Robust H_∞ Fuzzy Output-Feedback Control With Multiple Probabilistic Delays and Multiple Missing Measurements

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***Abstract*—In this paper, the robust H_∞-control problem is investigated for a class of uncertain discrete-time fuzzy systems with both multiple probabilistic delays and multiple missing measurements. A sequence of random variables, all of which are mutually independent but obey the Bernoulli distribution, is introduced to account for the probabilistic communication delays. The measurement-missing phenomenon occurs in a random way. The missing probability for each sensor satisfies a certain probabilistic distribution in the interval $[0\ 1]$. Here, the attention is focused on the analysis and design of H_∞ fuzzy output-feedback controllers such that the closed-loop Takagi–Sugeno (T–S) fuzzy-control system is exponentially stable in the mean square. The disturbance-rejection attenuation is constrained to a given level by means of the H_∞-performance index. Intensive analysis is carried out to obtain sufficient conditions for the existence of admissible output feedback controllers, which ensures the exponential stability as well as the prescribed H_∞ performance. The cone-complementarity-linearization procedure is employed to cast the controller-design problem into a sequential minimization one that is solved by the semi-definite program method. Simulation results are utilized to demonstrate the effectiveness of the proposed design technique in this paper.**

***Index Terms*—Discrete-time fuzzy systems, fuzzy control, multiple missing measurements, multiple probabilistic time delays, networked-control systems (NCSs), robust H_∞ control, stochastic systems.**

I. Introduction

OVER the past few decades, the fuzzy-logic control has proven to be an effective approach when dealing with complex nonlinear systems [4], [8], [12], [26], [28], [29], [36]. In particular, the control technique based on the Takagi–Sugeno (T–S) fuzzy model has attracted considerable research attention. Such a fuzzy model, which has a convenient and simple dynamic structure, could approximate any smooth nonlinear function to any specified accuracy within any compact set. As such, it is possible for the existing results of traditional linear-systems theory to be extended for certain class of nonlinear systems [8]. As a result, a majority of literature has appeared on the control problems for T–S fuzzy systems [1]–[3], [7], [9], [10], [14]–[16], [18], [19], [21], [24], [34], [39], [41], [43]. Note that the output feedback T–S fuzzy control has received particular research interests, owing to the unmeasured partial state variables in practical applications. It is well known that parameter uncertainties are unavoidable due mainly to the modeling inaccuracies, variations of the operating point, and aging of the devices, etc. Therefore, the issue of robustness analysis has been taken into account in T–S fuzzy systems by many researchers. For example, in [33], the robust H_∞-control problem has been considered for a class of uncertain time-delay T–S fuzzy systems. The stability issue has been addressed in [32] for a class of T–S fuzzy dynamical systems with time delays and uncertain parameters. The robust stability and controller-design problems for networked-control systems (NCSs) with uncertain parameters have been studied in [13] and [39], respectively.

On another active research frontier, NCSs have attracted much attention because of their successful applications in modern complicated industry processes, e.g., aircraft and space shuttle, nuclear power stations, high-performance automobiles, etc. However, several practical factors, such as communication delays (also called network-induced time delays) and missing measurements (also called packet dropouts or probabilistic information missing), which constitute crucial issues in NCSs, would degrade the system performance or even cause instability. Hence, it is of vital importance to evaluate the effects of communication delays and missing measurements when designing NCSs in the industrial applications. Many results exist in the literature, which describe the communication delays in NCSs. For example, in [40], the stability of an NCS under the network-induced delay was studied by using a hybrid-system technique. The optimal stochastic-control method was proposed in [20] to control communication delays in NCS. A networked controller was designed in the frequency domain by using the robust-control theory proposed in [11], in which the network delays were

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considered as an uncertainty. However, most of the relevant literature mentioned earlier has focused on the *constant time delays*. Delays resulting from network transmissions are inherently *random* and *time varying*. This is particularly true when signals are transmitted over Internet, and therefore, existing control methods for constant time delay cannot be directly used [5], [31]. Recently, some researchers have started to model the network-induced time delays in multiform probabilistic ways, and accordingly, some initial results have been reported. For example, in [17], [39], and [42], the random communication delays were modeled as Markov chains, and the resulting closed-loop systems were represented as Markovian jump linear systems with two jumping parameters. In [37], two kinds of random delays, which happen in the channels from the controller to the plant and from the sensor to the controller, were simultaneously considered. The random delays were modeled in [37] as a linear function of the stochastic variable, which satisfies the Bernoulli-random binary distribution. Different from [37], the problem of stability analysis and stabilization-control design was studied in [38] for T–S fuzzy systems with probabilistic-interval delay, and the Bernoulli-distributed sequence was used to describe the probability distribution of the time-varying delay-taking values in an interval. It should be mentioned that, among others, the binary representation of the random delays has been fairly popular because of its practicality and simplicity in describing communication delays. However, most research attention has been centered on the *single* random delay having a *fixed* value if it occurs. This would lead to conservative results, or even degradation in the system's performance, since, at a certain time, the NCS could give rise to multiple time-varying delays but with different occurrence probabilities. Therefore, a more advanced methodology is needed to handle the time-varying-network-induced time delays in a closed-loop-control system.

In terms of the missing measurement (packet dropout) phenomena in the NCSs, which has also stirred a great deal of research interest (see, e.g., [10], [13], [22], [25], [30], and references therein), lots of results have appeared on this topic over the past few years. For example, in [10], the H_∞ controller was designed for a class of nonlinear systems with missing measurements, where the Bernoulli-random binary distribution was utilized to model the unreliable communication links. In [22] and [25], an improved model was presented to describe the packet dropout problems, where the latest packet received would be used if the current packet was lost during the transmission. The H_∞-control problem was studied in [23] for NCS with random-packet losses described as a Markov chain. However, most of the existing literature has implicitly assumed that the measurement signal is either completely intact (with missing probability 0) or completely missing (with missing probability 1), and all the sensors have the same missing probability. In fact, in some practical cases, where partial/multiple missing measurements take place for an array of sensors, the individual sensor would have different missing probability according to a certain probabilistic distribution in the interval $[0\ 1]$ (not just 0 or 1). Based on this fact, some new approaches should be developed to describe the missing-measurement phenomena.

Looking into the reported results on NCSs and fuzzy-control systems discussed earlier, it was observed that most of the literature about NCSs has been based on linear systems. Little attention has been focused on nonlinear NCSs, especially when the probabilistic delays and missing measurements are taken into consideration. In addition, the fact that nonlinear NCSs can be approximated by T–S fuzzy models has been largely overlooked due mainly to system complexity and mathematical difficulty. Motivated by the aforementioned discussion, in this paper, we aim to investigate the robust H_∞ fuzzy-control problem for a class of discrete-time uncertain-networked systems with multiple probabilistic time-varying communication delays and multiple missing measurements. A sufficient condition for the robustly exponential stability of the closed-loop fuzzy system is obtained, while a prescribed H_∞-disturbance-rejection-attenuation level is guaranteed. The explicit expression of the desired controller parameters is also derived. A numerical simulation example is used to demonstrate the effectiveness of the proposed control scheme in this paper. The main contributions of this paper, which lie primarily on the new research problems and new system models, are summarized as follows.

1) A model is proposed to describe the multiple probabilistic communication delays of different sizes, all of which could occur according to a specified Bernoulli distribution.

2) Multiple missing measurements corresponding to partial sensor outputs with missing probability over the interval $[0\ 1]$ are considered.

3) The investigation on the T–S fuzzy model is carried out for a class of complex systems that account for the modeling errors, disturbance rejection attenuation, probabilistic delay, and missing measurements within the same framework.

The rest of this paper is organized as follows. Section II formulates the problem under consideration. The exponentially stability condition and robust H_∞ performance of the closed-loop fuzzy system are given in Section III. The fuzzy-controller-design problem is solved in Section IV. An illustrative example is given in Section V, and we conclude the paper in Section VI.

Notation: The notation used in the paper is fairly standard. The superscript "T" stands for matrix transposition, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all real matrices of dimension $m \times n$; and I and 0 represent the identity matrix and zero matrix, respectively. The notation $P > 0$ means that P is real symmetric and positive definite, $\mathrm{tr}(M)$ refers to the trace of the matrix M, the notation $\|A\|$ refers to the norm of a matrix A defined by $\|A\| = \sqrt{\mathrm{tr}(A^T A)}$, and $\|\cdot\|_2$ stands for the usual l_2 norm. In symmetric block matrices or complex matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry, and $\mathrm{diag}\{\ldots\}$ stands for a block-diagonal matrix. In addition, $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|\,y\}$ will, respectively, mean expectation of x and expectation of x conditional on y. The set of all nonnegative integers is denoted by $\mathbb{I}^+$, and the set of all nonnegative real numbers is represented by $\mathbb{R}^+$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

II. Problem Formulation

In this paper, we consider the output feedback-control problem for discrete-time fuzzy systems in NCSs, where the framework is shown in Fig. 1. The sensors are connected to the

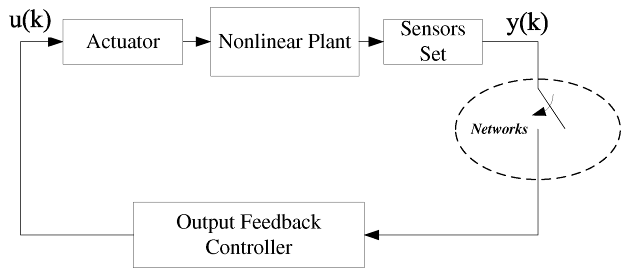

Fig. 1. Framework of output feedback-control systems over networks environments.

controller via a network, which is shared by other NCSs and subject to communication delays and missing measurements (packet dropouts). The fuzzy systems with multiple stochastic-communication delays and uncertain parameters can be described as follows:

$\triangle$ *Plant Rule i:* IF $\theta_1(k)$ is M_{i1}, and $\theta_2(k)$ is M_{i2} and, ..., and $\theta_p(k)$ is M_{ip} THEN

$$\begin{aligned} x(k+1) &= A_i(k)x(k) + A_{di}\sum_{m=1}^{h}\alpha_m(k)x(k-\tau_m(k)) \\ &\quad + B_{1i}u(k) + D_{1i}v(k) \\ \tilde{y}(k) &= C_i x(k) + D_{2i}v(k) \\ z(k) &= C_{zi}(k)x(k) + B_{2i}u(k) + D_{3i}v(k) \\ x(k) &= \phi(k)\ \forall k \in Z^-,\ i = 1,\dots,r \end{aligned} \tag{1}$$

where M_{ij} is the fuzzy set, r is the number of IF–THEN rules, and $\theta(k) = [\theta_1(k), \theta_2(k), \dots, \theta_p(k)]$ is the premise variable vector. It is assumed that the premise variables do not depend on the input variables $u(k)$, which is needed to avoid a complicated defuzzification process of fuzzy controllers. $x(k) \in \mathbb{R}^n$ represents the state vector, $u(k) \in \mathbb{R}^m$ is the control input, $\tilde{y}(k) \in \mathbb{R}^s$ is the process output, $z(k) \in \mathbb{R}^q$ is the controlled output, $v(k) \in \mathbb{R}^p$ is a vector of exogenous inputs, which belongs to $l_2[0,\infty)$ (e.g., reference signals, disturbance signals, sensor noise, etc.), $\tau_m(k)$ $(m = 1, 2, \dots, h)$ are the communication delays that occur according to the stochastic variables $\alpha_m(k)$, and $\phi(k)$ $(\forall k \in Z^-)$ is the initial state.

