Recent Advances in Forest Mensuration and Growth and Yield Research

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GROWTH MODEL FOR ITALIAN DOUGLAS FIR PLANTATIONS

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ABSTRACT

The fundamental elements of a growth model for Douglas fir plantations in central and southern Italy have been developed and tested. Growth and yield of Douglas fir in Italy is still estimated by yield tables. No variable density growth function has yet been published. This work concentrates on diameter and basal area growth as functions of site and stand characteristics.

The model can be used both as stand (average) model or as size-class model. The main function estimates annual basal area growth of the stand through age, current basal area and site characteristics. If current diameter distribution is also known, a second function estimates size-class allotment of overall stand growth enabling distribution projection.

The research is based on a set of 55 plots distributed over 3 Italian regions: Toscana, Puglia and Basilicata. In Toscana, where more than half of the plots are located, each plot has been remeasured 3 to 5 times during the last 15 years (la Marca, Scotti, 1986; Corona et al., 1990). In the other regions the plots have been established more recently (Scotti et al., 1995). The main factors influencing current stands status are: initial plantation density (ranging from 800 to over 3000 trees/ha) and thinning regime (no thinning, selective or combined, removing from 10 to over 35% of the basal area).

The model is expected to become a basic tool for developing decision support systems for silvicultural planning, optimizing plantation density, thinnings and rotation for specific forest sites and timber management objectives. Results include a basic evaluation of model performance at stand and size-class level. Stand basal area is accurately estimated over a wide range of conditions. Diameter distribution projections compare quite well with correspondent observed distributions: even the worst cases do not appear to be significantly biased.

1 INTRODUCTION

The fundamental elements of a growth model for Douglas fir plantations in central and southern Italy have been developed and tested. Growth and yield of Douglas fir in Italy is still estimated by yield tables. No variable density growth function has yet been published. This work concentrates on horizontal stand structure estimating breast height diameter (DBH) and basal area (G) growth as functions of site and stand characteristics. The model can be used both at stand (average) level or at size-class level. At stand level annual basal area growth is estimated as function of standing basal area, age, thinning, stand and site characteristics.

If current stand diameter distribution is also known, a second function estimates size-class allotment of overall stand growth enabling DBH distribution projection compatible with stand level projection.

2. MATERIALS AND METHODS

2.1 The database

The research is based on a set of 55 plots distributed over 3 Italian regions: Toscana, Puglia and Basilicata (figure 1). In Toscana, where more than half of the plots are located, 3 geographic locations are considered sampling a wide range of cultivation conditions. Each plot has been remeasured 3 to 5 times during the last 15 years. In the other regions the plots have been established more recently, considering 2 geographic locations each. Geographic position and main environmental characteristics are synthesized in Table 1.

After first tally, excluding few non-thinning reference plots, all the others have been thinned applying different regimes (systematic, selective or mixed thinning criteria) and intensities. DBH for all trees of all plots has been re-measured from 2 to 4 times. Some plots have already been thinned twice. Almost every year all plots have been surveyed for damages (uprooting, stem breakage, etc.) Damaged trees have generally been removed.

2.1.1 Definition of the variables

To perform stand level analysis aggregated statistics and indicators have been computed for every remeasurement of each plot. The following table defines the most relevant statistics considered. All accumulation values refer to the beginning of remeasurement period.

VAR.	*units*	*Definition*
SI	[m]	Mean height of the dominant trees at 30 years
PD	[n./ha]	Number of trees per hectare at plantation time
P_YEAR	[year]	Year of plantation
DBH	[cm]	Breast height diameter measured at from a fixed position
AGE	[years]	Measurement year - Plantation year
AGE-D	[years]	Age difference to successive remeasurement
N	[n./ha]	Trees/ha at beginning of remeasurement period after thinning
SBA or G	[m^2/ha]	Stand basal area/ha after thinning
DG	[cm]	DBH of average basal area tree
SBAI or G'	[m^2/(ha*year)]	Stand basal area increment per year
TNP	%	Thinning: n. of trees removed as % of total before thinning
T-SBA-P	%	Thinning: stand basal area removed as % of total before thinning
T-DG-RP	%	Thinning: (Dg. after - Dg. before) % of Dg. before thinning

2.1.2 Time invariant plots characteristics: Site

Static and initial plot characteristics like plantation year, plantation density and site index, are collectively referred to as site characteristics. Site index has been evaluated measuring a sample of heights of the plots dominant trees (100 biggest -DBH- trees per ha) and projecting the height of the average dominant tree to the reference age of 30 years via the function developed by Maetzke & Nocentini (1994).

2.1.3 Time dependant plot characteristics:

Dynamic characteristics tightly related to time like DBH, height, eventual thinning or damages, are called stand characteristics. Having identified each tree through its position on the schematic maps single tree records have been accumulated accurately controlling logical consistency of successive observations.

2.2 PRELIMINARY ANALYSIS

2.2.1 Site characteristics

The experimental material available is a collection of several small scale thinning trials. Uneven distribution of basic site characteristics is therefore unavoidable. The problem has to be analyzed and taken in due consideration during results interpretation and particularly for any eventual application.

Of the three time independent characteristics considered, plantation year (P-YEAR), plantation density (P-DENS) and site index (SI), P-DENS has by far the highest CV% and dominates the set (Table 2). Overall correlations are relatively low, the highest being that between site index and plantation year: -0.52.

