Variance-Constrained Dissipative Observer-Based Control for A Class of Nonlinear Stochastic Systems with Degraded Measurements

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Abstract

This paper is concerned with the variance-constrained dissipative control problem for a class of stochastic nonlinear systems with multiple degraded measurements, where the degraded probability for each sensor is governed by an individual random variable satisfying a certain probabilistic distribution over a given interval. The purpose of the problem is to design an observer-based controller such that, for all possible degraded measurements, the closed-loop system is exponentially mean-square stable and strictly dissipative, while the individual steady-state variance is not more than the pre-specified upper bound constraints. A general framework is established so that the required exponential mean-square stability, dissipativity as well as the variance constraints can be easily enforced. A sufficient condition is given for the solvability of the addressed multiobjective control problem, and the desired observer and controller gains are characterized in terms of the solution to a convex optimization problem that can be easily solved by using the semi-definite programming method. Finally, a numerical example is presented to show the effectiveness and applicability of the proposed algorithm.

Keywords

Nonlinear systems; Stochastic systems; Dissipative control; Variance-constrained control; Degraded measurements.

I. Introduction

In stochastic control problems, it is often the case that the performance requirements of engineering systems are expressed as upper bounds on the steady-state variances, see e.g. [14,24]. Current control design techniques, such as LQG and H_∞ design, do not seem to give a direct solution to this kind of design problem since they lack a convenient avenue for imposing design objectives stated in terms of upper bounds on the variance values. For example, the LQG controllers minimize a linear quadratic performance index without guaranteeing the variance constraints with respect to individual system states. The covariance control theory [14] developed in late 80's has provided a more direct methodology for achieving the individual variance constraints than the LQG control theory. Covariance control theory is capable of dealing with variance-constrained control problems and, at the same time, considering other multiple performance objectives due to its design flexibility. Therefore, the idea of covariance control theory has been widely applied in solving multiobjective control problems as well as filtering problems, see $[2, 16, 25, 33, 35]$ for instance.

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In recent years, there has been an increasing research interest on controller/filter design problems with variance constraints and several approaches have been developed. For example, in [16], a Riccati-equation method has been proposed to solve the filtering problem for linear time-varying stochastic systems with pre-specified error variance bounds. In [2], the sliding mode control (SMC) method has been applied to solve the robust controller design problems for linear parameter perturbed systems, since SMC has certain robustness to matched disturbances or parameter perturbations. It should be pointed out that most available literature has been concerned with *linear* stochastic systems [25, 26] with the linear matrix inequality (LMI) approach. As far as *nonlinear* stochastic systems are concerned, unfortunately, the relevant results have been very few mainly due to the complexity in dealing with *the existence and expression of the steady-state variance for nonlinear stochastic systems*.

In the context of multiobjective stochastic control, there have appeared several results in the literature. For example, a multiobjective filter has been designed in [21] for systems with Lipschitz-type nonlinearity, but the variance bounds cannot be pre-specified. An LMI approach has been proposed in [33] to cope with robust control and filtering problems for a class of stochastic nonlinear systems by achieving H_2 performance indices. In [24], for a special class of nonlinear stochastic systems, namely, systems with multiplicative noises (also called bilinear systems or systems with state/control dependent noises), a state feedback controller has been put forwarded in a unified LMI framework in order to ensure that the multiple objectives (including the variance constraint) are simultaneously satisfied.

On another research front, the theory of dissipative systems, which plays an important role in system and control areas, has been attracting a great deal of research interests and many results have been reported so far, see [4, 12, 18, 22, 30, 31]. Originated in [30], the dissipative theory serves as a powerful tool in characterizing important system behaviors such as stability and passivity, and has close connections with bounded real lemma, passitivity lemma and circle criterion. It is worth mentioning that, due to its simplicity in analysis and convenience in simulation, the LMI method has gained particular attention in dissipative control problems. For example, in [22, 31], an LMI method was used to design the state feedback controller ensuring both the asymptotic stability and strictly quadratic dissipativity. For singular systems, [4] established a unified LMI framework to satisfy admissibility and dissipativity of the system simultaneously. In [18], the dissipative control problem was solved for time-delay systems.

In engineering systems, it is always desirable for the controlled systems to achieve multiple performance indices such as stability, robustness, dissipativity and steady-state variance. Therefore, a seemingly natural research problem arises here: can we handle the robust variance-constrained dissipative control problem for uncertain stochastic systems with general nonlinearities? To the best of the author's knowledge, such a multiobjective research problem has not received any attention despite its theoretical significance, and this constitutes the main motivation of our current investigation. The main features/contributions of this paper can be described as follows. (1) For the first time, the stochastic dissipativity is combined with the steady-state variance in a stochastic control problem, which serve as two important performance requirements for a class of uncertain nonlinear stochastic systems. The trade-off between these two requirements are investigated in detail by means of a convex optimization approach. (2) Compared with existing literature that uses measurement outputs for dissipative control design, this paper considers the multiple packet-dropout model which describes the phenomenon of measurement degraded occurred frequently in practical applications (for example, the target tacking problem). As such, the system model studied reflects practical engineering systems in a more comprehensive and realistic way.

The rest of the paper is organized as follows. In Section II, the multiobjective control problem is formulated for a class of nonlinear stochastic systems. In Section III, the stability, dissipativity and steady state variance are analyzed one by one. In Section IV, an LMI algorithm is developed for controller design. In Section V, an illustrative example is presented to show the effectiveness of the proposed algorithm. In Section VI, concluding remarks are provided.

Notation The following notation will be used in this paper. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ matrices, and $\mathbb{I}^+$ denotes the set of nonnegative integers. The notation $X \geq Y$ (respectively $X > Y$) where X and Y are real symmetric matrices, means that $X - Y$ is positive semi-definite (respectively positive definite). $\mathbb{E}\{x\}$ stands for the expectation of stochastic variable x and $\mathbb{E}\{x|y\}$ for the expectation of x conditional on y. The superscript "T" denotes the transpose. $\rho(A)$ means the spectral radius of matrix A, while $\mathrm{tr}(A)$ the trace of matrix A. $\mathrm{vec}(A)$ represents stack or vector operator (operator which forms a vector out of the columns of a matrix) applied on A. $\otimes$ stands for the Kronecker product of matrices. $\mathrm{diag}\{F_1, F_2, \ldots, F_m\}$ denotes a block diagonal matrix whose diagonal blocks are given by F_1, F_2, $\ldots$, F_m.

II. Problem Formulation

Consider the following discrete-time nonlinear stochastic system:

$$\begin{cases} x(k+1) = Ax(k) + B_1u(k) + f(x(k)) + D_1\omega(k) \\ \quad\;\; z(k) = Lx(k) + B_2u(k) + D_2\omega(k) \end{cases} \tag{1}$$

with the measurement equation

$$\begin{aligned} y(k) &= \Theta Cx(k) + g(x(k)) + D_3\omega(k) \\ &= \sum_{j=1}^{m} \theta_j C_j x(k) + g(x(k)) + D_3\omega(k) \end{aligned} \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the system state, $u(k) \in \mathbb{R}^p$ is the control input, $z(k) \in \mathbb{R}^r$ is the controlled output, $y(k) \in \mathbb{R}^m$ is the measured output vector, $\omega(k) \in \mathbb{R}^r$ is a zero mean Gaussian white noise sequence with covariance $W > 0$. A, B_1, B_2, D_1, D_2, D_3, C, L are known real constant matrices with appropriate dimensions.

The stochastic matrix Θ describes the phenomenon of multiple measurement degraded in the process of information retrieval from the sensor output. Θ is defined as

$$\Theta = \mathrm{diag}\{\theta_1, \theta_2, \ldots, \theta_m\} \tag{3}$$

with $\theta_j (j = 1, 2, \ldots, m)$ being m independent random variables which are also independent from $\omega(k)$. It is assumed that θ_j has the probabilistic density function $p_i(s)$ on the interval $[0, 1]$ with mathematical expectation $\bar{\theta}_j$ and variance σ_j^2. $C_j \triangleq \mathrm{diag}\{\underbrace{0, \ldots, 0}_{j-1}, 1, \underbrace{0, \ldots, 0}_{m-j}\}C$.

Remark 1: It has been illustrated in [29] that the description of measurements degraded phenomenon given in (3) is much more general than those in most previous literature where the data missing phenomenon is simply modeled by a single Bernoulli sequence [26, 34]. In such a model, when $\theta_j = 1$, it means that the jth sensor is in good condition, otherwise there might be partial or complete sensor failure. To be specific, when $\theta_{jk} = 0$, the sensor is totally out of work and the measurements are completely missing, while $0 < \theta_{jk} < 1$ means that we could only measure the output signals with reduced gains, namely, degraded measurements. In this sense, the model (1)–(2) offers a comprehensive means to reflect systems complexity such as nonlinearities, stochasticity and data degraded from multiple sensors.

