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Working Paper 128
May 2020

FACULTY OF MANAGEMENT AND BUSINESS

FI-33014 TAMPERE UNIVERSITY, FINLAND

ISSN 1458-1191
ISBN 978-952-03-1594-8 (pdf)

Instrument choice in the case of multiple externalities

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May 22, 2020

Abstract

We study market-based regulation in a polluting industry that produces two externalities at the same time. There is a negative externality (emissions) to which every firm in the industry contributes, and a positive externality (technological spillover), so that an additional application of green technology becomes easier as the number of appliers increases. An optimal policy is shown to consist of a uniform emission price across polluting firms and a subsidy to early users of green technology. We also show that the presence of the second externality strongly affects the instrument choice under uncertainty between taxes and tradable permits, and that the influence depends on the design of the instruments. More specifically, it depends on whether early users of green technology are subsidized or not.

Keywords: Green production, emission taxation, internalizing externalities, spillover effect, tradable emission permits, uncertainty

JEL Codes: D62, D81, H23, Q58

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1 Introduction

The subject of this study is the market-wide transition from brown toward green production and the role of the market-based implementations in this transition. We assume that the speed of transition is affected by knowledge spillovers inside the polluting industry. We have two main research questions. First, we ask how the simultaneous presence of negative extenality (emissions) and positive externality (technological spillover) affects the market-based instrument designs.[1] Second, we ask how adding the positive externality on the top of the negative one affects instrument choice between market-based instruments (tradable permits and environmental taxes) under uncertainty. After all, the traditional formula of instrument choice (Weitzman [12]) is derived under a single negative externality.

Our results show that the traditional emission pricing rule is not affected as the expected emission price should equal the expected marginal damages of emissions. Moreover, the positive externality should be internalized by subsidizing firms that produce the externality and leave the externality receivers totally unsubsidized. Regarding the second question, we show that instrument choice is affected by the presence of the second externality. However, the magnitude and the direction of the impact is case-specific.[2] First, the impact depends on the scope of the policy, so it matters whether the positive externality is priced or not. Second, if the positive externality is priced, the impact depends on the specific design of the instruments. The subsidy may either be fixed (zero or non-zero) or the size of the subsidy may follow the changes in the permit price. Altogether, regardless of which of these options is chosen, the derived comparative statistic does not reduce back to the original formula as represented by Weitzman [12] in the year of 1974.

We summarize our findings in instrument choice (Weitzman [12]) into three main categories. First, adding knowledge spillovers into a polluting industry reduces marginal abatement costs in an externality receiving sector, so aggregate marginal costs are reduced as well. Importantly, this takes place in such a way that efficiency is main-

[1] Jaffe, Newell, and Stavins [4] discuss the simultaneous presence of knowledge spillovers and environmental externalities.

[2] We assume that the second externality is positive for most of the time. At the end of the work, we briefly discuss the effects of a negative technological externality in our model.

tained. A similar effect can be found from Mendelsohn [7], who finds that endogenous technical change favors the quantity instrument. This happens practically for the same reason as positive spillovers favors the quantity instrument: Investments in R&D reduces the slope of the marginal abatement cost curve.[3]

The second effect evolves as technological externality gives rise to correlation between sector-specific technology adoptions (Meunier [8]). The correlation is an endogenous phenomenon as the random variables themselves are independently distributed in our model. The correlation is positive as there exists cost complementarity in aggregate costs, so that the benefits from a green investment increases as the number of green technology users in another sector increases. As in Meunier [8], positive correlation implies that the tax instrument is favored in instrument choice.

The third category is associated with the nature of constraints that we encounter in our analysis. First, there is a constraint on information in our model. It means in practice that the regulatory agency is unable to change the policy *ex-post* to reflect the changes in business environment. This type of constraint is already analyzed in Weitzman [12], where a constraint on information causes regulatory instruments to implement wrong level of emissions *ex post.* However, in the present context, there is an additional externality to be internalized and another constraint on information to be suffered. Consequently, the policy not only implements wrong level of emissions but also wrong size of spillovers *ex post.*

However, consequences of the informational constraint depend on the prevailing subsidy regime. In a Weitzman-type approach, the second externality is internalized by following the rules from expected social welfare optimization. Alternatively, the agency may have a mandate to internalize only the negative externality but not the positive one.

We will analyze these issues in instrument choice by studying three different subsidy rules. In the first case, permit implementation uses a subsidy rule, where the size of the subsidy follows movements in the permit price according to a pre-announced formula ("the pegged-rule"). In the second case, both permit and tax implementa-

[3]Requate [9] reviews the performance of various environmental policy instruments in the adoption and the development of green technology. Among other things, he subsumes spillovers under technology innovation (in contrast of technology adoption).

tions apply the same fixed subsidy.[4] In the the third case, the rule is that there are no subsidies. We show that the fixed rule has an advantage over pegged rule over a range of spillover effects. Furthermore, we show that a regime change from zero-subsidization toward fixed-subsidization invariably favors the quantity instrument, tradable permits.

Earlier, Shinkuma and Sugeta [10] has studied market-based regulations under positive and negative externalities. They study instrument choice in the long run when entry itself creates a positive externality. Decision-making is sequential, so that a polluting firm learns its productivity and chooses the level of production only after it has decided to enter the market. Consequently, the aggregate number of firms does not react to the outcomes of uncertainty. More recently, Meunier [8] has studied changes that a second externality induces to market-based instruments under uncertainty. It is similar to our study as the second externality does not enter the model endogenously but is exogenously given. However, the second externality remains unpriced throughout the analysis, whereas our analysis especially studies the pricing of it. We will include the unpriced case in our framework as well partly because it gives us the change to learn the consequences of subsidization.

Our work complements the study of technology change in a Weitzman [12] framework by incorporating technological spillovers into it.[5] Earlier studies include D'Amato and Dijkstra [2], Krysiak [5], and Mendelsohn [7]. The model of D'Amato and Dijkstra [2] can be considered as a pure model of adoption. Conversely, In Mendelsohn [7], there is an explicit (uncertain) relationship between technological advances and R&D expenses. Specifically, for Mendelsohn [7], innovation is a pure private good, so the firm under investigation has proper private incentives and no subsidization is required.

We start by defining a polluting industry. It consists of two sectors, an externality-generating sector and an externality-receiving sector. We then show that a negative externality and a positive externality can be simultaneously internalized by market-based instruments in a socially efficient manner. Both environmental taxes and trad-

[4]In this context, a "quantities vs. quantities" arises as the permit system can be implemented either by a fixed or by a pegged rule.

[5]For further discussion about knowledge spillovers and environmental regulation, see Heal and Tarui [3] and Smulders and DiMaria [11].

able permits are able to implement the social optimum, but the expected levels of social welfare are shown to differ between the instruments. We will review this issue under two specifications, namely, under a pegged rule and under a fixed rule. We also study consequences of using a zero rule, where the agency is constrained not to subsidize the externality-generating sector at all. We will provide a summary of the main results in the concluding section.

1.1 The Polluting Industry

The polluting industry consists of two sectors, labeled as g and r. The sectors, in turn, consists of numerous infinitesimally small firms. The firms b_j are uniformly distributed over $[0, 1]$ in the both sectors. Every firm in the industry may choose between two production technologies, labeled as 0 and 1. A firm b_j that uses incumbent technology 0 in sector j has variable costs

$$A_{0j}(e_{0j}; b_j) = \frac{1}{2}\left(\bar{b}_j - b_j - e_{0j}\right)^2,$$

where $\bar{b}_j > 0$ and $j = g, r$. Variable costs are a function of emissions e_{0j}. Denoting the unit price of emissions by s, we have

$$e_{0j}(b_j, s) = \bar{b}_j - b_j - s, \tag{1}$$

so the costs as a function of emissions price are

$$C_{0j}(s) = A_{0j}(s) = \frac{1}{2}s^2. \tag{2}$$

Alternatively, the firm b_j may use new technology 1 in sector j. It has variable costs

$$A_{1j}(e_{1j}; b_j) = \epsilon_j + \frac{1}{2}\left(\bar{b}_j - b_j - q_j - e_{1j}\right)^2,$$

which means that the costs A_{1j} depend on emissions e_{1j} and on an additive random variable ϵ_j that satisfies $E(\epsilon_j) = 0$. In particular, it holds that

$$e_{1j}(b_j, s) = e_{0j}(b_j, s) - q_j, \tag{3}$$

so the investment disposes of a part q_j of the emissions away. Consequently, we will call technology zero as brown (polluting) technology and technology one as green (clean) technology. We also write

$$A_{1j}(s) = \epsilon_j + \frac{1}{2}s^2, \tag{4}$$

where $j = g$, r.

If the firm b_j invest in technology 1, the total production costs are

$$C_{1j} = A_{1j}(s) + I_j(b_j) + X_j, \tag{5}$$

where $j = g$, r. An investment creates an investment cost equal to $I_j(b_j)$. It can be written as

$$I_j(b_j) = \overline{F}_j + \eta_j + c_j b_j. \tag{6}$$

It holds that $\overline{F}_j > 0$ and $c_j > 0$ are sector-specific constants while η_j is a random variable, $j = g$, r. It further holds that $E(\eta_j) = 0$ and that η_j and ϵ_j are identically and independently distributed. Moreover, for future needs, we define

$$\theta_j = \varepsilon_j + \eta_j.$$

By the properties of ε_j and η_j, it holds that $E(\theta_j) = 0$ and $Var(\theta_j) = Var(\varepsilon_j) + Var(\eta_j) = \sigma^2$, where $j = r, g$.

