DEVELOPING MATHEMATICAL GIFTEDNESS WITHIN PRIMARY SCHOOLS

A STUDY OF STRATEGIES FOR EDUCATING CHILDREN WHO ARE GIFTED IN MATHEMATICS

A thesis submitted for the degree of Doctor of Philosophy

by

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Abstract

This thesis explores the range of strategies used for educational provision for gifted children in mathematics in a group of schools in England. A review of literature relating to international theory and existing research in gifted education and empirical work into the teaching of gifted mathematicians were carried out. The literature review examined the dominant theories of intelligence and giftedness in general, including the historical background of definitions of giftedness and methods for its measurement, before specifically focusing on the concept of mathematical giftedness.

The study was located in primary schools within Greater London, where schools are required to implement the ‘Gifted and Talented’ policy of the UK government. The research was conducted in two stages during the school years 2007-2008 and 2008-2009. The first stage involved a questionnaire survey sent to primary schools within five Local Educational Authorities. For the second stage of the research, which constituted the main study, a case study approach was used. The main methods of data collection employed within the case study were observations of mathematics lessons, semi-structured interviews with children nominated as able or gifted mathematicians and their teachers, as well as analysing documentary evidence (i.e., school policy, teacher’s planning, children’s assessment records and children’s written work).

It was found that schools were responding to the policy in pragmatic terms, although no specific training was provided for practising teachers or school co-ordinators as part of the national training programme in making provision for mathematically gifted children. In practice, in classrooms, it was found that teachers’ level of confidence and expertise, the level of focused attention given to gifted children, the level of support and extension through higher-order questioning, as well as the size of the class and the nature of the work set were factors which affected the progress, perceptions and attitudes of children who were nominated to be able mathematicians.

There is a paucity of research which has investigated aspects of provision for gifted and talented children, particularly in mathematics, in the UK. By specifically addressing this topic, this study makes a distinct contribution to current literature in both understanding aspects of mathematical giftedness and the range of provision used. This study makes a particular contribution to finding out how practising teachers in England are responding to a government initiative, which should be of interest to both policy-makers and practitioners. This thesis also presents examples for organising and teaching mathematics to gifted children at higher cognitive levels, within regular classrooms; this may be of interest to audiences internationally, including countries where there are no policies of provision for mathematically gifted children.

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Abbreviations

APP: Assessing Pupils' Progress
Becta: British Educational Communications and Technology Agency
BACE Centre: Brunel Able Children's Education Centre
CQS: Classroom Quality Standards
DCSF: Department for Children, Schools and Families
DfEE: Department for Education and Employment
DfES: Department for Education and Skills
Di.Ha.Ta.P.E.: *Elliniki Eteria gia tin Proagogi tis Ekpedefsis ton Dimiourgikon Harismatikon Talantouhon Pedion & Efivon: Di.Ha.Ta.P.E.* (Greek Association for the Promotion of the Education of Gifted and Talented Children and Youths)
EiC: Excellence in Cities
GCSE: General Certificate of Secondary Education
GMNERA: Greek Ministry of National Education and Religious Affairs
GNC: Greek National Curriculum
GRIE: Greek Institute of Education
HMI: Her Majesty's Inspectorate of Schools
ICT: Information and Communication Technology
IQS: Institutional Quality Standards
LSE: London School of Economics and Political Science
NACE: National Association for Able Children in Education
NAGC: National Association for Gifted Children
NAGTY: National Academy for Gifted and Talented Youth
NCETM: National Centre for Excellence in the Teaching of Mathematics
NCTM: National Council of Teachers of Mathematics
NFER: National Foundation for Educational Research in England and Wales
Ofsted: Office for Standards in Education
P.A.G.T.C.: Parents Association of Gifted and Talented Children
QCA: Qualifications and Curriculum Authority
QCDA: Qualifications and Curriculum Development Agency
SATs: Standard Attainment Tests
YGT: Young Gifted and Talented
ZPD: Zone of Proximal Development

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1 Chapter One: Introduction

This thesis reports the strategies used for provision for mathematically gifted children in a group of primary schools in England. It also presents my findings of how the needs of these children are addressed in these schools located in Greater London, during the period 2008-2009. Based on a questionnaire survey, as well as on classroom observations and discussions with teachers and children, this study attempts to illuminate emerging issues in practice that will be useful for school teachers to face the challenge of educating gifted mathematicians and policy makers in England and elsewhere. It aims at contributing to the understanding and awareness of these children's special needs.

The research methodology used for the study is mainly qualitative and involves the use of questionnaires for teachers consisting of closed and open-ended questions, documentary evidence, classroom observations and interviews with both teachers and pupils.

Before I explain my personal motivation for embarking on this study and outline the aims and the research questions, I will set the context of the education of gifted and talented children in the UK and internationally, in general and specifically in mathematics. A more detailed background on the concept of giftedness and the education of mathematically gifted children will be presented in Chapter Two.

1.1 Setting the scene

> The student most neglected, in terms of realizing full potential, is the gifted student of mathematics. (NCTM, 1980, p. 18)

There has been a myth that children with higher abilities in mathematics do not require special attention, because they seem to cope well with their studies independently (Cockcroft, 1982; Johnson, 2000; NCTM, 1980). This myth and other common views, such as of those that want people to draw attention only to children with lower academic abilities and high risk for failure in school or disabilities led, according to Koshy and Robinson (2006), educational systems internationally to leave academically gifted children neglected for a long time.

This situation started changing at the turn of the twentieth century when, according to the US National Association for Gifted Children (NAGC, 2005), advancements in education and psychology changed the way of viewing giftedness and talent and the publications of relevant research studies by some new pioneers, such as Lewis Terman (1916, 1925 cited in NAGC, 2005), became widely known. A growing interest in the education of gifted children started then from the USA first and extended elsewhere afterwards. An historical review of the concept of giftedness and talent development is presented in more detail in Chapter Two.

Mathematically gifted children then became part of that interest, but they attracted more attention when studies from the former USSR (Krutetskii, 1976) and the USA (NCTM, 1980) highlighted the importance of nurturing gifted mathematicians in countries that wanted to have a leading role in the technological world. The US National Council of Teachers of Mathematics (NCTM), for instance, in *An Agenda for Action: Recommendations for School Mathematics of the 1980s* stated: "Outstanding mathematical ability is a precious societal resource, sorely needed to maintain leadership in a technological world" (NCTM 1980, p. 18).

Further advancement in studying brain functioning and human behaviour produced broader conceptions of giftedness involving multiple talents or intelligences and models of talent development. Some examples include Gardner's (1983) theory of *Multiple Intelligences*, Sternberg's (1985) *Triarchic Theory of Intelligence* and Gagne's (1985) *Differentiated Model of Giftedness* which influenced perceptions about gifted children and their education. Gardner's (1983) MI theory, particularly, according to VanTassel-Baska (1998), raised awareness about the education of gifted children and inspired many educators to start thinking about educational provision within curriculum. Gardner's (1983) views about the existence of a specific mathematical ability (or *logical-mathematical intelligence*), which is associated with mathematical giftedness have, also, influenced this present study and, thus, have become the cornerstone. More details about this and other modern theories of giftedness and talent development will be presented in Chapter Two.

Since Gardner's (1983) MI theory, many experts in the field of gifted education (Johnson, 2000; Koshy, 2001; McClure, 2001; Sheffield, 1999, 2003; Tomlinson, 1995) have acknowledged the importance of nurturing of mathematically gifted children and argue that these pupils require differentiated instruction according to their needs and suggest provision within the curriculum for such children. Further details about specific provision within curriculum for mathematically gifted children are given in Chapter Two.

The literature review of this study (presented in Chapter Two) has shown that the most known methods of provision are ‘acceleration’, ‘differentiation’ and ‘enrichment’ and can be implemented in combination with other methods, such as ‘grouping by ability’ or ‘curriculum compacting’. Additionally, the review of current international policies has shown that many countries — such as the USA, Australia, New Zealand, the UK and other European countries — have included provision for gifted children in general and also in mathematics in their national educational policies and in their latest curriculum frameworks.

The following sections will briefly present the development of the education of gifted children and specifically in mathematics with special reference to the UK educational system as well as internationally. At this point, it should be noted that the term ‘gifted and talented’ is used by the UK policy (DCSF, 2008a, 2008b, 2008c) and therefore is used by this study when it is referred to the UK educational system. However, additional terms are used throughout this thesis, such as ‘gifted’ or ‘promising’, or ‘able’ (with variations, such as ‘very able’, ‘more able’, or ‘exceptionally able’), when it is referred to international context.

1.1.1 Gifted and talented education in England (UK)

In the UK the problem of the lack of sufficient provision for the gifted (or very able) children in schools became widely known during the last decades of the twentieth century, when the report from Her Majesty’s Inspectorate of Schools (HMI) in 1978 highlighted the fact that the most able children’s work was not well-matched (HMI, 1978). This remained unchanged after 14 years when a new report of Her Majesty’s Inspectorate of Schools (HMI, 1992) showed that very able pupils in maintained schools were often insufficiently challenged during their daily work.

With regards to children with high ability in mathematics, there was in the UK, like elsewhere until the early 1980s, a common view that mathematically gifted children did not need any special attention, as they could take care of themselves. That view was challenged in 1982 by the Cockcroft Report (1982), which stated:

> The statement that able children can take care of themselves is misleading, it may be true that such children can take care of themselves better than the less able, but this does not mean that they should be entirely responsible for their own programming, they need guidance, encouragement and the right kind of opportunities and challenges to fulfil their promise. (par. 332)

Experts supporting the need for meeting the needs of gifted children within education added their voices into the argument. Straker (1983), for instance, who later directed the National Numeracy Strategy in the UK, argued that "Gifted pupils have a great deal to contribute to the future well-being of the society, provided that their talents are developed to the full extent during their formal education." (p. 7)

In 1999, the House of Commons Select Committee (House of Commons, 1999) framed a new policy regarding 'highly able children' by making recommendations for targeted funding to support gifted and talented children, monitoring the progress of the schools and initial teacher training. Recommendations required the education of gifted children to be financially supported through generic funding of schools. It was decided that the Office for Standards in Education (Ofsted) should inspect schools and Local Educational Authorities (LEAs), so as to obtain data on provision for gifted and talented children. Initial teacher training should include the education of gifted and talented children as a high priority and all schools should incorporate a co-ordinator especially appointed for gifted and talented education. Enrichment and extension of the normal curriculum, out of school provision and systematic use of ICT, as well as partnerships between schools and other bodies, such as universities, were recommended.

Following the above recommendations, the government launched some new initiatives, such as the *Excellence in Cities* (EiC) (DfEE, 1999a) and *Excellence Clusters* (EC) (2001, cited in NFER & LSE, 2004). Although these were initially launched to provide help for gifted disadvantaged pupils in the most deprived cities, towns and rural areas, they have, since then, raised awareness for the need of the education of gifted and talented children within all mainstream schools in England and Wales. More specifically, the initiatives required schools within a significant number of inner city

Local Education Authorities to identify 5 to 10 percent of their pupils as ‘gifted’ and provide them with a distinct teaching and learning programme.

In 2001, the Qualifications and Curriculum Authority (QCA) of the UK produced a guide for primary schools, in order to help teachers identify academically gifted children (see Table 1-1 for a list of key principles in the identification of gifted and talented children, as they appear on the webpages of DCSF, former DfES). At the same time, a Green Paper, *Schools: Achieving Success*, committed the Department for Education and Skills (DfES) to include support for gifted and talented students in all national strategies of school education (DfES, 2001).

Table 1-1: The key principles in the identification of gifted and talented children

- Emphasis should be on providing an appropriate, challenging and supportive environment rather than on labelling any particular child;
- There should be open communication between educators, pupils and parents/carers as part of the identification process — parents know their children best and should be engaged as partners in their child's learning;
- Parents/carers should be made aware that being on the gifted and talented register does not automatically guarantee academic success;
- Identification is a continuous process. Some pupils will be easy to identify at a very early age, while some will emerge later;
- Identification should be systemised within the school so that it is continuous, rather than a battery of specific tests at a set time of year;
- Schools need to be particularly vigilant for the 'hidden gifted' or under-represented groups, such as underachievers, those for whom English is not their first language, those with learning or physical disabilities or those from different cultural or socio-economic groups;
- Identification should be based on a portfolio approach, utilising a range of both qualitative, quantitative and value-added measures;
- The identified group should broadly represent the school's population;
- Teachers should be continually 'talent spotting'.

Source: The National Strategy website (The Standards Site, DfES, 2004a, webpage)

In mathematics, specific frameworks were developed in parallel with the general National Numeracy Strategy (DfEE, 1999b), by the Department for Education and Employment (DfEE) initially and the Department for Education and Skills (DfES) afterwards, to help teachers identify and provide extra support for mathematically able children within schools. Some examples include the *National literacy and numeracy strategies: Guidance on Teaching Able Children* (DfEE, 2000b), *Mathematical Challenges for Able Pupils in Key Stages 1 and 2* (DfEE, 2000a), the *Excellence and Enjoyment*: *A Strategy for Primary Schools* (DfES, 2003) and the *Problem solving: A CPD Pack to Support the Learning and Teaching of Mathematical Problem Solving* (DfES, 2004c).

In 2002, the government, hoping to further support the education of gifted children through the DfES — now called the Department for Children, Schools and Families (DCSF) — set up the National Academy for Gifted and Talented Youth (NAGTY) in partnership with the University of Warwick, supported by Johns Hopkins University in Baltimore, which had had long experience of programmes for gifted and talented children in the USA since 1979. The NAGTY (which no longer exists) then took the initiative to provide services and support for the top 5 percent of gifted and talented students, who met the eligibility criteria that the NAGTY had set. Selected children could then take part in special programmes in summer schools or 'master classes'. This included support for mathematically gifted pupils.

However, that practice of providing for selected children caused a lot of criticism that they wasted a lot of money by funding expensive high-profile summer schools for middle class families, usually white, whilst the daily practice in the normal classroom was neglected and, thus, the education standards were not raised (NAGTY, 2004). Additionally, an Ofsted report on the EiC initiative (Ofsted, 2004) revealed that one in five schools in the inner city areas did not identify or systematically measure the achievements of bright pupils.

Other independent bodies, also, offer support to teachers and parents confronted with the needs of pupils with higher abilities, such as: the NAGC (National Association for Gifted Children), NACE (National Association for Able Children in Education) and QCA (Qualifications and Curriculum Authority, which recently changed into Qualifications and Curriculum Development Agency, QCDA), which all collaborate with the DCSF (Department for Children, Schools and Families) and Ofsted (Office for Standards in Education). There are also universities, such as the Oxford Brookes University, which, through the Westminster Institute, runs the National Gifted and Talented Co-ordinators' Training Programme for schools in EiC areas and for

Excellence Clusters and Brunel University, which, through the Brunel Able Children's Education (BACE) Centre, offers support to professionals for effective provision for pupils who are gifted and talented. The government has also funded summer schools and after-school activities in all Local Educational Authorities, where mathematically gifted children have the opportunity to take 'world-class' tests as well as participate in pilot projects that test the feasibility of children taking GCSE (General Certificate of Secondary Education) examinations in mathematics earlier than usual (i.e., at the age of 11) (Koshy, 2001).

During the past few years, the scene on gifted education in the UK has rapidly changed. The educational provision for gifted and talented children and, thus, for mathematically gifted children, became an integral part of general education policy, which is expressed by the National Strategy and regulated by the National Quality Standards of the Department for Children, Schools and Families (DCSF, former DfES).

The rapid changes have started since the publication of the 2005 White Paper, *Higher Standard, Better Schools for All* (DfES, 2005), and the 2006 renewed Primary National Strategy for Literacy and Mathematics (DfES, 2006). The White Paper stated that gifted children in mathematics have the right to reach the limits of their ability, and the 2006 National Strategy stated that they should learn through appropriate challenges set by teachers whose role will be to tackle any barriers to progress they may possibly face.

Recently, the Department for Children, Schools and Families (DCSF) has launched three new initiatives for provision for gifted and talented children within primary and secondary schools: the *Institutional Quality Standards* (IQS), the *Classroom Quality Standards* (CQS), and new guidance on the identification of gifted and talented learners (DCSF, 2008a). These initiatives aim to improve the education of gifted and talented children within different domains in education, including mathematics.

In addition, the DCSF took on the responsibility of the national Gifted & Talented (G&T) strategy (funding and supporting any relevant programme) and, through two new publications, provided updated guidelines on the identification of gifted and talented learners (DCSF, 2008a) and on provision for them within primary schools

(DCSF, 2008b). The first encourages all schools to identify their gifted and talented pupils in any domain (e.g., academic, sports, arts) and maintain their own register for them (schools are free to determine the size of their gifted and talented populations, providing that they are able to justify it). Schools are advised that there is no single perfect instrument for the identification of gifted and talented pupils. Therefore, they are advised to use a 'best fit' model that will draw on a range of evidence, including teacher assessments, diagnostic tests and national key stage tests provided by the Qualification Curriculum Authority (QCA). They are also advised to find ways to involve pupils and parents/carers in the identification process. The second, which is an updated version of the initial guidance published in 2006 (DfES & NAGTY, 2006), provides guidance for effective provision mainly in the classrooms with suggestions for differentiation through grouping for enrichment and assessment of pupils' progress and for provision outside the classroom by engaging the families and communities. Along with these specific guides for the education of gifted and talented children, there are those general ones for everyday assessment, such as *The Assessment for Learning Strategy* (DCSF, 2008a), which suggest a range of practices through a programme for tracking of individual progress called Assessing Pupils' Progress (APP).

At this point, I should explain that all schools in England (and Wales) follow a general plan provided by the National Curriculum for the subjects that they have to teach in different stages, according to the age of pupils and the level of attainment the pupils should achieve at the end of each stage, but they are free to develop their own strategies to achieve their targets.

The age groups are divided in four key stages: Key Stage 1 for Years 1–2 (age 5–7), Key Stage 2 for Years 3–6 (age 7–11), Key Stage 3 for Years 7–9 (age 11–14) and Key Stage 4 for Years 10–11 (age 14–16). The levels of attainment that the pupils may be awarded throughout these stages range from 1 to 7, with 1 being the lowest. For special cases, where some children achieve higher, a Level 8 or 'Exceptional Performance' level may be added. The levels are subdivided in three subgroups (e.g., 4a, 4b and 4c), with 4c indicating the lower place within Level 4. Table 1-2 presents the Key Stages up to the age of 14 with expected attainment. According to this plan, a pupil at age 11 in Year 6 with Attainment Level 5 should be considered a higher achiever.

Finally, I should add that these recurrent changes in education in the UK have continued until the writing of this thesis. For instance, since the beginning of 2010, a new electronic version of the National Curriculum (QCDA, 2010a), which includes many interactive tools for immediate support for all the subjects, has been available online.

Table 1-2: Key Stages and Attainment Levels

Key Stages	The majority of children expected to work between levels	At end of key stage, the majority of children expected to attain level
Key Stage 1	1-3	2 (age 7)
Key Stage 2	2-5	4 (age 11)
Key Stage 3	3-7	5 or 6 (age 14)

Source: Qualifications and Curriculum Development Agency (former QCA) website (QCDA, 2010b)

1.1.2 Gifted and talented education internationally

The need for a national educational programme that addresses the needs of children with higher abilities in mathematics has been recognised over recent years in many countries, which have included provision for those children in their educational policies.

In the USA, for instance, after a long period of research projects and evaluations, the National Council of Teachers of Mathematics (NCTM) has provided schools with the *Principles and Standards for School Mathematics* (NCTM, 2000) and *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM, 2006). These outline a new framework for teaching and learning mathematics that is traditionally considered appropriate for the gifted only. *Principles and Standards* now expect all students to follow a curriculum that places emphasis on “problem solving, reasoning and proof, connections, communication and representation” (NCTM, 2004a, webpage). Although they support the idea that all children can benefit from a challenging mathematics curriculum, they do, however, make it clear that this does not mean that every child should be treated in the same way (NCTM, 2004b). Their suggestion is that students with exceptional abilities in mathematics should be kept challenged and engaged by using additional resources or following enrichment programmes and that “the talent and interest of these students must be nurtured so that they have the opportunity and guidance to excel in mathematics" (NCTM, 2004b, webpage).

In Europe, most EU countries have taken measures to promote giftedness through their educational systems, as the Eurydice (2006) study has revealed. According to this study, which took place among thirty European countries — the 25 EU members on 1st May 2004 plus the 3 countries of the European Free Trade Association (Iceland, Liechtenstein, Norway) and the two, at that time, candidate countries (Bulgaria and Romania) — almost all countries (the UK included) incorporate specific instructional methods within their schooling system for young people with higher potential in specific domains (e.g., mathematics), though some, like Greece, cover only the needs of those with higher abilities in sports, music and art. These methods are applied within schools ('in-school provision') or outside schools ('non-school provision') by using 'more advanced' and 'varied activities' or 'differentiated provision' as well as 'fast tracking' procedures for those who have higher abilities (Eurydice, 2006). This, naturally, does not mean that the aforementioned methods of provision are either unique or applicable in the same manner in every country. Malta and Norway, for example, which did not provide any data related to specific measures, address the needs of very able children through a general educational policy, which offers a differentiated approach to the individual child within a mainstream class. In other countries — such as Greece, France, Spain, Portugal, Slovakia, Czech Republic, Slovenia, Estonia, Ireland, and Scotland (from the UK) — gifted and talented pupils are seen as children with special educational needs in a similar way as those with learning difficulties or disabilities are. The result of this is the existence of schools that specialise in different fields (art, sports, or music) (Eurydice, 2006). Some European countries — such as Austria, Netherlands and Romania — also operate special centres, which offer support for gifted children. There are also institutes, academies and networks, which offer not only support for gifted children, but also in-service teacher training such as the National Education Institute in Slovenia, the National Academy for Gifted and Talented Youth (NAGTY) in England, Wales and Northern Ireland (UK), the Scottish Network for Able Pupils in Scotland (UK) and the Network of Teacher Training Institutes in Hungary (Eurydice, 2006). It should be noted that these findings of Eurydice (2006) represent the situation existing at that time (2006), thus, some of these institutes, academies or networks may not exist anymore (e.g. NAGTY in the UK, mentioned in previous section). However, these findings help us understand the attention of some European countries on the education of gifted and talented children.

Similar programmes are also incorporated within the Australian education system. Although each state has varied educational policies, they have all developed programmes for the education of gifted and talented children (including those with higher abilities in mathematics) within their state schools. These programmes incorporate a combination of grouping, enrichment, acceleration, and counselling strategies, according to the 2001 report *The Education of Gifted and Talented Children* issued by the Parliament of Australia-Senate-Committee (Australian Senate Employment, Workplace Relations, Small Business and Education Committee, 2001).

In New Zealand, also, the Ministry of Education released *The Schooling Strategy, Making a Bigger Difference for all Students* (New Zealand Ministry of Education, 2005) in 2005, which aimed to improve the education of gifted and talented children within state schools for the five years from 2005 to 2010.

1.1.3 Why is this study significant?

It is hoped that this study will make a significant contribution to mathematics education, especially in relation to providing effective learning provision for mathematically able children internationally, but within in the UK in particular.

Most of the previous research, carried out in the USA, investigated the application of methods for differentiation, acceleration or enrichment, but mainly through particular programmes specifically designed to offer support for gifted children from economically disadvantaged backgrounds, children with limited English proficiency and children with disabilities (U.S. Department of Education & Office of Communications and Outreach, 2007). There has also been a large amount of research into the effects of different grouping strategies on children’s achievement in the USA (e.g., Kulik, 1992; Kulik & Kulik, 1982, 1987, 1991, 1992; Slavin, 1986, 1987) and some research recently in the UK (Hallam, Ireson & Davies, 2004; Davies, Hallam & Ireson, 2003; Boaler, Wiliam & Brown, 2000). However, there has not been so much research (internationally and also in the UK) into whether and how the needs of these children, and particularly mathematically gifted children, are met in practice within maintained primary schools and what the perceptions and attitudes of both teachers and pupils are.

Amongst the little amount of empirical studies in the UK there was an action research entitled *Mathematics Enrichment Project* (MEP) conducted by the Brunel Able Children's Education (BACE) Centre (Koshy & Casey, 2005) on ways of actualising mathematical promise within 11 Local Education Authorities in inner London primary schools. This study revealed that many teachers felt uncomfortable teaching very able mathematicians and were not aware of available supporting resources. The study furthermore showed that the key point regarding provision is to raise teachers' awareness of how the needs of gifted mathematicians can be met. Personal development of the teacher was, also, found to be a key factor in raising their self-confidence in teaching mathematics to very able pupils. Similar findings, which draw attention to the lack in teacher expertise but, also, in providing higher-level mathematics lessons for gifted mathematicians in UK schools, are highlighted by reports published earlier (Smith, 2004) and later on (Williams, 2008).

The Smith (2004) report, for instance, highlights the decline of numbers of young people continuing to study mathematics post-16. The report draws attention to possible factors underlying the decline issue as the 'poor quality of teaching and learning' (p. 3) and the 'failure of the curriculum to excite interest and provide appropriate motivation' (p. 4) leading many young people to perceive mathematics as 'boring'. These statements do demonstrate the need to educate the young talented mathematicians in primary schools, so that they find mathematics an interesting and enjoyable subject to learn.

The Williams (2008) report, more recently, further highlights that in-class provision was not always stretching the gifted mathematicians in primary schools — something which has also been highlighted as a concern by Ofsted (2009). Consequently, the Williams (2008) report recommends, amongst other measures, better and ongoing training for the teachers, more targeted help for children who have difficulties through an intensive programme of intervention: 'Every Child Counts', better collaboration with parents, and a mathematics specialist teacher for every primary school within the next ten years. For the latter recommendation, particularly, it suggests that this specialist may also “advise on the provision for Gifted and Talented pupils in his or her school” (p. 20).

Interestingly, the National Centre for Education in the teaching of Mathematics (NCETM) of the UK indicates that there is very little UK based research in the area of teaching mathematically gifted children and recommends the article by Koshy, Ernest and Casey (2009) for reading on their website (NCETM, 2009). In this particular article, Koshy et al. (2009) reflecting on the findings from the MEP project (Koshy & Casey, 2005), earlier mentioned, conclude that the education of mathematically gifted children in the UK is at a critical crossroad. They contend that some issues — such as the relative merits of pull-out groups for gifted children in mathematics, the search for effective teaching methodologies and the role of teachers’ professional development in relation to students’ attitudes and motivation — are still unresolved and, therefore, call for further research.

Through undertaking a comprehensive literature review and explaining what happens in a sample of UK primary schools, this study should make a contribution to knowledge.

The following section will now explain my personal interest and motivation for embarking on this study.

1.2 Personal interest and motivation

Combined with my personal interest in developing and improving my own professional expertise in education, my motivation has also evolved on account of the following:

- My teaching experience with mixed-ability children in Greek primary schools, which raised my awareness of the need for differentiation
- My studies for a Masters Degree in Education at Brunel University, which concerned approaches for ‘teaching thinking’ and strategies for teaching mathematically able children
- The introduction of the new Curriculum Framework and related schoolbooks for primary schools in Greece

My teaching experience consists of a 21-year practice in different state primary schools (16 years in Greece and 5 years in a Greek primary school in the UK that also follows the Greek National Curriculum (GNC)). All state primary schools in Greece have mixed-ability classrooms and, thus, I have always worked within a differentiated environment. A common problem that I always encountered in the classroom was the challenge of providing for children who worked at a faster pace in mathematics. The children who constantly finished their tasks earlier than the others did became bored or, even worse, disruptive. When that happened, I felt as if I had missed those children and as if I had not covered their needs. I then wondered the following:

- Why do these children always finish their tasks earlier than the others do?
- Is it possible that those children who seem more advanced may have extra help at home? Or do they seem to be more advanced because the others do not pay as much attention to the lesson?
- What could I do to keep them challenged and engaged in their learning?
- What could I do to extend them without leaving the others behind?
- How could I differentiate the lessons for these children within the normal curriculum?

These questions motivated me to start looking for further strategies that would help me challenge those children beyond the regular lesson. An excellent opportunity for me to start looking for answers was, without doubt, my postgraduate studies for a Masters Degree in Education at Brunel University (Dimitriadis, 2005), which provided a starting point for my doctoral studies.

My studies during the MA course at Brunel University gave me an opportunity to explore many interesting aspects of classroom teaching, such as strategies for teaching higher order thinking and specific methods for teaching mathematically able children through studying for two modules. The first was entitled *Teaching Learning and Assessment*, and the second, *Teaching Mathematically Able Children*.

The first module gave me the opportunity to study strategies for more effective teaching and learning, such as strategies for 'teaching thinking' (Fisher, 1992, 1995, 1998, 1999; McGuinness, 1999). During that module, I learnt to identify pupils' thinking skills and implement methods into my practice for challenging their higher-order thinking.

The second module gave me a new orientation in relation to my old concerns about those children who always finish their tasks in mathematics earlier than others, because it introduced me, for the first time, to the concept of 'giftedness' in mathematics. It was a module with practical guidance for teaching mathematically able children, part of the *Mathematics Enrichment Project* (Casey & Koshy, 2002, 2003) offered by Brunel University and funded by the UK Department of Education and Skills (DfES). We were offered teaching frameworks specifically designed for able mathematicians such as the *Key Concepts Model* (Casey, 1999) for teaching mathematics to able children, Koshy's (2001) proposal for the use of principals of Bloom's (1956) *Taxonomy of Educational Objectives* and Sheffield's (1999) proposal for a programme of three dimensions of learning: *Depth or complexity, Breadth* and *Rate* for teaching mathematically able pupils. These strategies influenced my personal practice and increased my awareness that those children, who work faster than others within the normal curriculum, need special attention and probably something more than what the usual curriculum provides.

After finishing my MA studies, I went back to Greece and continued teaching for one year. During that time, I tried to implement some of the teaching approaches I had learnt whilst completing my postgraduate studies. I felt better equipped to challenge and motivate pupils who seemed promising in mathematics, by implementing 'teaching thinking' skills and using 'mathematics enrichment' activities.

I realized that keeping portfolios, writing 'achievement reports' (Koshy, 2001) and carrying out problem-solving strategies helped the children develop greater ability to identify themselves as mathematically able and motivated them to be engaged with more complicated problems. I then began looking for further activities to challenge those children who were able to work at higher cognitive levels. I found suitable problems with open-ended solutions and puzzles in an English publication for mathematics: *Mathematical challenges for able pupils in Key Stages 1 and 2* (DfEE, 2000a) and I gave these, translated in Greek, as extension work to those children who completed tasks earlier than their peers. I found that this practice increased the interest of my pupils and their motivation to do more mathematics and more challenging activities. However, all these were based on my own initiatives rather than on what the normal curriculum provided to meet the needs of more able children.

I should explain, at this point, that in Greece there has not been any specific programme of provision for gifted mathematicians or gifted and talented children in general apart from children with talent in music or sports. This is despite the growing interest, which is mainly expressed by independent bodies, such as the Greek Association for the Promotion of the Education of Gifted and Talented Children and Youths (in Greek: Elliniki Eteria gia tin Proagogi tis Ekpedefsis ton Dimiourgikon Harismatikon Talantouhon Pedion & Efivon) (Di.Ha.Ta.P.E.), the Parents Association of Gifted and Talented Children in Greece (PAGTC) (in Greek: Syllogos Goneon kai Kidemonon Harismatikon kai Talantouhon Pedion) and individual educationalists like Kinigos (1991, 1993).

The news of the entry of a new Curriculum Framework (GMNERA & GIE, 2004) with the new schoolbooks for primary education (GMNERA & GIE, 2006b) for the following school year (2006–2007) gave me hopes for something better in regards to the challenge of the children with higher abilities. I was hoping to find useful directions and suggestions to meet my concerns through the seminars that were organised for the introduction of the new books and the new curriculum framework.

A seminar for mathematics was organized by the 23rd Educational Region of Attica (in Greek: 23^{h} Ekpaideftiki Perifereia Attikis, similar to a UK Local Educational Authority) in February 2006 in Athens on behalf of the Greek Ministry of National Education and Religious Affairs and the Greek Institute of Education (GMNERA & GIE, 2006a). During the seminar, I was impressed, at first glance, as I found a large variety of teaching materials within a sample of the student textbooks appropriate for teaching using higher-order levels (e.g., open-ended problems and graphs). The sample from the teacher's handbook also showed that there was an emphasis on problem-solving in mathematics and on new ideas, such as the 'cross thematic' programme that connects different subjects (e.g., science, history, literature, etc.) within a lesson (e.g., mathematics). I was enthusiastic, expecting to hear recommendations and ideas about teaching mathematics at different levels, especially at higher levels, appropriate for gifted children.

Much to my surprise, issues relating to the education of children with higher abilities and strategies for handling differentiation within primary schools were not mentioned in any of the seminars. This left me unsatisfied, with the impression that there is no interest in promoting mathematical ability within primary schools and that provision for mathematically gifted children in the Greek educational system still remains absent. This feeling motivated me to embark on an investigation of my own, beyond the Greek education system, to find out how the needs of children, who are very able or gifted in mathematics, are addressed within primary schools in countries that have developed specific policies for the education of gifted children. I was hoping that this would make me able to make recommendations that would help teachers in Greece effectively address the needs of gifted mathematicians within primary schools and also influence policy makers to start thinking about developing a policy of provision for these children.

I decided then that the best place for me to embark on a study on strategies for the education of mathematically gifted children would be the UK, where I was first introduced to that issue and where specific frameworks and innovations for gifted and talented children have been initiated within primary education during the last decade.

I will now present the aims of this study and the research questions.

1.3 Aims and research questions

By undertaking this study, I aim:

- To explore the strategies used by teachers for educational provision for gifted and talented children, particularly in mathematics. This will also involve a study of international theory and research into the teaching of gifted mathematicians.
- To carry out a questionnaire survey, interviews and classroom observations in English primary schools to explore how the needs of mathematically gifted children are addressed in practice.
- To make an assessment of what is happening in primary schools — where there is specific attention given to gifted and talented children — by comparing the

findings of aims 1 and 2 against recommendations for specific provision and effective teaching for mathematically gifted children.

With these aims in mind, the following research questions were developed.

1) What strategies are schools using, if any, regarding the education of gifted and talented children in general and specifically in mathematics?
2) What are the teachers' perceptions of and attitudes towards mathematically gifted children, their education and the methods used by their schools?
3) How are the needs of mathematically gifted children met within classrooms in everyday practice?
4) What is the impact of the schools' strategies on pupils' achievement and attitudes?

I expect this study will generate useful findings for school teachers faced with the challenge of effectively educating children with higher abilities in mathematics and for policy makers involved in gifted and mathematics education. The findings of this study should have implications in both the UK, where provision for gifted and talented children already exists to see how it is applied in practice and its impact on pupils' performance and behaviour, and other countries like Greece, where there is not such kind of provision to raise awareness on the education of these children.

1.4 The structure of the thesis

The thesis is presented in seven chapters and it consists of the following structure.

Chapter One presented the aims of my investigation and a background to the study. It explained my personal interest and motivation for embarking on the investigation and set the national scene on the education of gifted and talented children in the UK and internationally, with a focus on the education of mathematically gifted children.

Chapter Two sets the theoretical context for the study based on international theory and research. It presents a historical background on the concept of giftedness and a review of the most well-known theories of giftedness and talent development. It discusses the concept of specific mathematical ability together with identification issues and strategies of provision for mathematically gifted children. Practical strategies and organisational structures for everyday teaching employed within primary schools and the teacher's role are also discussed.

Chapter Three presents the research methodology used for collection of the data and justifies the choices made.

Chapter Four presents the first stage of the research, the preliminary phase and its findings, which were derived from a questionnaire survey.

Chapter Five presents four case studies, which comprised the second stage of the research, the main study. The data collected from all case studies were based on observations, documentary evidence and interviews with the teachers and pupils who were identified as able or gifted mathematicians.

Chapter Six presents a discussion on my findings and an evaluation of how the original aims of the study have been met.

Chapter Seven presents the conclusions of the study, its implications for both teachers and policy makers in relation to mathematics education and its possible limitations. It also presents my personal learning achieved from this investigation and outlines the contribution of this study to aspects of educating mathematically gifted children along with possible topics for future research.

2 Chapter Two: Review of the Literature

The previous chapter explained my personal interest and motivation for embarking on this study along with the aims, the research questions and expectations of my investigation. It set the scene on the education of gifted and talented children in general as well as specifically in mathematics with more details about the UK educational system within which this study took place.

This chapter presents the theoretical background of the study based on international theory and research. Its contents are presented in three parts. The first part presents a background on conceptions of giftedness with a focus on multifaceted views and broader theories of giftedness and talent. The second part presents the concept of specific mathematical ability. It discusses issues relating to the nature of mathematical ability with a focus on theories and research findings about who mathematically gifted children are and how we can identify them within primary schools. The third part presents methods of provision for the education of gifted children in general and in mathematics in particular. After each part, a short summary of the main issues discussed is given.

2.1 Conceptions of giftedness

A systematic and scientific approach to the concept of giftedness began in the late 19th century through the study of intelligence. This approach became more widespread during the 1920s and 1930s, when advancements in psychology and education brought elements of credibility to the field of gifted education (NAGC, 2005). The first half of the 20th Century was characterised by efforts to create valid IQ (Intelligence Quotient) tests for measuring intelligence, which could give reliable results (e.g., Binet, 1905 and Terman, 1916, 1925, all cited in NAGC, 2005).

However, there were some researchers, who, during that period, argued that intelligence could not be measured with single IQ tests, suggesting new theories that involved more multifaceted approaches to intelligence and its measurement. Thurstone (1938) was one of the first who proposed that intelligence should not be seen as a single or general ability and suggested his theory of *Primary Mental Abilities*, which he defined as: (a) Verbal comprehension, (b) Reasoning, (c) Perceptual speed, (d)

Numerical ability, (e) Word fluency, (f) Associative memory, and (g) Spatial visualization.

During the following years the growth of educational psychology brought out new insights into the meaning of outstanding talent among children and contributed to the development of new theories of giftedness — such as Renzulli's (1978) *Three Ring Conception of Giftedness* — that include other factors, which constitute a gifted and talented personality.

After the 1980s, further research findings on brain function and recent theories coming from cognitive psychology have suggested that intelligence is only one key to understanding giftedness in a child and interest in studying all characteristics of very able children and nurturing their special abilities has increased. Gardner's (1983, 1993, 1999) theory of *Multiple Intelligences* (MI), Sternberg's (1985) *Triarchic Theory of Intelligence*, Gagne's (1985, 2003) *Differentiated Model of Giftedness and Talent* and Renzulli's (1978, 1986, 1998) *Three Ring Conception of Giftedness* are some examples of theories and models which, drawing on findings that come from both cognitive and educational psychology research, have contributed to forming new conceptions of giftedness and talent. These theories and models are presented in the following sections, begin with Renzulli's (1978) *Three Ring* model — which, even though it was published before 1980, was renewed and revised later, to include new research evidence, too (e.g., Renzulli, (1986, 1998) — and end with Gardner's (1983) MI theory.

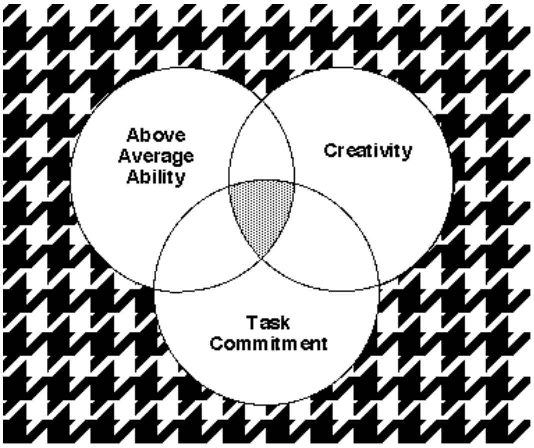
Renzulli's Three Ring Conception of Giftedness

The *Three Ring Conception of Giftedness* (Renzulli, 1978, 1986, 1998) suggests that giftedness involves an interaction between three basic sets of human traits — *above average ability*, *creativity* and *task commitment* — which, like three overlapping rings, create a common area where the most gifted behaviour is displayed (see Figure 2-1).

Above average ability, according to Renzulli (2002), is considered a top level of performance (not necessarily exceptional) in any particular area of human endeavour at a percentage of 15-20 percent. This definition differs from that of others such as Terman (1926, cited in Renzulli, 1998), Ogilvie (1972, cited in George, 2003), the UK

Standards (DfEE, 1999a), and Gagne (1985), who consider high ability as the top 1 percent, 3 percent, 5–10 percent, or 10 percent, respectively, of performance among children of the same age in the same field (or fields). This ability, according to Renzulli (2002), can be categorised in relation to general and specific ability.

General ability refers to numerical and verbal reasoning, word fluency, memory and spatial connections, which can be expressed in areas, such as: mathematics, science, languages, religion and arts. Specific ability refers to the capability to obtain competence or knowledge skills useful in special areas, such as the special skills a mathematician or an archaeologist needs in order to become successful.



Source: Renzulli (1998, p. 11)

Figure 2-1: Three-Ring Conception of Giftedness

The latter view of Renzulli (2002) follows Gardner's (1983) theory of *Multiple Intelligences* that there are independent and distinct intelligences, such as the 'logical-mathematical intelligence' and 'linguistic intelligence' and each one is associated with a specific type of giftedness. If we focus on mathematics, which is the main interest of this study, we will find that Renzulli's (2002) view about a specific ability also agrees with Krutetskii (1976), a Russian psychologist who, after an extensive study of children with high abilities in mathematics, suggested that those children have a unique ability, which he called 'mathematical cast of mind'. More details about Krutetskii's (1976) views will be presented in the third part of this chapter, which discusses the nature of specific mathematic ability and methods of its development.

Creativity refers to the ability to produce creative accomplishments or generate interesting and practicable ideas at a highly valued level appropriately designed to suit one or more target addresses (Renzulli, 1999, 2002).

Task commitment refers to motivation towards a specific problem or performance and is linked with the terms: "perseverance, endurance, hard work, practice and the confidence in one's ability to engage in important work" (Renzulli, 2002, p. 72).

Looking for effective ways to identify giftedness at early stages, Renzulli (1999) divided giftedness into two broad categories: (a) Lesson learning or 'schoolhouse' giftedness, and (b) Creative productive giftedness.

The first is related to the success in learning and test-taking in schools. This type of giftedness, according to Renzulli (1999), is easily identified by standardised ability tests or informal techniques of assessment such as test scores, teacher ratings, previous grades or accomplishments that he called 'status information'. Therefore, we can identify learners that are more able and provide differentiated learning for them. For this, Renzulli and his colleagues developed a technique for modifying the regular curriculum, which they named *Curriculum Compacting* (Reis, Burns & Renzulli, 1992; Renzulli, Smith & Reis, 1982). With this method, learners that are more able skip parts of the normal curriculum that they have already mastered and work within an alternative curriculum with more challenging content, appropriate for their needs. However, more details about instructional practices will be discussed in the third part of this chapter.

The second category, in contrast to the first, does not have any connection with the scores in IQ tests or other measures of cognitive ability, because, as Renzulli (1999) argues, these cannot predict creative-productive giftedness. Creativity and productivity, he maintains, have a temporal and contextual nature and are not always displayed at the maximum level. Instead, they have 'peaks and valleys' of high-level output (Renzulli, 1999) and, therefore, demand 'action information' which should involve identification approaches under circumstances in which gifted behaviours are displayed and encouraged. Renzulli and his colleagues have developed two models for provision for gifted and talented children — such as the *Enrichment Triad Model* (Renzulli, 1977) and *The Schoolwide Enrichment Model* (Renzulli & Reis, 1985, 1997)

— which they believe may encourage the creative productivity of these children to be displayed. However, models of provision for the education of gifted children will be discussed in the third part of this chapter.

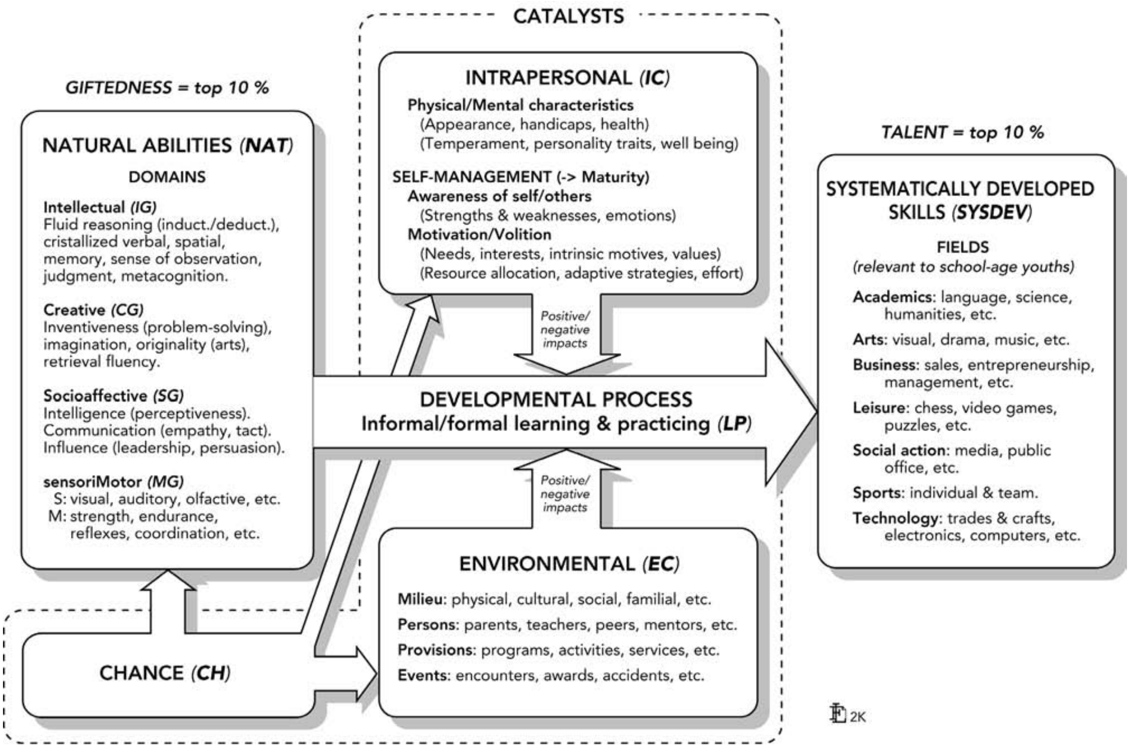
A very interesting point relating to the gift and talent development in Renzulli's (1998, 1986, 1978) view is that a balance of all three sets of human traits is needed along with the appropriate opportunities or experiences which, however, are only provided through special educational programmes and not through the regular instructional ones.

Recently, Renzulli (1998, 2002) emphasised the interaction between personality and environmental factors, which influences gifted behaviour. He represented this interaction as a 'houndstooth' background in his latter revision of his *Three Ring* model (Renzulli 1998, see Figure 2-1). This interaction is referred to as *Operation Houndstooth* (Renzulli & Reis, 2003; Renzulli, 2002). It attempts to draw the attention to how gifted education can help bright students not only develop their talents for their own good, but also raise their awareness about how they can contribute to our society by using their intellectual abilities, motivation and creative talents so as to improve the lives of others.

However, there are some critics of Renzulli's theory of giftedness. One criticism that Renzulli's (1978, 1986, 1998) *Three Ring* has mostly faced is that his model of giftedness does not include 'gifted underachievers', because of the criterion of motivation that those people usually do not display (Gagne, 2004a) or those children who do not show evidence of creativity (Gagne, 2004a; VanTassel-Baska, 1998). His definition of giftedness also has limited applicability to high-achieving children only (George, 2003). Despite the above critique, Renzulli's work is recognised for its theoretical and practical contribution to the field of gifted education (Colangelo & Davis, 2003), especially for those individual schools that aim to choose the children who will attend a special programme that they implement (George, 2003). Renzulli (1998) also maintains that a large number of research studies have been carried out in school programmes which have used an identification system based on his *Three-Ring* model.

Gagne’s Differentiated Model of Giftedness and Talent (DMGT)

Another model of giftedness that describes above average abilities in different domains and the interaction between them, like Renzulli’s (1978) *Three Ring* model, is Gagne’s (1985, 2003) *Differentiated Model of Giftedness and Talent* (Figure 2-2). The difference between Gagne’s model and that of Renzulli is that Gagne’s (1985, 2003) model suggests a clear distinction between ‘talent’ and ‘giftedness’. He describes what characterises a gift and what characterises a talent and attempts to explain the factors that influence the transformation of a natural gift into a talent, which involves outstanding mastery of skills and knowledge.



Source: Gagne (2004b, p. 121)

Figure 2-2: Differentiated Model of Giftedness and Talent

Giftedness is connected to untrained or natural human abilities called *aptitudes* or *gifts*, which are displayed at a scale that places a person at least amongst the top 10 percent of same-age peers in each of the following four domains: (a) Intellectual, (b) Creative, (c) Socio-affective, and (d) Sensorimotor.

These aptitudes, which are natural abilities and have genetic roots, can be observed in children during their schooling in every task where they are undertaken (Gagne, 1985). *Intellectual aptitudes*, for instance, are needed when children learn to read, speak a foreign language, or understand a new mathematical concept. *Creative aptitudes* are used to solve various technical problems or produce original work in literature, art and science. *Socio-affective aptitudes*, which refer to social abilities, are used by the children in their daily interactions with classmates, teachers, or parents. Finally, *Sensorimotor aptitudes*, which refer to physical abilities, are displayed in carpentry, sports or music (Gagne, 2004a, 2004b).

On the contrary, *talent* involves not only natural ability but also outstanding mastery of methodically developed skills and knowledge in at least one field of human activity on a scale that places a person's achievement at least amongst the top 10 percent of peers their age who are active in that field (or fields). According to Gagne's model, such fields of talent, relevant for children and youths, are: (a) Academics, (b) Arts, (c) Business, (d) Leisure, (e) Social action, (f) Sports, and (g) Technology.

Talents, according to Gagne (1985), are developed from human aptitudes through training and the process of education. Therefore, talent requires the existence of natural abilities to an above average degree, but in order for these natural abilities to be developed and recognised as talents in a child or an adult, systematic education and practice is needed, either formal (in schools or athletic teams) or informal (self-taught).

Several times, Gagne (2003, 2004a, 2004b) refined his initial model (Gagne, 1985) and its components until he proposed his *Differentiated Model of Giftedness and Talent (DMGT)* as a developmental theory, which represents the process of transforming natural abilities (aptitudes or gifts) into skills (talents).

The latest version of DMGT (Gagne, 2004a, 2004b) includes three main factors (*catalysts*), which influence the talent development process. These are *intrapersonal* (e.g., physical/mental characteristics and self-management), *environmental* (e.g., milieu, persons, provisions and events) and *chance* (see Figure 2-2). These factors work in two dimensions: directions (positive or negative) and strength. Therefore, the talent development process can be facilitated, inhibited, or accelerated according to the direction or the strength of each catalyst and, thus, every catalyst should be considered relatively. In addition, *Chance* (the third catalyst) can directly influence the other catalysts (intrapersonal and environmental) as well as natural abilities (see Figure 2-2). Most importantly, chance determines, through the recombination of hereditary factors (intrapersonal catalysts), which types of giftedness a child possesses and to what extent (Gagne, 2003, 2004a, 2004b). Some examples of chance, as a factor that influences the talent development process, could be considered as the area in which the children are born, the quality of parenting they experience, as well as the 'accidents of birth and background' (Atkinson, 1978, cited in Tannenbaum 1983, p. 221), because children have no control over any of them.

Gagne (2004a, 2004b) suggests that talents can be measured directly by observing the outstanding performance of individuals using their specific skills in any field. Thus, if we refer to children within the school system, the measurement of talent could be based on normative assessments including teacher exams, achievement tests, scholarships, competitions and so on. Regarding the measurement of giftedness, Gagne (2004a, 2004b) proposes the use of IQ tests as the most appropriate way of measuring only the general cognitive functions (i.e., his model's intellectual domain of giftedness). For the other domains, he does not reject the use of IQ tests, but he suggests further assessment instruments, such as self-assessments or peer judgments, for more valid results. Regarding giftedness in children, Gagne (2004a, 2004b), like Renzulli (1999), suggests that this will be observed only if the children have the opportunity to be in an environment that allows them to display their abilities.

