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Abstract

We apply a real options approach to develop a general characterization of a firm's optimal organizational mode. We find that the optimal exercise threshold for the establishment of (partial) in-house production is an increasing function of the underlying market uncertainty. However, contrary to common business wisdom, we show that increased market uncertainty induces a higher optimal proportion of in-house production once the investment threshold is reached and once this threshold prescribes partial in-house production.

Keywords: Outsourcing, real options, production mode.

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1 Introduction

On a global scale outsourcing is a highly popular method for firms to organize their production in order to achieve competitiveness. Many scattered observations support such a view and, as Grossman and Helpman (2002b) argue

> "firms seem to be outsourcing an ever expanding set of activities, ranging from product design to assembly, from research and development to marketing, distribution, and after-sales services".

Indeed, for example, Domberger (1999), Grossman and Helpman (2002a) and Shy and Stenbacka (2003) illustrate the significance of the phenomenon of outsourcing by listing a large number of characteristic examples from an extensive spectrum of different industries. A recent cover story in Business Week (Business Week, February 3, 2003) presents a long list of upscale jobs, including basic research, chip design, engineering, financial analysis, which are outsourced on a large scale to, for example India, China and Eastern Europe.

It is not at all uncommon that brand building takes place based on outsourced capacity expansions. For example, entry of the Swedish operator Telia into the Finnish mobile phone industry was based on leased network capacity from a Finnish competitor, Radiolinja. In fact, in telecommunications most western countries apply a policy whereby incumbent operators are required to grant access to their network capacity at "fair and reasonable" terms for entrants. Similarly, in many cases brand-producing firms often sign contracts with established dealer networks rather than building their own brand-specific retailer network of dealers.

For each of the large number of activities or components comprising their products firms have to decide whether to produce these in-house or whether to outsource. An influential line of research in organization theory, originated with the famous contribution by Coase (1937), has offered important insights for explaining outsourcing practices in the attempt to define the boundaries of the firms. Following this approach, for example, Williamson (1985) as well as Grossman and Hart (1986) characterize the factors determining the boundaries of firms in terms of transaction costs, asset specificity and incomplete contracts. Holmström and Roberts (1998) and Spulber (1999, chapter 11) offer updated surveys of the research about the boundaries of firms and, in particular, Holmström and Roberts summarize the past two decades of research as having emphasized the importance of "hold-up" problems between transacting parties.

The contract theoretical approach to understanding outsourcing has recently been complemented with research contributions presenting perspectives based on industrial organization with firms competing with their design of organizational production mode as strategic instrument. Grossman and Helpman (2002a) analyze the equilibrium mode of organization when inputs are fully or partially specialized. Their model exhibits a tradeoff between vertical integration and outsourcing such that in-house production represents a less specialized form of production (higher input costs), whereas outsourced production has to overcome search frictions and contractual imperfections. Shy and Stenbacka (2003) focus on the strategic incentives for outsourcing within the framework of an oligopoly model. In that model the competing oligopolists face a tradeoff between whether to make an irreversible investment in a production facility for supplying a component in-house or to accept buying the component from a subcontractor adding a profit-maximizing markup in reflection of its market power in the input-producing industry. Shy and Stenbacka (2001) extend this approach by investigating what fraction of components to produce internally and what fraction to outsource if in-house production generates monitoring costs, which increase as a convex function of the number of production lines managed in-house. All the research contributions outlined above focus on the design of production mode under the highly restrictive assumption that each component has to be completely outsourced or produced completely in-house.

In the present article we will adopt a real options approach to analyze the optimal organizational mode of production within a context where the firm can decide for each component which proportion to produce in-house and which proportion to outsource. We exemplify the empirical

significance of such an approach by reference to the mobile phone industry. In this industry Nokia, Motorola and Ericsson all engage in substantial outsourcing, but at different degrees. It is estimated that 15-20 % of Nokia's production of mobile handsets is outsourced, 30-40 % of Motorola's production, whereas Ericsson outsources basically all of its production. In contrast, the German rival Siemens is known to apply outsourcing to a very limited extent. In fact, in a recent analysis of Nokia The Economist describes outsourcing as offering "a reference, enabling Nokia to ensure that its own facilities stay competitive" (The Economist, 23 Nov. 2002 (p. 68)). This argument, commonly used by business analysts, views outsourcing as a disciplining device with the purpose of generating a benchmark against which to foster the competitiveness of in-house production. But, of course, in light of the literature emphasizing "hold-up" problems as a generator of contractual inefficiencies this argument can also be used the other way around: By maintaining a certain proportion of in-house production the firm can reduce the degree to which it can be made hostage of a subcontractor. In addition, through in-house production the firm gets updated signals about the costs of producing components, which should be expected to affect the mark-up, which a subcontractor can extract.

In the present article we apply a real options approach to formulate the firm's problem of how to optimally design its organizational production mode. The firm has access to an industry of outside producers of the necessary input, the component. By outsourcing the firm has to pay a mark-up, determined by, for example, the bargaining power of the subcontractor or the degree of competition between the subcontractors. Alternatively, the firm has the option of undertaking an irreversible investment in order to establish its own production facility so that it can avoid the mark-up. Thus, the firm's decision of if/when to exercise the option of investing into the establishment of in-house production represents a tradeoff which is affected by the required fixed investment, the mark-up on outsourced production as well as the parameters characterizing the stochastic properties of the revenue flow. In such a context we particularly analyze the organizational production modes with partial subcontracting, where the firm decides to produce only a certain proportion in-house. To the best of our knowledge this offers a new angle to the theoretical literature on outsourcing.

Within the framework of the real options approach applied in this study we address the following questions: How can we characterize the threshold above which the firm finds it optimal to adopt in-house production? What is the firm's optimal share of outsourcing and how does it behave as a function of the state variable? Which are the structural components determining this share? In particular, in light of the common perception according to which outsourcing is viewed as a highly valuable risk-shifting device for the firm we analyze the relationship between the optimal share of outsourcing and the volatility of the firm's revenue flow. Is the common perception that more uncertainty induces a higher degree of outsourcing consistent with an adequate analysis of the underlying real option values associated with in-house production? Furthermore, our study will predict exclusive in-house production or exclusive outsourcing as special cases and we are able to precisely delineate those circumstances under which each of these "pure" organizational forms will emerge.

