DIMENSIONAL INSTABILITY OF CEMENT BONDED PARTICLEBOARD:

Modelling CBPB as a composite of two materials

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Summary. Previous papers have quantitatively indicated that the total movement of cement bonded particleboard (CBPB) is equal to the sum of the movement of its components. This paper examined the efficacy of the law of mixtures when applied to the movement of a wood-cement composite under internal swelling or shrinkage stresses. Abundant data generated in companion papers were first manipulated to develop the universal formulae for predicting the movement of components. In conjunction with previous numerical results from image analysis of the structure of CBPB, and the orientated elasticity and stress algorithms, the models for theoretically predicting mass and dimensional changes of CBPB were derived. Validation studies were conducted and these demonstrated an excellent agreement of the theoretical predictions with experimental data for both mass and dimensional changes of CBPB due to internal swelling or shrinkage stresses during adsorption and desorption. The success also implied that CBPB can be treated as a composite and its properties can be well derived by the law of mixtures even though CBPB is an unusual type of composite having a very high volume fraction of wood chips, but a very high mass fraction of cement paste.

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Introduction

Cement bonded particleboard (CBPB) is viewed as a composite material or mixture comprising two phases: cement paste and wood chips. Now that the work on the behaviour of CBPB and its components over the various conditions has indicated the additive behaviour between CBPB and its components (Fan, *et al*. 1998a; 1998b; 1998c; 1999), the next task is to characterise quantitatively the material behaviour of CBPB based on these two constituents, possibly using the law of mixtures.

The mixture theory has been applied to various composites to predict their mechanical properties, e.g. concrete (Ortiz, 1985), organic wood based panels (Hoover, *et al*. 1992; Poblete, 1992; Mundy, *et al.* 1996) and fibre reinforced cement mortar (Pakotiprapha, *et al*. 1974; Nathan, *et al.* 1977); it was also found to be applicable to assess wave propagation (Benveniste and Aboudi, 1976). In the application of the law, a certain number of assumptions are made, and mathematical models are basically derived from Hooke's Law using one, or a combination of three geometrical arrangements of two–phase models. For a short fibre composite under external loading, two factors are concerned–the orientation and length of the fibres. The stress in the fibre varies along the length of the fibre, being zero at the fibre ends and building up to a maximum value at the centre of the fibre as a result of shear at the fibre/matrix interface (Pakotiprapha, *et al*. 1974). The behaviour of this short fibre composite cannot be described, unlike an unidirectional long fibre composite, and correction factors must be introduced to account for the stress distribution and changing orientation of the fibres (Broutman and Krock, 1967; Cox, 1952).

In this paper, data generated in previous papers (Fan, *et al*. 1998c; 1999) were manipulated to develop universal formulae to predict the behaviour of the dissected chips and cement paste over various conditions. By applying the law of mixtures and treating the wood chips as "fibre" and the cement paste as "matrix", explicit expressions for predicting the dimensional and mass changes of CBPB were derived from these universal formulae, together with the numerical results of image analysis (Fan, *et al*. 1998e). Experimentally determined and predicted values have been compared in order to verify first the validity of the developed mathematical expressions, and second that CBPB could be treated as a composite and that the law of mixtures was applicable. It is hoped that the availability of analytical expressions for predicting mass and dimensional changes of CBPB will lead to a better understanding of the behaviour of CBPB and encourage its use under appropriate conditions.

Notation

E_{RT}	Mean transverse modulus of elasticity of wood
E_L	Longitudinal modulus of elasticity of wood
E_p	Modulus of elasticity of cement paste
E_{wa}	Modulus of elasticity of embedded wood chips at α angle
E_θ	Modulus of elasticity of wood chips at θ direction
E_φ	Modulus of elasticity of wood chips at φ direction
G_{LRT}	Mean transverse shear modulus of wood
$\Delta L(T)_{cp\alpha}$	Length/width (thickness) change of CBPB at α angle
$\Delta L(T)_p$	Length (thickness) change of cement paste
m_{pf}	Mass fraction of cement paste in unit mass of CBPB
m_{wf}	Mass fraction of wood chips in unit mass of CBPB
ΔM_{cpj}	Mass change of CBPB at the various conditions tested
ΔM_{pj}	Mass change of cement paste at corresponding conditions
ΔM_{wj}	Mass change of wood chips at corresponding conditions