The factor X_j in the production cost formula (Equation (5)) represents influences external to the sector j. Consequently, we denote $X_j \equiv X_j(b_k)$, so the investment costs in sector j are influenced by the investments in sector k, where $j \neq k$. In principle, externalities can flow between and within the technologies and sectors and they can be either negative or positive. However, we substantially restrict the externality flows from the outset. First, we assume that there are externalities only within the green technology. Second, we set

$$X_g = 0. \tag{7}$$

This means that externality flows from sector g to sector r, not *vice versa.* We say that sector g is an externality generator while sector r is an externality recipient.

Third, we assume that

$$X_r > 0. \tag{8}$$

Positive effect implies positive spillovers: An investment creates not only private but also public benefits.[6] The investments in green technology in sector g will increase the productivity of green technology in sector r.[7] As always, the most important feature of an externality is that it causes real effects, but it has no price.

Finally, in addition to the emission price s, we incorporate another regulatory instrument into analysis. We assume technology- and sector-specific subsidies. We denote these by S_{ij}, where $i = 0, 1$ and $j = g, r$. Furthermore, for future purposes, we define a subsidy difference as

$$\Delta S_j = S_{1j} - S_{0j}.$$

1.2 The Externality-Generating Sector

In sector g, total costs for a firm b_g after choosing technology 0 are

$$\Pi_{0g}(b_g, s) = C_{0g}(s) + se_{0g}(b_g, s) - S_{0g}.$$

Alternatively, total costs for the same firm b_g after choosing technology 1 are

$$\Pi_{1g}(b_g, s) = C_{1g}(b_g, s) + se_{1g}(b_g, s) - S_{1g}.$$

The firm b_g chooses to invest if

$$\Pi_{1g}(b_g, s) < \Pi_{0g}(b_g, s).$$

After incorporating the various cost components from Equations (1)−(8), this is equivalent that a condition

$$b_g < \frac{sq_g + \Delta S_g - \overline{F}_j - \theta_g}{c_g}$$

[6]The negative effect in turn implies that some scarce resources are congested. Product market rivalry may also explain the negative sign.

[7]Moreover, every green firm in sector r enjoys the same size of the externality effect, X_r.

holds. Furthermore, there exists a firm b_g^* that is indifferent between technologies 0 and 1. Consequently, it holds

$$\Pi_{1g}(s, b_g^*) = \Pi_{0g} s, (b_g^*),$$

so that

$$b_g^* = \frac{sq_g + \Delta S_g - \overline{F}_g - \theta_g}{c_g}. \tag{9}$$

In our model, firms in sector g are divided by the technology they use. The firms $b_g \in \left[0, b_g^*\right]$ apply green technology while the firms $b_g \in \left[b_g^*, 1\right]$ use brown technology. Note, in particular, how increasing strictness of environmental policy ($ds > 0$) or increasing support for green technology ($d\Delta S_g > 0$) increases investments in green technology ($db_g^* > 0$).

1.3 The Externality-Receiving Sector

In sector r, total costs for a firm b_r after choosing technology 0 are

$$\Pi_{0r}(b_r, s) = C_{0r}(s) + se_{0r}(b_r, s) - S_{0r} = \frac{1}{2}s^2 + s\left(\overline{b}_r - b_r - s\right) - S_{0r} \tag{10}$$

or, after choosing technology 1, aggregate costs become

$$\begin{aligned} \Pi_{1g}(b_r, b_g, s) &= C_{1r}(b_r, b_g, s) + se_{1r}(b_r, s) - S_{1r} \qquad (11) \\ &= \varepsilon_r + \frac{1}{2}s^2 + \overline{F}_r + \eta_r + c_r b_r - \phi b_g + s\left(\overline{b}_r - b_r - q_r - s\right) - S_{1r}. \end{aligned}$$

Importantly, we assume a linear externality effect (Meunier [8])

$$X_r = \phi b_g,$$

where $\phi > 0$ and b_g is the number of firms that use green technology in sector g. Externality effect is a knowledge spillover: A green investment in sector g improves relative profitability of green technology in sector r and the externality disappears at $b_g = 0$. If $\phi < 0$, instead, then a green investment in sector g congests applications of green technology in sector r. In what follows, we derive our results under positive technological externality. We discuss shortly the other possibility at the end of the study.

Next, we derive the use of green technology b_r^* in sector r for a given number of investments b_g^* in sector g. By definition, a firm b_r^* is indifferent between brown and green technologies which means that

$$\Pi_{0r}(b_r^*, s) = \Pi_{1r}(b_r^*, b_g^*, s).$$

By Equations (10) and (11), we calculate that

$$b_r^*(b_g^*) = \frac{sq_r + \Delta S_r - \overline{F}_r - \theta_r + \phi b_g^*}{c_r}.$$

We insert the firm b_g^* (Equation (9)) into $b_r^*(b_g^*)$, so

$$\begin{aligned} b_r^* &= \frac{sq_r + \Delta S_r - \overline{F}_r - \theta_r + \phi\left(\frac{sq_g + \Delta S_g - \overline{F}_g - \theta_g}{c_g}\right)}{c_r} \\ &= \left(\frac{c_g q_r + \phi q_g}{c_r c_g}\right) s + \frac{\Delta S_r}{c_r} + \frac{\phi \Delta S_g}{c_r c_g} - \frac{\overline{F}_r + \theta_r}{c_r} - \phi \frac{\overline{F}_g + \theta_g}{c_r c_g}. \end{aligned} \tag{12}$$

Sector g has an indirect effect on investment choices in sector r through the externality effect.

2 Instrument design

2.1 Social Optimum

In this chapter, we derive socially optimal environmental policy. It consists of optimal price of emissions and of optimal subsidies that together determine the optimal emission levels and technologies for firms in the polluting industry. By studying the sector-specific responses in Equations (9) and (12), the choices are seen to depend on the differences between the individual subsidy levels (ΔS_j). As $\Delta S_j = S_{1j} - S_{0j}$, a positive subsidy means that green technology is subsidized more heavily than brown technology. In principle, the policy can even set a negative subsidy for brown technology ($S_{0j} < 0$) to support the use of green technology.

The optimization amounts to a minimization of aggregate expected societal costs, i.e., the problem can be written as

$$\min_{s,\Delta S_r,\Delta S_g} ESC = E(C_g + C_r) + ED(e),$$

where C_g and C_r are the total production costs in sectors g and r, respectively. We denote damages of emissions by $D(e)$, where

$$e = e_g + e_r.$$

We further assume that $D'(e) > 0$ and $D''(e) > 0$.

We solve the cost minimization in Appendix A. Based on the optimization, we write

Lemma 1 *Optimal design applies a strictly non-zero emission price.*
Proof. *By Equation (55) in Appendix A, the first order condition for the price variable can be written as*

$$\frac{dESC}{ds} = E\left[2s + q_r\frac{dSC}{\Delta S_r} + q_g\frac{dSC}{\Delta S_g} - 2D'(e)\right] = 0. \tag{13}$$

Consequently, if both $\frac{dSC}{\Delta S_r}$ *and* $\frac{dSC}{\Delta S_g}$ *vanish, the differential* $\frac{dESC}{ds}$ *does not vanish.* ■

Intuitively, Lemma 1 follows as our framework includes both variable and fixed costs, and the unit price is needed in the regulation of the variable costs. Regarding the content of the optimal environmental policy, we have

Lemma 2 *Optimal policy design implements the emission price*

$$Es = ED'(e), \tag{14}$$

and the subsidies

$$\Delta S_g = \phi E b_r^* \tag{15}$$

and

$$\Delta S_r = 0. \tag{16}$$

Proof. Optimality requires that $\frac{dSC}{\Delta S_r} = \frac{dSC}{\Delta S_g} = 0$. The optimal emission price follows as we insert these conditions into Equation (13). As for the subsidy policy, the rules in Equations (15) and (16) will follow as we manipulate first order conditions $\frac{dSC}{\Delta S_r} = 0$ and $\frac{dSC}{\Delta S_g} = 0$ to yield Equations (57) and (58) in Appendix A. ■

Altogether, two types of externalities simultaneously operate in our framework. In addition to technological spillovers, there is a negative externality (emissions) to which every firm in the polluting industry contributes. The calculations above yield a familiar rule that an optimal policy should equate the emission price with the expected marginal damage (Equation (14)). This rule applies both in the presence and absence of spillover effect. On the other hand, subsidization tool should be used only in the presence of technological spillovers (Equations (15) and (16)). As the value $\phi = 0$ means that zero spillovers work in the polluting industry, then no subsidization is needed at all. If $\phi > 0$ instead, the optimal policy should subsidize the sector that generates the externality and leave the externality receiving sector totally unsubsidized. As only green technology produces the externality, it should be favored over brown technology[8] in sector g. Furthermore, as the benefits of the externality are

$$TX_r = \int_0^{b_r^*} \int_0^{b_g^*} x_r(b_g) db_g db_r, \tag{17}$$

then

$$\frac{dTX_r}{db_g^*} = \int_0^{b_r^*} \phi db_r = \phi b_r^*.$$

Thus, by Equation (15), the size of the subsidy is equal to the change in the expected benefits of externality in sector r that follows from increasing the green investments in sector g slightly. The reason for the zero subsidy in sector r is that the firms have proper private incentives to internalize the externality in there. For a reference, Baumol and Oates ([1], Chapters 3 and 4) discuss this result in length.

[8]Specifically, the policy can set $S_{0g} = 0$, so $\Delta S_g = S_{1g}$, and only green technology firms receive subsidy payments.