Our study proceeds as follows. In section 2 we present a general real options model characterizing the dynamics of the optimal production mode. Section 3 offers an illustrative example in which we pay particular attention to the impact of increased uncertainty on the organizational mode. Finally, some concluding comments are found in section 4.

2   A General Real Option Approach to Outsourcing

We formulate the firm's problem of how to optimally design its organizational production mode within the framework of a real options model. The firm has access to an industry of outside producers of the necessary input, the component. In addition, the firm has the option of undertaking an irreversible investment in order to establish its own production facility so that it can avoid the mark-up charged by a subcontractor. Furthermore, we extend the firm's action space by

allowing the firm to decide which fraction, $\alpha \in [0,1]$, of its output it wants to produce in-house. Thus, we analyze organizational production modes which include partial subcontracting, where the firm decides to keep only the proportion $1 - \alpha$ of its output outsourced.

In order to redesign its production mode and to introduce complete in-house production, or to shift at least some proportion of the production to take place in-house, the firm has to make an irreversible investment. We assume the organizational redesign costs of redirecting the proportion α of the production towards in-house production to be a strictly increasing and convex function of this proportion. These investment costs include the establishment of production capacity in-house as well as other types of switching and adjustment costs. For the formal analysis and for reasons of tractability we introduce the cost function

$$I(\alpha) = k + c\alpha + \frac{K}{\beta}\alpha^{\beta} \tag{2.1}$$

to measure the sunk cost incurred when exercising the option of shifting the production mode. The parameters of this cost function are assumed to satisfy that $k > 0, K > 0, c \geq 0$ and $\beta > 1$. We can, for example, make the interpretation that the convex term $K\alpha^{\beta}/\beta$ captures the costs associated with the establishment of the capacity to produce the proportion α in-house, whereas αc denotes the costs associated with terminating the outsourcing contracts and shifting the proportion mode. Finally, k denotes the fixed cost component associated with any organizational redesign.

The firm's design of organizational mode, i.e. the selection of which proportion α to produce in-house, will affect the firm's profit flow in an essential way. We let $\pi_I : \mathbb{R}_+ \mapsto \mathbb{R}$ and $\pi_0 : \mathbb{R}_+ \mapsto \mathbb{R}$ denote the state-dependent profit flows with in-house production and outsourcing, respectively. These profit flows are assumed to be increasing and continuously differentiable functions with $\lim_{x \downarrow 0} \pi_i(x) \leq 0$ $(i = I, 0)$. We make the natural assumption that

(A1) $\pi_I(x) \geq \pi_0(x)$ for all $x \in \mathbb{R}_+$.

Further, we define the mapping $\Delta : \mathbb{R}_+ \mapsto \mathbb{R}_+$ as $\Delta(x) = \pi_I(x) - \pi_0(x)$ and assume that

(A2) $\Delta(x)$ is non-decreasing and satisfies the condition $\Delta(x) > r(K + c)$ as $x \to \infty$, where $r > 0$ denotes the discount rate.

Assumption (A1) means that there is a benefit in terms of the generated profit flow from the establishment of in-house production. Of course, if this were not the case the firm would have no incentives whatsoever to switch from a production mode with outsourcing. Assumption (A1) could, for example, capture the idea that by in-house production the firm achieves benefits from vertical integration, which could make it possible to improve customer-specific services, to design customer-specific bundles or to quicker transmit customer responses into required modifications in the production of components. Similarly, we can reasonably expect outsourcing to be associated with a disadvantage with respect to the flow costs of production. Such a disadvantage could stem from imperfect competition in the component-producing industry, which generates mark-ups in the pricing of components. This could be illustrated by the telecommunications industry, where an expansion based on leasing network capacity from a competitor clearly includes such a mark-up. In this particular industry the regulatory policy typically specifies a reasonable rate of return on the investments represented by the network capacity. In general, the mark-up associated with outsourcing could also have other sources than imperfect competition. In line with Grossman and Helpman (2002a) the mark-up could capture costs for search frictions and contractual incompleteness associated with outsourced production[1].

Assumption (A2) means that the profit flow benefit of in-house production is larger the more favorable is the state of nature. Furthermore, this benefit becomes significant enough for

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[1]However, we do not want to emphasize such an alternative interpretation too much, because with a focus on informational asymmetries such search frictions and contractual imperfections have to be balanced against the agency costs associated with in-house production.

sufficiently favorable states of nature. It is worth pointing out that in many examples, like in the one we construct in Section 3, this benefit grows without bounds as the state of nature approaches infinity.

We assume that the stochastic dynamics modelling the underlying state of nature evolves on $\mathbb{R}_+$ according to the dynamics described by the stochastic differential equation

$$dX_t = \mu(X_t)dt + \sigma(X_t)dW_t, \quad X_0 = x, \tag{2.2}$$

where the drift $\mu : \mathbb{R}_+ \mapsto \mathbb{R}$ and the diffusion coefficient $\sigma : \mathbb{R}_+ \mapsto \mathbb{R}_+$ are assumed to be sufficiently smooth mappings so as to guarantee the existence of a solution for (2.2) (cf. Borodin and Salminen 2002, pp. 47–48). For simplicity, we assume that $\sigma(x) > 0$ for all $x \in \mathbb{R}_+$ and that both 0 and ∞ are natural boundaries for X_t. Consequently, although the underlying diffusion may tend towards its boundaries it is never expected to attain these boundaries in finite time. In accordance with standard notation, we denote as

$$\mathcal{A} = \frac{1}{2}\sigma^2(x)\frac{d^2}{dx^2} + \mu(x)\frac{d}{dx}$$

the differential operator associated to the underlying diffusion X_t.

Given the underlying diffusion X_t, denote as $\mathcal{L}^1(\mathbb{R}_+)$ the class of measurable mappings satisfying the *absence of speculative bubbles condition*

$$E_x \int_0^\infty e^{-rt}|f(X_t)|dt < \infty$$

and define for any $f \in \mathcal{L}^1(\mathbb{R}_+)$ the functional $R_r f : \mathbb{R}_+ \mapsto \mathbb{R}$ as

$$(R_r f)(x) = E_x \int_0^\infty e^{-rt}f(X_t)dt.$$

In other words, $(R_r f)(x)$ is the expected cumulative present value of the revenue flow $f(x)$ from the present up to an infinite future.