The stochastic variables $\alpha_m(k) \in \mathbb{R}(m = 1, 2, \dots, h)$ in (1) are mutually uncorrelated Bernoulli-distributed-white sequences, and a natural assumption on the sequence $\{\alpha_m(k)\}$ can be made as follows:

$$\begin{aligned} \text{Prob}\,\{\alpha_m(k) = 1\} &= \mathbb{E}\,\{\alpha_m(k)\} = \bar{\alpha}_m \\ \text{Prob}\,\{\alpha_m(k) = 0\} &= 1 - \bar{\alpha}_m. \end{aligned}$$

The following assumption is needed on the random-communication time delays, which are considered.

Assumption 1: The communication delays $\tau_m(k)$ $(m = 1, 2, \dots, h)$ are time varying and satisfy $d_t \le \tau_m(k) \le d_T$, where d_t and d_T are constant positive scalars, which represent the lower and upper bounds on the communication delays, respectively.

Remark 1: The description of the communication delays in (1) includes two features. 1) The communication delays are allowed to occur in any fashion, either discretely, successively, or even distributely. 2) Each possible delay could occur independently and randomly according to an individual probability distribution, which can be specified *a prior* through statistical test.

The matrices $A_i(k) = A_i + \Delta A_i(k)$, $C_{zi}(k) = C_{zi} + \Delta C_{zi}(k)$, and $A_i, A_{di}, B_{1i}, B_{2i}, C_i, C_{zi}, D_{1i}, D_{2i}$, and D_{3i} are known as constant matrices with compatible dimensions. The matrices $\Delta A_i(k)$ and $\Delta C_{zi}(k)$ represent time-varying norm-bounded parameter uncertainties that satisfy

$$\begin{bmatrix} \Delta A_i(k) \\ \Delta C_{zi}(k) \end{bmatrix} = \begin{bmatrix} H_{ai} \\ H_{ci} \end{bmatrix} F(k)E \tag{2}$$

where H_{ai}, H_{ci}, and E are constant matrices of appropriate dimensions, and $F(k)$ is an unknown matrix function satisfying

$$F^T(k)F(k) \le I \quad \forall k. \tag{3}$$

The parameter uncertainties $\Delta A_i(k)$ and $\Delta C_{zi}(k)$ are said to be admissible, if both (2) and (3) hold.

In this paper, the missing-measurement (packet dropout) phenomenon constitutes another focus of our present research. The multiple missing measurements are described by

$$\begin{aligned} y(k) &= \Xi C_i x(k) + D_{2i}v(k) \\ &= \sum_{l=1}^{s}\beta_l C_{il}x(k) + D_{2i}v(k) \end{aligned} \tag{4}$$

where $y(k) \in \mathbb{R}^s$ is the *actual* measurement signal of (1), $\Xi := \text{diag}\{\beta_1, \dots, \beta_s\}$ with β_l $(l = 1, \dots, s)$ being s unrelated random variables, which are also unrelated with $\alpha_m(k)$. It is assumed that β_l has the probabilistic-density function $q_l(s)$ $(l = 1, \dots, s)$ on the interval $[0\ 1]$ with mathematical expectation μ_l and variance σ_l^2. C_{il} is defined by

$$C_{il} := \text{diag}\{\underbrace{0,\dots,0}_{l-1}, 1, \underbrace{0,\dots,0}_{s-l}\}C_i.$$

Note that β_l could satisfy any discrete probabilistic distributions on the interval $[0\ 1]$, which includes the widely used Bernoulli distribution as a special case. In the sequel, we denote $\bar{\Xi} = \mathbb{E}\{\Xi\}$.

Remark 2: In real systems, the measurement data may be transferred through multiple sensors. On one hand, for a different sensor, the data-missing probability may be different. On the other hand, due to various reasons, such as sensor aging and sensor temporal failure, the data missing at one moment might be partial, and therefore, the missing probability cannot be simply described by 0 or 1. In (4), the diagonal matrix Ξ accounts for the probabilistic-missing status of the array of sensors, where the random variable β_j corresponds to the jth sensor. In addition, β_j can take value on the interval $[0\ \ 1]$ and the probability for β_j to take different values may differ from each other. It is easy to see that the widely used Bernoulli distribution is included here as a special case.

For a given pair of $(x(k), u(k))$, the final output of the fuzzy system is inferred as follows:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{r} h_i(\theta(k)) \Bigg[A_i(k)x(k) + B_{1i}u(k) \\ &\qquad + A_{di} \sum_{m=1}^{h} \alpha_m(k)x(k-\tau_m(k)) + D_{1i}v(k) \Bigg] \\ y(k) &= \sum_{i=1}^{r} h_i(\theta(k)) \left[\Xi C_i x(k) + D_{2i}v(k) \right] \\ z(k) &= \sum_{i=1}^{r} h_i(\theta(k)) \left[C_{zi}(k)x(k) + B_{2i}u(k) + D_{3i}v(k) \right] \end{aligned} \tag{5}$$

where the fuzzy-basis functions are given by

$$h_i(\theta(k)) = \frac{\vartheta_i(\theta(k))}{\sum_{i=1}^{r} \vartheta_i(\theta(k))}, \quad \vartheta_i(\theta(k)) = \Pi_{j=1}^{p} M_{ij}(\theta_j(k))$$

with $M_{ij}(\theta_j(k))$ represents the grade of membership of $\theta_j(k)$ in M_{ij}. Here, $\vartheta_i(\theta(k))$ has the following basic property:

$$\vartheta_i(\theta(k)) \geq 0, \; i = 1, 2, \ldots, r, \quad \sum_{i=1}^{r} \vartheta_i(\theta(k)) > 0 \quad \forall k$$

and, therefore

$$h_i(\theta(k)) \geq 0, \; i = 1, 2, \ldots, r, \quad \sum_{i=1}^{r} h_i(\theta(k)) = 1 \quad \forall k.$$

In what follows, we define $h_i := h_i(\theta(k))$ for brevity.

In this paper, by the parallel distributed compensation (PDC), we consider the following *fuzzy dynamic output-feedback controller* for the fuzzy system (5):

$\triangle$ *Controller Rule i:* IF $\theta_1(k)$ is M_{i1} and $\theta_2(k)$ is M_{i2} and, $\ldots$, and $\theta_p(k)$ is M_{ip} THEN

$$\left. \begin{aligned} x_c(k+1) &= A_{ki}x_c(k) + B_{ki}y(k) \\ u(k) &= C_{ki}x_c(k), \;\; i = 1, 2, \ldots, r \end{aligned} \right\} \tag{6}$$

where $x_c(k) \in R^n$ is the controller state, and A_{ki}, B_{ki}, and C_{ki} are controller parameters to be determined. Then, the overall fuzzy output-feedback controller is given by

$$\left. \begin{aligned} x_c(k+1) &= \sum_{i=1}^{r} h_i \left[A_{ki}x_c(k) + B_{ki}y(k) \right] \\ u(k) &= \sum_{i=1}^{r} h_i C_{ki}x_c(k), \;\; i = 1, 2, \ldots, r \end{aligned} \right\}. \tag{7}$$

From (5) and (7), the closed-loop system can be obtained as

$$\begin{aligned} \bar{x}(k+1) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \Bigg[\left(A_{ij}(k) + B_{ij}\right)\bar{x}(k) + D_{ij}v(k) \\ &\qquad + \sum_{m=1}^{h} (\bar{A}_{dmi} + \tilde{A}_{dmi})\bar{x}(k-\tau_m(k)) \Bigg] \\ z(k) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \left[\bar{C}_{ij}(k)\bar{x}(k) + D_{3i}v(k) \right] \end{aligned} \tag{8}$$

where

$$\begin{aligned} \bar{x}(k) &= \begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix} \; A_{ij}(k) = \begin{bmatrix} A_i(k) & B_{1i}C_{kj} \\ B_{ki}\bar{\Xi}C_j & A_{ki} \end{bmatrix} \\ B_{ij} &= \begin{bmatrix} 0 & 0 \\ B_{ki}(\Xi - \bar{\Xi})C_j & 0 \end{bmatrix} D_{ij} = \begin{bmatrix} D_{1i} \\ B_{ki}D_{2j} \end{bmatrix} \\ \bar{A}_{dmi} &= \begin{bmatrix} \bar{\alpha}_m A_{di} & 0 \\ 0 & 0 \end{bmatrix} \; \tilde{A}_{dmi} = \begin{bmatrix} \tilde{\alpha}_m(k)A_{di} & 0 \\ 0 & 0 \end{bmatrix} \\ \bar{C}_{ij}(k) &= \left[\, C_{zi}(k) \quad B_{2i}C_{kj} \,\right] \end{aligned}$$

with $\tilde{\alpha}_m(k) = \alpha_m(k) - \bar{\alpha}_m$. It is clear that $\mathbb{E}\{\tilde{\alpha}_m(k)\} = 0$ and that $\mathbb{E}\{\tilde{\alpha}_m^2(k)\} = \bar{\alpha}_m(1 - \bar{\alpha}_m)$.

Before formulating the problem to be investigated, we first introduce the following definition.

Definition 1: For the system (8) and every initial conditions ϕ, the trivial solution is said to be exponentially mean square stable if, in the case of $v(k) = 0$, there exist constants $\delta > 0$ and $0 < \kappa < 1$ such that

$$\mathbb{E}\left\{\|\bar{x}(k)\|^2\right\} \leq \delta\kappa^k \sup_{-d_M \leq i \leq 0} \mathbb{E}\left\{\|\phi(i)\|^2\right\} \quad \forall k \geq 0.$$

Our aim in this paper is to develop techniques to deal with the robust H_∞ dynamic output feedback-control problem for the discrete-time fuzzy system (8) such that, for all admissible multiple stochastic communication delays, multiple missing measurements, and uncertain parameters, the following two requirements are satisfied simultaneously.

$R1$): The fuzzy system (8) is exponentially stable in the mean square.

$R2$): Under zero-initial condition, the controlled output $z(k)$ satisfies

$$\sum_{k=0}^{\infty} \mathbb{E}\left\{\|z(k)\|^2\right\} \leqslant \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{\|v(k)\|^2\} \tag{9}$$

for all nonzero $v(k)$, where $\gamma > 0$ is a prescribed scalar.