2.2 Implementation of the Optimal Policy

We denote

$$\Delta S_g \equiv S$$

and write

$$S \quad = \quad \phi E b_r^* = \phi\left(\left(\frac{c_g q_r + \phi q_g}{c_r c_g}\right) s + \frac{\phi S}{c_r c_g} - \frac{\overline{F}_r}{c_r} - \phi\frac{\overline{F}_g}{c_r c_g}\right)$$

by using Equations (12), (15), and (16). After arrangements, we may also write

$$S(s) = \phi\left(\frac{c_g q_r + \phi q_g}{c_r c_g - \phi^2}\right) s - \phi\left(\frac{c_g \overline{F}_r + \phi \overline{F}_g}{c_r c_g - \phi^2}\right). \tag{18}$$

We call this the subsidy rule.[9] The rule shows that the subsidy can be written as a function of the emissions price s, so the optimal policy ultimately depends on the emission price alone.

So far, we have treated the instrument s in general terms by calling it the price of emissions. In our framework, a regulation can be implemented either by environmental taxes or by tradable permits, so the emission price is either a tax rate or a price of a permit. In what follows, we will denote the tax rate by τ and set

$$Es = \tau \tag{19}$$

in Equation (14). Regarding the permit implementation, the regulatory agency merely fixes the number of permits. The equilibrium permit price equates supply and demand of permits in the permit markets, i.e., it holds that

$$l = e(p, S), \tag{20}$$

where l is the number of permits and p is the permit price. In particular, the agency will follow the optimal policy as it incorporates the subsidies $\Delta S_g = S(p)$ and $\Delta S_g = 0$, into the equilibrium condition in Equation (20). By Equation (73) in Appendix B,

[9] In a meaningful framework, the positive externality cannot be too strong. In our study, it must hold that $c_r c_g > \phi^2$.

the equilibrium price is

$$p = \gamma(x - l + x_1(\theta)), \tag{21}$$

where $\theta = (\theta_g, \theta_r)$ and

$$\gamma = \left(2 + \frac{\phi q_r q_g + c_g q_r^2 + c_r q_g^2}{c_r c_g - \phi^2}\right)^{-1} > 0. \tag{22}$$

As

$$x_1(\theta) = \left(\frac{q_g c_r + q_r \phi}{c_r c_g}\right)\theta_r + \frac{q_g}{c_g}\theta_g, \tag{23}$$

then $Ex_1(\theta) = 0$ and

$$Ep \equiv \overline{p} = c(x - l). \tag{24}$$

Consequently, the agency sets

$$Es = \overline{p}, \tag{25}$$

and Equation (24) provides the link between $\overline{p}$ and l.

We conclude that both the tax and permit instruments can implement the optimal policy. The permit implementation is based on the design, where the size of the subsidy explicitly depends on the permit price. According to the subsidy rule introduced in Equation (18), values of random variables that will increase (decrease) the permit price will also increase (decrease) the size of the subsidy. In the tax policy, instead, the tax rate τ remains genuinely fixed, so the subsidy remains fixed as well.

In addition to these full-blooded price and quantity instruments, there is third option available, namely, a hybrid instrument that applies the tax subsidy but allows the permit price to adapt. We will return to this implementation later.

3 Instrument Choice

We studied above the implementation of a policy that regulates two externalities at the same time. In comparing the suggested instruments, we have to further investigate the social costs of the regulation. Regarding the abatement costs, the uncertainty enters the costs through three distinct channels. It may enter through emission price, independently of the emission price, and through the interaction of uncertainty and emission price.[10] We do the decomposition in Appendix B. Accordingly, we write aggregate abatement costs in two parts as

$$C(s, S(s), \theta) = \Psi^1(s) + \Psi^2(s, S(s), \theta). \tag{26}$$

In this representation, Ψ^1 is independent of random variables while Ψ^2 is not. We calculate that

$$\frac{d\Psi^1(s)}{ds} = \frac{C(s, S(s), \theta)}{\partial s} + \frac{C(s, S(s), \theta)}{\partial S}\frac{dS(s)}{ds} = \frac{s}{\gamma}$$

(see Equation (69)), where γ is introduced in Equation (22) above. Thus,

$$\Psi^1(s) = \overline{\Psi}^1 + \frac{s^2}{2\gamma}, \tag{27}$$

where $\overline{\Psi}^1$ is a constant. The factor $\Psi^2(s, S(s), \theta)$, in turn, is written as

$$\Psi^2(s, S(s), \theta) = y(\theta) + y_1(\theta)\left(S(s) + q_g s\right) \tag{28}$$

(see Equation (70)), where

$$y_1(\theta) = \phi\left(\frac{c_g\theta_r + \phi\theta_g}{c_r c_g}\right).$$

In what follows, the factor $y_1(\theta)$ turns out to be of major importance. Note, in particular, that $y_1(\theta) = 0$ in the absence of the technological externality ($\phi = 0$). It also holds that $Ey_1(\theta) = 0$.

The other part in societal costs consists of damages of emissions. In Appendix B, we write aggregate emissions as

$$e(s) = x + x_1(\theta) - x_S S(s) - x_s s, \tag{29}$$

or after inserting the rule $S(s)$, we write the emissions as a function of the emission

[10] The last category emerges due to the presence of technological externality.

price as

$$e(s) = x + x_1(\theta) - \frac{s}{\gamma}. \quad (30)$$

(see Equations (72) and (73)). The factors $x_1(\theta)$ and γ were defined above in the context of permit markets (Equations (22) and (23), respectively). In particular, it holds that $\gamma > 0$ and $Ex_1(\theta) = 0$.

It is time to move to the main question of our study, namely, to the instrument choice under uncertainty. An integral part of the choice is the so-called Weitzman assumption. Accordingly, the agency has inferior information concerning the abatement costs. Based on the expected response function of the polluting industry, the agency implements either a price or a quantity instrument. In the present context, the agency may choose between an environmental tax and tradable permits. The novel feature in our framework concerns the presence of the technological externality and the internalization of it under uncertainty.

The instrument choice is based on the comparative advantage (Weitzman [12]). In choosing between instruments τ and p, we write

$$\Delta(\tau, p) = E\left[C(\tau, \theta) + D(e(\tau, \theta))\right] - E\left[C(p(\theta), \theta)) + ED(l)\right]. \quad (31)$$

Accordingly, a strictly positive (negative) $\Delta(\tau, p)$ implies that the agency prefers quantity (price) instrument. Referring to our abatement cost calculations above (Equations (27) and (28)), we write

$$\begin{aligned} EC(s(\theta), \theta) &= E\Psi^1(s(\theta))) + E\Psi^2(s(\theta), \theta) \quad (32) \\ &= \overline{\Psi}^1 + \frac{1}{2\gamma}Es^2 + Ey(\theta) + E\left[y_1(\theta)\left(S(s) + q_g s\right)\right]. \end{aligned}$$

In particular, if we incorporate the rule $S(p)$ into expected costs, then (by Equation (71) in Appendix B)

$$EC(p(\theta), \theta) = \overline{\Psi}^1 + \frac{1}{2\gamma}Ep(\theta)^2 + EY(\theta) + Y(\theta) + \Gamma E\left(y(\theta)s\right), \quad (33)$$

where

$$\Gamma = c_g \frac{\phi q_r + c_r q_g}{c_r c_g - \phi^2} > 0. \tag{34}$$

Furthermore, we have $\overline{p} = \tau$, so (by Equations (21) and (24)),

$$p(\theta) = \tau + \gamma x_1(\theta)).$$

We may conclude that

$$E\left[C(\tau,\theta)\right] - E\left[C(p(\theta),\theta))\right] = -\left(\frac{\gamma}{2} E\left[x_1(\theta)\right]^2 + \gamma \Gamma E\left[x_1(\theta) y_1(\theta)\right]\right). \tag{35}$$

Note that the factor $E\left[x_1(\theta)y_1(\theta)\right]$ evolves entirely due to technological externality. We calculate that

$$E y_1(\theta) x_1(\theta) = \phi \left(\frac{c_g^2 q_r + \phi^2 q_r + c_g c_r q_g}{c_r c_g^3}\right) \sigma^2 > 0. \tag{36}$$

The aggregate emissions in permit system remains fixed and are equal to the permit allocation l. As compared to tax system, it holds that $E\left[e(\tau,\theta)\right] = l$, so (by Equation (30))

$$e(\tau,\theta) = l + x_1(\theta).$$

Regarding the damages of emissions, we further assume that[11]

$$D(e) = \frac{d}{2} e^2,$$

so

$$E\left[D(e(\tau,\theta))\right] - E\left[D(l))\right] = \frac{d}{2} E\left[x_1(\theta)\right]^2.$$

Altogether, we have

Proposition 1 *Let $\gamma > 0$ be the slope of the marginal abatement function, $d > 0$ be the slope of marginal damage function, and $\phi > 0$ be the amount of externality that an*

[11] Note that the damage function includes no uncertainties. This assumption does not reduce the generality of our analysis for as long as damage and abatement cost uncertainties are independent. If this assumption does not hold, then we should incorporate damage uncertainties into the analysis as well. See, Weitzman [12].

additional investment in sector g generates. Furthermore, assume that the efficient subsidy rule S(p) in Equation (18) internalizes the technological externality. Then, the comparative advantage between the price and the tax instrument is

$$\Delta(\tau, p) = \frac{d-y}{2} E(x_1(\theta))^2 - \gamma\Gamma E(y_1(\theta)x_1(\theta)), \tag{37}$$

where $\gamma\Gamma E(y_1(\theta)x(\theta)) > 0$.