Gagne's (1985) DMGT model with the latest refinements (Gagne, 2004a, 2004b) has faced some critique from other recent researchers. Feldhusen (2004), for instance, claims that Gagne insists so much on defining 'giftedness', while this term has very little use outside of the specialised field of gifted education. He claims that there are other terms — such as high ability, natural ability, aptitude, or precocity — that should be considered. Furthermore, Feldhusen (2004) criticises Gagne in that he does not support his arguments about the intrapersonal catalysts with current research findings, but instead uses old references.

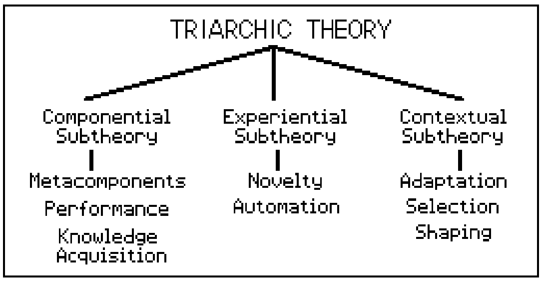
However, Feldhusen (2004) essentially recognises that Gagne has done great work in the field of gifted education, as his DMGT (Gagne, 1985) model has contributed to a better understanding of the concept of talent and influenced many programs within schools that now aim to help students identify, understand and develop their talents.

State educational programmes, such as the revised 2004 *Policy and Implementation Strategies for the Education of Gifted and Talented Students* for the State of New South Wales in Australia (NSW Department of Education and Training, 2004), have also been influenced by Gagne's work and have included ideas for talent development in their national curriculum.

Sternberg's Triarchic Theory of Intelligence

Psychologist Robert Sternberg, like Gardner (1983), studied human intelligence and described processes, structures and factors that produce intelligent behaviour rather than factors that influence the development of an existing gift into a talent, like Gagne (1985), or definitions of gifted behaviour, like Renzulli (1978).

Sternberg's (1985) *Triarchic Theory of Intelligence* is one of the most well-known theories of human intelligence. It includes three facets of higher ability that can be viewed as three sub-theories: *componential*, *experiential* and *contextual*, as the following diagram demonstrates (Figure 2-3).



Source: Sternberg (1985, cited in Kearsley, 2007, webpage)
Figure 2-3: Triarchic Theory of Intelligence

The *componential* sub-theory describes the structures and processes, which together produce intelligent behaviour. These processes are categorised as *metacomponents*, in other words, metacognitive abilities, which manage and monitor processing; *performance components*, which help to carry out plans; and *knowledge acquisition components*, which collect and encode new knowledge. This last component of intelligence refers to problem-solving ability.

The *experiential* sub-theory proposes that there is a relationship between the behaviour in a given task or in a particular situation and the amount of experience one has in that task or situation. According to Sternberg, intelligence is better expressed when the task or situation is relatively novel or unfamiliar. This aspect of intelligence refers to one's ability to deal with new situations by utilising past experiences and present skills.

The *contextual* sub-theory connects intelligent behaviour with the sociocultural context in which it takes place. Sternberg proposes that intelligent behaviour in a given culture or context requires the ability to reshape or adapt to the present environment, or even select a new environment to which one is better suited. This aspect of intelligence refers to one's ability to adapt to the external world or to a changing environment.

Sternberg (1985) also highlights that there is no evidence to suggest that the degree of success in a person's future career is dependent on possessing a specific number of these intelligences, but rather that success involves the interaction of all three facets or sub-theories.

Later, Sternberg (1997, 1999) proposed the theory of *Successful Intelligence*, the overall ability to succeed in life. According to the theory of *Successful Intelligence* (Sternberg 1997, 1999), there are three types of intelligences in human cognition that contribute to future success: (a) Analytical (i.e., the ability to analyse and evaluate ideas, solve problems and make decisions), (b) Creative (i.e., the ability of going further than what is currently known to generate both novel and valuable ideas) and (c) Practical (i.e., the ability to solve everyday problems and work-related challenges by using their personal experiences).

Recently, the three types of intelligence, or, as Sternberg also refers to them, abilities, have been connected with 'giftedness' (Sternberg, 2003a) and have been translated into *analytic giftedness*, *synthetic giftedness* and *practical giftedness*. The new term 'synthetic' refers to creative people as well as to those who are insightful, intuitive or simply adept at dealing with novel situations.

The pursuit of a better definition of giftedness led Sternberg (1998, 2000b, 2004) to introduce one further aspect of giftedness that he called 'wisdom'. According to the *Balance Theory of Wisdom* (Sternberg, 1998), wisdom is based upon the application of intelligence and creativity aimed at a common good and is mediated by a balance of interests as follows: intrapersonal (one's own), interpersonal (others') and extrapersonal (organisational, institutional), over the long and short-term. It is believed that wisdom leads to the attainment of balance when shaping or adapting to existing environments, and the selection of new environments. The theory of wisdom was the basis for the development of a model of giftedness called 'WICS' (Sternberg, 2003b, 2003c, 2005a, 2005b) — the term is an acronym of: Wisdom, Intelligence, Creativity, Synthesised — which has been suggested as key to the development of the gifted leaders of the future.

With regards to the identification of different types of giftedness, Sternberg (2003b) proposes that each type of giftedness should be measured with a different kind of assessment. He distinguishes ability tests from achievement tests and suggests the use of the former rather than the latter, arguing that standardised tests measure achievement more than ability and that this is not sufficient when attempting to evaluate all types of giftedness. More specifically, he suggests the use of ability tests which are 'fluid' (e.g., matrix or series-completion problems) rather than 'crystallized' (e.g., vocabulary and reading tests) and principally novel. Particularly, for the measurement of practical intelligence, he suggests the use of sub-tests of 'tacit knowledge' as well as 'scenario-based' measures. The first involves 'managing oneself', 'managing others' and 'managing tasks'. The second includes 'real-life scenarios' involving problems to be solved.

Finally, Sternberg argues that giftedness can be developed by improving weakness and capitalising on potency and that this is important in gifted education, which should be based on modern programmes with a broader orientation (Sternberg, 2003a). These modern educational programmes for gifted children should aim at the development of gifted children's wisdom by teaching these children how to use their talents to help others. In addition, these programmes should be based on teamwork and on the collaboration of institutions. The first overcomes the diversity in teachers' abilities or talents. Each person can find the best possible role, which enables him/her to contribute to the cooperative endeavour within the programme or institution. The second may help to develop common methods and a common model of assessment for the measurement of different kinds of giftedness (Sternberg, 2000b).

Sternberg's theories of intelligence, together with those developed by Gardner, are recognised as those that have broadened the current definitions of intelligence and contributed to "one revolution in education in the past two decades" (Colangelo & Davis, 2003, p. 42). The following section will now present Gardner's (1983) theory of Multiple Intelligences that focuses on a specific ability, mathematical ability, which is related to mathematical intelligence.

Gardner's theory of Multiple Intelligences

Gardner (1983) continued Thurstone's (1938) theory of *Primary Mental Abilities* and, based on systematic studies of brain function as well as on observations of human behaviour, initially proposed that there are seven independent intelligences: (a) Linguistic, (b) Logical-mathematical, (c) Spatial, (d) Bodily-kinaesthetic, (e) Musical, (f) Interpersonal, and (g) Intrapersonal.

Each one represents a relatively autonomous set of problem-solving abilities, having a distinctive basis within the nervous system and brain, and each is associated with a specific type of giftedness.

- *Linguistic intelligence,* for example, is involved in writing, reading, talking and listening;
- *Logical-mathematical intelligence* in making calculations, solving puzzles and developing proofs;
- *Spatial intelligence* in moving from one place to another or designating orientation in space;
- *Bodily-kinaesthetic intelligence* in using the body to perform skilled movements (e.g., useful for athletes, dancers, surgeons);
- *Musical intelligence* in singing, playing music and composing;
- *Interpersonal intelligence* in understanding other individuals and their relationships (useful for psychologists, teachers and politicians); and
- *Intrapersonal intelligence* is involved in self-understanding (recognising one's own thoughts, emotions and actions).

Gardner later put forward an eighth intelligence called *naturalist intelligence* and proposed the consideration of a ninth type for future inclusion called *existential intelligence* (Gardner, 1999).

The later-added eighth intelligence (*naturalist*) is involved in understanding the natural world and working successfully within it. It includes capacities displayed in everyday life such as those that people deploy when they select one car or a pair of sneakers or gloves rather than another (Gardner, 2006).

The idea of a ninth intelligence (*existential intelligence*) will concern (if confirmed by Gardner) one's ability to raise questions about his/her place in the world. At the moment, Gardner argues that only the initial seven intelligences together with the eighth (*naturalist*) meet the criteria that he has set for an 'intelligence' and, thus, he speaks about '8½ Intelligences' (Gardner, 1999, 2006).

Gardner built his theory and established particular criteria to consider 'candidate intelligences' (Gardner, 1999, 2006) based upon evidence coming from various sources, such as biological sciences, neuropsychology, developmental psychology, traditional psychology and psychometrics. He studied the development of both ordinary and gifted children, the breakdown of cognitive skills after brain damage and exceptional populations, such as prodigies, idiot savants and autistic children. He also utilised data from psychometric studies, which included examination of correlations along with tests and psychological training studies, such as measures of transfer and generalisation across tasks (Gardner, 2006). Gardner (1999) himself has acknowledged the great inspiration that two particular and distinct populations provided for his MI theory: (a) stroke victims who were suffering from aphasia at the Boston University Aphasia Research Center, and (b) children, ordinary and gifted, at Harvard's Project Zero laboratory. By studying the first group, for instance, he found patients who were left with some forms of 'intelligence' intact in spite of the damage to other cognitive abilities such as speech, something that led him to argue that there are different distinct 'intelligences' and each one can be dissociated from others (Gardner, 1983). His work with ordinary and gifted children gave him the opportunity to study their cognitive development in relation to educational implications.

Gardner (1983, p. 60) initially defined an ‘intelligence’ as the ability “to resolve genuine problems or difficulties” — including “the potential for finding or creating problems” (p. 61) — and create effective products, which are valued within one or more cultural settings. Later on, however, he realised that by looking at problem solving only, someone could assume that intelligence “would be evident and appreciated everywhere, regardless of what was (and was not) valued in particular cultures at particular times.” (Gardner 1999, p. 33) He then suggested a new definition, which conceptualises an ‘intelligence’ as “a biopsychological potential to process information that can be activated in a cultural setting to solve problems or create products that are of value in a culture” (Gardner 1999, pp. 33-34). With this refined definition, Gardner (1999, p. 34) suggests that:

> Intelligences are not things that can be seen or counted. Instead, they are potentials — presumably, neural ones — that will or will not be activated, depending upon the values of a particular culture, the opportunities available in that culture, and the personal decisions made by individuals and/or their families, school teachers, and others.

Considering Gardner’s (1983) MI theory from an educational perspective, we should highlight his latest suggestions about the factors that influence the development of an ‘intelligence’, the way that it is demonstrated, as well as his suggestions about the assessment of multiple intelligences within schools.

More specifically, Gardner (1998, 1999) recognises the experiences of individuals as factors that influence the degree to which each of the intelligences can be demonstrated and that the intelligences, even though independent of one other, can work in concert within a domain. In mathematics, for instance, logical-mathematical and linguistic intelligences are required for solving complex word problems. He also argues that these distinct intelligences are based on abilities and skills, which can be developed in different ways for different people, depending on both heredity and training. This, where children are concerned, is affected by the parents’ role and teachers’ work within schools, as it connects the development of intelligence with the appropriate education. He also suggests, through his revised (Gardner, 1999) definition of intelligence, that independent intelligences are valued differently within different cultures, meaning that different countries, different districts within the same country, or even different teachers may emphasise certain intelligences whilst minimising others.

In relation to the assessment of multiple intelligences, Gardner (1992, 2006) and Karolyi, Ramos-Ford and Gardner (2003) suggest the development of new forms of assessment that could be used within primary schools and may have implications for the practical aspect of this study. Although he acknowledges the conventional intelligence tests for measuring three types of intelligence (i.e., linguistic, logical-mathematical and spatial), he proposes that these tests cannot measure effectively the remaining five and suggests the use of several forms of assessment for gifted children, which are not to be based on a single test score. Particularly, for children in primary schools, Gardner (1992, 2006) and Karolyi et al. (2003) suggest teachers employ assessment practices that take into account observations of children's behaviour and their working styles, such as the way a child reacts to different materials, the ability for planning an activity and for reflection on a task, as well as the level of persistence (Karolyi et al., 2003).

Gardner (2006) and Karolyi et al. (2003) distinguish assessment from tests and argue that while tests are only for gathering bits of information, assessment has a broader quality, being more flexible and requiring a continuous process of observation and reflection. They also propose that assessment of the gifted must be an ongoing, unobtrusive component of the child's natural learning environment and *intelligence fair*. In order for this to be successful, Karolyi et al. (2003) suggest that any assessment methodology should try to access intelligence 'in-operation' but not confound intelligences. An *intelligence fair* approach for measuring spatial intelligence, for instance, should not take as fact the existence of linguistic and logical-mathematical intelligences, but focus on spatial abilities involving both understanding the task and producing responses. Furthermore, it will be more beneficial for a teacher, who wants to assess a child's mathematical ability to provide an interactive board game that helps demonstrate the understanding of numbers and then observe the child at play, rather than trying to assess his/her verbal response to individual word problems. He argues that assessing a child's abilities by observing interaction within the classroom is more *intelligence fair* and evaluations become more useful "when they occur in situations closely resembling actual working conditions" (Karolyi et al., 2003, p. 105). These recommendations agree with those that have been suggested by educational researchers from the field of mathematics (e.g., Eyre, 2001; Johnson, 2000; Koshy, 2001; Koshy & Casey, 1997a), who recommend a continuous identification process through provision for mathematically gifted children within schools. This is discussed in more detail in the next sections.

One way to assess someone effectively, according to Karolyi et al. (2003), can be the observation of his/her progress or success through the use of portfolios or 'process folios'. Portfolios, which include students' progress in a specific domain as well as samples of their best work, allow teachers and students the opportunity to reflect on their goals and ways in which to achieve them; the portfolio also gives both of them the chance to revise and rethink goals and processes if necessary (Gardner, 1992; Karolyi et al., 2003). The use of portfolios is also recommended especially for identifying gifted mathematicians (Eyre, 2001; Koshy, 2001).

Gardner, Krechevsky, Feldman and colleagues developed a program called 'Project Spectrum' (Karolyi et al., 2003), which includes methods of assessment of gifted children, such as those mentioned above and an instructional approach for the education of gifted children based on *Multiple Intelligence* theory. This program, according to them, is the 'MI theory in action', providing a rich child-friendly environment with a wide range of opportunities. This allows them to demonstrate different abilities (or intelligences) and also leads to teachers being able to observe and assess them in a meaningful context. All the collected information is used to build a 'Spectrum Profile' that becomes the basis for all future decisions about the kind of experiences each student is to be presented with in the classroom (Karolyi et al., 2003).

The contribution of Gardner's (1983) MI theory to the field of gifted education has been recognised by many educational researchers. VanTassel-Baska (1998), for instance, asserts that this theory has led to a shift of interest towards emphasis on talent development within schools and inspired many educators to apply this idea to the classroom environment within the curriculum. Furthermore, Colangelo and Davis (2003, p. 42) argue that: "MI theory has opened the eyes of many educators regarding conceptions of intelligence and giftedness and the teaching of all students."

At the end, it should be noted that Gardner's (1983) MI theory about the existence of a specific mathematical intelligence, which is a factor of mathematical giftedness, has influenced my investigation and guides my thesis. From the time I adopted this theory, I have recognised the existence of mathematically gifted children and, thus, my research in provision methods for addressing their needs within primary schools has acquired a real target. In the next section, I am going to discuss this target in more detail, including the nature of mathematical ability, identification issues and methods of provision for gifted mathematicians.

This part presented a background of conceptions of giftedness. Broader conceptions of intelligence and giftedness developed by four of the most known researchers in the field (Gardner, Sternberg, Gagne and Renzulli) were discussed. Comparing the theories presented, we can conclude that although each one had begun from a different starting point with a different focus — e.g., on domains where intelligences are demonstrated (Gardner, 1983), on mental processes (Sternberg, 1985), on the factors that influence the transforming of gifts into talents (Gagne, 1985), or on the definition of gifted behaviour and identification practices (Renzulli, 1978) — they all seem to finally contribute in the same way to gifted education and to the benefit of society (the common good, according to Renzulli (2002) and Sternberg (2003b)). Identification methods of giftedness in childhood now demand a wider approach that will not be based on the results from some cognitive ability or IQ tests only (which are not rejected), but on more information from different sources. Identification issues are considered integrated issues in gifted educational programmes that aim to help students identify, understand and develop their talents in any domain through the appropriate opportunities or experiences provided for this reason.

The table on the following page (Table 2-1) represents some of the most characteristic viewpoints from the broader conceptions of giftedness discussed in this section. The next sections will discuss the main interest of this study, which is specific mathematical ability, identification of such an ability and methods of provision for mathematically gifted children.

Table 2-1: Broader conceptions of intelligence and giftedness at a glance

	Gardner	Sternberg	Gagne	Renzulli
Types of Intelligences	Logical-mathematical, Linguistic, Musical, Spatial, Bodily-kinaesthetic, Interpersonal, Intrapersonal, Natural	Analytical, Creative, Practical		
Types of Giftedness	Each one of the different intelligences is associated with a specific type of giftedness.	Analytic, Synthetic, Practical, Wisdom	Intellectual, Creative, Socio-affective, Sensorimotor	Above average ability, Creativity, Task commitment
Developmental Factors	Experiences, Different cultures, Interaction among distinct intelligences within a domain, Heredity, Training	Experiences, Interaction among the three facets of intelligence (or the 'synthesis' of the different types of giftedness according to WICS model)	Heredity and training combined with the following 3 catalysts: Intrapersonal (physical, mental, motivation, volition, self-management, personality), Environmental (milieu, persons, provisions, events) and Chance	Experiences (or appropriate opportunities), Motivation, Personality in connection with environmental factors ("Operation Houndstooth"), Interaction among the three types of giftedness

Identification & Assessment	IQ tests supported by evidence from different sources (e.g., neuropsychological data). Several forms of assessment (e.g., observations, portfolios), which must be ‘in-operation’ and ‘intelligence fair’ (Spectrum Approach).	IQ and achievement tests for measuring analytic giftedness. Multiple techniques of assessment for measuring each type of giftedness according to WICS model (e.g., interviews, recommendation letters, grades, ability tests with fluid and novel context and real-life scenarios).	‘Metric-Based’ (MB) model of levels of giftedness. IQ tests for measuring the intellectual giftedness. Self-assessments and peer judgements for the other types of giftedness. Observations on tasks during schooling for identifying and measuring talents.	IQ and ability tests for measuring above average ability (and the ‘schoolhouse ability’). Status and action information for identifying and measuring creativity and task commitment.
Educational Provision	Classrooms with a rich child-friendly environment, appropriate for the education of gifted children in different types of intelligences (“Project Spectrum”).	Gifted educational programmes according to the WICS model with an emphasis on assessment of each type of giftedness and development of wisdom.	Provision involving strategies of: Enrichment (or Differentiation), Grouping and Acceleration	Curriculum compacting, Enrichment strategies (“Enrichment Triad Model”), Special instructional programs with emphasis on raising awareness about how gifted children can contribute to our society and improve the lives of others.

2.2 Mathematical ability and mathematically gifted children

The previous sections discussed the general concept of giftedness. Prominent theories of giftedness and talent development, which have been brought out recently, were reviewed. It became clear that this study is based on Gardner's (1983) MI theory and the existence of a specific mathematic ability or *logical-mathematical intelligence*, which is associated with mathematical giftedness.

This section discusses the nature of mathematic ability, which characterises children who are gifted in mathematics, explaining what we mean when we talk about mathematically gifted children and how we can recognise them.

2.2.1 Definitions

In 1976 Krutetskii, a Russian psychologist who studied the behaviour of able mathematicians for twelve consecutive years (1955-1966), highlighted the difficulties of giving a unique definition regarding mathematical ability. He concluded that all attempts to define mathematical ability until then did not lead to a common definition that could satisfy everyone. However, he noted that there was indeed an agreement among all researchers, as they distinguished 'school' mathematical ability, in other words the ability in mastering mathematical information and doing mathematical tests or problems, from 'creative' mathematical ability, which relates "to the independent creation of an original product that has a social value" (Krutetskii 1976, p. 21). This view has not changed much since then as it agrees with recent models of giftedness that were developed later on and discussed in the previous sections — e.g., Gagne's (1985) DMGT , Renzulli's (1978) *Three Ring*, and Sternberg's (2003b) WICS Model.

However, Gardner (1983), with his MI theory discussed earlier, has added that mathematical ability, which forms mathematical giftedness, is associated with a distinct intelligence that is the *logical-mathematical intelligence*, which can be developed differently for different people and across different situations (e.g., parents' role and teachers' work in schools). Mathematical intelligence, according to Gardner (1998, 1999), can be displayed within a domain as a separate entity or in correlation with other intelligences (e.g. *logical-mathematical intelligence* and *linguistic intelligence* in order to solve a complex word problem). Orton (1987, p. 116), based on the latter view of Gardner has added: "Mathematical ability can take many forms, each form derived from a different mix of other abilities…numerical ability, spatial ability, verbal and non-verbal reasoning, convergent and divergent thinking abilities and so on."

Koshy, Ernest and Casey (2009) and also the DfES (2004b) and the US Department of Education and Office of Educational Research and Improvement (1993) highlight a further dimension of mathematical ability, which is a potential or future-oriented skill; in other words the capacity to master new mathematical facts and skills and also to solve non-routine and novel problems. This attribute, according to Koshy et al. (2009), is the reason why mathematical ability is not easily observable and, thus, difficult to be assessed.

Furthermore, there are greater difficulties in recognising real ability in early stages of primary school because, as McClure (2001, p. 65) argues:

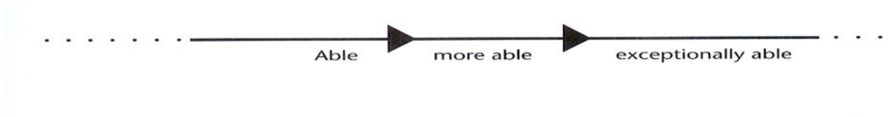
> One of the things we know about primary children is that their abilities change and are expressed differently over time. Identifying able mathematicians at 5 is different from identifying them at say 11, partly because they have fewer skills to exhibit their abilities and partly because their abilities may change.

McClure (2001) also argues that in some cases, the exceptional ability in numeracy that a child shows from the age of five may be because of parental involvement and that this child may not differ from his/her peers if they have had similar help.

The latter view highlights the difficulties in the identification of mathematical ability in childhood and also raise the following question:

Who are the mathematically gifted children and how can we recognise them?

As mentioned earlier, there is no simple and unique way to define mathematic ability, so as to find a universal definition of mathematically gifted children to satisfy everyone. Some experts, such as Krutetskii (1976), prefer to define them by referring to a selection of particular characteristics of mathematically gifted children, drawing on a wide list. Such lists of characteristics, of course, differ from individual to individual (McClure, 2001). Others, such as Koshy and Casey (1997b), refer to gifted mathematicians by using a 'continuum' model, where children's abilities can range from able to exceptional (see Figure 2-4). According to Koshy (2001), such a model for referring to more able children allows flexibility in both the identification of mathematical ability and making provisions. The following section will discuss these issues in more detail.



Source: Koshy and Casey (1997b, p. 5)

Figure 2-4: The Ability Continuum

Recent authors (e.g., Kennard, 2001; Koshy, 2001; Koshy, Ernest & Casey, 2009; McClure, 2001) refer to Krutetskii's (1976) work when they speak about characteristics of mathematically able or gifted children. They recognise that Krutetskii was one of the first researchers who analysed the ways of working and thinking of children who had been identified as gifted in mathematics and provided a list of particular characteristics of mathematically gifted children that "have guided many researchers ever since" (Koshy, 2001, p. 20). Kruteskii's (1976) list suggests that the following characteristics could be observed in mathematically gifted children:

- *The formalised perception of mathematical material*. In other words, this is the ability to understand the terms of a mathematical problem, compare its data, find relationships and categorise mathematical problems according to their structure.
- *The generalisation of mathematical material*. In other words, this is the ability of very able mathematicians to perceive a general rule from a particular task that they have worked on and apply it to solving other problems.
- *The curtailment of thinking*. This is the characteristic of mathematically gifted children to omit intermediate steps in a logical argument and think in abbreviated structures during a mathematical activity.
- *Flexibility of mental processes*. In other words, this is the ability to look for several different ways to solve a single problem and the flexibility to switch from one method to another during a problem-solving process.
- *Striving for economy of mental effort, rationality ('elegance') in a solution*. In other words, this is the tendency of a gifted child to evaluate different possible solutions of a mathematical problem and choose the simplest, clearest, most economical and most rational of them. However, Krutetskii found that this

ability was not clearly displayed in the primary grades, but it started being noticeable only in the intermediate grades.

- *Mathematical memory*. In other words, this is the ability to memorise generalised mathematical relationships, problem types, solutions and problem-solving approaches from previous experience. This ability also is developed as the child matures. Gifted pupils in the primary grades usually remember concrete data and relationships very well but, as they get older, "the general and the particular, the relevant and the irrelevant, the necessary and the unnecessary are retained side by side in their memories" (Adapted from Krutetskii, 1976, pp. 332-339).

Krutetskii (1976) also suggested that mathematically gifted children persevere in doing mathematical tasks without tiring or losing their capabilities and they have the ability to see the world 'through mathematical eyes'. The latter characteristic, which was named by Krutetskii (1976) 'mathematical cast of mind', is a unique organisation of mind that makes the phenomena of the environment mathematical. Gifted children therefore tend to pay attention to the mathematical aspect of phenomena, to detect quantitative and spatial relationships, bonds, and practical dependencies everywhere. Krutetskii (1976) identified three main types of individuals with a mathematical cast of mind:

- the *analytic* type, who tends to think in verbal-logical ways;
- the *geometric* type, who tends to think in visual-pictorial ways; and
- the *harmonic* type, who combines both of the previous types, and because of that, is suggested as having the best mathematical skill.

In the USA, Sheffield (2003) has recently suggested a set of similar characteristics, but this refers to mathematically promising students rather than to mathematically gifted ones. Sheffield (1999, 2003) prefers the term 'promising' instead of 'gifted' and reminds us of the results from the *Report of the NCTM Task Force on Mathematically Promising Students* (Sheffield et al., 1995, cited in Sheffield, 2003). In that report, mathematically promising students are defined "as those who have the potential to become the leaders and problem solvers of the future." (Sheffield, 2003, p. 2) Mathematical promise is described as a function of *ability*, *motivation*, *belief*, and *experience or opportunity*.

Sheffield (2003) argues that teachers should be aware of some of the characteristics that mathematically promising students demonstrate in order to be successful in developing ever-increasing numbers of those students. For this reason, she has collected a set of *Characteristics of a Mathematically Promising Student* (see Table 2-2), including many of the characteristics on Krutetskii's (1976) list.

Table 2-2: Sheffield's list of characteristics of a mathematically promising student

Source: Sheffield (2003, pp. 3-4)

CHARACTERISTICS OF A MATHEMATICALLY PROMISING STUDENT

MATHEMATICAL FRAME OF MIND

1. Loves exploring patterns and puzzles
2. Sees mathematics and structure in a variety of situations
3. Recognizes, creates, and extends patterns
4. Organizes and categorizes information
5. Has a deep understanding of simple mathematical concepts, including a strong number sense

MATHEMATICAL FORMALIZATION AND GENERALIZATION

1. Generalizes the structure of a problem, often from only a few examples
2. Uses proportional reasoning
3. Thinks logically and symbolically with quantitative and spatial relations
4. Develops proofs and other convincing arguments

MATHEMATICAL CREATIVITY

1. Processes information flexibly — switches from computation to visual to symbolic to graphic representations as appropriate in solving problems
2. Reverses processes — can switch from a direct to a reverse train of thought
3. Has original approaches to problem solving — solves problems in unique ways, tries unusual methods
4. Strives for mathematical elegance and clarity in explaining reasoning

MATHEMATICAL CURIOSITY AND PERSEVERANCE

1. Is curious about mathematical connections and relationships — asks "why" and "what if"
2. Has energy and persistence in solving difficult problems
3. Digs beyond the surface of a problem, continues to explore after the initial problem has been solved

The following characteristics may be useful in a mathematics class but are not necessary for a student to be mathematically promising:

1. Speed and accuracy with computation
2. Memory for formulas and facts
3. Spatial ability

Sheffield (2003) categorised the characteristics of mathematically gifted children into four types of ability so that they become easily detectable to ordinary teachers: 'mathematical frame of mind', 'mathematical formalization and generalization', 'mathematical creativity', 'mathematical curiosity and perseverance' (Table 2-2). These types of ability are considered attributes of mathematically gifted children. Sheffield (2003) proposes that neither all nor most of these characteristics are exhibited in all promising students, but they are only indicators of potential mathematically promising talent, which should be developed in all students, if it is possible. She also suggests that teachers, who are usually happy when they see some of their students computing rapidly and accurately, should not consider this characteristic as a clear indicator of mathematical giftedness. This is because many gifted children often care more about the process that they may follow to solve a problem rather than about the computation, which they consider an unimportant detail.

Characteristics lists, such as those presented in this section, the first from Krutetskii's research in Russia and the second from Sheffield's research in the USA, can be useful, according to Koshy (2001), for teachers — not only in raising their awareness about mathematical giftedness but also in thinking about provision. Sheffield (2003) also highlights the value of such lists as motivation for teachers and parents to try harder to find attractive mathematical challenges at all levels, which will engage pupils in the development of mathematical power. It is not a coincidence that both Koshy and Sheffield refer to such lists of characteristics as factors that help teachers and parents raise their awareness of the concept of mathematical giftedness, but not as the sole means of identification of mathematically gifted or promising students. This may be because the effectiveness of a characteristics list is mostly dependent on the teacher's observation and because, as mentioned earlier, mathematical ability is not always easily observable.

Consequently, the next question posed is:

> *Are there any practical methods that help teachers identify mathematically gifted children?*

2.2.2 Identification of mathematically gifted children

Today, it is widely recognised that the identification of gifted children in a specific domain, such as mathematics, is a very complex task and, therefore, multiple criteria and information sources should be used in combination to identify gifted children in mathematics (McClure, 2001), as in any other context, as discussed in section 2.1of this chapter (Gagne, 2004b; Gardner, 1992; Karolyi, Ramos-Ford & Gardner, 2003; Renzulli, 1999, 2004; Renzulli & Reis, 1985; Sternberg, 2003b). These sources of information might include nominations (from parents, teachers, or peers), tests, performance-based assessment and diagnostic assessments (e.g., observations, work produced by pupils and characteristics checklists).

Nominations

Nominations coming from either parents or teachers or peers can be very helpful in the identification of mathematical ability (Eyre, 2001; Feldhusen, 2001; McClure, 2001). According to Straker (1983), ability in mathematics is often revealed in early childhood and, therefore, parents usually notice their child's special abilities in mathematics before their child starts school. Because of this, Koshy (1997b, 2001) further contends that it is logical for teachers to consult parents at the time their child starts school. Perhaps a communication with parents about the aptitudes and abilities of their child may reveal some indicators of mathematical ability such as a fascination for numbers or an ability to spot number patterns or make sophisticated constructions (Koshy, 2001). Koshy (2001) suggests that schools can acquire parent's involvement in the search for the talent in practice if, for example, they provide a space on a school admission form where parents can describe their child's interests, interview the parents to see their perceptions about any area of their child's expertise or employ parent questionnaires and surveys.

There are, however, many cases where parents' reports are often disregarded as biased or as evidence of an ambitious parent, either because most parents think that their own child is gifted (Davis & Rimm, 1985) or because others, especially some well-educated parents, underestimate rather than overestimate their children (Chitwood, 1986). Nevertheless, there are research findings that value parents' nominations as a source of information for the identification of a child as gifted. Louis and Lewis (1992), for example, found that the majority of parents (61 percent) were correct in identifying their children as gifted, while the rest (39 percent) were not totally wrong because their children were indeed advanced, but they did not meet the criteria for giftedness. Gardner (2006) also found, through ‘Project Spectrum’, that there was a discrepancy between parents’ and teachers’ perceptions of whether or not a child was gifted, probably because the parents were biased and/or had fewer opportunities (in contrast with the teachers) to observe the strengths of a large number of children. In ‘numbers’, however, Gardner (2006) found that the identification was easier regardless of whether the child was at school or at home.

Peer nominations — or ‘peer judgments’, according to Gagne (2004a, 2004b) — can be another helpful source of identification. Koshy (2001) writes that experience has shown that if the teacher asks children to nominate pupils who they think are gifted in mathematics and compare their answers with his/her own list, he/she will find that the peer group assessment is quite accurate. Sometimes, indeed, it is more useful in identifying some pupils who tend to hide their ability from the teacher because of some reasons, such as the fear of ‘extra’ work. Jenkins (1979, cited in Gagne, 1989) has suggested that such nominations from peers could take the form of a game of ‘make-believe’, in which pupils are asked to imagine that they are in a difficult situation (stranded on a desert island for instance). They are then asked to imagine that they need to call for help from a classmate who they believe is the best organiser (i.e., persuader, leader), best ‘fixer’ (i.e., someone able to improve things), best inventor (i.e., creator, discoverer), entertainer and so on.

Teachers’ nominations, of course, are a key point for the identification of a specific ability, such as mathematical ability and, because of this, a large amount of work has been done to consider the effectiveness of teachers in identifying gifted and talented children. For example, Freeman’s (1998) research review showed that teachers would be able to identify gifted and talented children effectively only if they had been trained in what to search for. Otherwise, teachers without training could possibly confuse gifted children with tidy, neat and conforming children. As Eyre (2001, p. 14) says, their “assessment[s] can be inaccurate and dangerous”.

Tests

Cognitive tests and IQ tests are useful in giving information about a child's potential, but they are limited in one dimension, mainly relating to verbal and analytic skills (Renzulli, 2004). Furthermore, standardised tests are often insufficient in identifying gifted minority and low-income students (VanTassel-Baska, Feng & Evans, 2007; Naglieri & Ford, 2003), because they require children to answer verbal and quantitative questions to receive high scores, whereas many smart minority children lack the reading, writing and arithmetic skills (Naglieri & Ford, 2003). Naglieri and Ford (2003) argue that such children would have demonstrated high performance if they had been assessed in a nonverbal test of general ability. Nevertheless, IQ tests are still used widely as an indicator of future success in school examinations (Koshy, 2001; Freeman, 1998). Cognitive abilities tests are also used by many Local Educational Authorities (LEAs) in the UK as indicators of potential (Koshy, 2001).

Achievement tests are widely conducted within schools, in the UK and elsewhere. In the UK, for instance, the National Curriculum provides maintained schools with Standard Assessment Tasks (SATs) while, at the same time, schools are able to use more tests, such as the standardised tests produced by the National Foundation for Educational Research (NFER) (Koshy, 2001; McClure, 2001). In the USA, the best-known assessment for gifted children is the Scholastic Aptitude Test (SAT). This is used in the USA and elsewhere to select gifted children to participate in special programmes where they have the opportunity through acceleration to succeed and gain early entry into university (Renzulli, 2004; Koshy, 2001). Achievement tests give information about children's academic performance and they are very useful for the teacher because, according to George (2003), they can confirm teachers' initial judgements, encourage him/her to be adaptable when new evidence comes out and provide him/her with unbiased results as evidence of high ability. However, none of the abovementioned writers have argued that the use of cognitive or achievement tests can predict adult achievement from childhood or that they should be used as the only source of information to identify high ability.

Performance-based assessment

Recently, VanTassel-Baska, Feng and Evans (2007) carried out a research study in South Carolina (USA) to identify gifted students with low-income and minority backgrounds by employing a new dimension of identification called 'performance-based assessment'. This method of identification combined 'traditional' measures, such as intelligence and achievement test scores, with a variety of 'non-traditional' methods, such as:

- observations of students interacting with a range of learning opportunities — as Gardner suggested (Karolyi, Ramos-Ford & Gardner, 2003);
- *dynamic assessment*, which combines instruction with identification based on a 'test-intervene-retest' format. This approach involves instructing children on how to perform on certain tasks with the mediated assistance of an adult and then measuring their performance on how to master the testing tasks and their progress in learning how to solve similar problems (Kirschenbaum, 1998); and
- *nonverbal tests* (Naglieri & Ford, 2003; Naglieri & Kaufman, 2001; Bracken & McCallum, 1998), which focus on the ability observed through spatial or logical organisation, by utilising shapes or geometric designs for example, but not on the answers given on verbal and quantitative questions, which normally characterise IQ tests. Children are not required to read, write, or speak (Naglieri & Ford, 2003).

VanTassel-Baska, Feng and Evans (2007) argue that the identification protocol has demonstrated efficacy of 'performance-based assessment' in identifying more low-income and minority gifted students.

Diagnostic assessments

Eyre (2001) refers to 'diagnostic assessment' as one of the three broad forms of information available to schools ('tests' and 'opinion' are the other two). She suggests that this kind of assessment can overcome problems connected to tests and help teachers to recognise some aspects of high attainment, when it is supported by the work that the children produce (through the use of portfolios, for example); observations of the classroom teacher (especially in question and answer sessions); and characteristics checklists.

Portfolios

One practice that can help teachers collecting evidence of high levels of performance is the use of portfolios (Eyre, 2001; Feldhusen, 2001; Karolyi, Ramos-Ford & Gardner, 2003; Koshy, 2001; Renzulli & Reis, 1985). Feldhusen (2001) suggests that portfolios could contain child's projects, problem-solving activities, as well as creative productions. Koshy (2001) recommends that teachers encourage their pupils to keep their 'best' work up-to-date so that they can assess their progress anytime, even when they are too busy during the school day. She argues that teachers can use children's portfolios to make judgements about their special aptitudes and "assess the reasoning behind children's output." With the help of some interviews, they can arrange for the appropriate provision to be delivered to them in the future by adjusting the tasks (Koshy, 2001, p. 27).

Eyre (2001) also suggests that, in some cases, portfolios can give sufficient evidence of the achievement of a child for inclusion on a gifted register. In their *Schoolwide Enrichment Model*, Renzulli and Reis (1985) suggest the use of a 'total talent portfolio', which will assess three dimensions of the learner: abilities, interests and learning styles. They argue that the benefit of keeping portfolios is that they contain information that focuses on strengths rather than weaknesses. Others believe that portfolios allow teachers and students the opportunity to reflect on their goals and ways in which to achieve them. They give both of them the chance to revise and rethink goals and processes if necessary (Gardner, 1992; Karolyi et al., 2003). A portfolio-based approach for the identification of high ability is also recommended by the UK Standards (DfES, 2004a). In addition, I personally experienced the successful use of portfolios during my previous research in *Teaching Thinking in Primary Schools through Mathematics* for my masters dissertation, which showed that pupils were motivated to work harder when they saw their best work collected in their portfolios (Dimitriadis, 2005).

Observations

Teacher's systematic observations are considered one of the most effective ways to identify gifted children in a specific domain, such as mathematics (Gardner, 1992, 2006; Karolyi et al., 2003; Koshy, 2001). However, systematic observation to identify the gifted does not only mean observing what naturally occurs in the classroom —

because, as earlier mentioned, mathematical ability is not always visible — but, also, creating the right opportunities that allow pupils to show their real potential as well as systematic monitoring of their work. For instance, Koshy (2001) suggests that teachers must provide open-ended mathematical problems that encourage investigations, observe the pupils undertaking mathematical tasks, listen to them and monitor them by keeping written outputs. Furthermore, it should be reiterated that Karolyi, Ramos-Ford and Gardner (2003), mentioned in section 2.1 of this chapter, have suggested observations of children's interaction within classrooms as an 'intelligence-fair' assessment method of giftedness.

Characteristics checklists

In previous sections, some examples of lists of characteristics of mathematically gifted children were presented, highlighting their role in raising awareness of the education of these children, as they help teachers, parents and anyone involved in education to understand better some kinds of behaviour (Eyre, 2001; Feldhusen, 2001; Freeman, 1998; Koshy, 2001; Sheffield, 2003). Additionally, there is evidence from recent research that teachers feel more confident to recognise mathematically gifted children when they use a checklist of characteristics of mathematical gifted behaviours. This is because they feel that the systematic observation of children's behaviours during the daily mathematics lessons allows them to distinguish more easily the gifted pupils (Koshy & Casey, 2005).

However, as mentioned earlier, checklists cannot be used as a sole method to identify mathematically gifted children, because they are only based on teachers' observations, which may be affected by a variety of factors. Some of these factors are presented below.

Barriers to the identification of mathematically gifted children in schools

Teachers may fail to recognise real mathematical ability for many reasons. One reason may be the learning disorders that some gifted children may have, because traditional giftedness and learning disorders can mask each other and it may be impossible to identify any of them (Karolyi et al., 2003).

Enthusiasm is another matter that needs attention, because it does not necessarily reveal skill in a particular domain. Therefore, if we want to assess children's giftedness, we should take into consideration the distinction between capacities and preferences. For example, according to the theory of *Multiple Intelligences* (Gardner, 1983), a child can be gifted with logical-mathematical intelligence but not in bodily-kinaesthetic. So, if we attempt to assess his/her giftedness through an environment in which athletics are better organised according to the students' level than mathematics or in which athletic prowess is more valued by other children or the community than mathematical skill, we will possibly not have reliable and valid results. This is because a child who is gifted with logical-mathematical intelligence may engage enthusiastically in sports activities and yet avoid mathematics (Karolyi et al., 2003).

Certain fears can disguise mathematical ability, forming a barrier to the identification of gifted mathematicians. For instance, fear of having to do 'extra' work (Koshy, 2001) is many times a reason for gifted children to hide their abilities, which occurs when teachers give extra work (mostly unnecessary and repetitive) to those finishing quickly, or leave them to work alone on more exercises from their textbook (Koshy & Casey, 1997a). Fears of being labelled as 'nerd' or 'geek' (Sheffield, 2003) and of being unpopular in a normal classroom sometimes hold back their success (Fielker, 1997; Koshy, 2001). Underachievers with high potential often prefer to hide their exceptionality in order to make friends and blend in with other children in a mixed-ability class (Freeman, 1998).

Language problems many times do not allow teachers to see the difficulty a child may have in communicating mathematics, and this difficulty may hold that child back and prevent him/her from expressing his/her ability (Koshy, 2001). An example of this could be a child for whom English is not his/her first language. For such a child, who may not have a good command of the English language, it is possible to feel no confidence to contribute to mathematical discussion in the class. This lack of confidence may increase when there are other gifted children around who, as Freeman (1998) contends, use the appropriate technical language (in that case, mathematical language) rather than a simplified version. Fielker (1997) furthermore argues that children who are gifted in mathematics but poor at language are prone to facing difficulties even with numerical ideas if the mathematical task demands a focus on description and explanation rather than just calculation. This also reminds us of Gardner's suggestions, mentioned previously, about an 'intelligence fair' assessment which should not confound intelligences (Gardner, 2006; Karolyi et al., 2003)

Social problems between gifted children and their classmates — caused because of their family background (Koshy, 2001) or because they tend to organise others by enforcing rules to them (Webb, 1993) — may sometimes mislead their identification, because when this happens, they lose their confidence and become unable to reach their full potential (Freeman, 1998; Koshy, 2001).

Lack of appropriate provision is another factor that may hold mathematical potential hidden. Koshy and Casey (1997a) argue that the assumption that identification must always be a basic condition for provision and that provision exclusively targets selected children is wrong. They present examples from their research project entitled *Bright Challenge* (Koshy & Casey, 1997a), where the teachers who participated in that project, which offered a range of challenging opportunities for all students, were pleasantly astonished by the appearance of cases of students who, until that time, were not identified as 'very able'.

Drawing upon Krutetskii's (1976) suggestions that we do not know how far the mathematical ability may go unless it is continuously challenged, recent educational researchers (e.g., Freeman, 1998; Kennard, 2001; Koshy, 2001; Koshy & Casey, 1997a), agree that provision can help mathematical ability to be demonstrated and, thus, identified. This means, for example, that a child, who is able in detecting patterns and making generalisations, will do this only if the appropriate activities are provided Koshy (2001). In addition, Freeman (1998) suggests a 'Sports Approach', which combines both identification and provision. The main idea of this approach comes from the field of sports where, for example, we do not know how high a child can jump unless we progressively increase the height of the bar.

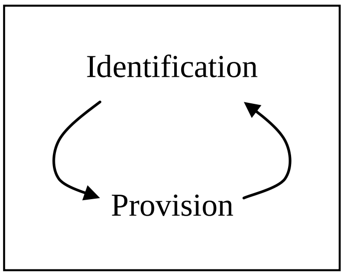
The last issue to be discussed in this section is closely related to this study, which concerns provision methods for mathematically gifted children and, so, it is further discussed in the following section.

2.2.3 Flexibility of identification and provision

The review of the literature has shown that identification is now recognised as a basic element in any attempt at serving the needs of children with higher abilities in a specific domain such as mathematics. It has also shown that this should be flexible in combination with the appropriate provision, which would be incorporated by every school on a daily basis (DfES, 2004a; Eyre, 2001; Freeman, 1998; Johnson, 2000; Koshy, 2001). Eyre, for instance, argues that: "...effective identification will be a combination of the assessment of precocious achievement or behaviour plus an emphasis on creating the conditions which will allow giftedness to develop and reveal itself." (Eyre, 2001, pp. 10-11)

However, there is still disagreement about the purpose of identification and its place in everyday schools. Renzulli (2004), for example, suggests that the purpose of identification is to select a group of pupils with high abilities to participate in special programmes and, thus, identification is useful if only it has been designed according to the offered programmes (Renzulli, 2004). On the other hand, others, like Coleman (2003) and VanTassel-Baska (1998), do not see identification as an instrument to construct specific programmes for a group of children only, but suggest identification as a guide for the curriculum provision and instruction that concerns all students. Eyre (2001) has furthermore recognised that providing well for the group of pupils already identified as gifted and simultaneously creating opportunities for those not identified yet to demonstrate their abilities, is the key point for the schools to face the dilemma of making effective provisions for gifted pupils.

The latter view of Eyre, which highlights the importance of providing opportunities for all pupils, is supported and amplified by Koshy (2001), Koshy and Casey (1997a), and Freeman (1998), who furthermore argue that identification should not always come first, but should sometimes follow the appropriate provision. For example, Freeman (1998) argues for identification by provision and suggests for this a 'Sports Approach' mentioned earlier, where children are offered a range of opportunities of progressively increased difficulty, aiming to help hidden abilities to be revealed and identified. Koshy (2001), also mentioned earlier in this section, has suggested that suitable provision is an essential factor for effective identification. She highlighted the importance of the nature of work that is being offered to the pupils whose mathematical ability is being assessed. She proposes an approach similar to 'dynamic assessment' (Kirschenbaum, 1998), mentioned earlier, but argues that the 'test-intervene-retest' format should be flexible and follow a circular process in practice (Figure 2-5).



Source: Koshy (2001, p. 24)

Figure 2-5: Koshy's circular process for identification-provision-identification

For example, a group of gifted mathematicians could be identified at the time they started their school, at an early age, based on parent nominations, intelligence, or cognitive tests. The school then should not only provide appropriate opportunities for those pupils to display their abilities, but also be ready to revise the first judgements. On the other hand, a range of 'right' opportunities should be offered not only to the identified group, but to all children with a view to discovering other pupils with high ability who were not identified earlier. For these pupils teachers should be ready to rethink and rearrange instruction. Therefore, identification can follow provision and vice versa, as Figure 2-5 shows.

This part discussed the nature of mathematic ability, the characteristics of mathematically gifted children and practical methods for their identification. It was made clear that there is not a single and unique way to define mathematic ability and mathematically gifted children or to securely identify children with higher mathematical potential. Characteristics lists, systematic observations and portfolios combined with tests and nominations from teachers, parents and peers may be useful information sources for teachers to identify their gifted pupils. In any case, however, it is suggested that the identification should be an ongoing process, connected with provision and flexible in a way that allows a circular process between identification-provision-identification and amendments at any stage of this process. The next part of this chapter discusses methods of provision specifically for mathematically gifted children.

2.3 Provision for Mathematically Gifted Children

If we subscribe to Gardner's (1983, 1999) theory of Multiple Intelligences, then we have more reasons to focus our interest on specific ability such as mathematical ability and consider provision that will not be general, but 'subject-specific' (Koshy, 2001) or 'content-specific' (VanTassel-Baska, 1992) so as to be able to meet the particular needs of mathematically gifted children more effectively. The following sections present that type of provision. This involves a range of selected strategies, appropriate for teaching everyday mathematics lessons to gifted children in 'depth' and 'complexity' within primary schools. These strategies include constructivist theories of learning which, I should note, have influenced my personal practice in teaching mathematics to both more able children and less able children for many years in primary schools; methods, which encourage pupils' higher-order thinking and their motivation to learn higher level mathematics; a framework for planning and teaching mathematics lessons according to Bloom's (1956) *Taxonomy of Educational Objectives*; and organisational structures. At the end, the role of the teacher is discussed.

2.3.1 Teaching and learning strategies

This study draws on constructivist learning theories (e.g., Bruner, 1960; Vygotsky, 1962, 1978), theories that emphasise the role of 'affect' and motivation in learning (e.g., Ernest, 1985), and theories for learning at higher cognitive levels (e.g., Bloom, 1956). These theories describe approaches that can enhance the effectiveness of teachers in teaching mathematics to gifted children.

2.3.1.1 Constructivist approaches - Learning within ZPD

As previously mentioned, modern theories of giftedness suggest that the development of a gift and/or talent is affected by the experiences and opportunities offered. As this thesis focuses on finding effective ways of developing mathematical ability within schools, the role of opportunities and experiences has to be seen in the context of learning process, something that has been mainly discussed in constructivist theories of the learning.

Constructivist theories of learning suggest that experiences structure the basis on which learners construct their own learning. Children’s experiences within their social environment, along with social interaction, determine the process of their learning (Bruner, 1960; Vygotsky, 1962). This, when learning mathematics, means that pupils gain new mathematical knowledge through reflection on their own learning and resolve problematic situations through self-questioning and teachers’ questions, a process of learning that suits pupils with higher abilities as discussed in the previous sections.

In addition, Vygotsky (1962) argued for the cultural notion of learning through active participation in contribution with others and the appropriate support from the teacher, who often provides a ‘scaffolding’ approach to teach problem solving. According to this approach, the support of an adult is needed to help children accomplish their tasks, but with a view to gradually withdrawing the control and support whilst the children increase their mastery of the task.

An application of the ‘scaffolding’ model was suggested by Vygotsky (1978) for use within the *Zone of Proximal Development* (ZPD). This was defined as: “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance.” (p. 86)

In other words, the ZPD is a learning zone between the real level of development and the potential level of development, where the child needs the help of a teacher, peer or parent in order to accomplish a difficult task that appears to be beyond his/her abilities. Awareness of the principle of ZPD can help a teacher to support children’s learning and this is true in the context of teaching children who are gifted in mathematics. The teacher needs to plan and provide challenging tasks and instructions within the ZPD, as highlighted by Koshy, Ernest and Casey (2009).