The nonlinear stochastic functions $f(x(k))$ and $g(x(k))$ are assumed to have the following first moments for all $x(k)$:

$$\mathbb{E}\left\{\left[\begin{array}{c} f(x(k)) \\ g(x(k)) \end{array}\right]\middle| x(k)\right\} = 0, \tag{4}$$

with the covariance given by

$$\mathbb{E}\left\{\left[\begin{array}{c} f(x(k)) \\ g(x(k)) \end{array}\right]\left[\begin{array}{cc} f^{\mathrm{T}}(x(j)) & g^{\mathrm{T}}(x(j)) \end{array}\right]\middle| x(k)\right\} = 0, \qquad k \neq j \tag{5}$$

and

$$\mathbb{E}\left\{\left[\begin{array}{c} f(x(k)) \\ g(x(k)) \end{array}\right]\left[\begin{array}{cc} f^{\mathrm{T}}(x(k)) & g^{\mathrm{T}}(x(k)) \end{array}\right]\middle| x(k)\right\} = \sum_{i=1}^{q} \Pi_i x^{\mathrm{T}}(k)\Gamma_i x(k), \tag{6}$$

where Π_i and Γ_i $(i = 1, 2, \dots, q)$ are known positive-definite matrices with appropriate dimensions.

Remark 2: As discussed in [33, 36], the stochastic nonlinearity described by (4)–(6) accounts for several classes of well-studied nonlinear systems, such as the system with state-dependent multiplicative noises and the system whose state has power dependent on the sector-bounded (or sign) of the nonlinear state function of the state.

For system (1), consider the following observer-based controller:

$$\hat{x}(k+1) = A_f\hat{x}(k) + H_f y(k) \tag{7}$$

$$u(k) = K\hat{x}(k) \tag{8}$$

where A_f and H_f (observer parameters) and K (controller parameter) are to be determined.

From (1), (7) and (8), we obtain the following augmented system:

$$\begin{cases} \xi(k+1) = \bar{A}\xi(k) + \bar{H}h(x(k)) + \bar{D}\omega(k) \\ z(k) = \bar{L}\xi(k) + D_2\omega(k) \end{cases} \tag{9}$$

where

$$\xi(k) = \left[\begin{array}{c} x(k) \\ \hat{x}(k) \end{array}\right], \quad \bar{A} = \left[\begin{array}{cc} A & B_1K \\ H_f\Theta C & A_f \end{array}\right], \quad \bar{H} = \left[\begin{array}{cc} I & 0 \\ 0 & H_f \end{array}\right], \quad \bar{D} = \left[\begin{array}{c} D_1 \\ H_fD_3 \end{array}\right],$$

$$\bar{L} = \left[\begin{array}{cc} L & B_2K \end{array}\right], \quad h(x(k)) = \left[\begin{array}{c} f(x(k)) \\ g(x(k)) \end{array}\right].$$

Definition 1: [24] System (9) is said to be exponentially mean square stable if, with $\omega(k) = 0$, there exist constants $\rho \geqslant 1$ and $\tau \in (0, 1)$ such that

$$\mathbb{E}\{\|\xi(k)\|^2\} \leqslant \rho\tau^k\mathbb{E}\{\|\xi(0)\|^2\}, \quad \forall\xi(0) \in \mathbb{R}^{2n}, \quad k \in \mathbb{I}^+ \tag{10}$$

for all possible degraded measurements.

We are now in a position to introduce the performance of dissipativity. Let the energy supply function of system (1) be defined by

$$G(\omega, z, T) = \langle z, Qz\rangle_T + 2\langle z, S\omega\rangle_T + \langle \omega, R\omega\rangle_T, \quad \forall T \geqslant 0 \tag{11}$$

where Q, S and R are real matrices with Q, R symmetric, $T \geqslant 0$ is an integer and $\langle a, b\rangle_T \triangleq \sum_{k=0}^{T} a^{\mathrm{T}}(k)b(k)$. Without loss of generality, we assume that $Q < 0$ and denote $\bar{Q} = \sqrt{-Q}$.

Definition 2: [4] Closed-loop system (9) is said to be strictly (Q, R, S) dissipative if, for some scalar $\gamma > 0$, the following inequality

$$G(\omega, z, T) \geqslant \gamma\langle\omega, \omega\rangle_T, \quad \forall T \geqslant 0 \tag{12}$$

holds under zero initial condition.

If system (1) is asymptotically stable, its steady-state covariance is defined as follows:

$$\bar{X} \triangleq \lim_{k\to\infty} \mathbb{E}\{x(k)x^{\mathrm{T}}(k)\}. \tag{13}$$

Assumption 1: The matrices Π_i and Γ_i in (6) have the following form:

$$\Pi_i = \bar{\pi}_i\bar{\pi}_i^{\mathrm{T}} = \begin{bmatrix} \pi_{1i} \\ \pi_{2i} \end{bmatrix}\begin{bmatrix} \pi_{1i} \\ \pi_{2i} \end{bmatrix}^{\mathrm{T}}, \quad \Gamma_i = \eta_i\eta_i^{\mathrm{T}} \tag{14}$$

where π_{i1}, π_{2i} and η_i $(i = 1, 2, \ldots, i)$ are known vectors with appropriate dimensions.

This paper aims to determine the observer parameters A_f, H_f and the feedback controller parameter K for the system (1) such that, for all possible degraded measurements , the following three objectives are achieved simultaneously:

(R1) Augmented system (9) is exponentially mean square stable;

(R2) Augmented system (9) is strictly (Q, S, R) dissipative;

(R3) The steady-state variance for each individual state of system (1) satisfies

$$\bar{X}_s \leqslant \delta_s^2, \quad s = 1, 2, \ldots, n \tag{15}$$

where $\bar{X}_s$ stands for the steady-state variance for the sth state, and δ_s^2 denotes the pre-specified steady-state variance constraint on the sth state.

III. Stability, Dissipativity and Variance Analysis

Before giving our preliminary results, let us introduce some useful lemmas. For presentation convenience, we denote

$$\hat{A} \triangleq \begin{bmatrix} A & B_1K \\ H_f\bar{\Theta}C & A_f \end{bmatrix}, \quad \breve{A} \triangleq \begin{bmatrix} 0 & 0 \\ H_f(\Theta - \bar{\Theta})C & 0 \end{bmatrix},$$

$$\tilde{A}_i \triangleq \begin{bmatrix} 0 & 0 \\ H_fC_i & 0 \end{bmatrix}, \quad \bar{\Gamma}_i \triangleq \begin{bmatrix} \eta_i \\ 0 \end{bmatrix}\begin{bmatrix} \eta_i \\ 0 \end{bmatrix}^{\mathrm{T}} \triangleq \bar{\eta}_i\bar{\eta}_i^{\mathrm{T}}, \quad \bar{\Theta} \triangleq \mathbb{E}\{\Theta\}.$$

Lemma 1: [23] Let $V(\xi(k)) = \xi^{\mathrm{T}}(k)X\xi(k)$ be a Lyapunov functional where $X > 0$. If there exist real scalars λ, $\mu > 0$, $\nu > 0$ and $0 < \psi < 1$ such that both

$$\mu\|\xi(k)\|^2 \leqslant V(\xi(k)) \leqslant \nu\|\xi(k)\|^2 \tag{16}$$

and

$$\mathbb{E}\{V(\xi(k+1))|\xi(k)\} - V(\xi(k)) \leqslant \lambda - \psi V(\xi(k)) \tag{17}$$

hold, then the process $\xi(k)$ satisfies

$$\mathbb{E}\{\|\xi(k)\|^2\} \leqslant \frac{\nu}{\mu}\|\xi(0)\|^2(1-\psi)^k + \frac{\lambda}{\mu\psi}. \tag{18}$$

Lemma 2: (Schur Complement Equivalence) Given constant matrices $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$ where $\mathcal{S}_1 = \mathcal{S}_1^{\mathrm{T}}$ and $0 < \mathcal{S}_2 = \mathcal{S}_2^{\mathrm{T}}$, then $\mathcal{S}_1 + \mathcal{S}_3^{\mathrm{T}}\mathcal{S}_2^{-1}\mathcal{S}_3 < 0$ if and only if

$$\begin{bmatrix} \mathcal{S}_1 & \mathcal{S}_3^{\mathrm{T}} \\ \mathcal{S}_3 & -\mathcal{S}_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\mathcal{S}_2 & \mathcal{S}_3 \\ \mathcal{S}_3^{\mathrm{T}} & \mathcal{S}_1 \end{bmatrix} < 0. \tag{19}$$

Lemma 3: Given the parameters A_f, H_f and K. The following statements are equivalent:

1)

$$\rho\left(\hat{A}^{\mathrm{T}} \otimes \hat{A}^{\mathrm{T}} + \sum_{j=1}^{m} \sigma_j^2 \tilde{A}_j^{\mathrm{T}} \otimes \tilde{A}_j^{\mathrm{T}} + \sum_{i=1}^{q} \mathrm{vec}(\bar{\Gamma}_i)\mathrm{vec}^{\mathrm{T}}(\bar{H}\Pi_i\bar{H}^{\mathrm{T}})\right) < 1 \tag{20}$$

or

$$\rho\left(\hat{A} \otimes \hat{A} + \sum_{j=1}^{m} \sigma_j^2 \tilde{A}_j \otimes \tilde{A}_j + \sum_{i=1}^{q} \mathrm{vec}(\bar{H}\Pi_i\bar{H}^{\mathrm{T}})\mathrm{vec}^{\mathrm{T}}(\bar{\Gamma}_i)\right) < 1 \tag{21}$$

2) There exists a positive definite matrix $X > 0$ such that

$$\hat{A}^{\mathrm{T}} X \hat{A} + \sum_{j=1}^{m} \sigma_j^2 \tilde{A}_j^{\mathrm{T}} X \tilde{A}_j + \sum_{i=1}^{q} \bar{\Gamma}_i \mathrm{tr}[X\bar{H}\Pi_i\bar{H}^{\mathrm{T}}] - X < 0 \tag{22}$$

3) There exists a positive definite matrix $Y > 0$ such that

$$\hat{A} Y \hat{A}^{\mathrm{T}} + \sum_{j=1}^{m} \sigma_j^2 \tilde{A}_j Y \tilde{A}_j^{\mathrm{T}} + \sum_{i=1}^{q} \bar{H}\Pi_i\bar{H}^{\mathrm{T}} \mathrm{tr}[\bar{\Gamma}_i Y] - Y < 0 \tag{23}$$

4) System (9) is exponentially mean square stable.

The proof of Lemma 2 can be carried out along the similar line of that of Lemma 2 of [19] and is therefore omitted. The main difference between this lemma and Theorem 1 of [36] is that the state matrix of system (9) involves the stochastic variable Θ which describes probabilistic degraded measurements.

The following theorem gives a sufficient condition for the exponential mean-square stability as well as strictly (Q, S, R) dissipativity of the system (9).

Theorem 1: Given the parameters A_f, H_f, K, symmetric matrices Q, R and a matrix S. The closed-loop system (9) is exponentially mean-square stable and strictly (Q, S, R) dissipative if there exists a matrix $X > 0$ such that the following matrix inequality holds:

$$\Omega \triangleq \begin{bmatrix} \Omega_{11} & \hat{A}^{\mathrm{T}} X \bar{D} - \bar{L}^{\mathrm{T}} Q D_2 - \bar{L}^{\mathrm{T}} S \\ * & \bar{D}^{\mathrm{T}} X \bar{D} - D_2^{\mathrm{T}} Q D_2 - D_2^{\mathrm{T}} S - S^{\mathrm{T}} D_2 - R \end{bmatrix} < 0 \tag{24}$$

where

$$\Omega_{11} \triangleq \hat{A}^{\mathrm{T}} X \hat{A} + \sum_{j=1}^{m} \sigma_j^2 \tilde{A}_j^{\mathrm{T}} X \tilde{A}_j + \sum_{i=1}^{q} \bar{\Gamma}_i \mathrm{tr}[X\bar{H}\Pi_i\bar{H}^{\mathrm{T}}] - X - \bar{L}^{\mathrm{T}} Q \bar{L}.$$

Proof: First, it follows from (24) that

$$\hat{A}^{\mathrm{T}} X \hat{A} + \sum_{j=1}^{m} \sigma_j^2 \tilde{A}_j^{\mathrm{T}} X \tilde{A}_j + \sum_{i=1}^{q} \bar{\Gamma}_i \mathrm{tr}[X\bar{H}\Pi_i\bar{H}^{\mathrm{T}}] - X < \bar{L}^{\mathrm{T}} Q \bar{L} < 0. \tag{25}$$

Therefore, from Lemma 3, system (9) is exponentially mean-square stable.

When $\omega(k) \neq 0$, we obtain from (24) that

$$\begin{aligned}
&\mathbb{E}\left\{V(\xi(k+1))\big|\xi(k)\right\} - V(\xi(k)) - z^{\mathrm{T}}(k)Qz(k) - 2z^{\mathrm{T}}(k)S\omega(k) - \omega^{\mathrm{T}}(k)R\omega(k) \\
=&\mathbb{E}\left\{(\bar{A}\xi(k) + \bar{H}h(x(k)) + \bar{D}\omega(k))^{\mathrm{T}}X(\bar{A}\xi(k) + \bar{H}h(x(k)) + \bar{D}\omega(k))|\xi(k)\right\} - \xi^{\mathrm{T}}(k)X\xi(k) \\
&-(\bar{L}\xi(k) + D_2\omega(k))^{\mathrm{T}}Q(\bar{L}\xi(k) + D_2\omega(k)) - 2(\bar{L}\xi(k) + D_2\omega(k))^{\mathrm{T}}S\omega(k) - \omega^{\mathrm{T}}(k)R\omega(k) \\
=&\begin{bmatrix} \xi(k) \\ \omega(k) \end{bmatrix}^{\mathrm{T}} \Omega \begin{bmatrix} \xi(k) \\ \omega(k) \end{bmatrix} < 0.
\end{aligned} \tag{26}$$

Obviously, there always exists a sufficiently small positive scalar $\gamma > 0$ such that

$$\Omega + \mathrm{diag}\{0, \gamma I\} < 0 \tag{27}$$

and therefore

$$\mathbb{E}\left\{V(\xi(k+1))\big|\xi(k)\right\} - V(\xi(k)) + \gamma\omega^{\mathrm{T}}(k)\omega(k) < z^{\mathrm{T}}(k)Qz(k) + 2z^{\mathrm{T}}(k)S\omega(k) + \omega^{\mathrm{T}}(k)R\omega(k). \tag{28}$$

Summing (28) from 0 to T with respect to k on both sides, and noticing that $V(\xi(T+1)) > 0$ and $V(\xi(0)) = 0$, it can be obtained that

$$G(\omega, z, T) \geqslant \gamma\langle\omega, \omega\rangle_T \tag{29}$$

which implies that the system (9) is strictly (Q, S, R) dissipative. The proof is complete. ■

Now, let us proceed to analyze the steady-state covariance of the system (9). Define the state covariance of system (9) as

$$Y(k) \triangleq \mathbb{E}\left\{\xi(k)\xi^{\mathrm{T}}(k)\right\}. \tag{30}$$

The evolution of $Y(k)$ can be derived as follows:

$$Y(k+1) = \hat{A}Y(k)\hat{A}^{\mathrm{T}} + \sum_{j=1}^{m}\sigma_j^2\tilde{A}_jY(k)\tilde{A}_j^{\mathrm{T}} + \sum_{i=1}^{q}\bar{H}\Pi_i\bar{H}^{\mathrm{T}}\mathrm{tr}[\bar{\Gamma}_iY(k)] + \bar{D}W\bar{D}^{\mathrm{T}}. \tag{31}$$

Furthermore, define the steady-state covariance as

$$\bar{Y} \triangleq \lim_{k\to\infty} Y(k). \tag{32}$$

The following theorem presents a sufficient condition that guarantees the exponentially mean-square stability of system (9) and, at the same time, gives an upper bound of the steady-state covariance.

Theorem 2: Given the parameters A_f, H_f and K. If there exists a matrix $Y > 0$ such that

$$\hat{A}Y\hat{A}^{\mathrm{T}} + \sum_{j=1}^{m}\sigma_j^2\tilde{A}_jY\tilde{A}_j^{\mathrm{T}} + \sum_{i=1}^{q}\bar{H}\Pi_i\bar{H}^{\mathrm{T}}\mathrm{tr}(\bar{\Gamma}_iY) - Y + \bar{D}W\bar{D}^{\mathrm{T}} < 0, \tag{33}$$

then system (9) is exponentially mean square stable. Moreover, the steady-state covariance defined in (32) exists and satisfies $\bar{Y} \leqslant Y$.