Given our assumptions above, the optimal timing of when to initiate a switch in production mode is captured by the optimization problem

$$V(x) = \sup_\tau E_x\left[\int_0^\tau e^{-rs}\pi_0(X_s)ds + e^{-r\tau}G(X_\tau)\right], \tag{2.3}$$

where

$$G(x) = \sup_{\alpha \in [0,1]}\left[\alpha(R_r\pi_I)(x) + (1-\alpha)(R_r\pi_0)(x) - I(\alpha)\right]. \tag{2.4}$$

The definition (2.4) means that the firm engages in an optimal organizational redesign, i.e. an optimal selection of which proportion of the production to shift in-house, at the timing when the switch in production mode takes place. Since

$$E_x \int_0^\tau e^{-rs}\pi_0(X_s)ds = (R_r\pi_0)(x) - E_x\left[e^{-r\tau}(R_r\pi_0)(X_\tau)\right]$$

we can make use of the strong Markov property of diffusions and the relationship

$$G(x) = (R_r\pi_0)(x) + \sup_{\alpha \in [0,1]}\left[\alpha(R_r\Delta)(x) - k - \alpha c - \frac{K}{\beta}\alpha^\beta\right],$$

to find that the optimal timing problem (2.3) can be re-expressed as

$$V(x) = (R_r\pi_0)(x) + E_x\left[e^{-r\tau}F(X_\tau)\right], \tag{2.5}$$

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where

$$F(x) = \sup_{\alpha \in [0,1]} \left[\alpha(R_r\Delta)(x) - k - \alpha c - \frac{K}{\beta}\alpha^\beta \right]. \tag{2.6}$$

We can now establish the following.

Theorem 2.1. *(A) If $\Delta(0) \leq rc$ then the optimal proportion of in-house production reads as*

$$\alpha^*(x) = \begin{cases} 1 & x \geq \tilde{x}_2 \\ ((R_r\Delta)(x) - c)^{\frac{1}{\beta-1}} K^{\frac{1}{1-\beta}} & \tilde{x}_1 < x < \tilde{x}_2 \\ 0 & x \leq \tilde{x}_1 \end{cases} \tag{2.7}$$

and the state-contingent value of the option generated by an optimal proportion of in-house production reads as

$$F(x) = \begin{cases} (R_r\Delta)(x) - K/\beta - c - k & x \geq \tilde{x}_2 \\ \left(\frac{\beta-1}{\beta}\right)((R_r\Delta)(x) - c)^{\frac{\beta}{\beta-1}} K^{\frac{1}{1-\beta}} - k & \tilde{x}_1 < x < \tilde{x}_2 \\ -k & x \leq \tilde{x}_1 \end{cases} \tag{2.8}$$

where $\tilde{x}_1$ is the unique root of equation $(R_r\Delta)(x) = c$ and $\tilde{x}_2$ is the unique root of equation $(R_r\Delta)(x) = c + K$.
(B) If $rc < \Delta(0) < r(c + K)$ then the optimal proportion of in-house production reads as

$$\alpha^*(x) = \begin{cases} 1 & x \geq \tilde{x}_2 \\ ((R_r\Delta)(x) - c)^{\frac{1}{\beta-1}} K^{\frac{1}{1-\beta}} & x < \tilde{x}_2 \end{cases} \tag{2.9}$$

and the state-contingent value of the option generated by an optimal proportion of in-house production reads as

$$F(x) = \begin{cases} (R_r\Delta)(x) - K/\beta - c - k & x \geq \tilde{x}_2 \\ \left(\frac{\beta-1}{\beta}\right)((R_r\Delta)(x) - c)^{\frac{\beta}{\beta-1}} K^{\frac{1}{1-\beta}} - k & x < \tilde{x}_2, \end{cases} \tag{2.10}$$

where $\tilde{x}_2$ is defined as in part (A).
(C) If $\Delta(0) \geq r(c+K)$ then the optimal proportion of in-house production reads as $\alpha^*(x) = 1$ and the state-contingent value of the option generated by an optimal proportion of in-house production reads as $F(x) = (R_r\Delta)(x) - K/\beta - c - k$ for all $x \in \mathbb{R}_+$.

Proof. See Appendix A. ☐

Theorem 2.1 (A) focuses on situations where the net present value of the profit gains associated with in-house production is sufficiently small relative to the adjustment cost, c, for sufficiently unfavorable states-of-nature. Under such circumstances the choice of optimal production mode partitions the state space into three disjoint regions. For states of nature below $\tilde{x}_1$, defined by the equation $(R_r\Delta)(x) = c$, the firm has no incentive whatsoever to introduce any shift in its production mode. For an intermediate range of states of nature $\tilde{x}_1 < x < \tilde{x}_2$, where $\tilde{x}_2$ is defined by the equation $(R_r\Delta)(x) = c + K$, it is optimal for the firm to adopt a production mode with partial outsourcing and thereby to shift an intermediate share $(0 < \alpha^*(x) < 1)$ of its production to take place in-house. Furthermore, in this range the optimal proportion of in-house production is increasing in the prevailing state of nature. Finally, for sufficiently good states of nature, i.e. $x \geq \tilde{x}_2$, it is optimal for the firm to switch all its production activities to take place in-house. In an analogous way we can interpret the nature of the optimal production mode for situations where the net present value of the profit gains associated with in-house production are at an

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intermediate or sufficiently high level relative to the costs of establishing partial or complete in-house production. These cases are captured by Theorem 2.1 (B) and 2.1 (C), respectively.

In light of Theorem 2.1 we can conclude that we have to separate two crucially different considerations when we characterize the optimal production mode. On the one hand, the firm has to decide the timing of when to initiate an organizational redesign. On the other hand, the firm has also to determine the proportion of the activities which are shifted from the outsourced mode to in-house production. In Theorem 2.1 we delineate the boundaries determining the partition of the parameter space into disjoint regions with complete outsourcing, partial outsourcing, and complete in-house production, respectively. These boundaries are essentially determined by the costs of redesigning the organizational mode, by the expected net present value of the mark-up associated with outsourced production and by the nature, for example the volatility, of the stochastic process describing the state of nature. We will return in greater detail to these characterizations later on.