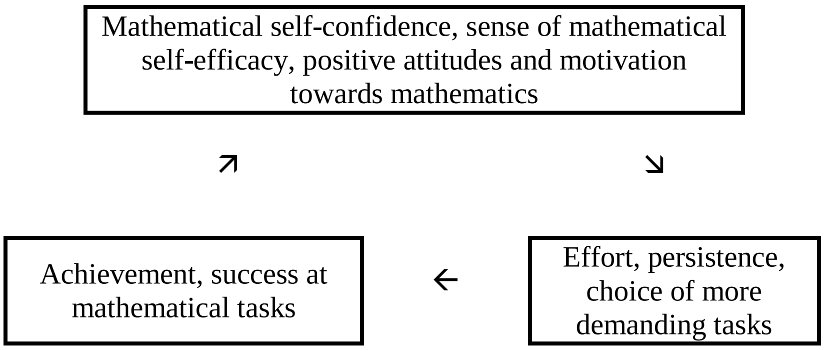
2.3.1.2 The role of ‘affect’ in learning mathematics

There is also the role of ‘affect’ in the learning of mathematics. Motivation and attitudes are also important in the learning process of a mathematically gifted child. Koshy, Ernest and Casey (2009) consider this affective dimension using Ernest’s (1985) success cycle as follows.

The success cycle has three components:

1. Positive affect including attitudes and motivation towards mathematics
2. Effort, persistence and engagement with cognitively demanding tasks
3. Achievement and success at mathematical tasks

(Koshy et al., 2009, p. 5)



Source: Koshy, Ernest and Casey, 2009, p. 6

Figure 2-6: The success cycle

As can be seen in Figure 2-6, the success cycle has no real beginning because these three components are linked cyclically each impacting positively on its successor. The authors describe the cycle as by stating that as a starting point we can say that students who have positive attitudes and beliefs about mathematics will have high self-confidence and mathematical self-efficacy, they will enjoy challenging tasks and be motivated which leads to increased effort, persistence and more time on tasks. Increased effort and work will give rise to continued success at mathematical tasks and overall achievement in mathematics and this will further enhance positive attitudes and so on, completing the success cycle.

The success cycle has no real beginning because these three components are linked cyclically, and each impacts positively on its successor.

2.3.1.3 Providing opportunities for higher-order thinking

Gifted pupils in mathematics need to have opportunities to be engaged in higher-level cognitive activities. These activities must challenge pupils' higher-order thinking skills (Casey 1999, 2002; Ernest, 1998, 2000; Koshy 2001; Sheffield, 1992, 2003).

Resnick (1987) has offered a list of higher-order thinking skills, which could be involved in a mathematics curriculum and should be taken into account for planning mathematical activities. This can also be used for planning activities for pupils' that are gifted in mathematics. According to this list, higher-order thinking skills:

- Are non-algorithmic and not fully known in advance
- Are complex
- Utilise multiple criteria, which may conflict with one another
- Yield multiple solutions and viewpoints
- Involve uncertainty
- Involve a process of making meaning
- Are effortful and require mental work
- Involve fine distinction in judgement and interpretations, not predetermined
- Involve self-regulation

Others, like Schoenfeld (1992), Ernest (1998), and Koshy (2001), consider thinking skills abilities for 'reasoning', 'hypothesising', 'communicating', 'decision making', 'refining ideas', 'creating new and original thoughts', 'problem-solving' and 'metacognition'.

Fisher (1992, 1995, 1998, 1999), has proposed some very interesting ideas for 'teaching thinking' through the curriculum to all students and suggestions for schools to be transformed into 'thinking schools'. Many of his suggestions could be implemented within mathematics for challenging gifted pupils' higher-order thinking. Fisher (1995), for example, has suggested that teachers, during the lesson, should ensure 'thinking time' for their pupils and use the appropriate questioning to challenge their thinking effectively. An appropriate line of questioning, according to Fisher

(1995), may include questions such as ‘What do you think?’, ‘What else?’, ‘What if?’, ‘What more?’, ‘Can you explain it?’

Focus on problem-solving procedures

Problem-solving is considered ‘the heart of mathematics’ by the Cockcroft Report (Cockcroft, 1982). It is also considered a thinking skill by other educational researchers (e.g., Ernest, 1998; Koshy, 2001; Schoenfeld, 1992), as earlier mentioned. However, as problem-solving requires the combination of other skills, such as ‘reasoning’, ‘hypothesising’, ‘decision making’ (Koshy, 2001), as well as ‘metacognitive skills’ (i.e., self-regulation and monitoring, reflection and verbalisation) (Schoenfeld, 1992), it may offer more opportunities for gifted pupils to be engaged in higher levels of thinking and display their capabilities. Furthermore, as Koshy (2001) argues “children who are very able in mathematics demonstrate a flair for problem solving. Some have become addicted to solving puzzles and activities which involve logic and reasoning.” (Koshy, 2001, p. 50) Therefore, a programme for mathematically gifted children should focus on problem-solving procedures.

Focus on metacognition

Metacognition was introduced first by Flavell (1976, 1981) and referred, according to the psychological literature, to the area of self-knowledge. Wenden (1991, p. 34) writes: “…metacognitive knowledge includes all facts learners acquire about their cognitive processes as they are applied and used to gain knowledge and acquire skills in varied situations.” Metacognition is also referred as a thinking skill that is used for planning, monitoring and evaluating the learning activity (Schoenfeld, 1992) or, as “the human capacity…to be self-reflective, not just to think and know but to think about their own thinking and knowing” (Fisher, 1998, p.2).

Recent theories and models of giftedness, earlier described, show that gifted children have higher abilities for metacognition. Therefore, a programme of provision specifically designed for mathematically gifted children should give them opportunities to use and improve their metacognitive skills.

Ernest (1998) suggested a list of ‘metacognitive activities’ that involve higher-order thinking and can improve pupils’ metacognitive skills. These are activities for ‘planning’, ‘monitoring progress’, ‘making effort calculations’, ‘decision making’, ‘checking work’, and ‘choosing strategies’. Such metacognitive activities are also involved in problem-solving procedures, as mentioned earlier.

In addition, Koshy (2001) proposes that metacognitive activities should be offered, not only during the problem-solving process or after, but also with every opportunity given within mathematics lessons. She proposes, for example, that pupils should keep portfolios of their work and compile achievement records so that they have the opportunity to develop abilities for ‘self-assessment’ and ‘self-regulation’.

Casey and Koshy (2002, 2003), through their *Mathematics Enrichment Project* in Brunel University, offered further guidance to both teachers and students for developing a portfolio of mathematical achievement (see Appendix 1), which may contain selected pieces of ‘best work’, ‘assessed tests’ and ‘sample self-evaluations’ of their own learning coming from their own achievement records.

Koshy (2001), however, has suggested that in order to teach mathematics effectively to gifted children, we need to organise the good practice under a more specific programme of provision. For this reason, she has developed a framework for planning mathematics in a way that encourages pupils’ higher-order levels of thinking, based on Bloom’s (1956) *Taxonomy of Educational Objectives*. As a teacher, I found this framework very useful to stimulate the interest of the gifted pupils and, therefore, I have included it in this study.

2.3.1.4 A framework for planning and teaching mathematics at higher cognitive levels according to Bloom’s Taxonomy

Bloom (1956), in his *Taxonomy of Educational Objectives: the Cognitive Domain*, made a classification of six levels of thinking, which starts with ‘knowledge’, goes higher with ‘comprehension’, ‘application’, ‘analysis’ and ‘synthesis’, and finishes with ‘evaluation’ at the top. Koshy (2001) has acknowledged that Bloom’s Taxonomy has been widely used for conceptualising levels of thinking and has developed a framework, which follows its six levels to plan mathematics for very able children. A summary with some examples of Koshy’s (2001) suggestions for planning mathematics according to Bloom’s Taxonomy is presented in Table 2-3.

Table 2-3: Planning mathematics according to Bloom's Taxonomy

Adapted from Koshy (2001)

Educational Objectives - Thinking levels	Students' Outcomes and Activities
Knowledge	Pupils must be able to ▪ name and identify squares, triangles and circles; ▪ know the measurement units (cm, km, etc.); ▪ recollect the multiplication table bonds; ▪ recall and use the symbols of the four operations, percentages, equals, etc.
Comprehension	Pupils should be able to ▪ describe shapes and their properties; ▪ perform algorithms; ▪ analyse a two-digit number in tens and units; ▪ solve a simple problem that involves the use of a known algorithm; ▪ form a number pattern; ▪ measure a straight line by using a ruler; ▪ collect simple data.
Application	Pupils may be able to ▪ invent number stories; ▪ use methods for solving simultaneous equations; ▪ apply rules to situations; ▪ choose methods to solve word problems; ▪ organise data and produce graphs.
Analysis	Pupils should be able to ▪ identify and use patterns; ▪ explore and indicate the underlying structure of mathematical theories; ▪ realize the relationship between concepts such as decimals, percentages and fractions; ▪ attempt conjectures, hypotheses and generalizations; ▪ try establishing proofs; ▪ analyse textbooks; ▪ write a text regarding a mathematical topic.
Synthesis	Pupils should be able to ▪ create a new system of numbers; ▪ ask 'What if?' and create new situations; ▪ build new and unique solutions; ▪ write some pages for their own mathematics textbook; ▪ challenge existing methods and suggest different solutions; ▪ work in several bases.
Evaluation	Pupils should learn to ▪ evaluate solutions and judge their relative effectiveness; ▪ evaluate the way the textbook treats the topics (i.e., probability); ▪ self-assess learning; ▪ record steps of their thinking; ▪ examine proofs introduced by oneself or someone else; ▪ judge the supremacy of certain algorithms or procedures in comparison with others.

By following the above presented framework, teachers can design and offer mathematics lessons at higher cognitive levels, suitable for gifted mathematicians. Koshy (2001), furthermore, argues that any model of talent development in mathematics must engage pupils in higher-order levels of thinking and encourage them to be 'effective thinkers' and 'creative problem solvers':

> Able pupils, like all other pupils, need to have opportunities to be engaged in higher levels of thinking. Gifted pupils often demonstrate superior cognitive capabilities. Therefore, it is our duty to encourage them to be effective thinkers and creative problem solvers. (Koshy 2001, p. 45)

2.3.2 Organisational strategies

2.3.2.1 Differentiation

Differentiation as a form of provision for gifted children is supported by many educational researchers, such as VanTassel-Baska (1985b, 2004) and Tomlinson (1995, 2004). VanTassel-Baska (2004, 2007) argues for the need for differentiation of the curriculum for gifted children and suggests a full set of curriculum options across domains appropriate to nurturing the social-emotional needs of diverse gifted students. In order for this to be successful, VanTassel-Baska (2007) argues that any curriculum differentiations have to initially identify appropriate goals and outcomes together with what is important for these students to learn and at what stages of development they will be able to do this. A differentiated curriculum then must provide experiences adequately different from the norm through a trained teacher of the gifted. Such experiences must be characterised by 'depth' and 'complexity' and be able to challenge gifted students, who have the ability of thinking in-depth and 'conceptualising abstractively'. Both VanTassel-Baska (2007) and Tomlinson (2004) agree that the selection of teaching materials, which should go beyond the single textbook, for use in the classroom is very important to serve high ability students. Tomlinson (1995), for instance, argues that a class is not differentiated when the work is the same for all students and the adjustments only consist of giving certain pupils questions of different levels of difficulty, grading some pupils higher than others, or allowing pupils who finish earlier to play games. Speaking of mathematics, Tomlinson (1995), Sheffield (1999), Koshy and Casey (2005) suggest that it is not appropriate for gifted mathematicians to do only extra mathematical problems from their textbooks or extension assignments about what they have already learnt after completing their ‘regular’ work. When we ask them to do such work, this is perceived as a punishment (Tomlinson, 1995). Sheffield (1999), Koshy and Casey (2005) argue, for this reason, that the work which gifted children must do should be part of a carefully designed plan for mathematics enrichment. These ideas, which may offer an answer to the concerns presented earlier about what kind of work truly differentiates the lessons for gifted children, are discussed in more detail in a following section (2.3.3).

Sheffield (1999) and Koshy (2001) who did a great deal of work and research-based study around the education of mathematically gifted children, agree with the need for differentiating instruction within the curriculum. In addition, they suggest that even amongst pupils who are able or promising in mathematics there are differences in the degree of their ability and attitudes and thus they may need a different approach or treatment by the teacher, within the regular classroom or outside in special small groups. For example, an able child who loves mathematics and enjoys problem solving may not need much teacher intervention because he/she can work alone on challenging work. The opposite can happen if an able child is lacking in motivation. In that case, the teacher must intervene by offering a great amount of encouragement and even a structured programme to the point that the child will start getting motivated. In this case also, Koshy (2001) suggests that an enrichment programme can help with the need for differentiation within a class to be served more effectively. Furthermore, Sheffield (1999, 53) contends that there may be “severely and profoundly promising, gifted, talented, and motivated students”, who need more individualised services (in small classes of one or two) from highly trained specialists who have a background in both advanced mathematics and gifted education.

Most researchers conclude that a successful, differentiated curriculum must involve differentiated instruction, ongoing assessments and flexible grouping (Koshy, 2001; Tomlinson, 1995; VanTassel-Baska, 2007). On the contrary, there has not been an agreement about the use of acceleration or enrichment as a form of educational provision for gifted children, as the historical debate regarding acceleration versus enrichment still stands (Freeman, 1998; Kennard, 2001; Koshy, 2001; Sheffield, 1999).

2.3.2.2 Acceleration

Acceleration is a practice that allows students either early entrance and early exit at various stages of development or more rapid movement through traditional organisations (VanTassel-Baska, 2007; Freeman, 1998).

In the case of mathematics, 'rapid movement' or 'fast-tracking' could mean, according to Koshy (2001), that gifted pupils can learn the same content at a faster pace than their peers in the same class or group. This is done by giving such pupils opportunities within the class to learn mathematics from a higher level, by providing them with a special programme that includes more advanced subjects than the normal programme, by having them practice mathematics topics with older pupils outside the class, or by having them skip years. 'Early entrance and early exit' could mean that some promising or gifted pupils can enter kindergarten, first grade, or college (i.e., in the USA) (VanTassel-Baska, 2004) earlier or take early examinations for the General Certificate of Secondary Education (GCSE) (i.e., in the UK) (Koshy, 2001).

There has been an extended discussion about the value (or lack thereof) of acceleration as a strategy of provision and there have been many who have argued either for or against this approach. VanTassel-Baska (1985a, 1998) and Freeman (1998), for example, consider acceleration one of the most important practices that should be included in a differentiated curriculum policy for addressing the needs of gifted students and the most economic way to provide special provision for exceptionally able children, particularly in mathematics. VanTassel-Baska (1998), furthermore, contends that the most important justification for acceleration is that readiness to learn certain kinds of knowledge is not dependent on chronological age. She also contends that gifted students are capable of learning advanced subjects at a significantly faster rate than their same-age peers. This makes acceleration useful for fast learners because they are bored with the work they have already mastered.

On the other hand, there are others, like Fielker (1997), Casey (1999), and Sheffield (1999), who criticise acceleration practices and consider them ineffective methods of provision for mathematically gifted children. Fielker (1997, p. 9), for example, writes that through acceleration, more able children "do not learn more about mathematics… What they do is merely learn the same mathematics sooner" and that "this does not seem to fulfil the needs of the more able, who deserve something better". Casey (1999) agrees with Fielker and adds that those who favour acceleration assume that the difference between a higher ability learner and others is simply chronological, but this is not right because such a view connects variations of ability with the age of the learner only. Sheffield (1999, p. 45) also argues that programmes that only accelerate in mathematics usually "do not allow students the opportunity to enjoy the beauty of mathematics or to explore the mathematics deeply enough to become real mathematicians." In addition, a group of mathematics educators in the UK (UK Mathematics Foundation, 2000, p, 7) has contended that acceleration may have "serious disadvantages for pupils' long term development" and, therefore, needs to be "handled with great caution."

The most common criticism relates to the emotional effects of acceleration and its practical application. Fielker (1997) believes that if some schools decide to move some very able children up a year, they will face some serious problems (or 'dangers', according to Fielker). These are to do with the following:

- the homogeneity in ability, which is not achieved when younger bright pupils are simply left to study the same materials with older pupils of mixed ability;
- the maturity of those who are moved up, because for younger children, even if they are brighter, it is difficult to be mixed with children who are probably physically, emotionally and socially more mature; and
- the resentment felt by those average or less able children who will see their classmates moving up a year.

In her introduction to the *Essential Readings in Gifted Education* series, Reis (2004, cited in VanTassel-Baska, 2004) writes that acceleration is not the favourite practice in the USA. It is often dismissed by the teachers and administrators who do not consider it so practical, mostly because of scheduling problems and concerns related to the social effects of grade skipping. That explains why most school districts do not permit early entry to kindergarten or the first grade, grade skipping, or early entry to college. In the UK it is also relatively rare to see acceleration involving 'fast-tracking' and 'early entry' (Kennard, 2001; Koshy, 2001), apart from some secondary schools that allow students to enter a year early in GCSE mathematics examinations (Kennard, 2001). This may be because such practices have a strong relation with governmental policy (Koshy, Ernest & Casey, 2009) or because there is a notion that pupils who are moved up a year will experience social or emotional drawbacks, as mentioned earlier.

However, the emotional effect of acceleration is still an issue of dispute. For instance, Gross (1993), on the contrary with what has been previously mentioned, has written that exceptionally gifted children often suffer emotional and social drawbacks when they are not accelerated. Jones and Southern (1991) also suggest that it is not ethical to ignore gifted pupils once they have been identified and that there is not enough evidence to support the theory that acceleration brings social and emotional consequences to gifted pupils. Furthermore, research carried out by the Centre for Talented Youth (CTY) at Johns Hopkins University in the USA (1994, cited in Koshy 2001) and by Gross (1993) in Australia for over ten years has shown that gifted students who undertook accelerated work as preteens continued to reap benefits later on.

Recently, in 2004, the Belin-Blank International Center for Gifted Education and Talent Development at the University of Iowa, endorsed by the National Association for Gifted Children in the USA, in collaboration with the Gifted Education Research, Resource and Information Centre (GERRIC) at the University of New South Wales in Australia, published the report *A Nation Deceived: How Schools Hold Back America's Brightest Students* (Colangelo et al., 2004) indicating the advantages of acceleration for gifted children. According to this report:

> Acceleration does not mean pushing a child. It does not mean forcing a child to learn advanced material or socialize with older children before he or she is ready. Indeed, it is the exact opposite. Acceleration is about appropriate educational planning. It is about matching the level and complexity of the curriculum with the readiness and motivation of the child (Colangelo et al., 2004, p. 1).

However, despite any positive or negative effects of acceleration on gifted children's development, it would be futile to speak about the use of a form of radical acceleration, such as year skipping, for use in countries like Greece, where this type of acceleration is not currently allowed by educational law. In such countries, however, the type of acceleration that could be used is what Koshy (2001) has suggested: opportunities for acceleration within the classroom that emerge when some able children learn more than their classmates following a successful programme of mathematics enrichment.

2.3.2.3 Enrichment

Enrichment is suggested as an alternative strategy to both differentiation and acceleration for mathematically gifted children (Koshy, 2001; Koshy & Casey, 2005). Differentiation, for instance, is involved when gifted pupils are provided with enriched and challenging activities according to their diverse needs (Koshy, 2001). Acceleration is involved when a successful enrichment programme motivates them to work with more complex problems and in-depth investigations, because then gifted pupils gain new and advanced knowledge sooner than when they do the regular work (Koshy & Casey, 2005).

Renzulli's enrichment models

Renzulli, a leading expert in the field of gifted education, has developed (alone or with his colleagues) complete educational programmes for enrichment — such as the *Enrichment Triad Model* (Renzulli, 1977) and *The Schoolwide Enrichment Model* (Renzulli & Reis, 1985) — which also involve differentiation and acceleration through ability grouping and 'curriculum compacting' for gifted children. However, these models of enrichment demand big changes to the school programme in practice and, as VanTassel-Baska (2007) asserts, when a programme requires big changes and extra finance, it is difficult to be adopted by every school. This is the reason that these models have not been included in this study. Nevertheless, there are some very interesting ideas within Renzulli's enrichment models, which could be very useful in everyday practice and thus could be implemented within a mathematics programme for gifted children within ordinary schools. Some of these ideas are presented below.

'Individual and small group investigations of real problems', for example, from the *Enrichment Triad Model* (Renzulli, 1977), where children are able to select the problem or the area of their study by themselves, is something that can increase their interest and task commitment. This is considered an element of gifted behaviour in Renzulli's *Three Ring Conception of Giftedness* (Renzulli, 1978). Real problems and investigations have also been suggested as ideas of good practice for challenging higher abilities in mathematics by experts in the field such as Sheffield (1999), Casey (1999), and Koshy (2001).

‘Curriculum modification techniques’ from *The Schoolwide Enrichment Model* (Renzulli & Reis, 1985) can also be very useful to address the needs of gifted mathematicians in regular schools. These techniques are based on the ‘curriculum compacting’ process (Renzulli, 1994; Reis, Burns & Renzulli, 1992). They give instructions about how the schools can adapt the regular curriculum by either removing work that has already been mastered or by reorganizing and optimising work that can be learnt easily so the gifted children will not lose their motivation by working on repetitive or simple work. Others who found ‘compacting’ a useful strategy, such as Feldhusen (2001), have suggested children’s involvement in planning the programme. This practice includes elements of acceleration as it allows gifted children to learn new topics faster (‘accelerated content approach’, according to Renzulli (1999)).

Renzulli (1999, p. 26) also has proposed that enrichment works better if schools implement “a multi-age cluster group in mathematics for high achieving students…and curriculum compacting for students who have already mastered the material to be covered in an upcoming unit of study.”

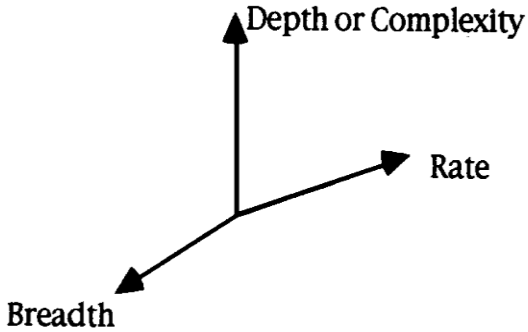
Enrichment for adding depth or complexity in mathematics lessons

Enrichment in mathematics means broadening of the knowledge base and the process of learning, giving the opportunity for gifted mathematicians to learn mathematics in more depth and, thus, ‘go beyond the acquisition of facts and skills’ (Koshy, 2001), independently of the grouping strategies that may be used. The question is how this can be achieved in practice.

Some educators suggest enrichment for ‘added breadth’ in mathematics, explaining that this can be achieved through “extension work, which enriches the official curriculum by requiring a deeper understanding of standard material (for example, by insisting on a higher level of fluency in working with fractions, ratio, algebra or in problem solving.” (UK Mathematics Foundation, 2000, p. 7)

However, ‘added breadth’ in mathematics lessons, as described above, although useful, is restricted to acquisition of facts and skills from a larger number of mathematical topics giving no opportunities to go beyond these, as described at the beginning of this section.

Sheffield (1999), the Chairperson of the US Task Force on *Mathematically Promising Students*, developed a three-dimensional model (Figure 2-7) for teaching mathematics to promising or gifted students. This model suggests that mathematically gifted children should be provided with challenges towards at least three dimensions of learning defined as *Breadth*, *Rate,* and *Depth or Complexity*.



Source: Sheffield (2003, p. 8)

Figure 2-7: Sheffield's three-dimensional model for adding depth and complexity to the mathematics curriculum

According to this model, a successful programme of provision should not only look at the number of the topics (*breadth*) or the *rate* at which these topics are provided, but must look at changing the *depth or complexity* of the mathematical investigations. This means that gifted pupils must have the time and encouragement to 'explore the depth and complexities of problems'.

However, this does not mean that teachers have to look for a large number of difficult puzzles and problems, but they need to find ways to engage pupils in higher-level cognitive activities, which, as mentioned earlier, challenge their higher-order thinking skills.

For instance, Sheffield (1999) suggests that the original problem, which teachers give to the pupils at the beginning, does not need to be a difficult one, but it does need to be an interesting, challenging and open one, which will give opportunities for investigations and discoveries. For example, in the case where children have to practice adding two-digit numbers with regrouping, instead of being asked to complete a page of exercises such as:

a)	56	b)	38	c)	69
	+ 47		+ 78		+ 46

or some more difficult ones with 3- or 4-digit numbers, they could be asked to "find three consecutive integers with a sum of 162." (Sheffield, 1999, p. 47)

Such an activity allows children to continue practicing adding two-digit numbers with regrouping, but it also gives them the opportunity to make interesting discoveries along the way. It also gives the opportunity to the teacher to challenge gifted pupils by asking them to find the answer in as many ways as possible, posing related questions, investigating possible patterns, making hypotheses, evaluating their observations and discussing their findings. Such practices, according to Sheffield (1999), give opportunities to the students 'to do some real mathematics'. Furthermore, she maintains, there were examples of students who had been taught to explore problem patterns and connections, posing new problems and creating their own solutions, starting to approach mathematics differently from others. When they used to work by following only the one way that the teacher or the textbook proposed, the most able pupils started feeling frustrated by the 'one right method'.

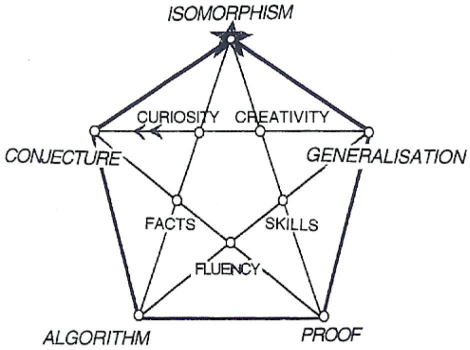
In addition, Casey (1999), through his *Key Concepts Model* (Figure 2-8) for teaching mathematics to very able children, suggests examples for learning algorithms in depth by engaging their higher-order thinking skills without just asking them to do 'more of the same'. For instance, when pupils have been taught an algorithm to add two numbers and they are able to use it correctly, instead of doing more of the same, the teacher could ask them to add different pairs of odd numbers and try to see if they could find some common features in the sums obtained. Such an activity can involve 'conjectures' and 'generalisations'. A conjecture, for example, may be:

> Is the sum of two odd numbers always even? (Casey, 1999, p. 14)

Trials with many pairs of odd numbers will provide supporting evidence, which usually leads to generalisations, such as:

> The sum of two odd numbers is always an even number. (Casey, 1999, p. 14)

Such a procedure can stimulate pupils' 'curiosity' and, with the appropriate guidance by the teacher, pupils can investigate other features of the four basic operations on numbers and attempt generalisations based on a ‘creative’ personal ‘proof’ (Casey, 1999).



Source: Casey (2002, p. 128)
Figure 2-8: Casey’s Key Concepts Model

Recent research in teaching mathematics to gifted children (Koshy & Casey 2005) found that teachers who participated in a project for mathematics enrichment “felt more at ease with the enrichment strategy” (p. 299). This finding, along with the fact that enrichment does not require big changes to governmental policies like acceleration (Koshy, Ernest & Casey, 2009), suggests that the implementation of enrichment as a new strategy for teaching gifted mathematicians within an educational system like the Greek one where, as mentioned in Chapter One, there is not any specific curriculum framework for gifted children, will probably not be met with resistance. Furthermore, the review of the literature has shown that enrichment is suggested as a strategy of provision in mathematics for gifted children by current educational policies in many countries, such as in the UK (DfES & NAGTY, 2006; DfEE, 2000b), the USA (NCTM, 2000) and Australia (NSW Department of Education and Training, 2004).

At this point, the focus of this study should be reiterated, to find effective methods that serve the diverse needs of mathematically gifted children within the curriculum. These methods should be applicable in any primary school, here in the UK and internationally, without the need for big changes in governmental policies. At the same time, they should have the best chances of a wider acceptance from both teachers and students. After very careful consideration of the abovementioned strategies (‘acceleration’, ‘differentiation’ and ‘enrichment’), it seems that the strategy that better meets this study’s requirements is ‘enrichment’ combined with ‘differentiation’. In practice, this means that children that are more able should work at higher levels than their same-age peers, doing mathematics in 'depth' and 'complexity', either as part of a higher- ability group or individually, but under differentiated instructions in any case.

The following section will now discuss grouping practices that may help teachers achieve differentiation and enrichment, and in some cases acceleration too.

2.3.2.4 Grouping practices

Grouping, as mentioned earlier in this chapter, has been recognised in the USA, in the UK and elsewhere as an organisational structure that helps teachers provide differentiated instruction within the curriculum, appropriate to the diverse needs of every child, especially those with higher abilities (Koshy, 2001; Tomlinson, 1995; VanTassel-Baska, 2007).

According to the Eurydice report (Eurydice, 2006) on how the needs of gifted and talented young people are addressed through the educational systems in the European Union, almost all European countries implement grouping strategies within their schools (primary and general secondary). These strategies are usually carried out through arrangements which are similar to those used in the USA, as they have been described in Slavin's (1986) report (despite some differences in terminology) and they are 'mixed-ability grouping' ('heterogeneous grouping' in the USA) and separate 'homogeneous grouping'. The latter can be 'within-class grouping', 'streaming' ('tracking' in the USA), 'setting' ('regrouping' in the USA), or 'pull-out' grouping.

Mixed ability grouping

Mixed-ability grouping (or 'heterogeneous grouping' according to American terminology) within the class is the only type of grouping that is recommended by the Greek National Curriculum (GMNERA & GIE, 2004) for primary schools and, therefore, the type of grouping of which I have personal experience. The idea behind this suggestion is that pupils will have the opportunity to do collaborative work by working in mixed-ability groups within the regular classroom. This will help them develop collaboration skills and positive attitudes towards diversity. Because of this, teachers are also encouraged to make sure that each child will have the opportunity to collaborate with all children throughout the year (GMNERA & GIE, 2005). The success of such an arrangement, of course, is strongly correlated with the professional development of the teacher, because he/she should know when and how the groups must be arranged and prepared to provide the appropriate opportunities. My experience, from teaching in various schools in Greece has shown me that allowing children with different abilities and attitudes towards mathematics to be able to work effectively together on the same task is not an easy enterprise and that it is difficult to gain positive results for all pupils without the teacher's intervention. Furthermore, there is always the temptation that the teacher, for his/her own convenience, may permit the less able pupils to just copy the work that the others have done or may let the more able pupils work alone to finish the task quickly.

Research findings on the effects of mixed-ability grouping on pupils' attainment are equivocal. Although some researchers have shown that total mixed-ability grouping may have a negative impact on both the achievement and motivation of high ability pupils (Reid, Clunies, Goacher & Vile, 1981, cited in McClure, 2001), there are others, such as Boaler, Wiliam and Brown (2000), who found positive results on pupils' attitudes towards mixed-ability grouping. Some experts in the field of primary education, such as Fielker (1997), consider this organisational practice appropriate for teaching mathematics to all children, including the able ones.

Fielker (1997), however, highlights the administrative difficulties that teachers may face to meet the needs of all children through whole class teaching and the danger of not meeting the needs of either able children or less able ones when teachers use just a textbook.

Homogeneous grouping

Homogeneous grouping is used in primary schools in order to place children in groups according to their ability and, because of this, it is well known as 'ability grouping'. Ability grouping has been implemented in US schools, elementary (primary) and secondary, since the beginning of the twentieth century, and "hundreds of studies" have been carried out in the USA in relation to the effects of various forms of ability grouping, according to Slavin (1986). Most of these studies suggest that all forms of ability grouping have positive effects for gifted students (Kulik, 1992). Especially, it has been suggested that ability grouping for a specific area of the curriculum produces substantial academic benefits in achievement, improves pupils' attitudes towards school and strengthens self-efficacy in specific domains, such as mathematics (Kulik, 1992; Kulik & Kulik, 1991, 1992). Therefore, gifted pupils should be separated from their peers of the same age at least for a part of their schooling day (Kulik, 1992; Kulik & Kulik, 1982, 1991, 1992).

In the UK, also, there is a long tradition of grouping by ability, which mainly started being implemented in primary schools after the Second World War, according to Hallam, Ireson and Davies (2004). Recent research in the UK (e.g., Boaler, Wiliam & Brown, 2000; Davies, Hallam & Ireson, 2003; Hallam, Ireson & Davies, 2004) has shown that the pressure for raising standards and delivering a curriculum which will meet the needs of all pupils including the 'able', has forced schools to adopt practices of ability grouping for at least part of the curriculum. However, there has not been much research on the effects of ability grouping in the UK so far.

Over time, ability grouping has taken different forms, as schools have aimed to provide a differentiated curriculum to suit the needs of all children. 'Streaming', 'setting', 'within-class grouping' and 'pull-out' grouping are the best-known forms of ability grouping that are implemented in schools for meeting the different needs of each child.

Within-class ability grouping

Many primary schools in the UK and elsewhere implement within-class ability grouping for particular subjects. This type of ability grouping is implemented within each year class where pupils are mainly taught in mixed-ability groups, but are regrouped by ability during the day for a particular subject, usually for mathematics (Harlen & Malcolm, 1999; Kennard, 2001; McClure, 2001).

There is research that shows benefits for all children from within-class grouping, especially in mathematics, for both achievement and behaviour (Boaler et al., 2000; Harlen & Malcolm, 1999). More specifically, Harlen and Malcolm's (1999) research review on grouping methods in schools has shown internationally that most researchers agree that within-class grouping according to the subject gives more advantages to all pupils — compared with setting and streaming, which mainly focus on gifted children — providing that teaching materials and pace are suited to the needs of the children. The same research review has revealed that the greatest advantages of within-class ability grouping were found in mathematics. Boaler, Wiliam and Brown's (2000) research on different grouping methods used for teaching mathematics in primary schools in the UK concludes that within-class ability grouping is a much more flexible grouping practice, compared to others, which allows opportunities for the whole class to do the same work and allows pupils who are considered weaker 'to shine in some areas'.

However, although there is no doubt that the research revealed positive results from the use of within-class ability grouping in mathematics for all children including the 'weaker' (e.g., Boaler et al., 2000), there are no relevant findings on the effects of this type of grouping on mathematically able children or on exceptionally able children in mathematics that may exist in a 'mixed-ability' class.

Streaming

Streaming is one of the oldest strategies of grouping where children are segregated by ability independently of their age and they are taught in a class for all subjects (Boaler et al., 2000). After the Second World War, streaming was the dominant form of ability grouping in primary schools in the UK, but over time it became unpopular, as it was found that it had negative effects on pupils who were in the lower streams (Hallam et al., 2004). More specifically, Barker Lunn and Ferri's (1970) study of streamed and non-streamed primary schools in the UK showed that streaming could lead to low self-esteem and social alienation while the effects on pupils' achievement, even on those in the highest streams, were equivocal. In the USA, relative research on the effects of ability grouping on the achievement of elementary (primary) schoolchildren, carried out by Slavin (1987), revealed that no academic achievement advantages came from streaming. However, a research study carried out by Lee and Croll (1995) on streaming in two local authorities in the UK showed that there are some headteachers who support streaming. This may be, as Harlen and Malcolm (1999, p. 55) explain, "because of the difficulties teachers have with mixed-ability classes, particularly when they are increasing in size".

Setting

Setting is a practice of ability grouping where pupils from different years are set in groups that work as parallel classes on a specific subject, such as mathematics. There are usually three classes, described as ‘top’, ‘middle’ and ‘lower’ sets, but there can be more, depending on the availability or the number of members of staff (Koshy, 2001).

Setting, initially used by secondary schools in order to fulfil an increasing demand, started in the UK in the 1990s to meet the needs of gifted children and to help them achieve ‘academic success’ (Boaler et al., 2000). The literature review (Boaler et al., 2000; Davies et al., 2003; Hallam et al., 2004) has shown that the interest in implementing setting within primary schools in the UK started in 1997. At that time, the Government White Paper *Excellence in Schools* (DfEE, 1997) valued the use of ability grouping, especially setting, within the curriculum as a strategy that helps teachers meet the needs of gifted children. One year later, the Office for Standards in Education (Ofsted, 1998) published a survey on setting in primary schools, which showed an increase in this particular practice.

Setting is suggested as a strategy that, according to Koshy (2001), can help teachers face the difficulties arising from the existence of the gap between those who are struggling and those who can do more and need more opportunities, something that teachers often face in mathematics. Koshy (2001) suggests that setting helps teachers better serve the needs of very able children in mathematics, because such children are placed in a top set where the ability range is narrower and teachers can manage differentiation more easily by selecting and providing tasks that can be tackled by most children. Furthermore, it is easier for teachers to provide challenging activities, conduct discussions and ask questions that involve higher levels of thinking and increase the pace of instruction without having to consider whether there are pupils who have understood what has been taught. However, it is possible for some teachers to lack confidence to teach pupils in the top sets. In this case, a mathematics co-ordinator, or someone who loves teaching mathematics and has the expertise, can help them to teach even the highest set.

Research on the effects of setting has shown equivocal outcomes. For instance, it was found that children who are more able typically benefit in terms of achievement, providing that advanced curriculum materials for differentiation have been used (Harlen & Malcolm, 1999). In contrast, there have been different findings regarding pupils' attitudes towards setting (Boaler et al., 2000; Hallam et al., 2004).

Boaler et al. (2000), for instance, who studied mixed-ability grouping and setting practices in mathematics in the UK schools, found that almost all of the pupils from 'setted groups' (including those in the highest sets) were unhappy with their position. Boaler et al. (2000) suggest that pupils' negative attitudes towards setting are associated with curriculum polarisation characterised by a lack of opportunities for pupils in the lower sets and the fast pace (incompatible with understanding for many students) and high pressure in the highest sets. The other reason may be, according to Boaler et al. (2000), that setting influences the teaching approach. Teachers, for example, in 'setted' classes usually see pupils in high sets as 'mini-mathematicians' who could work at a continuous fast pace through high level tasks, whereas they see pupils in the low sets as failures who could accomplish only low level tasks or, even worse, copy off the board.

In their own school-based research, Hallam et al. (2004) found that pupils' attitudes towards school were not affected by grouping practices, but by other factors, such as the size, ethos and expectations of the school and the attitudes of parents and teachers. Additionally, pupils were aware of how they were grouped, as well as of the purpose of grouping, and they accepted the rationales provided.

However, there are indeed difficulties in organising setting in ordinary schools, like those mentioned earlier, and many more, such as those relating to the identification and allocation of children into the right group (set) (Davies et al., 2003). A possible solution to these problems could be the use of variations in setting such as those proposed by the UK National Numeracy Strategy (DfEE, 2000b) for teaching gifted children:

- temporary setting (during revision sessions, for instance);
- part-time setting (two or three times per week, for example);
- setting of the pupils as they are getting older;
- combination of setting with mixed-ability grouping (by having a top set while teaching the rest of the pupils in mixed-ability groups, for example).

Pull-out grouping

Pull-out grouping is a programme designed for gifted pupils who are in the regular classroom setting for most of the time. These pupils are pulled-out for a limited time to go to a special class designed to meet their needs. The idea is that gifted pupils have the opportunity through these programmes to leave the everyday, regular classroom, which, as Renzulli (1987) advocated, often does not take their interest into consideration. They also have the opportunity to be engaged in more challenging activities.

The effectiveness of pull-out grouping programmes in meeting the needs of gifted children has received much criticism. Most of this has been voiced by VanTassel-Baska (1987, 2009). VanTassel-Baska (1987) criticised the limited time that the children spend in these programmes, the lack of focused instruction for an allotted daily period, as well as the lack of communication and articulation that often exists between the special class and the regular classroom. Recently, she added that these programmes, which separate gifted children from the regular classroom, make gifted education look like 'an analogue to special education, and giftedness as an elitist enterprise' (VanTassel-Baska, 2009, p. 266). Davis and Rimm (1985) have furthermore criticised the quality of work that is offered through these programmes, in that pull-out programmes often bring about "too much fun and games and too little valuable, theory-based training" (p. 122).

The research carried out on the effectiveness of pull-out programmes, however, has revealed positive elements of this practice. For instance, in their review and meta-analysis of nine research studies on pull-out programmes in gifted education, Vaughn, Feldhusen and Asher (1991) concluded that pull-out grouping programmes in gifted education have significant positive effects for the variables of achievement, critical thinking, and creativity. They also concluded that they could be a feasible programming option for gifted pupils if they address the criticism voiced by VanTassel-Baska (1987), at that time, and strive to meet the seven criteria for quality cited by Belcastro (1987): (a) integration with regular curriculum, (b) identification of students, (c) daily programme experience, (d) placement with intellectual peers, (e) pace of program matched with students' learning rates, (f) complex and higher level curriculum, and (g) excellent teachers (Vaughn, Feldhusen & Asher, 1991, p. 93).

It seems that pull-out grouping and setting, despite the criticism that they have received, are the most appropriate grouping strategies for meeting the diverse needs of more able or exceptionally able children and, thus, gifted mathematicians. Of course, in practice, we should consider that — as Davies et al. (2003, p. 57) advocate — each school has to determine the most appropriate type of grouping according to “the physical layout of the school, staffing levels and structures, the availability and quality of resources and year cohort size”.

2.3.3 Other recommended practices

It was previously said that setting able pupils into ability groups only is not enough to address all of their needs, because even among able pupils, there are variations in their abilities and interests. Therefore, teachers must be prepared to utilise as many organisational options as possible. Koshy (2001) and Kennard (2001) have proposed some additional options that schools can incorporate to address the needs of mathematically gifted children more effectively, such as work with a mentor or co-ordinator, co-teaching, and the use of further tools and resources (e.g., the ICT).

Employing a mentor

Koshy (2001) and Sheffield (1999) suggest that schools should use a mentor to work with pupils demonstrating exceptional mathematical ability. A mentor could be an adult who is able to provide support, in the classroom or outside, depending on the pupils’ abilities. Such a person must be very enthusiastic about mathematics and act as a role model while at the same time, as a subject expert (Koshy, 2001), he/she must be able to teach higher level mathematics and understand and challenge exceptional students (Sheffield, 1999). Such a person as a mentor is, therefore, able to identify a pupil’s true potential — according to Vygotsky’s (1978) Zone of Proximal Development, as mentioned earlier — and enhance his/her intellectual development.

Incorporating a co-ordinator

A co-ordinator can alleviate the lack of confidence of some teachers not having the subject expertise to teach mathematically gifted children (Koshy, 2001). Kennard (2001) furthermore suggests that a co-ordinator can contribute to the development, implementation, monitoring and evaluation of any special provision programme for enhancing mathematical ability. They can also lead in any method of identification of high ability, produce medium-term planning and guidance, as well as liaise with other specialists and phases to ensure progress. In addition, the Williams (2008) report on teaching mathematics in early years settings (nurseries) and primary schools (mentioned earlier) suggests that any primary school needs to have a 'mathematics specialist' with advanced knowledge of the subject and pedagogy who will be able "to maximise impact on standards and to narrow attainment gaps." (p. 7)

Co-teaching

In the first part of this chapter (section 2.2.1), it was mentioned that teamwork between teachers within a school can help the diversity in teachers' abilities to be overcome so that each person could find the best possible role in relation to gifted education (Sternberg, 2000b). Similarly, Kennard (2001) has suggested 'co-teaching' for mathematically gifted children. This could be achieved if schools could arrange a long-term programme that would cover short periods (even for an hour only). In such a programme, teachers would be able to collaborate with their colleagues and, with the support of the headteacher, they would look for evidence of high ability and ways for effective teaching or would test new teaching materials. Alternatively, Kennard (2001) suggests that schools could schedule regular review meetings at 'half-termly' or 'termly' intervals to discuss characteristics of mathematically able pupils' remarkable work, as well as the effectiveness of planning and any provision programme that the school implemented. In this case, the presence of a specialist, as mentioned earlier, would be a big advantage for both planning and monitoring special educational programmes.

Use of further tools and resources

Using calculators

Calculators can be used for more able pupils on more complex mathematical problems which require extended time for thinking. For instance, the UK National Numeracy Strategy (DfEE, 2000a) suggests that calculators are valuable when they are used for extended tasks, challenging problems and 'investigative activities' because they reduce computation time and provide immediate feedback. This in turn helps pupils focus on understanding their methods and justifying their results (Hembree & Dessart, 1986).

Using computers and multimedia

Computers, along with video recorders, tape recorders and all multimedia resources, are part of what we usually call ICT (Information and Communication Technology). Computers and multimedia can provide fast and reliable feedback that is non-judgemental and impartial, and, therefore, encourage pupils “to make their own conjectures and to test out and modify their ideas” (Becta, 2009b, p. 2) — in other words, to engage in higher-order cognitive activities which, as mentioned earlier, is suitable work for mathematically gifted children.

Recent educational policies, such as those in the USA (NCTM, 2000) and the UK (DfES, 2006), have suggested the use of computers and other ICT resources as an essential principle in teaching and learning mathematics. For example, the US Framework *Principles and Standards for School Mathematics* (NCTM, 2000) suggests the use of both calculators and computers in teaching mathematics. This is because they can help students learn mathematics at a deeper level by making and testing conjectures and working at higher levels of generalisation or abstraction (which, as mentioned earlier, are on the top levels of Bloom’s *Taxonomy of Educational Objectives*).

Similarly, in the UK, the 2006 *Primary Framework for Literacy and Mathematics* (DfES, 2006) emphasises the use of ICT aiming to provide more help with planning, teaching and assessment. It suggests the use of ICT in handling data (e.g., to create a simple bar chart or to organise, present, analyse and interpret the data in diagrams, tables and tally charts), in understanding shapes and as an extra resource for extending mathematics within or outside the curriculum. More recently, the government, as mentioned in Chapter One, has introduced a new electronic format of the *National Curriculum* that includes many interactive tools for immediate support for any subject and links to a wide range of teaching and learning resources available through the Primary National Strategy (QCDA, 2010c). Additionally, the British Educational Communications and Technology Agency (Becta, 2009b), a government body that promotes technology in learning in the UK, has provided examples for using ICT resources to effectively support pupils’ learning within primary mathematics through five major opportunities: ‘learning from feedback’; ‘observing patterns and seeing connections'; 'exploring data'; 'teaching the computer', and 'developing visual imagery'.

Research on the effects of using ICT in schools has revealed evidence of the benefits of using computers. It was found that the use of computers increases teachers' enthusiasm for their work; increases motivation; provides teachers with more opportunities for effective assessment and target setting; facilitates communication between teachers as well as between teachers and pupils; provides pupils with real-life experiences and audiences; and provides links between home and school, which in turn help to raise pupils' self-esteem (Becta 1998, 2002). In addition, 'time savings and positive impact on attainment' have been recently reported as further benefits of technology in pupils' learning (Becta, 2009a).

However, teachers like me, who may feel enthusiastic about the benefits of using computers in schools, should be aware of what two national frameworks for mathematics, one in the UK (DfEE, 1999b) and another in the USA (NCTM, 2004b have suggested. According to the former, the computers should be used only "if it is the most efficient and effective way to meet the objectives of the lesson…an aimless exploration of an 'adventure game' or repetitive practice of number bonds already mastered, is not a good use of lesson time." (p. 32) According to the latter:

> Technology cannot replace the mathematics teacher, nor can it be used as a replacement for basic understandings and intuitions. The teacher must make prudent decisions about when and how to use technology and should ensure that the technology is enhancing students' mathematical thinking. (NCTM, 2004b, webpage)

In addition, there are arguments that come from a broader field, including physics and mechanics (e.g., Morin, 2008), suggesting that heavy reliance on ICT in mathematics may prevent mathematical development. Morin (2008, p. 13), for instance, has written: "People tend to rely a bit too much on computers and calculators nowadays, without pausing to think about what is actually going on in a problem." Adding at the end the following poem:

> The skill to do math on a page
> Has declined to the point of outrage.
> Equations quadratic

Are solved on Math'matica[1],
And on birthdays we don't know our age.
(Morin, 2008, p. 13)

2.3.4 The role of the teacher

Teaching and learning strategies presented in this chapter (section 2.3.1) have highlighted the role of the teacher as a key point in delivering provision in classrooms effectively. Selecting suitable materials beyond a single textbook, planning cognitive activities, encouraging higher-order thinking, using higher-order questioning, and teaching within gifted pupils' ZPD require a well-trained teacher of the gifted with subject expertise. Sheffield (1999), for instance, has suggested that mathematically promising children need a 'challenging mathematics teacher' who will know how to distinguish real enrichment in mathematics — which can be enjoyable — from 'fun' through playing with the latest computer games or through filling in puzzles with pieces to plan enrichment lessons according to the goals and objectives of the curriculum, with a mathematical purpose beyond fun. She, also, has suggested that the teacher of gifted or exceptionally able mathematicians must be a highly trained specialist with "a background both in higher level mathematics and in understanding exceptional students" (p. 53). Other educationalists, who agree with the need for well-trained teachers, have also suggested that teachers need support in recognising mathematically gifted children, and that teachers themselves need to be interested in their personal development (Freeman, 1998; Johnson, 2000; Koshy, 2001).

Focusing on the UK, where this study is taking place, we can see that there is more that needs to be done in relation to teachers' development. This has been concluded by the Williams (2008) report, which suggests higher entry requirements in mathematics for those willing to become teachers (criticising the existing at that time criteria as low) as well as an improved Initial Teacher Training with more advanced mathematics.

Personal development of the teacher was found, through empirical research (Koshy & Casey, 2005), to be a basic factor for raising teachers' self-confidence in teaching mathematics to very able pupils. This is very important in gifted education, because, as

[1] It is "Mathematica", a computerised programme for mathematics (Wolfram Research, 2010), very popular in the USA

Koshy (2001) argues, teachers' self-confidence and subject expertise can generate enthusiasm for the subject, thus inspiring the pupils.

This last part discussed practical methods for schooling provision for mathematically gifted children along with the role of the teacher. Organisational strategies for differentiation, acceleration and enrichment and specific strategies for teaching mathematics to gifted children were presented and discussed. Differentiation and enrichment, or differentiation through enrichment, seem to be the strategies that most educational researchers suggest as the most suitable for provision for mathematically gifted children. To achieve differentiation within the curriculum the following methods were suggested: differentiated work, differentiated instructions, ongoing assessment and flexible grouping. To help the teacher provide enrichment lessons, Koshy's (2001) framework for planning and teaching mathematics to very able children according to Bloom's Taxonomy was suggested, because it provides good examples for challenging these children's higher-order cognitive skills. Regarding the most appropriate grouping method for addressing the needs of mathematically gifted children, the review of the existing research and literature showed that 'setting' and 'pull-out' programmes in mathematics appear to be more effective in boosting gifted pupils' achievement. Finally, it was made clear that, as with identification discussed earlier, there is not any single strategy or method to meet the needs of mathematically gifted children. It depends on the school to determine the best and most appropriate strategy for provision, including grouping practices and other activities beyond the main curriculum for use within or outside school, through a well-trained teacher with subject expertise and confidence to teach mathematics at higher cognitive levels.

2.4 Summary

This chapter presented a review of literature and provided a background for the study. It discussed conceptions of high ability and specifically the nature of mathematical ability along with ways to develop it within schooling.

There is an agreement nowadays that mathematical ability, independently of whether it is innate or not, can be developed through experiences, instruction, training and continuous challenge, and, therefore, through the appropriate educational programme. It has emerged from the literature review that there has been an increasing interest in the education of mathematically gifted children since the beginning of the 1980s and especially since Gardner's (1983) MI theory became widely known. Educational systems in many countries, including the UK, have recognised the need to identify gifted mathematicians at early stages and effectively nurture their mathematical ability within schools.

However, the literature review has revealed a gap between the large amount of theories and recommendations of good practice for nurturing mathematical ability and the relatively little empirical support from real school-life. This study, therefore, aims to contribute to the field of gifted and mathematics education by bringing out new empirical evidence of what is actually happening in primary classrooms regarding the development of mathematical talent. Theories and models of talent development and teaching mathematically gifted children, discussed in this chapter, provided an educational context and a framework to present and analyse the empirical data. Therefore, the choice of what was included in this study, although it was personal in essence, was influenced by Gardner's (1983, 1999, 2006) MI theory about the existence of specific mathematical ability and its assessment, constructivist theories for teaching and learning within pupils' ZPD (Vygotsky, 1978), the role of 'affect' in learning mathematics (Ernest, 1985) and subject-specific frameworks (Casey, 1999; Koshy, 2001; Sheffield, 1999) for teaching mathematics at higher cognitive levels.