Proof: First of all, matrix inequality (33) indicates that

$$\hat{A}Y\hat{A}^{\mathrm{T}} + \sum_{j=1}^{m}\sigma_j^2\tilde{A}_jY\tilde{A}_j^{\mathrm{T}} + \sum_{i=1}^{q}\bar{H}\Pi_i\bar{H}^{\mathrm{T}}\mathrm{tr}(\bar{\Gamma}_iY) - Y < -\bar{D}W\bar{D}^{\mathrm{T}} < 0 \tag{34}$$

and therefore it follows from Lemma 3 that system (9) is exponentially mean-square stable. Rewrite equation (31) into the following form:

$$\mathrm{vec}(Y(k+1)) = \left(\hat{A}\otimes\hat{A}+\sum_{j=1}^{m}\sigma_j^2\tilde{A}_j\otimes\tilde{A}_j+\sum_{i=1}^{q}\mathrm{vec}(\bar{H}\Pi_i\bar{H}^{\mathrm{T}})\mathrm{vec}^{\mathrm{T}}(\bar{\Gamma}_i)\right)\mathrm{st}(Y(k))+\mathrm{st}(\bar{D}W\bar{D}^{\mathrm{T}}). \tag{35}$$

From Lemma 3, the exponentially mean-square stability of system (9) ensures that the inequality (21) holds, which implies the convergence of the covariance $Y(k)$ to the constant matrix $\bar{Y}$ when $k\to\infty$, that is

$$-\bar{Y}+\hat{A}\bar{Y}\hat{A}^{\mathrm{T}}+\sum_{j=1}^{m}\sigma_j^2\tilde{A}_j\bar{Y}\tilde{A}_j^{\mathrm{T}}+\sum_{i=1}^{q}\bar{H}\Pi_i\bar{H}^{\mathrm{T}}\mathrm{tr}(\bar{\Gamma}_i\bar{Y})+\bar{D}W\bar{D}^{\mathrm{T}}=0. \tag{36}$$

Subtracting (36) from (33), we obtain

$$-(Y-\bar{Y})+\hat{A}(Y-\bar{Y})\hat{A}^{\mathrm{T}}+\sum_{j=1}^{m}\sigma_j^2\tilde{A}_j(Y-\bar{Y})\tilde{A}_j^{\mathrm{T}}+\sum_{i=1}^{q}\bar{H}\Pi_i\bar{H}^{\mathrm{T}}\mathrm{tr}[\bar{\Gamma}_i(Y-\bar{Y})]<0. \tag{37}$$

In the following stage, we need to prove that $\tilde{Y}\triangleq Y-\bar{Y}\geqslant 0$. For this purpose, let us first prove the fact that if system (9) is exponentially mean square stable and there exists a symmetric matrix $\tilde{X}$ such that

$$\hat{A}^{\mathrm{T}}\tilde{X}\hat{A}+\sum_{j=1}^{m}\sigma_j^2\tilde{A}_j^{\mathrm{T}}\tilde{X}\tilde{A}_j+\sum_{i=1}^{q}\bar{\Gamma}_i\mathrm{tr}(\tilde{X}\bar{H}\Pi_i\bar{H}^{\mathrm{T}})-\tilde{X}<0, \tag{38}$$

then $\tilde{X}\geqslant 0$. In fact, if (38) holds, then there always exists a matrix $\Xi>0$ satisfying

$$\hat{A}^{\mathrm{T}}\tilde{X}\hat{A}+\sum_{j=1}^{m}\sigma_j^2\tilde{A}_j^{\mathrm{T}}\tilde{X}\tilde{A}_j+\sum_{i=1}^{q}\bar{\Gamma}_i\mathrm{tr}(\tilde{X}\bar{H}\Pi_i\bar{H}^{\mathrm{T}})-\tilde{X}=-\Xi. \tag{39}$$

Using the functional $V(\xi(k))=\xi^{\mathrm{T}}(k)\tilde{X}\xi(k)$ for (9), we obtain

$$\begin{aligned}&\mathbb{E}\left\{V(\xi(k+1))\middle|\xi(k)\right\}-V(\xi(k))\\ =&\xi^{\mathrm{T}}(k)\left[\hat{A}^{\mathrm{T}}\tilde{X}\hat{A}+\sum_{j=1}^{m}\sigma_j^2\tilde{A}_j^{\mathrm{T}}\tilde{X}\tilde{A}_j+\sum_{i=1}^{q}\bar{\Gamma}_i\mathrm{tr}(\tilde{X}\bar{H}\Pi_i\bar{H}^{\mathrm{T}})-\tilde{X}\right]\xi(k)=-\xi^{\mathrm{T}}(k)\Xi\xi^{\mathrm{T}}(k).\end{aligned} \tag{40}$$

Taking sum on both sides of (40) with respect to k from 0 to ∞ results in

$$\lim_{n\to\infty}\mathbb{E}\left[\xi^{\mathrm{T}}(n)\tilde{X}\xi(n)\right]-\xi^{\mathrm{T}}(0)\tilde{X}\xi(0)=-\lim_{n\to\infty}\sum_{k=0}^{n}\xi^{\mathrm{T}}(k)\Xi\xi(k). \tag{41}$$

Since system (9) is exponentially mean square stable, we have

$$\lim_{n\to\infty}\mathbb{E}(\xi^{\mathrm{T}}(n)\tilde{X}\xi(n))\leqslant\|\tilde{X}\|\lim_{n\to\infty}\xi^{\mathrm{T}}(n)\xi(n)=0. \tag{42}$$

Therefore, for any nonzero initial state $\xi(0)$, it can be deduced from (41) that

$$\xi^{\mathrm{T}}(0)\tilde{X}\xi(0)=\lim_{n\to\infty}\sum_{k=0}^{n}\xi^{\mathrm{T}}(k)\Xi\xi(k)\geqslant 0, \tag{43}$$

which means $\tilde{X}\geqslant 0$.

Now, let us construct an auxiliary system as follows:

$$\bar{\xi}(k+1) = \bar{A}^{\mathrm{T}}\bar{\xi}(k) + \bar{h}(\bar{\xi}(k)) \tag{44}$$

where $\bar{h}(\bar{\xi}(k))$ satisfies

$$\begin{aligned} \mathbb{E}\left\{\bar{h}(\bar{\xi}(k))|\bar{\xi}(k)\right\} &= 0, \\ \mathbb{E}\left\{\bar{h}(\bar{\xi}(k))\bar{h}^{\mathrm{T}}(\bar{\xi}(j))|\bar{\xi}(k)\right\} &= 0, \qquad k \neq j \\ \mathbb{E}\left\{\bar{h}(\bar{\xi}(k))\bar{h}^{\mathrm{T}}(\bar{\xi}(k))|\bar{\xi}(k)\right\} &= \sum_{i=1}^{q}\bar{\Gamma}_i\bar{\xi}^{\mathrm{T}}(k)\bar{H}\Pi_i\bar{H}^{\mathrm{T}}\bar{\xi}(k). \end{aligned} \tag{45}$$

It follows from the exponentially mean-square stability of system (9) and Lemma 3 that

$$\rho\left(\hat{A}^{\mathrm{T}} \otimes \hat{A}^{\mathrm{T}} + \sum_{j=1}^{m}\sigma_j^2\tilde{A}_j^{\mathrm{T}} \otimes \tilde{A}_j^{\mathrm{T}} + \sum_{i=1}^{q}\mathrm{vec}(\bar{\Gamma}_i)\mathrm{vec}^{\mathrm{T}}(\bar{H}\Pi_i\bar{H}^{\mathrm{T}})\right) < 1. \tag{46}$$

Thus, auxiliary system (44) is also exponentially mean-square stable. Then, from the previously proven fact, if there exists a symmetric matrix $\tilde{Y}$ such that

$$(\hat{A}^{\mathrm{T}})^{\mathrm{T}}\tilde{Y}\hat{A}^{\mathrm{T}} + \sum_{j=1}^{m}\sigma_j^2(\tilde{A}_j^{\mathrm{T}})^{\mathrm{T}}\tilde{Y}\tilde{A}_j^{\mathrm{T}} + \sum_{i=1}^{q}\bar{H}\Pi_i\bar{H}^{\mathrm{T}}\mathrm{tr}(\bar{\Gamma}_i\tilde{Y}) - \tilde{Y} < 0, \tag{47}$$

it can be concluded that $\tilde{Y} \geqslant 0$. The proof is complete. ■

Based on the results we have obtained so far concerning the exponentially mean-square stability, dissipative property as well as steady-state covariance, we are now ready to cope with the addressed multiobjective controller design problem.

IV. Observer-Based Controller Design

In this section, we will first propose a sufficient condition for the solvability of the addressed problem in terms of the feasibility of certain constrained LMIs. Then, an algorithm is presented via cone complementarity linearization method to solve the addressed non-convex optimization problem.

A. Solvability of Multiobjective Control Problem

To begin with, a corollary is given that combines the exponentially mean-square stability, system dissipativity and steady-state covariance constraints.