Site index values range is roughly between 20 m and 30 m (base age=30 years); plantation densities are mainly between 1000 and 2500 trees per hectare. The distribution by geographic location is particularly affected by the "Casentino" area characteristics where plantation density range is very high and highly correlated to site index (corr.=0.79, N=13, p=0.0013)

2.2.2 Stand level - thinning characteristics

The ability to predict stand response to thinning constitutes a major objective for the development of the growth and yield model. Particularly at aggregated level, descriptive or predictive ability of the model is limited to the range of thinnings experimented. It is therefore essential to characterize the different thinnings that have been performed. Defined thinning statistics synthesize in quantitative form treatment type and intensity (Table 3). The least correlated variables are T-SBA-P and T-DG-RP. Stand basal area percent reduction (T-SBA-P) expresses intensity, while percent increase of mean basal area DBH (T-DG-RP) is related to thinning type, since it reflects the distribution of basal area reduction within the stand. Given T-SBA-P and T-DG-RP values the percentage of number of trees removed (TNP) is fully determinated.

From this quantitative point of view, thinning characteristics do not differ significantly between the geographic areas considered. Stand basal area removal percentage varies from 10% to over 35%. Percentage modification of mean basal area DBH ranges from very low levels (2%) for practically non selective thinnings to 10% for selective thinnings from below. Only three plots in "Consuma" area have received particularly selective treatments: mean basal area DBH increased 25 to 30% with percentage number of trees removed ranging over 50%.

2.3 MODELLING METHODOLOGY

Objective of the work is to develop a model for compatible stand-basal-area (G) and DBH-distribution projections applicable either only at stand level or eventually also at size-class level. Compatibility is achieved assuming stand basal area growth function main component of the model and estimating class DBH development through stand basal area growth distribution. Applying following definitions,

$$\begin{cases} G = \text{Stand\_Basal\_Area} \\ G' = \text{Stand\_BA\_Increment} \\ A = \text{Stand\_Age} \\ N = \text{Number\_of\_Trees / ha} \\ g = \text{Individual\_BA} \\ g' = \text{Ind.\_BA\_Increment} \\ \overline{g} = \text{Mean\_Ind.\_BA} \\ \overline{g'} = \text{Mean\_Ind.\_BA\_Inc.} \\ \alpha, \beta, \gamma = \text{Model\_Parameters} \end{cases}$$

stand level growth function has been developed based on the "power decline equation" (Zeide 1988):

$$G' = \gamma * \frac{G}{A} * [\ln(\alpha) - \ln(G)]$$

and size-class distribution of stand basal area growth is modeled estimating individual increments as linear function of individual basal area, assuming stand basal area increment and initial distribution, hence the number of living trees, is known:

$$\begin{cases} \overline{g} = G/N \\ \overline{g'} = G'/N \\ g' = \overline{g'} + (g - \overline{g}) * \beta \end{cases}$$

At both levels functions parameters have been estimated through a multistage procedure:
a) first independent plot by plot parameter estimates have been computed,
b) then regression analysis is applied to relate parameter variations
between plots (and remeasurements) to site and stand characteristics,
c) finally model coefficients are calibrated by simultaneous estimation.

2.3.1 Stand level

Stand level function includes two parameters α and γ.α represents the asymptotic value of G for age approaching infinity. Although the age range is relatively limited (15 to 35 years) a rough estimate of plot α value has been computed for all plots.

The estimates have been calculated via linear regression introducing in the stand basal area increment function following definitions:

Hence:

and

$$\begin{cases} \varphi_1 = \gamma * \ln(\alpha) & X_1 = \dfrac{G}{A} \\ \varphi_2 = \gamma & X_2 = -\dfrac{G}{A} * \ln(G) \end{cases}$$

$$G' = \varphi_1 * X_1 + \varphi_2 * X_2$$

$$\alpha = \exp\left(\frac{\varphi_1}{\varphi_2}\right)$$

Then basal area asymptotic values have been related to stand characteristics expressing growth potential: SITE INDEX and PLANTATION DENSITY. Once α is defined, the analysis concentrates on γ, evidencing that γ represents a scale factor converting reduced average increment X to current increment G'. Expressing γ as linear combination of plots characteristics, main relevant variables are selected through regression analysis.

In the reference equation γ reflects the growing conditions of the stand as a time-independent expression; i.e. γ value changes only if the number of trees changes significantly. The right hand side of stand level function equation, excluding γ, represents an expression of average increment progressively reduced as G approaches its maximum size α Indeed, setting

function expression reduces to $X = \dfrac{G}{A} * [\ln(\alpha) - \ln(G)]$

$$G' = \gamma * X$$

Therefore only site characteristics and non-growth stand variables (i.e. excluding diameter related variables) will be considered. Thinnings are expected to evidence a significant influence on this parameter.

2.3.2 Size-class level

Size-class level function $$g' = \bar{g}' + (g - \bar{g}) * \beta$$

has only one parameter: β. An independent estimate of β is provided by each remeasurement of each plot. Again, main factors are identified through regression analysis expressing β as linear combination of site and stand variables. At this level all variable categories will be considered.

3. RESULTS

First ordinary least squares estimates of the coefficients of the linear combinations evaluating model parameters α, γ and β are discussed. Successively simultaneous estimation results are presented.

3.1 Ordinary least squares estimates

3.1.1 Stand level

Estimation of α
As it is quite frequently the case with the estimation of growth function parameters, the reliability of asymptotic value estimation is particularly low although it has a relevant influence on function output. Fitting the "power decline function" to single plots is only expected to produce very rough α estimates given the short time span single plots measurements cover in relation to the stands potential life time. As foreseen, for some plots the estimated asymptotic basal area value was far out of range. However, over half of the plots exhibited reasonable fits with α values within an acceptable range. figure 2 presents two examples at lower ($\alpha \approx 100$ m^2/ha) and upper ($\alpha \approx 200$ m^2/ha) limits of the range comparing observed and predicted values of stand basal area and basal area increment. Excluding out of range cases, the remaining set had a sufficient coverage of the experimental conditions considered. Regression analysis results are summarized in Table 4. The best reasonable estimation function identified, considered ln (α) as dependent and only the site index (SI) and plantation density (PD) interaction term as independent variable with coefficient's significance level barely within 0.15 level.