Lemma 2.2. *The state-contingent real option value $F(x)$ is non-decreasing and continuously differentiable on $\mathbb{R}_+$. Moreover, $F'(x) \leq (R_r\Delta)'(x)$ for all $x \in \mathbb{R}_+$.*

Proof. See Appendix B. □

In light of Lemma 2.2 we can conclude that the option value associated with an organizational redesign is higher the more favorable is the state of nature. Furthermore, this option value increases at a pace which cannot exceed the pace at which the expected cumulative present value of future benefits associated with the shift to in-house production increases as a function of the state of nature.

In order to determine the optimal investment strategy and, especially, the optimal threshold at which an organizational redesign is irreversibly performed, we now consider the behavior of the mapping $L : \mathbb{R}_+ \mapsto \mathbb{R}$ defined as

$$L(x) = \frac{F'(x)}{S'(x)}\psi(x) - \frac{\psi'(x)}{S'(x)}F(x),$$

where $\psi : \mathbb{R}_+ \mapsto \mathbb{R}_+$ denotes the increasing fundamental solution of the ordinary second order differential equation $(\mathcal{A}u)(x) = ru(x)$ and

$$S'(x) = \exp\left(-\int \frac{2\mu(x)dx}{\sigma^2(x)}\right)$$

denotes the density of the scale function of the underlying diffusion process X_t. Given these definitions, we can now prove the following general result highlighting the significance of the function $L(x)$ defined above.[2]

Theorem 2.3. *Assume that equation $L(x) = 0$ has a unique root x^*, that $L(x) \gtreqqless 0$ for $x \lesseqqgtr x^*$, and that $L(x)$ is non-increasing on $x > x^*$. Then, $x^* = \operatorname{argmax}\{F(x)/\psi(x)\} \in (\tilde{x}_1, \infty)$ is the optimal threshold at which the irreversible organizational redesign should be undertaken, $\tau^* = \inf\{t \geq 0 : X_t \geq x^*\}$ is the optimal exercise date, and*

$$V(x) = (R_r\pi_0)(x) + \psi(x)\sup_{y \geq x}\left[\frac{F(y)}{\psi(y)}\right] = \begin{cases} (R_r\pi_0)(x) + F(x) & x \geq x^* \\ (R_r\pi_0)(x) + \psi(x)F(x^*)/\psi(x^*) & x < x^* \end{cases}$$

is the value of the firm, which operates with an optimal production mode.

Proof. See Appendix C. □

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[2]Alvarez and Stenbacka 2001, 2003 apply a related approach to analyze optimal technology and risk adoption in the presence of irreversibility and uncertainty.

Theorem 2.3 specifies a general condition under which it is optimal to introduce at least a partial switch in the production mode. Once such an organizational redesign is executed in an optimal way, i.e. so that an optimal proportion of the production is shifted to take place in-house, we can express the real option value associated with an optimal management of the firm's production mode according to $V(x)$ as characterized in the Theorem. However, this general formulation does not characterize the optimal proportion of in-house production as a function of the prevailing state of nature.

Next we characterize those circumstances under which the optimal organizational redesign is drastic insofar as it involves a switch from complete outsourcing to complete in-house production.

Theorem 2.4. *If $\Delta(0) \geq r \max(c + K, c + k + K/\beta)$ complete in-house production should immediately be established (i.e. $\tau^* = 0$) thereby resulting in the real option value $V(x) = (R_r \pi_I)(x) - K/\beta - c - k$. However, if $r(c + K) \leq \Delta(0) \leq r(c + k + K/\beta)$ then there is a unique optimal exercise threshold x^* satisfying the ordinary first order condition $F'(x^*)\psi(x^*) = F(x^*)\psi'(x^*)$. In that case the optimal adoption date is $\tau^* = \inf\{t \geq 0 : X_t \geq x^*\}$ and the real option value reads as*

$$V(x) = (R_r \pi_0)(x) + \psi(x) \sup_{y \geq x} \left[\frac{F(y)}{\psi(y)}\right] = \begin{cases} (R_r \pi_I)(x) - K/\beta - c - k & x \geq x^* \\ (R_r \pi_0)(x) + \frac{(R_r \pi_I)'(x^*)}{\psi'(x^*)}\psi(x) & x < x^*. \end{cases}$$

Especially, at τ^* the firm changes its production mode from complete outsourcing to complete in-house production.

Proof. See Appendix D. □

Theorem 2.4 demonstrates that the organizational re-design is drastic when the benefit associated with the shift to in-house production is sufficiently significant for all states of nature.

Finally, we turn to a characterization of those circumstances under which the optimal organizational redesign is partial so that an interior proportion of the production is shifted to take place in-house. In this respect we can report

Theorem 2.5. *Assume that*

$$\hat{x} = \operatorname{argmax}\left\{\frac{(R_r \Delta)(x) - c - k - K/\beta}{\psi(x)}\right\} < \tilde{x}_2$$

and that $L(x)$ is decreasing on $(\tilde{x}_1, \infty)$. Then, there is a unique optimal exercise threshold $x^* = \operatorname{argmax}\{F(x)/\psi(x)\} \in (\tilde{x}_1, \tilde{x}_2)$ at which the organizational redesign involves partial outsourcing so that the proportion $0 < \alpha^*(x^*) < 1$ is shifted to be performed as in-house production. Moreover, in this case the real option value associated with the optimal production mode is

$$V(x) = \begin{cases} (R_r \pi_I)(x) - K/\beta - c - k & x \geq \tilde{x}_2 \\ (R_r \pi_0)(x) + \left(\frac{\beta-1}{\beta}\right)((R_r \Delta)(x) - c)^{\frac{\beta}{\beta-1}} K^{\frac{1}{1-\beta}} - k & x^* \leq x < \tilde{x}_2 \\ (R_r \pi_0)(x) + \psi(x)\left(\frac{(R_r \Delta)(x^*) - c}{K}\right)^{\frac{1}{\beta-1}} \frac{(R_r \Delta)'(x^*)}{\psi'(x^*)} & x \leq x^*. \end{cases} \quad (2.11)$$

Proof. See Appendix E. □

So far we have characterized the optimal production mode under very general conditions, with a particular emphasis on the issues related to existence. From this general analysis two central considerations stand out as particularly crucial. An optimal dynamic scheme of organizational redesign is characterized by (a) the timing at which the shift in organizational mode takes place and (b) the proportion of the activities, which are shifted from the outsourced production mode to in-house production. In light of Theorems 2.3 - 2.5 we can generally infer that three types of forces determine the dynamics of optimal production mode management. These forces are (1) the costs

of redesigning the production mode, (2) the net present value of the mark-up flow associated with outsourced production and (3) the properties of the stochastic process determining the state of nature. In order to gain further insights into these issues we next illustrate our model by focusing on an example with particularly simple profit flows and with a standard geometric Brownian motion.