Corollary 1: Given the parameters A_f, H_f, K, matrices Q, R and S with Q and R being symmetric. Denote $Y_0 \triangleq \mathrm{diag}\{\delta_1^2, \delta_2^2, \ldots, \delta_n^2\}$ where δ_s^2 $(s = 1, 2, \ldots, n)$ are the pre-specified upper bounds of the steady-state variance of each individual state. If there exist a matrix $Y > 0$ and scalars $\alpha_i > 0$, $\beta_i > 0$ satisfying $\alpha_i\beta_i = 1$ $(i = 1, 2, \ldots, q)$ such that

$$\begin{bmatrix} I & 0 \end{bmatrix} Y \begin{bmatrix} I \\ 0 \end{bmatrix} - Y_0 < 0 \tag{48}$$

$$\begin{bmatrix} -\alpha_i^{-1} & \bar{\pi}_i^{\mathrm{T}}\bar{H}^{\mathrm{T}} \\ * & -Y \end{bmatrix} < 0 \tag{49}$$

$$\begin{bmatrix} -\beta_i^{-1} & \bar{\eta}_i^{\mathrm{T}}Y \\ * & -Y \end{bmatrix} < 0 \tag{50}$$

$$\begin{bmatrix} -Y & \hat{A}Y & \bar{\mathcal{A}}\mathcal{Y} & \hat{H} & \bar{D} \\ * & -Y & 0 & 0 & 0 \\ * & * & -\mathcal{Y} & 0 & 0 \\ * & * & * & -\bar{\beta} & 0 \\ * & * & * & * & -W^{-1} \end{bmatrix} < 0 \tag{51}$$

$$\begin{bmatrix} -Y & -Y(\bar{L}^{\mathrm{T}}QD_2 - \bar{L}^{\mathrm{T}}S) & Y\hat{A}^{\mathrm{T}} & Y\mathcal{A}^{\mathrm{T}} & Y\hat{\eta} & Y\bar{L}^{\mathrm{T}}\bar{Q} \\ * & -D_2^{\mathrm{T}}QD_2 - D_2^{\mathrm{T}}S - S^{\mathrm{T}}D_2 - R & \bar{D}^{\mathrm{T}} & 0 & 0 & 0 \\ * & * & -Y & 0 & 0 & 0 \\ * & * & * & -\mathcal{Y} & 0 & 0 \\ * & * & * & * & -\bar{\alpha} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{52}$$

where

$$\mathcal{A}^{\mathrm{T}} = \begin{bmatrix} \sigma_1\tilde{A}_1^{\mathrm{T}} & \sigma_2\tilde{A}_2^{\mathrm{T}} & \cdots & \sigma_m\tilde{A}_m^{\mathrm{T}} \end{bmatrix}, \qquad \bar{\mathcal{A}} = \begin{bmatrix} \sigma_1\tilde{A}_1 & \sigma_2\tilde{A}_2 & \cdots & \sigma_m\tilde{A}_m \end{bmatrix},$$
$$\mathcal{Y} = \mathrm{diag}\{Y, Y, \ldots, Y\}, \quad \hat{H} = \begin{bmatrix} \bar{H}\bar{\pi}_1 & \bar{H}\bar{\pi}_2 & \cdots & \bar{H}\bar{\pi}_q \end{bmatrix}, \quad \hat{\eta} = \begin{bmatrix} \bar{\eta}_1 & \bar{\eta}_2 & \cdots & \bar{\eta}_q \end{bmatrix},$$
$$\bar{\alpha} = \mathrm{diag}\{\alpha_1 I, \alpha_2 I, \ldots, \alpha_q I\}, \qquad \bar{\beta} = \mathrm{diag}\{\beta_1 I, \beta_2 I, \ldots, \beta_q I\},$$

then the system is exponentially mean-square stable and strictly (Q, S, R) dissipative, while its individual steady-state variance is not more than the corresponding pre-specified upper-bound.

Proof: Based on the results we have obtained in Theorem 1 and Theorem 2, it suffices to prove that inequality (50) with (51) guarantee (33) holds, and inequality (49) with (52) imply (24).

First, by Lemma 2, we can see that (51) is equivalent to

$$\hat{A}Y\hat{A}^{\mathrm{T}} + \sum_{j=1}^{m}\sigma_j^2\tilde{A}_jY\tilde{A}_j^{\mathrm{T}} + \sum_{i=1}^{q}\bar{H}\Pi_i\bar{H}^{\mathrm{T}}\beta_i^{-1} - Y + \bar{D}W\bar{D}^{\mathrm{T}} < 0. \tag{53}$$

Then, by Schur Complement Equivalence, it is not difficult to see that inequality (50) indicates $\bar{\eta}_i^{\mathrm{T}}Y\bar{\eta}_i < \beta_i^{-1}$ or, equivalently, $\mathrm{tr}(\Gamma_iY) < \beta_i^{-1}$, and therefore

$$\hat{A}Y\hat{A}^{\mathrm{T}} + \sum_{j=1}^{m}\sigma_j^2\tilde{A}_jY\tilde{A}_j^{\mathrm{T}} + \sum_{i=1}^{q}\bar{H}\Pi_i\bar{H}^{\mathrm{T}}\mathrm{tr}(\bar{\Gamma}_iY) - Y + \bar{D}W\bar{D}^{\mathrm{T}} < 0. \tag{54}$$

It follows directly from Theorem 2 that the system (9) is exponentially mean-square stable and the steady-state covariance defined by (32) exists and satisfies $\bar{Y} < Y$ where $\bar{Y}$ satisfies (36). Moreover, from (48), we can see that the steady-state covariance of the system (1) defined in (13) satisfies

$$\bar{X} = \begin{bmatrix} I & 0 \end{bmatrix}\bar{Y}\begin{bmatrix} I \\ 0 \end{bmatrix} < \begin{bmatrix} I & 0 \end{bmatrix}Y\begin{bmatrix} I \\ 0 \end{bmatrix} < Y_0, \tag{55}$$

which means that the steady-state covariance constraint is also achieved. Similarly, it is not difficult to prove that the exponentially mean-square stability and system dissipativity can be ensured simultaneously by inequality (49) together with (52). The proof is complete. ■

Theorem 3: Given pre-specified steady-state variance upper bounds δ_1^2, δ_2^2 ,..., δ_n^2, matrices Q, S and R with Q and R being symmetric, and scalars $\epsilon > 0$ and $\zeta > 0$. If there exist matrices $M > 0$, $N > 0$, real matrices $\bar{A}_f$, $\bar{H}_f$, $\bar{K}$, and scalars $\alpha_i > 0$, $\beta_i > 0$ $(i = 1, 2, \ldots, q)$ such that

$$\alpha_i\beta_i = 1 \qquad (i = 1, 2, \ldots, q) \tag{56}$$

$$e_s^{\mathrm{T}} M e_s - \delta_s^2 < 0 \qquad (s = 1, 2, \dots, n) \tag{57}$$

$$\begin{bmatrix} -\beta_i & \pi_{1i}^{\mathrm{T}} N + \pi_{2i}^{\mathrm{T}} \bar{H}_f^{\mathrm{T}} & \pi_{1i}^{\mathrm{T}} \\ * & -N & -I \\ * & * & -M \end{bmatrix} < 0 \tag{58}$$

$$\begin{bmatrix} -\alpha_i & \eta_i^{\mathrm{T}} & \eta_i^{\mathrm{T}} M \\ * & -N & -I \\ * & * & -M \end{bmatrix} < 0 \tag{59}$$

$$\begin{bmatrix} -N & -I & NA + \bar{H}_f \bar{\Theta} C & \bar{A}_f & \hat{C} & \Phi_{16} \\ * & -M & A & AM + B_1 \bar{K} & 0 & \Phi_{26} \\ * & * & -N & -I & 0 & 0 \\ * & * & * & -M & 0 & 0 \\ * & * & * & * & -\bar{\mathcal{Y}} & \Phi_{56} \\ * & * & * & * & * & \Phi_{66} \end{bmatrix} < 0 \tag{60}$$

$$\begin{bmatrix} -N & -I & L^{\mathrm{T}} \bar{S} & A^{\mathrm{T}} N + C^{\mathrm{T}} \bar{\Theta} \bar{H}_f^{\mathrm{T}} & A^{\mathrm{T}} & \bar{C} & \Upsilon_{17} \\ * & -M & (ML^{\mathrm{T}} + \bar{K}^{\mathrm{T}} B_2^{\mathrm{T}}) \bar{S} & \bar{A}_f^{\mathrm{T}} & MA^{\mathrm{T}} + \bar{K}^{\mathrm{T}} B_1^{\mathrm{T}} & 0 & \Upsilon_{27} \\ * & * & -\bar{R} & D_1^{\mathrm{T}} N + D_3^{\mathrm{T}} \bar{H}_f^{\mathrm{T}} & D_1^{\mathrm{T}} & 0 & 0 \\ * & * & * & -N & -I & 0 & 0 \\ * & * & * & * & -M & 0 & 0 \\ * & * & * & * & * & -\bar{\mathcal{Y}} & \Upsilon_{67} \\ * & * & * & * & * & * & \Upsilon_{77} \end{bmatrix} < 0 \tag{61}$$

where

$$e_s = [\underbrace{0 \quad \cdots \quad 0}_{s-1} \quad 1 \quad \underbrace{0 \quad \cdots \quad 0}_{n-s}]^{\mathrm{T}},$$