Having these in mind, I framed the main research questions, which I have presented in Chapter One (section 1.3), and a conceptual framework for planning the research (Figure 2-9). The detailed research plan is presented in the next chapter. I hope that, through a questionnaire survey in London primary schools and carrying out some in-depth case studies I will find interesting answers on how the needs of mathematically gifted children are being addressed in everyday practice within ordinary primary schools, what methods of specific provision are being implemented, and what the effects are on pupils' achievement and behaviour.

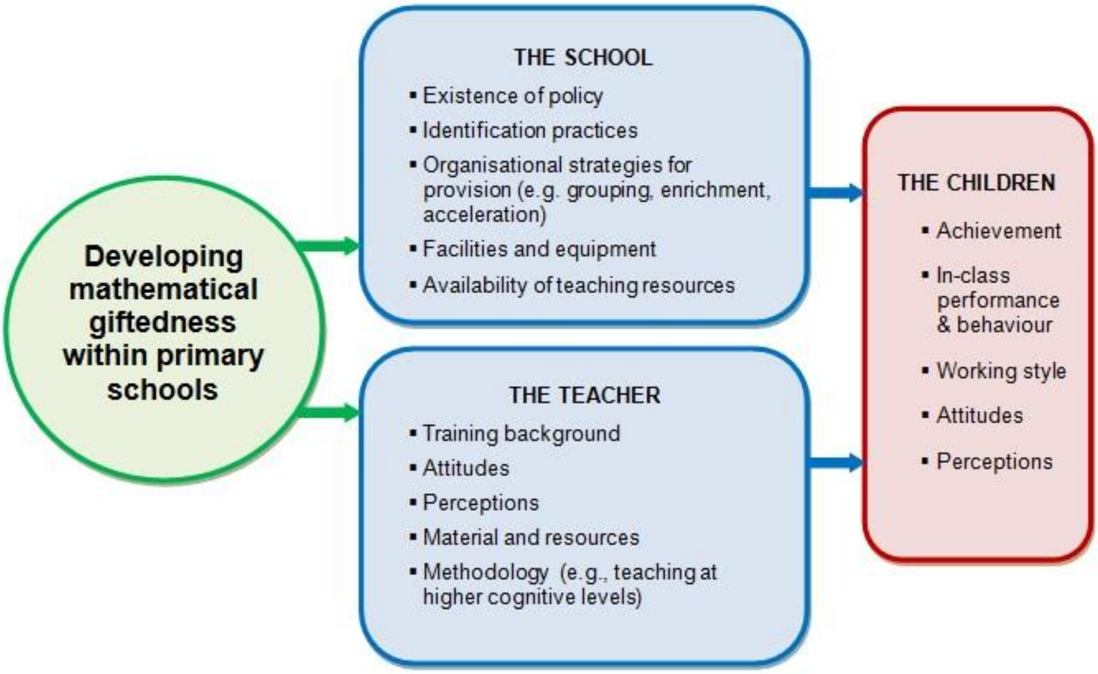


Figure 2-9: Conceptual framework for the research

The following chapter presents the research methodology used for this study, the methods of data collection, the methodological instruments and the rationale of choice.

3 Chapter Three: Research Design

This chapter presents the research methodology and methods employed for collecting and analysing the data required for this study. It initially explains the difference between method and methodology, presents the general characteristics of qualitative and quantitative methods and positivist and interpretivist methodologies and justifies the rationale for using mixed methods for this investigation. Then, it presents the research design for this study.

3.1 Methodological considerations

3.1.1 Methods and methodologies

Before I planned my research project, I needed to distinguish the research method from the methodology, the qualitative from quantitative methods and the positivist from interpretivist methodologies.

Many writers distinguish method from methodology (Cohen, Manion & Morrison, 2007; Hitchcock & Hughes, 1995; Scott, 1996). Method refers to "the instruments by which data are collected" (Scott, 1996, p. 61), such as observation, interviews and questionnaires. These enable the researcher to listen to subjects, observe what people do and say, and collect and examine documents people construct. They are classified into qualitative methods, which mainly use words (e.g., interviews, observations, or documents) (Miles & Huberman, 1994) and quantitative methods, which mainly use numbers or closed-ended questions (e.g., quantitative hypotheses) (Creswell, 2009).

Methodology refers to the whole range of questions about the assumed appropriate ways of going about social research. It is a broad and complex area of ideas, concepts, frameworks and theories, which surrounds the use of various methods for gathering data on the social world. There are two clear perspectives or traditions in social research: positivism and interpretivism. Researchers from each of these perspectives may use the same data-collection instrument, but structure this instrument in different ways and analyze the data provided differently. Positivism seeks to uncover patterns in social life by collecting facts, usually using quantitative methods, about the world. The data collection instrument acts in a neutral capacity and does not play a part in the determination of the truth-value of the data (Scott & Usher, 1999). On the other hand, interpretive approaches choose mostly qualitative methods and are considered qualitative methodologies. These approaches enable researchers to learn, first hand, about the social world they are investigating by means of involvement and participation in that world through a focus upon what individual actors say and do (Hitchcock & Hughes, 1995).

Experiment and survey approaches, according to Scott (1999), are considered positivist methodologies, while ethnography and case study are considered interpretivist (Marshall & Rossman, 1999, 2006). Therefore, the question that an educational researcher needs to consider is, "Which is the most appropriate methodology for my research study? Positivism, which relates to quantitative methodologies, or interpretivism, which relates to qualitative methodologies?"

3.1.2 Considering qualitative versus quantitative methodology for researching education

Usher (1996) and Silverman (1993, 2006) are sceptical about the use of a positivist framework to research education. Usher (1996) considers the adoption of a positivist view contestable in the social/educational domain, because it emphasises the perception that there is a certain truth, which must be known. It emphasises 'determinacy' (there are no contradictory explanations but a single one), 'rationality' (that objectivity is better than subjectivity) and 'impersonality'. Silverman (1993, p. 21) argues that "purely quantitative researchers may neglect the social and cultural constructions of the 'variables' they seek to correlate." On the contrary, qualitative researchers, according to Miles and Huberman (1994, p. 10), "focus on naturally occurring, ordinary events in natural settings", so that they have "a strong handle on what *real life* is like", take into account the influences of the local context and, thus, gather richer data, which increases the potential for revealing complexity. Researchers who collect qualitative data:

> …can go far beyond 'snapshots' or 'what?' or 'how many?' to just how and why things happen as they do — and even *assess causality* as it actually plays out in a particular setting. And the inherent *flexibility* of qualitative studies (data collection times and methods can be varied as a study proceeds) gives further confidence that we've really understood what has been going on. (Miles & Huberman, 1994, p. 10)

What I was attempting to investigate was complex and occurring in natural settings, as I was trying to find how the needs of a particular group of children, who are gifted in mathematics, are met in reality of the classroom. Therefore, a qualitative approach seemed most appropriate. Furthermore, Hitchcock and Hughes (1995, p. 12) suggest that particularly for a teacher (like me), the qualitative method is the most suitable method to research education: “Qualitative as opposed to quantitative research is more amenable and accessible to teachers and has the considerable advantage of drawing both the researcher and the subjects of the research closer together.”

To those who consider quantitative methodology the only legitimate research approach, Scott and Usher's (1999) arguments are worthy of consideration. They contend that the days where the sense that quantitative research was ‘better’ and more ‘legitimate’ than qualitative — which was considered ‘soft’, ‘unrigorous’, ‘subjective’ and, because of that ‘totally discounted’ — have passed. Qualitative research is now more accepted and researchers who follow this methodology are not obliged to work harder in order to ‘prove’ the validity of their results as they used to be in the past. They also argue that it no longer matters whether qualitative research is legitimate or not and that the real question now is to what extent qualitative research is compatible with quantitative research and whether or not is characterised by ‘epistemology’. ‘Epistemology’, according to Scott and Usher (1999), makes a distinction between what is legitimate knowledge and what is a simple opinion or belief — a crucial issue for a teacher, like me, who aspires to research other teachers’ practices.

However, ‘epistemology’ does not mean that I have to exclusively follow a scientific approach for my investigation in order to secure legitimate knowledge. As Scott and Usher (1999) argue, of course, ‘epistemology’ is based on science, as a model for investigations that can be measured and tested, but it is also based on ‘empiricism’, which, through observations, gains knowledge coming from sense experience. Moreover, Scott and Usher (1999, p. 10) argue, “research is not a technology, but practice, that it is not individualistic, but social, and there are no universal methods to be applied invariantly”. In other words, methods from different methodologies may be combined depending on what is being investigated.

3.1.3 Mixed methods of research

There has been an increasing interest in the research literature on using mixed methods, which combine qualitative and quantitative approaches of research. Bell (2005), for instance, explains that some approaches may depend heavily on one method of data collection but this does not mean that they are exclusive to one method. Therefore, a questionnaire, which is inevitably considered quantitative, may include qualitative features, and “case studies, which are generally considered to be qualitative studies, can combine a wide range of methods including quantitative techniques” (p. 115).

Creswell (2009) has adopted Newman and Benz’s (1998, cited in Creswell 2009) views that qualitative and quantitative research approaches should not be considered as polar opposites or dichotomies, but as different ends of a continuum. Creswell argues that “a study tends to be more qualitative than quantitative or vice versa” and that somewhere in the middle of this continuum there are mixed research methods that “incorporate elements of both qualitative and quantitative approaches” (Creswell, 2009, p. 3). He considers a mixed methods approach a result of continuous development and evolution of qualitative and quantitative methodology in the human and social sciences and, therefore, “another step forward”, because they utilise “the strengths of both qualitative and quantitative research” (Creswell, 2009, p. 203). Creswell reminds us what Tashakkori and Teddlie (1998, cited in Creswell, 2009, p. 14) had said about the usefulness of mixed methods when connecting qualitative and quantitative data: “the results from one method can help identify participants to study or questions to ask for the other method”. That means, according to Creswell (2009), that the researcher can begin with a qualitative method, such as interviews, and follow up with a quantitative survey with a large sample in order to expand on the findings. Or the opposite, beginning with a quantitative method, such as a survey, in order to collect data from a larger sample about a concept or theory and follow up with a qualitative method, such as an in-depth case study, in order to explore the theory or concept in practice and in greater detail.

At this point, I need to reiterate the purpose of my study and the research questions, mentioned earlier in Chapter One (section 1.3). The purpose of the study was to explore and study the strategies used for educating children who are gifted in mathematics within primary schools and the research questions were the following:

1) What strategies are schools using, if any, regarding the education of gifted and talented children in general and specifically in mathematics?
2) What are the teachers' perceptions of and attitudes towards mathematically gifted children, their education and the methods used by their schools?
3) How are the needs of mathematically gifted children met within classrooms in everyday practice?
4) What is the impact of the schools' strategies on pupils' achievement and attitudes?

Therefore, taking into account all the aforementioned, I decided that the most appropriate methodology for the purpose of this study would be qualitative in nature. This would allow me, according to Miles and Huberman (1994), mentioned earlier, to focus on naturally occurring events, such as teaching and learning mathematics, taking place in natural settings, namely in primary classrooms. In addition, I felt that I needed to support my methodology with a range of methods for data collection which would involve both qualitative and quantitative methods, as Creswell (2009) recommends. This would help me improve the quality of information from different resources and enhance the credibility and validity of the research. Therefore, I decided to conduct the research in two phases following Creswell's (2009) suggestions of using a questionnaire survey for the first phase and in-depth case studies for the second phase (the main study).

3.2 Research design of the study

3.2.1 The first phase: a questionnaire survey

"A survey design provides a quantitative or numeric description of trends, attitudes, or opinions of a population by studying a sample of that population." (Creswell, 2009, p. 145) It also allows the researcher to gather information from a wide field of issues, populations or programmes in a fast and economic way. A researcher can gather standardised information through a large-scale survey, by using the same instruments and questions for all participants, analyse the data statistically and observe patterns of responses that help to make generalisations about the targets of focus (Morrison, 1993).

In my case, the questionnaire survey I designed (see Appendix 2) aimed to collect not only numeric data, such as how many schools keep a separate register for gifted children in mathematics or how many schools make specific provision for mathematically gifted children, but also qualitative data relating to teachers' views through the use of open-ended questions. These questions would allow me to obtain authentic personal data within the research themes in that teachers would be able to demonstrate their individual and unique understanding regarding mathematically gifted children and their education, giving an exploratory nature to the questionnaire where the possible answers are unknown (Bailey, 1994).

However, developing a questionnaire was not easy. A questionnaire needs extra attention and a lot of time for refinements through pilot testing, which, according to Creswell (2009, p. 150), "is important to establish the content validity of an instrument and to improve questions, format, and scales". If the open-ended questions, for instance, are not clear or are too open-ended to a level that the respondent cannot understand what kind of information is being sought, they may lead to irrelevant and redundant information, or discourage people to complete them (Cohen, Manion & Morrison, 2007). To avoid this and to increase the validity of the research, I piloted the questionnaire before I finalised it with four teachers, colleagues of mine. This helped me reduce ambiguity in wording, identify misunderstood items, and acquire feedback on the appropriateness of the questions to the purpose of the study.

Therefore, in the first stage for the preliminary phase of the research, 224 questionnaires were distributed during the summer term (April – July) of school year 2007-2008 to all maintained primary schools within five London Local Educational Authorities (LEAs) in Greater London. One questionnaire per school was sent. Each questionnaire was addressed to the headteacher asking him/her to make sure that a mathematics co-ordinator or a classroom teacher involved in teaching mathematics to gifted children would complete it. The aim of using questionnaire was to help the organization of the main study, which would involve an in-depth look at primary classrooms to find out the nature of provision for mathematically gifted children in everyday practice.

As the study is mainly qualitative, a few numerical data was only sought through the questionnaire. Therefore, some simple 'excel' analysis (see a sample in Appendix 3) was to be carried for the questionnaire without any further statistical analysis.The analysis of teachers' responses gave me a first insight about how primary schools in a large area of London addressed the needs of mathematically gifted children and useful data about how teachers perceive mathematically able or gifted children as well as their level of confidence in teaching these children. The analysis of the collected data helped me choose the case study schools (see section 3.2.4) and to focus on the next stage.

3.2.2 The second phase: Case studies

Following the questionnaire survey, case study schools were set up to study in-depth a selected sample (see section 3.2.5) of teachers and children, identified as more able or gifted mathematicians, in their classrooms to find out how the needs of these children are addressed in practice. The reasons for choosing a case study methodology were that it is described as the most appropriate method to explore in-depth classroom activities within a limited time scale (Bell, 2005; Yin, 2003) it can ideally combine different methods of data collection (Cohen et al., 2007) so that more information and different resources can be integrated, in this way enhancing the credibility and validity of the research; and it can present to the readers a more accurate and clear picture of what was studied (Adelman, Jenkins & Kemmis, 1980; Cohen et al., 2007; Hitchcock & Hughes, 1995). I also had personal experience of a successful use of a case study when I carried out an in-depth study, which was part of my masters dissertation (Dimitriadis, 2005) on teaching thinking through mathematics in my own class with a focus on five individuals. Therefore, I felt at ease with this methodology.

This part of the research constituted the main study and, therefore, the case study methodology played the main role for this thesis. I will now discuss in more detail how the case study approach was used, its strengths and relative weaknesses and explain how it served the purpose of this study.

3.2.3 Case study as a structure for research

Bromley (1990, p. 302) defines case study as a "systematic inquiry into an event or a set of related events which aims to describe and explain the phenomenon of interest".

Bell (1993, p. 8) suggests that the case study approach is particularly appropriate for individual researchers, like in my case, because on one hand, it "gives an opportunity for one aspect of a problem to be studied in some depth within a limited time scale". On the other hand, it "allows the researcher to concentrate on a specific instance or situation and to identify, or attempt to identify, the various interactive processes at work". It is an instance in action and it aims to provide, as accurately as possible, the fullest, most complete description of the case (Adelman, Jenkins & Kemmis, 1980).

Furthermore, Cohen et al. (2007, p. 253) maintain that a case study "provides a unique example of real people in real situations, enabling readers to understand ideas more clearly than simply by presenting them with abstract theories or principles." The reality of the contexts in which people or effects are observed, like the real mathematics classes in my case, is one of the strengths of case study methodology, which "can establish cause and effect…recognizing that context is a powerful determinant of both causes and effects." (p. 253)

Hitchcock and Hughes (1995) also suggest the case study as a suitable qualitative research methodology to investigate 'how' and 'why' questions and events within real-life contexts, in which the researcher has little control, such as those present in the real classrooms this study investigates.

A key issue in case study research that may help an individual researcher in education, like me, is the way of selecting information. Yin (2009), for instance, argues that the case study is more flexible than the other research methodologies and that the researcher does not need to have a minimum number of cases, or to randomly 'select' cases. Similarly, Cohen et al. (2007) argue that the researcher, who uses case study, does not always need to look for criteria of representativeness because there is the possibility that an infrequent or unrepresentative event or incident will be critical to the understanding of the case. In my case, for example, a particular behaviour might only be demonstrated once, but it might be very important. Thus, it should not be ignored simply because it happens only once.

Furthermore, Cohen et al. (2007, p. 258) argue that case studies do not have to seek frequencies of occurrences, but they "can replace quantity with quality and intensity, separating the 'significant few' from the 'insignificant many' instances of behaviour".

They also argue that "significance rather than frequency is a hallmark of case studies, offering the researcher an insight into the real dynamics of situations and people".

Case study meets the three tenets of the qualitative method: 'describing', 'understanding' and 'explaining' (Yin, 2009), which I consider as indispensable for my research and it appears to be open in different approaches, which it can combine. Adelman, Jenkins and Kemmis (1980) describe the case study as an 'umbrella' term for a family of research. Moreover, Hammersley (1992) argues that different research strategies such as ethnography, survey and experimental research utilise the concept of a case, but treat it differently.

A case study, therefore, may have the following characteristics:

- A concern with a rich and vivid description of events within the case.
- A chronological narrative of events occurring in the case.
- A combination of a description and analysis of events.
- A focus on actors, individual or groups, and their perceptions.
- A focus on specific events within the case.
- A researcher who is trying to be integrally involved in the case.
- A way to present the case so as to be able to reveal the richness of the situation.

(Hitchcock & Hughes, 1995, p. 317)

Stake (1978, 1995) and Yin (2003) have suggested techniques to successfully organise and conduct research. Stake (1978, 1995), for example, has studied the individual as the unit of analysis, and has used the case study method to develop rich and comprehensive understandings about people. He writes (1995) that the number and type of case study depend on the purpose of the inquiry and distinguishes an 'instrumental case study', which is used to provide insight into an issue; an 'intrinsic case study', which is undertaken to gain a deeper understanding of the case; and a 'collective case study', which is the study of a number of cases for inquiring into a particular phenomenon. Stake (1995) accepts that there are many other types of case studies based on their specific purpose, such as the 'teaching case study' or the 'biography'.

Yin (2003) also, has suggested examples in education for instructional use with appropriate design for case studies and recommendations for using three types of case studies: 'exploratory', 'descriptive' and 'explanatory (causal)'. Each of these three types can be either 'single-case' studies (with a focus on a single case only) or 'multiple-case' studies (with a focus on two or more cases, which are included in the same study). Yin (2003, p. 5) briefly described the characteristics of the main three types of case studies as follows:

> An *exploratory* case study (whether based on single or multiple cases) is aimed at defining the questions and hypotheses of a subsequent study (not necessarily a case study) or at determining the feasibility of the desired research procedures.
>
> A *descriptive* case study presents a complete description of a phenomenon within its contexts.
>
> An *explanatory* case study presents data bearing on cause-effect relationships — explaining how events happened.

Therefore, according to Yin (2003), this study may be considered an 'explanatory' case study because it tries to answer a 'how' question. It is also a 'multiple-case' study because it studies many cases with different characteristics. However, because this study aims to present an accurate and clear picture of each case in its context, it also includes elements of a 'descriptive' case study.

Limitations in case study approach

Two main limitations are the focus of criticism of the case study approach. The first is that it is incapable of providing a generalising conclusion because of its dependence on a single case or because its outcomes are not easy to be cross-checked. The second is that it may be subjective and biased. Regarding the first, for instance, Stake (1995, p. 4) contends: "We do not study a case primarily to understand other cases. Our first obligation is to understand this one case" and that "Case study seems a poor basis for generalization" (p. 7). Bassey (1999), for this reason, suggested the notion of 'fuzzy generalisations' as a means of disseminating the results of case study research. This means that something may happen without any measure of probability (e.g., it is possible to happen again but it is not certain).

However, Bassey's (1999) suggestions have been criticised by Hammersley (2001) for both their uniqueness and their validity that they do not take into account the nature of generalisation as a whole and the role of the researcher in validation. Furthermore, Yin (2003, 2009) and Hamel, Dufour and Fortin (1993) reject the criticism to the case study and argue that the relative size of the sample does not transform a multiple case into a macroscopic study. It does not matter, for example, whether 2, 10, or 100 cases are used. The goal of the study should form the parameters, and should then be applied to all research. Thus, even a single case could be accepted, provided it covered the established objective. In this research, each selected teacher with her class was studied as an individual case and each case was used "to provide insight into an issue" (Denzin & Lincoln, 2005, p. 445), which was a better understanding of how primary schools address the needs of mathematically gifted children.

Regarding the second limitation, it is obvious that a case study methodology, which, as mentioned earlier (section 3.1.1), is an interpretive approach to research, can be subjective and personal, and, thus, open to bias, which may be not easy to eliminate (Cohen, Manion & Morrison, 2007). However, my intention was to minimise any possible bias that may influence my interpretations, by leaving out personal views, such as those formed by my long teaching experience and taking into account issues relating to validity and reliability throughout the research process. Issues related to validity and reliability of this research, along with ethical issues, are discussed in the following sections (3.2.6, 3.2.7 and 3.2.10) dealing with the tools used for the collection of the data. Before this, I will present the sample of the schools and teachers selected for the main study.

3.2.4 Choosing a sample

After the analysis of questionnaire responses, four different schools were chosen to conduct the case studies in (see Table 3-1). The criteria for selection were that schools had to implement a range of provision for their gifted mathematicians including:

- In-classroom provision
- Setting
- Pull-out grouping, and
- Mentoring

I chose schools where different methods of provision existed so that I could study a range of provision offered to mathematically gifted children. This would help me understand what teachers were doing with reference to provision for these children. The findings, it is hoped, would be of benefit to teachers and educationalists with whom they will be shared.

Table 3-1: Description of case study schools

Data appeared on this table are based on teachers' responses to the questionnaire in the first phase of the research

THE SCHOOLS	IDENTIFICATION PRACTICES	SPECIFIC PROVISION FOR GIFTED MATHEMATICIANS
School A (Community school in Southwest Inner London, 350 children on roll, 3-11 year-olds)	Achievement tests and teacher assessments General register for G&T children	Pull-out groups for more able children who work outside the whole class doing additional lessons every two out of three weeks.
School B (Community school Northwest Inner London, 450 children on roll, 3-11 year-olds)	Achievement tests, teacher assessments, teacher nominations, parental nominations and discussions with children General register for G&T children	In-classroom provision "Enrichment weeks"
School C (Community school in West Outer London, 327 children on roll, 7-11 year-olds — a junior school)	Achievement tests, teacher assessments and teacher nominations General register for G&T children	Setting within each year class
School D (Voluntary aided school in West Outer London, 247 children on roll, 3-11 year-olds)	Achievement tests, teacher assessments and teacher nominations General register for G&T children Separate register for G&T children in mathematics	Pull-out groups for gifted children who bypass certain classes and do more advanced mathematics in parallel to the regular class. Mentoring for particular pull-out groups for gifted children

The participant schools were situated in four different LEAs and from areas that represented a range of socio-economic backgrounds, all in Greater London. All four schools were mixed in gender and had different population sizes. Three of them had students from a wide range of ethnic and cultural backgrounds, while one (School D) had a mainly white population.

3.2.5 Selection of participants

Within the sample of schools, four teachers with their group of children identified as able or gifted mathematicians were chosen. The choice of the teachers was based on suggestions of mathematics co-ordinators of each school. They were teachers who had undertaken responsibilities in teaching mathematics to able or gifted children. In two cases, these teachers were the mathematics co-ordinators themselves. Even though I was hoping to have both female and male teachers as participants, my sample consisted of female teachers only. The four teachers with their group of able or gifted children were, as mentioned earlier, individual cases and they are presented in Table 3-2. More details about the teachers, the children and the way of their identification are presented in Chapter Five, which presents the findings from the case studies.

There were two more teachers, deputy headteachers with responsibility in identification of and provision for gifted children from School B and School D. These teachers' participation pertained to clarify some answers given at the previous stage of the research, through the questionnaire, relating to their school policy. The second teacher (Julie from School D), who had responsibilities for co-ordinating mathematics and ICT for gifted children, participated in an interview as well. This provided more details about the school policy, because the case study teacher (Claire) in that school was an external teacher working occasionally as a mentor for a group of gifted mathematicians and, thus, was not familiar with the policy of the school.

Table 3-2: The case study teachers and their classes

CASE STUDY	TEACHER	POSITION IN SCHOOL	GROUP OF IDENTIFIED CHILDREN
First case study	Emma (School A)	Mathematics co-ordinator & Teacher in pull-out groups	Four children (three boys and two girls, all around ten years old)
Second case study	Sarah (School B)	Mathematics co-ordinator & Teacher in Year 2 (regular class)	Five children (four boys and one girl, all around seven years old)
Third case study	Kate (School C)	Teacher for Year 5 top mathematics set	Six children (two boys and four girls, all around ten years old)
Fourth case study	Claire (School D)	Key Stage 3 teacher for Year 6 mathematics pull-out group	Five children (one boy and four girls, all around eleven years old)

3.2.6 Tools used within the case study to collect the data

Yin (2009) suggests that the way to ensure construct validity in a case study is to use multiple sources of evidence. This study used multiple sources of evidence, which involved classroom observations, interviews with both teachers and children, as well as documentary evidence.

3.2.6.1 Classroom observation

Most of my data were collected through classroom observations because I wanted to obtain what Cohen, Manion and Morrison (2007, p. 396) described as the main strength of the observation method:

> The distinctive feature of observation as a research process is that it offers an investigator the opportunity to gather 'live' data from naturally occurring social situations. In this way, the researcher can look directly at what is taking place *in situ* rather than relying on second-hand accounts. The use of immediate awareness, or direct cognition, as a principal mode of research thus has the potential to yield more valid or authentic data than would otherwise be the case with mediated or inferential methods. And this is observation's unique strength.

I also wanted to explore issues that may not have been revealed through the questionnaire I employed in the first phase and things that might have been difficult to describe in an interview (Creswell, 2009), so as to increase the validity of the research. Cohen et al. (2007) argue that observational data may ensure greater validity than other methods, because the incidents that are observed are less predictable compared to the data collected through a questionnaire or a test, for instance, and because it enables researchers to:

- understand the context of programmes,
- be open-ended and inductive,
- see things that might otherwise be unconsciously missed,
- discover things that participants might not freely talk about in interview situations,
- move beyond perception-based data (e.g., opinions in interviews) and
- access personal knowledge.

(Adapted from Cohen, Manion & Morrison, 2007, p. 396)

Possible disadvantages in observation as a method

Disadvantages that have been highlighted in observational research are a lack of control when observing within natural settings; ethical issues relating to anonymity, to the fact that some private information observed cannot be reported and that the researcher may be seen as intrusive in people's space (Creswell, 2009; Cohen et al., 2007); as well as difficulties in recording and analysing the data (Bell, 2005).

Despite these difficulties, I decided to observe participants in their natural setting to find out what actually happened. I will now explain how my research addressed the aforementioned limitations.

In relation to the lack of control, I did not really want to have any control over the situation. On the contrary, what I was intending to achieve was "to be as unobtrusive as possible so that observed behaviour was as close to normal as possible" Bell (2005, p. 189). This means that I chose to adopt the non-participant style for my observations, which would give me the additional benefit of not being biased, something that is difficult for a participant observer to achieve (Bell, 2005).

In regards to the ethical issues (referred to 3.2.8 section), this research does not use the real names of the teachers or the children, in order to ensure confidentiality and anonymity. However, all the pseudonyms that I use are representative of the participants' gender and origin. For instance, a girl of Arabic origin with an Arabic name was given an Arabic name, which was found from a list of Arabic girls' names. The same was done for an Indian boy, a British girl and so on. The Internet was a very useful source of names from a wide range of origins. Furthermore, the observations were arranged in collaboration with each teacher — who, it should be noted were very keen to help and tried to do something different in each of the lessons observed — during my first visit to the school. The teachers were promised that my presence would be unobtrusive and that my study would not cause any inconvenience to them or their pupils. Before I started the observations, I asked the teachers to suggest a place for me, which would not draw the attention of the children or obstruct the lessons in any case. This usually consisted of a corner next to where my focus group children were sitting, close enough to observe and hear them better, but not so close to produce any distraction.

Recording and analysing the observational data is a difficult part, especially if you want to observe the content of a lesson through an unstructured observation (Creswell, 2009). Cohen, Manion and Morrison (2007) suggest that the observer needs to decide in advance what should be considered, the observation evidence and how to enter the data. Bell (2005) advocates that we need to create our own system of coding symbols that we should memorise, decide how often to record the happening (e.g., all the time? every 3 seconds? 5 minutes? 20 minutes?) and whom we are recording (e.g., the entire group? individuals?). In my case, the first stage of the research helped me to identify the issues I wanted to observe in the class, but I had my observation form open for events as they happened in the classroom. I used an observation form (see Appendix 4.1), which I separated into sections in order to record what was happening every 10 minutes. I also used a checklist in order to avoid missing something important (see Appendix 4.2). During the lesson, I was trying to record what I was hearing and observing without making judgements or interpretations at that time (see a sample of a ten-minute observation notes in Appendix 4.3).

3.2.6.2 Interviews

According to Burns (2000, p. 467), interviews in case study research "are essential, as most case studies are about people and their activities." Similarly, there are arguments that support the use of interviews in qualitative research in general. For instance, Cohen et al. (2007, p. 349) argue that:

> The interview is a flexible tool for data collection, enabling multi-sensory channels to be used: verbal, non-verbal, spoken and heard. The order of the interview may be controlled while still giving space for spontaneity, and the interviewer can press not only for complete answers but also for responses about complex and deep issues. In short, the interview is a powerful implement for researchers.

Therefore, interviews were a suitable tool to gather data to answer the research questions of this study. During the case studies, I conducted semi-structured interviews with children suggested as being more able mathematicians and their teachers. In the case where the teacher was an external part-time teacher, I also interviewed the mathematics co-ordinator, who was responsible for organising and running the provision programme. Twenty-five interviews were conducted in total.

Semi-structured interviews, according to Drever (2003), mean that there is a general structure by deciding in advance the area that will be covered and the main questions that will be asked. The detailed structure is left to be worked out during the interview. The interviewee can answer in his or her own words to some extent, but the interviewer responds using follow-up questions, prompts and probes to encourage the interviewee to clarify or expand on the answers. However, this kind of interaction between interviewer and interviewee should not be confused with an ordinary conversation. Instead, interviewers should know that an interview is a ‘formal encounter’, with a specific purpose about which both parties are aware (Drever, 2003). Therefore, I had prepared some initial questions to help me focus on the purpose of my study (a schedule of interview questions for both teachers and children is presented in Appendix 5.1). These initial questions, however, sometimes changed during the interview according to the answer and when a child remained silent unable to answer (see in Appendix 5.2 a small sample of a child interviewing where such changes happened). All the interviews were tape recorded, transcribed, and then analysed.

The interviews with each teacher and the children were conducted before the observation of their lessons. The children were interviewed one-by-one in order to avoid the possibility of them copying each other in a group interview. Therefore, extra care was taken for them to feel confident with me, the interviewer. Before the interviews, the teacher introduced me to them and explained (or let me to explain) why I was there and what I needed from them. I also explained to them that the questions that I would ask did not have right or wrong answers, that whatever they said would remain strictly confidential, and that nobody, not even their teacher, would know what they said. I requested the interviews with each child to be done in a quiet place, close to the teacher or in the presence of another teacher nearby in order for the child to feel secure, confident and not isolated with me. I should say, at this point, that all the children were very keen to take part. Moreover, the choice for the interviews to take place before the observations proved to be a very good idea, because I had the opportunity to get to know the children while I was talking to them and then I was able to recognise them within the class and observe them better.

Possible disadvantages of interviews

Interviews, as they provide indirect information that is filtered through the views of interviewees, have been criticised as open to bias (Creswell, 2009; Cohen, Manion & Morrison, 2007). "The resources of bias are the characteristics of the interviewer, the characteristics of the respondent, and the substantive content of the questions." (Cohen et al., 2007, p. 150) Therefore, I had to consider how to achieve validity in my interviews by minimising any possible bias that could occur. This was achieved by not allowing, as much as possible, my personal opinions and expectations to affect the process. I also had to ensure that the responses of the interviewees were not misunderstood. This was achieved through unbiased prompting and probing. However, as it was, also, important to ensure reliability (Drever, 2003), these prompts and probes were aiming at the encouragement of the respondents, especially in the cases of child interviewing, to tell their own story, within the agenda, without leading their response.

3.2.6.3 Documentary evidence

Documents pertaining to the subject of study were collected. They were school documents about policy and planning, tracking sheets of children's achievement, records of their assessment and photocopies of their work. These helped me to acquire and present a more clear and accurate picture of each case. According to Burns (2000), documents are important for supporting evidence derived from other sources. However, this was done having in mind that documentary evidence may not be accurate or may be biased and subjective, especially these written by the schools, and that some of them were written for a specific purpose for a particular audience in mind (Burns, 2000).

3.2.7 Ethical considerations

Before the research began, the approval of the Research Ethics Committee of Brunel University London was obtained along with the informed consent of the teacher, the headteacher and the children's parents who gave their consent for both the interviews and the observation of the lessons. Parents also consented for their children's written work to be included, anonymously, in publications (see copies of the participation information sheets and consent forms sent to headteachers, classroom teachers and parents in Appendix 6). The teachers were given the opportunity to review their interview transcripts and to read the observation notes before the analysis took place. Issues of confidentiality and anonymity were taken into account during the whole research project. During the first phase of the research, for instance, the names of schools were changed and codes were used instead (e.g., School 01) while during the main phase extra measures were taken for the observed lessons (see section 3.2.6.1) and the interviews (see section 3.2.6.2) and, therefore, I believe that all ethical requirements for the research were met.

3.2.8 Time frame

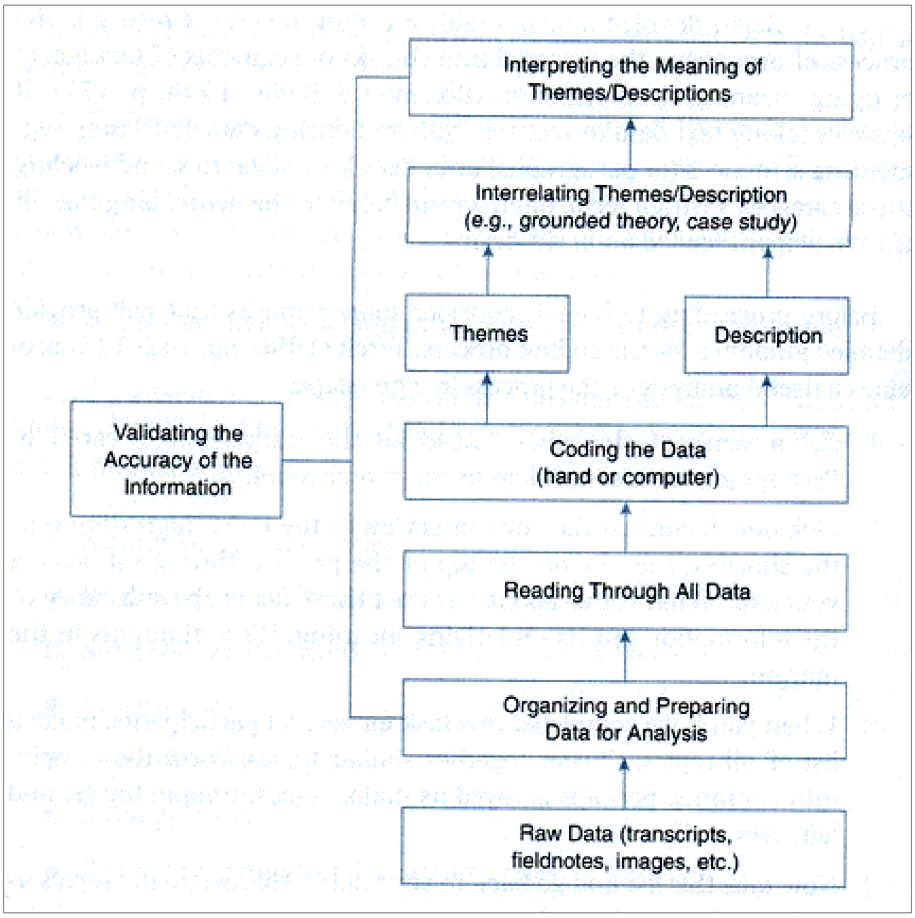
The following table (Table 3-3) presents the key dates of both stages of the research.

Table 3-3: A time frame of the research

KEY DATES	ACTIONS
June 2008	- Distributing the questionnaires by post (2nd June 2008) - Start organising and preparing the analysis of the questionnaire replies - The summer holidays were used to analyse the questionnaires
September 2008	- Contact with the teachers from chosen schools - Initial meetings with participating teachers to arrange a plan for the interviews and observations
October 2008 – January 2009	- Research conducted in School A and started in School B - The holiday term was used to examine the data collected from School A (Dimitriadis, in press), which was used as a pilot study.
January – February 2009	Field work completed in School B and started in School C
February – March 2009	Field work completed in School C and started in School D
March – April 2009	Field work completed in School D
May – June 2009	Transcripts of the interviews and copies from my field notes were delivered to each school for perusal by participating teachers

3.2.9 The analysis of data

The analysis of the data in qualitative research “is an ongoing process involving continual reflection about the data, asking analytic questions, and writing memos throughout the study … the researcher collects qualitative data, analyzes it for themes or perspectives, and reports 4–5 themes.” (Creswell, 2009, p. 184) Similarly, for the case study research Creswell (2009) argues that it involves “a detailed description of the setting or individuals, followed by analysis of the data for themes or issues” (p. 184) and suggests a process for the analysis of any type of qualitative research as the following diagram illustrates (Figure 3-1).



Source: Creswell (2009, p. 185)

Figure 3-1: Creswell’s data analysis process in qualitative research

Organising the data

Following the first steps of Creswell’s (2009) model for organising and preparing the data for analysis, I decided to organise the findings in three parts:

a) Planning. This included data gathered through school documentation, teachers’ responses to the questionnaire (carried out during the previous stage), as well as teachers’ clarifications and further explanations that they gave in the interviews about how they plan and organise the in-class provision, the resources that they use, and their methods of assessment.

b) Perceptions and attitudes of the teacher and the children. This required transcribing of the interviews of both teachers and children. The data gathered through the interviews would bring out details about teachers' confidence levels in teaching mathematics to gifted children, their perceptions of the effectiveness of provision that they offer and the possible support that they need. The analysis of the children's interviews would reveal how the children see mathematics and the lessons that they do, their working habits, as well as what they would like the teachers to do or change in their lessons.
c) How the teachers meet the needs of mathematically gifted children within classroom and the impact of their methodology on children's behaviour and performance. This included data from what was observed in classrooms (field notes) through close observation of their lessons as well as pupils' written work and assessment results.

All schools that responded to the questionnaire were coded with numbers between 1–44, according to the order that their responses were received and analysed (e.g., School 01, School 02 and so on). From them, the four schools that were chosen for the case studies were given a different name as follows (the order A, B, C and D was, also, the order that the case studies took place):

School 01 → School A (First case study)
School 09 → School B (Second case study)
School 04 → School C (Third case study)
School 35 → School D (Fourth Case study)

Thematic analysis

The data gathered from interviews, observations and documentation, in this research, were thematically analysed. This involved categorising the data by initially having in mind the research questions and the main issues explored through the literature review and then the emerging themes from the field work. The first stage of the research (the questionnaire survey) and the issues that emerged gave the first themes (e.g., 'existence of a policy', 'organisational strategies', 'teaching resources and materials'), which became the guide for the analysis of data collected through the interviews and documentation. The data collected from the observation of the lessons, which concerned teaching and learning strategies, were analysed and categorised under the titles formed by what was suggested as methods of good practice from the literature (e.g., ‘analysis’, ‘synthesis’ and ‘evaluation’ from the higher levels of Bloom’s Taxonomy). This was a process demanding continuous review of the analysis of the interview transcripts and observation notes and refinement of the themes, as Creswell (2009), mentioned above, has suggested.

My interpretations

Interpretations are the final steps in the process of data analysis in qualitative research in Creswell’s (2009) model (Figure 3-1). This means that the interpretation of my findings is a very important issue for the study. Although, the intent was to leave out any personal views, as previously explained, it is possible that my interpretations have been influenced by my 21 years of teaching experience in primary schools. For instance, many times during the observations, I caught myself putting myself into another teachers’ place. Therefore, my own expectations and beliefs could possibly have influenced the interpretation of the lessons I observed and, in turn, what I left out or what I included in my study. However, in order to ensure the most accurate picture of my case studies, I shared both the interview transcripts and my field notes with the teachers before I attempted any analysis and interpretation of them, as mentioned earlier. I should add, at this point, that there were occasions where some minor changes were made.

At the end a comparison was made to see whether and how recommended ‘good practice’ for meeting the needs of mathematically gifted children, described in Chapter Two, was being implemented in practice. This, according to Creswell (2009, p. 189), allows ‘authors [to] suggest that the findings confirm past information or diverge from it’ and also to ‘suggest new questions that need to be asked — questions raised by the data and analysis that the inquirer had not foreseen earlier in the study.’

3.2.10 Validity, reliability and trustworthiness

In section 3.2.3, where the case study as a structure for the research was discussed, it was mentioned that issues of validity and reliability should be taken into account during the research project and addressed in the best possible level. In this section I will explain how this was achieved in the present research, beginning with an explanation of the terms.

Validity usually refers to internal and external validity. Internal validity links the cause and effect and shows how the findings of the study match what is studied — appropriate for 'explanatory' cases, according to Yin (2009) — while external validity is concerned with the ability of applying the findings in general, in other words whether the results are generalisable beyond the immediate case (Yin, 2009). Reliability refers to the quality of measurement. Whether, for instance, by using the same methodology repetitively the results of the study are the same. If so, the research instrument is considered to be reliable. Because of this, reliability is, also, similar to 'consistency' or 'repeatability'. However, because the terms validity and reliability are traditionally used by the positivists, many qualitative researchers have redefined these terms for qualitative research linking them both with the ability of generalising the findings of qualitative research and referring to both of them as 'trustworthiness' (Bassey, 1999; Lincoln & Guba, 1985; Mishler, 2000). For instance, Mishler (2000) argues that the idea of discovering truth through measures of reliability and validity is replaced by the idea of trustworthiness. In addition, Lincoln and Guba (1985) maintain that trustworthiness is distinct from the standard experimental precedent of trying to show validity, soundness, and significance and that the aim of trustworthiness in a qualitative research is to support the argument that the findings of the research are "worth paying attention to" (p. 290).

Therefore, the question "How to test or maximize the validity and reliability of a qualitative study?" can be redefined as "How to test or maximize the trustworthiness of the qualitative research?" Bassey (1999), for this reason, has suggested eight questions that work as a checklist to demonstrate the trustworthiness of the research. Looking at Bassey's (1999, adapted from p. 75) criteria for trustworthiness (Table 3-4), I can explain in the following table how they were addressed in my research.

Table 3-4: Bassey's criteria for trustworthiness addressed by the present research

Eight questions of trustworthiness	How these are addressed in this research
Has there been prolonged engagement with data sources?	[Yes] The main study was conducted within a full school year and the time with participants (teachers and pupils) appeared to be sufficient.
Has there been persistent observation of emerging issues?	[Yes] The core of the case study involved persistent and unobtrusive observations of classroom lessons, thus recording what was actually happening in there.
Have raw data been adequately checked with their sources?	[Yes] Teachers had the opportunity to check both interview transcripts and field notes as explained earlier.
Has there been sufficient triangulation of raw data leading to analytical statements?	[Yes] As explained in the following section.
Has the working hypothesis, or evaluation, or emerging story been systematically tested against the analytical statements?	[Yes] The main argument about whether and how mathematical giftedness is promoted within primary schools originated from the raw data.
Has a critical friend thoroughly tried to challenge the findings?	[Yes] My supervisor was continuously reviewing my research as it progressed, giving me constructive criticism.
Is the account of the research sufficiently detailed to give the reader confidence in the findings?	[Yes] The argument of the study has been justified by an analytical presentation of the lessons observed, including descriptive parts and illustrations, as well as an analytical account of what teachers and pupils said in the interviews, supported by extracts of their responses.
Does the case record provide an adequate audit trail?	[Yes] Detailed field notes were kept in each lesson observed and the interviews were recorded and transcribed, as explained in previous sections, so as to provide an adequate audit trail.

3.2.11 Triangulation

Triangulation is a technique to improve the validity and reliability (or the trustworthiness) of qualitative research by using more than one methods of data collection in studying aspects of human behaviour (Burns, 2000; Cohen, Manion, Morrison, 2007). It is a process of "using multiple perceptions to clarify meaning, verifying the repeatability of an observation or interpretation" (Denzin & Lincoln, 2005, p. 454). Scott and Usher (1999) also argue that the observation of classrooms from more than one school and the use of different forms of data can increase the credibility of the qualitative study, as the conclusions drawn from the use of one form of data can be triangulated with those taken from others.

The flexibility of case study methodology, which, as mentioned earlier, can combine different research methods, gives opportunities for triangulation, which in turn increases its validity and reliability. A case study, therefore, which does not rely on just one kind of data but utilises multiple sources of evidence through the comparison of data gathered from these different sources can achieve 'triangulation' (Yin, 2009). In my case, this was achieved through the use of a questionnaire, interviews, classroom observations, school documentation and pupils' written work.

3.3 Summary

This chapter examined the different approaches to research and justified the reasons for choosing particular methods to carry out this study. It presented the selected methods of data collection and came to a research design with the following key features:

- A questionnaire survey at the first stage of the research
- Four in-depth case studies incorporating:
    - semi-structured interviews,
    - documentary evidence, and
    - non-participant unstructured classroom observations

The following chapter now reports on the first phase of the research.

4 Chapter Four: Preliminary Phase – The Questionnaire

The previous chapter presented the methodology employed in this study along with my justifications for choosing the methods I used to collect the data. I explained that the study was conducted in two stages. This chapter presents the first stage and briefly discusses its findings. The main study is presented in Chapter Five.

The main areas that the questionnaire (Appendix 2) aimed to investigate were framed at the end of Chapter Two (see Figure 2-9) after a comprehensive review of the literature. These areas were:

- Existence of a policy for identification of and provision for gifted and talented children in general and specifically in mathematics
- Identification of gifted children in mathematics
- Provision strategies specifically for gifted mathematicians
- Classroom practices in teaching mathematics to gifted children
- Teachers' perceptions, and attitudes regarding children with higher abilities in mathematics and their education

Teachers from 44 schools responded to the questionnaire. Thirty-one of them were mathematics co-ordinators, who in some cases had a second role, such as headteacher, deputy headteacher, or classroom teacher (see Table 4-1). Thirty out of 31 mathematics co-ordinators had training in teaching mathematics in general, and some (12) of them had specific training in teaching mathematically able children. Most of them responded that they had training in the identification of and provision for gifted and talented (G&T) children through some courses organised by the Local Educational Authority or by their school.

Table 4-1 presents the role and training background of all participating teachers. The analysis and a brief discussion of all teachers' responses are presented in the following sections.

Table 4-1: Participating teachers

THE TEACHERS		TRAINING BACKGROUND		
Position in the school	*Number*	*Training in identification of & provision for G&T children*	*Training in teaching mathematics to able children*	*Training in teaching mathematics in general*
Headteacher	4	4	2	3
Deputy headteacher (DHT)	3	1	1	3
Assistant headteacher or assistant DHT	2	2	2	2
Classroom teacher	4	3	3	3
Maths co-ordinator	20	15	7	20
Maths co-ordinator & headteacher	1	1	1	1
Maths co-ordinator & DHT	3	1	1	2
Maths co-ordinator & assistant Head or assistant DHT	1	1	1	1
Maths co-ordinator & classroom teacher	6	3	2	6
TOTAL	44	31	20	40

4.1 Existence of policy for identification and provision

The following presentation is based on the analysis of teachers’ responses to the questions on whether they had:

- A policy for the identification of and provision for gifted and talented children in general and specifically in mathematics,
- A gifted and talented register (which is a list of children who are identified as gifted), a general one and/or a specific one for mathematics, as well as if they review the register and when,
- Co-ordinators for planning and running programmes for gifted and talented children, as well as asking for which subjects

Almost all schools (43 out of 44) have a policy for the identification of gifted and talented children. Fewer schools (34), but the majority of those which responded (77 percent), have a policy of provision for gifted and talented children in general (see Table 4-2). Thirty-six schools have co-ordinators for planning and running special programmes for gifted and talented children, while 34 schools (not necessarily the same as the 34 that have a general policy of provision) make specific provision for gifted children in mathematics (Table 4-2).

Table 4-2: School policies for addressing the needs of gifted and talented children in general and specifically in mathematics

n=44

POLICIES	SCHOOLS
Have a policy of identification of gifted and talented (G&T) children	43
Maintain a register for gifted and talented (G&T) children	43
Keep a separate register for mathematically gifted children	13
Have a policy of provision for G&T children	34
Have a policy of provision and co-ordinators for planning and running special programmes for G&T children	31
Do not have a policy of provision but have co-ordinators for planning and running special programmes for G&T children	5
Have a policy of provision and make special provision for G&T children in mathematics	31
Do not have a policy of provision but make special provision for G&T children in mathematics	3

All 43 schools with a policy for identification also keep a register of gifted and talented children in general, while thirteen of those schools keep a separate register particularly for gifted mathematicians. Teachers indicated a range of percentages, from 1 percent to 20 percent, of gifted and talented children in their classes who are on the register, with most frequent answers being 7-10 percent for gifted and talented children in general and 4-6 percent for gifted children in mathematics (see Figure 4-1).

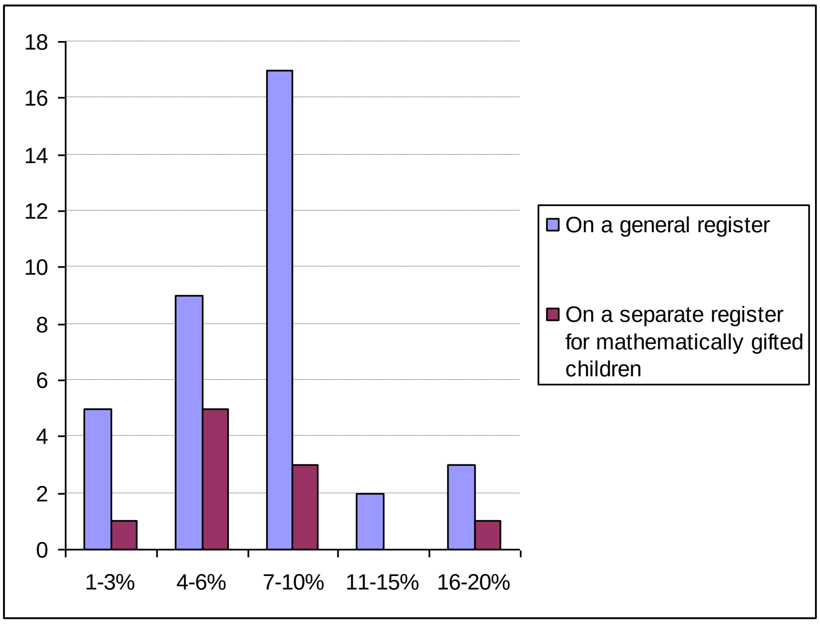


Figure 4-1: Percentage distribution of children registered as gifted and talented (G&T)

Twenty-one of those schools review their register either annually or once a term, while there was a number of teachers (five) who, although indicated that their schools review the register, did not know when (see Figure 4-2).

