12}(⌰)	1.170
DP_{13}(◎)	+∞
DP_{14}(⌒)	4.322
DP_{15}(⌓)	4.322

Note： Values of the parameters are listed as follow. $c_{vc} = 5, c_m = 7, c_{rm} = 20, c_{cmm} = 100, m_{vc} = 0.01, m_m = 0.002, m_{rm} = 0.002, m_{rmt} = 0.003, m_{cmm} = 0.001, V = 0.004,$ and $I_p = 0$.