$$\bar{S} = S - QD_2, \quad \bar{R} = D_2^{\mathrm{T}} Q D_2 + D_2^{\mathrm{T}} S + S^{\mathrm{T}} D_2 + R, \quad \tilde{C} = \begin{bmatrix} \sigma_1 M C_1^{\mathrm{T}} & \sigma_2 M C_2^{\mathrm{T}} & \cdots & \sigma_m M C_m^{\mathrm{T}} \end{bmatrix},$$

$$\hat{C} = \begin{bmatrix} \sigma_1 \bar{H}_f C_1 & 0 & \sigma_2 \bar{H}_f C_2 & 0 & \cdots & \sigma_m \bar{H}_f C_m & 0 \end{bmatrix}, \quad \tilde{H}_f = \begin{bmatrix} \zeta_1 \bar{H}_f & \zeta_2 \bar{H}_f & \cdots & \zeta_m \bar{H}_f \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} \sigma_1 C_1^{\mathrm{T}} \bar{H}_f^{\mathrm{T}} & 0 & \sigma_2 C_2^{\mathrm{T}} \bar{H}_f^{\mathrm{T}} & 0 & \cdots & \sigma_m C_m^{\mathrm{T}} \bar{H}_f^{\mathrm{T}} & 0 \end{bmatrix}, \quad \tilde{\eta} = \begin{bmatrix} \eta_1 & \eta_2 & \cdots & \eta_q \end{bmatrix},$$

$$\hat{\Pi} = \begin{bmatrix} N\pi_{11} + \bar{H}_f \pi_{21} & N\pi_{12} + \bar{H}_f \pi_{22} & \cdots & N\pi_{1q} + \bar{H}_f \pi_{2q} \end{bmatrix}, \quad \tilde{\Pi} = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1q} \end{bmatrix},$$

$$\Phi_{16} = \begin{bmatrix} \tilde{H}_f & 0 & \hat{\Pi} & ND_1 + \bar{H}_f D_3 \end{bmatrix}, \quad \Phi_{26} = \begin{bmatrix} 0 & 0 & \tilde{\Pi} & D_1 \end{bmatrix}, \quad \Phi_{56} = \begin{bmatrix} 0 & \mathcal{C} & 0 & 0 \end{bmatrix},$$

$$\Phi_{66} = \mathrm{diag}\{-Z, -Z, -\bar{\beta}, -W^{-1}\}, \quad Z = \mathrm{diag}\{\zeta_1 I, \zeta_2 I, \dots, \zeta_m I\}, \quad \Upsilon_{17} = \begin{bmatrix} 0 & 0 & \tilde{\eta} & L^{\mathrm{T}} \bar{Q} \end{bmatrix},$$

$$\Upsilon_{27} = \begin{bmatrix} \tilde{C} & 0 & M\tilde{\eta} & (ML^{\mathrm{T}} + \bar{K}^{\mathrm{T}} B_2^{\mathrm{T}}) \bar{Q} \end{bmatrix}, \quad \Upsilon_{67} = \begin{bmatrix} 0 & -\mathcal{H} & 0 & 0 \end{bmatrix},$$

$$\Upsilon_{77} = \mathrm{diag}\{-\bar{Z}, -\bar{Z}, -\bar{\alpha}, -I\}, \quad \bar{Z} = \mathrm{diag}\{\vartheta_1 I, \vartheta_2 I, \dots, \vartheta_m I\},$$

$$\mathcal{C} = \mathrm{diag}\{0, \sigma_1 M C_1^{\mathrm{T}}, 0, \sigma_2 M C_2^{\mathrm{T}}, 0, \dots, 0, \sigma_m M C_m^{\mathrm{T}}\}, \quad \mathcal{H} = \mathrm{diag}\{\vartheta_1 \bar{H}_f, 0, \vartheta_2 \bar{H}_f, 0, \dots, \vartheta_m \bar{H}_f, 0\},$$

$$\bar{\mathcal{Y}} = \mathrm{diag}\left\{ \begin{bmatrix} N & I \\ I & M \end{bmatrix}, \begin{bmatrix} N & I \\ I & M \end{bmatrix}, \dots, \begin{bmatrix} N & I \\ I & M \end{bmatrix} \right\}$$

then system (9) is exponentially mean-square stable and strictly (Q, S, R) dissipative and, meanwhile, the individual steady-state variance constraint is also satisfied. Moreover, the desired estimator parameters and feedback controller parameter can be obtained by

$$\begin{aligned} K &= \bar{K}(U^{\mathrm{T}})^{-1} \\ H_f &= V^{-1}\bar{H}_f \\ A_f &= V^{-1}(\bar{A}_f - (NA + VH_f\bar{\Theta}C)M - NB_1KU^{\mathrm{T}})(U^{\mathrm{T}})^{-1} \end{aligned} \tag{62}$$

where the nonsingular matrices U and V satisfy

$$UV^{\mathrm{T}} = I - MN \tag{63}$$

which can be determined by the singular value decomposition of $I - MN$.

Proof: First, under the conditions of this theorem, it is easy to see that

$$\begin{bmatrix} -N & -I \\ -I & -M \end{bmatrix} < 0, \tag{64}$$

which, by Schur Complement Equivalence, gives that $-N + M^{-1} < 0$ implying the non-singularity of $I - MN$. Therefore, there always exist nonsingular matrices U and V such that (63) is true.

Introduce the following construction of Y,

$$Y = \begin{bmatrix} M & U \\ U^{\mathrm{T}} & \Xi_1 \end{bmatrix}, \quad Y^{-1} = \begin{bmatrix} N & V \\ V^{\mathrm{T}} & \Xi_2 \end{bmatrix}, \quad \Xi_1 = -U^{\mathrm{T}}NV^{-\mathrm{T}}, \quad \Xi_2 = -V^{\mathrm{T}}MU^{-\mathrm{T}}, \tag{65}$$

and define

$$\Psi_1 = \begin{bmatrix} N & I \\ V^{\mathrm{T}} & 0 \end{bmatrix}, \qquad \Psi_2 = \begin{bmatrix} I & M \\ 0 & U^{\mathrm{T}} \end{bmatrix}. \tag{66}$$

Then, we have

$$Y\Psi_1 = \Psi_2, \qquad UV^{\mathrm{T}} = I - MN. \tag{67}$$

Next, let us prove that the inequality (49) is equivalent to (58). To start with, performing the congruence transformation to (49) on both sides by $\mathrm{diag}\{I, \Psi_1^{\mathrm{T}}\}$, we obtain

$$\begin{aligned} &\begin{bmatrix} 1 & 0 \\ 0 & \Psi_1^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} -\alpha_i^{-1} & \bar{\pi}_i^{\mathrm{T}}\bar{H}^{\mathrm{T}} \\ * & -Y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \Psi_1 \end{bmatrix} < 0 \\ \Leftrightarrow &\begin{bmatrix} -\alpha_i^{-1} & \bar{\pi}_i^{\mathrm{T}}\bar{H}^{\mathrm{T}}\Psi_1 \\ * & -\Psi_1^{\mathrm{T}}Y\Psi_1 \end{bmatrix} < 0 \\ \Leftrightarrow &\begin{bmatrix} -\beta_i & \pi_{1i}^{\mathrm{T}}N + \pi_{2i}^{\mathrm{T}}\bar{H}_f^{\mathrm{T}} & \pi_{1i}^{\mathrm{T}} \\ * & -N & -I \\ * & * & -M \end{bmatrix} < 0. \end{aligned} \tag{68}$$

where $\beta_i \triangleq \alpha_i^{-1}$ $(i = 1, 2, \ldots, q)$. Therefore, inequality (49) is equivalent to (58). Similarly, we can prove that inequality (50) holds if and only if inequality (59) holds. It is worth pointing out that, here we use the equality constraints $\alpha_i\beta_i = 1$ $(i = 1, 2, \ldots, q)$ to avoid the presence of the variable α_i and its reciprocal α_i^{-1} in the same set of LMIs.

In the following, we will show that the inequalities (51) and (52) are implied by inequalities (60) and (61), respectively. Performing the congruence transformation to (51) on both sides by $\mathrm{diag}\{\Psi_1^{\mathrm{T}}, \Psi_1^{\mathrm{T}}, \Psi_1^{\mathrm{T}}, \ldots, \Psi_1^{\mathrm{T}}, I, I\}$

results in

$$\begin{bmatrix} -\Psi_1^{\mathrm{T}} Y \Psi_1 & \Psi_1^{\mathrm{T}} \hat{A} Y \Psi_1 & \Psi_1^{\mathrm{T}} \bar{\mathcal{A}} \mathcal{Y} \bar{\Psi}_1 & \Psi_1^{\mathrm{T}} \hat{H} & \Psi_1^{\mathrm{T}} \bar{D} \\ * & -\Psi_1^{\mathrm{T}} Y \Psi_1 & 0 & 0 & 0 \\ * & * & -\bar{\Psi}_1^{\mathrm{T}} \mathcal{Y} \bar{\Psi}_1 & 0 & 0 \\ * & * & * & -\bar{\beta} & 0 \\ * & * & * & * & -W^{-1} \end{bmatrix} < 0, \tag{69}$$

where $\bar{\Psi}_1 = \mathrm{diag}\{\Psi_1, \Psi_1, \ldots, \Psi_1\}$.