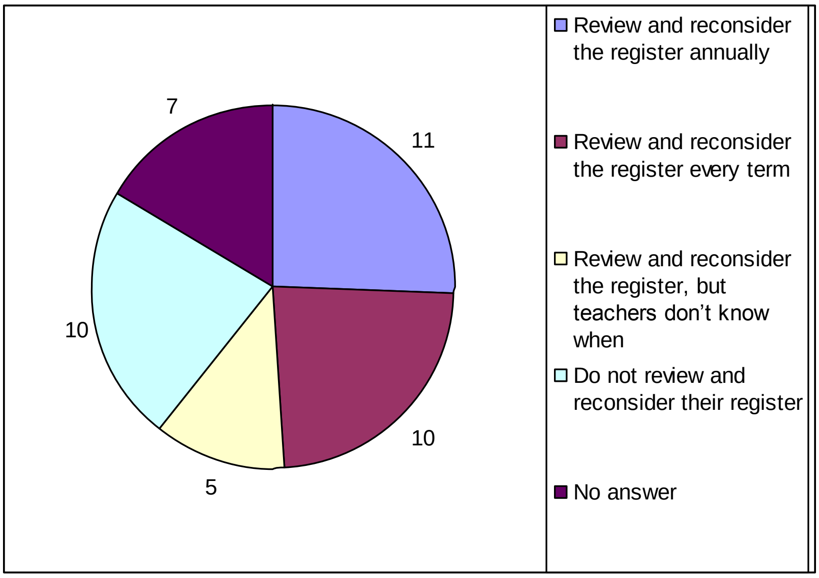


Figure 4-2: Reviewing and reconsidering the register for mathematically gifted children

Mathematics appeared to be the favourite subject for making provision for gifted and talented children, followed by English and sports (see Figure 4-3).

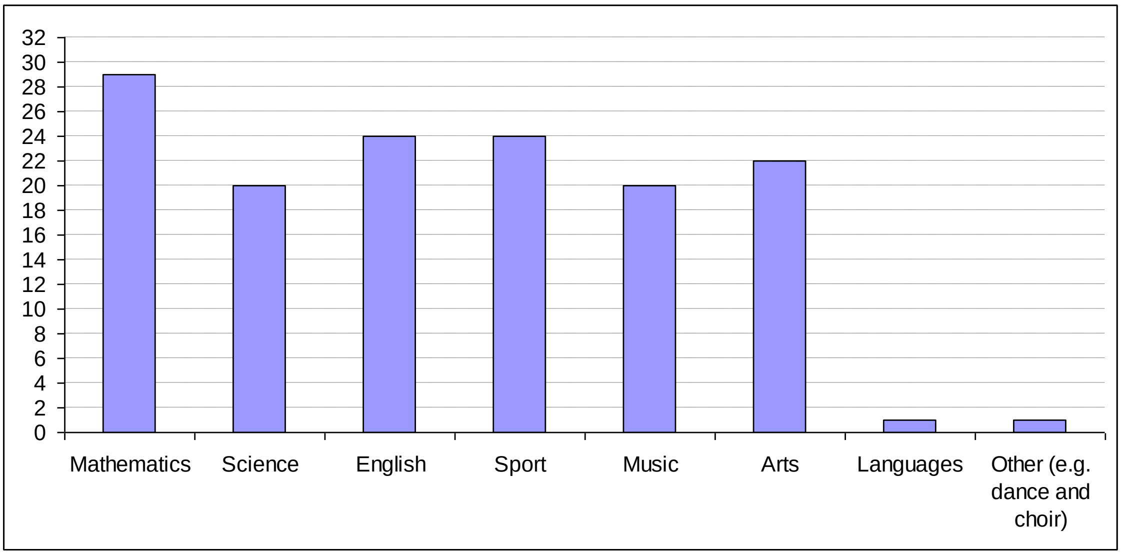


Figure 4-3: Subjects for which the schools assign co-ordinators to plan and run special programmes for gifted and talented children

The following presentation is based on the analysis of teachers' responses to the question asking what methods they use to identify gifted mathematicians. Some choices were given, such as 'IQ tests', 'cognitive tests', 'achievement tests', 'nominations from parents', 'nominations from teachers', and spaces were provided to further explain their choices or to add another method that they use (see Appendix 2).

4.2 How schools identify gifted children in mathematics

It was previously said that almost all schools (43 out of 44) identify gifted and talented children. Teachers, who replied that their school identifies gifted and talented children, were asked to indicate their main methods for the identification of mathematically gifted children. The methods were separated into three categories: a) Tests, b) Teacher assessments, and c) Nominations. Teachers first indicated which of these categories they use. Table 4-3 presents the teachers' answers.

It appears that over half of schools (24 out of 44) use a combination of all three methods. If we look at each category separately, we will see that "teacher assessments" and "tests" appear more frequently in the list of methods that the schools use, the former appearing slightly more frequently (24+10+3+2=39 against 24+10+2+2=38).

Table 4-3: Identification methods, by school, for mathematically gifted children

n=44

METHODS	SCHOOLS
Tests, teacher assessments and nominations	24
Tests and teacher assessments	10
Teacher assessments and nominations	3
Tests and nominations	2
Tests only	2
Teacher assessments only	2
TOTAL	43

When the teachers explained their methods further, we see that the tests they mainly use (36 out of 44, see Figure 4-4) are achievement tests based on criteria referenced by the National Curriculum (e.g., SATs and QCA tests) and almost all nominations come from class teachers. "Teacher assessments" include a variety of practices, which involve observations, discussions, individual work, pupils' written work, informal tests and use of the National Curriculum Level Descriptors or special assessment programmes, such as the Assessing Pupils' Progress (APP) and the Intensifying Support Programme (ISP).

Teachers' descriptions of their assessment methods were categorised into seven groups as they appear in Figure 4-4. It seems that most teachers' assessments are based on the observation of pupils' performance during the lessons (23 teachers, Figure 4-4). It is useful, therefore, to see in Figure 4-5 what exactly the teachers observe in order to assess children's mathematic ability.

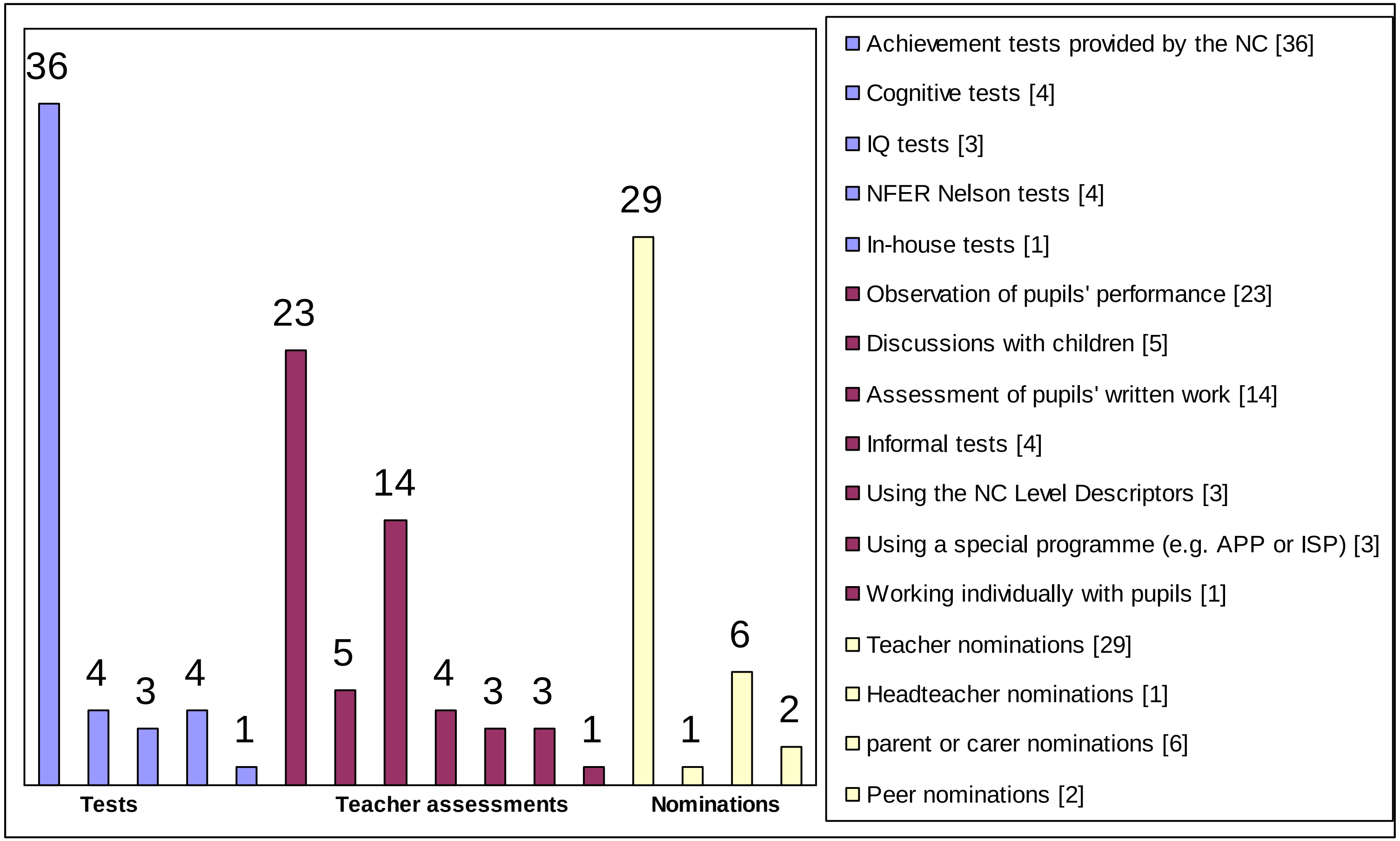


Figure 4-4: Distribution of identification methods for mathematically gifted children

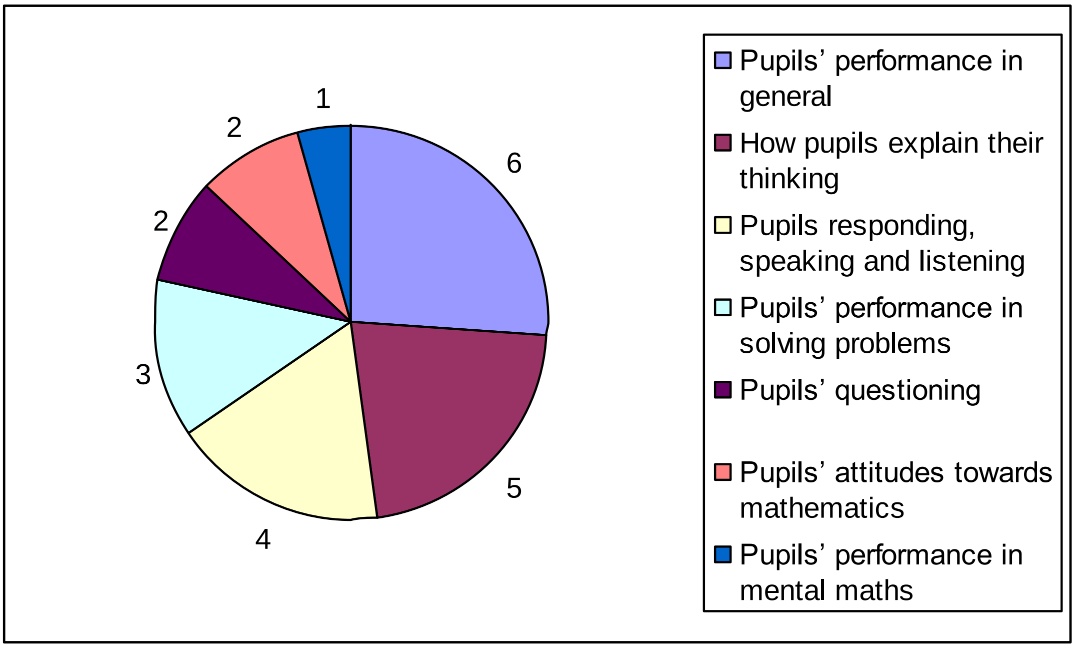


Figure 4-5: What teachers observe to assess children's mathematical ability

The following presentation is based on the analysis of teachers' responses to the question about what methods of provision they use specifically for mathematically gifted children. Some choices were given, such as 'skipping years', 'bypassing certain classes', 'early entry to the school', 'early exit from the school', 'differentiated tasks for children within classroom', 'grouping children by ability within class', 'setting', 'working with a mentor' and space was provided to add something different and explain it (see Appendix 2).

4.3 How schools address the needs of mathematically gifted children

Thirty-four schools, as previously mentioned, recorded that they make special provision for mathematically gifted children. Almost all these schools (33) provide differentiated work for more able mathematicians within classrooms and use grouping arrangements (see Table 4-4).

Table 4-4: Practices of specific provision for mathematically gifted children

n=44 (schools), n=34 (specific provision for mathematically gifted children)

METHODS	SCHOOLS
Differentiation	
Differentiated tasks within classroom	33
Grouping	
Grouping by ability within class	18
Setting	3
Pull-out groups	1
Grouping by ability within class and setting	9
Grouping by ability within class and pull-out groups for mathematically gifted children	1
Grouping by ability within class, setting and pull-out groups for mathematically gifted children	1
Acceleration	
Bypassing certain classes	4
Working with higher year groups	1
Extra support	
Working with a mentor	5
Out-of-class provision	
Outside classroom activities	14
Outside school activities	10
Other	
Challenge mornings once a term	1
Booster lessons	1
Use of commercial ICT programmes (e.g., RM Maths Learning and Abacus Talk Maths Programme)	1

It can also be seen in Table 4-4 that a small number of schools (five) use a mentor to work with these children. Many schools (24) offer out-of-class provision by giving opportunities for either during school hours (e.g., during the break time) or after-school activities (e.g., participation in national or local mathematics challenges and competitions). A few schools (five) use acceleration as a method of provision for their gifted mathematicians in a form that they allow gifted students to bypass certain classes and attend different mathematics classes with more advanced lessons in either special groups for gifted mathematicians (pull-out groups or top sets), or higher-year classes.

The type of grouping that most of the schools use appears to be “grouping by ability within classrooms”, while there are some schools (eleven) that combine this type of grouping with “setting” or/and “pull-out” grouping for gifted pupils (Table 4-4). Looking at the distribution of grouping arrangements by type (Figure 4-6), we see a strong preference, again, for “grouping by ability within classroom”, a small preference for “setting” and less of a preference for pull-out groups for gifted children. Other types of grouping, such as “streaming”, were not mentioned by any school.

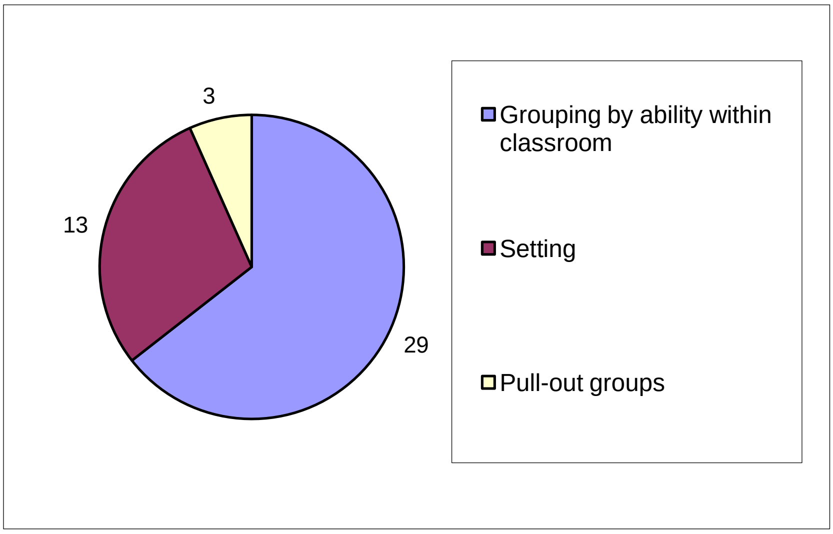


Figure 4-6: Distribution of grouping type amongst schools

4.4 Teachers’ classroom practices

This section examines the practices teachers implement every day in their classrooms in order to address the needs of mathematically gifted children. The presentation is based on the analysis of teachers’ responses about the organisational structures and teaching materials they use in their classrooms.

The following presentation is based on the analysis of teachers’ replies to the questions on whether they use extra support materials; from which resources; for whom (e.g., more able or less able pupils?); how often; and if not, why?

4.4.1 Extra support material and resources

Thirty-one out of 44 teachers use extra support materials in mathematics (see Figure 4-7).

n=44 (5 headteachers and deputy headteachers did not give information about classroom practices)

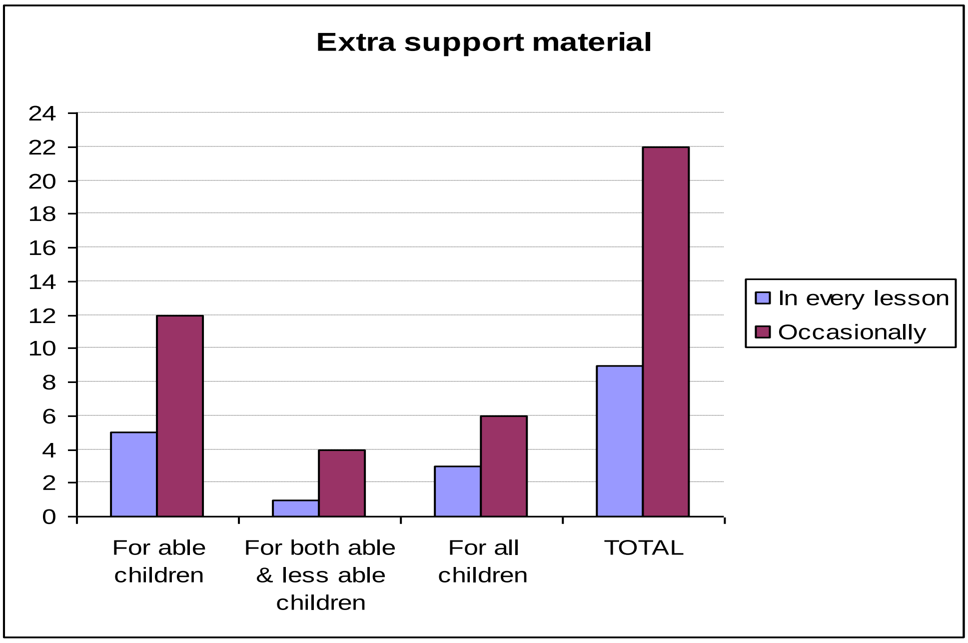


Figure 4-7: Use of extra support material

Taking into account that five teachers, who were either headteachers or deputy headteachers, did not give any information about classroom practices, we can say that only eight classroom teachers do not use extra support material. Twenty-two teachers use extra support materials occasionally and give most of them to more able students (Figure 4-7).

The favourite resources for the teachers, who use extra support material, appear to be those that teachers can find online, such as NRICH, Curriculum Online and Testbase, and publications referenced by the National Strategy (e.g., DfES/DCSF and QCA, see Figure 4-8).

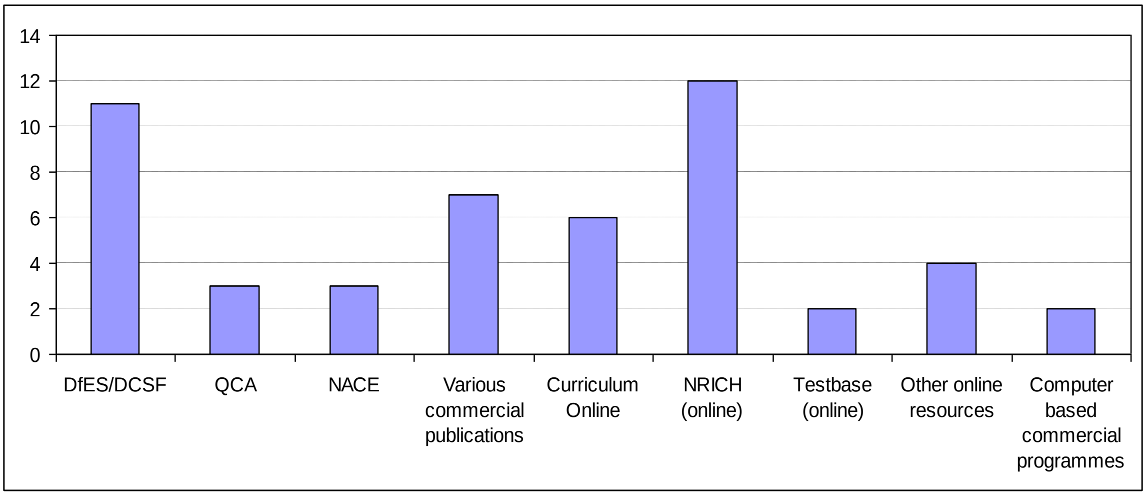


Figure 4-8: Resources used by teachers for extra support

The eight teachers who answered that they do not use extra support materials explained their reasons. Their reasons were categorised and are now presented in Table 4-5.

Table 4-5: Reasons for not using extra support materials

TEACHERS DO NOT USE EXTRA SUPPORT MATERIAL BECAUSE	TEACHERS
They use a wide range of materials (but did not specify which ones)	3
They routinely differentiate for all pupils, including gifted and talented	1
There is no time	2
They do not know where they can find appropriate resources or need greater support in sourcing material	2
TOTAL	8

It seems that half of them believe that they are already doing enough for their students: "[I] differentiate routinely for all abilities, including G&T", a mathematics co-ordinator and classroom teacher stated (School 16). There are some who feel that they are not able to find the appropriate resources without help ("Need greater support in sourcing material", a mathematics co-ordinator, School 29) or do not have enough time for something extra during the lessons.

The following presentation is based on the analysis of teachers' replies to the questions on whether they use grouping arrangements in their mathematics classes; how often; what kind of grouping; as well as if they change their groupings and why.

4.4.2 Grouping arrangements

Thirty-eight out of 44 teachers answered that they organise groupings within their classrooms (30 in every lesson and eight occasionally, see Figure 4-9).

n=44 (5 headteachers and deputy headteachers did not give information about classroom practices)

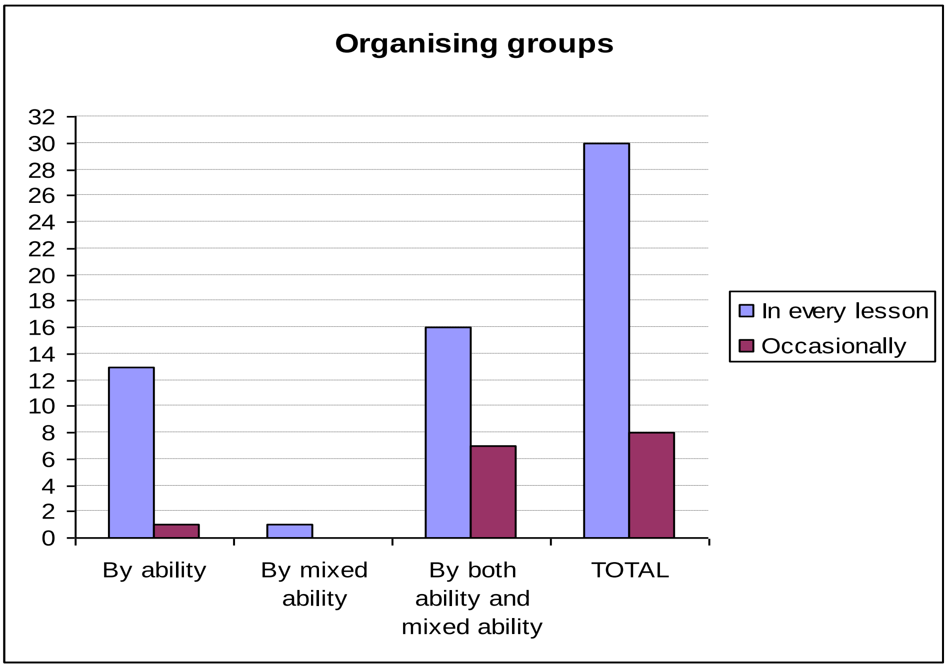


Figure 4-9: Teachers' grouping practices

Taking into account that five teachers (headteachers or deputy headteachers) did not give any information about classroom practices, we can say that only one classroom teacher does not use groupings within the classroom.

The majority of classroom teachers (23) appear to group their pupils in two ways, by ability and mixed-ability grouping. Fewer teachers (14) use grouping by ability as a sole method and only one organises his or her class solely in mixed-ability groups in every lesson (Figure 4-9). The latter teacher however appears to give differentiated tasks to more able pupils.

All 38 teachers, who use grouping arrangements in their classrooms, indicated that they revised and rearrange the groups throughout the year either by changing the type of grouping (e.g., from ability to mixed-ability) or by swapping the children between different groups. Table 4-6 presents teachers' explanations, categorised in groups, about the reasons that make them reorganise their groupings throughout the school year.

Table 4-6: Reasons for rearranging the groupings throughout the year

REARRANGE THE GROUPS THROUGHOUT THE YEAR	TEACHERS
Depending on the activity or the topic	14
Depending on children's progress*	15
Depending on both children's progress and the activity or the topic	3
In order for the gifted and talented children to act as peer-tutors in different groups or for all pupils to learn from each other	2
In order for the children to maintain confidence and achievement	1
Because when children change groups periodically, they work better	1
In order to enhance their social skills	1
No explanation given	1
TOTAL	38

* Two of these teachers referred to cases in which children make some progress and then go to a higher ability group, but not the opposite.

It can be seen that most of the teachers reorganise their groupings depending on the activity or the topic and their children's progress. There are a few teachers who swap their pupils between different groups either for more able children to act as "peer-tutors" to the others; for all the children to maintain confidence, achievement and social skills; or because they believe that children work better when they change groups periodically. Following is a sample of some teachers' explanations.

> When a child shows good understanding of all concepts taught or when he/she achieved a good score in tests. At another time, if a child shows signs of struggling to cope we will reduce the amount of stress by moving him/her to a lower group. (Mathematics co-ordinator, School 03)
>
> There are times children (G&T) need to be peer-tutors to extend their thinking and justify skills. (Mathematics co-ordinator, School 02)
>
> To take into account individual learning styles. Often, children excel in one area of maths while [struggling] in another, so grouping needs to be flexible. (Mathematics co-ordinator, School 42)
>
> When pupils are achieving very well in their own groups, they are given the work of the next group which raises self-esteem. If they have continued success, they can be moved to another table. If pupils are finding work too challenging in their group, movement is less likely, but work of a more appropriate nature is given. (Mathematics co-ordinator, School 30)

The latter explanation brings out another issue: How easy (or difficult) it is for a teacher to move a child from a higher-ability group down to a middle or lower-ability one. I decided to further investigate this by talking with selected teachers and studying their practices during the next stage.

4.5 Teachers' perceptions, attitudes and perceived needs

This section examines teachers' perceptions of the nature of mathematical ability and the most suitable term that denotes children with higher mathematical ability, teachers' attitudes towards teaching gifted mathematicians and provision that their school makes for them, teachers' needs for delivering effective provision for mathematically gifted children in their classrooms, and the possible need for further training and support. It is separated into four sub-sections: a) definitions and terminology, b) teachers' attitudes towards mathematically gifted children, c) teachers' attitudes towards the school's methods for addressing the needs of mathematically gifted children, and c) teachers' needs.

The next section is based on the analysis of teachers' replies to the questions asking to describe children with higher abilities in mathematics and identify the most suitable term to describe these children. Some choices were given, such as 'gifted', 'able', 'very able', 'exceptionally able', 'promising', and space was provided to suggest another term.

4.5.1 Definitions and terminology

Definition

Thirty-six teachers gave their own description of mathematically able or gifted children through the questionnaire. These descriptions were categorised into groups and are now presented in Table 4-7 in descending order, according to their popularity.

Looking at the top of Table 4-7, we can see that most of the teachers describe able mathematicians as those who are able to use a range of problem solving methods and strategies, who ask questions (some say higher-order questions), and who grasp new mathematical concepts quickly and easily. Table 4-7 also shows that many teachers consider mathematically able children those who are proficient in number calculation and mental maths, those who think logically, reason and explain their methods, those who see patterns in numbers and those who enjoy challenging tasks. We can also see the terms "methods", "problem(s)", and "quickly" appearing more than one time, which means that most of the teachers consider children's performance in problem solving and their speed in understanding, responding and completing tasks main indicators of mathematic ability.

Table 4-7: Characteristics of mathematically able children according to the teachers

CHARACTERISTICS OF MATHEMATICALLY ABLE/GIFTED CHILDREN	TEACHERS
Uses a range of problem-solving methods and strategies	12
Asks questions*	10
Grasps new mathematical concepts quickly and easily	8
Reasons and explains their methods	7
Able in number calculation and mental maths	7
Thinks logically	7
Sees patterns in numbers	6
Enjoys challenging tasks	6
Applies previous knowledge to different situations	5
Works above level expected for age group	5
Thinks, calculates and completes tasks quickly	5
Shows stamina and perseverance	4
Shows enthusiasm	3
Thinks "outside the box"	3
Eager	2
Keen to learn	2
Thinks of alternative ways to answer a question or find a solution to a problem	2
Good at working out problems	2
Loves to share their findings	1
Systematic	1
Displays strong spatial awareness	1
Enjoys the process to find the answer	1
Makes connections	1
Confident and precocious	1
Makes generalisations	1

* Three of those answered "higher-order questions"

Following is a sample of some interesting descriptions given by the teachers.

> Mathematically able children are those who enjoy solving mathematical problems, offer solutions, enquire about a mathematical situation, show enthusiasm and perseverance or those who explore challenging problems and communicate their findings clearly in both writing and speaking. (Mathematics co-ordinator, School 03)
>
> [Mathematically able children are] enthusiastic learners, enjoy challenging tasks, [are] good at using strategies for problem solving, [and are] quick at completing tasks. (Classroom teacher, School 08)
>
> [Mathematically able children are] able to grasp new concepts, spot mathematical relationships and patterns, [and] ask questions, which are followed through by further investigation. (Mathematics co-ordinator, School 10)

[Mathematically able children are] children who are able to see the process as a valuable journey in achieving the answer. These children also view the “journey” as one that can and should be altered to have a fuller understanding of the concept (Headteacher, School 31)

[Mathematically able children are] children who are working above the level expected of the average child in the year group…who are able to understand and use concepts with ease…who can apply their mathematical knowledge and skills to a variety of situations/problems. (Mathematics co-ordinator, School 38)

[Mathematically able children are] interested, [are] eager, want to know more, can be methodical and persistent, but sometimes they are the opposite and jump stages. They can quickly apply thinking. (Mathematics co-ordinator & deputy headteacher, School 44)

Terminology

Regarding the most suitable term that denotes children with higher mathematic ability, the majority of teachers (29 out of 44) answered that they prefer the term “able” with variations such as “very able”, “more able” or “exceptionally able” to denote children who show higher mathematical ability than their peers (see Figure 4-10).

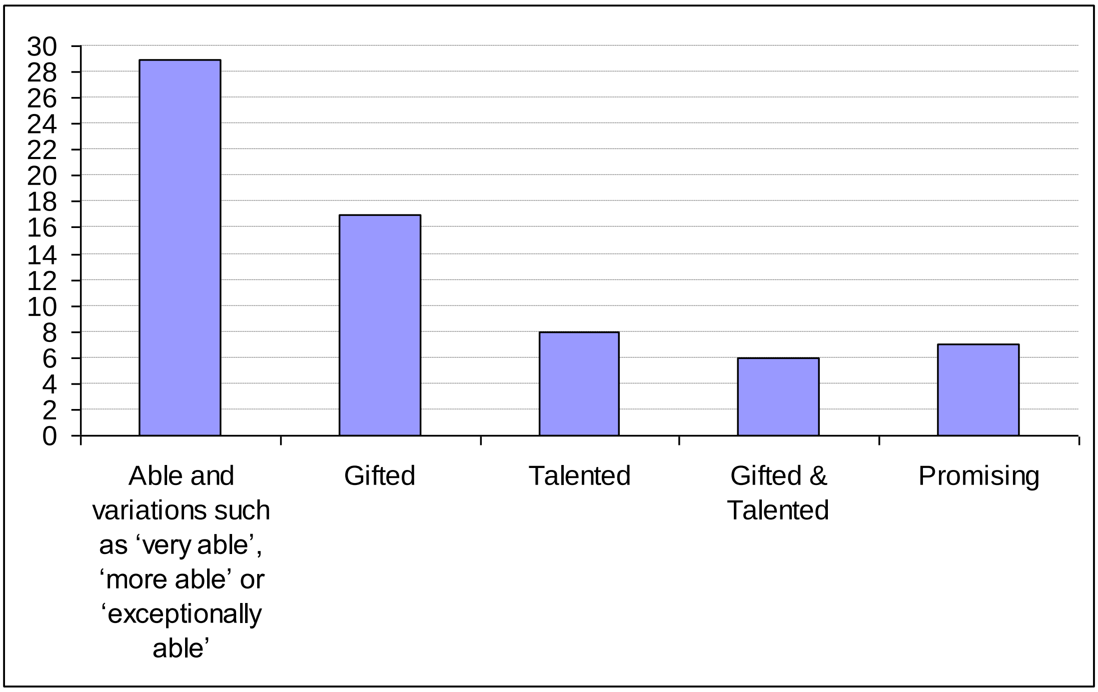


Figure 4-10: Distribution of terms used by teachers to denote children with higher mathematic ability

However, two teachers made a distinction between the terms "gifted" and "able" and one teacher made a distinction between "gifted and talented" and "exceptionally able":

> I think this phrase [mathematically able children] is more appropriate than "gifted", which suggests an extremely unusual ability that would make a child stand out from all his/her peers. [Mathematically able children are] able to grasp new concepts, spot mathematical relationships and patterns, [and] ask questions, which are followed through by further investigation. (Mathematics co-ordinator, School 10)
>
> Able: well above average consistently - Gifted: goes beyond this (Mathematics manager, School 24)
>
> Gifted and talented: highest achievers in their class - Exceptionally able: working at least one level n/c [national curriculum] above class (Mathematics consultant/co-ordinator, School 13)

The next section is based on the analysis of teachers' replies to the questions on whether they feel that the presence of 'gifted' or 'able' mathematicians in their classroom makes their work 'easy', 'very easy', 'neutral', 'difficult', or 'very difficult', and why. Teachers were also asked to indicate how comfortable they feel teaching mathematics to gifted children by choosing one of the options ranging from 'very comfortable' to 'very uncomfortable', and explaining why.

4.5.2 Teachers' attitudes towards teaching mathematically gifted children

Regarding the question about how easy or difficult their work as a teacher becomes when there are "gifted" or "able" mathematicians in their classroom, a first consideration of the teachers' answers shows that most of them feel that their work becomes neither easy nor difficult (20 teachers indicated "neutral") with the presence of some gifted children in their mathematics class. However, a noticeable number of teachers (15) feel that the presence of such children makes their work "difficult" or "very difficult" (see Figure 4-11).

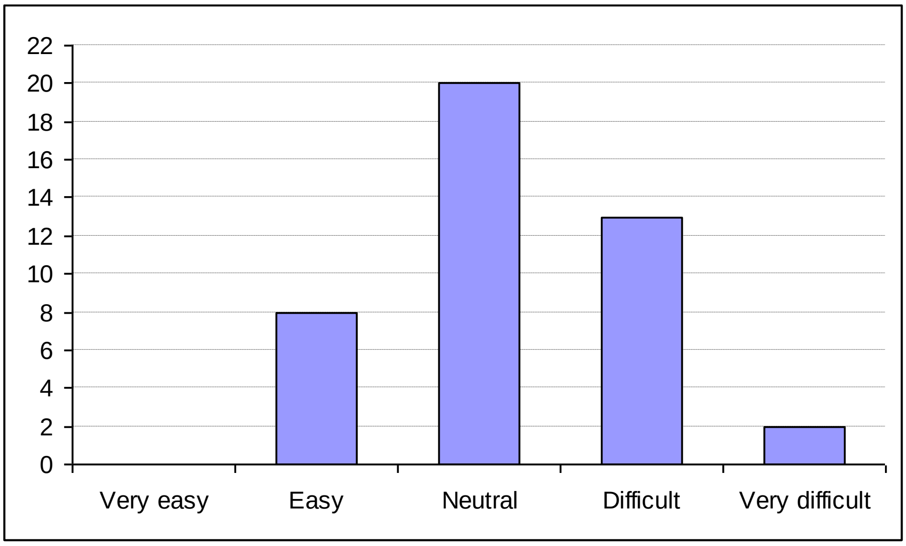


Figure 4-11: Levels of easiness or difficulty that teachers feel in teaching mathematics to a class in which there are some gifted mathematicians

It is very interesting to see how teachers, who chose the option "neutral", explained their answer in the follow-up open-ended question (i.e., 'explain why'). It seems that most of them, by choosing this answer, meant that this is "neither easy nor difficult" because catering for all pupils is a part of their everyday teaching for which they feel well prepared and confident, as the following answers show:

> It is a part (no more/no less) of teaching a mixed-ability class and trying to enable all children to achieve their learning potential (Deputy headteacher & G&T Co-ordinator, School 09)
>
> "I have to differentiate work accordingly anyway and as I am confident in teaching maths it doesn't matter whether a child is able or special needs." (Mathematics co-ordinator & Deputy headteacher, School 44)

However, there were others (five teachers) who, by choosing "neutral", meant that it is "both easy and difficult" depending on the range of levels of ability in the class and the number of the gifted children, as the following answer shows:

> "It depends on the range of the rest of the group and how many able children are promising/able/gifted" (Mathematics co-ordinator, School 02)

Or because as another teacher wrote:

> Easy: They act as mentors in group work; they raise the bar for the rest, especially the rest of the top group. Difficult: [It is] time consuming to find the right challenges for them. We are discouraged from going to standard textbooks of [the] year above. Specific resources may be too expensive (Mathematics co-ordinator & Classroom teacher, School 16)

Therefore, the initial picture represented in Figure 4-11 should be accordingly reframed (see Figure 4-12). Considering teachers' further explanations as a more accurate indicator of what they really believe, we can say that there are more teachers than initially appeared who believe that the presence of some able or gifted children makes their work difficult or very difficult.

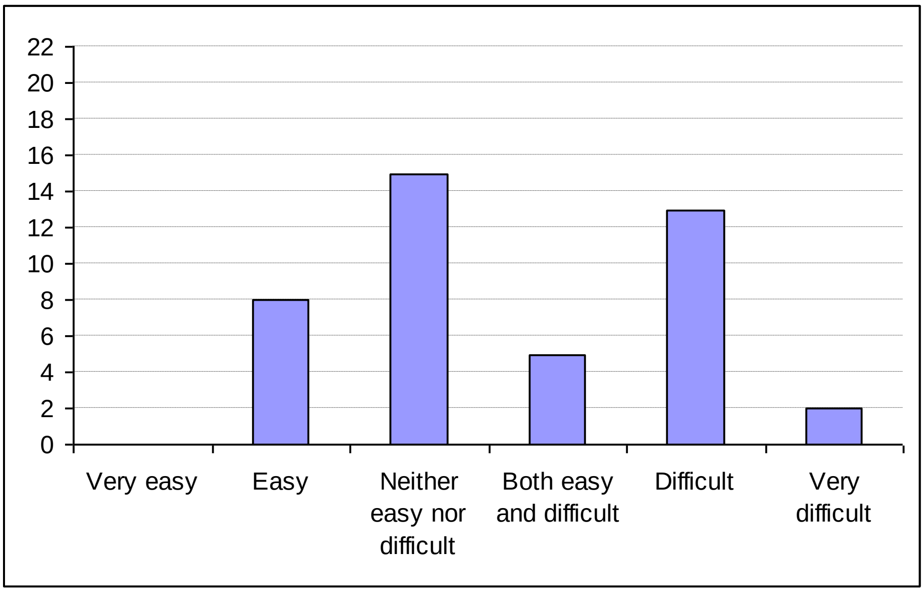


Figure 4-12: Levels of easiness or difficulty reframed

Regarding this difficulty, the main reasons mentioned were the extra work that the teachers need to do in order to find the right materials and plan challenging lessons for gifted children, the special needs that these children have and the need to cater for the less able children at the same time:

> [It is] time consuming to find the right challenges for them (Mathematics co-ordinator & Classroom teacher, School 16)
>
> Because you need to plan in order to truly extend them not just give them harder number sentences. (Mathematics co-ordinator, School 14)
>
> It is difficult to pitch work at a level suitable for them without confusing the rest of the group (Mathematics co-ordinator, School 38)

On the other hand, regarding the ease of teaching gifted children, the main reasons highlighted were that these children can be good role models that will raise the standards in the class; they can reach the expected targets easier and faster and act as peer mentors in group work:

> [They] provide good role models, but [it] can sometimes be difficult to ensure challenging and interesting tasks (Mathematics co-ordinator & Classroom teacher, School 25)
>
> Age related expectations are easier for them to reach. They can adopt new concepts/methods too much quicker. (Mathematics co-ordinator, School 01)
>
> I use them for peer mentoring across the groups in the year group or in the lower school. (Mathematics co-ordinator & senior manager, School 04)

Table 4-8 presents all the answers categorised in descending order according to their popularity.

Table 4-8: What makes teachers' work in a class with gifted mathematicians difficult or easy

IT IS DIFFICULT BECAUSE	TEACHERS
Teacher needs to work harder to find the right materials and plan specifically for gifted children	9
Gifted children need a special approach	4
Teacher needs to care about the rest of the class, including the less able children	4
IT IS EASY BECAUSE	
They can be good role models and raise the standards	3
They understand new concepts very quickly	2
They can act as peer mentors	2
They enjoy the challenges	1
The school implements provision	1

Teachers were also asked to indicate how comfortable they feel in teaching mathematics to gifted children. The analysis of their responses showed that the great majority of the teachers feel very comfortable or moderately comfortable, as Figure 4-13 indicates.

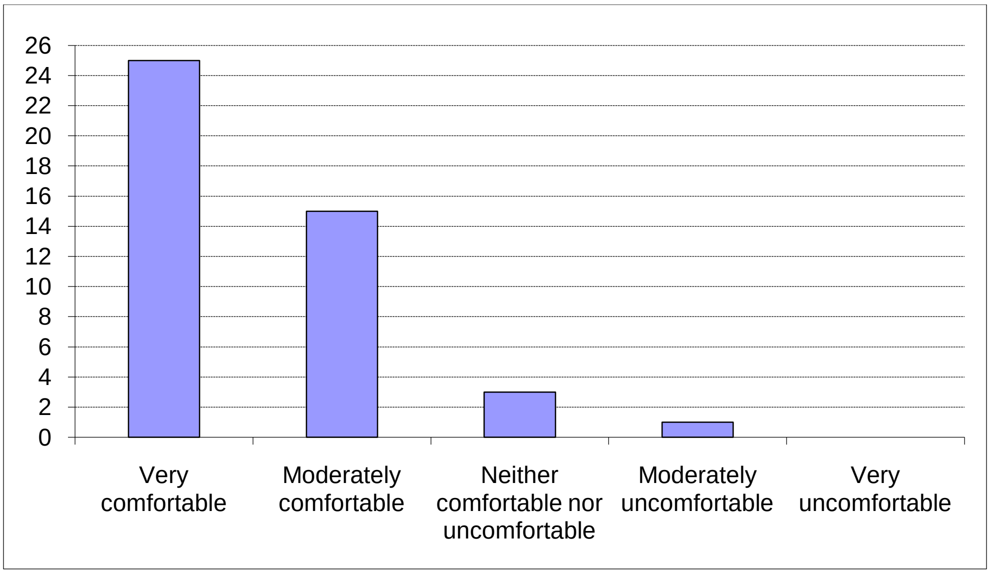


Figure 4-13: Levels of comfort that teachers feel in teaching mathematics to gifted children

If we now compare the findings from the questions about the ease (or difficulty) and the comfort against the training background of the teachers, we will see some very interesting findings, as Figure 4-14 indicates.

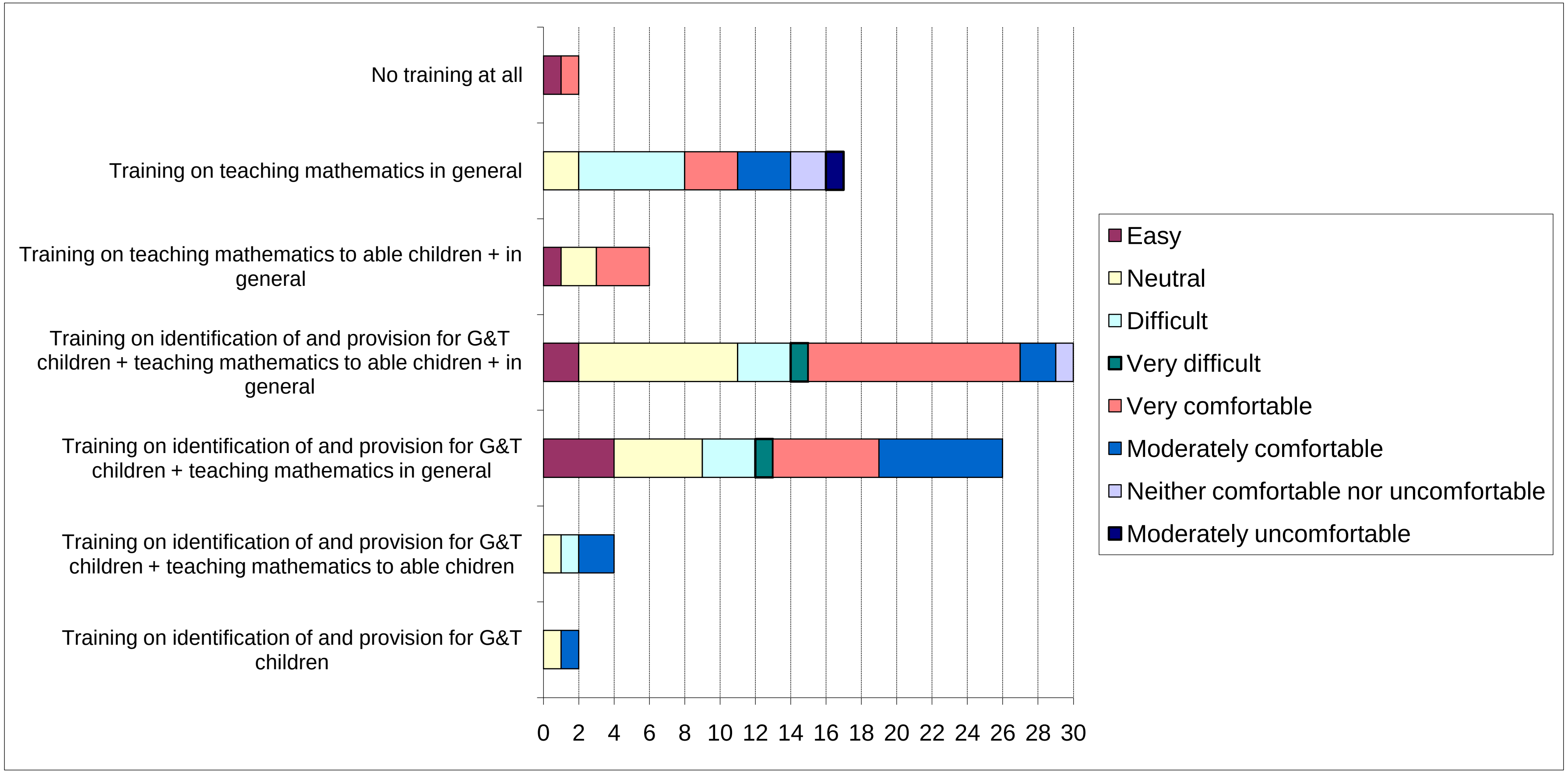


Figure 4-14: Teachers' attitudes towards teaching mathematics to gifted children compared with their training

If we compare the distribution of each answer on the level of difficulty and comfort with the total across the categories of training as they appear in Figure 4-14, we will see that the option “very comfortable” has been mostly chosen by teachers with training, not only in teaching mathematics in general, but also specifically in teaching mathematically able children and in the identification of and provision for gifted and talented children. Many teachers from this category have, also, chosen the option “neutral” (in response to the question “how easy?”). This means that teachers in this category may not see the work with mathematically gifted children as easy, but the great majority of them feel very comfortable in teaching them. This, in turn, indicates a greater level of self-confidence in teachers of this category, because, on the one hand, they recognise that meeting the needs of mathematically gifted children is not an easy task, but, on the other hand, they feel comfortable teaching them.

It is also interesting to see that when general training in mathematics is combined with specific training for mathematically able children, it gives better indicators of the level of comfort, independently of the levels of difficulty or ease, than when it is combined with training in identification of and provision for gifted and talented children in general. This indicates that specific training in teaching mathematically gifted children may yield better results regarding teachers’ ability to teach mathematics to gifted children than general training in identification of and provision for gifted and talented children.

The next section is based on the analysis of teachers’ replies to the questions that asked teachers to evaluate the methods their schools implement for the identification of and provision for gifted mathematicians. Some choices were given ranging from ‘very simple’ to ‘very difficult’ and from ‘very reliable’ to ‘very unreliable’, for the identification; from ‘very effective’ to ‘very ineffective’, for provision; and from ‘very well’ to ‘very poorly’ for the level to which the needs of gifted mathematicians are met. Space was also provided for further information.

4.5.3 How teachers evaluate their school’s methods for identification and provision

Teachers’ replies showed that most of them find the identification methods that their school implement “relatively simple” and “relatively reliable” (see Figure 4-15).

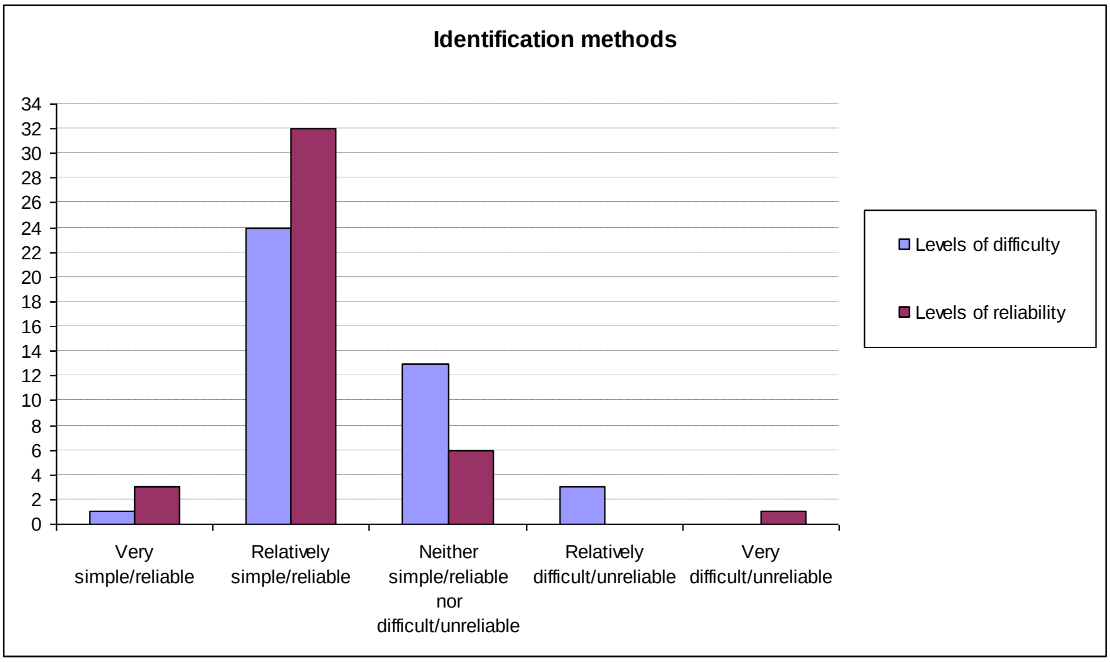


Figure 4-15: Levels of difficulty and reliability of identification methods

Most of the teachers also appear to believe that the provision their school offers for gifted children in mathematics is "moderately effective" and that they address the needs of mathematically gifted children "well" (Figure 4-16).