III. Robust H_∞ Fuzzy-control-performance Analysis

Before proceeding further, we give the following lemmas, which will be used in establishing our main results.

Lemma 1 (Schur complement): Given constant matrices $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$, where $\mathcal{S}_1 = \mathcal{S}_1^T$ and $0 < \mathcal{S}_2 = \mathcal{S}_2^T$, then $\mathcal{S}_1 + \mathcal{S}_3^T \mathcal{S}_2^{-1} \mathcal{S}_3 < 0$ if and only if

$$\begin{bmatrix} \mathcal{S}_1 & \mathcal{S}_3^T \\ \mathcal{S}_3 & -\mathcal{S}_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\mathcal{S}_2 & \mathcal{S}_3 \\ \mathcal{S}_3^T & \mathcal{S}_1 \end{bmatrix} < 0. \tag{10}$$

Lemma 2 (S-procedure): Letting $L = L^T$ and H and E be real matrices of appropriate dimensions with F satisfying $FF^T \leq I$, then $L + HFE + E^T F^T H^T < 0$ if and only if there exists a positive scalar $\varepsilon > 0$ such that $L + \varepsilon^{-1} HH^T + \varepsilon E^T E < 0$ or, equivalently

$$\begin{bmatrix} L & H & \varepsilon E^T \\ H^T & -\varepsilon I & 0 \\ \varepsilon E & 0 & -\varepsilon I \end{bmatrix} < 0. \tag{11}$$

Lemma 3: For any real matrices X_{ij} for $i, j = 1, 2, \ldots, r$, and $n > 0$ with appropriate dimensions, we have [21]

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r} h_i h_j h_k h_l X_{ij}^T \Lambda X_{kl} \leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j X_{ij}^T \Lambda X_{ij}. \tag{12}$$

For convenience of presentation, we first discuss the nominal system of (8) (i.e., without parameter uncertainties $\Delta A_i(k)$ and $\Delta C_{zi}(k)$) and will eventually extend our main results to the general case. We have the following analysis result that serves as a theoretical basis for the subsequent design problem.

Theorem 1: Let us consider the nominal fuzzy system of (8) with given controller parameters and a prescribed H_∞ performance $\gamma > 0$. Then, the nominal fuzzy system of (8) is exponentially stable with disturbance attenuation level γ, if there exist matrices $P > 0$ and $Q_k > 0$ $(k = 1, 2, \ldots, h)$ satisfying

$$\begin{bmatrix} \Pi_i & * \\ 0.5\Sigma_{ii} & \Lambda \end{bmatrix} < 0 \tag{13}$$

$$\begin{bmatrix} 4\Pi_i & * \\ \Sigma_{ij} & \Lambda \end{bmatrix} < 0 \tag{14}$$

where

$$\Pi_i = \mathrm{diag}\left\{\Upsilon_k, \digamma_i, -\gamma^2 I\right\}, \Lambda = \mathrm{diag}\left\{-\check{P}, -P, -I\right\}$$

$$\Sigma_{ij} = \begin{bmatrix} \Phi_{11ij} & \Phi_{12ij} \\ \Phi_{21ij} & D_{3i} + D_{3j} \end{bmatrix}$$

$$\Phi_{11ij} = \begin{bmatrix} \check{C}_{ij} + \check{C}_{ji} & 0 \\ P(A_{ij} + A_{ji}) & P(\hat{Z}_{mi} + \hat{Z}_{mj}) \end{bmatrix}$$

$$\Phi_{12ij} = \begin{bmatrix} 0 \\ P(D_{ij} + D_{ji}) \end{bmatrix} \Phi_{21ij} = \begin{bmatrix} \bar{C}_{ij} + \bar{C}_{ji} & 0 \end{bmatrix}$$

$$\Upsilon_k = \sum_{k=1}^{h} (d_T - d_t + 1)\, Q_k - P$$

$$\hat{C}_{lij} = \begin{bmatrix} 0 & 0 \\ B_{ki}C_{jl} & 0 \end{bmatrix} A_{ij} = \begin{bmatrix} A_i & B_{1i}C_{kj} \\ B_{ki}\bar{\Xi}C_j & A_{ki} \end{bmatrix}$$

$$\check{C}_{ij} = \left[\sigma_1 \hat{C}_{1ij}^T P, \ldots, \sigma_s \hat{C}_{sij}^T P\right]^T \hat{A}_{di} = \begin{bmatrix} A_{di} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\check{P} = \mathrm{diag}\{\underbrace{P, \ldots, P}_{s}\},\ \breve{P} = \mathrm{diag}\{\underbrace{P, \ldots, P}_{h}\}$$

$$\breve{A}_{di} = \mathrm{diag}\{\underbrace{\hat{A}_{di}, \ldots, \hat{A}_{di}}_{h}\}, \hat{Z}_{mi} = \left[\bar{A}_{d1i}, \ldots, \bar{A}_{dhi}\right]$$

$$\bar{C}_{ij} = \begin{bmatrix} C_{zi} & B_{2i}C_{kj} \end{bmatrix}$$

$$\hat{\alpha} = \mathrm{diag}\left\{\bar{\alpha}_1(1 - \bar{\alpha}_1), \ldots, \bar{\alpha}_h(1 - \bar{\alpha}_h)\right\}$$

$$\digamma_i = \hat{\alpha}\breve{A}_{di}^T \breve{P} \breve{A}_{di} - \hat{Q},\ \ \hat{Q} = \mathrm{diag}\left\{Q_1, \ldots, Q_h\right\}. \tag{15}$$

Proof: Let

$$\Theta_j(k) := \{x(k - \tau_j(k)), x(k - \tau_j(k) + 1), \ldots, x(k)\}$$

$$\chi(k) := \{\Theta_1(k) \cup \Theta_2(k) \cup \cdots \cup\ \Theta_h(k)\} = \bigcup_{j=1}^{h} \Theta_j(k)$$

where $j = 1, 2, \ldots, h$.

In order to show that the nominal system of (8) is exponentially stable with disturbance-attenuation level γ under conditions (13) and (14), we choose the following Lyapunov functional for the nominal system of (8):

$$V(\chi(k)) = \sum_{i=1}^{3} V_i(k)$$

where

$$V_1(k) = \bar{x}^T(k) P \bar{x}(k)$$

$$V_2(k) = \sum_{j=1}^{h} \sum_{i=k-\tau_j(k)}^{k-1} \bar{x}^T(i) Q_j \bar{x}(i)$$

$$V_3(k) = \sum_{j=1}^{h} \sum_{m=-d_M+1}^{-d_m} \sum_{i=k+m}^{k-1} \bar{x}^T(i) Q_j \bar{x}(i)$$

with $P > 0,\ Q_j > 0\ (j = 1, 2, \ldots, h)$ being matrices to be determined. Then, along the trajectory of system (8), we have

$$\begin{aligned} \mathbb{E}\{\Delta V | x(k)\} &\overset{\triangle}{=} \mathbb{E}\{V(\chi(k+1)) | \chi(k)\} - V(\chi(k)) \\ &= \mathbb{E}\{(V(\chi(k+1)) - V(\chi(k)) | \chi(k)\} \\ &= \sum_{i=1}^{3} \mathbb{E}\{\Delta V_i | \chi(k)\}. \end{aligned} \tag{16}$$

From (8) and Lemma 3, we can obtain

$$\begin{aligned} &\mathbb{E}\{\Delta V_1 | \chi(k)\} \\ &\quad = \mathbb{E}\{(\bar{x}^T(k+1) P \bar{x}(k+1) - \bar{x}^T(k) P \bar{x}(k)) | \chi(k)\} \\ &\quad = \mathbb{E}\Bigg\{ \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r} h_i h_j h_s h_t \Bigg[(A_{ij} + B_{ij})\, \bar{x}(k) \\ &\qquad + \sum_{m=1}^{h} \left(\bar{A}_{dmi} + \tilde{A}_{dmi}\right) \bar{x}(k - \tau_m(k)) + D_{ij} v(k) \Bigg]^T \\ &\qquad \times P \Bigg[(A_{st} + B_{st})\, \bar{x}(k) \sum_{m=1}^{h} \left(\bar{A}_{dms} + \tilde{A}_{dms}\right) \end{aligned}$$

$$
\begin{aligned}
&\times \bar{x}(k-\tau_m(k)) + D_{st}v(k)\Big]\Big|\chi(k)\Big\} - \bar{x}^T(k)P\bar{x}(k) \\
&\le \mathbb{E}\Big\{\sum_{i=1}^r\sum_{j=1}^r\sum_{s=1}^r\sum_{t=1}^r h_ih_jh_sh_t\Big[\bar{x}^T(k)\left(A_{ij}^TPA_{st} - P\right. \\
&\left. + B_{ij}^TPB_{ij}\right)\bar{x}(k) + 2\bar{x}^T(k)A_{ij}^TPD_{st}v(k) + 2\bar{x}^T(k) \\
&\times A_{ij}^TP\left(\sum_{m=1}^h \bar{A}_{dms}\bar{x}(k-\tau_m(k))\right) + v^T(k)D_{ij}^TPD_{st} \\
&\times v(k) + 2\left(\sum_{m=1}^h \bar{x}^T(k-\tau_m(k))\bar{A}_{dmi}^T\right)PD_{st}v(k) \\
&+ \left(\sum_{m=1}^h \bar{x}^T(k-\tau_m(k))\bar{A}_{dmi}^T\right)P\left(\sum_{m=1}^h \bar{A}_{dms}\right. \\
&\left. \times \bar{x}(k-\tau_m(k))\right) + \sum_{m=1}^h \bar{x}^T(k-\tau_m(k)) \\
&\times \tilde{A}_{dmi}^TP\tilde{A}_{dmi}\bar{x}(k-\tau_m(k))\Big]\Big\}
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
&\mathbb{E}\left\{B_{ij}^TPB_{ij}\right\} \\
&= \sum_{l=1}^s \sigma_l^2\begin{bmatrix} 0 & 0 \\ B_{ki}C_{jl} & 0\end{bmatrix}^T P \begin{bmatrix} 0 & 0 \\ B_{ki}C_{jl} & 0\end{bmatrix} \\
&= \sum_{l=1}^s \sigma_l^2 \hat{C}_{lij}^TP\hat{C}_{lij}
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
&\mathbb{E}\left\{\tilde{A}_{mdi}^TP\tilde{A}_{mdi}\right\} \\
&= \mathbb{E}\left\{\begin{bmatrix} \tilde{\alpha}_m(k)A_{di} & 0 \\ 0 & 0\end{bmatrix}^T P \begin{bmatrix} \tilde{\alpha}_m(k)A_{di} & 0 \\ 0 & 0\end{bmatrix}\right\} \\
&= \bar{\alpha}_m(1-\bar{\alpha}_m)\begin{bmatrix} A_{di} & 0 \\ 0 & 0\end{bmatrix}^T P \begin{bmatrix} A_{di} & 0 \\ 0 & 0\end{bmatrix} \\
&= \bar{\alpha}_m(1-\bar{\alpha}_m)\hat{A}_{di}^TP\hat{A}_{di}.
\end{aligned} \tag{19}
$$