For the term $\Psi_1^{\mathrm{T}} \bar{\mathcal{A}} \mathcal{Y} \bar{\Psi}_1$ in (69), we conduct the following calculation:

$$\begin{aligned} \Psi_1^{\mathrm{T}} \bar{\mathcal{A}} \mathcal{Y} \bar{\Psi}_1 &= \Psi_1^{\mathrm{T}} \begin{bmatrix} \sigma_1 \tilde{A}_1 & \sigma_2 \tilde{A}_2 & \cdots & \sigma_m \tilde{A}_m \end{bmatrix} \begin{bmatrix} Y & 0 & 0 & 0 \\ 0 & Y & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & Y \end{bmatrix} \begin{bmatrix} \Psi_1 & 0 & 0 & 0 \\ 0 & \Psi_1 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \Psi_1 \end{bmatrix} \\ &= \begin{bmatrix} \sigma_1 \Psi_1^{\mathrm{T}} \tilde{A}_1 Y \Psi_1 & \sigma_2 \Psi_1^{\mathrm{T}} \tilde{A}_2 Y \Psi_1 & \cdots & \sigma_m \Psi_1^{\mathrm{T}} \tilde{A}_m Y \Psi_1 \end{bmatrix} \\ &= \begin{bmatrix} \sigma_1 \bar{H}_f C_1 & \sigma_1 \bar{H}_f C_1 M & \sigma_2 \bar{H}_f C_2 & \sigma_2 \bar{H}_f C_2 M & \cdots & \sigma_m \bar{H}_f C_m & \sigma_m \bar{H}_f C_m M \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}. \end{aligned} \tag{70}$$

Notice that the matrix variables $\bar{H}_f$ and M are not linear in the term $\sigma_i \bar{H}_f C_i M$. Here, for arbitrary scalars $\zeta_i > 0$ $(i = 1, 2, \ldots, m)$, it is true that

$$\begin{bmatrix} 0 & \bar{H}_f C_i M \\ M^{\mathrm{T}} C_i^{\mathrm{T}} \bar{H}_f^{\mathrm{T}} & 0 \end{bmatrix} \leqslant \begin{bmatrix} \zeta_i \bar{H}_f \bar{H}_f^{\mathrm{T}} & 0 \\ 0 & \zeta_i^{-1} M^{\mathrm{T}} C_i^{\mathrm{T}} C_i M \end{bmatrix}. \tag{71}$$

Then, it follows directly from (69) with (71) that the matrix inequality (51) is true if (60) is true. Similarly, we could easily prove that the inequality (61) implies (52). Therefore, according to Corollary 1, system (9) is exponentially mean-square stable and strictly (Q, R, S) dissipative, and the steady-state covariance exists, satisfying $\bar{X} \leqslant M$ by (55). Next, it is obvious that (48) is equivalent to

$$M - Y_0 < 0. \tag{72}$$

Thus, $\bar{X} \leqslant M < Y_0$. Now, from the n LMIs in (57), we can see that the individual variance of each system states is not more than the pre-specified value. In other words, the design requirements $(R1)$, $(R2)$ and $(R3)$ are simultaneously satisfied. The proof is complete. ■

B. Computational Algorithm

It is worth mentioning that the obtained conditions in Theorem 3 are not all strict LMIs which, as a result, cannot be solved directly by applying Matlab LMI-Toolbox. However, with the so-called cone complementarity linearization (CCL) method proposed in [20], we can convert the original non-convex feasibility problem of certain LMIs into some sequential optimization problems subject to LMI constraints. To this end, we introduce a new condition by $\alpha_i \beta_i \geqslant 1$ which, by Schur Complement Equivalence, is equivalent to

$$\begin{bmatrix} -\alpha_i & 1 \\ 1 & -\beta_i \end{bmatrix} \leqslant 0, \qquad i = 1, 2, \ldots, q. \tag{73}$$

Then, using CCL method, we suggest the following minimization problem involving LMI conditions instead of the original non-convex problem formulated in Theorem 3.

Problem MCD (Multiobjective Controller Design)

$$\min \quad \sum_{i=1}^{q} \alpha_i \beta_i \quad \text{subject to} \quad (57)-(61) \quad \text{and} \quad (73). \tag{74}$$

If the solution of the above minimization problem is q, that is, $\min(\sum_{i=1}^{q} \alpha_i \beta_i) = q$, then the condition in Theorem 3 is solvable. It is should be pointed out that this algorithm does not guarantee finding a global optimal solution for the problem above. Nevertheless, the proposed minimization problem is much easier to be solved than the original non-convex feasibility problem.

Algorithm MCD

Step 1. Find a feasible set $(M^{(0)}, N^{(0)}, \bar{A}_f^{(0)}, \bar{H}_f^{(0)}, \bar{K}^{(0)}, \alpha_i^{(0)}, \beta_i^{(0)})$ satisfying (57)–(61) and (73). Set $d = 0$.

Step 2. Solve the following optimization problem

$$\min \sum_{i=1}^{q} \left(\alpha_i^{(d)} \beta_i + \alpha_i \beta_i^{(d)} \right)$$
$$\text{subject to} \quad (57)-(61) \quad \text{and} \quad (73).$$

and denote g^* as the optimized value.

Step 3. Substitute the obtained matrix variables $(M, N, \bar{A}_f, \bar{H}_f, \bar{K}, \alpha_i, \beta_i)$ into (57)–(61). If conditions (57)–(61) are satisfied with

$$|g^* - 2q| < \upsilon$$

where υ is a sufficiently small positive scalar, then output the feasible solutions $(M, N, \bar{A}_f, \bar{H}_f, \bar{K}, \alpha_i, \beta_i)$ and obtain the desired parameters A_f, H_f and K by (62) and (63). EXIT.

Step 4. If $d > N$ where N is the maximum number of iterations allowed, EXIT.

Step 5. Set $d = d + 1$, $(M^{(d)}, N^{(d)}, \bar{A}_f^{(d)}, \bar{H}_f^{(d)}, \bar{K}^{(d)}, \alpha_i^{(d)}, \beta_i^{(d)})$=$(M, N, \bar{A}_f, \bar{H}_f, \bar{K}, \alpha_i, \beta_i)$, and go to Step 2.

V. Numerical Example

In this section, we present an illustrative example to demonstrate the effectiveness of the proposed algorithm. Consider the following discrete-time stochastic nonlinear system:

$$\begin{cases} x(k+1) = \begin{bmatrix} 0.2 & -0.05 \\ -0.1 & 0.08 \end{bmatrix} x(k) + \begin{bmatrix} 0.03 \\ -0.5 \end{bmatrix} u(k) + f(x(k)) + \begin{bmatrix} 0.1 \\ 0.03 \end{bmatrix} \omega(k) \\ z(k) = \begin{bmatrix} 0.05 & -0.07 \end{bmatrix} x(k) + 0.04u(k) + 0.25\omega(k) \end{cases} \tag{75}$$

with the measured output equation:

$$y(k) = \Theta \begin{bmatrix} -0.4 & 0.3 \\ 0.2 & -0.1 \end{bmatrix} x(k) + g(x(k)) + \begin{bmatrix} 0.02 \\ 0.01 \end{bmatrix} \omega(k). \tag{76}$$