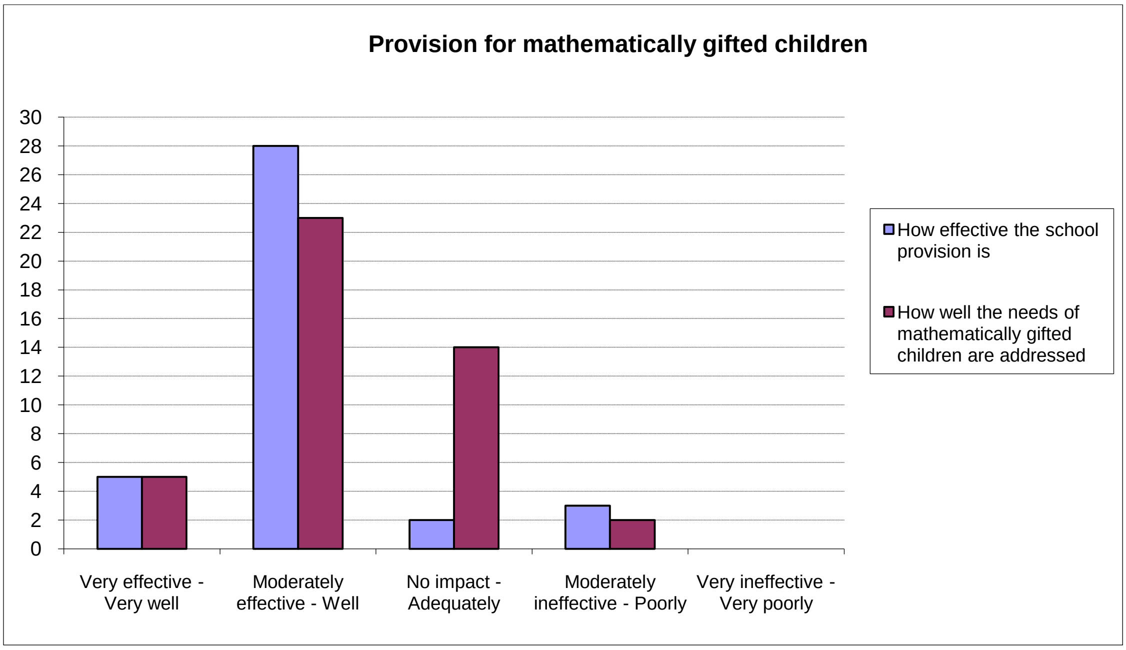


Figure 4-16: Levels of effectiveness and success of school provision for gifted mathematicians

Furthermore, teachers were asked to provide some more information about their school provision. Their answers showed that there are some few teachers who feel that:

- Provision for mathematically gifted children in their schools is an area for development:

> This is still an area for development. We intend to continue to employ a member of staff to specifically work with mathematically able children across age range[s] (e.g., combining year groups). (Mathematics co-ordinator & inclusion manager, School 10)
>
> I feel this is an area for development in this school, which I am addressing. (Mathematics co-ordinator, School 14)

- They need more exemplars of good practice and help:

> I would like to have some exemplars of good practice in other schools. Maybe you can help. (Mathematics co-ordinator & senior manager, School 04)

- The impact of provision is not the same for all children:

> Some classes [are] better at differentiation etc than others. (Mathematics co-ordinator & classroom teacher, School 25)

- The size of the school affects the ability to provide staff in order to address the needs of mathematically gifted children:

> We are a small school so [we] only one form entry. I find this affects how much we are able to provide/staff [are able] to provide etc. (Mathematics co-ordinator, School 36)

- Their school may not offer a very effective provision for mathematically gifted children, but in general, all gifted and talented children are very effectively catered to:

> Those who are G&T in maths are often the ones who are involved in sport, music, etc. This means they are thoroughly involved in lunchtime and afterschool activities for these too. This can be 3 times a week after school alone. Maths and Literacy have to compete with these so provision is definitely effective, but not really very effective. G&T pupils, as a whole, are very effectively catered for. (Mathematics co-ordinator, School 30)

- It is difficult to evaluate the effectiveness of provision:

> It feels the right thing to be doing, but how do we measure it? (Mathematics co-ordinator & classroom teacher, School 16)

The next section is based on the analysis of teachers' replies to the question, which asked if they feel that they need more support or training to effectively address the needs of gifted mathematicians and in which areas. For the latter, some choices were given, such as 'identification', 'provision in classroom', teaching materials', ‘monitoring children’s progress’, as well as extra space to add more needs and explain their answers.

4.5.4 Teachers’ needs

The majority of teachers (28 out of 44) indicated that they feel that they need more support or training to address the needs of mathematically gifted children more effectively. These teachers suggested one or more areas in which they would like to have more support and/or training. Table 4-9 presents these suggestions. It can be seen that most of the teachers appear to ask for more support in classroom provision and in teaching materials, while there are also many teachers who ask for support in monitoring pupils’ progress, in outside classroom provision and in identification of gifted mathematicians.

Table 4-9: Teachers’ suggestions for further support or training

THEY NEED MORE SUPPORT OR TRAINING	TEACHERS
In provision within classroom	20
In teaching materials	23
In monitoring children's progress	12
In provision outside classroom	11
In identification	9
In working with parents (of children both on and not on the register)	1
In groups/riots and clubs	1

Teachers were also asked to give any additional thoughts about the education of mathematically gifted children. The following presentation, therefore, is based on the analysis of teachers’ replies to the aforementioned open question.

4.5.5 Teachers’ additional thoughts

There were not many replies to this particular question. Only seven teachers answered and expressed their thoughts and beliefs. From their answers some issues important for the education of mathematically gifted children were highlighted. The following are some examples:

- Teacher’s background, qualifications and confidence level:

> A teacher's own background and qualification [are] of great relevance. Their confidence level is also significant in challenging the most able. (Mathematics co-ordinator, School 34)

- Staff stability:

> Training when staff movement is high means it can be difficult to keep high quality provision. (Headteacher, School 05)

- Reliable identification of the truly gifted children:

> The LEA has asked us to have a specific quota for G&T pupils. I worry this is artificial and would doubt that all those on the register are truly gifted. Gifted and able are not the same in my book (Mathematics co-ordinator, School 42)

- School budgeting and resources:

> Teaching of maths to gifted pupils is not difficult. The issue lies in budget, resources and knowledge, [and] teachers’ experience. Over time, teachers get better and more skilled in their teaching. Hence, it is at this point when able children’s needs are met depending on resources [and] budget. (Assistant headteacher, School 43)

- Parental involvement in identification and provision:

> [There should be] more about the involvement of parents/carers in identification and provision (quite rightly, really all parents/carers believe their children to be gifted and/or talented in some way!) (Deputy headteacher & G&T Co-ordinator, School 09)

The analysis of all the answers showed that the issues that concerned most of the teachers were firstly “teachers’ qualifications and background” and secondly “teachers’ confidence level” and “staff stability”. Figure 4-17 shows the distribution of each issue amongst all the answers.

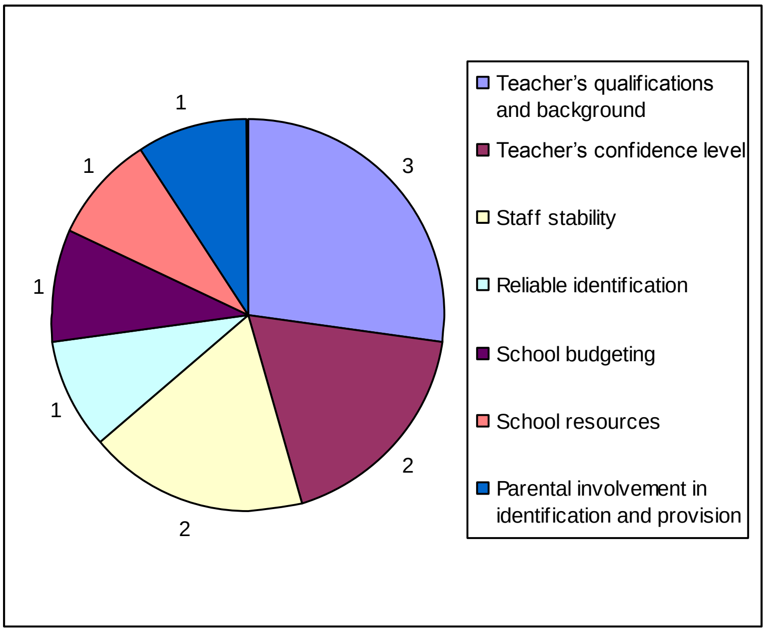


Figure 4-17: Important issues in the education of mathematically gifted children

4.6 Emerging issues

i. Who are the mathematically able or gifted children?

Most of the teachers who took part in this stage of the research described mathematically able children as those who display ability in solving problems, as well as those who quickly understand new concepts and respond to and complete tasks. Most of the teachers prefer the term "able" with variations, such as "very able", "more able" or "exceptionally able" to describe children with higher mathematic ability than their peers. However, some teachers made a distinction between the terms, considering "able" those who stand above the average of the class and "gifted" or "exceptionally able" those who go even higher or work at least one national curriculum level above the class.

ii. Existence of policy for identification and provision

Almost all the schools that took part in this stage of the research have a policy for the identification of gifted and talented children (43 out of 44). However, not all of those schools also have a policy on provision in general or in particular subjects. In mathematics, in particular, 34 schools appear to make specific provision for mathematically gifted children and 29 schools appear to have co-ordinators for planning and running programmes for gifted mathematicians.

iii. Methods for the identification of mathematically gifted children

It seems that a significant number of schools (24 out of 44) use a combination of "tests", "teacher assessments" and "nominations" for the identification of mathematically gifted children. Furthermore, teacher assessments and tests appear to be the favourite identification methods amongst schools. Teacher assessments, according to the teachers' responses, are mainly based on the observation of pupils' performance during the lessons and on assessment of pupils' written work. The tests that are mainly used for identification are standardised achievement tests provided by the National Curriculum (e.g., SATs and QCA tests), and almost all nominations come from the class teachers. Most teachers find the identification methods of their school "relatively simple" and "relatively reliable".

iv. The gifted and talented register

All 43 schools that have identification policies also maintain a register for gifted and talented children by following the guidance on the identification of gifted and talented learners of the National Programme for Gifted and Talented Education (DCSF, 2008a; DfES & NAGTY, 2006). Thirteen of these schools also keep a separate register for gifted mathematicians. Seven to ten percent and 4-6 percent of all children are on the register for gifted and talented children in general and gifted children in mathematics, respectively. It should be noted, at this point, that there are some teachers who feel that the children who appear on the gifted and talented register, which the Local Educational Authority asked them to keep, are "not truly gifted".

v. Acceleration

Acceleration methods for "rapid movement" from primary to secondary school, such as those discussed in Chapter Two, are not used by any of the schools that took part in the first stage of the research. The only type of acceleration that is implemented for the gifted children is that of learning new content at a faster pace than their peers through either special programmes or working with older pupils.

vi. Differentiation

All schools with policies for provision offer differentiated work for children within classrooms, something referred to as "differentiation" in Chapter Two of the literature review. I thought that differentiation practices were important for my study and decided to investigate them further at the next stage.

vii. Enrichment

There are some teachers who do not use extra support materials for their more able pupils. This is because they feel that they do not need to use anything different than what they already use (four teachers), cannot find suitable resources without help (two teachers), or that there is not enough time during everyday lessons for extra activities (two teachers). In contrast, most of the teachers (twenty-one) use extra support materials, which they mainly find through resources that are available online (e.g., NRICH) or those provided by the National Curriculum (e.g., DCSF/DfES and QCA). However, most of the teachers appeared to use these extra materials occasionally and not systematically for every lesson, as the relevant literature suggests for 'mathematics enrichment'. Furthermore, the question that needs to be answered is: What kind of materials do the teachers choose? (e.g., challenging open-ended problems and investigations, or activities that are easy and funny to keep more able pupils quiet in order for the teacher to work with those who have difficulties?) I thought this was also important for my study and decided to investigate it further at the next stage.

viii. Grouping structures

Most of the schools that took part in this stage of the research use "within-class ability grouping". Fewer schools use "setting", which was suggested in Chapter Two as most suitable for meeting the needs of mathematically able children. None of them use "streaming", which was discussed in Chapter Two as a method specifically developed for meeting the needs of gifted children in various academic areas. Some of the schools use another type of grouping, which involves small special groups only for identified gifted mathematicians in particular years (e.g., Year 6 or Year 5). These are referred to as 'Pull-out' programmes in Chapter Two.

ix. Movement between groups

All the teachers who use grouping arrangements in their mixed-ability classrooms indicated that they review and rearrange the groups throughout the year either by changing the type of grouping (e.g., from ability to mixed-ability) or by swapping the children between different groups. Some teachers swap the children between mixed-ability groupings to use the gifted children as “peer-tutors” in different groups. Others swap them because they believe that all the children maintain confidence, achievement and social skills, or work better when they change groups periodically. The main reason for moving a child between different ability groups appeared to be their progress, displayed through their performance in class, and their results in achievement tests. It seems, however, that there is a problem in moving a pupil from a higher to a lower-ability group within the year even when the child faces difficulties. Some teachers prefer to keep the child in the same group and give him or her different work. I thought that it would be very useful for my study if I further investigated grouping practices at the next stage. It would be interesting to see, for instance, how the “flexible grouping” works in practice.

x. Other strategies for provision

Many of the respondent schools (24 out of 44) also offer out-of-class provision by giving opportunities for either inside-school (e.g., during the break time) or afterschool activities (e.g., participation in national or local mathematics challenges and competitions). Few schools (five) offer additional support for mathematically able children by using a mentor to work with them. Moreover, there are some schools (two), which organise special lessons for able mathematicians periodically (e.g., ‘Challenge mornings once a term’ and ‘Enrichment weeks’).

xi. The effectiveness of school provision and the factors that influence its success

Most of the teachers appear to believe that their provision for gifted children in mathematics is “moderately effective” and that they address the needs of mathematically able children “well”. There are, however, some teachers who feel that provision for mathematically able or gifted children is an area that requires development. Some feel that it does not have the same impact on all children and that the effectiveness and quality of provision is mainly influenced by teachers’ personal background, qualifications and confidence level, as well as by the size of the school, its budgeting and the resources that it may provide to the teachers, and the stability and quality of the staff.

xii. Teachers’ attitudes towards teaching mathematically gifted children

Many teachers feel that their work as a teacher becomes difficult or very difficult when there are gifted mathematicians in their class, because they have to work harder to find the right materials and plan challenging lessons for these children or because it is difficult to have very different levels of ability in the class.

xiii. Teachers' training backgrounds and confidence levels

Teachers with training not only in teaching mathematics in general, but also in teaching mathematics to able children and in the identification of and provision for gifted and talented children displayed more confidence in teaching mathematics to very able children. Moreover, specific training for mathematically able children seems to have a better impact on teachers' self-confidence than training in identification and provision in general.

xiv. Teachers' needs

A large number of teachers (28 out of 44) feel that they need more support or training in order to effectively address the needs of mathematically gifted children. The areas in which teachers appear to ask for more support are mainly teaching materials and in-classroom provision. Secondly, teachers also appear to ask for support with monitoring pupils' progress, out-of-classroom provision, identification of gifted mathematicians, and working with parents.

4.7 Summary

This chapter presented and briefly discussed the findings from the questionnaire survey, which was the first stage of the research. I collected details about school policies, materials, resources and organisational practices from 44 primary schools within five Local Educational Authorities in Greater London. This gave me first insights about how primary schools in a large area of London address the needs of mathematically gifted children. Useful data were also collected about teachers' perceptions of who the mathematically gifted children are and their attitudes towards teaching them. The findings from this stage, however, will be further discussed in Chapter Six together with the findings from the next stage, the main study.

The analysis of the collected data helped me to focus on the next stage, which involved in-depth case studies of selected teachers and children identified as more able mathematicians. Details on how I conducted the next stage of the study and how I chose the teachers and the children are presented as the "main study" in the next chapter.

5 Chapter Five: Case Studies – The Main Study

The previous chapter presented a brief analysis of the findings from the first stage of the research, the preliminary study, which involved a questionnaire survey on how primary schools within five London Local Educational Authorities address the needs of mathematically gifted children. That stage aimed at collecting as many responses as possible from mathematics co-ordinators and/or classroom teachers in primary schools. The analysis of the findings helped me to acquire first insights into what methods of identification and provision the schools use and teachers' attitudes towards teaching gifted mathematicians. This also helped me choose the case study schools and the teachers for the next stage of the research, the main study, as I explained in Chapter Three. This chapter presents the findings from the main study of the research, from the four case studies. My aim is to present an accurate and complete picture of the four teachers, their perceptions and attitudes, their resources, their organisational structures and their methodology in teaching mathematics to more able and gifted children. I also want to present an accurate picture of the selected children, their perceptions and attitudes, their behaviour during the lessons, and their achievements.

Finding the best way of presenting the case studies took me a lot of time, as many reconstructions were needed. I wanted to present the best possible picture of the participants, their teaching practice, as well as pupils' performance and behaviour in the classroom in order to tell the story as realistically as possible. Therefore, it was important to include some of the descriptive parts based on my field notes. However, in order to make the presentation brief and to the point, only the parts of the lessons that help to illuminate the important issues considered relevant to the study are presented. These are the parts that show relevant organisational matters and aspects of teaching mathematics at higher cognitive levels, according to the theory presented in Chapter Two. Quotations and illustrations are also used in order to support the presentation. The same has been done with the data collected from the interviews; only the key issues regarding teachers' and pupils' perceptions and attitudes are presented with some samples of their responses.

I also felt that it would be better if there were uniformity in presentation of all the case studies. Therefore, taking into account the issues that emerged from both the literature review and the first phase of the research (as I explained in Chapter Three, section 3.2.9), I have chosen the following headings and order for all the case studies:

- Background
- School policy for identification and provision
- Teaching resources
- Summary of teacher's perceptions and attitudes
- Summary of children's perceptions and attitudes
- Description of lessons observed
- Children's progress
- Children's behaviour and performance in class

After the presentation of each case study, some initial comments and key issues relating to each of the cases are highlighted and briefly discussed. The main issues emerged from all case studies will be brought together and will be discussed in more detail in Chapter Six along with those that emerged from the previous phase of the research. These issues will be the main themes for discussion relating to the strategies for provision for mathematically gifted children presented in Chapter Two.

It should be noted that all the information presented here is based on the evidence that was available at the time the research was conducted in each school. It should also be noted that everything included in the presentation is what was considered pertinent by me only, in order to bring out as full and representative a picture as possible of the teachers and their methodology and the children and their behaviour in the classroom.

5.1 First case study: Emma's class

5.1.1 Background

Emma is an experienced teacher with ten years of teaching experience. She had her initial training in New Zealand, where, as she explained, much emphasis is placed on the education of gifted children. Emma has also attended training courses on identification of and provision for gifted and talented children and on teaching mathematics in general offered by the Local Educational Authority in the UK, where she is working. She has been working at this school for three years, all of them as a mathematics co-ordinator. This year, along with her responsibilities as a mathematics co-ordinator, Emma is teaching special small groups of mathematically able children from each year class.

The selected group of able mathematicians initially consisted of five children — three boys and two girls, all around ten years old (Table 5-1, section 5.1.5). However, one of the boys decided to leave the group later on because, as Emma explained, he found it difficult to cope with the lessons. That boy was withdrawn from the research as well.

The children of this group had been identified through a Qualifications and Curriculum Authority (QCA[2]) test at the end of the previous school year. Emma told me that she selected the children who achieved the highest results in that test:

> At the end of the year, so at the Year 4 they did a QCA test, which is basically a 45-minute test, maths test. I've got the highest results in the year groups and I've got the highest outcomes in a test.

5.1.2 School policy for identification and provision

The school has a policy of identification of and provision for gifted and talented children. Co-ordinators for science and mathematics are responsible for running identification and provision procedures. They identify gifted and talented children through achievement tests, usually QCA tests, which take place at the end of each term. They keep a general register for gifted and talented children, in which 6 percent of all children (this includes gifted mathematicians) are registered for the school year.

They also have a policy of provision for gifted students in mathematics. This involves pull-out grouping for more able pupils. The pull-out programme was organised for the first time this school year. The children had been identified through an achievement QCA test at the end of the previous school year and allocated in four special small groups: Year 2 group, Year 3 & 4 group, Year 5 group and Year 6 group. According to Emma, they have a plan for Year 1 able children to join Year 2 group after the first term. Emma, the mathematics co-ordinator, organises and teaches all the special groups. Each group works outside the regular class once a fortnight doing extra hours of mathematics lessons, additional to those they do in the regular class. The selected

[2] The Qualifications and Curriculum Authority (QCA) is a Public Body of the Department for Children, Schools and Families (DCSF). Its purpose is to support the secure delivery of the public exam system, as well as to develop and deliver high-quality national curriculum tests and assessments in collaboration with key stakeholders including schools, colleges, local authorities and awarding bodies.

children do five hours of mathematics per week in their regular class plus 45 minutes extra every two out of three weeks in the pull-out group. The lessons in the group take place on different days and at different hours, so the children miss different parts of the lessons from the regular class every time (but never mathematics or English, as Emma explained). Emma, furthermore, said that when these children are in the regular classroom, they do differentiated work from higher levels.

5.1.3 Teaching resources

Emma had answered in the questionnaire that she occasionally uses extra support materials, which she gives to both less able and more able pupils. In the interview, she explained that for this particular group of able mathematicians she regularly uses the DfEE handbook *Mathematical Challenges for Able Pupils in Key Stages 1 and 2* (DfEE, 2000a), which she considers “very useful for those more able children” and other online programmes such as the ‘NRICH’ programme. She said that she does “lots of research and searching for different resources” all the time and, because of this, she believes that her “resource knowledge is up to date”.

The materials that she used in the lessons I observed were worksheets produced by the school, photocopies from an educational pack of Bank of England and commercial leaflets (i.e., a shopping catalogue) (see Table 5-2, in section 5.1.6 for more details).

5.1.4 Summary of teacher’s perceptions and attitudes

About having able or gifted mathematicians in the class

Emma believes that mathematically able children may make the work of a classroom teacher easier, because:

> You can use very able children in your class to come out and explain in their own words how they have learnt something or concept on how they work out something else. You can also use more able children to teach small groups for small aspects...

Such a practice, she maintained, may help both more able and less able children improve themselves because “more able children learn more by teaching someone else” and, at the same time, when they explain their outcomes, they “help the other children with their understanding and…broaden the children who aren’t as able”.

Emma said that she feels very comfortable in teaching mathematics to able or gifted children and that she believes that she is competent "to cater for these children and extend them". She explained this as a result of both her initial training in New Zealand which, according to her, "is very much catered to [teaching] more able pupils", and her long experience as a mathematics co-ordinator, which, as she said, provided her with good knowledge of available resources.

About organising and teaching special groups and possible difficulties or problems

Emma recognised that there are difficulties in organising and teaching top ability groups and that even though she tried to select the top mathematicians only from each year group, there are cases in which some children cannot follow what the other children do in the top groups. She referred to children in the younger age groups and said that they cannot be easily moved from the groups during the year:

> I would say there are a couple of children in my younger groups, where a couple of them are not as good in maths as the others. That's slightly problematic, because they don't always understand the task, whereas the others do…I am giving them a bit longer to settle in, because I am not going to say, "You are not coming to the group anymore". But I have taken them off a list of who did well the previous year.

About the effectiveness of provision offered

Considering the differentiated lessons that all the teachers do in their classrooms and the pull-out groups that she organises and teaches, Emma said that her school addresses the needs of mathematically able children "adequately". Five weeks after these groups were organised, Emma appeared to believe that the children who took part displayed very positive attitudes towards mathematics. However, she appeared uncertain about the progress of these children at that time:

> At the moment their attitudes are extremely positive. All of the children are really keen to come out to the group to the more inspired and more interesting maths, because it's beyond just what they are doing in classes, extra stuff that is really making them think. So it is challenging them, whereas the lessons (the regular ones) are not necessarily always challenging them. But in terms of impact on progression, I don't know yet. I have only been doing it for five weeks. It's too soon to tell…They are really motivated; they're really enthusiastic about it. So it's definitely got their interest in maths up.

About further support or training

When Emma was asked what kind of support she would find helpful, she replied that it would be better if they had more support staff (adults) to work with more able pupils within classrooms. This is because sometimes the teacher does not pay enough attention to these children, as they usually find a lot of tasks easy, and this leads to these children not reaching their full potentials. She specifically said:

> The higher ability children often get forgotten, because they find a lot of tasks easier, but they don't find all tasks easy. It depends on the task. Sometimes they aren't stretched; sometimes they do what they can already do, but they aren't stretched get better at something…actually, they need some adult support to lead them in the right way.

5.1.5 Summary of children's perceptions and attitudes

Before starting this section the profiles of the children are presented (Table 5-1). This also includes their results in two consecutive assessments in order to have a more complete view of them. Their achievement is discussed later in section 5.1.7.

Table 5-1: The profile of selected children, School A

CHILD'S NAME	GENDER	AGE	1st ASSESSEMENT *End of the previous school year*	2nd ASSESSMENT *End of the first term this school year*
Emel	Girl	10	3B	3A
Ahmed	Boy	10	3B	4C
Daurama	Girl	10	3B	4C
Thanh	Boy	10	3A	4A

Mathematics as a subject

All four children perceive mathematics as numbers, sums, divisions and other operations:

> It is something to do with numbers. You can times and subtract and divide. (Emel)
>
> [It is] sums. (Ahmed)
>
> [It is] dividing and times-ing, adding and things like that. (Daurama)
>
> [It is] fractions, division. (Thanh)

In addition, they all had something positive to say about mathematics such as mathematics "is fun" (Emel, Daurama and Thanh), it is "enjoyable and most of the kids like it" (Thanh), it is a useful lesson to "get good jobs" (Emel), it is a useful lesson "for everyday life" (Daurama), or just "I like all of them [mathematics lessons]" (Ahmed). One of the children furthermore said that he usually does mathematics at home in his free time: "It is quite fun. I usually do it at home, when there's nothing to do." (Thanh)

The work that they do in the group

All four children agreed that the work that they do in the special group is more difficult or challenging than the work they do in the regular class. However, three out of four children said that they enjoy being part of this group because:

- They mostly work alone and there is not a lot of distraction as in the regular class, where there are children who "wait for me to do their question". (Emel)
- "In here it's a kind of make a bit of fun…we do tests, which are not real tests, but we do activities that involve maths and show how maths helps in everyday life." (Daurama)
- "I feel popular…It [the work in the special group] makes me more focused on what I am supposed to do." (Ahmed)

The fourth child, Thanh, agreed that they do more challenging work in the group and that this work "can be fun", but he appeared to be sceptical about the amount of the work they do in the group: "In the other class, they start their work quicker, so they get more done than us and in our class we stay doing one part quite a lot."

Work habits

Two children, Ahmed and Thanh, said in the interviews that they prefer working as a group to complete a task. Thanh explained this further:

> Because if you are alone and if you suggest an answer, it can be wrong. If you are in a group, they will all think different things and you can check if you're closer to the answer — if your answer is right.

Daurama appeared to agree that it is good to work in groups, but only for the difficult tasks and not for the simple ones where she prefers working alone:

> If the task requires group work, I like to work in a group. If it's possible to do it on my own, I like to do it on my own…If it's hard, I like to work in a group. It is because you can hear from others' ideas. If it is kind of easy, it's better to work alone.

In contrast, Emel seems to be certain that it is better for her to always work alone because she does not like being distracted by others:

> If I work with partners, they like to wait for me to do their question, because sometimes they do this with other people and that's why I get distracted…If I have a partner, sometimes this distracts me, and I don't get any work done; that's why I like working alone.

Making mathematics lessons more interesting

When the children were asked to suggest some ideas to make mathematics lessons more interesting, two of them had something to propose about either teacher's methodology (Emel) or the kind of activities in mathematics (Daurama).

Emel took the opportunity to complain about the way the teacher chooses who will say the answer every time. This does not allow her to show that she knows the answer: "When the teacher asks questions, sometimes when people don't put their hands up…then the teacher just waits…and you are really desperate to tell the answer, but the teacher picks someone else." Then she suggested a different way for the teacher to check children's answers: "The teacher could tell the class to say it out…or write it on the whiteboard and show it or just do it on your work."

Daurama suggested that mathematics would become more interesting if it was being linked with everyday life and children's hobbies. She suggested, for instance, maths problems that would involve swimming competitions:

> I think we should think of maths as everyday…like link it with your hobbies, because I like swimming, so maybe if you wanted to teach a maths lesson, you can maybe link it with swimming somehow…maybe like…I swam 15 miles and she swam other miles and we were racing…and make it more interesting.

5.1.6 Lessons observed in Emma's class

5.1.6.1 Organisation of the lessons

The selected children were gathering in the library, a small room with a block of four tables and a whiteboard. In all three lessons, the children were sitting around that block. Emma, apart from the time that she was writing on the board or offering individual help, was sitting amongst them. There were cases in which the children did not work in that room. Half of the first lesson, for instance, took place in the school corridors, where the children were moving freely, in order to find ideas to construct a "numeracy trail". The first five to seven minutes of the second lesson took place in a different room where the children watched a video about "money".

The lessons were shorter than regular classes (45 minutes), and Emma did not do a starter activity, but rather did a quick reiteration of the past lesson before she introduced the new one.

The three lessons that were observed, the materials, and the resources that they used are presented in Table 5-2.

Table 5-2: The lessons observed, the materials and the resources used in Emma's class

TITLE OF THE LESSON & LESSON OBJECTIVE	MATERIALS USED	RESOURCES USED
"Constructing a Numeracy Trail" Pupils had to find and pose their own problems, which would come from the school environment	Samples of 'Numeracy Trail'	School-produced worksheets
"Money & Prices" Pupils had to learn what money is, how it works and think about real-life problems involving money and prices	Video: 'pounds & pence' Activity card: 'Money & prices' A shopping catalogue	Bank of England (2008a) Bank of England (2008b) Market store
"Triangles, quadrilaterals & polygons" Pupils had to reinforce their knowledge about the properties of specific kinds of polygons, quadrilaterals (e.g., rectangular) and triangles (e.g., isosceles); learn the names of more polygons (e.g., heptagon, octagon, etc.); and be able to construct or draw particular shapes.	Pin boards and rubber bands A worksheet with dots for drawing shapes	Commercial products Worksheet created by the teacher

5.1.6.2 Aspects of teaching mathematics at higher cognitive levels

Analysis

In every lesson, Emma provided opportunities for analysis using challenging questions that demanded reasoning (e.g. 'Can you tell me why?'). In most of the cases, when children were asked to further explain some of their answers, they seemed able to provide explanations. Sometimes they were asked to explain the solution to a problem, such as in the following dialogue with Daurama, when Emma asked her to explain how she found the answer to the question: "How many minutes in 5½ hours?"

> Emma: Why did you multiply by 60?
> Daurama: Because one hour has 60 minutes.
> Emma: Why did you add 30 at the end?
> Daurama: Because half an hour has 30 minutes.

Synthesis

Emma provided many opportunities for synthesis in all three lessons observed by asking pupils to create their own problems, by changing the facts and creating new situations, by relating knowledge from several areas, and by using past knowledge to create new ideas, as can be seen below.

Creating their own mathematics problems

In the first lesson, Emma asked pupils to create their own 'numeracy trail' questionnaire and prompted them to take ideas from the school environment in order to do it. She then let them walk along the school corridors and, working as a team, look for ideas. The children then came up with their own problems, which they presented in the class, discussed, and, with the help of the teacher, categorised into three levels of difficulty (easy, medium, difficult).

Creating new situations

In the second lesson, Emma initially asked questions about a video – "What is money and how does it work?" (Bank of England, 2008a) – which the pupils watched. She asked them to explain what they would do if they were in the same position as the people in the story, but also what they would do in a different situation:

> What would you choose? …Why? …What if there was a cheque for £20?

Next, she asked questions about the use of money in everyday life and questions about imaginary situations:

> Could you imagine what would happen if there wasn't any money?
> …Imagine if you were a shopkeeper… What problems would you have?
> …How would you get paid for work?

The children were giving answers quickly; they seemed confident and were apparently enjoying what they were doing. They also easily completed a relevant exercise on paper (Appendix 7.1).

Generating knowledge from existing knowledge

In the third lesson, which was about learning and applying mathematical facts (e.g. properties of polygons) and skills (e.g. drawing particular shapes, calculating angles of polygons), Emma asked the question, "How many degrees are all the angles of a triangle in total?" When she saw that all the children remained silent, she attempted to show them a way to discover the answer using their previous knowledge. She drew a rectangle on the whiteboard, pointed at one right angle and asked how many degrees that angle was:

> '90o,' they all answered together.

She then asked them to calculate the total of four right angles, reminded them that a triangle is half of a rectangle, and asked them at the end to find how many degrees a triangle has in total:

> '180o,' Thanh said.

Evaluation

Emma prompted the children for self-evaluation and evaluation of others' work such that, in all three lessons, she was encouraging them to discuss their answers. In the third lesson, where children seemed to face difficulties in drawing specific polygons, Emma gave them more time. For instance, when the children finished an activity of drawing as many different polygons as they could, she suggested that they check their own work by rotating the sheets of paper on which they had drawn the shapes. This method proved very effective, as all the children found that many of their shapes were the same, but facing a different direction (Figure 5-1). She then asked pupils to pass their work to the others clockwise so that everyone would check what the others had done. At each round, Emma asked the children to explain their ideas. At the end of the lesson, she asked them to say how what they had learnt about shapes could help them in real life.

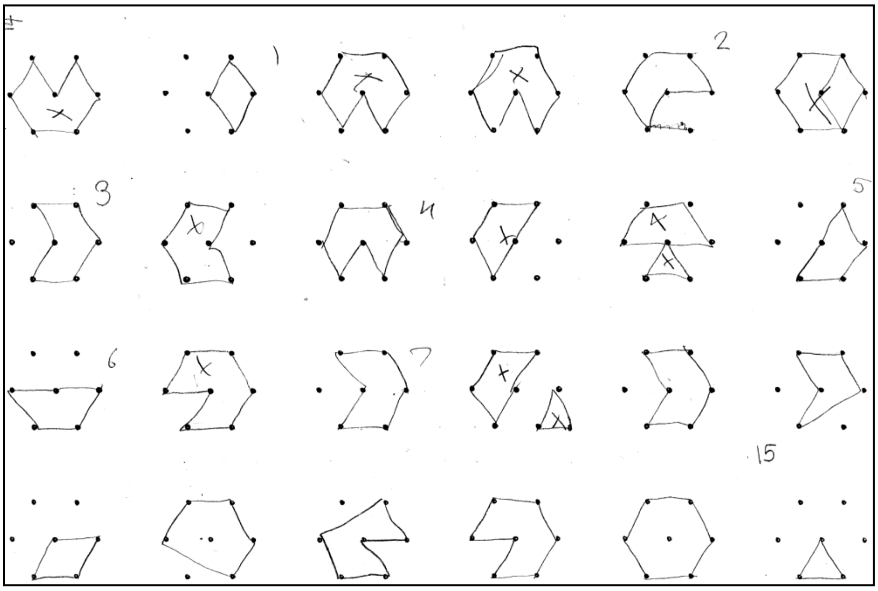


Figure 5-1: Emel's work, School A, Day 3

Monitoring, supporting and encouraging

In all three lessons, Emma was checking children's questions and answers, mediating when necessary. For instance, in the second lesson, Emma gave a shopping catalogue to each child and asked them to choose five presents for friends worth £50 in total. The children could choose any combination using a variety of products, but they could not exceed the given amount. However, half of the pupils seemed to have difficulties starting. Emma, then, helped them by asking:

> Imagine that you have only £50 to spend for five presents. How much do you think you can spend for every item on average?

And then she continued mediating by asking:

> How much so far? (to each child)
>
> Are you sure that you want to buy tuna for a present? (to Ahmed, when she saw what he had chosen from the catalogue)

In all lessons, Emma also seemed to have created a pleasant atmosphere, as she approached the children with a friendly manner and also with humour (e.g. 'You are cheap! Try to buy something good!' she told Ahmed when he chose a tin of tuna as a present to a friend) and had good communication and interaction with the children. She was also encouraging and praising them at every opportunity (e.g. 'That's a very good example!', 'Good idea!', 'Well done!', 'Fantastic!', 'Excellent!').

5.1.7 Children's progress

The results of children assessment (Table 5-2, section 5.1.5) showed that all four children made significant progress during the short period that they had been attending the special group. One of them (Thanh) had the most impressive improvement. It should be noted, though, that this child also had lot of practice at home, as he said in the interview.

The achievement of the selected children, which is presented in Table 5-2, is based on two assessments. The first (which was the main indicator, according to the teacher, for identifying these children) took place at the end of the previous school year and it was a QCA test, a standardised achievement test from Qualifications and Curriculum Authority (QCA, 2009) resources. The second took place at the end of the first term of the current school year and it was a GL Assessment from Granada Learning Group (GL Assessment, 2009) resources.

5.1.8 Children's performance and behaviour in class

Children's performance during the lessons was not always high, while diverse abilities amongst the children were observed. For instance, two of the four children (Emel and Ahmed) demonstrated a lack of knowledge about basic facts related to polygons and triangles in the last lesson and a lack of skills in number calculations (especially Ahmed) during the previous two lessons. Also, the overall in-class performance of these two children did not involve participation in discussions about a problem or a solution, despite the small size of the group which facilitated more interaction between them and the teacher. Their participation was restricted to answers to some simple questions only. Additionally, Ahmed had difficulties, in many cases, to understand the questions. He therefore needed Emma's help every time to complete his tasks. In contrast to these two children, Daurama and Thanh both displayed reasoning skills in many cases and showed knowledge about real-life problems beyond the school books such as the ‘credit crunch’, and they had answers to the questions about ‘Who makes the cheques?’ and ‘Who puts tax?’

All four children seemed to enjoy the activities that they did, especially constructing a ‘numeracy trail’, which took place outside the classroom, and those activities connected with real-life problems relating to ‘money and prices’. In all activities, they worked individually, even in the first where they were supposed to work as a team to find appropriate problems with correct answers.

The following section presents an initial analysis of what I saw in the lessons and what I heard from both the teacher and the children. These are my first perceptions and interpretations. These findings will be discussed in more depth in Chapter Six.

5.1.9 Initial comments on the first case study

Provision for mathematically able children in School A is offered through pull-out groups comprised by pupils of the same age who are identified through QCA tests. The selected pupils are offered extra lessons, additional to those they do in the regular classroom.

The lessons observed in the Year 5 pull-out group were different from the usual lessons, as Emma had said in the interview. Emma chose the activities from a variety of resources, including occasional materials such as commercial leaflets, worksheets created by herself and supporting materials such as pin boards and rubber bands. The activities were not difficult or complicated, but they involved investigations, real-life and open-ended problems appropriate for challenging able mathematicians, as suggested by the literature discussed in Chapter Two. Emma also appeared aware of available resources and the methods of challenging pupils’ higher-order thinking. Pupils were given the opportunity to be engaged in higher-order levels of thinking through higher-order questioning (e.g., “Why?”, “What if…?”, “Imagine if you were… how…?”), problem-solving, communicating, refining ideas, creating their own mathematical problems and attempting evaluations.

In all three lessons observed, Emma appeared very comfortable in teaching the group and well prepared for the lessons. It seems that both her experience as a mathematics co-ordinator and the subject-specific training (in both mathematics and gifted education) helped with this. In addition, the small size of the group seemed to make her work easier and more effective. Emma herself had acknowledged that in the interview where she had said: "When I work with my children in a small group, they learn loads".

All the children, after four months of working in the special group, made significant progress, according to their assessment at the end of the first term. Even Ahmed, who seemed not to cope well, moved from 3B to 4C, something that, I should say, surprised Emma as well, when she was reading the results to me.

5.1.10 Emerging Issues

- The children were identified using a single QCA achievement test, from which Emma chose the higher results. This seems not enough to identify the truly able mathematicians. For example, one of the selected children left the group after a month, after my first observation, because, as Emma said, he found it too difficult. Emma also said in the interview that the main difficulty in teaching special groups is that there are some children in these groups who do not have the same abilities as others and cannot follow what the others do. The observation of the lessons also found differences in pupils' abilities, as two of them showed lack of knowledge about basic facts related to polygons and triangles and lack of skills in number calculations.
- Changes in grouping arrangements seemed to pose difficulties when children needed to be moved out of the group. For example, in the interview, Emma said that even though she knows that a child has difficulties and cannot follow the group, she does not feel comfortable taking him or her out of the group during the year, but prefers to wait. However, this seems to affect the pace of the lessons and makes others, like Thanh, feel unsatisfied because they stayed on one part for too long.
- During the activities, pupils were working alone, which was something that seemed to keep them satisfied. Some of them also appeared in the interviews to prefer working alone unless the activity was very difficult. Emel, in particular, had made clear that one of the reasons she preferred working in the special

class rather than in the regular one was that there were no other children around who were just waiting for her to find the answers.

- All four pupils revealed positive attitudes towards mathematics and the lessons in the special group. The observations of their lessons also found that the children were working with a high interest in all activities and that they especially enjoyed those involving real-life problems, something that is amplified by Daurama’s suggestions about more mathematics problems that are connected with children’s hobbies.
- All four children appeared, in the interviews, to perceive mathematics as ‘sums’, ‘divisions’ and other operations. One of them also added that mathematics is a useful lesson to “get good jobs”.

I used Emma’s case study as a pilot for the rest of my studies. This helped me collect more data from my observations, because, on the one hand, I had more experience in observing and keeping field notes and, on the other hand, I learned that I could fill some gaps in my observations by talking with the teacher before and after the lessons.

Now I will present the other three case studies.

5.2 *Second case study: Sarah's class*

5.2.1 Background

Sarah is an experienced teacher and mathematics co-ordinator. She has been teaching for 15 years, all of them in this school. The last 11 years, she has been a mathematics co-ordinator. She has attended an in-service training course on identification of and provision for gifted and talented children as well as courses specifically for mathematics co-ordinators provided by the Local Educational Authority. This year, along with her responsibilities as a mathematics co-ordinator, Sarah is teaching a Year 2 class (a regular mixed-ability class) with 29 pupils. In parallel, she is studying for a Masters Degree in Education at a London university. At this point, it should be noted that Sarah was not amongst the teachers who took part in the first stage of my research (the questionnaire survey). The deputy headteacher of the school and co-ordinator for gifted and talented children was the one who responded to the questionnaire and gave me the first picture of their school and their policy. I also met with him first, and then he introduced me to Sarah, who, I should add, was very enthusiastic about the research and keen to participate.

Five children were selected — four boys and one girl, all around seven years old (see Table 5-3, section 5.2.5). They were suggested by Sarah as being mathematicians that were more able. Sarah made clear that they were "not necessarily gifted", but just better in mathematics than the others in the class were. The identification of these children was "purely based" on Sarah's own judgement. As she explained, she took into account:

> whether the children are willing to talk about maths, stand up, give explanations, say what they did…children who can make it in their own way; they don't always stick to what I tell them to do; they are secure enough to say, "I can do it in this way"…children who tend to ask questions.

5.2.2 School policy for identification and provision

The school has a policy of identification of and provision for gifted and talented children. In order to identify their gifted and talented children, they use, according to their policy, a range of sources, such as achievement tests (e.g., SATs and QCA tests); teachers' nominations (both current and previous teachers); child tracking and assessment of work; parental information regarding gifts, talents and out-of-school activities; and discussions with children. They keep a general register for gifted and talented children that they review every year. The year that the research took place, 8 percent of all children were placed on the register (this includes gifted mathematicians). The decision about who will be in the register, according to their policy, is made by the school after parental consultation.

However, the latter idea is not welcome by the deputy headteacher, who wrote in the questionnaire, and repeated in our discussion, that the biggest problem regarding identification of and provision for more able children in the school is the parental involvement and that parents often believe that their children are gifted: "...really all parents/carers believe their children to be gifted and/or talented in some way!"

In practice, and according to what Sarah said, it seems that the identification of mathematically gifted children is merely based on teachers' assessment — as it happened in the case of the five children mentioned earlier — and on achievement tests. Sarah, for instance, talking about how she is generally assessing her pupils explained that she uses formal assessment tests. These tests are carried out every term and they are different in nature each time. The first is about mental maths, the second is about 'using and applying' and involves problem-solving tasks and tests on the AT1 skills (National Curriculum Attainment Target 1: using and applying skills), and the third is the Standard Attainment Tests (SATs) provided by the National Curriculum, which assesses the overall work during the year. Children were also given an unaided task at the end of every Unit (Unit A, B, C, D and E, according to the National Curriculum) or at the end of every two weeks in order. As Sarah said, this was for the teachers to be able to "judge whether they've got the concept or whether they can apply those skills" and be sure that they "got the children right".

The school has subject co-ordinators who run particular programmes in sports, music and foreign languages for gifted and talented children and one general gifted and talented co-ordinator who has the special responsibility of co-ordinating the identification of and provision for gifted and talented children. That person, at the time of the research, was the deputy headteacher.

Provision for mathematically able children is offered through the regular classrooms. Their policy of provision suggests that all children should be provided with a challenging and enriched curriculum and differentiated work. Parents are also suggested to be involved in delivering enrichment activities as a part of homework and in special projects, which are carried out within classrooms.

Their policy includes guidance for classroom teachers on how to identify children whose achievement is above average and organise for them an individual programme of open-ended work assignments and challenging activities for enrichment and extension such as investigations, problem-solving, codes and quizzes. A mathematics co-ordinator — in this case, Sarah — is responsible for advising and keeping all the teachers up-to-date with available activities to support teaching of gifted and talented children. Teachers are also encouraged to organise flexible groupings by ability within the classroom, where the children can move between groups. The school, according to the deputy headteacher, is currently exploring the possibility of introducing 'setting' as an ability-grouping arrangement for mathematics and literacy in the future, depending on their budget.

5.2.3 Teaching resources

Sarah said that she uses resources that she can find in school or on the Internet. The school, she explained, does "not have a scheme to follow or something like that". Therefore, resourcing is a part of her planning and, as she said, she always tries to provide each ability group with different resources and suitable differentiated work. She explained this as follows:

> When we do planning [of] our lessons, we plan for differentiated lessons. We plan what we are going to do with the middle achieving, what we are going to [do to] support lower achieving, [and] what we are going to do to extend the higher achievers. So, we do through planning, we might do it through resourcing — so how to give them different resources. Depending on their reading ability, perhaps you give them more investigative type of work and problem-solving type of work.

In the lessons observed, Sarah used materials that she selected from past SATs questions and from resources for older children (age 11, level 1) through both the National Strategy and commercial resources (see Table 5-4, in section 5.2.6 for more details).

5.2.4 Summary of teacher's perceptions and attitudes

About having able or gifted mathematicians in the class

Sarah believes that having some very able mathematicians in her class makes her work as a teacher neither easy nor difficult. This is because able children need differentiated lessons as much as the less able children and, as she explained, she always differentiates the lessons for all of the children. Furthermore, she said that she finds it easier to work with able children than the less able ones: "I think teachers generally find it easier to extend an activity to the higher achievers rather than trying to support the lower achievers… [It] is naturally easier to extend it."

Sarah explained that she feels very comfortable in teaching mathematics to very able children, because, on one hand, she likes mathematics and, on the other hand, she has attended many courses every year as mathematics co-ordinator:

> It is because I like it; I like maths. So, that is number one and number two, I have done so many courses and, as a maths co-ordinator, every year I am going on courses. So, I've got a lot of knowledge. I think that's all.

About organising and teaching ability groups within the regular classroom and possible difficulties or problems

Sarah organises her pupils into three ability groups: the lower achievers, the middle achievers and the higher achievers. As she explained, "children can move between those" groups depending on the learning objective and may form pairs of mixed-ability as follows:

> At some tasks, maybe in one pair [there would be] a child who is a good reader but not necessarily a good mathematician; someone may be good at maths but not good at reading. So, they might be paired up in that way…

The ability groupings are re-evaluated and reorganised every half term (every six weeks) after either formal assessments, as they were earlier described or informal observations during lessons. At the time that the research took place (during the second term), Sarah had changed the groups once. She explained this as follows:

> [A]s you go through the year you might notice some children have a learning spurt, so you might move them even though you haven't moved anyone else around or you might find that children perhaps in a way missed out and they are pulled back in some other group for support.

According to Sarah, the problems that sometimes occur in groupings have to do with the wrong judgement of the teacher, who may put pupils in the wrong groups and the kind of talk that the children are doing in the group. Sarah explained this as follows:

> Perhaps the wrong personality is sitting near each other and they are doing confrontational talk rather than exploratory talk… [T]hen the only other difficulty might be…perhaps you misjudged someone and so you weren't aware of a particular skill they had in an area or you thought they did have a skill and they don't. So, that's a usual complication to come across.

In the case of regrouping, sometimes there are problems from parents when a child has to change groups and move to a lower one. Sarah had such experience in the past year with one parent who was angry and complained because her child moved to a lower-ability group within the class:

> [H]er mum was very upset. She couldn't understand how her daughter's ability had changed that those six weeks. I was trying to explain that's not the ability as the experience and that she had been ill and we needed to support her. She went to the headteacher and complained about it…She didn't talk me for the rest of the year, she was very unhappy...

However, Sarah believes that the children themselves always know who is good at maths and what each pupil's place in the class is:

> I think children definitely know where they are in the class. They know who is good at maths; they know who they think isn't. This does not mean that they are correct, but they know where they sit in the hierarchy of the class, even when we change the name of the group…whatever, they are not cheated; they know that.

About the effectiveness of provision

Sarah believes that her school addresses the needs of mathematically able children "very well" and that she is preparing them for independent learning. She also believes that these children need not just more work but challenging work, which makes them think. She concretely said:

> I think the most important aspect is to not necessarily expect them to do more work just because they are better at maths, but to give them work that requires them to think, to make [it] a bit challenging and…not necessarily going to get the right answer…I try to prepare them for independent learning, which isn't just a page of sums or isn't just more than everyone else; it's using the knowledge and telling me what the child does.

About further support or training

When Sarah was asked what kind of support she would find helpful, she replied that she would not ask for more support or training regarding the education of gifted mathematicians, but that she would like to have:

a) More time to prepare the lessons rightly, because:

> I think time is a great problem for teachers because we spend all the evening at school preparing lessons for the next day and you can't do that physically. So, that is number one…

b) More support staff in the class to support the children in order to do more challenging work, because:

> [I]f you are giving those children challenging work that means you then have to be around even if it is just to help them a bit into the activity or to say something that makes them think something else, and I think in that case, you have to give the rest of class stuff they can get on by themselves. Similarly, that group has to have stuff that they can get on by themselves. So, you cannot always make those lessons as challenging as you might have wanted, because physically, you are not able to be around to support them.

5.2.5 Summary of children's perceptions and attitudes

Before starting this section the profiles of the children are presented (Table 5-3). This also includes their results in two consecutive assessments in order to have a better picture of them. Their achievement is discussed later in section 5.2.7.

Table 5-3: The profile of selected children, School B

CHILD'S NAME	GENDER	AGE	1st ASSESSEMENT	2nd ASSESSMENT
Alvin	Boy	7	2B	2A
Nevil	Boy	7	2B	2A
Amy	Girl	7	2B	2A
Jake	Boy	7	2B	2A
Jason	Boy	7	2B	2A

Mathematics as a subject

Talking about what mathematics was about was not easy for all of these children, maybe because of their age. I had to find different ways to ask the same question (e.g., "What does mathematics mean to you?" or "When you hear the word 'mathematics', what comes to your mind first?" or "What makes mathematics different from other subjects like history, for example?"), be encouraging (e.g., "That is very interesting!") and prompting (e.g., "Like?" "But you can't really give me an example?").