Taking (17)–(19) into consideration, we have

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_1|\chi(k)\} \\
&\le \sum_{i=1}^r\sum_{j=1}^r\sum_{s=1}^r\sum_{t=1}^r h_ih_jh_sh_t\Big[\bar{x}^T(k)\Big(A_{ij}^TPA_{st} - P \\
&\quad + \sum_{l=1}^s \sigma_l^2\hat{C}_{lij}^TP\hat{C}_{lij}\bar{x}(k)\Big) + 2\bar{x}^T(k)A_{ij}^TPD_{st}v(k) \\
&\quad + 2\bar{x}^T(k)A_{ij}^TP\left(\sum_{m=1}^h \bar{A}_{dms}\bar{x}(k-\tau_m(k))\right) \\
&\quad + \left(\sum_{m=1}^h \bar{x}^T(k-\tau_m(k))\bar{A}_{dmi}^T\right)P\Big(\sum_{m=1}^h \bar{A}_{dms} \\
&\quad \times \bar{x}(k-\tau_m(k))\Big) + \sum_{m=1}^h \bar{\alpha}_m(1-\bar{\alpha}_m) \\
&\quad \times \bar{x}^T(k-\tau_m(k))\hat{A}_{di}^TP\hat{A}_{di}\bar{x}^T(k-\tau_m(k)) \\
&\quad + 2\left(\sum_{m=1}^h \bar{A}_{dmi}\bar{x}(k-\tau_m(k))\right)^T PD_{st}v(k) \\
&\quad + v^T(k)D_{ij}^TPD_{st}v(k)\Big].
\end{aligned} \tag{20}
$$

Next, it can be derived that

$$
\begin{aligned}
&\mathbb{E}\left\{\Delta V_2\Big|\chi(k)\right\} \\
&\quad \le \mathbb{E}\left\{\sum_{j=1}^h\Big(\bar{x}^T(k)Q_j\bar{x}(k) - \bar{x}^T(k-\tau_j(k))Q_j\right. \\
&\quad \left. \times \bar{x}(k-\tau_j(k)) + \sum_{i=k-d_M+1}^{k-d_m}\bar{x}^T(i)Q_j\bar{x}(i)\Big)\Big|\chi(k)\right\} \\
&\mathbb{E}\left\{\Delta V_3\Big|\chi(k)\right\} \\
&\quad = \mathbb{E}\Big\{\sum_{j=1}^h\Big((d_T - d_t)\bar{x}^T(k)Q_j\bar{x}(k) - \sum_{i=k-d_M+1}^{k-d_m}\bar{x}^T(i) \\
&\quad \times Q_j\bar{x}(i)\Big)\Big|\chi(k)\Big\}.
\end{aligned}
$$

For notational convenience, we denote the following matrix variables:

$$
\begin{aligned}
\xi(k) &= [\,\bar{x}^T(k) \quad \bar{x}^T(k-\tau) \quad v^T(k)\,]^T \\
\hat{\xi}(k) &= [\,\bar{x}^T(k) \quad \bar{x}^T(k-\tau)\,]^T \; \Omega_{ij} = [\,\bar{A}_{ij}^T \quad \tilde{C}_{ij}^T\,]^T \\
\bar{A}_{ij} &= [\,A_{ij} \quad \hat{Z}_{mi} \quad D_{ij}\,] \; \tilde{A}_{ij} = [\,A_{ij} \quad \hat{Z}_{mi}\,] \\
\bar{Z}_{ij} &= [\,\check{P}^{-1}\check{C}_{ij} \quad 0 \quad 0\,] \; \tilde{Z}_{ij} = [\,\check{P}^{-1}\check{C}_{ij} \quad 0\,] \\
\tilde{C}_{ij} &= [\,\bar{C}_{ij} \quad 0 \quad D_{3i}\,] \; \tilde{P} = \mathrm{diag}\{P, I\} \\
\bar{x}(k-\tau) &= [\,\bar{x}^T(k-\tau_1(k)), \ldots, \bar{x}^T(k-\tau_h(k))\,]^T \\
\hat{P} &= \mathrm{diag}\{\Upsilon_k, \digamma_i\} \quad \bar{P} = \mathrm{diag}\left\{\Upsilon_k, \digamma_i, -\gamma^2 I\right\}.
\end{aligned} \tag{21}
$$

In the following, we first prove the exponential stability of the nominal fuzzy system of (8) with $v(k) = 0$. It follows from Lemma 3 that

$$\begin{aligned}
&\mathbb{E}\{\Delta V|x(k)\}\\
&\leq \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r} h_i h_j h_s h_t \hat{\xi}^T(k)\big(\tilde{A}_{ij}^T P \tilde{A}_{st} + \tilde{Z}_{ij}^T \check{P} \tilde{Z}_{ij}\\
&\quad + \hat{P}\big)\hat{\xi}(k)\\
&\leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \hat{\xi}^T(k)\big(\tilde{A}_{ij}^T P \tilde{A}_{ij} + \tilde{Z}_{ij}^T \check{P} \tilde{Z}_{ij} + \hat{P}\big)\hat{\xi}(k)\\
&= \sum_{i=1}^{r} h_i^2 \hat{\xi}^T(k)\big(\tilde{A}_{ii}^T P \tilde{A}_{ii} + \tilde{Z}_{ii}^T \check{P} \tilde{Z}_{ii} + \hat{P}\big)\hat{\xi}(k)\\
&\quad + \frac{1}{2}\sum_{i,j=1,i<j}^{r} h_i h_j \hat{\xi}^T(k)\Big[\big(\tilde{A}_{ij} + \tilde{A}_{ji}\big)^T P \big(\tilde{A}_{ij} + \tilde{A}_{ji}\big)\\
&\quad + \big(\tilde{Z}_{ij} + \tilde{Z}_{ji}\big)^T \check{P} \big(\tilde{Z}_{ij} + \tilde{Z}_{ji}\big) + 4\hat{P}\Big]\hat{\xi}(k). \qquad (22)
\end{aligned}$$

By the Schur complement, we know that $\mathbb{E}\{\Delta V|x(k)\} < 0$ if and only if (13) and (14) are true. Furthermore, from [30, Th. 1], it can be concluded that the discrete-time nominal fuzzy system of (8) with $v(k) = 0$ is exponentially stable.

Let us now deal with the H_∞ performance of the nominal fuzzy system of (8). Let us assume zero-initial condition and introduce the following index:

$$J(n) = \mathbb{E}\sum_{k=0}^{n}\left[z^T(k)z(k) - \gamma^2 v^T(k)v(k)\right]$$

where n is a nonnegative integer. Obviously, our aim is to show $J(n) < 0$ under the zero-initial condition. From (8) and (17)–(22), we have

$$\begin{aligned}
J(n) &= \mathbb{E}\sum_{k=0}^{n}\left[z^T(k)z(k) - \gamma^2 v^T(k)v(k) + \Delta V(\chi(k))\right]\\
&\quad - \mathbb{E}V(\chi(n+1))\\
&\leq \mathbb{E}\sum_{k=0}^{n}\left[z^T(k)z(k) - \gamma^2 v^T(k)v(k) + \Delta V(\chi(k))\right]\\
&\leq \mathbb{E}\sum_{k=0}^{n}\Bigg\{\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r} h_i h_j h_s h_t \Bigg(\Bigg[(A_{ij} + B_{ij})\\
&\quad \times \bar{x}(k) + \sum_{m=1}^{h}\left(\bar{A}_{dmi} + \tilde{A}_{dmi}\right)\bar{x}(k - \tau_m(k))\\
&\quad + D_{ij}v(k)\Bigg]^T P\Bigg[(A_{st} + B_{st})\,\bar{x}(k) + D_{st}v(k)\\
&\quad + \sum_{m=1}^{h}\left(\bar{A}_{dms} + \tilde{A}_{dms}\right)\bar{x}(k - \tau_m(k))\Bigg]\\
&\quad + \left[\bar{C}_{ij}\bar{x}(k) + D_{3i}v(k)\right]^T\left[\bar{C}_{st}\bar{x}(k) + D_{3s}v(k)\right]\Bigg)\\
&\quad - \bar{x}^T(k)P\bar{x}(k) + \bar{x}^T(k)\left(\sum_{m=1}^{h}(d_T - d_t + 1)Q_m\right)\\
&\quad \times \bar{x}(k) - \sum_{m=1}^{h}\bar{x}^T(k - \tau_m(k))Q_m\bar{x}(k - \tau_m(k))\\
&\quad - \gamma^2 v^T(k)v(k)\Bigg\}\\
&= \sum_{k=0}^{n}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r} h_i h_j h_s h_t \xi^T(k)(\Omega_{ij}^T \tilde{P}\Omega_{st}\\
&\quad + \bar{Z}_{ij}^T \check{P}\bar{Z}_{ij} + \bar{P})\xi(k)\\
&\leq \sum_{k=0}^{n}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \xi^T(k)\big(\Omega_{ij}^T \tilde{P}\Omega_{ij}\\
&\quad + \bar{Z}_{ij}^T \check{P}\bar{Z}_{ij} + \bar{P}\big)\xi(k)\\
&= \sum_{k=0}^{n}\sum_{i=1}^{r} h_i^2 \xi^T(k)\left(\Omega_{ii}^T \tilde{P}\Omega_{ii} + \bar{Z}_{ii}^T \check{P}\bar{Z}_{ii} + \bar{P}\right)\xi(k)\\
&\quad + \frac{1}{2}\sum_{k=0}^{n}\sum_{i,j=1,i<j}^{r} h_i h_j \xi^T(k)\Big[(\Omega_{ij} + \Omega_{ji})^T\,\tilde{P}\\
&\quad \times (\Omega_{ij} + \Omega_{ji}) + \left(\bar{Z}_{ij} + \bar{Z}_{ji}\right)^T \check{P}\left(\bar{Z}_{ij} + \bar{Z}_{ji}\right)\\
&\quad + 4\bar{P}\Big]\xi(k). \qquad (23)
\end{aligned}$$

From Schur complement Lemma, we can conclude from (13) and (14) that $J(n) < 0$. Letting $n \to \infty$, we obtain

$$\sum_{k=0}^{\infty}\mathbb{E}\left\{\|z(k)\|^2\right\} \leq \gamma^2\sum_{k=0}^{\infty}\mathbb{E}\left\{\|v(k)\|^2\right\}$$

which completes the proof of Theorem 1. ■

Remark 3: In Theorem 1, with given controller gain and disturbance attenuation level γ, we obtain the stochastic-stability conditions of the nominal fuzzy system (8), which are represented via a set of matrix inequalities in (13) and (14). We will show later in the following section that such inequalities can be converted into linear matrix inequalities (LMIs) when designing the actual controllers. Note that the feasibility of LMIs can be easily checked by using the MATLAB LMI toolbox.