The positive sign of the factor $\gamma\Gamma E(y_1(\theta)x(\theta))$ follows from Equations (22), (34), and (36).

The measure $\Delta(\tau, p)$ consists of two additive terms. The first of these, $\frac{d-y}{2}E(x(\theta))^2$, is the traditional Weitzman effect. Accordingly, the instrument choice depends on the slopes of the abatement cost function (γ) and the damage function (d). The second term depends on the expected cross-product $Ey_1(\theta)x_1(\theta)$. It results from the presence of technological externality. If $\phi = 0$, then no technological externality exists, and the expected cross-product $Ey_1(\theta)x_1(\theta)$ disappears.

Besides producing the cross-product term, the technological externality changes the traditional instrument comparison in another way.[12] To see the effect, we write first the slope of the marginal abatement function in the absence of spillovers as

$$\gamma_w = \left(2 + \frac{c_g q_r^2 + c_r q_g^2}{c_r c_g}\right)^{-1} > 0. \tag{38}$$

Then, a new version of the comparative advantage will be written as

$$\Delta(\tau, p) = \frac{1}{n}\left(\frac{nd - \gamma_w}{2} E(x_1(\theta))^2 - \gamma_w \Gamma E(y_1(\theta)x_1(\theta))\right), \tag{39}$$

where[13]

$$n = \frac{\gamma_w}{\gamma} \geq 1.$$

[12] Consequently, this effect will also disappear form the instrument choice formula as the technological externality disappears.

[13] If we write

$$\frac{\gamma_w}{\gamma} = \frac{2 + \frac{\phi q_r q_g + c_g q_r^2 + c_r (q_g)^2}{c_r c_g - \phi^2}}{2 + \frac{c_g q_r^2 + c_r (q_g)^2}{c_r c_g}} = \frac{2 + i_w}{2 + i},$$

In particular, it holds that $n > 1$ as long as $\phi > 0$.

The two effects that we discovered arise from the changes in abatement costs. The first of these changes is incorporated into the parameter $n > 1$. It reflects the fact that, *ceteris paribus,* positive externality flattens the slope of the aggregate marginal abatement function. The spillover effect is a positive externality that reduces abatement costs in the externality receiving sector, so the aggregate marginal costs are reduced as well. We interpret the multiplier $n > 1$ as the additional weight given to the marginal damage in the instrument choice. Following the basic principles of instrument choice (Weitzman [12]), more weight will be given to the damages as marginal cost curve gets flatter. This favors the quantity instrument that keeps the emissions at a predetermined level.[14]

The second effect is related to the expected cross-product $Ey_1(\theta)x_1(\theta)$. The basic explanation for it is the positive correlation that emerges endogenously due to the cost complementarity in the abatement costs function.[15] As in Meunier [8], complementary favors the price instrument, environmental taxation.

We discussed in Introduction that our case is more complicated than the original Weitzman [12] study in that the instruments implement not only the wrong levels of emissions, but also implement the wrong level inefficiently *ex post*. This effect will arise even though the permit policy applies the subsidy rule $S(p)$ in the implementation. The inefficiency would disappear if the permit implementation could apply a rule

$$\widetilde{S} = S(p) - \phi \frac{c_g \theta_r + \phi \theta_g}{c_r c_g}$$

instead. The difference between $\widetilde{S}$ and $S(p)$ illustrates the informational asymmetries and the constraint that the policies cannot use state-contingent policies. We will explore the inefficiency issue further in the next chapter, as we introduce another

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then

$$\frac{i_w}{i} = \frac{\phi q_r q_g + c_g q_r^2 + c_r (q_g)^2}{c_g q_r^2 + c_r (q_g)^2} \frac{c_r c_g}{c_r c_g - \phi^2} > 1.$$

It follows that $\gamma_w > \gamma$.

[14]The term $\frac{1}{n}$ outside the brackets is strictly larger than zero, so it only strengthens but does not change the sign of the comparative advantage.

[15]See the benefit formula in Equation (17) above. The correlation is an endogenous phenomenon as the random variables themselves are independently distributed in our model.

subsidy rule into permit implementation.

4 Extensions

4.1 Hybrid Implementation

Our presentation above was based on a specific permit design. It introduced a novel feature into the implementation of subsidy policies. The subsidy was assumed to adjusts to the realizations of uncertainty through changes in the permit price. In this section, we will review an alternative feasible permit design that relies on a fixed subsidy. It is a hybrid instrument in the sense that it uses tradable permits to reduce the negative externality and uses a fixed subsidy to increase the production of positive externality.

Denote the equilibrium price in the hybrid system by p_h and the number of permits by l_h. Hybrid system applies the same expected subsidy as the pegged implementation[16] above, so (by Equation (24)) it holds that

$$Ep_h = \gamma(x - l_h).$$

As optimality requires that $Es = \overline{p}$, then (by Equation (24) again) the policy should set $l_h = l$. Thus, the permit policy applies the same design as before. However, after the uncertainty has revealed itself, the implementations will diverge. Using emission formula in Equation (29), it holds that

$$p_h = \overline{p} + \gamma_h x_1(\theta),$$

where

$$\gamma_h = x_s^{-1} = \left(2 + \frac{c_r q_g^2 + c_g q_r^2 + q_r \phi q_g}{c_r c_g}\right)^{-1} \tag{40}$$

In specific, it holds that

$$\gamma_h > \gamma. \tag{41}$$

[16]The name originates from the currency markets, where a currency peg represents one kind of exchange rate policy. Under the policy, typically one (small) currency follows another (big) one in a fixed relationship.

In response to realizations of uncertainty $x_1(\theta)$, the hybrid system will induce more intense price movements than the pegged system

In summary, the hybrid and the pegged permit systems produce identical expected emissions and expected emission prices under optimal policy designs. As the implementations will differ *ex-post*, a question arises about the choice between these systems. Note that the comparison between permit systems depends entirely on the differences in abatement costs. The emissions are fixed by the same permit allocation under both implementations, so the expected damages are identical as well. We then write

$$\Delta(p_h, p) = E\left[C(p_h(\theta), \theta)\right] - E\left[C(p(\theta), \theta))\right].$$

Specifically, by Equation (32),

$$E\left[C(p_h(\theta), \theta)\right] = \overline{\Psi}^1 + \frac{1}{2\gamma}E\left[p_h\right]^2 + Ey(\theta) + E\left[y_1(\theta)\left(S(\overline{p}) + q_g p_h\right)\right].$$

Then, (by the expected abatement costs in Equation (33))

$$\Delta(p_h, p) = \frac{\gamma_h^2 - \gamma^2}{2\gamma}E\left[x_1(\theta)\right]^2 + \frac{q_g\gamma\gamma_h - \Gamma\gamma^2}{\gamma}E\left[y_1(\theta)x_1(\theta)\right] \tag{42}$$

Based on this comparison, we may state that

Proposition 2 *Assume that the regulator may decide whether the subsidy in the permit implementation is fixed or is not. Our model gives no unequivocal answer to this problem as the answer depends on the parameters of the abatement cost functions.*

Basically, this results follows as the two differences in comparative advantage pull in opposite directions. By Equation (41),

$$\frac{\gamma_h^2 - \gamma^2}{2\gamma} > 0, \tag{43}$$

and, by straightforward calculations, it holds that

$$\frac{q_g\gamma\gamma_h - \Gamma\gamma^2}{\gamma} < 0. \tag{44}$$

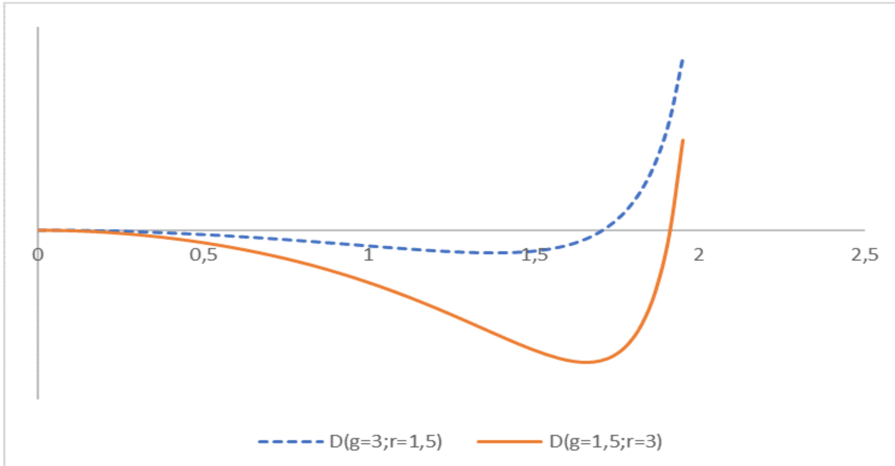

Figure 1: Instrument Choice between Permit Implementations. Negative (positive) values imply that fixed-rule (variable-rule) in subsidization should be used.