$\Delta M(L; T)_{w/P}$	Mass, length or thickness changes of wood chips or cement paste at various conditions
t	Duration of exposure
ν_{LRT}	Mean transverse Poisson's ratio of wood
V_{pf}	Volume fraction of cement paste in unit mass of CBPB
V_{wf}	Volume fraction of wood chip in unit mass of CBPB
ρ_{cp}	Density of CBPB
ρ_k	Density of wood chip or cement paste
$\sigma_{cp\alpha}$	Overall stresses of CBPB at α angle
σ_L	Stress in the longitudinal direction of wood
σ_{RT}	Mean stress in the transverse direction of wood
σ_p	Stress of cement paste
$\sigma_{w\alpha}$	Stress of the wood chips at α angle
σ_θ	Stress of the wood chips at θ direction
σ_φ	Stress of the chip at φ direction
$\varepsilon_{cp\alpha}$	Strain in CBPB
ε_p	Strain of cement paste
ε_{WL}	Strain in the length of wood chips
ε_{WT}	Strain in the thickness of wood chips
$\varepsilon_{w\alpha}$	Strain in wood chips
α	Angle between the longitudinal direction of wood chips and surfaces or edges of CBPB
θ	Angle between wood chips and edges (length direction) of CBPB
φ	Angle between wood chip and vertical coordinate

A, B, C Coefficients related to the feature of materials and exposure conditions

Theoretical considerations

Assumptions

The following assumptions are made in predicting the stress – strain relation of the CBPB:

1) All the chips are straight and have identical mechanical properties;
2) The length direction of the wood chips is in the longitudinal direction of wood;
3) The radial and tangential directions of the wood chips are uniformly orientated through 360^0 and the mean values of radial and tangential properties represent the overall transverse properties;
4) The wood chips are uniformly distributed with equal probabilities of orientation in the length and width direction of CBPB and can be represented by an equivalent volume or effective number of wood chips. This assumption was proved by the results of image analysis (Fan, *et al*. 1998e);
5) The strain between the wood chips and cement paste matrix is compatible.

According to the law of mixtures, it can be proposed that volume fractions should be used in the determination of mechanical properties and volume changes, and mass fractions in the determination of mass change. Both fractions could be applied in density calculations, but by virtue of the unnatural compressed condition of cement paste and wood chips in CBPB it is more reasonable to employ the mass fraction.

Density prediction

An equation for the density of CBPB was developed by applying to the law of mixtures:

Case 1, by using volume fraction. The density of CBPB is

$$\rho_{cp} = \sum \rho_k V_{kf} \tag{1}$$

Case 2, by using mass fraction. The density of CBPB can be calculated as:

$$\rho_{cp} = \sum \rho_k m_{kf} \tag{2}$$

In the equations, k denotes p or w representing cement paste or wood chips.

Mass change prediction

Applying the law of mixtures, the change in mass of CBPB during sorption can be expressed as:

$$\Delta M_{cpj} = \Delta M_{pj} m_{pf} + \Delta M_{wj} m_{wf} \tag{3}$$

On adsorption, a coefficient should be included to account for the overlap condensation on the cement paste within the dissected chips. The "correction coefficient" is approximately equal to the ratio of mass change of dissected chips on desorption to that on adsorption, assuming similar behaviour in the adsorption and desorption of wood under normal environment. In this study the value was calculated to be about 0.66.

Predicting dimensional changes by using values relating to its constituents

When CBPB is subjected to changing environments, it is the swelling or shrinking stresses which produce dimensional (volume) changes. This sharply departs from most theories proposed to date which consider performance of composites as relating to externally applied stresses. It is necessary to devise an appropriate law for dimensional changes due to internal stresses by considering the relationship of stresses between the wood chip and cement paste, the nature of cement paste and wood chips, and the structure of CBPB.