Estimation of γ
Theoretical constraints on γ parameter limit the set of potential predictor variables to site characteristics and stand characteristics not expressing growth. Regression analysis evidenced that: site index exhibited the most relevant effect, and thinning effects are expressed directly through TNT (%n. of trees removed by thinning), and indirectly through N (n. of standing trees).

Having identified the structure of the function estimating γ parameter, α coefficients have been recalibrated via non-linear regression on G'. Final regression residuals are very well distributed (figure 3) although a slight negative trend exists. All statistical indicators are quite satisfactory (Table 5)

3.1.2 Size-class level

Estimation of β
First stage independent estimation of β values for each remeasurement is heavily affected by the high random variability of single tree increments. Beside natural variability, also measurement approximation alters observed variance. Although average coefficient of determination is relatively low (average R2=0.36) for only very few non significant regression coefficients were observed. Second stage analysis, relating individual β values to site and stand characteristics, is not constrained by any theoretical consideration.

β expresses average difference of basal area increment between trees with unit difference in size. Many variables appear to be significantly correlated to this parameter: average basal area (r=-.63), age (r=-.58), stand basal area (r=-.41) and site index (r=-.39) are negatively correlated (with $p \leq .001$) while number of trees correlation (r=+.18) has positive sign (with p=.025). Variables directly expressing thinning characteristics didn't evidence any effect on β. Due to the significant correlation among site and stand variables, linear combinations with more than two terms would produce less reliable coefficient estimates. N. of trees (N) and average basal area diameter (DG), being relatively independent, appeared to form the most effective combination. Regression analysis results are presented in Table 4. Although the slope of the size-class increment regression line is subject to very large within plot random variations from one remeasurement to the other, residuals are quite well distributed (figure 4.). Root mean square error is very small compared to predictions range. Few observations with extreme residuals were not dropped as outliers since they had little influence on coefficients estimates.

3.2 Seemingly unrelated regression estimates

The three parameters α, γ and β characterizing model structure are expressed as functions of site and stand characteristics. α representing asymptotic basal area value, is relatively independent. While γ and β are both increment values estimated by regression on the same observations set. Their dependent variables are different but, to some extent, correlated. Some of the independents are in common. Using the model at size-class level, both functions are involved. In this case, independent ordinary least squares (OLS) estimates can be biased due to residuals-regressors correlations.

To reduce the effect of functions interdependence 'Seemingly Unrelated Regression' (SUR) method of SYSLIN SAS procedure has been applied (SAS, 1993). Results are synthesized in Table 7. Although the differences between SUR and OLS coefficients estimates are all within standard error limits, the confidence range of SUR estimates are narrower.

4. DISCUSSION

Final model variables, equations and coefficient estimates are summarized.
Predictor variables:

$$\begin{cases} SI = Site\_Index = H\_dom@30years[m] \\ PD = Plantation\_Density[N\_trees/ha] \\ A = Age[years]\_from\_plantation \\ N = Standing\_Trees[N\_trees/ha] \\ TNP = Thinning\_N\_removed[\%]\_of\_N\_Before\_T. \\ G = Stand\_Basal\_Area[m^2/ha] \end{cases}$$

Stand level - estimation of basal area current increment (G'):

$$\begin{cases} G' = \gamma * \dfrac{G}{A} * [\ln(\alpha) - \ln(G)] \\ \ln(\alpha) = \alpha_0 + \alpha_1 * SI * PD \\ \gamma = \gamma_1 * SI^{-1} + \gamma_2 * N^2 / 1E6 + \gamma_3 * \sqrt{TNP} \end{cases}$$

Size-class: estimation of tree current basal area increment (g') given tree basal area (g)

$$\begin{cases} g' = g' + (g - \bar{g}) * \beta \\ g' = G'/N \\ \bar{g} = G/N \\ \beta = \beta_0 + \beta_1 * N + \beta_2 * D_g \\ D_g = \sqrt{\frac{4}{\pi} * \bar{g}} \end{cases}$$

Model coefficients:

Coefficient	*Estimated Value*	*Standard Error*	*t*	*Prob. > |T|*
α_0	4.6482767	0.16778012		(*)
α_1	0.00000814	0.00000112		(*)
γ_1	27.535735	0.668269	41.205	0.0001
γ_2	-0.073966	0.012427	-5.952	0.0001
γ_3	-0.008192	0.001152	-7.114	0.0001
β_0	0.214653	0.013219	16.238	0.0001
β_1	-0.000020355	0.000004758	-4.278	0.0001
β_2	-0.005236	0.000443	-11.815	0.0001

(*) non-linear regression

In the previous section regression results have been evaluated comparing observed and predicted values of the specific dependent variables considered. A first evaluation of model performance will be presented in this section using the basic functions to estimate original observed values, not standardized for regression analysis purposes.

4.1 Comparing observed & predicted stand basal area remeasurements

The basic stand level function estimates yearly basal area increment given basal area of standing trees, site index, stand age, number of live trees and percent number of trees removed by thinning. Observed remeasurement periods range from 1 to 6 years. Basal area values corresponding to original remeasurement periods have been estimated.

Plotting the difference, predicted minus observed basal area, by plot (figure 5) and versus remeasurement period length (figure 6) provides a first level test of model application. Prediction errors are very small compared to stand basal area values, only few large residuals can be observed. Plot biases tend to have constant sign and some site effect can be noticed (site is identified by the first character of plots labels).

Remeasurement period length, as expected, negatively influences prediction error (as in the previous graph, residuals for the same plot are connected) but signs appear to be sufficiently well distributed almost compensating any bias if averages across plots were estimated.