IV. Robust H_∞ Fuzzy-controller Design

In this section, we aim to solve the robust H_∞ fuzzy output-feedback controller design problem for the system (8). That is, we are interested in determining the controller parameters in (7) such that the closed-loop fuzzy system in (8) is exponentially stable with a guaranteed H_∞ performance. The following theorem provides sufficient conditions for the existence of such an H_∞-fuzzy controller for the nominal fuzzy system of (8).

Theorem 2: Let us consider the nominal fuzzy system of (8). For a prescribed constant $\gamma > 0$, if there exist positive definite matrices $P > 0$, $L > 0$, $Q_k > 0$ $(k = 1, 2, \ldots h)$, and matrices K_i and $\bar{C}_{ki}$ such that

$$\Gamma_1 \triangleq \begin{bmatrix} \Pi_i & * \\ 0.5\bar{\Sigma}_{ii} & \bar{\Lambda} \end{bmatrix} < 0 \quad i = 1, 2, \ldots, r. \tag{24}$$

$$\Gamma_2 \triangleq \begin{bmatrix} 4\Pi_i & * \\ \bar{\Sigma}_{ij} & \bar{\Lambda} \end{bmatrix} < 0 \quad 1 \le i < j \le r. \tag{25}$$

$$PL = I \tag{26}$$

hold, then the nominal system of (8) is exponentially stable with disturbance attenuation γ, where Π_i is defined in Theorem 1, and

$$\begin{aligned}
\bar{\Lambda} &= \operatorname{diag}\left\{-\bar{L}, -L, -I\right\} \\
\bar{\Sigma}_{ij} &= \begin{bmatrix} \bar{\Phi}_{11ij} + \bar{\Phi}_{11ji} & \bar{\Phi}_{12ij} + \bar{\Phi}_{12ji} \\ \bar{\Phi}_{21ij} + \bar{\Phi}_{21ji} & D_{3i} + D_{3j} \end{bmatrix} \\
\bar{\Phi}_{11ij} &= \begin{bmatrix} \check{C}_{ij} & 0 \\ \bar{A}_i + \bar{E}K_i\bar{R}_j + \bar{B}_{1i}\bar{C}_{kj} & \hat{Z}_{mi} \end{bmatrix} \\
\bar{\Phi}_{12ij} &= \begin{bmatrix} 0 \\ \bar{D}_{1i} + \bar{E}K_i\bar{D}_{2j} \end{bmatrix} \\
\bar{\Phi}_{21ij} &= \left[\, \bar{C}_{zi} + B_{2i}\bar{C}_{kj} \quad 0 \,\right] \\
\bar{A}_i &= \begin{bmatrix} A_i & 0 \\ 0 & 0 \end{bmatrix} \; \bar{E} = \begin{bmatrix} 0 \\ I \end{bmatrix} \; \bar{B}_{1i} = \begin{bmatrix} B_{1i} \\ 0 \end{bmatrix} \\
R_{il} &= \begin{bmatrix} 0 & 0 \\ C_{il} & 0 \end{bmatrix} \bar{D}_{1i} = \begin{bmatrix} D_{1i} \\ 0 \end{bmatrix} \bar{D}_{2i} = \begin{bmatrix} 0 \\ D_{2i} \end{bmatrix} \\
K_i &= \left[\, A_{ki} \quad B_{ki} \,\right] \; \bar{C}_{ki} = \left[\, 0 \quad C_{ki} \,\right] \\
\bar{C}_{zi} &= \left[\, C_{zi} \quad 0 \,\right] \quad \bar{L} = \operatorname{diag}\{\underbrace{L, \ldots, L}_{s}\} \\
\bar{R}_i &= \begin{bmatrix} 0 & I \\ \bar{\Xi}C_i & 0 \end{bmatrix} \quad \check{C}_{ij} = \begin{bmatrix} \sigma_1\bar{E}K_i\hat{R}_{j1} \\ \cdots \\ \sigma_s\bar{E}K_i\hat{R}_{js} \end{bmatrix}.
\end{aligned} \tag{27}$$

Furthermore, if $(P, L, Q_k, K_i, \bar{C}_{ki})$ is a feasible solution of (24)–(26), then the controller parameters in the form of (7) can be easily obtained by K_i and $\bar{C}_{ki}$.

Proof: In order to avoid partitioning the positive define matrices P and Q_k, we rewrite the parameters in Theorem 1 in the following form:

$$\begin{aligned}
A_{ij} &= \bar{A}_i + \bar{E}K_i\bar{R}_j + \bar{B}_{1i}\bar{C}_{kj}, \quad \hat{C}_{lij} = \bar{E}K_i\hat{R}_{jl} \\
\bar{C}_{ij} &= \bar{C}_{zi} + B_{2i}\bar{C}_{kj}, \quad D_{ij} = \bar{D}_{1i} + \bar{E}K_i\bar{D}_{2j}.
\end{aligned} \tag{28}$$

Noticing (28), (13) and (14) can be rewritten as

$$\begin{bmatrix} \Pi_i & * \\ -0.5\Lambda\bar{\Sigma}_{ii} & \Lambda \end{bmatrix} < 0 \tag{29}$$

$$\begin{bmatrix} 4\Pi_i & * \\ -\Lambda\bar{\Sigma}_{ij} & \Lambda \end{bmatrix} < 0. \tag{30}$$

Pre- and post-multiplying the inequalities (29) and (30) by $\operatorname{diag}\{I, I, I, \check{P}^{-1}, P^{-1}, I\}$ and letting $L = P^{-1}$, we can obtain (24) and (25) readily, and the proof is then complete. ■

Having established the main results for nominal systems, we are now in a position to show that the robust H_∞ controller parameters can be determined based on the results of Theorem 2.

Theorem 3: Let us consider the uncertain fuzzy system (8). For a prescribed constant $\gamma > 0$, if there exist positive definite matrices $P > 0, L > 0, Q_k > 0$ $(k = 1, 2, \ldots, h)$, and matrices $K_i, \bar{C}_{ki}$ and a positive constant scalar $\varepsilon > 0$ such that

$$\begin{bmatrix} \Pi_i & * & * \\ 0.5\bar{\Sigma}_{ii} & \bar{\Lambda} & * \\ \hat{\Sigma} & 0.5\tilde{\Sigma}_{ii} & \Xi \end{bmatrix} < 0 \quad i = 1, 2, \ldots, r \tag{31}$$

$$\begin{bmatrix} 4\Pi_i & * & * \\ \bar{\Sigma}_{ij} & \bar{\Lambda} & * \\ \hat{\Sigma} & \tilde{\Sigma}_{ij} & \Xi \end{bmatrix} < 0 \quad 1 \le i < j \le r \tag{32}$$

$$PL = I \tag{33}$$

hold, where Π_i is defined in Theorem 1 and

$$\begin{aligned}
\hat{\Sigma} &= \left[\, \hat{\Phi} \quad 0 \,\right] \;\; \hat{\Phi} = \begin{bmatrix} 0 \\ \varepsilon\hat{E} \end{bmatrix} \\
\tilde{\Sigma}_{ij} &= \begin{bmatrix} 0 & \bar{H}_{ai}^T + \bar{H}_{aj}^T & \bar{H}_{ci}^T + \bar{H}_{cj}^T \\ 0 & 0 & 0 \end{bmatrix} \\
\Xi &= \operatorname{diag}\{-\varepsilon I, -\varepsilon I\}
\end{aligned}$$

then the system (8) is exponentially stable with disturbance attenuation γ. Furthermore, if $(P, L, Q_k, K_i, \bar{C}_{ki})$ is a feasible solution of (31)–(33), then the controller parameters in the form of (7) can be obtained directly from K_i and $\bar{C}_{ki}$.

Proof: In (24) and (25), let us replace $\bar{A}_i, \bar{A}_j, \bar{C}_{zi}$, and $\bar{C}_{zj}$ with $\bar{A}_i + \Delta\bar{A}_i(k), \bar{A}_j + \Delta\bar{A}_j(k), \bar{C}_{zi} + \Delta\bar{C}_{zi}(k)$, and $\bar{C}_{zj} + \Delta\bar{C}_{zj}(k)$, respectively, where

$$\Delta\bar{A}_i(k) = \begin{bmatrix} \Delta A_i(k) & 0 \\ 0 & 0 \end{bmatrix} \Delta\bar{C}_{zi}(k) = \left[\, \Delta C_{zi}(k) \quad 0 \,\right].$$

Then, we rewrite (24) and (25) in terms of Lemma 2 as follows:

$$\begin{aligned}
\Gamma_1 + \check{H}_1F(k)\check{E} + \check{E}^TF^T(k)\check{H}_1^T &< 0 \\
\Gamma_2 + \check{H}_2F(k)\check{E} + \check{E}^TF^T(k)\check{H}_2^T &< 0
\end{aligned}$$

where Γ_1 and Γ_2 have been defined in (24) and (25), and

$$\begin{aligned}\check{H}_1 &= [0 \quad 0 \quad 0 \quad 0 \quad \bar{H}_{ai}^T \quad \bar{H}_{ci}^T\,]^T\\ \bar{H}_{ai} &= [\,H_{ai}^T \quad 0]^T \;\; \bar{H}_{ci} = H_{ci}\\ \check{E} &= [\,\hat{E} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \;\; \hat{E} = [\,E \quad 0]\\ \check{H}_2 &= [0 \quad 0 \quad 0 \quad 0 \quad (\bar{H}_{ai}+\bar{H}_{aj})^T \quad (\bar{H}_{ci}+\bar{H}_{cj})^T\,]^T.\end{aligned}$$

From Lemma 1 and Lemma 2, we can easily obtain (31) and (32), and the proof is then complete. ■

Note that there is a matrix equality in Theorem 3, which gives rise to significant difficulty in numerical computation. Nevertheless, such difficulty can be overcome by using the cone-complementarity linearization (CCL) algorithm proposed in [6] and [9]. As described in [6] and [9], the CCL algorithm can convert the LMIs with equality constraints to strict LMIs under mild conditions, which can then be easily solvable via the MATLAB LMI toolbox. To be specific, the key point of the CCL algorithm is that if the LMI $\begin{bmatrix} P & I \\ I & L \end{bmatrix} \geq 0$ is feasible in the $n \times n$ matrix variables $L > 0$ and $P > 0$, then $\mathrm{tr}(PL) \geq n$, and $\mathrm{tr}(PL) = n$ if and only if $PL = I$. Based on such a statement, it is likely to solve the equalities in (33) by using of CCL algorithm. In view of this observation, we put forward the following nonlinear minimization problem involving LMI conditions instead of the original nonconvex feasibility problem formulated in Theorem 3.