3 Optimal Organizational Mode: An Illustration

In order to illustrate explicitly our general results we now assume that $\pi_I(x) = a_I x$, $\pi_0(x) = a_0 x$, and that $a_I > a_0 > 0$. Moreover, as typically in the literature on real options, we assume that the underlying stochastic dynamics evolve on $\mathbb{R}_+$ according to a standard geometric Brownian motion described by the stochastic differential equation

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad X_0 = x, \tag{3.1}$$

where $\mu \in \mathbb{R}_+$ and $\sigma \in \mathbb{R}_+$ are exogenously determined parameters and W_t is a standard Brownian motion.

Given our assumptions regarding the revenue flows associated with the chosen organizational mode we find that $(R_r \pi_I)(x) = a_I x/(r-\mu)$ and $(R_r \pi_0)(x) = a_0 x/(r-\mu)$, if the standard absence of speculative bubbles condition $r > \mu$ is satisfied. Hence, the value associated with an adoption of the optimal production mode reads as

$$V(x) = \frac{a_0 x}{r - \mu} + E_x\left[e^{-r\tau} F(X_\tau)\right], \tag{3.2}$$

where the state-contingent value of the option generated by an optimal proportion of in-house production is

$$F(x) = \sup_{\alpha \in [0,1]} \left[\alpha \frac{(a_I - a_0)x}{r - \mu} - k - \alpha c - \frac{K}{\beta}\alpha^\beta\right]. \tag{3.3}$$

We can now establish the following result.

Lemma 3.1. *The optimal proportion of in-house production reads as*

$$\alpha^*(x) = \begin{cases} 1 & x \geq \tilde{x}_2 \\ \left(\frac{(a_I - a_0)x}{(r-\mu)} - c\right)^{\frac{1}{\beta-1}} K^{\frac{1}{1-\beta}} & \tilde{x}_1 < x < \tilde{x}_2 \\ 0 & x \leq \tilde{x}_1 \end{cases}$$

and the state-contingent value of the real option associated with the optimal production mode is

$$F(x) = \begin{cases} \frac{(a_I - a_0)x}{r-\mu} - \frac{K}{\beta} - c - k & x \geq \tilde{x}_2 \\ \left(\frac{\beta-1}{\beta}\right)\left(\frac{(a_I - a_0)x}{(r-\mu)} - c\right)^{\frac{\beta}{\beta-1}} K^{\frac{1}{1-\beta}} - k & \tilde{x}_1 < x < \tilde{x}_2 \\ -k & x \leq \tilde{x}_1 \end{cases}$$

where $\tilde{x}_2 = (r-\mu)(c+K)/(a_I - a_0)$ and $\tilde{x}_1 = (r-\mu)c/(a_I - a_0)$.

Proof. The alleged result is an implication of part (A) of Theorem 2.1. □

Given the results of Lemma 3.1, we can now establish the following set of results characterizing the dynamics of the optimal production mode.

Theorem 3.2. *The real option value associated with an optimal management of the firm's production mode reads as*

$$V(x) = \frac{a_0 x}{r - \mu} + x^\kappa \sup_{y \geq x} \left[y^{-\kappa} F(y) \right] = \begin{cases} \frac{a_0 x}{r - \mu} + F(x) & x \geq x^* \\ \frac{a_0 x}{r - \mu} + F(x^*) \left(\frac{x}{x^*} \right)^\kappa & x < x^* \end{cases} \tag{3.4}$$

where

$$\kappa = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} > 1$$

denotes the positive root of the characteristic equation $a(a-1)\sigma^2 + 2\mu a - 2r = 0$. Moreover, if $(\beta - \kappa(\beta - 1))K + \beta(c + \kappa k) \geq 0$ then

$$x^* = \frac{\psi}{(\psi - 1)} \frac{r - \mu}{(a_I - a_0)} \left(k + c + \frac{K}{\beta} \right) \in [\tilde{x}_2, \infty).$$

Otherwise, the optimal threshold $x^* \in (\tilde{x}_1, \tilde{x}_2)$ constitutes the unique root of the equation $F'(x^*)x^* = \kappa F(x^*)$.

Proof. The alleged result is direct implication of Theorem 2.3 and Theorem 2.4. □

For the example under consideration we are able to explicitly carry out comparative statics with respect to the threshold of when to initiate an organizational redesign and with respect to the optimal proportion of in-house production. We find it particularly interesting to explore the relationship between the volatility of the stochastic process describing the state of nature and the optimal production mode. In this respect we can report

Corollary 3.3. *The optimal exercise threshold for the establishment of complete or partial in-house production is strictly increasing as a function of the market volatility, i.e. $\partial x^*/\partial\sigma > 0$, and the optimal proportion of in-house production is also increasing as a function of the market volatility, i.e $\partial\alpha^*(x^*)/\partial\sigma \geq 0$.*

Proof. See Appendix F. □

From the first part of Corollary 3.3 we can conclude that increased market uncertainty, measured by the volatility coefficient σ, will postpone the timing of when to initiate a switch to in-house production. In fact, since

$$E_x[\tau^*] = \frac{\ln(x^*/x)}{\mu - \frac{1}{2}\sigma^2}$$

we can explicitly see how the threshold-increasing effect, $\partial x^*/\partial\sigma > 0$, translates into a pronounced extension in expected value terms of the phase of complete outsourcing, whenever $\mu > \sigma^2/2$. This condition guarantees the almost sure attainability of the threshold x^* in finite time. In this respect our model tends to support the conventional business wisdom according to which increased uncertainty promotes outsourcing as it increases the value of postponing irreversible investments for the establishment of in-house production. In fact, this common business argument was formalized through the real options approach applied in Van Mieghem (1999). He calculated the option value of subcontracting in situations where capacity investments for in-house production have to be made in the presence of substantial uncertainty. In such a context he found that the option value of subcontracting increases as markets are more volatile. Such a perspective however does not seem to be sustainable from the point of view of a structural long-term perspective where also the contracts forming the basis for outsourcing are very sensitive to the underlying fluctuations in the economic environment.[3]

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[3]In fact, empirically it seems to be the case that the length of the subcontracting contracts has shortened drastically during the recent years. Thus, we have strong reasons to believe that these outsourcing contracts are frequently adjusted to changes in the underlying state of nature.