The analysis of children's responses showed that four out of five children perceived mathematics as "numbers". Alvin explained this further: "Well, it's about writing numbers. They are numbers. You need to know the numbers properly…numbers, like three, eight in the times table." And Jason added that maths is also signs, meaning the arithmetic operations: "[Mathematics] includes numbers and includes signs… signs, like divided by, subtractions, times tables."

The fifth child (Jake) was not able to describe what mathematics is and he kept saying, "I don't really know".

In addition, three out of five children responded that they like mathematics and enjoy the lessons:

> I like the lesson. (Nevil)
>
> [It's] fun… Because I like number work and I like doing learning objectives in maths. (Jake)
>
> I have lots of ideas and very good ideas in class while we are doing maths… I'm quite happy that I have good ideas… (Alvin)

The other two preferred to speak about the level of difficulty, saying that mathematics "it's difficult" (Amy) or "quite easy, but sometimes difficult" (Jason).

The work that they do in the class

When the children were asked to talk about the work that they do, they said that sometimes they do difficult tasks and that they have to try really hard:

> It's quite hard… sometimes it's quite easy… you have to really try… you have to practise. (Alvin)
>
> [It's] not easy, but quite easy. (Nevil)

> I feel that it's difficult. (Amy)
>
> Sometimes I struggle with them; sometimes it's quite easy. (Jake)
>
> It's quite easy for me. The only thing I find hard is times tables. I find some times tables easy, but some hard, and I find some divided bys easy and some hard. (Jason)

The 'difficult' work — which, as Sarah had said, comes from a higher level — seems to sometimes make some children, like Jake, feel uncomfortable or sad:

> Sometimes it's happy; sometimes it's sad… One time, I felt a little bit sad when Mrs. Sarah gave me some other people's work that I thought that I might be able to do, but the work she gave me was quite hard… I had to have her coming to me all the time. I didn't like it too much. (Jake)

Work habits

Alvin and Jason said that they prefer working alone unless they work with the peers that they want. They both agreed that working with other children who are good at mathematics is nice, but they both complained that when they work as a group, no one from the group helps them. Alvin, for instance, explained this as follows:

> I am usually working on my own, because I am quite good at working on my own. When I am working on my own, I have quiet and nice interactive bits because I use my own ideas, but with a partner, I sometimes have to use their own ideas, but I've got very good ideas usually… I am good on my own, because usually when I am working with someone else, I am actually telling them what to do. I'm doing all the work and then I let them copy me… When I had Jake yesterday and times tables, he was quite quick, so, all I had to do was check which ones he was doing and then I done [sic] it with him and then we just finished in the last seconds… It was quite nice, because when I was with Jake and Jason [two pupils of the higher-ability group], we all actually worked together… When they know the answer, I ask them, but they didn't [sic] usually actually tell me…

Jason also replied:

> I like working alone really... Well, there are still people on my table. I like working like that. I like working with people on my table, but nobody is helping me. That's why I like working…because sometimes I work with somebody that I don't really like and nobody likes him… I am happy when I am working with one of my friends.

The other three children said that they prefer working with other children as a group because:

> It's quite hard doing things on your own. (Jake)

If I get stuck on a number, the other person might get it. (Nevil)

I just don't like being alone. (Amy)

Making mathematics lessons more interesting

When the children were asked to suggest ideas to make mathematics lessons more interesting, two of them (Jake and Jason) suggested more exercises with arithmetic operations that they enjoy, such as "times tables" (Jake) or a range of operations: "…add a certain number and then take away a different number" (Jason).

Alvin suggested harder operations in combination with easier ones and the following process:

> I think to make maths more interesting you like to start from quite hard ones and then make them much…easier. Like you start from sixty-five take away thirty, now they [sic] would be thirty-five, and you start with the hard ones and go down. So, you try your best with the hard ones then you get to relax a bit after. You need to try more…like a bit more difficult lessons at the start, and then you get to relax a bit and do even harder lessons than before. So, you…do a little bit of hard work, get to relax a bit more, and then you have to do even more hard work.

Nevil and Amy did not answer this question.

5.2.6 The lessons in Sarah's class

5.2.6.1 Organisation of the lessons

Sarah organises the lessons on a weekly basis and keeps a relevant plan, such as the one in Figure 5-2, which was about the first lesson observed.

Block, Unit, Day	Objectives Success Criteria	Starter AfL	Direct Teaching AfL	Independent Tasks Including differentiation AfL	Plenary AfL
WEDS	I can solve simple word problems. I can solve mathematical problems and puzzles.	**Use example from test base. Model problem solving techniques.**	Discuss names and properties of 3d shapes. Discuss cubes in detail. Ask children to explain what they already know about cubes. Model independent activity and recording on table/chart.	**Investigate making cuboids from smaller cubes. Ext - making cubes**	**Ask children to share their work. Did anyone notice a pattern?**

Figure 5-2: Sarah's lesson plan, day 1

Each lesson lasted 60 minutes. It started with all the children sitting on the carpet in front of the whiteboards. From this place — sometimes randomly paired — they did a ‘starter’ lesson for 15-20 minutes. This involved mental mathematics and computerised activities through the interactive whiteboard with exercises for revision or introduction to the learning objective (see Appendix 7.2). After the starter lesson, pupils were divided into groups by ability and took their places around each one of the six blocks of tables that were in the classroom. Each group’s place changed from lesson to lesson (as the seating plans of the first two lessons indicate in Appendix 7.3). From these places, pupils did the main lesson, which lasted 30–35 minutes. During the last 10 minutes, all the children gathered on the carpet again for the closing lesson. They discussed their results with the teacher or did further examples on the board.

The lessons observed along with the materials and the resources that they used are presented in Table 5-4.

Table 5-4: The lessons observed, the materials and the resources used in Sarah’s class

TITLE OF THE LESSON & LESSON OBJECTIVE	MATERIALS USED	RESOURCES USED
“*3D Shapes: cuboids & cubes”* Pupils had to investigate the properties of the cuboids, learn the names, be able to construct cuboids from smaller cubes and understand how to count the cubes used to make a cuboid.	“Mirror images” computerised activity on symmetry (SA*) “3D shape problem” Plastic cubes	Testbase website (2008) National Strategy website: Mathematics Resource Library (DCSF, 2009)
“Shape & space” The group of able pupils had to solve a problem related to shape and space without the help of the teacher, while the other children worked on additions.	“Adding two and three one-digit numbers” computerised game (SA) Worksheet: “Shape & space problem-solving” (DA**)	National Strategy website: Mathematics Resource Library (DCSF, 2009) Commercial publication: Badger Maths Problem Solving: Years 1-2 (Nathan, 2007)
“Handling information/data” Pupils had to understand bar charts and learn how to read the information represented, how to organise their own data, compare their results and make generalisations.	“Graphs & charts” computerised activities on “data handling” (SA) A ‘data handling’ problem, a data recording form, and 1-6 dice	Testbase website (2008) Commercial publication: BEAM, Primary maths resources (Clarke, 2009)

*SA=Starter Activity **DA=Differentiated Activity for more able pupils

The lessons, including the main part that took place in groups by ability, mainly involved the same activities for all groups. Only one lesson involved differentiated work. This included a problem-solving activity for the group of able pupils and activities on adding two and three one-digit numbers for the rest of the class. It was a lesson, in which Sarah worked with children in lower and middle-ability groups, leaving the group of able pupils to work alone on a ‘space & shape’ problem (Appendix 7.4), which, as she explained, was easy and not something challenging, allowing them to work on it unaided.

In all three lessons observed, there was a teaching assistant who was helping the pupils while they worked in groups, mainly those in the lower-ability groups.

5.2.6.2 Aspects of teaching mathematics at higher cognitive levels

Analysis

Encouraging pupils to express their thoughts and explain their answers

Sarah in both starter and main lessons gave opportunities for all pupils to be engaged in higher-order thinking through questions that encouraged them to express their thoughts and explain their answers, such as the following that were about reading and explaining information provided by a graph in the third lesson (see Appendix 7.2.2):

> Can you explain what this means?
> Can you explain this more?
> What do you mean by ‘the highest number’?

Encouraging pupils to explain their methods and present different solutions

Sarah kept emphasising methodology, even when it was a simple addition, asking ‘why?’ and ‘how?’ questions, such as for the case of 8+9 in the second lesson:

> Tell me, why did you go to 17?
> Why didn’t you go to 15 or 20?
> What do you have to do to work out this?
> How do you find the difference?

She was, also, asking for “another way” or “a different method” every time. This gave opportunities to all pupils to display knowledge and comprehension, in number calculation for instance, but also to more able pupils to display a different way of thinking than the others. For example, in the second lesson again, talking about how they calculated 8 + 9, a pupil said that she put 9 in her head and then counted 8 more with her fingers, while Amy said: “I knew nine plus nine was eighteen. I took one away… seventeen.” Jason also presented a complicated but correct method for calculating the total of 7, 3 and 4: “I knew 6 + 4 is 10. I took one away from 7, and I did 6 + 4 = 10. Then I added the one I took with 3. That makes 4 + 10 = 14.”

Identifying and using patterns

Sarah’s questioning involved questions for all levels. In the first lesson, for instance, she asked the class to find the pattern in numbers that they had found counting up to 70 by fives, and then to use the pattern to find numbers bigger than 70 going up by fives. When she saw that most of the children easily answered these questions, she used more challenging questions, which, in some cases, were addressed to particular children, as the following examples show.

> Sarah: Can you find any bigger number [to all pupils]?
> A pupil: Ninety!
> Sarah: Well done!
> Sarah: Bigger [she asked Amy]?
> Amy: A hundred and ten!
> Sarah: Higher [she asked Jason]?
> Jason: A thousand one hundred and eighty!
> Sarah: Well done, Jason! Alvin, how [much] higher can you go?

Reasoning, hypothesising and generalising

When pupils were separated into ability groups for the main lesson, Sarah, even when she gave the same activities to all groups (in two out of the three lessons), extended these activities to higher cognitive levels for pupils in the higher ability group by asking them to further explore an issue and to identify structures of mathematical theories.

For instance, in the first lesson on cuboids and cubes, although the common question was to construct many different cuboids and to find a way of recording the cubes used for each cuboid, Sarah asked the five children in the higher-ability group to find number patterns while they were recording their cubes and attempt generalisations about the numbers that make a cube. Similarly, in the third lesson, in which each group was recording data by rolling two 1–6 dice and adding the two numbers every time, Sarah was asking questions during the process but while she asked pupils from lower and middle-ability groups to simply present their results and calculations, she asked able pupils more challenging questions demanding reasoning, hypothesising and generalising. The following dialogues of Alvin and Amy with Sarah are such examples.

The dialogue of Sarah with Alvin:

> Sarah: What did you find?
>
> Alvin: More fives.
>
> Sarah: Why? What do you think?
>
> Alvin: Because there are many ways to take it.
>
> Sarah: How many ways?
>
> Alvin: 3 + 2, 4 + 1… [He is thinking]… Two ways, but 11 will be most likely.
>
> Sarah: Why?
>
> Alvin: Because there are more ways to find it, like 7 + 4…
>
> Sarah: But there is no seven in the dice!
>
> Alvin: …Then, there is only one way: 5 + 6… Six has more ways: 5 + 1, 4 + 2, and 3 + 3.
>
> Sarah: That's right, well done!

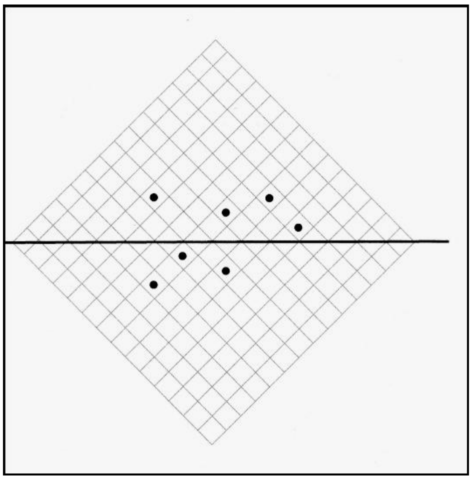
The dialogue of Sarah with Amy:

> Sarah: Does anybody have an idea why you mostly get 6s, 7s and 8s? Do you know why? [to Amy]
>
> Amy: Because we have the most ways to find them.
>
> Sarah: Can you explain this more? You are right.
>
> Amy: The dice have the most numbers to add up to these numbers.
>
> Sarah: Good girl! Well done!

Synthesis

Building new and unique solutions

Some good examples of the interesting ways of thinking an able child can display when he or she has the appropriate encouragement and time to think are the different and unique solutions that Alvin presented in two cases in the first lesson. The first was when he was encouraged to present his different way in a symmetry exercise 'mirror images' (Figure 5-3).



Resource: Test Base website (Testbase, 2008)

Figure 5-3: Sarah's second starter activity 'mirror images', day 1

Alvin, unlike the other children who were trying to find the symmetrical circles by counting the squares above and under the symmetry line, saw the small black circles (A, B, C) as vertices of specific triangles and drew symmetrical triangles on both sides of the symmetry line adding the reflection of given circles as vertices (e.g., the point D) of the new triangles (see Figure 5-4).

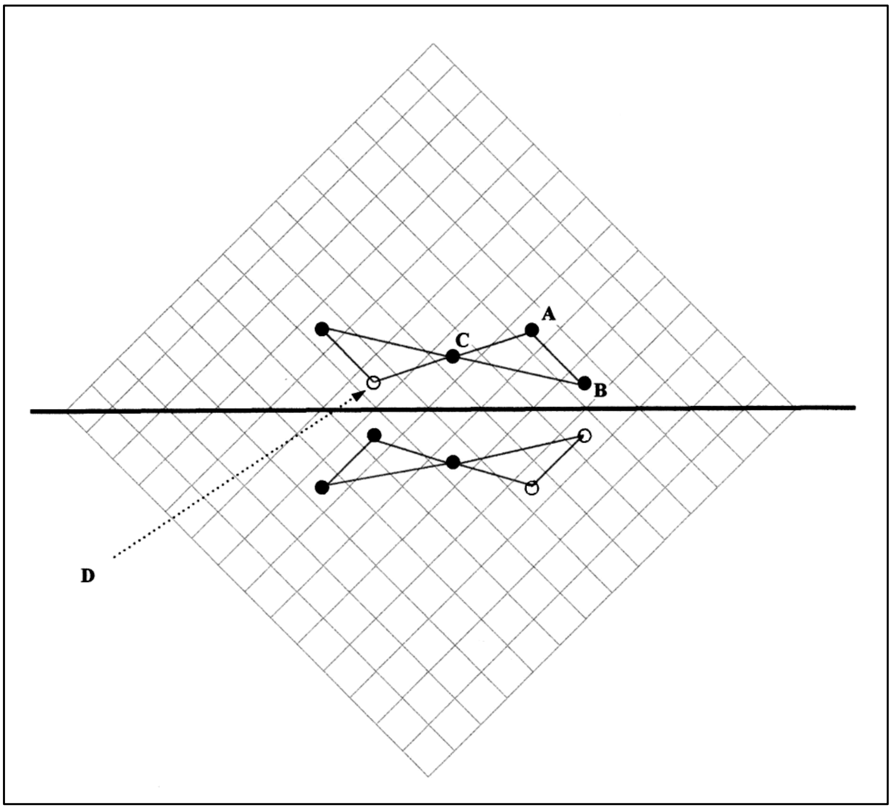


Figure 5-4: Alvin's reflection shapes, School B, Day 1

The second was when Sarah challenged him by asking how much higher he could go by counting up by fives, as mentioned earlier. This gave Alvin the opportunity to show his knowledge about numbers and self-confidence about the uniqueness of his answer. He said: "Nobody will beat my number!" Then, he presented his large number, challenging his teacher (as Sarah admitted):

> Alvin: Ninety-five googols six trillions ninety-five!
>
> Sarah: Let's see if I can write this number right… [After she wrote the number with some zeros missing] …That's the extra large number of Alvin! I don't know if I even wrote it correctly. Thank you for challenging me, Alvin!

Creating new situations and giving opportunities for further investigation

There were cases when Sarah intervened in able pupils' work in order to change the situation and create new ones using 'what if' questions. For instance in the first lesson, she asked "What if you have nine?" to Alvin, who had started making generalisations about even numbers (e.g., 'eight', 'six' and 'four'), making him start conjecturing about square numbers: "I think it works. Sometimes four works, because two times two makes four and sometimes nine, because three times three makes nine."

Evaluation

Examine proofs introduced by oneself or someone else

In every case when Sarah asked pupils to find patterns, she also asked them to check if they were right. For instance, after she heard the first attempts for generalisation from Alvin, Jason and Amy about the numbers that make cubes, she asked them to try their numbers and find out if they were right. When Alvin tried the number eight and saw that it produced a cube, he believed that he was right and ready to form his theory about 'even' numbers. At the same time, Jason was not able to make a cube with six, so he had questions that made Alvin rethink, as it appears in the following dialogue:

> Alvin: Even numbers make a cube.
>
> Jason: Why doesn't six work?
>
> Alvin: Because they are too little.

Again, Sarah asked Alvin to try more numbers and see if he was right. Alvin tried the numbers that he suggested and when he saw that they did not work, he tried to make different sized cubes, recording every time the pieces that he used. When Sarah asked him again about his previous generalisations, Alvin appeared sceptical. He discovered that the numbers four and nine ruined his theories about even and square numbers respectively. Instead, he found out that the numbers eight and twenty-seven can make cubes.

Evaluating solutions of others

When a pupil was trying to complete an exercise on the board, Sarah asked the rest of the class not to say the answers but to show thumbs up or down to indicate whether it was right or wrong respectively. Afterwards, she discussed the solution with both those who had their thumbs up and those who had their thumbs down. In this way, Sarah had all the pupils engaged in a process of evaluation of others' work and, at the same time, she was able to check at a glance how many knew the answer. In addition, she was commending those who had noticed a mistake ("Well done to those three who had their hands down!"), making the rest pay more attention to what was happening on the board.

Encouraging pupils to evaluate their own methods

Sarah's questions and encouraging words to pupils to present their methods even when they had made a mistake ("Don't give up! …we learn from our mistakes.") encouraged Alvin to speak about a different method that he had used to find the total of 7, 3 and 4 and a mistake that he had made. The interesting dialogue between Sarah and Alvin is presented below:

> Alvin: I did… I went to 9 + 9 = 18…
>
> Sarah: Why did you choose 9 + 9? There is no 9 here. What made you choose that number?
>
> Alvin: I did 9 + 9 and then I took away 4, but actually I made a mistake.
>
> Sarah: What mistake?
>
> Alvin: My mistake was… I took away 2.
>
> Sarah: Why did you choose 9 add 9?
>
> Alvin: Because I didn't know what 7 + 7 equals. I knew 9 + 9 and then I took away 2, but that was a mistake, because I had to take away 2 more because 9 + 9 has 2 + 2 more than 7 + 7.
>
> Sarah: That was a nice method, but too long, and because of that, you made a mistake. But you realised your mistake and that is fine, because we learn from our mistakes.

Monitoring, supporting and encouraging

Sarah, with the help of her teaching assistant, was monitoring the work of all pupils in the class most of the time, offering support when necessary. She only left the group of able pupils to work alone in one lesson (the third lesson), in order to support the other groups, as she explained. Sarah was mediating many times during the problem-solving process to help pupils be systematic (e.g., in the third lesson where they had to systematically record data in a way that would help them make comparisons) and checking their understanding by asking questions, such as:

> What does the question mean?
> What does 'total' mean?
> What does 'most likely' mean?

During all the lessons, Sarah maintained a good atmosphere and appropriate environment for learning. Every time, she commended and rewarded those who answered a question correctly:

> Very good explanation!
> …good boy! …good girl!
> That's right! Well done!

She was, also, encouraging those hesitating to talk (e.g., "That's fine! Just say it.").

5.2.7 Children's progress

The results in two consecutive assessments (achievement tests from QCA resources) show that all five pupils made significant progress (Table 5-3, section 5.2.5).

Talking with Sarah again at the end of the school year when I sent her a copy of my observation notes, I was informed that all five children achieved Level 3 in the SATs "with Alvin doing the best and Jason coming a close second".

5.2.8 Children's performance and behaviour in class

All five children demonstrated that they had the knowledge and the skills to perform mental calculations with fluency, to make cuboids and find the right way to calculate the pieces that they used. They all seemed confident in reading graphs and understanding the information represented, in that they put their hands up almost always and when they spoke they answered correctly. Furthermore, when they were given the opportunity to explain their methods, Amy, Jason and Alvin displayed reasoning and analytical skills.

Alvin's performance was especially interesting throughout all the lessons. Alvin was watching carefully all the action in the class and was active all the time, either by approving when someone did something well or by having his hand up to speak. He was keen to answer most of the teacher's questions and seemed confident when he was speaking. He also displayed creative thinking when he found a different way to draw the reflection of the small circles in the second symmetry activity (Figure 5-4). Also, his willingness to express his ideas and the way that he presented them ("I have a different way… That's easy for me…look if we do this…) showed that he enjoyed doing that and that it possibly was not the first time that he had done it. Talking with Sarah, after the lesson, I learnt that he always likes doing this. Sarah also said that he is very articulate and quite egocentric and because of the latter, he is not so good at working with other children.

Alvin furthermore showed a disposition for finding different ways to calculate numbers and for self-evaluation when he said the mistake that he made in a mental calculation. Although he had realised his mistake and found the way to reach the right answer, he expressed his initial thoughts and highlighted his mistake. Speaking with Sarah after the lesson about the method that Alvin presented, I learnt that Alvin likes to find different and sometimes difficult ways to a solution ("He likes challenging himself", she said). I also learnt that he always reads the exercises very carefully and spends a lot of time thinking first before he starts writing: "He wants to have the whole answer in his mind first before he starts writing and saying it", Sarah explained.

Sarah's opinion about Alvin's work habits was confirmed during the group work when he always chose to work alone with high concentration and a systematic way, as someone can see in the sample of his work on data recording (Figure 5.5), where he recorded his numbers in a way that helped him to compare them very easily.

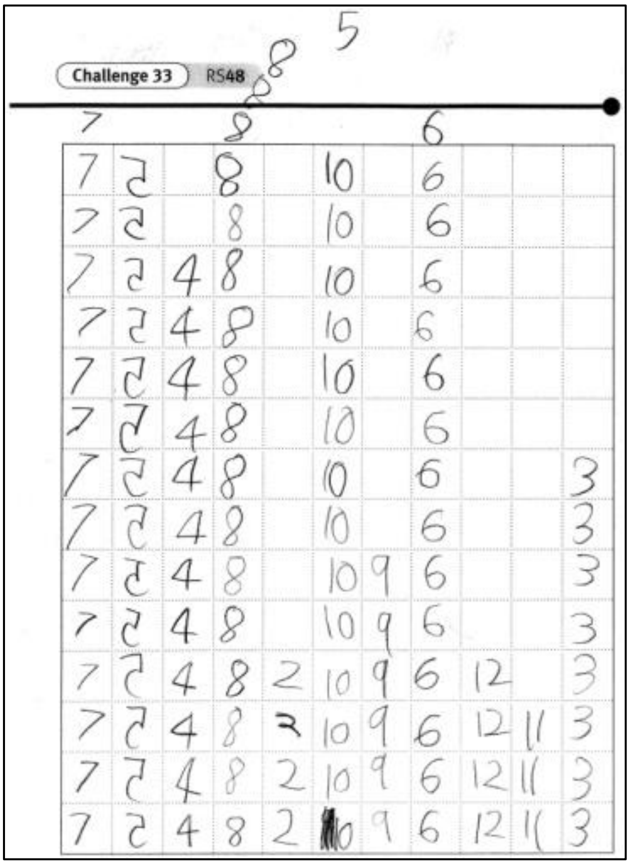


Figure 5-5: Alvin's 'recording data', Day 3

Alvin also showed an interest in my notes. When the lesson finished, he came to me and asked me if I was keeping notes of what they were doing in the class.

All five children worked with enthusiasm in activities that demanded investigations, such as those with the cuboids and data handling in the first and the second lesson respectively. The work in groups was mainly individual work apart from the cases when Sarah mediated by asking questions, raising short dialogues, such as the dialogue between Alvin and Jason, quoted earlier. Three of the five children (Alvin, Amy and Jason) always worked carefully on the task without losing their time, even when the teacher was not nearby. This was clearer in the second lesson, in which the five children worked totally by themselves in a 'shape & space' problem (see Appendix 7.4) that they did not manage to successfully solve. The abovementioned three only showed persistence and perseverance in answering all the questions, and one of them (Alvin) worked more systematically than the others. Most pupils seemed to miss basic details of the problem (i.e., that they had to move three sticks at a time to make a new shape). Even Alvin, who found out how to solve the problem correctly, did not understand that he had to find different shapes every time. Therefore, he made some similar shapes, but with a different direction. Because there was no discussion with Sarah about the solution at the end, no one realised the mistakes that he or she had made.

5.2.9 Initial comments on the second case study

Provision for mathematically able children in School B is offered in the regular classrooms. According to what was observed in Sarah's classroom, it is offered through grouping arrangements by ability, teaching materials from higher levels and differentiated instructions. The school provides teachers a range of resources from commercial publications and the opportunity to use online educational databases directly to the classroom through computers and interactive boards.

In the lessons observed, Sarah divided her pupils in three ability groupings (lower, middle and higher ability) for the main lesson, while, for the rest of the time, they worked as a whole class or in randomly created pairs. Apart from the second lesson, in which she gave a different activity to able children, Sarah used the same activities for the whole class. However, even with the same activities, Sarah offered differentiated instructions and extension for pupils that were more able. She asked more challenging questions to able children during the starter lesson and extended a common activity for the group of able children by asking them to explain more investigations, patterns, proofs and generalisations. The selected activities gave opportunities for such an extension because they were open-ended and involved investigations.

Sarah, in the interview, appeared aware that children with higher abilities in mathematics do not need just more work but challenging work that will make them think. In the lessons observed, Sarah emphasised pupils' ways of thinking and their methodology, asking them to be systematic, to reason, and to explain their methods. She appeared very confident in this and well prepared. Her questioning, with 'How?', 'Why?' and 'Is there another way?' questions, gave the opportunity to able pupils — like Alvin, Amy and Jason — to show their abilities in explaining their thinking, reasoning, hypothesising and generalising,. Alvin, in particular, found the opportunity through these questions to display creative thinking and abilities for evaluating of work of others and self-evaluation.

All five pupils made significant progress throughout the first terms of the year, according to their results in achievement tests, with Alvin, in particular, and Jason, secondly, doing the best in the SATs at the end of the school year.

5.2.10 Emerging Issues

- Although the school policy clearly suggests the involvement of parents and carers in the identification of and provision for gifted and talented children, this did not seem to happen in practice, at least in relation to mathematics. For instance, Sarah said that the identification of able mathematicians in her class is "purely based" on her own judgement and, more specifically, on what she is observing in the class. She also said that she is taking into account their results in the formal achievement tests. Parents and carers' involvement was only referred to as a problem, by both the deputy headteacher, who also was the G&T co-ordinator of the school at that time, and Sarah.
- Sarah said that she changes the grouping arrangements from ability grouping to mixed-ability depending on the learning objective to combine, for example, able mathematicians with others who are not able in mathematics, but able in literature. The research did not find evidence of this, as pupils were divided by ability in all three lessons observed. Maybe this was an intended outcome or a practice that Sarah had used in the past. On the other hand, it seems that she moves pupils between ability groupings depending on their achievement, based on formal assessments. For this, she keeps a tracking sheet with pupils' achievement in which the pupils are separated into groups by ability. Also, the example that Sarah described with the angry parent, whose child was moved to a lower-ability group, shows that such changes are not always easy.
- In the interview, Sarah said that she prepares different work for each ability group every time. In the lessons observed, however, she gave differentiated work only to the group of able pupils, in one of the three lessons, when she wanted to work with the less able children. Therefore, this different work was mainly to keep the group of able children quiet rather than to challenge or extend their abilities.
- Apart from the second lesson, in which Sarah wanted to work with lower-ability pupils, the materials that were used for all pupils, either for starter or

main activities, were for older children. These materials seemed to work well with the able children, offering challenge and extension, but kept the other children silent most of the time. This practice of using materials for older children was confirmed by all five pupils during the interviews when they talked about the "hard work" that they do, and especially by Jake, who said that he felt sad one time when he was not able to cope with such an activity.

- It seems that Sarah feels that the groups do not always work well and that the group of able children needs more attention and support, because she asked for more support staff in the classroom during the interview. She admitted that sometimes, when she wants to stay more with less able children to support them, she chooses easier activities for the able children, instead of truly challenging work, because they have to work by themselves. That happened in the second lesson, as mentioned earlier, and showed that the able pupils had difficulties figuring out how to work alone. Even Alvin, who found the way to solve the problem, did not realise that he had made some similar shapes with a different direction, because he did not have the opportunity to discuss his results with the teacher.
- Pupils suggested as being able mathematicians perceive mathematics as numbers and number operations.
- The five pupils interviewed consider teamwork differently. It seems that it is connected with the nature of activity that they have to do and with pupils' individual abilities. Pupils, who appeared more comfortable in the lessons observed, did not want to work with pupils who wait for them to find the answers, but only with those from the higher-ability group.

5.3 *Third case study: Kate's class*

5.3.1 Background

Kate has been teaching for six years, the last four at this school. She has worked as a classroom teacher in Year 3, 5 and 6 classes. This year she is teaching a Year 5 class and, for the first time in her career, a top mathematics set of 30 Year 5 pupils. Kate has not received any specific training in education of gifted and talented children, but only in teaching mathematics in general (one day inset mathematics training and three-day training on national strategy for mathematics provided by the Local Authority). Kate, like Sarah, was not amongst the teachers who took part in the first stage of my research. The mathematics co-ordinator and senior manager of the school was the one who responded to the questionnaire giving me the first picture of the school and their policy and suggested Kate for my study. When I personally met Kate, I found that she was very enthusiastic about the study and very eager to participate.

Six children were selected — two boys and four girls, all around ten years old (see Table 5-5, section 5.3.5). They had been in the top mathematics set since they were in Year 3 and were suggested by Kate, not as gifted mathematicians, but as more able than their peers in the top set, in which, as she explained, there are diverse abilities amongst the pupils. Three of them, however, are on the gifted and talented register, as Kate told me. The identification of these children was based on a QCA test at the end of the previous school year and on previous teacher's nominations. More details about the methods of identification used by the school are presented in the following section.

5.3.2 School policy for identification and provision

The school has an identification and provision policy for gifted and talented children and makes specific provision for mathematically able children through a 'setting' programme. They keep a general register for gifted and talented children (G&T register), where 5 percent of all children are registered (this includes gifted mathematicians). The identification of able or gifted mathematicians is mainly based on teacher nominations and teacher assessments, formal and informal. The formal assessments involve QCA tests at the end of each term. The identification is supported by a Target Tracker programme and a whole school assessment record. The Target Tracker is regularly set and reviewed by the class teacher, and it is available to all members of staff who want to have access to relevant information. Children are also aware of the targets and can monitor their own progress, along with the class teacher, on individual target sheets, which they stick in their books (see Amardeep's 'target sheet' in Figure 5-6). Children's assessment results are recorded on class tracking sheets in order to show their progress year after year.

Name:...

Maths Targets	1	2	3	4	5	Check with Teacher
1 Read the question carefully!	✓ 26/2/09	✓ 4/3/09	✓ 9/3/09			
2 Problem solving - puzzles and charts	✓ 25/2/09	✓ 4/3/09	✓ 9/3/09			
3 ratios - problem solving	✓ 4/3/09	✓ 9/3/09				

Figure 5-6: Amardeep's 'target sheet'

Every term, according to Kate, the teachers from different sets and the mathematics co-ordinator meet together, talk about the progress of their pupils and their needs, look at the results and group the pupils in ability sets (at the beginning of the year) or regroup them (after a term or a half-term), if some children need to be moved from one ability group to another. They also decide who has to be in the G&T register.

The ability sets in mathematics may be either an upper set and two differentiated middle-lower sets or three ability sets (an top, a middle, and a lower set), as Kate explained:

> Generally, in our school, we have one upper set and two middle-lower sets. In some circumstances, we have upper, middle and lower, but usually, we have two differentiated sets and one higher set.

Pupils in the sets are of the same age (e.g., Year 5 top mathematics set). There was only one case in the past in which a child, “an exceptional mathematician”, according to Kate, worked in a set a year above his age. However, this child did not finish school earlier, but in the last year, he worked with his same-age peers in the same class.

Changes to the G&T register and to the sets may be done either after every half term, if the teachers from the three ability sets in each year and the mathematics co-ordinator agree on this, or after the ‘test time’, which is at the end of each term. During the research, which took place in the spring term, one such change happened after the first half of the term, after my first observation. Pupils then moved from a lower set to the top set and vice versa. Four children, who achieved 4b, 4b, 4c and 3c in their assessments, joined Kate’s group, and three children from her group, who achieved 3c, 3b and 3b, went to the set below. Kate explained why children with the same grades (e.g., 3c) changed places between groups and why others from the lower set with similar or higher achievement (e.g., 3a) remained in the same place. She said that they decided to move up only those four because they “were identified by their maths teacher as working consistently at the higher level”. Similarly, the three children, who moved to a lower set, had been identified by Kate “as consistently needing/asking for additional explanations in relation to new concepts or not yet secure in a written method for the four operations”.

It should be noted at this point that, according to the mathematics co-ordinator (and senior manager) of the school, who replied to the questionnaire in the first phase of the research, the school appeared not to have either a policy of provision for gifted and talented children or co-ordinators for planning and running particular programmes for these children. This case study, however, discovered that the school has had a policy of both identification of and provision for gifted and talented children since 2006 at least, according to the date on the copy of the school policy that was given to me by Kate, the classroom teacher. The copy of their gifted and talented policy, also, shows that there has been a co-ordinator for gifted and talented children with specific responsibilities, as for example, to ensure that gifted and talented children are identified and tracked by classroom teachers and to attend relevant trainings so as to keep the staff updated, providing them in-school training every year.

5.3.3 Teaching resources

Kate said that she uses extra support materials that she occasionally gives to children that are more able. These materials are from the *Brain Academy* series published by Rising Stars UK Ltd. and from the handbook *Mathematical Challenges for Able Pupils in Key Stages 1 and 2* (DfEE, 2000). She also said that for the top mathematics set, she uses the *Hamilton Plans* (Hamilton Trust, 2009) from which she usually selects materials for older children ("from [the] Year 6 curriculum even though they're in Year 5"). This is because, as she explained, she wants to "extend them". She plans each lesson and keeps a diary on what she has done: "When I finish with them, I've got writing all around them, saying 'I did this. I did that, I changed that. These are the answers…'" She also explained that she uses computerised programmes through an interactive whiteboard for teaching particular lessons, such as the use of protractors: "…like today, I was teaching protractors. So, I do use a big on-screen protractor to measure angles. The children will come up and have a go at it themselves." The use of ICT was also confirmed by the children, who, however, mostly spoke about computerised games that they do in the starter lessons.

In the lessons observed, Kate used materials that came from commercial websites (e.g., www.fieryideas.com) or books (e.g., Brodie's *Mental Maths in Minutes* series) and the National Strategy website (www.nationalstrategies.standards.dcsf.gov.uk). However, in contrast with what Kate said, they were not always from the higher level (e.g., the mental maths test in minutes for ages 7–9 in the second lesson) (see Table 5-6, in section 5.3.6 for more details).

5.3.4 Summary of teacher's perceptions and attitudes

About having able or gifted mathematicians in the class

Kate believes that having more able children in the class makes her work as a teacher more difficult because particular planning is needed to challenge these children and she has "to work out in advance" for this. She also said that she does not feel so comfortable ("moderately uncomfortable", she replied) in teaching mathematics to them, because she is not so confident in her level of subject knowledge and she does not have experience in teaching mathematically able children: "I am not gifted in maths at all myself. This is the first year I have had a top set."

She also added that she does not have a problem to admit in front of her pupils that she does not know something and ask them to look for the answer by themselves or to wait until the next lesson. More concretely, she explained:

> If they ask me a question, I don't mind saying "I don't know that, but you go and find out and I go and find out and come back and see". So you have to be prepared to be flexible in that way. I think it's fine if they are quicker calculators than me [sic], because in the world, there are lots of people that can do mental maths much faster than me [sic], and that's fine. So you have to be relaxed about [the fact] that you're not in a competition with children. They are all going to be much better mathematicians than I am, but what I've got to offer them is things that perhaps they haven't come across before, and this is going to take them a couple of lessons to acquire it and then off we go. So that's what I'm here for… I'm not here to be a better mathematician than them; they're the super mathematicians. So I think…appreciating their skills and not being in competition with them is important.

Kate considers the help that she has from the mathematics co-ordinator very important for overcoming the aforementioned difficulties:

> I do get support from the numeracy co-ordinator. She is always there for me. If I say, "I am struggling with this; can you show me a way and explain that to me?" she will take time to explain it to me…

About organising and teaching the top set and possible difficulties or problems

Organising or reorganising the set usually does not create difficulties or problems, according to Kate. The only difficulty is finding those children who do not show their abilities. Kate, for instance, said that in this school, many girls, maybe because of their cultural background (Arabic), do not say the answers unless they are personally asked, so they are overpowered by the boys, who are "much more outspoken":

> [Y]ou have to be aware that the girls like Almirah, for example, she is very able, but it is not always obvious because she doesn't answer unless I call on her to answer a question. Whereas a boy like Abdullah, or especially Amardeep, who have hands up all the time, can't wait to give an explanation, can't wait to extend what is going on or ask more questions, whereas Almirah waits [for me] to ask for her. She won't extend her thinking out loud in front of anyone else. So there is an issue for girls in this school — whether it's less confidence or whether there is a cultural modesty.

Kate explained that she separates the 30 (or 31 after the first observation) pupils in the set in three ability groups (a higher, a middle and a lower), which are usually placed in separate places in the classroom ("basically, the high ability [group] is on one side of the room and the lower on the other, so the middle is in the middle"). However, Kate maintained that there are lessons in which she pairs an able pupil with someone who has difficulties, asking him/her to offer peer tutoring. The more able pupils also offer peer tutoring in the lower sets once a week, as Kate explained:

> [U]sually, I'd say probably more often, they are working with a partner with a similar ability, but in some lessons, we have some mixed-ability work so, they might go and I might say, "You swap, you swap… and find another partner". So the higher ability [children] are working with lower-ability persons to explain them maths, support them, or help the higher-ability children explain their processes and their strategies… And then once a week, these higher [-ability] children go to the other two maths groups and help children there, so they join in with the lesson and the other two maths groups and they are supporting individual children.

Kate believes that regrouping within the set and between sets, after the initial grouping, may benefit all the children for different reasons:

> Individual children may need to work with a different group just for one area of maths. Children in a group may not have 'gelled' and regrouping can help; initial groupings tend to be by test result and, after a time, some children may make additional progress. Children can benefit from and be motivated by working with alternative partners from time to time.

Kate furthermore said that pupils are prepared to be moved from one set to another every half term depending on their performance ("they accept that it is just how it works.") and that this keeps them working harder. This is because they do not want to be moved to a lower set, so, according to Kate, they say, "well, I have to work harder if I want to stay there". Also, parents, who sometimes worry so much about the placement of their children and ask for explanations, according to Kate, "usually accept that some children achieve more at the top of the middle rather than the bottom of the top and that boosts their confidence."

About the effectiveness of provision

Kate believes that her school addresses the needs of mathematically able children "well" and that the mathematics co-ordinator, who "is really enthusiastic about maths", plays an important role in this. Kate also explained that they focus on a different subject every year and that this year, their focus was on mathematics. Kate believes that the setting arrangements that they use gives opportunities to able pupils to do mathematics beyond the regular curriculum. "I think they have got plenty of opportunity… [We] do not stick to the Year 5 curriculum", she said. She asserted that the existence of the top ability set motivates pupils, making them feel proud ("I think the fact that there is a higher group that goes a bit faster, maybe takes up on some other areas of maths…it makes them feel proud", she said). However, she was not able to speak about the impact of the setting on pupils' achievement because she was working with such a method for the first time and little time had passed since she started.

However, it seems that not all the teachers in the school follow the strategies or use the materials suggested by the school policy for gifted and talented children, because according to Kate's final comments in the questionnaire:

> In primary school, there will be a mixture of teachers, who, while professional and competent, may be more or less confident/enthusiastic about the teaching of maths. In order to have a consistent quality of maths teaching to gifted children, similar strategies and materials need to be used across a school.

About further support or training

Kate feels that she needs more support and training in in-classroom provision, teaching materials and problem-solving methods, because she does not feel confident:

> I could certainly do with some more training, in investigations especially and puzzles and problem solving, you know the different ways for problem solving… I'm not very confident, so I think I need to set some time aside to work on that.

In addition, she would like to have either fewer children in the class or teaching assistants, because it is difficult for one teacher to check how all the children work and to keep them in order:

> In class… I don't have [a] teaching assistant in maths, and I always teach just on my own with a maths group. There are times when I wish I had another adult just [from] a behaviour[al] point of view. There are some children, you know, you really need to sit on to see if they are working and, in the top maths set that's mostly okay, but there are still one or two children who although [they] have achieved a place [in] this group, they need a little bit [of a] push to keep working. So…but I think every teacher would say that… [they] would like another pair of hands…or less [sic] children. You know, if I have less [sic] children in the class, I'll be twice as good. It's no secret.

5.3.5 Summary of children's perceptions and attitudes

Before starting this section the profiles of the children are presented (Table 5-5). This also includes their results in two consecutive assessments in order to have a better picture of them. Their achievement is discussed later in section 5.3.7.

Table 5-5: The profile of selected children, School C

CHILD'S NAME	GENDER	AGE	1st ASSESSEMENT	2nd ASSESSMENT
Amardeep	Boy	10	4A	5A
Bridget	Girl	10	4A	5C
Almirah	Girl	10	4A	4A
Asima	Girl	10	4B	4A
Rasheeda	Girl	10	4A	4B
Abdullah	Boy	10	4A	4A

Mathematics as a subject

All six pupils perceive mathematics as numbers and number operations. Three of them, furthermore, added that mathematics is "shapes" and "space" (Amardeep), "algebra" (Asima), and "ratios" and "percentages" (Abdullah).

All six pupils also replied that they like mathematics either because it is a useful lesson or because it is "fun" (Bridget & Abdullah). Pupils, who find mathematics useful, explained that they are inspired by parents (Amardeep) or by what people do in life (Asima) and believe that it helps to solve everyday problems and to get better jobs:

> My dad is an inspiration to me because he is really quick at maths. He is really good at this subject. So, he is always telling me to revise and revise in order to get a better job… (Amardeep)
>
> I think maths is good and it's good to do maths, because in your life when you grow up and you get a job, you will always be using maths; with time, when you go out and do shopping and all of that… It means a lot to me, because in the future, I want to do a big job… doctor (Asima)

Pupils, who see mathematics as fun, seem to have games in mind, according to what Rasheeda said:

> We have a lot of games that we can play that have to do with mathematics. That's quite good.

Only two of the pupils said that mathematics is "stressful" (Bridget) or makes them feel "confused" sometimes (Almirah).

All six pupils appeared to work a lot outside school, at home with their parents, not only to complete their regular homework but also to prepare themselves for the next lessons or even for the next year's mathematics. Below are two of the most characteristic answers:

> [A]t home I sit down with my mum for the homework and what I do is I figure them out…the answers, and then I just make more and then when that happens, it increases my level and, at school, it is easier because it helps me. (Asima)
>
> Well, my parents buy me maths…like books. I'm one year ahead, so at home, sometimes I do maths, so then at school, I'll know what to do. (Almirah)

The work that they do in the top set

All six pupils want to be in the top mathematics set and in the higher-ability group, but not all of them are happy with the work that they do and the way that they are working. Three of the six pupils appeared to enjoy the lessons because they have "fun" in the class (Bridget, Rasheeda and Almirah). Almirah explained the "fun" as follows:

> I like it because we don't have really easy work, but it's not too hard. We do, like, different stuff, like, sometimes we play games, maths games, or sometimes we have some sums we work out on the whiteboard.

Almirah is also happy with the teacher because she is "fun" and "nice" and because she lets them play games on the computer and work in groups:

> She is fun and she is nice and we play games on the computer. We can work in pairs, we can work in groups and Miss [Kate] lets us talk with other people, if we don't understand it.

However, the other three pupils expressed concerns about the kind of work and the way of working and even dissatisfaction and disappointment (Amardeep). For instance, Asima does not feel confident to say an answer and take part in the activities on the board, because the children laugh at those who say wrong answers. Even though this has never happened to her, she concretely said:

> When we play timetable games, if anybody says the wrong answer, the rest of the class laughs at them. They say, "Ha, ha! You don't know this; you don't know that!" …That's what they do sometimes. I have seen that happen to people, but it never happened to me. That's why sometimes I don't like doing the games, because that might happen to me.

Abdullah finds some of the work they do too easy and gets bored: "But sometimes it's easy. Then it gets boring…maybe when you're adding numbers."

Amardeep appeared the most concerned of all, unhappy and disappointed by both the work that they do, which he finds too easy, and the teacher, because he believes that she is not good at mathematics. He said that he would like to have more difficult mathematics ("I think that this book is too easy. I need more…harder books."). Many times during the interview he referred to his previous teacher to show that his current teacher is not good enough:

> I could have a better teacher. Do you know Mrs…? [He said the name of previous teacher] …Because she is quite good at maths. I was hoping to have her, because this teacher, Miss [Kate], is not.

He complained that his teacher is leaving him to work completely alone and she is working with less able children:

> I am working quite alone because the teacher is choosing other people that need help…but they are looking for answers. They need help with the answers, so I feel like shutting it out…

Or he complains that she is asking him to help others with simple work (something that Kate considers, according to what she earlier said, a benefit for able children):

> [W]e have gone to a part where Miss [Kate] has to help other people and she really gives…simple work and she just says, "Do your own work… Help other people a bit…"

Pupils explained that they occasionally work as a group on specific activities and confirmed what Kate said about changing groups sometimes:

> Sometimes we work in groups, sometimes [we work] with two people, sometimes individually. (Abdullah)
>
> Miss [Kate] swaps us around so we can work with different people. (Almirah)

Work habits

All six pupils replied that they like working as a group rather than individually, apart from Rasheeda, who prefers both "working with others" and "individually" because "there's a bit of peace and quiet". The following are reasons that make them prefer group work:

> [I]f you are stuck, then you can ask someone else. (Bridget)
>
> [Y]ou can talk with your partner or, like, you discuss it or if you get stuck you can talk and you can help each other. (Almirah)

> Because the people who are good at maths…it is actually good to know what they actually think about the problem and to compare the answers and if there is a disagreement, we could just figure it out again. (Asima)

> Because everybody gets involved and people don't get left out. Because some people don't do work if they are by themselves. (Abdullah)

> [I]f you get stuck on something you can ask them for help and you can discuss the question. (Rasheeda)

Also, Amardeep, who mostly works alone and is not happy ("that's just wrong"), added that he would prefer to work with other children, but only if they are clever. He concretely said: "Other children like clever people…not people that are here… Because we can discuss our ideas about…because we can work in a minute…and it's easy." He explained that he especially enjoys when he works with other children every Wednesday in a lower set, not "supporting individual children", as Kate said, but acting as a teacher:

> I go every Wednesday to Mrs… [he said the name of another teacher who teaches the middle maths set] class and help children… That's really good, because I have a chance to teach one of my own methods, the things I can do.

Furthermore, three of the pupils (Almirah, Rasheeda and Abdullah) said that they prefer swapping between groups and working with different partners. This is because "you can discuss it with different people, so, people with different ideas and you can listen to people with different ideas" (Almirah). Alternatively, it is just because they like "to change around a bit" (Rasheeda) and "know more people, so if they need any help, I can go help them" (Abdullah).

Making mathematics lessons more interesting

Most of the pupils suggested outside-classroom activities (Amardeep, Bridget and Rasheeda), mentioning those that they did in previous years and more "maths games" (Almirah and Abdullah). They also suggested more work with the computer (Bridget), more work in pairs (but with different partners every time) (Almirah), more exercises with time (Asima), and "harder maths" (Abdullah). Below are two examples of what the children said. The first is about the activities that they can do outside the classroom and the second about games, but not on the computer.

Ideas for outside-classroom activities by Amardeep:

> Go out for trips, learning about the history of maths…Going outside and doing activities… Probably when we do ratios, we can go to the playground and draw…and then go back to the classroom…It sticks in your brain more.

Ideas for playing maths games against people and not the computer by Almirah:

> I want to do more maths games, not only on the computer but with people…If you are on the computer, then you are just playing the game, and it's not work…On the computer, you just type in an answer, but if you are playing something and you are with people, like in "Space Invaders", then you have to quickly work it out in your head, and then if you are against other people, you have to take turns. So you don't just type it, but you work with other people.

5.3.6 Lessons observed in Kate's class

5.3.6.1 Organisation of the lessons

Kate organised the lessons by following the *Hamilton Plans* for Year 5/6 (see a sample in Appendix 7.5), from which she chose activities according to the Year 6 plan. The three lessons that were observed, the materials, and the resources that they used are presented in Table 5-6.

Table 5-6: The lessons observed, the materials and the resources used in Kate's class

TITLE OF THE LESSON & LESSON OBJECTIVE	MATERIALS USED	RESOURCES USED
"Area: rectangles & compound shapes " Pupils had to understand that the formula for the area of a rectangle is "length times breadth" and use it to calculate areas of rectangles and compound shapes.	"10 Maths Miracles: Ice breaker" computerised game (SA*) Worksheet: "area, compound shapes"	Fiery Ideas (2008) National Strategy: Mathematics Resource Library (DCSF, 2009)
"Patterns & sequences" Pupils had to understand what a sequence is and be able to identify the pattern, represent and interpret the sequence.	"10 Maths Miracles: Cell Mates" computerised game (SA) Worksheet: "Number sequences"	Fiery Ideas (2008) National Strategy: Mathematics Resource Library (DCSF, 2009)
"Ratios" Pupils had to learn how to represent a comparison of two numbers as a ratio and solve relevant problems.	Worksheet: "one-minute mental maths test" (SA) Worksheet: "Smartie Party" ('ratio' exercises) Real "Smarties" sweets	Commercial publications (Brodie, 2004) Primary Resources (2008)

*SA=Starter Activity

Each lesson lasted 60 minutes, including a starter activity, which lasted from 15 to 30 minutes. Mathematics lessons followed the English lessons and, thus, pupils had to change rooms. This caused some delays until all the children were in their places. The second lesson, for instance, started 5 minutes after the scheduled time, and the third lesson started after 10 minutes after the schedule time.

In each lesson, pupils of the same ability were gathered at the same side of the room, but some of them sat next to pupils of different ability (see the seating plan of the first two lessons in Appendix 7.6). Kate, in all lessons observed, worked without a teaching assistant. The teaching assistant always left the classroom after the English lessons finished.

5.3.6.2 Aspects of teaching mathematics at higher cognitive levels

Kate spent much time on starter activities, which were mostly funny computerised games and then on teaching mathematical facts and methods on the whiteboard using many examples. During that time, she asked questions to the whole class and encouraged pupils to express their thinking and present their methods. There were cases when higher-order questions engaged pupils in activities requiring analysis, synthesis and evaluations, as the following examples show. However, it should be noted, at this point, that none of the following examples concerned events that took place at the time that pupils worked in groups, because the pupils in my focus group worked always by themselves.

Analysis

Encouraging pupils to explain their thinking, present methods and different solutions

Kate many times during the whole-class lesson encouraged pupils, through questions, to present their methods and explain their thoughts. The following examples are mainly from the second lesson and some from the third. Pupils were, then, asked to:

Explain their thinking:

> Why did you choose 9?
> Why do you think it is 8? …Why 32?
> How many options [do] you think [there] will be? (In the third lesson about ratios)
> Think a little more. How many colours do you have?

Present their method:

> Which one way are you working on?
> How do you start?
> Do you use any particular strategy?

Present different solutions:

> Are there any different solutions?