The overall stresses of CBPB due to sorption is:

$$\sigma_{cp\alpha} = \sigma_p V_{pf} + \sigma_{w\alpha} V_{wf} \tag{4}$$

The α in the equation is related to the orientation of the wood chips, and

$$\sigma_p = E_p \varepsilon_p ; \sigma_{w\alpha} = E_{w\alpha} \varepsilon_{w\alpha} \tag{5}$$

In the literature, three models have been used for predicting the property of two–phase materials (e.g. Illston, *et al*. 1979). The model with the aggregate set within the cement paste appeared to be appropriate for the present study in view of the type of stresses due to moisture change, and the structure of CBPB (Fan, *et al*. 1998d), and this was selected.

Assuming the compatibility of strain in the wood chip and cement paste and by applying Hooke's Law the dimensional change of CBPB can be given as:

$$\varepsilon_{cp\alpha} = \sigma_{cp\alpha} \left(\frac{1 - \sqrt{V_{wf}}}{E_p} + \frac{1}{\left(\frac{1}{\sqrt{V_{wf}}} - 1 \right) E_p + E_{w\alpha}} \right) \tag{6}$$

Substituted $\sigma_{cp\alpha}$ with equation 4, σ_p with equation 5, and ε with $\Delta L(T)$, the dimensional change of CBPB can be calculated as:

$$\Delta L(T)_{cp\alpha} = [V_{pf} E_p \Delta L(T)_p + V_{wf} \sigma_{w\alpha}][\frac{1-\sqrt{V_{wf}}}{E_p} + \frac{1}{(\frac{1}{\sqrt{V_{wf}}}-1)E_p + E_{w\alpha}}] \quad (7)$$

$E_{w\alpha}$ and $\sigma_{w\alpha}$ depend upon the orientation of the wood chips (the angle between the grain of the wood chips and the surfaces or edges of CBPB). $E_{w\alpha}$ can be calculated by transforming the stress or strains from the rotated to the principal axes.

$$E_\varphi = \frac{1}{\frac{\cos^4\varphi}{E_L} + \frac{\sin^4\varphi}{E_{RT}} + (\frac{1}{G_{LRT}} - \frac{2\nu_{LRT}}{E_L})\sin^2\varphi\cos^2\varphi} \quad (8)$$

Image analysis showed that almost all chips lie flat in the CBPB (Fan, *et al*. 1999e). The effective angle between wood chips and the horizontal surface is 10.7^0. Thus, for the thickness direction of CBPB, $\varphi = 79.3^0$.

However, the effect of orientation of wood chips on the change in plane rotation of CBPB should be very similar due to the random distribution of wood chips in the horizontal surfaces (Fan, *et al.* 1998e). The mean effective angle cannot directly be used. $E_{w\alpha}$ should be produced by integration, θ ranging from 0 to π/2 radians.

$$\overline{E}_\theta = \frac{\int_0^{\frac{\pi}{2}} \frac{1}{\frac{\cos^4\theta}{E_L} + \frac{\sin^4\theta}{E_{RT}} + (\frac{1}{G_{LRT}} - \frac{2\nu_{LRT}}{E_L})\sin^2\theta\cos^2\theta} d\theta}{\int_0^{\frac{\pi}{2}} d\theta} \quad (9)$$

The Hankinson Formula was introduced to calculate the σ_{wa}. Therefore,

$$\sigma_{\varphi} = \frac{\sigma_L \sigma_{RT}}{\sigma_L \sin^n \varphi + \sigma_{RT} \cos^n \varphi}$$

$$= \frac{E_L E_{RT} \varepsilon_{Wl} \varepsilon_{wT}}{E_L \varepsilon_{wl} \sin^n \varphi + E_{RT} \varepsilon_{WT} \cos^n \varphi} \qquad (10)$$

As for the moduli of elasticity, so the modelling of length / width changes, $\sigma_{w\theta}$ should be integrated:

$$\sigma_{\theta} = \frac{\int_0^{\frac{\pi}{2}} \frac{\sigma_L \sigma_{RT}}{\sigma_L \sin^n \theta + \sigma_{RT} \cos^n \theta} d\theta}{\int_0^{\frac{\pi}{2}} d\theta}$$

$$= \frac{\int_0^{\frac{\pi}{2}} \frac{E_L E_{RT} \varepsilon_{wl} \varepsilon_{wT}}{E_L \varepsilon_{wl} \sin^n \theta + E_{RT} \varepsilon_{wT} \cos^n \theta} d\theta}{\int_0^{\frac{\pi}{2}} d\theta} \qquad (11)$$

Numerical tests

A general theory of the behaviour of CBPB during sorption has been outlined which would appear to exhibit considerable potential as a basis for a numerically systematic explanation of the observed material behaviour due to swelling or shrinkage stresses. To fulfil the objectives completely, it remains to check first, the ability of the theory to reproduce the available data and second, the suitability of the model for use in computation.