We illustrate the proposition in Figure 1. We draw $\Delta(p_h, p)$ as a function of the spillover parameter ϕ. The larger the ϕ, the more intensive is the spillover effect. We further assume that $c_r = q_r = r$ and $c_g = q_g = g$, so we incorporate some heterogeneity in an otherwise stripped-down illustration. Furthermore, the value of $\Delta(p_h, p)$ is drawn under two different set of parameter values. We have set $g = 1,5$ and $r = 3$ (the solid line) and $g = 3$ and $r = 1,5$ (the dashed line). As the vertical axis represents differences in expected abatement costs between fixed and variable subsidy rules, then negative (positive) values means that fixed (variable) subsidy rule should be applied.[17]

Figure 1 demonstrates that the sign of the comparative advantage depends on the parameters of the abatement cost function and is somewhat affected by the changes in the industry structure. Furthermore, the pegged-rule is seen to become the preferred rule as the spillover effect grows strong enough. This happens as the price volatility in the hybrid system grows large enough as compared to the pegged implementation. Naturally, both curves cut the horizontal axis at $\phi = 0$ as the spillover effect vanishes in there.

[17]The upper limit for the parameter ϕ is $\frac{\sqrt{3}}{2}$ in both illustrations as the condition $c_g c_r - \phi^2 > 0$ must be met, see Equation (22).

In the previous section, we discussed the inefficiency that the use simple instruments in subsidization creates *ex-post*. The comparative statistic in Equation (42) illustrates. A shift from one inefficient implementation to another affects both the variance-related factor ($E\left[x_1(\theta)\right]^2$) and the correlation-related factor ($E\left[y_1(\theta)x_1(\theta)\right]$), and that these changes (described in Equations (43) and (44)) pull the instrument choice in opposite directions.

The analysis above concerned the choice between two quantity instruments. They differ in that they apply different type of subsidies, but both instruments use tradable permits in the emission regulation. Regarding the choice between price and quantity instruments under fixed subsidy rule, we can calculate a comparative advantage (a counterpart of Equation (37)) as

$$\Delta\left(\tau, p_h\right) = \frac{d - \gamma\left(\frac{\gamma_h}{\gamma}\right)^2}{2} E\left[x_1(\theta)\right]^2 - \gamma\widetilde{\Gamma} E\left[x_1(\theta)y_1(\theta)\right], \tag{45}$$

where

$$\widetilde{\Gamma} = q_g\left(\frac{\gamma_h}{\gamma}\right). \tag{46}$$

We skip the derivation by noting that it closely follows the derivation of comparative advantage in the previous section.

4.2 Zero-Subsidies

So far, our analysis has concentrated on policies that internalizes both positive and negative externalities. In this section, we change this approach as we incorporate an additional restriction on the policies to be pursued. We assume that the agency is aware of the size of the positive externality but is unable to use subsidization to internalize it. Consequently, the agency internalizes only the negative externality and tries to keep an eye on the spillover effect as well. We will illustrate this issue by studying briefly both the individual instrument designs and the instrument choice.

The policy restriction in this section is equivalent to setting a zero subsidy in the instrument implementation. To incorporate this restriction into social welfare, we use our earlier decomposition as represented in Equation (26). In particular, after setting

$S = 0$, the abatement costs are

$$C_{sb}(s, \theta) = \Psi_{sb}^1(s) + \Psi_{sb}^2(s, \theta),$$

where Ψ_{sb}^1 is independent of random variables while Ψ_{sb}^2 is not. By Appendix B, Equations (66) and (67), it holds that

$$\frac{d\Psi_{sb}^1(s)}{ds} = z_{s_b} - \frac{1}{\gamma_{sb}} s, \tag{47}$$

where[18]

$$\gamma_{sb} = \left(2 + \frac{c_g q_r^2 + \frac{c_g c_r - \phi^2}{c_g} q_g^2}{c_r c_g}. \right)^{-1} \tag{48}$$

and $z_{s_b} > 0$. By the comparison of Equations (22) and (48), it holds that

$$\gamma_{sb} > \gamma. \tag{49}$$

If $\phi = 0$, then γ_{sb} becomes again the slope of the standard abatement cost function. Furthermore, after inserting the rule $S = 0$ into Equation (28), we have

$$\Psi_{sb}^2(s, \theta) = y(\theta) + y_1(\theta) q_g s. \tag{50}$$

Regarding the emissions, we incorporate $S = 0$ into Equation (29), so

$$e_{sb}(s, \theta) = x + x_1(\theta) - x_s s, \tag{51}$$

where $x_s > 0$.

The second-best design satisfies

$$\begin{aligned} \frac{dEW_{sb}}{ds_{sb}} &= \frac{dE\left[C_{sb}(s_{sb}, \theta) + D(e_{sb}(s_{sb}, \theta))\right]}{ds_{sb}} \\ &= E\left[\frac{d\Psi_{sb}^1(s_{sb})}{ds_{sb}} + \frac{\Psi_{sb}^2(s_{sb}, \theta)}{ds_{sb}} + de_{sb}\frac{de(s_{sb}, \theta)}{ds_{sb}}\right] = 0. \end{aligned}$$

[18] In order to maintain comparability in the main text, we will apply definitions $z_{s_b} = z_1 + Z_2$ and $\frac{1}{\gamma_{sb}} = z$ in this section. Note further that $\gamma_{sb} > 0$ as the condition $c_r c_g > \phi^2$ holds in the study.

By Equations (47), (50), and (51), this condition is equivalent to

$$Es_{sb} = (\gamma_{sb} x_s)\, dEe_{sb} - z_{sb}\gamma_{sb}.$$

As $\gamma_{sb} x_s \neq 1$ and $z_{sb}\gamma_{sb} > 0$, the emission price s_{sb} deviates from the first-best rule $Es = dEe(s)$.

The policy can be implemented either by environmental tax or by tradable permits. In the case of taxes, the agency sets

$$\tau_{sb} = s_{sb}.$$

In the case of permits, the agency knows (by Equation (51)) that equilibrium price satisfies

$$p_{sb} = \frac{1}{x_s}\left(x + x_1(\theta) - l_{sb}\right).$$

It sets

$$\overline{p}_{sb} = s_{sb},$$

so that

$$p_{sb} = \overline{p}_{sb} + \frac{x_1(\theta)}{x_s}.$$

Furthermore, the emissions levels are linked between instruments by the relation

$$e_{sb}(\tau_{sb}, \theta) = l_{sb} + x_1(\theta). \tag{52}$$

The analysis shows that an additional restriction on policy affects the overall instrument design. However, both instruments share identical policy parameters. Regarding the choice between the instruments, we state

Lemma 3 *Assume that the agency is aware of the presence of positive and negative externalities but it is unable to pay subsidies to firms that generate positive externality. Despite this, the instrument choice between price and quantity instruments is still affected by the positive externality.*

Thus, influences of positive externality on instrument choice are not entirely due to the use of subsidies.[19] The derivation of instrument choice follows the steps introduced earlier. In the present context, the difference between expected abatement costs, a counterpart of Equation (35), is

$$E\left[SC(\tau_{sb},\theta)\right]-E\left[SC(p_{sb}(\theta),\theta)\right]=-\left(\frac{q_g}{x_s}E(y_1(\theta)x_1(\theta))+\frac{1}{2x_s^2\gamma_{sb}}E\left(x_1(\theta)\right)^2\right),$$

while the difference between expected damages becomes

$$E\left[D(e(\tau_{sb},\theta))-D(l_{sb})\right]=\frac{d}{2}E\left(x_1(\theta)\right)^2$$

(see Equation (52)), so

$$\begin{aligned}\Delta\left(\tau_{sb},p_{sb}\right) &= E\left[SC(\tau_{sb},\theta)\right]-E\left[SC(p_{sb}(\theta),\theta)\right]-\left(E\left[D(e(\tau_{sb},\theta))-D(l_{sb})\right]\right)\\ &= \left(\frac{1}{2}\left(d-\frac{1}{x_s^2\gamma_{sb}}\right)E\left(x_1(\theta)\right)^2-\frac{q_g}{x_s}E\left(y_1(\theta)x_1(\theta)\right)\right).\end{aligned}$$

By Equations (40) and (46), we can also write the comparative advantage as

$$\Delta\left(\tau_{sb},p_{sb}\right)=\left(\frac{d-\gamma\frac{\gamma_h^2}{\gamma\gamma_{sb}}}{2}E\left(x_1(\theta)\right)^2-\gamma\widetilde{\Gamma}E\left(y_1(\theta)x_1(\theta)\right)\right), \tag{53}$$

where γ is the slope of the abatement function under the pegged-subsidy rule (Equation (22)). Specifically, this late representation allows us to write

Proposition 3 *Assume that there is a change of policy regime from the non-subsidized to the subsidized hybrid regime. The change favors the quantity instrument in the sense that* $\Delta\left(\tau_h,p_h\right)>\Delta\left(\tau_{sb},p_{sb}\right)$.

Proof. By Equations (45) and (53),

$$\Delta\left(\tau_h,p_h\right)-\Delta\left(\tau_{sb},p_{sb}\right)=\left(\frac{\gamma_{sb}-\gamma}{\gamma\gamma_{sb}}\right)\frac{\gamma_h^2}{2}E\left(x_1(\theta)\right)^2,$$

[19] Earlier, Meunier [8] has derived similar result.

where γ_{sb} and γ are both strictly positive. By Equation (49), $\gamma_{sb} > \gamma$, so

$$\Delta(\tau_h, p_h) - \Delta(\tau_{sb}, p_{sb}) > 0. \tag{54}$$

■ Consider a case, where $\Delta(\tau_{sb}, p_{sb}) = 0$, so that the agency is indifferent between the tax and the permit implementations in the second-best-regime. If the regime changes so that it allows substitution (and the design turns first-best with it), Proposition 3 says that $\Delta(\tau_h, p_h) > 0$, so the quantities become the preferred instrument.