The nonlinear minimization problem: min $\mathrm{tr}(PL)$ subject to (31) and (32) and

$$\begin{bmatrix} P & I \\ I & L \end{bmatrix} \geq 0. \tag{34}$$

If the solution of $\min \mathrm{tr}(PL)$ subject to (31) and (32) exists and $\min\mathrm{tr}(PL) = n$, then the conditions in Theorem 3 are solvable.

Finally, the following algorithm is suggested to solve the above problem.

Algorithm 1: HinfFC (HinfFC: H_∞ Fuzzy Control)

Step 1: Find a feasible set $(P_{(0)}, L_{(0)}, Q_{k(0)}, K_{i(0)}, \bar{C}_{ki(0)})$ satisfying (31), (32), and (34). Set $q = 0$.

Step 2: According to (31), (32), and (34), and solving the LMI problem, $\min \mathrm{tr}(PL_{(q)} + P_{(q)}L)$.

Step 3: Substitute the obtained matrix variables $(P, L, Q_k, K_i, \bar{C}_{ki})$ into (13) and (14). If conditions (13) and (14) are satisfied with $|\mathrm{tr}(PL) - n| < \delta$ for some sufficiently small scalar $\delta > 0$, then output the feasible solutions. Exit.

Step 4: If $q > N$, where N is the maximum number of iterations allowed, then output the feasible solutions $(P, L, Q_k, K_i, \bar{C}_{ki})$, and exit. Else, set $q = q + 1$, and go to Step 2.

V. Illustrative Examples

In this section, we present two illustrative examples to demonstrate the effectiveness of the developed method.

Example 1: Consider a T–S fuzzy model (1) with multiple communication delays and multiple missing measurements. The rules are given as follows:

Plant Rule 1: IF $x_1(k)$ is $h_1(x_1(k))$ THEN

$$\begin{cases} x(k+1) = A_1(k)x(k) + A_{d1}\sum\limits_{m=1}^{h}\alpha_m(k)x(k-\tau_m(k)) \\ \qquad\qquad + B_{11}u(k) + D_{11}v(k) \\ y(k) = \Xi C_1 x(k) + D_{21}v(k) \\ z(k) = C_{z1}(k)x(k) + B_{21}u(k) + D_{31}v(k). \end{cases} \tag{35}$$

Plant Rule 2: IF $x_1(k)$ is $h_2(x_1(k))$ THEN

$$\begin{cases} x(k+1) = A_2(k)x(k) + A_{d2}\sum\limits_{m=1}^{h}\alpha_m(k)x(k-\tau_m(k)) \\ \qquad\qquad + B_{12}u(k) + D_{12}v(k) \\ y(k) = \Xi C_2 x(k) + D_{22}v(k) \\ z(k) = C_{z2}(k)x(k) + B_{22}u(k) + D_{32}v(k). \end{cases} \tag{36}$$

The model parameters are given as follows:

$$A_1 = \begin{bmatrix} 1 & 0.2 & 0 \\ 0.1 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.2 \end{bmatrix} D_{11} = \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} 0.03 & 0 & -0.01 \\ 0.02 & 0.03 & 0 \\ 0.04 & 0.05 & -0.1 \end{bmatrix}$$

$$B_{11} = \begin{bmatrix} 1 & 1 \\ 0.4 & 1 \\ 0 & 1 \end{bmatrix} D_{31} = \begin{bmatrix} -0.1 \\ 0 \\ 0.1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0.8 & 0.7 \\ -0.6 & 0.9 & 0.6 \end{bmatrix} D_{21} = \begin{bmatrix} 0.15 \\ 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0.1 & 0.8 & 0.7 \\ -0.6 & 0.9 & 0.6 \end{bmatrix} D_{22} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$$

$$C_{z1} = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} B_{21} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$H_{a1} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix} \quad H_{c1} = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix} \quad H_{a2} = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix}$$

$$E = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}^T \quad H_{c2} = \begin{bmatrix} 0.1 \\ 0 \\ 0.5 \end{bmatrix} \quad D_{32} = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & -0.38 & 0 \\ -0.2 & 0 & 0.21 \\ 0.1 & 0 & -0.55 \end{bmatrix} \quad B_{12} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A_{d2} = \begin{bmatrix} 0 & 0.01 & -0.01 \\ 0.02 & 0.03 & 0 \\ 0.04 & 0.05 & -0.1 \end{bmatrix} D_{12} = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix}$$

$$C_{z2} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0.2 & 0 & 0.2 \\ 0 & 0.1 & 0.2 \end{bmatrix} \quad B_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}.$$

Let us assume that the time-varying communication delays satisfy $2 \le \tau_m(k) \le 6$ $(m = 1, 2)$ and

$$\bar{\alpha}_1 = \mathbb{E}\{\alpha_1(k)\} = 0.8, \quad \bar{\alpha}_2 = \mathbb{E}\{\alpha_2(k)\} = 0.6.$$

Let the probabilistic density functions of β_1 and β_2 in [0 1] be described by

$$q_1(s_1) = \begin{cases} 0, & s_1 = 0 \\ 0.1, & s_1 = 0.5 \\ 0.9, & s_1 = 1 \end{cases}$$
$$q_2(s_2) = \begin{cases} 0.1, & s_2 = 0 \\ 0.1, & s_2 = 0.5 \\ 0.8, & s_2 = 1 \end{cases} \tag{37}$$

from which the expectations and variances can be easily calculated as $\mu_1 = 0.95$, $\mu_2 = 0.85$, $\sigma_1 = 0.15$, and $\sigma_2 = 0.32$.

The membership function is assumed to be

$$h_1 = \begin{cases} 1, & x_0(1) = 0 \\ |\sin(x_0(1))|/x_0(1), & \text{else} \end{cases}$$
$$h_2 = 1 - h_1. \tag{38}$$

Our aim is to design a dynamic-output feedback paralleled controller in the form of (7) such that the system (8) is exponentially stable with a guaranteed H_∞ norm bound γ.

Letting $\gamma = 0.9$ and applying Theorem 3 with help from Algorithm 1, we can obtain the desired H_∞ controller parameters as follows (other matrices are omitted for space saving):

$$A_{k1} = \begin{bmatrix} -0.3671 & 0.0015 & 0.1389 \\ -0.2568 & 0.0028 & 0.1000 \\ -0.1402 & -0.0063 & 0.0417 \end{bmatrix}$$

$$A_{k2} = \begin{bmatrix} -0.5428 & -0.0071 & 0.2040 \\ -0.4209 & -0.0074 & 0.1557 \\ -0.0515 & 0.0069 & 0.0292 \end{bmatrix}$$

$$B_{k1} = \begin{bmatrix} 0.0245 & 0.1445 \\ 0.0187 & 0.1091 \\ 0.0031 & 0.0246 \end{bmatrix}$$

$$B_{k2} = \begin{bmatrix} 0.1098 & -0.1159 \\ 0.0426 & -0.1057 \\ 0.1741 & 0.0509 \end{bmatrix}$$

$$C_{k1} = \begin{bmatrix} -0.8083 & -0.0014 & 0.3029 \\ 0.2878 & 0.0006 & -0.1093 \end{bmatrix}$$

$$C_{k2} = \begin{bmatrix} -0.4780 & 0.0039 & 0.1819 \\ 0.6162 & 0.0004 & -0.2310 \end{bmatrix}.$$

For simulation purposes, we set the initial condition as

$$x(0) = [1 \quad 0 \quad -1]^T \quad x_c(0) = [0 \quad 0 \quad 0]^T \tag{39}$$

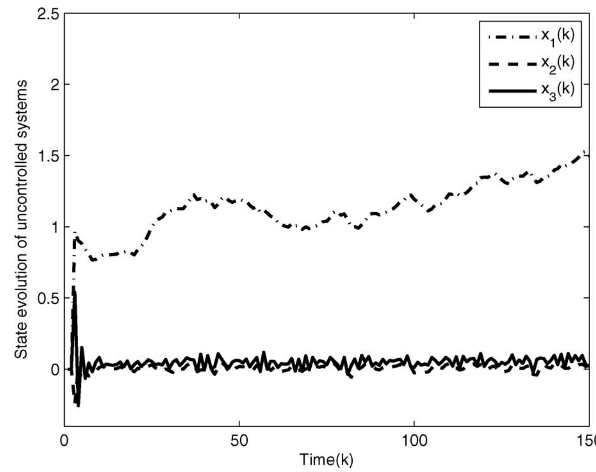

Fig. 2. State evolution $x(k)$ of uncontrolled systems.

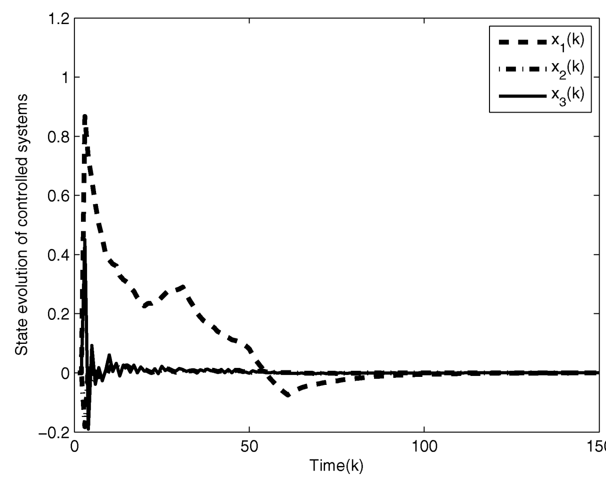

Fig. 3. State evolution $x(k)$ of controlled systems.

and the external disturbance as $v(k) \equiv 0$. Fig. 2 gives the state responses for the uncontrolled fuzzy systems, which are apparently unstable. Fig. 3 gives the state simulation results of the closed-loop fuzzy system, from which we can see that the closed-loop system is exponentially stable.

Next, to illustrate the disturbance-attenuation performance, the initial condition is chosen as $x(0) = 0$, $x_c(0) = 0$, and the external disturbance $v(k)$ is assumed to be

$$v(k) = \begin{cases} 0.3, & 20 \le k \le 30 \\ -0.2, & 50 \le k \le 60 \\ 0, & \text{else.} \end{cases} \tag{40}$$

Fig. 4 shows the controller output, Fig. 5 plots the controller state, and Fig. 6 depicts the disturbance input $v(k)$ and controlled output $z(k)$. Fig. 7 shows the time-varying delays $\tau_m(k)$ $(m = 1, 2)$. All the simulation results have confirmed our theoretical analysis for the robust H_∞ fuzzy-control problem for discrete-time fuzzy systems with multiple time-varying random communication delays and multiple missing measurements.