The second part of Corollary 3.3 establishes that $\partial \alpha^*(x^*)/\partial \sigma > 0$ for all those parameter configurations where it is optimal for the firm to apply partial outsourcing. Thus, when partial outsourcing is optimal it always holds true that increased market uncertainty will stimulate in-house production in the sense of increasing the optimal proportion of in-house production. Intuitively, this can be explained as follows. The introduction of a production mode with partial outsourcing means a switch from a profit flow associated with complete outsourcing to a profit flow with some degree of in-house production. This switch in organizational mode to partial outsourcing is undertaken when the discounted value of the cost savings associated with having a lower proportion of the production subject to mark-ups dominate relative to proportion-adjusted irreversible investment expenditure. An increase in market uncertainty will amplify the dominance of the discounted value of these cost savings, which will induce the firm to increase its proportion of in-house production. The commonly used business argument, whereby increased uncertainty is thought to support outsourcing, cited above, seems to basically focus on a situation where outsourcing is based on fixed contracts, where the fluctuations in the costs are not magnified by the mark-up factor.

Table 1 illustrates that the threshold justifying a switch in the production mode is increasing at the same time as the optimal proportion of in-house production is increasing. Table 1 is constructed under the assumptions that $r = 0.04, \mu = 0.01, a_I = 0.125, a_0 = 0.1, c = 0.1, K = 5, b = 2$, and $k = 0.1$ (implying that $\tilde{x}_1 = 0.12$ and $\tilde{x}_2 = 6.12$). It is worth emphasizing that the optimal proportion of in-house production at exercise $\alpha^*(x^*)$ is extremely sensitive to changes in the volatility of the underlying diffusion X_t.

σ	0.01	0.05	0.1	0.14
x^*	1.89	2.21	3.49	6.45
$\alpha^*(x^*)$	0.30	0.35	0.56	1

Table 1 Illustration of the threshold x^* and the optimal proportion of in-house production as functions of the volatility coefficient σ.

4 Concluding Comments

This article has presented a real options formulation of the firm's problem of how to optimally design it organizational production mode. We found that the optimal exercise threshold for the establishment of in-house production, partial or complete, is an increasing function of the underlying market uncertainty. Thus, increased uncertainty tends to postpone irreversible investments for the establishment of in-house production – a finding which is in line with common business wisdom. However, contrary to this business wisdom, we proved that increased market uncertainty induces a higher optimal proportion of in-house production once the investment threshold is reached and once this threshold prescribes partial in-house production. Intuitively, increased market volatility will amplify the discounted value of the cost savings whereby partial in-house production makes it possible for the firm to avoid paying a fluctuating mark-up to a subcontractor in case it were to apply the production mode of outsourcing.

Of course, our analysis can be generalized in several directions. One interesting extension would be to introduce elements of strategic competition, i.e. an oligopolistic market structure. In such a context the choice of organizational production mode would serve as a strategic instrument. With such considerations the firms would be strategic option holders with the exercise thresholds for (partial) in-house production and the proportion of in-house production as strategic instruments.

Our real options model has been formulated to capture the establishment of in-house production in a situation where there is an external market for the production of the component. An analogous approach could be developed to characterize the dynamics of optimal outsourcing starting from an initial production mode with in-house production. Such an analysis would be

relevant in cases where the firm's internal production capacity is unable to match a fast development in the competitiveness of the external component-producing industry. Such a scenario might emerge if the component-producing industry adopts a new technology at a pace faster than the firm itself.

References

[1] Alvarez, L. and R. Stenbacka *Adoption of Uncertain Multi-Stage Technology Projects: A Real Options Approach*, 2001, *Journal of Mathematical Economics*, **35**, 71–97.

[2] Alvarez, L. and R. Stenbacka *Optimal Risk Adoption: A Real Options Approach*, 2003, *Economic Theory*, **23**, 123–148.

[3] Borodin, A. and Salminen, P. *Handbook on Brownian motion - Facts and formulae*, 2nd edition, 2002, Birkhauser, Basel.

[4] Business Week *The New Global Job Shift*, Cover Story, 2003, 3 February.

[5] Coase, R. *The Nature of the Firm*, 1973, *Economica*, **4**, 386–405.

[6] Domberger, S. *The Contracting Organization: A Strategic Guide to Outsourcing*, 1999, Oxford University Press, Oxford.

[7] The Economist, *The Fight for Digital Dominance*, Special report Nokia vs Microsoft, 67–69, 2002, 23 November.

[8] Grossman, S. and O. Hart *The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration*, 1986, *Journal of Political Economy*, **94**, 691–719.

[9] Grossman, G. and E. Helpman *Integration versus Outsourcing in Industry Equilibrium*, 2002a, *Quarterly Journal of Economics*, **117**, 85–120.

[10] Grossman, G. and E. Helpman *Outsourcing in a Global Economy*, 2002b, National Bureau of Economic Research (NBER) Working Paper # 8728.

[11] Holmström, B. and J. Roberts *The Boundaries of the Firm Revisited*, 1998, *Journal of Economic Perspectives*, *12*, 73–94.

[12] Shy, O. and R. Stenbacka *Partial Subcontracting, Monitoring cost, and Market Structure*, 2001, The Yrjö Jahnsson Working Paper Series in Industrial Economics #2, Swedish School of Economics, Helsinki.

[13] Shy, O. and R. Stenbacka *Strategic Outsourcing*, 2003, *Journal of Economic Behavior & Organization*, **50**, 203–224.

[14] Spulber, D. *Market Microstructure: Intermediaries and the Theory of the Firm*, 1999, Cambridge University Press, New York.