Properties of wood chips and cement paste used in CBPB

The properties of wood were taken from previous data (Dinwoodie, 1981) in view of the identical wood species (Sitka spruce). It has been found that the elasticity of wood increases by 1.5 %/ unit % decrease in its moisture content (MC) within the range of moisture content 6 and 20 % (Gerhards, 1982); the values at moisture content of 12 % were converted to those at moisture contents 8.53 % and 19.59 % corresponding to the moisture content measured on exposure of CBPB to 35 % and 90 %RH (Table 1)

Table 1 Property of wood chips used in CBPB

Property	At MC of 12 % (N/mm^2)	At MC of 8.53 % (N/mm^2)	At MC of 19.59 % (N/mm^2)
E_L	11600	12204	10279
E_{RT}	700	736	620
ν_{LRT}	0.025	0.025	0.025
G_{LRT}	735	774	685

The elasticity of laboratory cement paste was measured. The mean value is 21200 N/mm^2.

Mass and dimensional change of dissected chips and of laboratory cement paste under a single change in RH

Only changes in mass and dimensions of dissected chips and laboratory cement paste in moving from 35 to 90% or from 90 to 35 %RH (Fan, *et al*. 1998c; 1999) were predicted in order to obtain the universal formulae for modelling CBPB as a composite in a later section.

As qualitatively discussed in previous papers (Fan, *et al*. 1998c; 1999), the trend of changes in dissected chips and laboratory cement paste, whether in mass or in dimensions, was similar to those of CBPB (Fan, *et al*. 1998a). A similar procedure was followed to that used in the prediction for CBPB to develop universal formulae. A summary of the models for the changes in mass and dimensions of dissected chips and laboratory cement paste is:

$$\Delta M(L;T)_{W/P} = A + Bt - Ae^{Ct} \tag{12}$$

The coefficients of the mathematical equation are given in Table 2.

Table 2 Coefficients of mathematical equations for mass and dimensional changes of cement paste and dissected chips

Model	$\Delta M(L;T)_{W/P} = A + Bt - Ae^{Ct}$						
	35-90%RH				90-35%RH		
Regression coefficients	A	B	C		A	B	C
Cement paste							
Mass	4.6918	$4.6\text{x}10^{-3}$	-0.0681		-1.6057	$4.0\text{x}10^{-3}$	-0.1432
Length	0.0593	$3.0\text{x}10^{-5}$	-0.1456		-0.1049	$1.0\text{x}10^{-4}$	-0.1050
Dissected chips							
Mass	7.5666	$4.4\text{x}10^{-3}$	-5.2688		-5.8263	$-1.2\text{x}10^{-4}$	-76.3860
Length	0.4993	$3.5\text{x}10^{-5}$	-6.0125		-0.4764	$-3.3\text{x}10^{-5}$	-5.7367
Thickness	4.4633	$3.1\text{x}10^{-4}$	-53.7450		-5.1712	$-3.6\text{x}10^{-4}$	-62.2694

Experimentally determined and calculated changes are presented in Figure 1 for the mass and length changes of cement paste, and in Figure 2 for the mass and dimensional changes of dissected chips.

Mass and volumetric fractions of wood chips and cement paste in CBPB

It is known that the CBPB tested was made using 65 % cement paste and 35 % wood chips (by mass). Therefore, the mass fractions of the components in unit mass of the CBPB can be calculated (Table 3).