4.2 Comparing observed & predicted remeasurement diameter distributions

At size-class level main application interest is in DBH-distribution frequencies.
Original frequencies have been computed and compared to frequency estimates based on model projections. For each remeasurement P_CHI value has been computed as the probability associated with the chi-square statistic comparing observed and estimated frequencies. The P_CHI values (figure 7), spreading across the whole 0-1 range, simply identify the best (CV1) and worst (F14) performing plots enabling a concise objective model evaluation by the inspection of observed and predicted frequency distributions of each remeasurement for those plots.

As evidenced by the graph for plot F14 (figure 8), even in the worst case predicted distributions do not exhibit any relevant bias, while the excellent predictions for plot CV1 (figure 9) are quite common.

5. CONCLUSIONS

An original approach to estimate the basic functions of a growth model operating at stand and size-class levels has been developed and successfully tested. Some constraints in the available experimental data conditioned models structure and parameters evaluation. Further research, but specially new data collection, is needed to extend geographic applicability field of the model. Future analysis effort should exploit the available tree-wise damage records.

The present paper focuses on the original methodological approach of basal area growth model structuring and on a first evaluation of the model itself. Results evidence significant potential for model application as basic tool in a decision support system, optimizing Douglas fir plantations planning, with multiple management objectives: multiple timber assortment productions, stand stability and management risks minimization, etc.

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Figure 1: Geographic locations of the plots

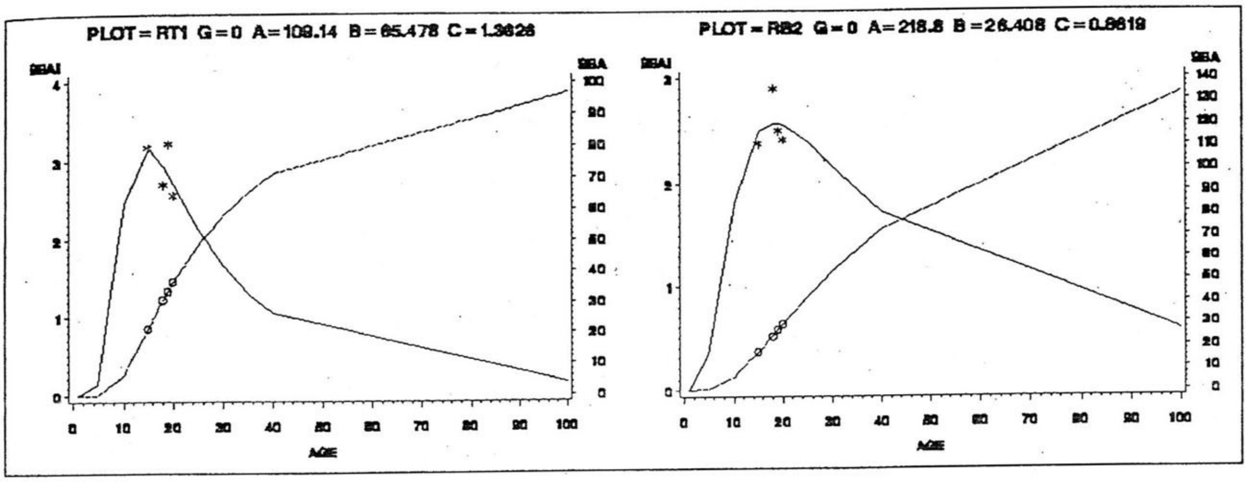


Figure 2: Fitting "power decline function" by plot: two extreme examples. A, B and C are function parameters α, β and γ. SBA (circles) is basal area, SBAI (stars) is basal area increment. Symbols are observed and lines are predicted values.