[15] Van Mieghem, J. *Coordinating Investment, Production and Subcontracting*, 1999, *Management Science*, **45**, 954–971.

[16] Williamson, O. *The Economic Institutions of Capitalism*, 1985, Free Press, New York.

A   Proof of Theorem 2.1

Proof. Consider the mapping

$$H(\alpha) = \alpha(R_r\Delta)(x) - k - \alpha c - \frac{K}{\beta}\alpha^\beta.$$

Standard differentiation yields

$$H'(\alpha) = (R_r\Delta)(x) - c - K\alpha^{\beta-1}$$

and

$$H''(\alpha) = -K(\beta-1)\alpha^{\beta-2} < 0$$

implying that if an interior proportion $\alpha^*(x)$ maximizing the mapping $H(\alpha)$ exists, it must satisfy the ordinary first order condition $H'(\alpha^*(x)) = 0$. Thus, we observe that

$$\alpha^*(x) = \begin{cases} 1 & x \in A_3 \\ ((R_r\Delta)(x) - c)^{\frac{1}{\beta-1}} K^{\frac{1}{1-\beta}} & x \in A_2 \\ 0 & x \in A_1, \end{cases}$$

where $A_1 = \{x \in \mathbb{R}_+ : (R_r\Delta)(x) \le c\}$, $A_2 = \{x \in \mathbb{R}_+ : c < (R_r\Delta)(x) < c + K\}$, and $A_3 = \{x \in \mathbb{R}_+ : (R_r\Delta)(x) \ge c + K\}$. Since $\lim_{x\downarrow 0}(R_r\Delta)(x) = \Delta(0)/r$ and $\lim_{x\to\infty}(R_r\Delta)(x) > K + c$ (since $\lim_{x\to\infty}\Delta(x) > r(K+c)$) the optimal proportions $\alpha^*(x)$ described in parts (A), (B), and (C) of our Theorem then follow from the continuity and monotonicity of the expected cumulative present value $(R_r\Delta)(x)$.

Establishing the form of the exercise payoff $F(x)$ in case $\alpha^*(x) = 0$ or $\alpha^*(x) = 1$ is straightforward. In order to derive the exercise payoff $F(x)$ in case $\alpha^*(x) \in (0,1)$ we observe that

$$H(\alpha) = \alpha\left[(R_r\Delta)(x) - c - K\alpha^{\beta-1}\right] + \frac{\beta-1}{\beta}K\alpha^\beta - k$$

from which the alleged result follows. $\qquad\square$

B   Proof of Lemma 2.2

Proof. The continuous differentiability of $F(x)$ in the case of part (C) of Theorem 2.1 is obvious. It is, therefore, sufficient to consider case (A) since case (B) is then entirely analogous. Standard differentiation yields

$$F'(x) = \begin{cases} (R_r\Delta)'(x) & x \ge \tilde{x}_2 \\ \left(\frac{(R_r\Delta)(x)-c}{K}\right)^{\frac{1}{\beta-1}} (R_r\Delta)'(x) & \tilde{x}_1 < x < \tilde{x}_2 \\ 0 & x \le \tilde{x}_1. \end{cases}$$

The continuous differentiability of $(R_r\Delta)(x)$ and condition $(R_r\Delta)(\tilde{x}_1) = c$ imply that $\lim_{x\downarrow\tilde{x}_1} F'(x) = 0$. On the other hand, since $(R_r\Delta)(\tilde{x}_2) = c + K$ the continuous differentiability of $(R_r\Delta)(x)$ implies that $\lim_{x\uparrow\tilde{x}_2} F'(x) = (R_r\Delta)'(\tilde{x}_2)$. Finally, since $(R_r\Delta)'(x) \ge 0$ and $0 \le ((R_r\Delta)(x)-c)/K \le 1$ for all $x \in (\tilde{x}_1, \tilde{x}_2)$ we find that $F'(x) \le (R_r\Delta)'(x)$ for all $x \in \mathbb{R}_+$. $\qquad\square$

C   Proof of Theorem 2.3

Proof. It is clear from the definition that $L(x) = k\psi'(x)/S'(x) > 0$ on $(0, \tilde{x}_1)$ and, therefore, that $F(x)/\psi(x)$ is increasing on $(0, \tilde{x}_1)$. Thus, we find that the root of equation $F(x) = 0$ necessarily

belongs to $(\tilde{x}_1, \infty)$. Moreover, our assumptions naturally imply that $x^* = \operatorname{argmax}\{F(x)/\psi(x)\}$. It now remains to establish that the proposed value function indeed constitutes the value of the considered optimal timing problem. To this end, define the mapping $J : \mathbb{R}_+ \mapsto \mathbb{R}_+$ as

$$J(x) = \sup_\tau E_x \left[e^{-r\tau} F(X_\tau) \right].$$

Since the proposed value function satisfies the identity

$$J_p(x) = \psi(x) \sup_{y \geq x} \left[\frac{F(y)}{\psi(y)} \right] = E_x \left[e^{-r\tau^*} F(X_{\tau^*}) \right]$$

we naturally have that $J(x) \geq J_p(x)$. In order to establish the opposite inequality we first observe that the proposed value function is continuously differentiable on $\mathbb{R}_+$ and dominates the mapping $\max(F(x), 0)$. Moreover, since $J_p''(x^*-) = \psi''(x^*)F(x^*)/\psi(x^*) < \infty$, $J_p''(x^*+) = F''(x^*+) < \infty$, $F''(\tilde{x}_2+) = (R_r\Delta)''(\tilde{x}_2) < \infty$, and $F''(\tilde{x}_2-) = (R_r\Delta)''(\tilde{x}_2) + (R_r\Delta)'^2(\tilde{x}_2)/((\beta-1)K) < \infty$ we find that the proposed value function is twice continuously differentiable outside a countable set. Finally, since $(\mathcal{A}J_p)(x) = rJ_p(x)$ on $(0, x^*)$ and $L'(x) = ((\mathcal{A}F)(x) - rF(x))\psi(x)m'(x) \leq 0$ on (x^*, ∞) by assumption, we find that $J_p(x)$ constitutes a r-excessive majorant of the exercise payoff $F(x)$. However, since $J(x)$ is the least of these majorants, we find that $J_p(x) \geq J(x)$ completing the proof of our theorem. □