In general, our analysis has shown that the positive technological externality has variance-related ($E(x_1(\theta))^2$) and covariance related ($((y_1(\theta)x_1(\theta)))$) effects. The change from zero-subsidy implementation to fixed-subsidy implementation does not change the covariance related effect at all. The explanation for this result is that both implementations apply fixed subsidy rules. However, the variance-related effect goes through changes. Basically, this occurs as fixed-subsidy implementation removes one constraint from the policy *ex-ante* as it internalizes the positive externality, the spillover effect, in an efficient manner.

4.3 Negative Technological Externality

Our analyses has concentrated on the transition from brown to green production. We have assumed that technological externality operates behind this transition, and that the sign of the externality is strictly positive. This seems plausible as green investments most likely utilize emerging technologies and the positive technological spillovers are typical among emerging technologies. However, in other types of regulation eras, we cannot rule out a possibility that a negative externality prevail amongst the polluting industry. In our framework, this assumption would mean that two negative externalities simultaneously operate.

We have briefly analyzed this possibility in our framework. Consequently, it holds that $\phi < 0$ in these analyses. As far as optimality requires that $S = \phi E b_r^*$ (see Lemma 2), then $S < 0$. This means that the optimal policy should tax firms in sector g because of their technological externality generation. We have also considered the consequences of setting $\phi < 0$ in one of the main results of this chapter, in Proposition 1. The results imply that the negative technological externality gives rise to a series of complex impacts. This is in contrast to positive technological externality, which yields much more predictable outcomes.

5 Conclusions

We extend the workings of market-based instruments (i.e., tradable permits and environmental taxes) to deal with the issue of subsidization. The extension is required as a positive externality (knowledge spillover) exists within the polluting industry. We show how pricing only the negative externality (pollution) and setting the price of positive externality to zero do not produce an optimal societal outcome. We also develop the study of instrument choice under uncertainty (Weitzman [12]) by incorporating a second externality into the framework.

The implementations apply payments that consist of an environmental part and a subsidy part. We study separately cases without and with the subsidy part of the payment. In the first case, the environmental agency has a mandate to regulate only the negative externality. In the second case, the optimality conditions promote subsidization but they do not explicitly state the form of it. We end up experimenting with two rules in the permit implementation, a linear rule that explicitly depends on the unit price of emissions, and a fixed rule that both permit and tax systems use in their implementations.

Our analysis shows that instrument choice is affected by the presence of the second externality. Technological externality produces dependency between the technology choices, which is further reflected as an additive correlation factor in the instrument choice formula. Under this new formula, we find diversity of case-specific effects. First, the effects are shown to depend on the scope of the policy, so it matters whether the positive externality is priced or not. Second, if the positive externality is priced, the effects depends on the specific design of the permit instruments, namely, whether the implementation applies linear or fixed rule. However, our analysis finds that a regime change from zero-subsidies toward fixed-subsidization invariably favors the quantity instrument, tradable permits.

Our interest lies in regulation in a situation where both negative and positive externalities exist simultaneously. We consider knowledge spillover as a positive externality. Regarding the properties of knowledge spillover itself, its existence is fairly intuitive and uncontroversial, at least in theory. The accurate identification and measurement of the effect is a bigger issue. In this respect, our approach does not question the identification of the effect but rather takes the spillover effect as a known parameter. A natural extension would be to incorporate an uncertain spillover effect into the problem of instrument choice. Instead of concentrating on uncertain marginal benefits and damages, we might focus on uncertain spillover effect. Our current framework would need considerable elaboration in order to conform to such an approach.

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Appendix

A Social Cost Minimization

Societal costs SC arise as a sum of of abatement costs C and damages of emissions D, i.e., they are

$$SC(s, \Delta S_r, \Delta S_g) = C(s, \Delta S_r, \Delta S_g) + D(e(s, \Delta S_r, \Delta S_g)).$$

We have

$$\begin{aligned} C(s, \Delta S_r, \Delta S_g) &= C_g(s, \Delta S_g) + C_r(s, \Delta S_r, \Delta S_g) \\ &= \left(\int_0^{b_g^*} A_{1g}(s) + I_g(b_g) + \int_{b_g^*}^1 A_{0g}(s) \right) db_g \\ &\quad + \int_0^{b_r^*} \left(A_{1r}(s) + I_r(b_r) - \int_0^{b_g^*} x_r(b_g) db_g + \int_{b_r^*}^1 A_{0r}(s) \right) db_r \end{aligned}$$

and

$$\begin{aligned} e(s, \Delta S_r, \Delta S_g) &= e_g(s, \Delta S_g) + e_r(s, \Delta S_r, \Delta S_g) \\ &= = \left(\int_0^{b_g^*} e_{0g}(b_g, s) + \int_{b_g^*}^1 e_{1g}(b_g, s) \right) db_g \\ &\quad + \left(\int_0^{b_r^*} e_{0r}(b_r, s) + \int_{b_r^*}^1 e_{1r}(b_r, s) \right) db_r, \end{aligned}$$

where $b_g^* \equiv b_g(s, \Delta S_g)$ and $b_r^* \equiv b_r^*(s, \Delta S_r, \Delta S_g)$ are sector-specific responses as represented in Equations (9) and (12), respectively. We denote the damages of emissions by $D(e(s, \Delta S_r, \Delta S_g))$, where $D'(e) > 0$ and $D''(e) > 0$.

In our approach, we derive the optimal values for the instruments s, ΔS_g and ΔS_r. The social cost minimum satisfies

$$\frac{dESC(s, \Delta S_g, \Delta S_r)}{ds} = \frac{dESC(s, \Delta S_g, \Delta S_r)}{d\Delta S_g} = \frac{dESC(s, \Delta S_g, \Delta S_r)}{d\Delta S_r} = 0.$$

We have

$$\begin{aligned}
\frac{dSC(s,\Delta S_g,\Delta S_r)}{d\Delta S_g} &= \frac{dC}{d\Delta S_g} + \frac{dD}{de}\frac{de}{d\Delta S_g} \\
&= \frac{\partial b_g^*}{\partial \Delta S_g}\left(A_{1g}(s) - A_{0g}(s) + I_g(b_g^*) - \int_0^{b_r^*} x_r(b_g^*)db_r\right) \\
&\quad + D'(e)\frac{\partial b_g^*}{\partial \Delta S_g}\left(e_{0g}(b_g^*,s) - e_{1g}(b_g^*,s)\right) = 0,
\end{aligned}$$

$$\begin{aligned}
\frac{dSC(s,\Delta S_g,\Delta S_r)}{d\Delta S_r} &= \frac{dC}{d\Delta S_r} + \frac{dD}{de}\frac{de}{d\Delta S_r} \\
&= \frac{\partial b_r^*}{\partial \Delta S_r}\left(A_{1r}(s) - A_{0r}(s) + I_r(b_r^*) - \int_0^{b_g^*} x_r(b_g)db_g\right) \\
&\quad + D'(e)\frac{\partial b_r^*}{\partial \Delta S_r}\left(e_{0r}(b_r^*,s) - e_{1r}(b_r^*,s)\right) = 0,
\end{aligned}$$

and

$$\frac{dSC(s,\Delta S_g,\Delta S_r)}{ds} = \frac{dC}{ds} + \frac{dD}{de}\frac{de}{ds} = 0,$$

where

$$\begin{aligned}
\frac{dC}{ds} &= \frac{\partial b_g^*}{\partial s}\left(A_{1g}(s) - A_{0g}(s) + I_g(b_g^*) - \int_0^{b_r^*} x_r(b_g^*)db_r\right) \\
&\quad + \left(\int_0^{b_g^*} \frac{dA_{1g}(s)}{ds} + \int_{b_g^*}^1 \frac{dA_{0g}(s)}{ds}\right) db_g \\
&\quad + \frac{\partial b_r^*}{\partial s}\left(A_{1r}(s) - A_{0r}(s) + I_r(b_r^*) - \int_0^{b_g^*} x_r(b_g)db_g\right) \\
&\quad + \left(\int_0^{b_r^*} \frac{dA_{1r}(s)}{ds} + \int_{b_r^*}^1 \frac{dA_{0r}(s)}{ds}\right) db_r
\end{aligned}$$

and

$$
\begin{aligned}
\frac{de(s,\Delta S_r,\Delta S_g)}{ds} &= \frac{\partial b_g^*}{\partial s}\left(e_{0g}(b_g^*,s)-e_{1g}(b_g^*,s)\right)\\
&+\left(\int_0^{b_g^*}\frac{de_{1g}(b_g,s)}{ds}+\int_{b_g^*}^1\frac{de_{0g}(b_g,s)}{ds}\right)db_g\\
&+\frac{\partial b_r^*}{\partial s}\left(e_{0r}(b_r^*,s)-e_{1r}(b_r^*,s)\right)\\
&+\left(\int_0^{b_r^*}\frac{de_{1r}(b_r,s)}{ds}+\int_{b_r^*}^1\frac{de_{0r}(b_r,s)}{ds}\right)db_r.
\end{aligned}
$$

Consider first the condition $\frac{dSC}{ds}=0$. By Equations (1), (2), (3), and (4) in the main text, it holds that

$$
\frac{dA_{1g}(s)}{ds}=\frac{dA_{0g}(s)}{ds}=\frac{dA_{1r}(s)}{ds}=\frac{dA_{0r}(s)}{ds}=s
$$

and

$$
\frac{de_{1g}(b_g,s)}{ds}=\frac{de_{0g}(b_g,s)}{ds}=\frac{de_{1r}(b_r,s)}{ds}=\frac{de_{0r}(b_r,s)}{ds}=-1.
$$