The stochastic nonlinear functions are taken to be

$$\begin{aligned} f(x(k)) &= \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix} (0.3 \cdot \mathrm{sign}[x_1(k)] \cdot x_1(k)\nu_1(k) + 0.4 \cdot \mathrm{sign}[x_2(k)] \cdot x_2(k)\nu_2(k)), \\ g(x(k)) &= \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix} (0.3 \cdot \mathrm{sign}[x_1(k)] \cdot x_1(k)\nu_1(k) + 0.4 \cdot \mathrm{sign}[x_2(k)] \cdot x_2(k)\nu_2(k)) \end{aligned} \tag{77}$$

where $x_i(k)$ is the ith component of $x(k)$. $\nu_i(k)$ is a zero mean, independent Gaussian white noise process with unity covariances, which is also assumed to be independent from $\omega(k)$. It is easy to check that $f(x(k))$ and $g(x(k))$ satisfy

$$\begin{aligned}
&\mathbb{E}\left\{\left[\begin{array}{c} f(x(k)) \\ g(x(k)) \end{array}\right]\middle| x(k)\right\} = 0, \\
&\mathbb{E}\left\{\left[\begin{array}{c} f(x(k)) \\ g(x(k)) \end{array}\right]\left[\begin{array}{cc} f^{\mathrm{T}}(x(j)) & g^{\mathrm{T}}(x(j)) \end{array}\right]\middle| x(k)\right\} = 0, \qquad k \neq j \\
&\mathbb{E}\left\{\left[\begin{array}{c} f(x(k)) \\ g(x(k)) \end{array}\right]\left[\begin{array}{cc} f^{\mathrm{T}}(x(k)) & g^{\mathrm{T}}(x(k)) \end{array}\right]\middle| x(k)\right\} \\
&= \left[\begin{array}{c} 0.2 \\ 0.3 \\ 0.1 \\ 0.4 \end{array}\right]\left[\begin{array}{c} 0.2 \\ 0.3 \\ 0.1 \\ 0.4 \end{array}\right]^{\mathrm{T}} x^{\mathrm{T}}(k)\left(\left[\begin{array}{c} 0.3 \\ 0 \end{array}\right]\left[\begin{array}{c} 0.3 \\ 0 \end{array}\right]^{\mathrm{T}} + \left[\begin{array}{c} 0 \\ 0.4 \end{array}\right]\left[\begin{array}{c} 0 \\ 0.4 \end{array}\right]^{\mathrm{T}}\right) x(k).
\end{aligned} \tag{78}$$

Hence,

$$\pi_{11} = \pi_{12} = \left[\begin{array}{c} 0.2 \\ 0.3 \end{array}\right], \quad \pi_{21} = \pi_{22} = \left[\begin{array}{c} 0.1 \\ 0.4 \end{array}\right], \quad \eta_1 = \left[\begin{array}{c} 0.3 \\ 0 \end{array}\right], \quad \eta_2 = \left[\begin{array}{c} 0 \\ 0.4 \end{array}\right]. \tag{79}$$

In addition, we assume that the probabilistic density functions of θ_1 and θ_2 in $[0,1]$ are described by

$$p_1(s_1) = \begin{cases} 0.8, & s_1 = 0 \\ 0.1, & s_1 = 0.5 \\ 0.1, & s_1 = 1 \end{cases} \qquad p_2(s_2) = \begin{cases} 0.7, & s_2 = 0 \\ 0.2, & s_2 = 0.5 \\ 0.1, & s_2 = 1 \end{cases} \tag{80}$$

from which the expectations and variances can be easily calculated as $\bar{\theta}_1 = 0.15$, $\bar{\theta}_2 = 0.2$, $\sigma_1^2 = 0.1025$ and $\sigma_2^2 = 0.11$. Select $Q = -1.2$, $S = 0.8$ and $R = 1.6$. Choose the required steady-state variance constraints as $\delta_1^2 = 0.36$ and $\delta_2^2 = 0.64$.

Applying standard numerical software to solve Problem *MCD*, we can obtain the observer and feedback controller parameters as follows:

$$\begin{aligned}
&A_f = \left[\begin{array}{cc} 0.6944 & 0.4181 \\ 0.9199 & 0.5465 \end{array}\right], \quad H_f = \left[\begin{array}{cc} 0.0155 & 0.0836 \\ 0.0221 & 0.1168 \end{array}\right], \quad K = \left[\begin{array}{cc} 0.3862 & 0.0120 \end{array}\right], \\
&\alpha_1 = 0.9586, \qquad \alpha_2 = 0.9664, \qquad \beta_1 = 1.0432, \qquad \beta_2 = 1.0348.
\end{aligned} \tag{81}$$

The time responses of the individual states $x_1(k)$, $x_2(k)$ and their estimates $\hat{x}_1(k)$, $\hat{x}_2(k)$ are shown in Fig. 1 and Fig. 2.

VI. Conclusion

In this paper, we have designed an observer-based controller for a class of nonlinear stochastic systems such that, for all possible degraded measurements , the closed-loop system is exponentially mean-square stable, the system dissipativity is achieved, and the steady-state variance of individual state components is not more than the pre-specified values. The nonlinearities considered here are characterized statistically, which can cover several classes of commonly encountered nonlinearities. The solvability of the addressed problem has been expressed as the feasibility of a set of LMIs with equality constraints. An algorithm has been proposed to convert the original non-convex feasibility problem into an optimal minimization problem which is much more easily to solve by standard numerical software. An illustrative example has been presented to demonstrate the effectiveness and applicability of the provided design method. Finally, we like to point out that the proposed dissipativity analysis method can be applied to more complex systems/networks such as networked control systems (NCS) [11], gene regulatory networks (GRN) [27], complex networks (CN) [28], neural networks (NN) [17] and fuzzy systems [37–39].

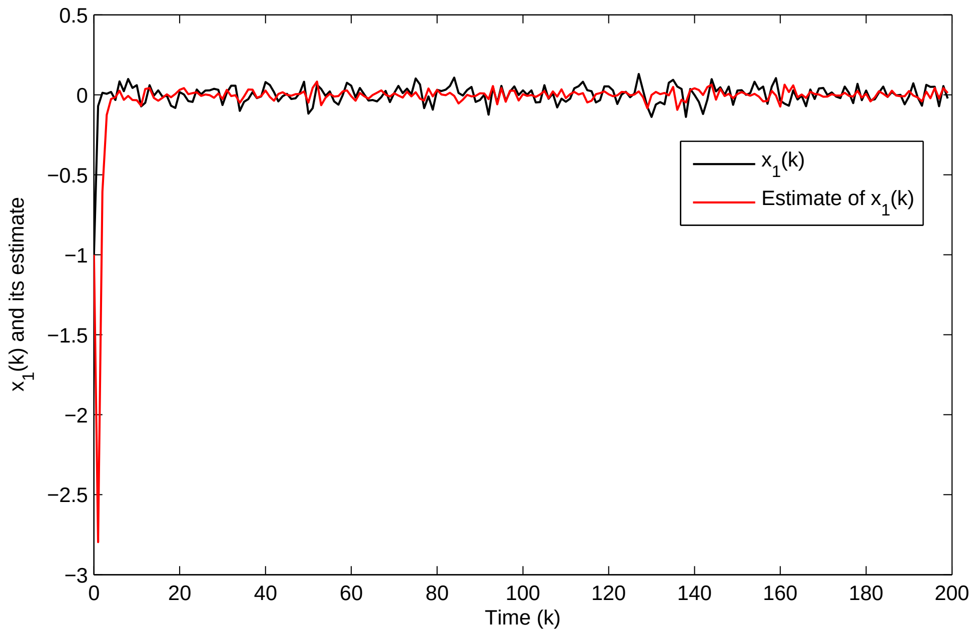

Fig. 1. System state $x_1(k)$ and its estimate $\hat{x}_1(k)$.

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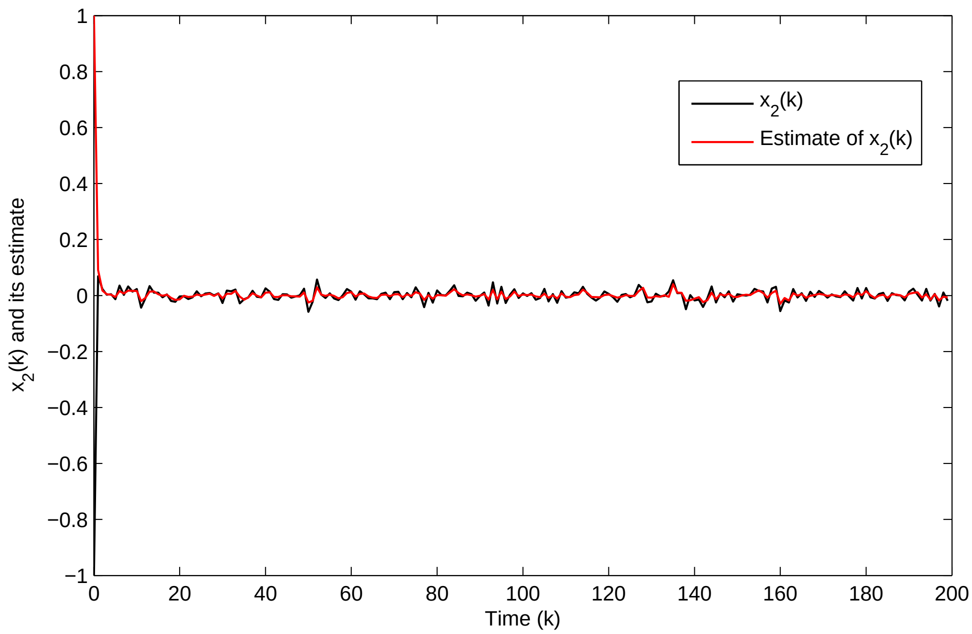

Fig. 2. System state $x_2(k)$ and its estimate $\hat{x}_2(k)$.

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