D   Proof of Theorem 2.4

Proof. As was demonstrated in part C of Theorem 2.1 inequality $\Delta(0) \geq r(c + K)$ implies that $\alpha^*(x) = 1$ for all $x \in \mathbb{R}_+$ and, therefore, that $F(x) = (R_r\Delta)(x) - K/\beta - c - k$. Standard differentiation now yields

$$(\mathcal{A}F)(x) - rF(x) = r\left(K/\beta + c + k\right) - \Delta(x)$$

which is non-increasing as a mapping of the underlying state x by the monotonicity of $\Delta(x)$. Thus, if inequality $(\beta - 1)K \geq \beta k$ holds, then

$$F(x) \leq r\left(K/\beta + c + k\right) - r(K + c) = r\left(\beta k - (\beta - 1)K\right)/\beta \leq 0$$

for all $x \in \mathbb{R}_+$. Moreover, since $F(x) \geq (\Delta(0) - r(K/\beta + c + k))/r \geq r((\beta - 1)K - \beta k)/\beta \geq 0$ we observe that $F(x)$ is r-excessive for the underlying diffusion X_t and, therefore, that $\tau^* = 0$ is optimal.

If, however, inequality $r(c + K) \leq \Delta(0) < r(c + k + K/\beta)$ holds then the set where $(\mathcal{A}F)(x) > rF(x)$ and, therefore, the set where waiting is optimal is non-empty. Denote now as $\bar{x}$ the root of equation $\Delta(x) = r(c + k + K/\beta)$ and consider the behavior of the functional $L(x)$. Given that the state-contingent value $F(x)$ can be re-expressed as $(R_r\tilde{\Delta})(x)$, where $\tilde{\Delta}(x) = \Delta(x) - r(c + k + K/\beta)$, we find that

$$L(x) = -\int_0^x \psi(y)\tilde{\Delta}(y)m'(y)dy$$

and, therefore, that $L(x) \geq 0$ as long as $x \leq \bar{x}$. If $x > t > \bar{x}$ then the mean value theorem for integrals implies that

$$L(x) = L(t) - \frac{\tilde{\Delta}(\xi)}{r} \frac{\psi'(x)}{S'(x)},$$

where $\xi \in [t, x]$. The assumed boundary behavior of the underlying diffusion implies that $\psi'(x)/S'(x) \to \infty$ as $x \to \infty$ and, therefore, that $L(x) \to -\infty$ as $x \to \infty$ since $\tilde{\Delta}(x) > 0$ for $x > \bar{x}$. The continuity and monotonicity of $L(x)$ then implies that equation $L(x) = 0$ has

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a unique root $x^* > \bar{x}$ and that $x^* = \operatorname{argmax}\{F(x)/\psi(x)\}$. It now remains to establish that the proposed value function coincides with the value of the optimal timing problem. To this end, let

$$J_p(x) = \psi(x) \sup_{y \geq x} \left[\frac{F(y)}{\psi(y)} \right]$$

denote the proposed value and observe that $J_p(x) = E_x \left[e^{-r\tau^*} F(X_{\tau^*}) \right]$, where $\tau^* = \inf\{t \geq 0 : X_t \geq x^*\}$. Thus, we find that $J_p(x) \leq J(x) = \sup_\tau E_x \left[e^{-r\tau} F(X_\tau) \right]$. To establish the opposite inequality we first observe that $J_p(x) \geq \max(F(x), 0)$ for all $x \in \mathbb{R}_+$ and that $J_p \in C^1(\mathbb{R}_+) \cap C^2(\mathbb{R}_+ \backslash \{x^*\})$. Moreover, since $(\mathcal{A}J_p)(x) = rJ_p(x)$ on $(0, x^*)$ and $(\mathcal{A}J_p)(x) - rJ_p(x) = r(c + k + K/\beta) - \Delta(x) \leq 0$ on (x^*, ∞) we find that $J_p(x)$ is a r-excessive majorant of $F(x)$ for the underlying diffusion X_t. However, since $J(x)$ is the least of these majorants, we find that $J_p(x) \geq J(x)$, completing the proof of our Theorem. $\qquad \square$

E   Proof of Theorem 2.5

Proof. As was demonstrated in Lemma 2.2 $F'(x) \leq (R_r\Delta)'(x)$ for all $x \in \mathbb{R}_+$. Moreover, since $F(x) \geq (R_r\Delta)(x) - c - k - K/\beta$ by the optimality of the chosen proportion $\alpha^*(x)$ we observe that

$$L(x) \leq \frac{(R_r\Delta)'(x)}{S'(x)} \psi(x) - \frac{\psi'(x)}{S'(x)} ((R_r\Delta)(x) - c - k - K/\beta)$$

implying that $F(x)/\psi(x)$ is decreasing on the set where $((R_r\Delta)(x) - c - k - K/\beta)/\psi(x)$ is decreasing. Combining this observation with the continuity and assumed monotonicity of $L(x)$ and the inequality $L(\tilde{x}_1) = k\psi'(\tilde{x}_1)/S'(\tilde{x}_1) > 0$ then implies that there is a unique optimal exercise threshold x^* on the set $(\tilde{x}_1, \tilde{x}_2)$. The rest of the proof is analogous to the proof of Theorem 2.3. $\qquad \square$

F   Proof of Corollary 3.3

Proof. The absence of speculative bubbles condition $r > \mu$ implies that $\kappa > 1$ and, therefore, that

$$\frac{\partial \kappa}{\partial \sigma} = \frac{2\kappa(1 - \kappa)}{\sigma(\kappa - \rho)} < 0,$$

where

$$\rho = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} < 0$$

denotes the negative root of the characteristic equation $a(a-1)\sigma^2 + 2\mu a - 2r = 0$. Consequently, since the state-contingent value of the option generated by an optimal proportion of in-house production is independent of the volatility coefficient σ and

$$\frac{\partial}{\partial \sigma} \left[F'(x)x - \kappa F(x) \right] = -\frac{\partial \kappa}{\partial \sigma} F(x) > 0$$

on the set where $F(x) \geq 0$, we find that increased volatility increases the optimal exercise threshold x^* satisfying the optimality condition $F'(x^*)x^* = \kappa F(x^*)$. $\qquad \square$

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