By the definitions of b_g^* and b_r^* (Equations (9) and (12)), the differentials $\frac{\partial b_g^*}{\partial \Delta S_g}$, $\frac{\partial b_g^*}{\partial \Delta S_r}$, and $\frac{\partial b_g^*}{\partial s}$ all are independent of the uncertainty. Furthermore, it holds that

$$
\frac{\partial b_g^*}{\partial s}=q_g\frac{\partial b_g^*}{\partial \Delta S_g}
$$

and

$$
\frac{\partial b_r^*}{\partial s}=q_r\frac{\partial b_r^*}{\partial \Delta S_r}.
$$

Consequently, we may write

$$
\frac{dESC}{ds}=E\left[2s+q_r\frac{dSC}{\Delta S_r}+q_g\frac{dSC}{\Delta S_g}-2D'(e)\right]=0. \tag{55}
$$

As $\frac{dESC}{\Delta S_r}=\frac{dESC}{\Delta S_g}=0$ at the social optimum, it holds that

$$
Es-ED'(e)=0. \tag{56}
$$

Consider next the condition $\frac{dESC}{d\Delta S_g} = 0$.We calculate that

$$\begin{aligned} I_g(b_g^*) &= \overline{F}_g + c_g b_g^* + \eta_g = \overline{F}_g + c_g \left(\frac{sq_g + \Delta S_g - \overline{F}_g - \theta_g}{c_g} \right) + \eta_g \\ &= sq_g + \Delta S_g - \epsilon_g. \end{aligned}$$

Furthermore, by Equations (2) and (4),

$$A_{1g}(s) - A_{0g}(s) = \epsilon_g,$$

and, by Equation (3),

$$e_{0g}(b_g^*, s) - e_{1g}(b_g^*, s) = q_g.$$

Taken together, we write

$$\frac{dESC}{d\Delta S_g} = \frac{\partial b_g^*}{\partial \Delta S_g} E \left(sq_g + \Delta S_g - \int_0^{b_r^*} x_r(b_g^*) db_r - D'(e) q_g \right) = 0$$

or, by Equation (56), and by the fact that $\frac{\partial b_g^*}{\partial \Delta S_g} \neq 0$, we write

$$E\Delta S_g - E\left[\int_0^{b_r^*} x_r(b_g^*) db_r \right] = 0.$$

As

$$\int_0^{b_r^*} x_r(b_g^*) db_r = \phi b_r^*,$$

we may write the optimal (expected) subsidy as

$$E\Delta S_g = \phi E b_r^*. \tag{57}$$

Regarding the subsidization in the externality-receiving sector, it holds that

$$\begin{aligned} I_r(b_r^*) &= \overline{F}_r + \eta_r + c_r b_r^* = \overline{F}_r + \eta_r + c_r \left(\frac{sq_r + \Delta S_r - \overline{F}_r - \theta_r + X_r}{c_r} \right) \\ &= sq_r + \Delta S_r + X_r - \epsilon_r, \end{aligned}$$

$$A_{1r}(s) - A_{0r}(s) = \epsilon_r,$$

and

$$e_{0r}(b_r^*, s) - e_{1g}(s, b_r^*, s) = q_r.$$

As

$$X_r = \int_0^{b_g^*} x_r(b_g) db_g,$$

we can write

$$\frac{dESC}{d\Delta S_g} = E\left[sq_r + \Delta S_r - D'(e)q_r\right] = 0.$$

It holds that $Es = ED'(e)$ at the social optimum, so the optimal (expected) subsidy satisfies

$$E\Delta S_r = 0. \tag{58}$$

B Instrument Choice

In this section, we derive various formulas for social welfare. In our approach, the social welfare will eventually be a function of emission price.

B.1

We start by deriving a decomposition for the total abatement costs. We will write is as

$$C(s, S(s), \theta) = \Psi^1(s) + \Psi^2(s, S(s), \theta), \tag{59}$$

where Ψ^1 is totally independent of the uncertainties while Ψ^2 is not. In what follows, we write the main stages of the derivations. Complete set of derivations is available from the author upon request.

In the abatement cost function, sector g-specific costs are

$$SC_g = \int_0^{b_g^*} C_{1g} db_g + \int_{b_g^*}^1 C_{0g} db_g$$

or, after incorporating the various types of costs (Equations (2), (4), (5), and (6)), and after integrating,

$$SC_g = \frac{1}{2}s^2 + \left(\overline{F}_g + \theta_g\right) b_g^* + c_g \frac{1}{2} b_g^{*2}.$$

In a similar fashion, we can write the sector r-specific costs

$$SC_r = \int_0^{b_r^*} C_{1r} db_r + \int_{b_r^*}^1 C_{0r} db_r$$

as

$$SC_r = \frac{1}{2}s^2 + \left(\overline{F}_r + \theta_r\right) b_r^* + c_r \frac{1}{2} b_r^{*2} - \phi b_g^* b_r^*. \tag{60}$$

In the following derivations, we find it convenient to write the cut-off firms as

$$b_g^*(\theta) = \frac{S(s) - \overline{F}_g}{c_g} - \frac{\theta_g}{c_g} + \frac{q_g}{c_g} s = \Omega_g - \frac{\theta_g}{c_g} + z_g s \tag{61}$$

and

$$b_r^* = \frac{\phi S(s) - \phi \overline{F}_g - c_g \overline{F}_r}{c_r c_g} - \frac{c_g \theta_r + \phi \theta_g}{c_r c_g} + \left(\frac{c_g q_r + \phi q_g}{c_r c_g} \right) s \tag{62}$$
$$= \Omega_r - \frac{c_g \theta_r + \phi \theta_g}{c_r c_g} + z_r s.$$

First, we derive the deterministic part of the costs, Ψ^1. By expanding the costs, we have (after setting all the uncertainty variables to equal zero)

$$\Psi^1(s) = z_0(S(s)) + z_1 s + z_2(S(s))s + \frac{z}{2} s^2, \tag{63}$$

where

$$z_0(S(s)) = \overline{F}_g \Omega_g + \frac{c_g}{2} \Omega_g^2 + \overline{F}_r \Omega_r + \frac{c_r}{2} \Omega_r^2 - \phi \Omega_g \Omega_r,$$

$$z_1 = \overline{F}_g z_g + \overline{F}_r z_r,$$
$$z_2(S(s)) = \left(c_g z_g - \phi z_r \right) \Omega_g + \left(c_r z_r - \phi z_g \right) \Omega_r,$$

and

$$z = 2 + c_r z_r^2 - 2\phi z_g z_r + c_g z_g^2. \tag{64}$$

After inserting Ω_g and Ω_r into z_0, it holds that

$$z_0(S(s)) = Z_0 + \frac{c_r \left(\phi^2 \overline{F}_g + c_g \phi \overline{F}_r \right)}{\left(c_r c_g \right)^2} S(s) + \frac{c_r \left(c_g c_r - \phi^2 \right)}{2 \left(c_r c_g \right)^2} \left(S(s) \right)^2$$
$$= Z_0 + Z_{00} S(s) - Z_{000} \frac{\left(S(s) \right)^2}{2}.$$

Moreover, it follows that

$$z_2(S(s)) = \frac{z_g}{c_r c_g} \left(\phi^2 - c_r c_g \right) \overline{F}_g - \frac{\left(c_r z_r - \phi z_g \right)}{c_r} \left(\overline{F}_r \right) + \frac{z_g}{c_r c_g} \left(c_r c_g - \phi^2 \right) \left(S(s) \right)$$
$$= Z_2 - Z_{22} S(s).$$

Next, we calculate that

$$\frac{d\Psi^1(s)}{ds} = \frac{dz_0}{dS} \frac{dS}{ds} + \left(z_1 + z_2(S(s)) \right) + \frac{dz_2}{dS} \frac{dS}{ds} s + zs.$$

By using the results just derived, we may also write

$$\frac{d\Psi^1(s)}{ds} = z_1 + Z_2 + (Z_{00} - Z_{000}S(s) - Z_{22}s)\frac{dS}{ds} - Z_{22}S(s) - zs. \tag{65}$$

In particular, it holds that

$$z_1 + Z_2 = \frac{q_g}{c_g}\frac{\phi}{c_r c_g}\left(c_g\overline{F}_r + \phi\overline{F}_g\right) > 0. \tag{66}$$

Furthermore, if we insert the values of z_g and z_r (Equations (61) and (62)) into the factor z (Equation (64)), it holds that

$$z = 2 + \frac{c_g q_r^2 + \frac{c_g c_r - \phi^2}{c_g} q_g^2}{c_r c_g}. \tag{67}$$

B.2

We will incorporate the linear rule

$$S(s) = \phi\left(\frac{c_g q_r + \phi q_g}{c_r c_g - \phi^2}\right)s - \phi\left(\frac{c_g\overline{F}_r + \phi\overline{F}_g}{c_r c_g - \phi^2}\right) \tag{68}$$