Example 2: In this example, we consider an uncertain nonlinear mass-spring-damper mechanical system [27], [35]

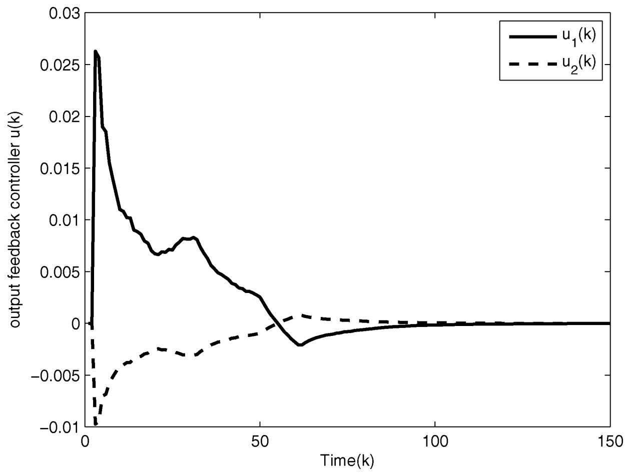

Fig. 4. Output feedback controllers $u(k)$.

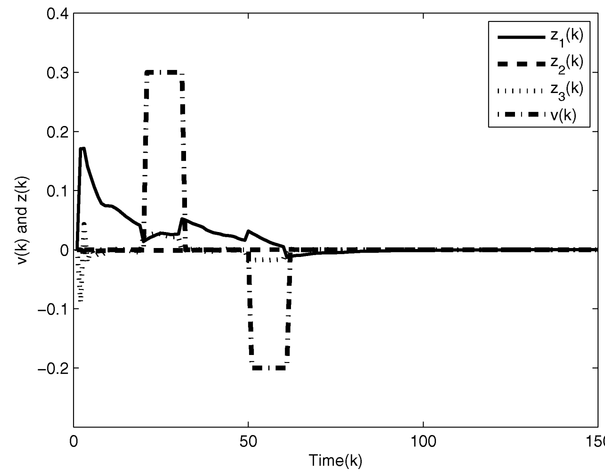

Fig. 6. Controlled output $z(k)$ and disturbance input $v(k)$.

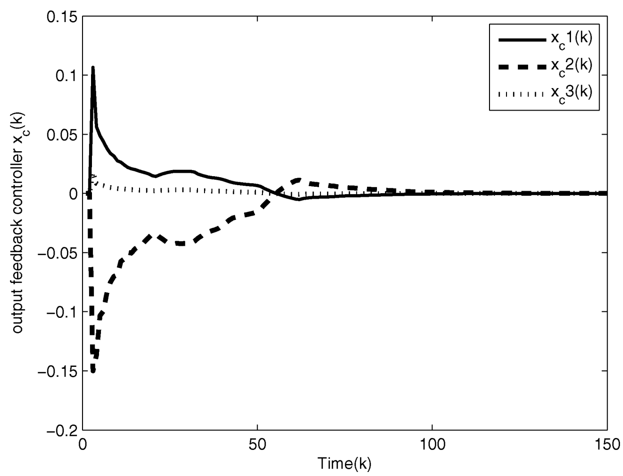

Fig. 5. Controller-state evolution $x_c(k)$.

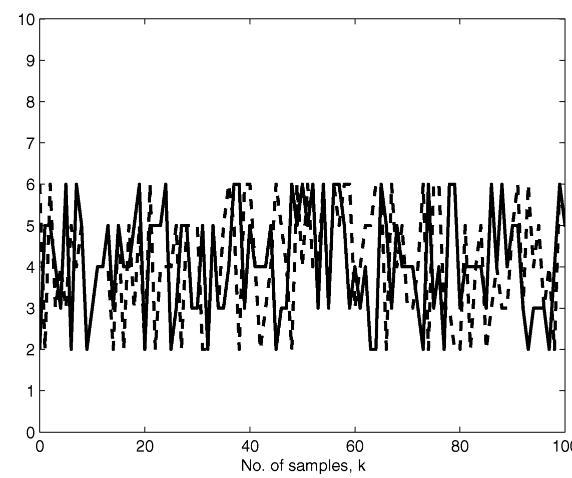

Fig. 7. Time-varying delays $\tau_i(k)(i = 1, 2)$.

controlled through a network, whose dynamic equation is given as follows:

$$\ddot{x}(t) = c(t)\dot{x}(t) - 0.02x(t) - 0.67x^3(t) + u(t). \tag{41}$$

Let us assume that $x(t) \in [-1.5, 1.5]$, $\dot{x}(t) \in [-1.5, 1.5]$ and that $c(t)\dot{x}(t) = -0.1\dot{x}^3(t)$, where $c(t)$ is the uncertain term, and $c(t) \in [-0.225, 0]$.

Let us consider the following controlled and measurement outputs:

$$\begin{aligned} z(t) &= [\, x^T(t) \quad u^T(t) \quad v^T(t) \,]^T \\ y(t) &= [\, x^T(t) \quad v^T(t) \,]^T . \end{aligned} \tag{42}$$

Using the same procedure as in [27], the nonlinear term $-0.67x^3(t)$ can be represented as

$$-0.67x^3 = M_1(x) \cdot 0 \cdot x - M_2(x) \cdot 1.5075x,$$

where $M_1(x), M_2(x) \in [0, 1]$, and $M_1(x) + M_2(x) = 1$. By solving the equations, $M_1(x)$ and $M_2(x)$ are obtained as follows:

$$M_1(x) = 1 - \frac{x^2(t)}{2.25},\ M_2(x) = \frac{x^2(t)}{2.25}.$$

M_1, M_2 can be interpreted as membership functions of fuzzy sets. By using these fuzzy sets and set sampling time $T = 0.02$, the uncertain nonlinear system (41) and (42) can be represented by the following T–S fuzzy model:

`Plant Rule 1`: IF $x(k)$ is $M_1(x)$ THEN

$$\begin{cases} x(k+1) = (A_1 + \Delta A_1(k))\, x(k) + B_{11}u(k) + D_{11}v(k) \\ \qquad + A_{d1} \sum_{m=1}^{h} \alpha_m(k) x(k - \tau_m(k)) \\ y(k) = \Xi C_1 x(k) + D_{21} v(k) \\ z(k) = (C_{z1} + \Delta C_{z1}(k))\, x(k) + B_{21}u(k) + D_{31}v(k). \end{cases} \tag{43}$$

`Plant Rule 2`: IF $x(k)$ is $M_2(x)$ THEN

$$\begin{cases} x(k+1) = (A_2 + \Delta A_2(k))\, x(k) + B_{12}u(k) + D_{12}v(k) \\ \qquad + A_{d2} \sum_{m=1}^{h} \alpha_m(k) x(k - \tau_m(k)) \\ y(k) = \Xi C_2 x(k) + D_{22} v(k) \\ z(k) = (C_{z2} + \Delta C_{z2}(k))\, x(k) + B_{22}u(k) + D_{32}v(k) \end{cases} \tag{44}$$

where

$$A_1 = \begin{bmatrix} -0.1125 & -0.02 \\ 1 & 0 \end{bmatrix} \; A_2 = \begin{bmatrix} -0.1125 & -1.527 \\ 1 & 0 \end{bmatrix}$$
$$C_{z1} = C_{z2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, C_1 = C_2 = [\,0 \quad 1\,]$$
$$A_{d1} = A_{d2} = 0,\; B_{21} = B_{22} = [\,0 \quad 1\,]^T$$
$$D_{21} = D_{22} = 0.5, B_{11} = B_{12} = [\,1 \quad 0\,]^T$$
$$D_{11} = D_{12} = 0,\; D_{31} = D_{32} = [\,-0.1 \quad 0\,]^T$$

and $\Delta A_1(k)$, $\Delta A_2(k)$ and $\Delta C_{z1}(k)$, $\Delta C_{z2}(k)$ can be represented in the form of (2) with

$$H_{a1} = H_{a2} = \begin{bmatrix} -0.1125 \\ 0 \end{bmatrix} \; E = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}^T$$
$$H_{c1} = H_{c2} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}.$$

Let the probabilistic density functions of β_1 in $[0\ 1]$ be described by

$$q(s_1) = \begin{cases} 0, & s_1 = 0 \\ 0.1, & s_1 = 0.5 \\ 0.9, & s_1 = 1 \end{cases} \tag{45}$$

from which the expectations and variances can be easily calculated as $\mu = 0.95$ and $\sigma = 0.15$, respectively.

Letting $\gamma = 0.8$ and applying Theorem 3 with help from Algorithm 1, we obtain the solution as follows:

$$A_{k1} = \begin{bmatrix} 0.6442 & 0.0125 \\ -0.2583 & 0.0457 \end{bmatrix} B_{k1} = \begin{bmatrix} 0.1205 \\ -0.0177 \end{bmatrix}$$
$$A_{k2} = \begin{bmatrix} -0.1098 & 0.1147 \\ -0.1765 & 0.0508 \end{bmatrix} B_{k2} = \begin{bmatrix} -0.1159 \\ 0.0509 \end{bmatrix}$$
$$C_{k1} = [\,-0.6145 \quad 0.2876\,]$$
$$C_{k2} = [\,0.1039 \quad -0.1765\,].$$

First, we assume

$$x(0) = [\,1 \quad 0\,]^T, \quad x_c(0) = [\,0 \quad 0.5\,]^T. \tag{46}$$

Fig. 8 gives the state responses of the closed-loop fuzzy system, when the external disturbance $v(k) = 0$, from which we can see that the two states converge to zero.

Next, to illustrate the disturbance-attenuation performance, we assume zero-initial condition and the external disturbance $v(k)$ is as in (40). Fig. 9 shows the changing curves of the state variables with which, according to Theorem 3, the addressed uncertain discrete-time fuzzy systems with multiple time-varying random communication delays and multiple missing measurements is exponentially stable in the mean square, and the effect of the disturbance input on the controlled output is constrained to the given level.

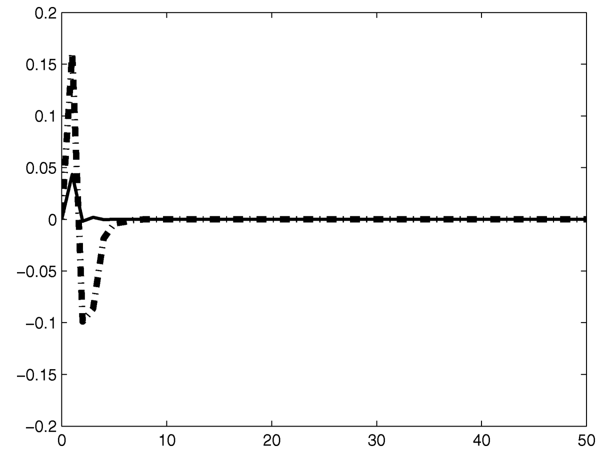

Fig. 8. Closed-loop state evolution $x(k)$ when $v(k) \equiv 0$.