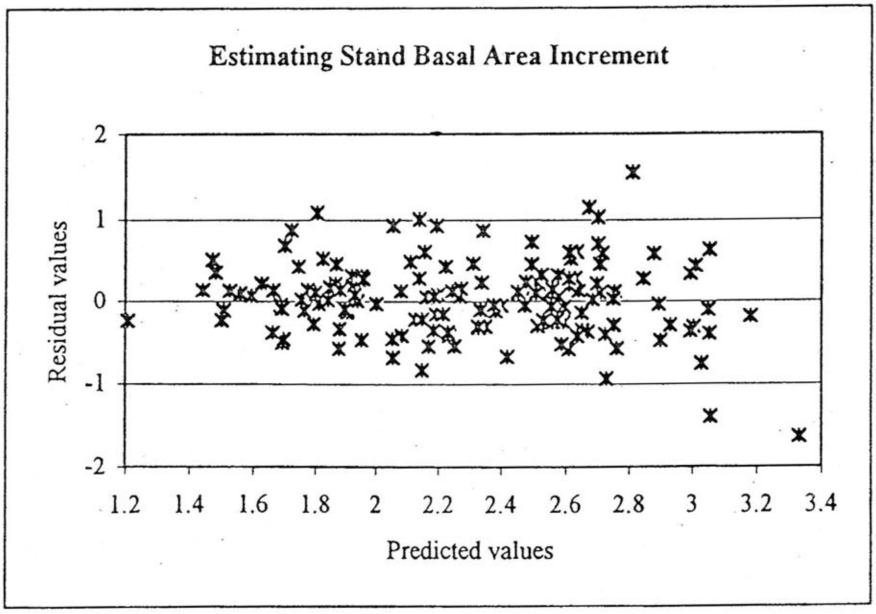


Figure 3. Residuals of stand basal area increment regression

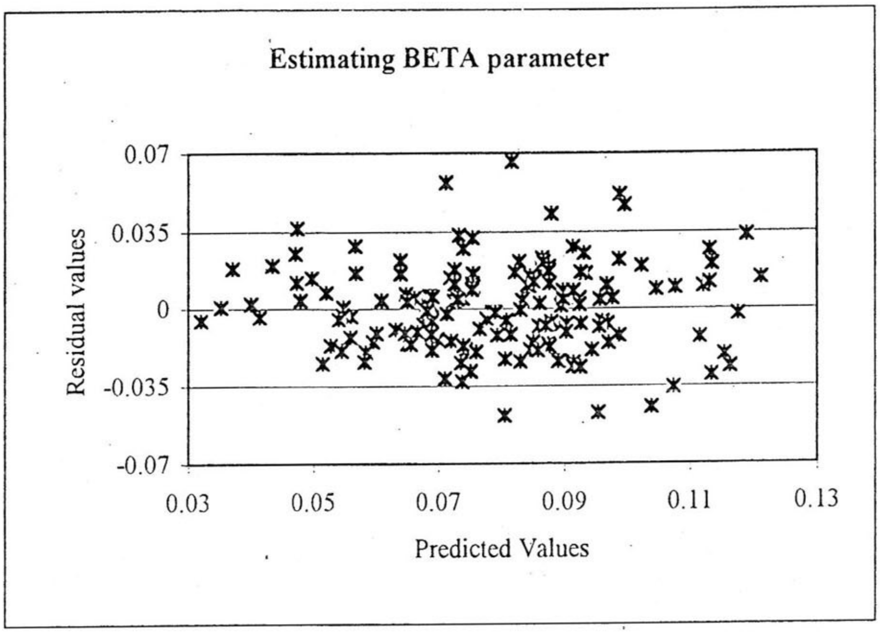


Figure 4. Residuals of β regression

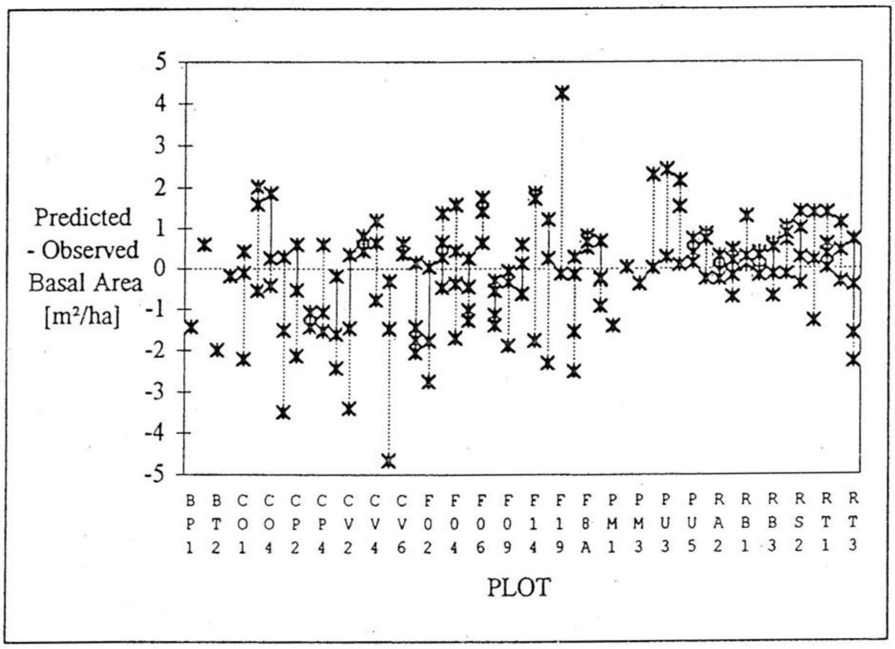


Figure 5. Estimating stand basal area remeasurement: residuals by plot

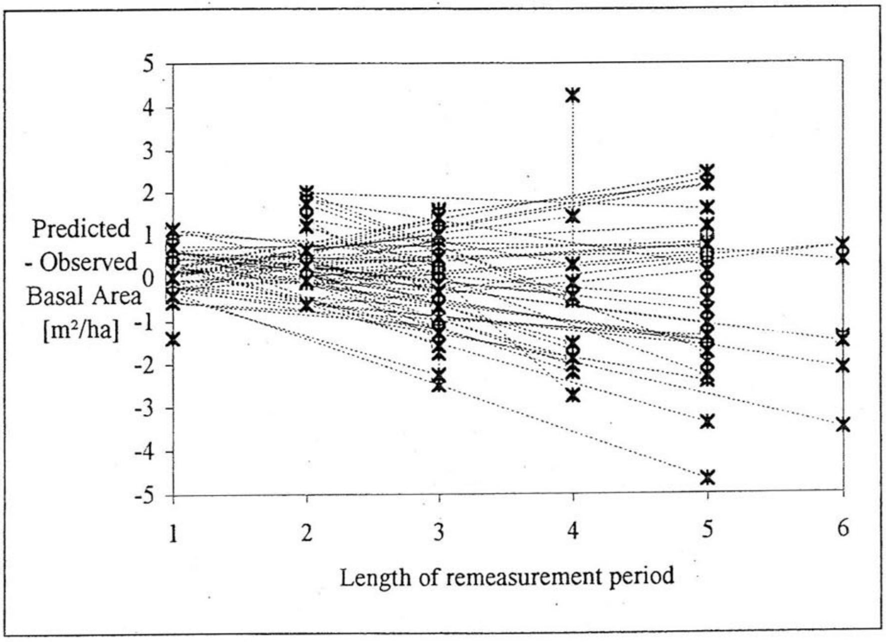


Figure 6. Estimating stand basal area remeasurement: residuals versus length of remeasurement period.