Table 3 Mass and volume fractions of cement paste and dissected chips in unit mass of CBPB used

Component	Mass fraction (m_{kf})*	Oven dry density (Kg/m^3)	Volume in unit mass of CBPB (m^3)	Volume fraction (V_{kf})
Cement paste	0.65	1857.78	3.5×10^{-4}	0.26
Wood chips	0.35	342.22	10.23×10^{-4}	0.74

*k = p or w

Using the relationship between mass, density and volume, the volume of the cement paste and wood chips in unit mass of the CBPB can be calculated, and thus the volume fractions of components are determined (Table 3).

Comparison of predicted density of CBPB with experimental values

Using equations 1 and 2, densities of CBPB are calculated and summarised in Table 4.

Table 4 Experimentally determined and theoretically predicted densities of CBPB at various conditions

Condition	Experimental (Kg/m^3)	Predicted by using volume fractions (Kg/m^3)	Predicted by using mass fractions (Kg/m^3)
At 20 ^{0}C/35 %RH	1342.08	774.6	1371.3
At 20 ^{0}C/65 %RH	1366.33	798.16	1408.2
At 20 ^{0}C/90 %RH	1430.29	875.28	1528.9

It is apparent that the densities predicted using the mass fractions were in good agreement with experimental data. The deviation between the values predicted and measured was only about 2 %, 3 % and 6 % at 35 %RH, 65 %RH and 90 %RH respectively. However, the deviations predicted by using the volume fraction range from 38.8 to 50.1 %. This confirms that the mass fraction should be chosen in the application of the raw of mixtures.

Application of the law of mixtures for mass change of CBPB

To evaluate the efficacy of the law of mixtures for mass change of CBPB, changes in the mass of CBPB on moving from 35 to 90 %RH and from 90 to 35 %RH were predicted by using equation 3. The theoretically predicted and experimentally determined values are compared and plotted in Figure 3A. As may be seen, mass increase and decrease were correctly predicted, the model accurately captured the relevant features of the response. The higher values predicted in the early stages were due to the size effect on the values arising from wood chip and cement paste (particles of wood chips and cement paste have higher exposed surfaces compared to CBPB in unit volume). The lower values predicted at later stage was due probably to a further hydration or higher carbonation of laboratory cement paste in terms of its shorter storage period prior to the test.

Application of the law of mixtures for dimensional change of CBPB

The final set of comparisons concerns the dimensional changes of CBPB. Equations 7 – 11 were used to compare the theoretically predicted and experimental results. In the prediction of length change, stress relief was considered in view of results obtained from image analysis in previous paper of this series (Fan, *et al*. 1998d), and a 0.5 coefficient was also included to account for the random distribution of wood chips in two directions (Illston, *et al*. 1994). The theoretically predicted and experimentally determined values are illustrated in Figures 3B and 3C. The close agreement between the experimentally determined and theoretically predicted values further validates the proposed models

A comparison of the predictions of thickness with those of length shows a slight higher correlation for thickness change than for length change. This may be attributable to a difference between the assumption (ideally random) and reality of the distribution of the wood chips in the plane of CBPB. Once again, there existed a higher deviation under desorption compared to adsorption due probably to further hydration or higher carbonation of laboratory cement paste.

Conclusions

1) It has been shown in this paper how the law of mixtures can be successfully applied to the constitutive behaviour of CBPB due to swelling and shrinkage stresses. This is a novel application of the law to this material. The approach gives a simplified method for theoretically predicting mass and dimensional changes of CBPB and a physical

interpretation of the mechanism of the movement of CBPB. Numerical tests demonstrated good agreement of the predictions using the law of mixtures with experimental data.

2) The success of the results also indicated that the properties of CBPB are directly related to the structure of CBPB (size distribution and orientation of chips, and percentage occupied by components). Thus, the product can be not only controlled by the percentage of components, but also by the placement of wood chips.
3) Mass fraction was more applicable for the prediction of mass and density, while volume fraction was more applicable for the dimension prediction, showing that in using the law of mixtures, the most closely related fraction should be chosen.
4) In using the values for the constituents in the models, caution must be exercised to make allowances where one material is integrated with another, e.g. when the cement paste is within the dissected wood chips.
5) The values of change predicted theoretically for early stages were higher than those determined experimentally, reflecting a size effect of the composite constituents.
6) The correlation between theoretical prediction and experimental determination was very high for mass and dimensional changes. Slightly higher deviation for length changes compared to thickness changes suggested that the accurate prediction of the distribution of the wood chips within CBPB is vital when applying models to the behaviour of CBPB.