(see Equation (18) in the main text) into the abatement costs. It holds that

$$\frac{dS}{ds} = \phi\left(\frac{c_g q_r + \phi q_g}{c_r c_g - \phi^2}\right) > 0,$$

so $\frac{dS}{ds}$ is a constant. Consequently, we write Equation (65) as

$$\frac{d\Psi^1(s)}{ds} = \left(Z_{00}\frac{dS}{ds} + z_1 + Z_2\right) - \left(Z_{000}\frac{dS}{ds} + Z_{22}\right)S(s) - \left(Z_{22}\frac{dS}{ds} + z\right)s$$

or, as

$$\frac{d\Psi^1(s)}{ds} = \Psi_0 - \Psi_1 S(s) - \Psi_2 s.$$

After inserting the value of the differential $\frac{dS}{ds}$ into $\frac{d\Psi^1(s)}{ds}$, we have

$$\Psi_0 = \frac{\phi}{c_r c_g}\left(\phi\overline{F}_g + c_g\overline{F}_r\right)\left(\frac{c_r q_g + \phi q_r}{c_r c_g - \phi^2}\right),$$

$$\Psi_1 = \frac{1}{c_r c_g}\left(\phi q_r + q_g c_r\right),$$

and

$$\Psi_2 = z + \frac{q_g}{c_g}\frac{1}{c_r c_g}\phi\left(c_g q_r + \phi q_g\right).$$

Specifically, after incorporating the factor z from Equation (67) into Ψ_2, it holds that

$$\Psi_2 = 2 + \frac{1}{c_r c_g}\left(\phi q_r q_g + c_g q_r^2 + c_r q_g^2\right).$$

After these derivations, the differential $\frac{d\Psi^1(s)}{ds}$ still depends on $S(s)$. Therefore, we incorporate the rule $S(s)$ (Equation (68)) into $\frac{d\Psi^1(s)}{ds}$. It follows that

$$\Psi_0 + \Psi_1 S(s) = \frac{c_r q_g + \phi q_r}{c_r c_g}\left(\phi\frac{c_g q_r + \phi q_g}{c_r c_g - \phi^2}\right)s.$$

Furthermore, if we write

$$\Psi_2 \equiv \frac{\Psi_2 c_r c_g - \Psi_2\phi^2}{c_r c_g - \phi^2},$$

then

$$\begin{aligned}\Psi_0 + \Psi_1 S(s) + \Psi_2 s = 2 + \frac{c_r q_g + \phi q_r}{c_r c_g}\left(\phi\frac{c_g q_r + \phi q_g}{c_r c_g - \phi^2}\right)s \\ + \frac{c_r c_g}{c_r c_g - \phi^2}\frac{1}{c_r c_g}\left(\phi q_r q_g + c_g q_r^2 + c_r q_g^2\right)s \\ - \frac{\phi^2}{c_r c_g - \phi^2}\frac{1}{c_r c_g}\left(\phi q_r q_g + c_g q_r^2 + c_r q_g^2\right)s\end{aligned}$$

After simplifying,

$$\Psi_0 + \Psi_1 S(s) + \Psi_2 s = \left(2 + \frac{\phi q_r q_g + c_g q_r^2 + c_r q_g^2}{c_r c_g - \phi^2}\right)s = \frac{s}{\gamma},$$

where the coefficient γ is introduced in the main text (Equation (22)). Altogether, we write

$$\Psi^1(s) = \overline{\Psi}^1 - \frac{1}{2\gamma}s^2. \tag{69}$$

B.3

In this part, we derive a representation for the factor $\Psi^2(s, S(s), \theta)$ in Equation (59). We start by collecting all the cost factors that were not included into $\Psi^1(s)$. This amounts to

$$\Psi^2(s,S(s),\theta) = \Psi^2(\theta) + \left(-c_g\frac{\theta_g}{c_g} + \phi\left(\frac{c_g\theta_r + \phi\theta_g}{c_rc_g}\right) + \theta_g\right)\Omega_g$$
$$+\left(-c_r\left(\frac{c_g\theta_r + \phi\theta_g}{c_rc_g}\right) + \phi\left(\frac{\Delta\theta^g}{\Delta c^g}\right) + \theta_r\right)\Omega_r$$
$$+\left(\theta_g z_g + \theta_r z_r - c_g z_g\frac{\theta_g}{c_g} - c_r\left(\frac{c_g\theta_r + \phi\theta_g}{c_rc_g}\right)z_r\right)s$$
$$-\rho\left(z_r\left(-\frac{\theta_g}{c_g}\right) + z_g\left(-\frac{c_g\theta_r + \phi\theta_g}{c_rc_g}\right)\right)s,$$

where $\Psi^2(\theta)$ includes all the uncertainty factors that do not contain the price variable s. Many factors will cancel out, leaving

$$\Psi^2(s,S(s),\theta) = \Psi^2(\theta) + \phi\left(\frac{c_g\theta_r + \phi\theta_g}{c_rc_g}\right)\left(\frac{S(s) - \overline{F}_g}{c_g}\right) + \phi z_g\left(\frac{c_g\theta_r + \phi\theta_g}{c_rc_g}\right)s.$$

We denote

$$y(\theta) = \Psi^2(\theta) - \phi\left(\frac{c_g\theta_r + \phi\theta_g}{c_rc_g}\right)\left(\frac{\overline{F}_g}{c_g}\right),$$

so

$$\Psi^2(s,S(s),\theta) = y(\theta) + \frac{\phi}{c_g}\left(\frac{c_g\theta_r + \phi\theta_g}{c_rc_g}\right)S(s) + \phi z_g\left(\frac{c_g\theta_r + \phi\theta_g}{c_rc_g}\right)s.$$

Remember that

$$z_g = \frac{q_g}{c_g},$$

so

$$\Psi^2(s,S(s),\theta) = y(\theta) + \frac{\phi}{c_g}\left(\frac{c_g\theta_r + \phi\theta_g}{c_rc_g}\right)\left(S(s) + q_g s\right).$$

We further denote

$$y_1(\theta) = \phi\left(\frac{c_g\theta_r + \phi\theta_g}{c_rc_g^2}\right),$$

so

$$\Psi^2(s, S(s), \theta) = y(\theta) + y_1(\theta)\left(S(s) + q_g s\right). \tag{70}$$

B.4

We insert the subsidy

$$S(s) = \phi\left(\frac{c_g q_r + \phi q_g}{c_r c_g - \phi^2}\right)s - \phi\left(\frac{c_g \overline{F}_r + \phi \overline{F}_g}{c_r c_g - \phi^2}\right)$$

into $\Psi^2(s, S(s), \theta)$. It follows that

$$\Psi^2(s, \theta) = Y(\theta) + y(\theta)\Gamma s, \tag{71}$$

where

$$Y(\theta) = y(\theta) + y_1(\theta)\phi\left(\frac{c_g \overline{F}_r + \phi \overline{F}_g}{c_r c_g - \phi^2}\right)$$

and

$$\Gamma = c_g \frac{\phi q_r + c_r q_g}{c_r c_g - \phi^2} > 0.$$

B.5

To conclude, we derive a formula for aggregate emissions. The amount of emissions in sector g is

$$e_g = \int_0^{b_g^*} e_{0g} db_g + \int_{b_g^*}^1 e_{1g} db_g$$

or, by Equations (1) and (3), and after integration,

$$e_g = \left(\overline{b}_g - s\right) - q_g b_g^*.$$

Similarly, the amount of emissions in sector g

$$e_r = \int_0^{b_r^*} e_{0r} db_r + \int_{b_r^*}^1 e_{1r} db_r$$

can be written as

$$e_r = \left(\overline{b}_r - s\right) - q_r b_r^*.$$

Thus, the aggregate level of emissions is

$$e = e_g + e_r = \left(\overline{b}_g - s\right) - q_g b_g^* + \left(\overline{b}_r - s\right) - q_r b_r^*.$$

We insert the b_g^* and the b_r^* (Equations (61) and (62)) into the emission formula. It holds that

$$\begin{aligned} e(s, S(s), \theta) \quad &= \quad \bar{b}_g + \frac{q_g}{c_g}\left(\overline{F}_g + \theta_g\right) + \bar{b}_r + q_r\left(\frac{\overline{F}_r - \theta_r}{c_r} + \phi\frac{\overline{F}_g + \theta_g}{c_r c_g}\right) \\ &\quad - \left(\frac{2c_r c_g + c_r q_g^2 + c_g q_r^2 + q_r \phi q_g}{c_r c_g}\right) s - \left(\frac{c_r q_g + \phi q_r}{c_r c_g}\right) S. \end{aligned}$$

Denote

$$x_0 = \bar{b}_g + \bar{b}_r + \left(\frac{c_r q_g + q_r \phi}{c_r c_g}\right)\overline{F}_g + \frac{q_r}{c_r}\overline{F}_r,$$

$$x_1(\theta) = \left(\frac{q_g c_r + q_r \phi}{c_r c_g}\right)\theta_r + \frac{q_g}{c_g}\theta_g,$$

$$x_S = \left(\frac{c_r q_g + \phi q_r}{c_r c_g}\right),$$

and

$$x_s = \frac{2c_r c_g + c_r q_g^2 + c_g q_r^2 + q_r \phi q_g}{c_r c_g},$$

so

$$e(s, S(s), \theta) = x_0 + x_1(\theta) - x_S S(s) - x_s s. \tag{72}$$

Next, we incorporate the rule $S(s)$ (Equation (68)) into emissions. Specifically, it holds that

$$\begin{aligned} x_S S(s) - x_s s &= \left(\frac{c_r q_g + \phi q_r}{c_r c_g}\right)\left(\phi\left(\frac{c_g q_r + \phi q_g}{c_r c_g - \phi^2}\right)s - \phi\left(\frac{c_g\overline{F}_r + \phi\overline{F}_g}{c_r c_g - \phi^2}\right)\right) - x_s s \\ &= \frac{1}{\gamma}s, \end{aligned}$$

where the coefficient γ is introduced in the main text (Equation (22)). Taken all things together, we have

$$e(s, \theta) = x_0 + x_1(\theta) - \frac{1}{\gamma}s. \tag{73}$$