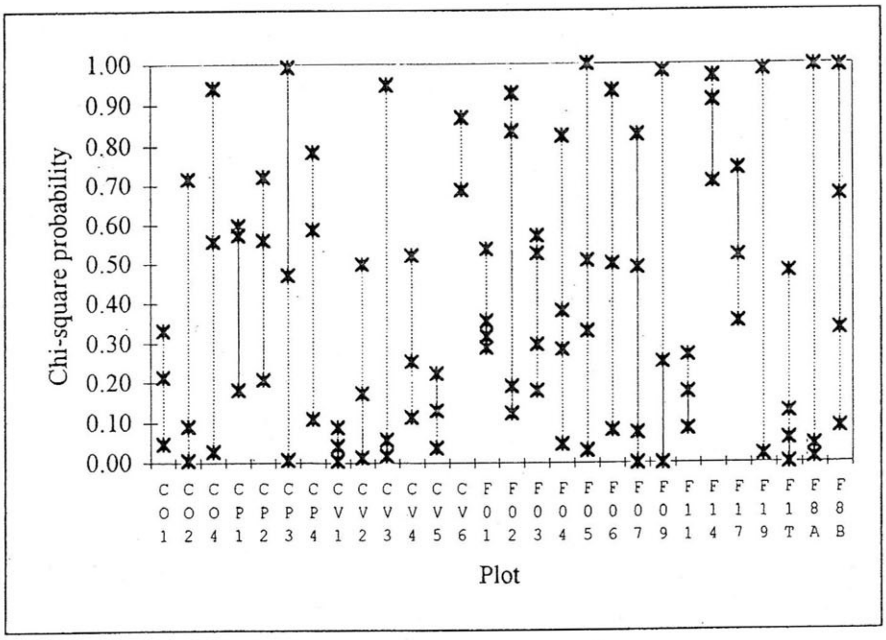


Figure 7. Estimation of remeasurement diameter distribution: chi-spuare probability comparing observed and predicted distribution.

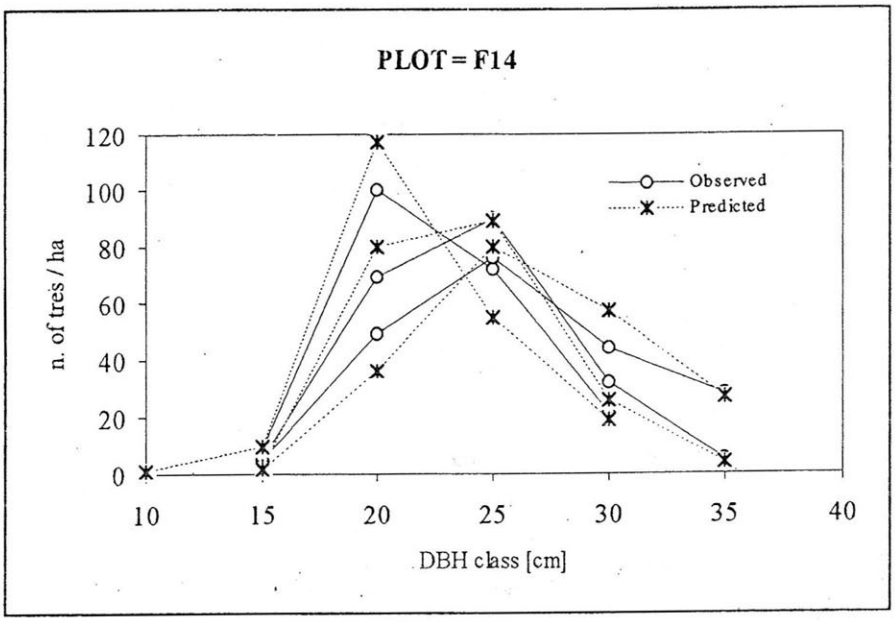


Figure 8. Estimation of remeasurement diameter distribution: worst case

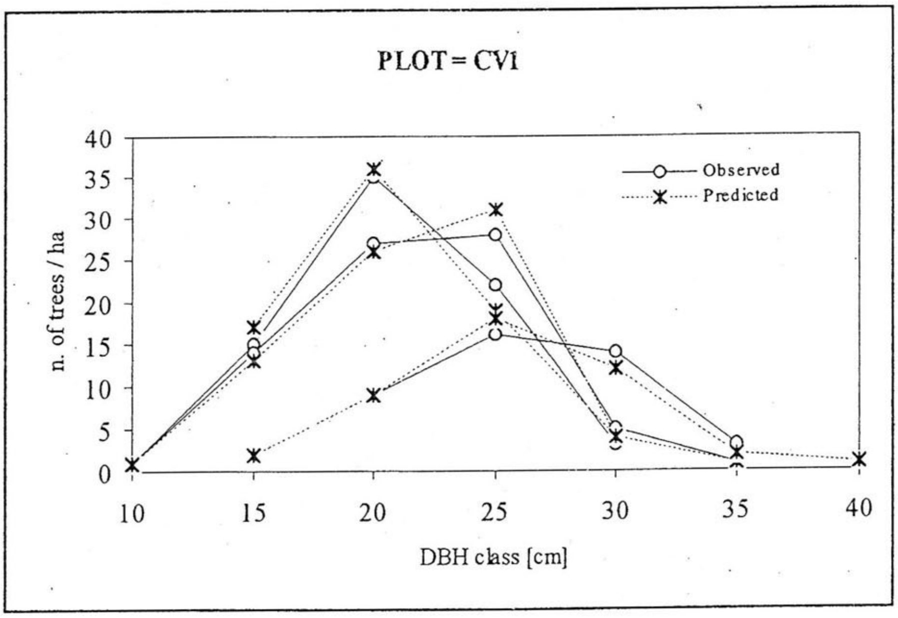


Figure 9. Estimation of remeasurement diameter distribution: best case

Table 1. Geographic and environmental characteristics of the sites where plots are located