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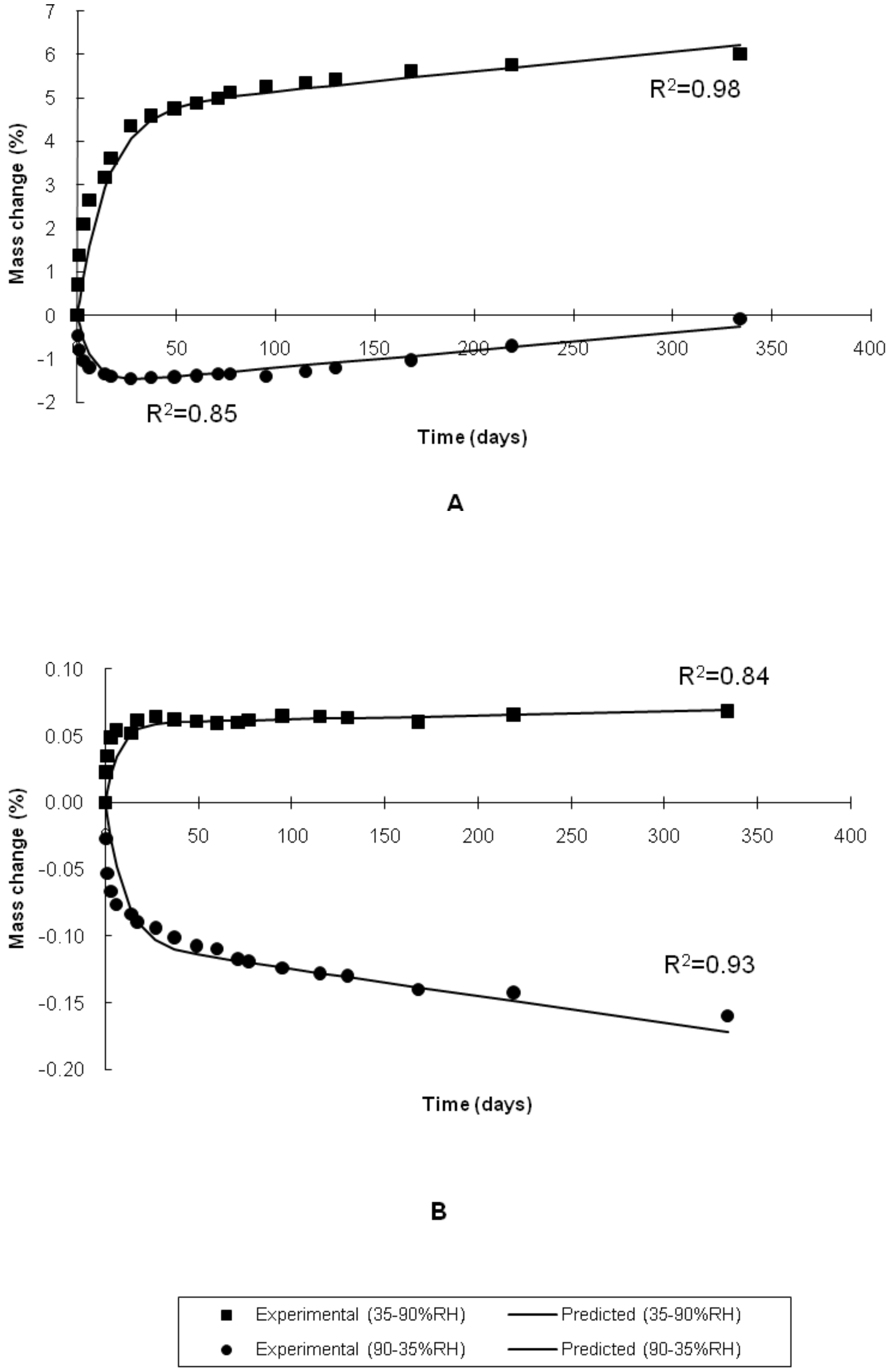