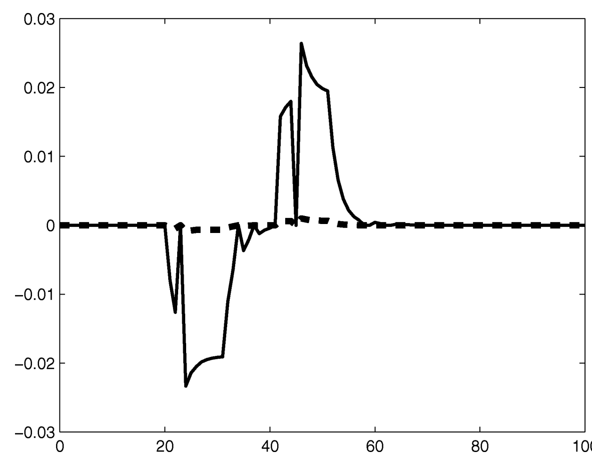

Fig. 9. Closed-loop state evolution $x(k)$ with $v(k)$ in (40) .

VI. Conclusion

In this paper, we have investigated the robust H_∞ fuzzy-output feedback-control problem for networked systems with multiple time-varying random communication delays and multiple missing measurements. The system model considered here involves parameter uncertainties, multiple stochastic time-varying delays, and multiple missing measurements. The control strategy takes the form of parallel distributed compensation. By using the CCL algorithm, sufficient conditions for the robustly exponential stability of the closed-loop T–S fuzzy-control system have been obtained, and at the same time, the prescribed H_∞-disturbance-rejection attenuation level has been guaranteed. Then, the explicit expression of the desired controller parameters have been derived. An illustrative simulation example has been provided to show the usefulness and effectiveness of the proposed design method.

REFERENCES

[1] W. Assawinchaichote, S. K. Nguang, P. Shi, and E. Boukas, "H_∞ fuzzy state-feedback control design for nonlinear systems with D-stability constraints: An LMI approach," *Math. Comput. Simul.*, vol. 78, pp. 514–531, 2008.

[2] Y. Y. Cao and P. M. Frank, "Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 2, pp. 200–211, Apr. 2000.

[3] B. Chen, X. Liu, S. Tong, and C. Lin, "Observer-based stabilization of T-S fuzzy systems with input delay," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 3, pp. 652–663, Jun. 2008.

[4] B. Chen, X. Liu, and S. Tong, "Adaptive fuzzy output tracking control of MIMO nonlinear uncertain systems," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 2, pp. 287–300, Apr. 2007.

[5] B. Chen and X. Liu, "Delay-dependent robust H_∞ control for T-S fuzzy systems with time delays," *IEEE Trans. Fuzzy Syst.*, vol. 13, no. 4, pp. 544–556, Aug. 2005.

[6] L. El Ghaoui, F. Oustry, and M. A. Rami, "A cone complementarity linearization algorithm for static output-feedback and related problems," *IEEE Trans. Autom. Control*, vol. 42, no. 8, pp. 1171–1176, Aug. 1997.

[7] G. Feng, "Controller synthesis of fuzzy dynamic systems based on piecewise Lyapunov function," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 5, pp. 605–612, Oct. 2003.

[8] G. Feng, "A survey on analysis and design of model-based fuzzy control systems," *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 5, pp. 676–697, Oct. 2006.

[9] H. Gao, Z. Wang, and C. Wang, "Improved H_∞ control of discrete-time fuzzy systems: A cone complementarity linearization approach," *Inf. Sci.*, vol. 175, no. 1–2, pp. 57–77, 2005.

[10] H. Gao, Y. Zhao, and T. Chen, "H_∞ fuzzy control of nonlinear systems under unreliable communication links," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 2, pp. 265–278, Apr. 2009.

[11] F. Gökas, "Distributed control of systems over communication networks," Ph.D. dissertation, Univ. Pennsylvania, Philadelphia, PA, 2000.

[12] C. Hua, Q.-G. Wang, and X. Guan, "Adaptive fuzzy output-feedback controller design for nonlinear time-delay systems with unknown control direction," *IEEE Trans. Syst., Man Cybern. B, Cybern.*, vol. 39, no. 2, pp. 306–317, Apr. 2009.

[13] X. Jiang and Q. Han, "On designing fuzzy controllers for a class of nonlinear networked control systems," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 4, pp. 1050–1058, Aug. 2008.

[14] C. Lin, Q.-G. Wang, T.-H. Lee, and Y. He, "Design of observer-based H_∞ control for fuzzy time-delay systems," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 2, pp. 534–543, Apr. 2008.

[15] C. Lin, Q.-G. Wang, and T.-H. Lee, "Stability and stabilization of a class of fuzzy time-delay descriptor systems," *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 4, pp. 542–551, Aug. 2006.

[16] X. Liu, "Delay-dependent H_∞ control for uncertain fuzzy systems with time-varying delays," *J. Comput. Appl. Math.*, vol. 68, no. 5, pp. 1352–1361, 2008.

[17] M. Liu, D. W. C. Ho, and Y. Niu, "Stabilization of markovian jump linear system over networks with random communication delay," *Automatica*, vol. 45, no. 2, pp. 416–421, 2009.

[18] S. K. Nguang and P. Shi, "Fuzzy H_∞ output feedback control of nonlinear systems under sampled measurements," *Automatica*, vol. 45, pp. 578–584, 2009.

[19] S. K. Nguang and P. Shi, "H_∞ fuzzy output feedback control design for nonlinear systems: An LMI approach," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 3, pp. 331–340, Jun. 2003.

[20] J. Nilsson, "Real-time control systems with delays," Ph.D. dissertation, Lund Inst. Technol., Lund, Sweden, 1998.

[21] X. Guan and C. Chen, "Delay-dependent guaranteed cost control for T-S fuzzy systems with time delays," *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 2, pp. 236–249, Apr. 2004.

[22] M. Sahebsara, T. Chen, and S. L. Shah, "Optimal H_∞ filtering in networked control systems with multiple packet dropouts," *Syst. Control Lett.*, vol. 57, pp. 696–702, 2008.

[23] P. Seiler and R. Sengupta, "An H_∞ approach to networked control," *IEEE Trans. Autom. Control*, vol. 50, no. 3, pp. 356–364, Mar. 2005.

[24] P. Shi and S. K. Nguang, "H_∞ output feedback control of fuzzy system models under sampled measurements," *Comput. Math. Appl.*, vol. 46, no. 5-6, pp. 705–717, 2003.

[25] S. Sun, L. Xie, and W. Xiao, "Optimal full-order and reduced-order estimators for discrete-time systems with multiple packet dropouts," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 4031–4038, Aug. 2008.

[26] K. Tanaka, H. Ohtake, and H. O. Wang, "Guaranteed cost control of polynomial fuzzy systems via a sum of squares approach," *IEEE Trans. Syst., Man Cybern. B, Cybern.*, vol. 39, no. 2, pp. 561–567, Apr. 2009.

[27] K. Tanaka, T. Ikeda, and H. O. Wang, "Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stabilizability, H_∞ control theory, and linear matrix inequalities," *IEEE Trans. Fuzzy Syst.*, vol. 4, no. 1, pp. 1–13, Feb. 1996.

[28] Y. Tang and J. Fang, "Synchronization of Takagi-Sugeno fuzzy stochastic delayed complex networks with hybrid coupling," *Mod. Phys. Lett. B*, vol. 23, no. 20–21, pp. 2429–2447, 2009.

[29] Y. Tang and J. Fang, "Robust synchronization in an array of fuzzy delayed cellular neural networks with stochastically hybrid coupling," *Neurocomputing*, vol. 72, no. 13–15, pp. 3253–3262, 2009.

[30] Z. Wang, D. W. C. Ho, Y. Liu, and X. Liu, "Robust H_∞ control for a class of nonlinear discrete time-delay stochastic systems with missing measurements," *Automatica*, vol. 45, no. 3, pp. 684–691, 2009.

[31] Z. Wang and F. Yang, "Robust filtering for uncertain linear systems with delayed states and outputs," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 49, no. 1, pp. 125–130, Jan. 2002.

[32] Z. Wang, D. W. C. Ho, and X. Liu, "A note on the robust stability of uncertain stochastic fuzzy systems with time-delays," *IEEE Trans. Syst., Man Cybern. A, Syst., Humans*, vol. 34, no. 4, pp. 570–576, Jul. 2004.

[33] G. Wei, G. Feng, and Z. Wang, "Robust H_∞ control for discrete-time fuzzy systems with infinite-distributed delays," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 1, pp. 224–232, Feb. 2009.

[34] H. Wu, "Delay-dependent H_∞ fuzzy observer-based control for discrete-time nonlinear systems with state delay," *Fuzzy Sets Syst.*, vol. 159, pp. 2696–2712, 2008.

[35] H. Wu and K. Cai, "H_2 guaranteed const fuzzy control for uncertain nonlinear systems via linear matrix inequalities," *Fuzzy Sets Syst.*, vol. 148, pp. 411–429, 2004.

[36] L. Wu and W.-X. Zheng, "L_2/L_∞ control of nonlinear fuzzy Ito stochastic delay systems via dynamic output feedback," *IEEE Trans. Syst., Man Cybern. B, Cybern.*, vol. 39, no. 5, pp. 1308–1315, Oct. 2009.

[37] F. Yang, Z. Wang, Y. Hung, and M. Gani, "H_∞ control for networked systems with random communication delays," *IEEE Trans. Autom. Control*, vol. 51, no. 3, pp. 511–518, Mar. 2006.

[38] D. Yue, E. Tian, Y. Zhang, and C. Peng, "Delay-distribution-dependent stability and stabilization of T-S fuzzy systems with probabilistic interval delay," *IEEE Trans. Syst., Man Cybern. B, Cybern.*, vol. 39, no. 2, pp. 503–516, Apr. 2009.

[39] H. Zhang, M. Li, J. Yang, and D. Yang, "Fuzzy model-based robust networked control for a class of nonlinear systems," *IEEE Trans. Syst., Man Cybern. A, Syst., Humans*, vol. 39, no. 2, pp. 437–447, Mar. 2009.

[40] W. Zhang, M. Branicky, and S. Phillips, "Stability of networked control systems," *IEEE Control Syst. Mag.*, vol. 21, no. 1, pp. 84–99, Feb. 2001.

[41] S. Zhou and T. Li, "Robust H_∞ control for discrete-time fuzzy systems via basis-dependent Lyapunov-Krasovskii functions," *Inf. Sci.*, vol. 174, no. 3-4, pp. 197–217, 2005.

[42] L. Zhang, Y. Shi, T. Chen, and B. Huang, "A new method for stabilization of networked control systems with random delays," *IEEE Trans. Autom. Control*, vol. 50, no. 8, pp. 1177–1181, Aug. 2005.

[43] Y. Zhao, J. Wu, and P. Shi, "H_∞ control of non-linear dynamic systems: a new fuzzy delay partitioning approach," *IET Control Theory Appl.*, vol. 3, no. 7, pp. 917–928, 2009.

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