Region	*Geographic location*	*ID*	*Latitude*	*Longitude*	*Elevation m a.s.l.*	*Slope %*	*Aspect ° North*	*Rainfall mm/year*	*Temp. year avg°C*
Toscana	Casentino	C	43°51'30''	11°43'	910	10	180	1126	10
Toscana	Casentino	C	43°50'30''	11°42'	960	0	225	1146	10
Toscana	Casentino	C	43°50'30''	11°42'40''	950	7.5	135	1142	10
Toscana	Consuma	F	43°48'10''	11°33'40''	715	0	180	1048	13
Toscana	Consuma	F	43°47'20''	11°37'20	960	7	26	1146	11
Toscana	Consuma	F	43°47'20''	11°36'	965	15	270	1148	11
Toscana	Consuma	F	43°47'25''	11°35'	932	0	210	1135	11
Toscana	Consuma	F	43°48'	11°36'20''	1000	35	260	1162	11
Toscana	Consuma	F	43°48'40''	11°34'	770	0		1070	12
Toscana	Consuma	F	43°47'10''	11°34'30''	870	0	45	1110	12
Toscana	Rincine	R	43°52'	11°38'	770	25	270	1237	11
Puglia	Inv. Cerri	PM	41°44'	15°51'	650	5	23	830	11
Puglia	Cappell.	PM	41°43'55"	15°52'	700	0		830	11
Puglia	Giovann.	PU	41°49'40"	15°58'58"	770	20	45	1200	11
Puglia	Valle Greci	PU	41°50'40"	15°55'38"	550	15	90	1200	11
Basilicata	Piano porc.	BP	40°34'09"	15°48'30"	1020	30	203	880	10
Basilicata	Montagna	BT	40°35'40"	15°42'51"	980	10	157	870	10

Table 2. Site characteristics

```
- Global statistics

          -------------------------------------------------------
          Variable   N     Mean  Std Dev  Minimum  Maximum      CV
          -------------------------------------------------------
          SITEIND   55    26.5      3.1     19.7     32.8     11.6
          PDENS     55  2107.8   1037.8    833.3   4216.6     49.2
          P_YEAR    55  1966.0      4.6   1959.0   1979.0      0.2
          -------------------------------------------------------

                     Pearson Correlation Coefficients
                   Prob > |R| under Ho: Rho=0 / N = 55
                ------------------------------------------
                                   PDENS            P_YEAR
                SITEIND          0.06887          -0.52224
                                  0.6173            0.0001
                PDENS                             -0.33768
                                                    0.0117
                ------------------------------------------


- Distribution by geographic location
----------------------------------------------------------------------
SUBSET
     N Obs  Variable   N     Mean  Std Dev  Minimum  Maximum        CV
----------------------------------------------------------------------
GEO = Toscana - Casentino
        13  SITEIND   13     27.0      1.3     24.6     29.2       4.9
            PDENS     13   3690.8    659.0   2603.1   4216.6      17.9
            P_YEAR    13   1963.0      1.2   1961.0   1964.0       0.1
GEO = Toscana - Consuma
        19  SITEIND   19     28.3      2.1     24.3     31.5       7.5
            PDENS     19   1721.6    592.9    833.3   2500.0      34.4
            P_YEAR    19   1963.2      1.8   1959.0   1965.0       0.1
GEO = Toscana - Rincine
        12  SITEIND   12     23.9      0.3     23.0     24.0       1.4
            PDENS     12   1470.0      0.0   1470.0   1470.0       0.0
            P_YEAR    12   1971.0      0.0   1971.0   1971.0       0.0
GEO = Puglia
         7  SITEIND    7     27.6      4.0     22.3     32.8      14.5
            PDENS      7   1292.9    179.0   1100.0   1600.0      13.8
            P_YEAR     7   1970.6      6.6   1965.0   1979.0       0.3
GEO = Basilicata
         4  SITEIND    4     22.3      5.3     19.7     30.2      23.5
            PDENS      4   2137.5    704.0   1150.0   2800.0      32.9
            P_YEAR     4   1966.5      5.0   1959.0   1969.0       0.3
----------------------------------------------------------------------
```

Table 3. Thinning characteristics (excluding no-thinning cases)

```
- Global statistics

          ---------------------------------------------------------------
          Variable           N        Mean     Std Dev     Minimum     Maximum
          ---------------------------------------------------------------
          T_SBA_P           58     22.8303      7.4821      8.6495     37.6081
          T_DG_RP           58      7.4738      5.3628      2.1603     29.9309
          TNP               58     32.6222      9.5735     16.8224     57.9618
          ---------------------------------------------------------------

                         Pearson Correlation Coefficients/
                            /Prob > |R| under Ho: Rho=0
                                   T_DG_RP                 TNP
                     -------------------------------------------
                     T_SBA_P        0.26484             0.84255
                                     0.0445              0.0001
                     T_DG_RP                            0.73677
                                                         0.0001
                     -------------------------------------------


- Distribution by geographic location

-------------------------------------------------------------------
SUBSET
  Variable           N        Mean     Std Dev  Minimum     Maximum       R(*)
                                                                          p(*)
-------------------------------------------------------------------
GEO = Toscana - Casentino
 T_SBA_P            21     19.8633      9.7976   8.6495     37.6081  0.27759
 T_DG_RP            21      5.9375      1.3228   3.9629      9.1491   0.2231
GEO = Toscana - Consuma
 T_SBA_P            24     24.4437      5.2112  15.2645     31.1326  0.35113
 T_DG_RP            24      9.2199      7.5162   2.5823     29.9309   0.0925
GEO = Toscana - Rincine
 T_SBA_P             9     23.4942      2.6730  20.6972     28.1420 -0.51636
 T_DG_RP             9      4.4782      1.7024   2.1603      6.7822   0.1547
GEO = Puglia
 T_SBA_P             3     24.6893      9.5612  17.0123     35.3991  0.51673
 T_DG_RP             3     11.7367      1.2684  10.3232     12.7754   0.6543
GEO = Basilicata
 T_SBA_P             1     34.8647           .  34.8647     34.8647   .
 T_DG_RP             1     12.0026           .  12.0026     12.0026   .
-------------------------------------------------------------------
                              (*)R = Pearson Correlation Coefficients
                                 p = Prob > |R| under Ho: Rho=0
```