Figure 1 Experimental and theoretically predicted change in mass (A) and length (B) of cement paste in moving from 35-90 or from 90-35%RH

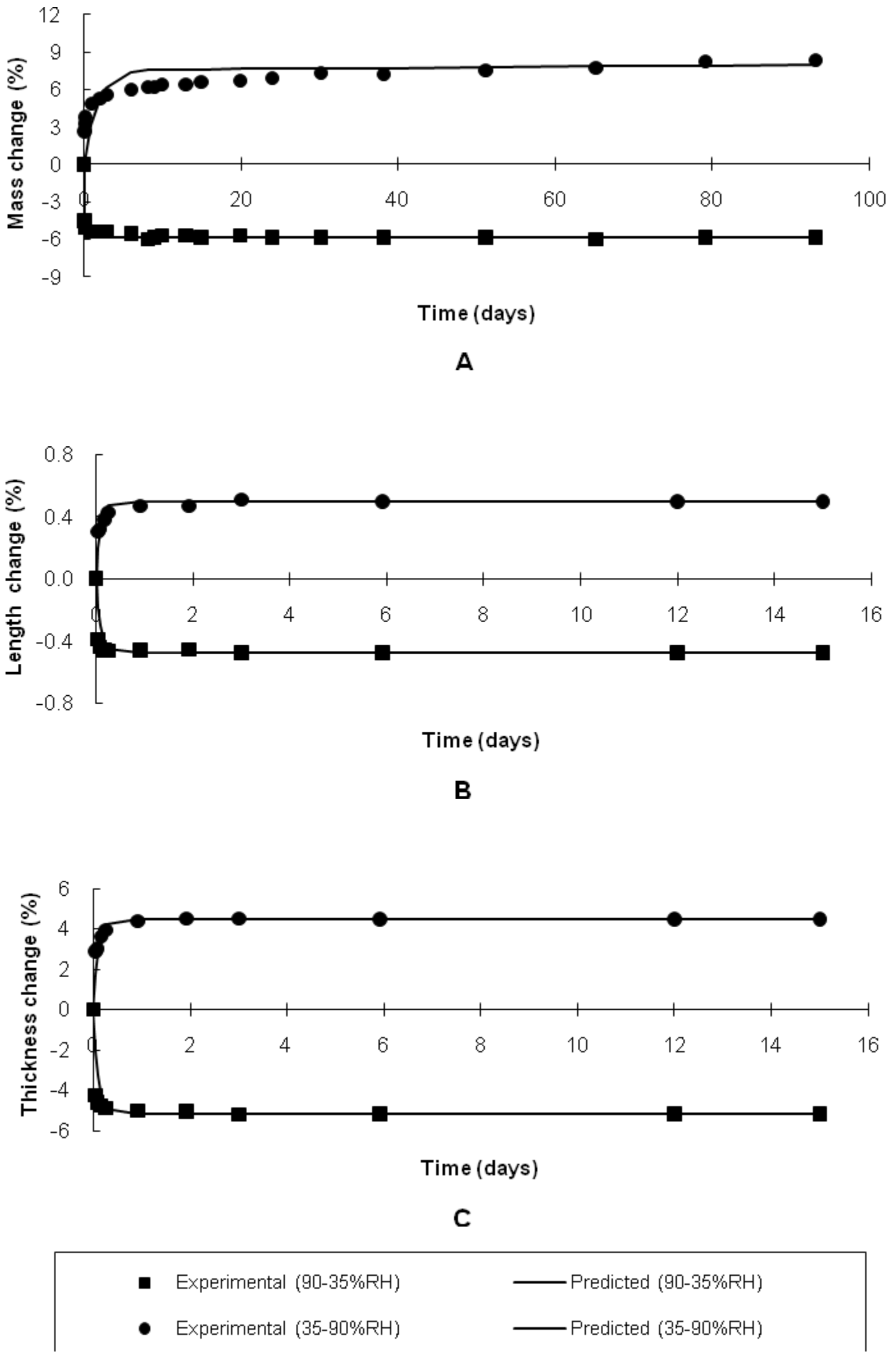

Figure 2 Experimental and theoretically predicted changes in mass (A), length (B) and thickness (C) of dissected chips in moving from 35-90 or from 90-35%RH

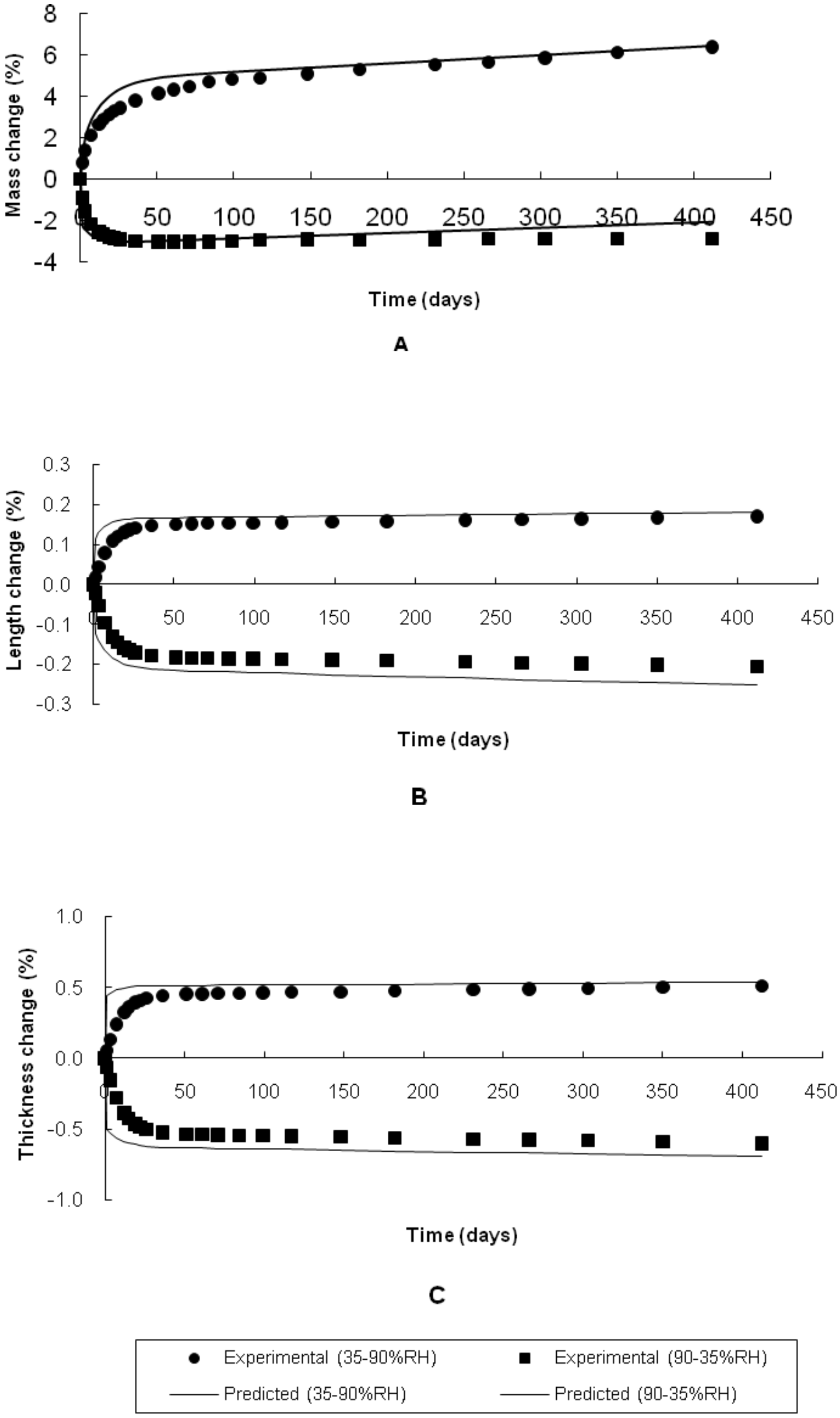

Figure 3 Theoretically predicted and experimental changes in mass (A), length (B) and thickness (C) of CBPB on both adsorption and desorption