Table 4. Estimation of α = G asymptotic value

```
Stepwise Procedure for Dependent Variable: ln(α)

Step 1   Variable SI*PD Entered      R-square = 0.09952027,  C(p) =  0.38378404

                 DF         Sum of Squares       Mean Square            F  Prob>F

Regression        1             0.42682548        0.42682548         2.43  0.1332
Error            22             3.86200436        0.17554565
Total            23             4.28882985

                 Parameter        Standard           Type II
Variable          Estimate           Error    Sum of Squares            F  Prob>F
INTERCEPT       4.58318590      0.19783837       94.21150812       536.68  0.0001
SI*PD           0.00000641      0.00000411        0.42682548         2.43  0.1332
```

Table 5. Estimation of stand basal area increment

```
Dependent variable: SBAI = G

                            Analysis of Variance
                              Sum of        Mean
   Source            DF       Squares      Square        F Value          Prob>F
   Model              3     832.42840   277.47613       1385.135          0.0001
   Error .          149      29.84831     0.20032
   U Total          152     862.27671

                    Root MSE        0.44758     R-Square          0.9654
                    Dep Mean        2.31094     Adj R-SQ          0.9647
                    C.V.           19.36774

NOTE: The NOINT option changes the definition of the R-Square statistic to:
        1 - (Residual Sum of Squares/Uncorrected Total Sum of Squares).

                               Parameter Estimates
                          Parameter        Standard    T for H0:
      Variable  DF         Estimate           Error    Parameter=0     Prob > |T|
      KT*SI^-1   1        27.882825        0.683831         40.774         0.0001
      KT*TNP^.5  1        -0.009455        0.001249         -7.569         0.0001
      KT*N^2/1e6 1        -0.073365        0.012541         -5.850         0.0001
Where:
   KT  =  (4.65 + 8.14e-6 * SI*PD - log(G))*G/A
```

Table 6. Estimation of basal area increment distribution

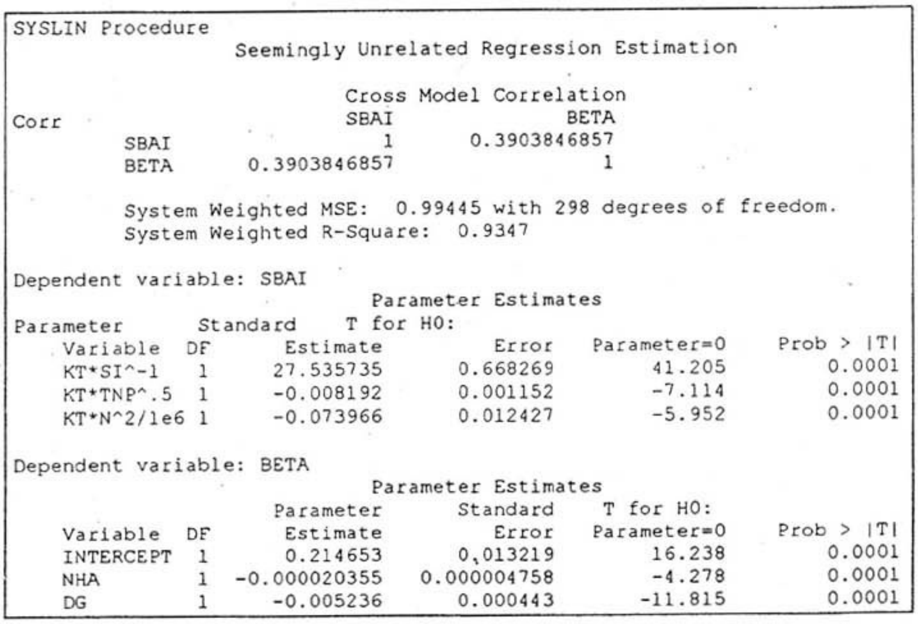
```
Dependent variable: β

                              Analysis of Variance
                                 Sum of         Mean
   Source             DF        Squares       Square        F Value          Prob>F
   Model               2        0.05340      0.02670         70.267          0.0001
   Error             149        0.05662      0.00038
   C Total           151        0.11002

                 Root MSE        0.01949      R-Square          0.4854
                 Dep Mean        0.07926      Adj R-SQ          0.4785
                 C.V.           24.59328

                                Parameter Estimates
                          Parameter         Standard     T for H0:
     Variable   DF         Estimate            Error     Parameter=0     Prob > |T|
     INTERCEPT   1         0.221845         0.014069          15.768         0.0001
     NHA         1     -0.000021521      0.000004859          -4.429         0.0001
     DG          1        -0.005494         0.000479         -11.469         0.0001
```

Table 7. Simultaneous calibration of γ and β coeffocients



```
SYSLIN Procedure
                  Seemingly Unrelated Regression Estimation

                           Cross Model Correlation
Corr                       SBAI              BETA
         SBAI                 1      0.3903846857
         BETA      0.3903846857                 1

         System Weighted MSE:  0.99445 with 298 degrees of freedom.
         System Weighted R-Square:  0.9347

Dependent variable: SBAI
                             Parameter Estimates
Parameter       Standard    T for H0:
    Variable  DF       Estimate           Error  Parameter=0     Prob > |T|
    KT*SI^-1   1      27.535735        0.668269       41.205         0.0001
    KT*TNP^.5  1      -0.008192        0.001152       -7.114         0.0001
    KT*N^2/1e6 1      -0.073966        0.012427       -5.952         0.0001

Dependent variable: BETA
                             Parameter Estimates
                      Parameter        Standard    T for H0:
    Variable  DF       Estimate           Error  Parameter=0     Prob > |T|
    INTERCEPT  1       0.214653        0,013219       16.238         0.0001
    NHA        1   -0.000020355     0.000004758       -4.278         0.0001
    DG         1      -0.005236        0.000443      -11.815         0.0001
```