Synthesis

Creating new situations – Relating knowledge with real life

In the first lesson, after pupils did some examples on calculating areas and after they identified the units of measurement of area — the name (e.g., *square centimetres*) and the symbols (e.g., cm^2) — Kate changed the situation from measuring geometrical shapes to measuring real things from everyday life. She asked, for instance, the pupils to think when they would need to use "a square meter" and "square decimetres", and received the following answers, respectively:

> Bridget: To measure the classroom.
> [........]
> Asima: For a book.

Similarly, she asked how they could measure "a CD cover" and "a table", receiving the following answers respectively:

> All the class: Square decimetres.
> [........]
> All the class: Square meters.

Generating knowledge from existing knowledge

In the same lesson, pupils initially discussed the characteristics of the rectangles; they did examples on calculating their area and learned the formula: $area = l \ X \ b$ (explaining that l = length and b = breadth). They were then asked to use what they had learned to calculate a compound area (Figure 5-7).

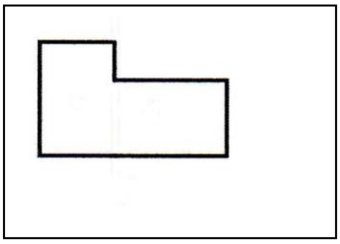


Figure 5-7: Kate's compound shape example (1)

Amardeep and Asima, then, took the opportunity to show their knowledge and understanding:

> Amardeep: We can split the shape into regular parts to make rectangles.
> Asima: Add them together.

Next, Asima separated the compound shape in two different ways (Figure 5-8).

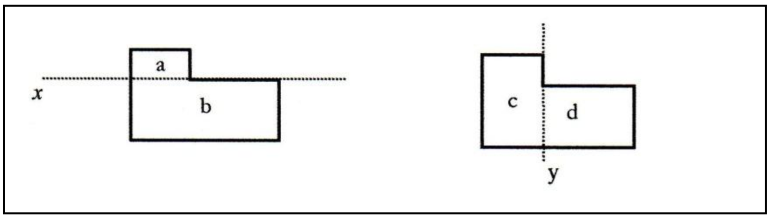


Figure 5-8: Asima's solutions

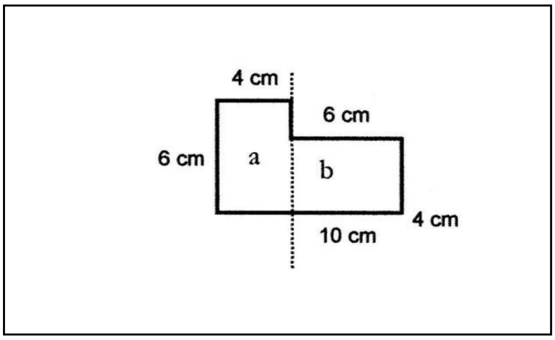


Figure 5-9: Kate's compound shape example (2)

At the end, Amardeep calculated the area of a similar compound shape (Figure 5-9), as follows:

> Amardeep: 6X4=24 cm^2 and then 24+24=48 cm^2

Evaluation

Evaluating activities and methods

Kate asked questions that gave pupils the opportunity to evaluate the activities that they did in the first starter lesson, such as:

> How difficult were the calculations?
>
> What made it difficult?

Evaluating solutions of others

In the third lesson, after the one-minute mental maths test, pupils discussed their results. They were then asked to check the answers that everyone presented. Kate was encouraging them to evaluate the solutions of others, saying:

> Let's put the hands up of those who don't agree with the answer… Pop your hands up.
>
> Remember, it's about checking answers, so check the answers.

Monitoring, supporting and encouraging

As mentioned at the beginning, Kate interacted with children mostly during the starter lessons and when she presented the lesson objective on the whiteboard. Then, (e.g., when number calculations were involved) Kate often needed to give clues (a number or an idea) to all pupils to go on.

During the group work, Kate was circulating amongst the children, checking what they were doing, but most of the time she stayed with those who had difficulties, while the able children were working alone. She only came a few times to the group of able children for a quick look and only stayed for a little longer in the third lesson, because some children had questions to ask. Abdullah, for example, who came to the class after they started the activity ("the teacher of the Middle Maths Set asked to borrow him in order for him to demonstrate a maths strategy to her class", as Kate said) needed some explanations to understand what he had to do. However, when there were more questions regarding the same issue, she wrote that issue on the board and explained it to the whole class. Such as for the 'Smartie Party' exercise where she presented examples about how children can be more systematic in order to find as many combinations of ratios between differently coloured sweets as possible.

Every time pupils found something right, she praised and rewarded them (e.g. “Fantastic!” “Excellent!” “Good job!” “Well done!”).

5.3.7 Children’s progress

The results of two formal assessments (tests from QCA resources) showed that three of the six pupils made progress throughout the first terms of the year, with Amardeep showing the most impressive progress. However, one pupil’s (Rasheeda) achievement decreased and two others achievement remained at the same level (Table 5-5, section 5.3.5).

5.3.8 Children’s performance and behaviour in class

During the work they did as a class, either on the starter activities or the examples that Kate presented on the whiteboard for the learning objective, five of the six pupils displayed attributes and characteristics that made them look more able than the others in the top mathematics set (the sixth pupil, Rasheeda, remained mostly silent and when she attempted to calculate an area, she made mistakes). They displayed knowledge and comprehension of mathematical facts. Four of them were keen to speak and answer questions, putting their hands up all the time (the fifth one, Bridget, was silent, but she answered the teacher’s questions correctly when she was asked). Two of them, Amardeep and Asima, had the opportunity to show skills of application, analysis and synthesis when they applied what they learnt about the area of a rectangle to the area of a compound shape and then explained their methods. Amardeep and Abdullah seemed more capable and confident with calculations. Amardeep, for example, was the only one who managed to complete the second level of a math game during the starter activity in the second lesson (Figure 5-10). Then, it appeared that Amardeep’s abilities were recognised by the others, because when, after many trials, Kate continued looking for the right answers but asking children who had wrong answers, many pupils asked her to pick Amardeep, saying: “Miss, Amardeep has the answer”. When Amardeep presented his solution (Figure 5-10), he received a big clap from the whole class as a reward.

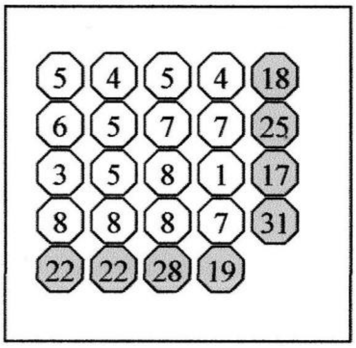
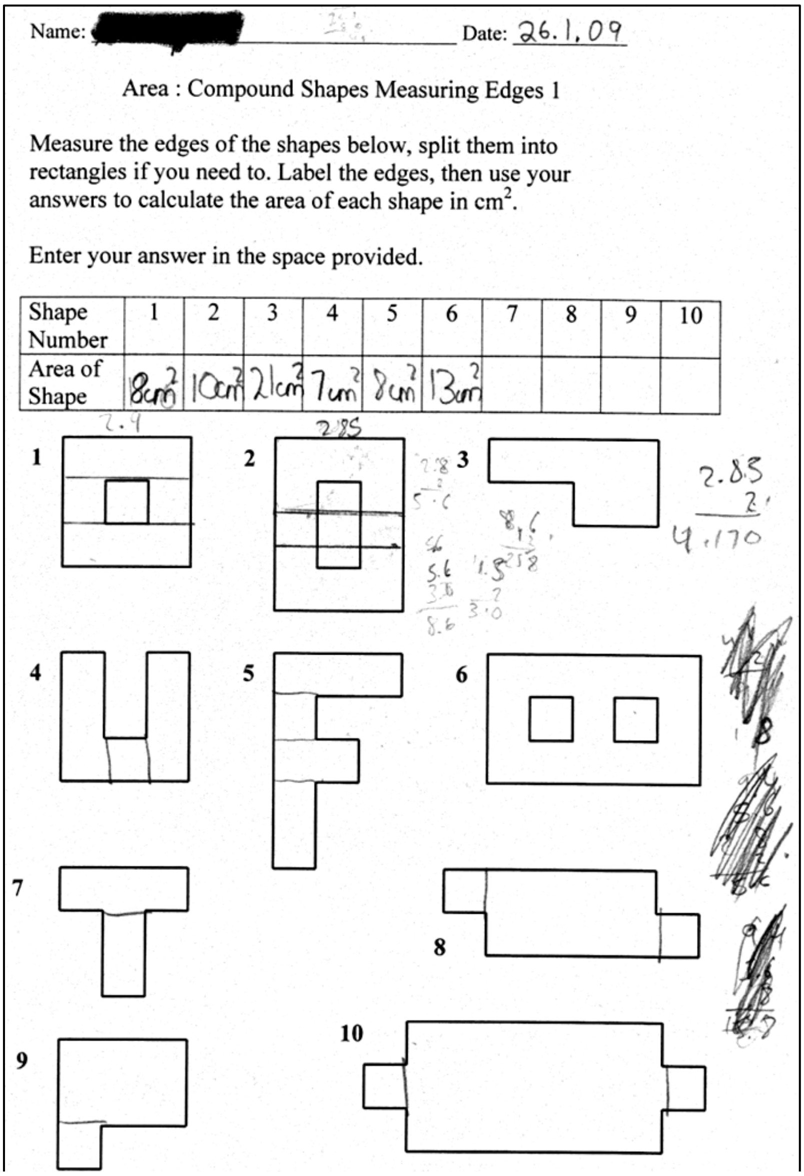


Figure 5-10: Amardeep's solution in the second-level game, 'starter' activity, Day 2

Furthermore, Amardeep appeared especially able in explaining his thinking and method fluently by using mathematical language. Amardeep also seemed to seek a leading role in any opportunity given. When, for instance, in the first lesson, he had the opportunity to guide the starter game together with Asima, he always did what he wanted without taking into account what the other child or the class wanted. He chose the level of difficulty, and he chose division, while the whole class wanted multiplication. He was the one who was always choosing who would answer the questions.

In the work in the group, however, where these children worked completely by themselves, they did not do it so well. None of the six children managed to complete correctly the tasks undertaken in all three lessons. For instance, in the first lesson no one managed to correctly apply the rules, which they previously discussed, on their worksheet. Apart from the mistakes in the method, there were also mistakes in calculations. What surprised me the most was Amardeep's, Asima's and Abdullah's work, because earlier, in the whiteboard activities, these three children appeared very confident and showed that they knew how to calculate compound shapes. In contrast to this, they were not able to do the same on their worksheet for similar shapes. Below is a sample of Amardeep's work (Figure 5-11), in which we can see that he did not label all the edges of the shapes (as the exercise required) and he did not always choose the right way to divide the compound shapes (e.g., Figure 5-11, shape 2). The answers for the area of the shapes appearing in his table (Figure 5-11) are not those that he found by himself but answers that he heard at the end of the lesson during the discussion of the results. Because of this, the answers on the table do not match with his measurements and his calculations, some of which are wrong (Figure 5-11).



Name: [redacted] Date: 26.1.09

Area : Compound Shapes Measuring Edges 1

Measure the edges of the shapes below, split them into rectangles if you need to. Label the edges, then use your answers to calculate the area of each shape in cm^2.

Enter your answer in the space provided.

Shape Number	1	2	3	4	5	6	7	8	9	10
Area of Shape	8cm²	10cm²	21cm²	7cm²	8cm²	13cm²				

Figure 5-11: Amardeep's work, Day 1

While they were working on the activity, the children were moving around freely. This sometimes caused a kind of disorder in some children, whom Kate gathered on the carpet later on. Most of the time, Asima and Abdullah seemed to collaborate well; they talked during the task, measured the shapes together and checked their findings, while Almirah (sometimes) and Amardeep (more often) were walking in the classroom and talking to other children. A similar picture and results were observed in the following two lessons. Furthermore, in the second lesson it was noticed that two pupils of the higher-ability group (Asima and Almirah) were hiding their work from pupils of other ability groups while they cooperated with those from the same ability group (e.g., Abdullah).

All pupils seemed to enjoy the starter activities on the computer and from the rest the last one on ‘ratios’, the “Smartie Party’ activity (see Appendix 7.7), which involved real ‘Smartie’ sweets. Abdullah, for instance, was heard saying: “This is fun. The best lesson we have had in ages!”

5.3.9 Initial comments on the third case study

Provision for mathematically able children in School C is offered through setting, which, however, consists of pupils of the same age only. The children are identified through QCA tests and teachers’ nominations. A Target Tracker programme and assessment records are also used to support the identification. There was evidence that children are moved between ability sets following the results of a term or a half-term assessment and teachers’ nominations, something that, according to Kate, makes them work harder. The lessons in the Year 5 top mathematics set follow the *Hamilton Plans* for Year 5/6, and the activities are usually for Year 6 pupils. However, it seems that a great amount of time is spent on starter activities, which involve funny games on the computer, according to what the children said in the interviews and what was observed in two out of three lessons.

Kate was keen to do her best in the lessons observed. She prepared the lessons and she always explained her aims to me before she started. She used examples on the whiteboard to teach facts and skills and asked questions to check pupils’ knowledge and understanding. While they were working on examples on the whiteboard, she gave pupils opportunities for analysis, synthesis, application and evaluation. She had prepared more activities for each lesson, extended work for more able children, but she did not have the time and opportunity to use them. Furthermore, a close look at the pupils’ writing showed that they did not manage to finish even the main activities in any of the lessons.

Not all the six children made a progress during the time they worked in Kate’s top mathematics set, while the achievement of one child decreased.

5.3.10 Emerging Issues

- The setting used by this school was different from the models presented in Chapter Two, in the literature review. This method of setting pupils from one year class only does not ensure homogeneity in the set, which, consequently, is separated further into three ability groups (higher, middle and lower). This lack of homogeneity, combined with the large size of the set (30 pupils, which became 31 later on) and the absence of a teaching assistant (who present in the English lessons, but leaves the class when mathematics begins), makes Kate's work more difficult. In the lessons observed, apart from the games in the starter lessons, Kate had problems keeping all children engaged and monitoring their work. Therefore, her wish to have teaching assistants or fewer children in her class, as expressed in the interview, sounds reasonable.

- Kate did not receive any training on gifted education or specifically for teaching mathematically able children. This may be an additional reason why Kate feels uncomfortable in teaching this group and insecure about her knowledge level in mathematics.
- Kate believes that more able pupils do not need help to do their work. However, the mistakes that the six pupils made and the unfinished exercises on their worksheets when they worked without help — in contrast with the perfect job that they did with similar activities on the whiteboard, where they had the teacher's interaction and immediate feedback — indicate that these children, although more able than the others, need more attention when they work on their tables.
- It seems that there is an emphasis on games and fun, mostly through the computer.
- Although pupils enjoy the games on the computer, they ask for something different that can be either more difficult mathematics, outside-classroom activities, or maths games against people and not the computer.
- One child appeared unhappy with his teacher, saying that she was not good at mathematics.
- Kate rightly asserts that the children feel proud of being part of the top mathematics set, but it seems that parental help and extra preparation at home have more impact on pupils' progress than the school and the lessons in the top set, which are described as 'fun' by most of the pupils or 'too easy' and 'boring' by two of them. All six pupils consider the preparation and advanced work that they do at home the reasons for their progress in mathematics.

- Unlike Kate's beliefs, peer tutoring does not seem to satisfy an able child, like Amardeep, when this is about helping less able children to do easy exercises only. On the contrary, he enjoys acting like a teacher and presenting his methods to a lower set.
- Kate said that the girls, in contrast with the boys, remain silent even when they know the answer and ascribed this to their cultural background. However, according to what Asima said, it seems that some of them are afraid of being ridiculed by other children in case they say something wrong. This brings out a question about the quality of the learning environment.
- The mathematics co-ordinator, who seems to play the main role in planning the programme, appeared to ignore the school policy of provision for gifted and talented children and the responsibility of the G&T co-ordinator that the school has had since at least 2006.

5.4 Fourth case study: Claire's class

5.4.1 Background

Claire is a Key Stage 3 mathematics teacher. She has been teaching for seven years. The first five years, she worked in a secondary school. The last two years, she has been teaching in three London schools through a company called 'Education London' preparing Year 11 students for the mathematics GCSE examinations. In parallel, this year, she is working in this primary school as a part-time teacher (twice a week for an hour each time) for a group of Year 6 able mathematicians. It is the first time that she has worked with primary school children, and she did not have any specific training for this. In addition, she has not received any special training in relation to the education of gifted and talented children.

Claire, like Sarah and Kate, was not amongst the teachers who took part in the first stage of my research. The acting assistant headteacher, who was responsible for the identification of and provision for able mathematicians, was the one who responded to the questionnaire, but at the time of the research, she was not at the school. Julie, the deputy headteacher and inclusion manager for mathematics and ICT, who undertook the responsibility for gifted mathematicians, accepted my research plan and suggested Claire's class for my case study. Julie also agreed to be interviewed in order to give me some more details about the policy of the school regarding the education of mathematically able children, something that I could not find from Claire, whose involvement in the school life was limited to teaching the particular group of able students, twice a week. Both Julie and Claire were very keen to participate and enthusiastic about the study.

Five children were selected — one boy and four girls, all around eleven years old (see Table 5-7, section 5.4.5). Initially, however, the pull-out group consisted of ten pupils. Three of them went back to the regular class because it was found that they needed more work on the basics rather than the extended work they did in the group, according to Claire. From the seven pupils who remained in the group, two did not bring the consent forms required for the participation to the research and, therefore, no data were collected on them.

All the children were identified, according to Julie, “through [a] teacher’s assessment and a QCA test at the end of the previous year”. They were the highest achievers of their year group, but not necessarily gifted. Only one of them, Matthew, is on the G&T register that Julie keeps. More details about the methods of identification used by the school are presented in the following section.

5.4.2 School policy for identification and provision

The school does not have co-ordinators for planning and running programmes for gifted and talented children, but the inclusion manager (who is also the deputy headteacher) has the responsibility of keeping a register for gifted children in mathematics and ICT. The identification of gifted and talented children is based on QCA tests, teacher assessments and teacher nominations, using the Level Descriptors of children’s attainment set by the National Curriculum. They assess the children every term. Julie explained that they do internal tests and APP — Assessing Pupils’ Progress (DCSF, 2008a) — in autumn and spring term and QCA tests and APP in summer term. After these, they focus on particular groups, such as the more able children in mathematics. They keep a general register for gifted and talented children, in which 10 percent of all children are registered, and a separate register for mathematically gifted children, with 8 percent of the children whom they review every term. Speaking about the register for gifted mathematicians, Julie explained that they try to have on this the gifted children only and therefore they need to keep it tight and updated:

> Yes, [we are] just modifying it because we made it tight, much too tight. It’s a register of the children who are coming above above [sic] average. They are not just children who are able. They are gifted.

They also keep a register for the Young Gifted and Talented programme (YG&T, 2009).

Although they do not have any policy of provision for gifted and talented children, they make specific provision for gifted children in mathematics. This, according to the teacher who responded to my questionnaire, involves “setting”, “bypassing certain classes” and “working with a mentor”. However, this research found that the grouping arrangements, referred as “setting” by the former responsible teacher, are actually pull-out groups for more able children that work in parallel with the regular classroom. Julie explained that they employ higher level teaching assistants to work with particular groups outside the classrooms but in collaboration with the classroom teachers. The pull-out groups for higher ability students in mathematics may include pupils of different ages. For instance, they currently have one group with Nursery, Reception and Year 1 pupils; one group with identified children from Years 3, 4, and 5; and one group with identified children from Year 6. According to Julie, they decided to employ, for the first time this year, a Key Stage 3 mathematics teacher for the Year 6 group in particular, which consisted of pupils already at level 5. Julie explained that this teacher was her choice, because she personally knew her and "wanted to use her expertise to stretch [their] children." Julie also said that although the involvement of the secondary teacher was an independent issue, the school is planning to build connections with some secondary schools to organise special programmes and activities for gifted and talented children and that at the moment they "are just drawing on a Key Stage 3 teacher's knowledge expertise of the key stage of the curriculum" to introduce it to their more able children.

Julie explained how the pull-out group works in parallel with the regular classroom as follows:

> They're carrying on into the Key Stage 2 curriculum and the children, who are in the top group, sometimes they do parallel work, enrichment and extension work, just taken into a different level. Sometimes it's different; it depends. They are able to take three lessons and, with two extra lessons, they can get to a higher level…The two teachers continually share planning with each other…
>
> [When the able pupils return to the regular classroom]…they do the same work but in the higher level. We have got a higher-level teacher assistant, who works specifically under the teacher's instructions…They start all together and then they do a separate piece of work.

More details about the work that the selected pupils do in the pull-out group were given by Claire and are presented in the following sections. Here, we should add Julie's final comments that the pull-out programme works very well with positive effects on pupils' attitudes towards mathematics and that the only problem is the "funding", as they needed to fund an external teacher for this.

5.4.3 Teaching resources

Claire uses her own resources, which are commercial books, bought by her, for various levels (from Level 3 to Level 9) and for GCSE (Foundation, Intermediate, and Higher

Tier). From these books, she chooses materials for Key Stage 3 students, because as she explained, “These are Level 5 students, so it’s good for just extending them.” In the lessons observed, Claire used materials from the books mentioned earlier and materials created by herself (see Table 5-8 in section 5.4.6 for more details).

5.4.4 Summary of teacher’s perceptions and attitudes

About having able or gifted mathematicians in the class

Claire explained that she did not have such an experience in the past because the secondary school pupils with whom she has worked were not higher or lower-ability students. This particular group, which consists only of gifted Year 6 mathematicians, does not cause any difficulty because there are no different levels and, thus, she does not need to differentiate the lessons. She only needs, as she said, to have some extra work for one or two pupils who finish earlier than the others do.

Although Claire has not received any specific training on teaching gifted and talented children, she said that she feels comfortable in teaching these primary school children because she is a Key Stage 3 mathematics teacher and has experience from teaching secondary school pupils. Furthermore, she added that she likes working with this particular group and that it is “fun” for her.

About organising and teaching the pull-out group and possible difficulties or problems

Claire explained that she is in contact with the classroom teacher and she always knows what he is going to do next. She also has the worksheets that he has prepared for the whole class which she uses sometimes as ideas for starters. However, more often, she gives them to her pupils as homework. The latter, however, means that the pupils of the group may have double homework per week, one from each teacher. Nevertheless, apart from this, Claire does not do anything similar to the lessons in the regular classroom. She is doing her own programme, which however does not follow a long-term plan, as she said:

> So, this is me kind of keeping an eye on what he is doing, but it’s not really anything more than that. The rest of the time, to be honest, I kind of follow where it takes us. We are doing some work; I can’t remember how we went to Pythagoras. We were doing different types of triangles and I introduced Pythagoras to them, so we did a few lessons on that…

Furthermore, she added that she always tries to do “what would be fun” but keeping in mind linking every new lesson with the previous ones. For instance, she said that she is planning “to reinforce the algebra using the structure of Pascal’s triangle, because they had the formula for triangular numbers.” She also uses the starter lessons to reinforce the basics about percentages, fractions, and operations with negative numbers, where she has found that they still make mistakes.

According to Claire, there are no problems in organising and teaching this special group, as all the children in the group are at attainment Level 5 of the National Curriculum. If someone cannot cope with the lessons or needs to work more on the SATs, then he or she goes back to the regular classroom without any problem. Such a movement happened this year after the first assessments, and three pupils returned to the other class because, as Claire explained:

> They’ve gone back because it was felt they needed a bit more help in that area — not because they weren’t able to do well in this area, [but] because this is more a kind of a “give it a go” form.

In this group, therefore, according to Claire, there remained only pupils who seemed able to achieve high grades in SATs examinations:

> [T]hey are all doing it fine. They are all on track and have a solid performance in the SATs, because otherwise they would be taken back into the class at this stage, so these are the ones that the school is quite happy [with]. They are provided enough basics [that] they are secure in getting big grades in the SATs exams. So, they’ve got spare time.

The only small problem Claire faced had to do with the behaviour of some children who, because of the informal nature of the sessions, believed that they could make jokes all the time instead of working. However, this problem was solved quickly with the help of the school, as Claire explained:

> The only very-very minor difficulty we had was because of the nature of the session. It’s very informal, and it’s quite lively, and they get a bit excited sometimes. I did send a girl back to the class on one occasion because she was being a bit cheeky, but, you know, very-very minor and the school is actually very-very helpful and kept her after the lesson to consider if she was allowed to continue. I would never [have] wanted her not to continue, but it certainly made the point that although this is an “out of the main classroom” setting and it’s fine to have a little bit of a laugh and a little bit of a joke, we are on task and we are going to work without messing around…

About the effectiveness of provision

Although Claire did not have the whole picture of how the school provides for mathematically able children, she was able to speak about this particular group of able mathematicians. She appeared certain that the work that they do in the group is groundwork for their future, hoping that it will produce positive results in secondary school. She concretely said:

> I very much hope [that] when they get on into secondary school, they are going to be in a very strong position, because they understand a little bit better about methods of working. They are going to be more confident when they come across difficult algebra or a topic like Pythagoras, because they have seen these before and they know that [they] can do it. So, I think they are going to be a lot more confident and, hopefully, that will mean that they do better later on. I think what we are doing, it is doing groundwork to make them more confident later.

Claire also added that the gifted mathematicians need extension rather than doing more of the same that they have already learnt and that this group gives them such opportunities through different and more advanced topics:

> I think it's important and it's interesting that it's not just more of the same. There is no point getting them to do thirty questions on the same topic that everybody else is doing ten, just because they can do them quicker. Obviously, you can grade the questions to be a bit harder, but it's not always possible to really extend people within that topic. If the topic is fairly straightforward, and they have got it, then they have got it. Leave it alone and do something else…You have to have, like, an extension activity to get on with, and that is one of the things that hopefully they get out of this. They can go back and look at the work of Pythagoras or they can go back and complete Pascal's triangle, if they finished the rest of the work.

About further support or training

Claire feels that she does not need any further support or training in order to effectively teach this group of able mathematicians.

5.4.5 Summary of children's perceptions and attitudes

Before starting this section the profiles of the children are presented (Table 5-7). This also includes their results in two consecutive assessments in order to have a better picture of them. Their achievement is discussed later in section 5.4.7.

Table 5-7: The profile of selected children, School D

CHILD'S NAME	GENDER	AGE	1st ASSESSMENT	2nd ASSESSMENT
Matthew	Boy	11	5B	5B
Lily	Girl	11	5C	5C
Cathy	Girl	11	4A	5C
Daisy	Girl	11	5C	5B
Zoe	Girl	11	4A	5C

Mathematics as a subject

All five pupils perceive mathematics as numbers and number calculations and as a subject that requires hard work. Some pupils added that mathematics is "algebra and percentages, area…" (Cathy) and a subject that requires logic and thinking ("you need to be logical and sometimes you have to think a lot" (Matthew)) or "a gift to do some sort of things" (Daisy). Two pupils also perceive mathematics as a useful tool for someone to succeed in real life, from doing a simple job in a shop to something harder, like becoming an astronaut:

> It's a subject that you would use later in your life most. Like you need to use [it] at work…If you work in a shop you have to work out change, but you need to be pretty good at maths to be, like, an astronaut. (Matthew)

> [I]t helps you a lot in real life, like when you are doing sort of like…invoicing and something like that. (Daisy)

All pupils also appeared to like mathematics, even though they find it hard sometimes, maybe because, as Lily said, "[it] is good for your mind".

The work that they do in the pull-out group

All five pupils said that they like being part of this group because they do mathematics that is more advanced and, thus, the usual work in the regular classroom looks easier. Cathy furthermore added that she feels more confident now and ready to face more difficult mathematics in secondary school: "I think it gives me more confidence at maths and it helps me as well in many things, because when I go to the secondary school, I will know a bit about subjects, and they will be easy."

They all used words like "hard" and "challenging" to describe the work that they do in the group. Lily furthermore described it as work out of the form, which she looks forward to doing every week, and Matthew talked enthusiastically about the different numbers that they learn to do, as we can see in their words presented below:

> [I]n the Year 6 class, we kind of just stay on the format, while with Miss [Claire], we are just going above the level. I like it better…I look forward to Fridays and Mondays. (Lily)

> Miss [Claire] is showing me…like…ratios and how to make our own one and different numbers than Fibonacci. (Matthew)

However, Lily appeared less confident with the work that they do in the group when she said, "I tend to feel alone quite a bit, because every time I learn something different...I don't think that I'm that good."

Work habits

All five pupils like group work but four of them responded that they like working alone if it is something easy, because they work more quickly without distractions. Reasons for preferring group work include the following:

> We have got [sic] more minds thinking on the…problem. (Matthew)

> [Y]ou can get everyone's opinion and it makes you think… (Cathy)

> I'd like someone to come and help me. (Daisy)

Zoe furthermore described the group work as follows:

> We kind of share each others' answers. Not telling straight away the answers…but the way we worked it out.

Making mathematics lessons more interesting

The pupils suggested that mathematics would become more interesting if there were more games (Lily and Daisy); challenging, but new subjects "that we've never heard of" (Cathy); and more hard problems for group work ("more hard problems that we can work together on. Problem solving that we can work on in pairs…" (Zoe). Moreover, Matthew suggested differentiated work, suitable for everyone's level, giving the impression that sometimes he finds the work he does too easy:

> [I]nstead of doing simple problems, for some people, you could give…an easier one to the people who don't really understand maths, and a medium to hard level to the people who are good, and a really challenging one to the really good people [in mathematics].

5.4.6 Lessons observed in Claire's class

5.4.6.1 Organisation of the lessons

The lessons in the pull-out group take place every Monday and Friday. The selected pupils leave the regular mathematics class and they are gathered in a small room next to the library, where there is a block of four tables and a small whiteboard. The two teachers, Claire and the classroom teacher, were talking to each other before the lessons about the work that they would do. During the lessons, pupils mostly used their personal plastic boards to write their calculations and their answers. Because of this, there are not many samples of their in-class written work.

Each lesson lasted 60 minutes, including a 20-minute starter activity. However, the actual lesson time in the first two lessons was shorter. The first lesson, for instance, was interrupted twice for 10 minutes each time, because the children had to be assembled for special events (Red nose day and cake sale), while in the second lesson, the children came to class after the first 10 minutes. The starter lessons mainly involved exercises that Claire had written on the whiteboard before pupils came in the classroom. These were questions about their previous learning in the pull-out group (e.g., Pythagoras' theorem and Fibonacci numbers) or questions about mathematical facts that they learned in the regular classroom and are considered basic for their progress (e.g., percentages, square roots, fractions, decimals and negative numbers) (see Figure 5-12). In one case, in the third lesson, a maths-domino game with negative numbers, made by Claire, was given.

The three lessons that were observed, the materials, and the resources that they used are presented in Table 5-8.

Table 5-8: The lessons observed, the materials and the resources used in Claire's class

TITLE OF THE LESSON & LESSON OBJECTIVE	MATERIALS USED	RESOURCES USED
"Pascal's Triangle" Pupils should explore Pascal's Triangle and find patterns.	Questions on the whiteboard (previous knowledge & a decoding game) (SA*) Worksheet: "Sum Fun" (Fibonacci numbers) (SA) Worksheet: "Pascal's Triangle: Blackline Master 2"	Activities created by the teacher Commercial publications (Garland, 1997) Commercial publications (Colledge, 1997)
"Working on Pascal's Triangle" Based on what they had learnt in the previous lesson, the pupils would become able to use the patterns from Pascal's Triangle in practice.	Questions on the whiteboard (previous knowledge & mathematics from the regular classroom) (SA) Worksheet: "Pascal's Triangle: Blackline Master 3" A jigsaw puzzle with algebra (DA**)	Activities created by the teacher Commercial publications (Colledge, 1997) Activities created by the teacher
"Algebra: finding & using formulas" Pupils had to understand the difference between a pattern and a formula in a number sequence. Then they had to become able to find the sequence formula and use it to find particular terms.	A maths-domino game (SA) Number sequences on the whiteboard	Activities created by the teacher

*SA=Starter Activity **DA=Differentiated activity for those who finished earlier

Claire's 'starter' activities on the whiteboard

Day 1

1. Look at Pythagoras theorem. Are you confident with this topic?
2. Look at Sum Fun. Can you explain how working systematically could help here?
3. Decipher this message:
   3 5 5 22
   11 5 7 19 14 19 3
   13 9 13 7 10 5 19 13

Day 2

Start with 12% of 300

1. √
2. add −4
3. double it
4. subtract 5
5. multiply by −3
6. decrease by ⅓
7. subtract −8
8. find 25%
9. subtract 8
10. add this to the start number

Figure 5-12: Claire's 'Starter' questions on the whiteboard

5.4.6.2 Aspects of teaching mathematics at higher cognitive levels

Analysis

Looking for patterns

Finding and using patterns was what they mostly did in both the starter and the main lessons. For example, in the first lesson, pupils were encouraged to look for patterns in Pascal's Triangle through discussions with Claire. Below is an example of their discussion about the pattern of the total of each row.

> Zoe: 1, 2, 4, 8…
>
> Claire: They are…what?
>
> Zoe: Square numbers.
>
> Claire: They are not square numbers. You are not saying the right thing.
>
> Lily: They keep timesing by two.
>
> Claire: Yes, that's good. So, what are they?
>
> Matthew: Rectangular numbers!
>
> Claire: No, it's something else.
>
> Cathy: The power of 4.
>
> Claire: You are close. They are powers but…Keep timesing by two is the key, but…What is the other way to say four?
>
> […………]
>
> Zoe: Powers of two.
>
> Claire: Powers of two! Well done!

Encouraging pupils to conjecture and attempt generalisations

Discussions about patterns and "why" questions encouraged pupils to attempt conjectures, make generalisations, and display their reasoning skills. Matthew, Cathy and Zoe appeared confident in this, in both Fibonacci numbers and Pascal's Triangle exercises, even though they were mistaken sometimes. For example, Matthew for a moment believed that he found a pattern amongst rows in Pascal's Triangle. He noticed that the digits in the second row make the number 11, in the third, 121, and in the fifth, 14641, and that each one of these numbers is the square of the previous number. Enthusiastic, he said this to the class. However, his pattern was not correct and it was not what Claire was expecting to hear, but despite this, Claire praised Matthew for his thinking: "very interesting and good thinking".

Encouraging pupils to present their solutions and explain their thinking

Pupils were encouraged to explain their thinking through "why" questions, as the following example from the domino game, in the third lesson, shows:

> Cathy: −2 [giving the answer for the first domino card: *"I am 8 divided by –4"*, see Appendix 7.9.1]
>
> Claire: Why?
>
> Cathy: Because it's divided by a negative number.

They were also encouraged to present their solutions, even when they were wrong. For instance, in the third lesson again — when they were trying to find formulas from number sequences — although Claire saw the right formula that Matthew had written on his board for the sequence: 2, 6, 12, 20, 30…, she left other pupils to present their wrong formulas. When she let Matthew present the right one: $n(n+1)$, at the end, she asked pupils to explain why he put $n + 1$ in brackets, giving the opportunity to Cathy to display her knowledge:

> Cathy: To do that first.
>
> Claire: That's right!

Synthesis

Generating knowledge from existing knowledge

The first pattern that the pupils discovered in Pascal's Triangle, in the first lesson, was that the number that is under two numbers equals their total. Claire wanted pupils to use what they learnt from algebra to represent the pattern with a formula. Therefore, she drew a chunk of the Triangle, in which she replaced the numbers in the first two boxes with the letters **a** and **b** (Figure 5-13). Next, she asked pupils to use what they had learnt in algebra to represent the number below them:

> Claire: You think algebra. What will be there [she points the third box below *a* and *b*]?
>
> Cathy: **a** plus **b**.
>
> Claire: **a** plus **b**! Absolutely right! That is how it works! [Writing at the same time, a + b in the third box, Figure 5-13]

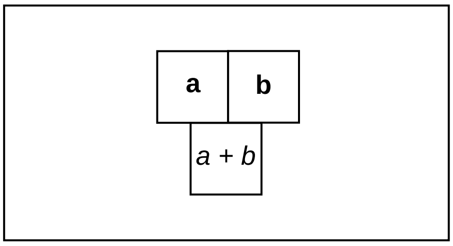


Figure 5-13: Writing a pattern from Pascal's Triangle using algebra

Encouraging pupils to be systematic and work in several bases using different skills

In the decoding exercise "Decipher this message", in the first starter lesson (see Figure 5-12), pupils had to combine logic, memory and linguistic skills to see connections between the numbers given and their places. A double letter, for example, which Lily noticed ("Five needs to be a vowel because it's in the middle…and it is double"), helped them to find the first word which was "G**OO**D". Claire's suggestions to think which common letter can go at the beginning, at the end and in the middle of a word, like number 13, helped later on to find the last word ("**E**V**E**RYON**E**") and then the whole phrase: "GOOD MORNING EVERYONE". Pupils also had the opportunity to use their communication and reasoning skills to present and explain their thoughts throughout this activity.

Evaluation

The first starter activity in the first lesson (Figure 5-12, Day 1) mainly aimed at engaging pupils in self-assessment of their own learning, asking them to reflect on their homework and evaluate firstly their confidence level on a specific topic and secondly their method (e.g., whether and how systematic work helped).

Monitoring, supporting and encouraging

Claire was continuously monitoring pupils' work, mediating when necessary. For example, in the second lesson, while pupils were trying to find missing numbers from Pascal's Triangle in their worksheets using the patterns they previously learned, Claire was checking everyone's work indicating their mistakes (e.g., "This is not right…Careful about this!") and giving ideas (e.g., "If we have 9 here, what will we have here?"). She was, also, asking them to explain why (e.g., "why 5 and 5 and not any other two numbers, which make 10”) and prompting them to work harder (e.g., “Come on brains!”).

Many times in all three lessons Claire during an activity was asking questions to check their knowledge and understanding, praising them, at the same time, when they expressed their thoughts (“Good thinking!”, “Good discussion!”, “Well done!”) and when they completed their task (e.g., “Well done! I am impressed! That was great! Very good teamwork! I enjoyed that!” after they completed the maths-domino game in the third lesson).

Claire’s approach to those who looked confused or had difficulties in particular questions

Sometimes Claire appeared impatient and critical with pupils who had difficulties during the lessons. For instance, during the maths-domino game Claire appeared impatient with Lily, who seemed confused and unable to follow what the others were doing, saying to the class: “She is going to wake up in a minute. It’s all right.”, or towards her, “Lily, switch on!” Later on, however, at the end of the same lesson, when Claire saw Lily working correctly on a formula, she commended her: “Well done for sorting it!” and “That’s right! Well done!”

There were, also, cases when most of the questions posed and answered by Claire while pupils’ responses were limited mainly to a simple agreement (i.e. “Yeah!”). The following dialogues are examples from the first lesson when pupils were working on an problem based on Pythagoras’ theorem and needed to find the square root of 576 (the result of) for the final answer:

> Claire [indicating number 576]: That’s more than a hundred, do you agree?
>
> Pupils: Yeah.
>
> Claire: Twenty by twenty is four hundred, do you agree?
>
> Pupils: Yeah.
>
> Claire: That number must be 21 squared, 22 squared, 23 squared or 24 squared, agree?
>
> Pupils: Yeah.
>
> Claire: Which of those squares end in a six?
>
> […………….]
>
> Claire: What is one times one? … One…So, 21 squared ends in one. What’s two times two?

> Pupils: Four.
>
> Claire: So, 22 squared will end in a 4. What's three times three?
>
> Pupils: Nine.
>
> Claire: So, 23 squared will end in a nine…Four times four is?
>
> Pupils: Sixteen.
>
> Claire: So, 24 squared ends in a six, and this must be 576. So, the missing side is 24.

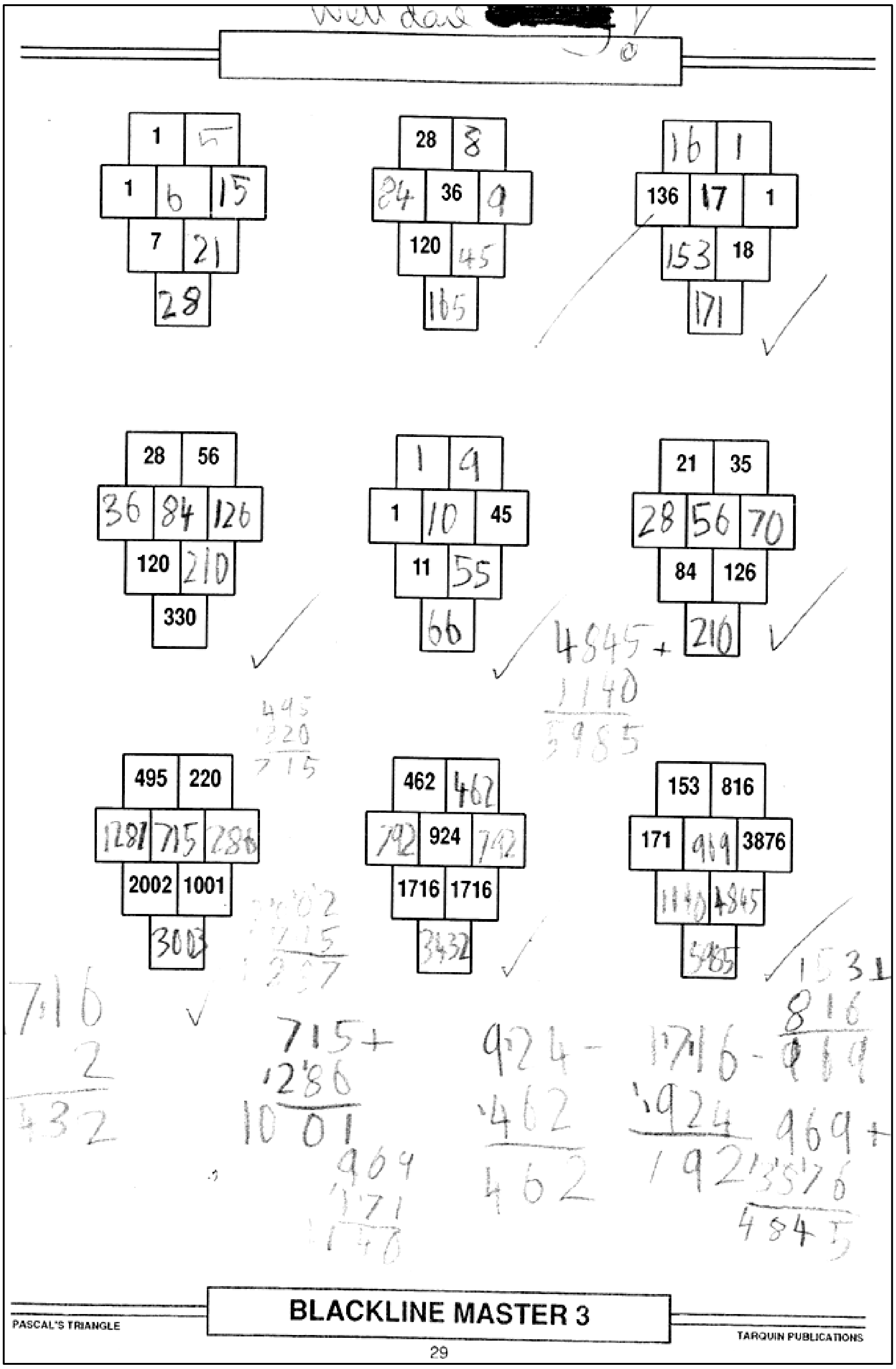
5.4.7 Children's progress

The results of two consecutive assessments (Table 5-7) showed that all five children at the end of the Spring Term have reached Level 5 of attainment. Three of them, made a significant progress moving from Level 4 to Level 5 or from Level 5C to Level 5B, while the other two of the group remained at the same high level (5B and 5C).

5.4.8 Children's performance and behaviour in class

Children's performance during the lessons revealed different levels of ability, especially when new and more advanced subjects were given (e.g., algebra, in the third lesson). For instance, in subjects that they were previously taught, in both the regular classroom and the pull-out group, all pupils seemed to know the basic mathematical facts, but some of them (e.g., Lily) had problems in application.

In Pythagoras' theorem, for example, while they all knew the theory, they were not all able to apply it in a problem and calculate the missing side of a right triangle. The same with Fibonacci numbers as they knew the pattern to find the Fibonacci sequence, but they were not able to replace the number 41 with Fibonacci numbers (Appendix 7.8). In using the patterns they, also, learned from Pascal's Triangle, although all five pupils appeared enthusiastic with a relevant exercise (especially Daisy, who said "I feel confident with these numbers!"), in practice most pupils had difficulties and needed individual help. Matthew was more effective, doing his work correctly and quickly (see a sample of Matthew's work in Figure 5-14). Because he finished earlier, he was given a jigsaw puzzle involving algebra to do as extra work. It seemed that he had done this before, because he expected it, and when he took the cards, he knew exactly what he had to do.



Source: "Pascal's Triangle: Blackline Master 3" (Colledge, 1997)

Figure 5-14: Matthew's work, Day 2

The pupils' performance in the maths-domino game proved that four out of five (Matthew, Cathy, Zoe and Daisy) understood the rules for negative number calculations and were able to apply them in the maths-domino game with success (see a representation of the game completed in Appendix 7.9.2).

In the algebra lesson, it was observed that almost all pupils had difficulties understanding the difference between the pattern in a number sequence and the sequence formula and were not able to find the latter. After many examples on the board from Claire, there was no proof that pupils, apart from Matthew, really understood the concept of a formula and its use. However, despite these difficulties that caused negative reactions, such as Zoe's ("I don't get it. I am really stuck!"), the four of them (Matthew, Zoe, Daisy and Cathy) showed that they knew the basics to calculate a mathematical expression like $\frac{1}{2}n(n+1)$ for different values of n and readiness in expressing and discussing their thoughts. Matthew, in particular, appeared more able than the others did. He found some formulas and was able to apply them properly to find different terms in a sequence. He was, also, able to convert fractions to the whole or mixed numbers and quickly calculate a mathematical expression such as this in Figure 5-15, which was about finding the 3^{rd} term of the 4^{th} diagonal of Pascal's Triangle, using the formula:

Figure 5-15: Matthew's calculations (a representation of what Matthew wrote on his board), Day 3

During the lessons, all the pupils were talking freely to the class and to each other. However, in some cases, Daisy, Cathy and Zoe were found to discuss other things instead of the lesson. On the contrary, Matthew's talking was always on task. Most pupils seemed comfortable to ask questions and talk freely to the teacher, like Daisy, who appeared to disagree with Claire about some mistakes, arguing that she had done them correctly.

Daisy also appeared to be competing with others when she said, "I'm the first one done!" and when she rushed to finish the main activity (in the second lesson) right after Matthew did to join him and work on the extra activity.

All children seemed to enjoy investigating Pascal's Triangle to identify patterns but they especially enjoyed the activities that were created by Claire, such as the decoding game in the first lesson and the domino game in the third lesson. They, also, appeared to put more effort to finish their tasks in the second lesson to join those who had finished earlier and started working on a jigsaw puzzle with algebra, also created by Claire.

5.4.9 Initial comments on the fourth case study

Provision for mathematically able children in School D is offered through a pull-out programme, which works differently from the one in School A. Firstly, the groups include pupils of different ages, so more emphasis is placed on the ability rather than age. Secondly, the pull-out groups work in parallel with the regular mathematics class, and this means that the selected children should be truly able because they have to cope with their regular mathematics within less time and do extra lessons in the pull-out group. Thirdly, the only group with pupils of the same age, the Year 6 group of my case study, works with a Key Stage 3 mathematics teacher twice a week on more advanced lessons, usually from the KS-3 curriculum.

The idea to employ a secondary school teacher to work with mathematicians that are more able is one of the strategies similar to the mentoring programme for exceptionally able children, suggested by the literature in Chapter Two, for enhancing their intellectual development and accelerating their learning. This brings out three questions: Are all the pupils in the pull-out group exceptionally able so that they need more advanced mathematics from a higher Key Stage? Is a secondary school teacher suitable to teach primary school pupils even if they are gifted or talented? What is the plan for the next step for these pupils who will be ahead?

Regarding the first question, both teachers, Julie and Claire, asserted that their pupils in the Year 6 pull-out group — who have been identified through QCA tests, internal tests, APP, and teacher’s nominations — are at Level 5 of attainment. Therefore, they are able to cover all the work in the regular classroom in three lessons, instead of five like the other children, and then with the two extra lessons in the group, they can be moved to a higher level. Indeed, in terms of attainment, their results in their assessments indicated no problems but improvement for more pupils, who now appear to all be at the 5th Level. However, the observation of the lessons showed that four of the five pupils (in the whole group) appeared confident with most of the work in the group, displaying knowledge, comprehension and application skills on basic mathematics of Year 6. In addition, in the case in which more advanced work was given, such as the work with formulas in the third lesson, only one pupil (Matthew) appeared able to cope with it.

Regarding the second question, Claire had said in the interview that even though she did not have any specific training in gifted and talented education or in teaching primary school children, her basic training in and experience from teaching secondary school students made her feel comfortable to teach this group of able pupils. Furthermore, the lessons she teaches are from the Key Stage 3 curriculum and, thus, as she said, familiar to her. Indeed, in the lessons observed, Claire was confident with the subject and well prepared. However, there were cases in which she appeared impatient when pupils were not able to answer immediately. For example, in the first lesson, she gave most of the answers in the starter exercises or was very critical, making negative comments when someone looked confused, like Lily in the domino game in the third lesson.

The third question cannot be answered by people from this school only, as it needs a broader plan that will secure these pupils' acceleration at the next stage, in the following years (i.e., secondary school). However, this is a question for future research.

In the lessons observed, Claire connected the learning objectives with knowledge gained in previous lessons, as she had said in the interview. She also used challenging activities, which made pupils think (e.g., the decoding game, investigations on Pascal's Triangle) and be engaged in discussions in which they had the opportunity to display their knowledge and skills of analysis and synthesis. Along with the materials that she chose to use, Claire, also, used materials taken from the regular classroom teacher for either starter activities or homework. In this way, she was connecting pupils with their regular classroom and the main lessons so that no gaps would appear in their progress. It should be noted that the two teachers, the regular classroom teacher and the group teacher, had a continuous collaboration and they both knew what each one was doing in his or her class. This was also confirmed by Julie, the mathematics co-ordinator, as their usual practice.

5.4.10 Issues emerged

- It seems that there is confusion about the term ‘setting’ as a grouping method. In the previous case study (School C), they called ‘setting’ a method of grouping by ability within each year group, while in this school, they called their grouping method ‘setting’, when it was actually a ‘pull-out’ programme.
- The main aim of the Year 6 pull-out programme, as it was expressed by both Julie and Claire, is to introduce pupils to the KS-3 curriculum and prepare them for secondary school. This is also what the children in the group believe. They all also seem to have as priority the success in the SATs examination. This sometimes works against the need, as it was expressed by Claire in the interview, for continuously providing the able pupils with new and more advanced mathematics, because they have to repeat lessons considered basics to secure high grades in the SATs.
- There is a mismatch between Lily’s results in achievement tests, which show higher grades (5C), and her performance in the group, in which she often appeared to be struggling and confused. In addition, her comments in the interview that she feels alone within the group when she has to do something new with which she lacks confidence prove that she is not feeling comfortable in the group. Furthermore, they show that sometimes achieving higher grades in tests, especially when there is a lot of preparation on similar ones in school or at home, does not necessarily prove mathematical giftedness.
- Although all pupils said that they prefer to work as a group, in the lessons observed, apart from the domino game, they mostly worked individually. When they had difficulties, they immediately asked Claire, who, because of the small size of the group, was able to monitor each one’s work.
- Mathematics is perceived by all five pupils as numbers and number calculations, while some of them consider mathematics a subject that involves logic and thinking or requires some kind of gift. Furthermore, two pupils believe that mathematics is a useful tool to get a good job and succeed in life.
- Most of the pupils said that they would like more games in their lessons, new and more challenging subjects and more teamwork on hard problems, while Matthew asked for differentiated work.

- All pupils expressed positive attitudes towards mathematics and the lessons that they do in the group, and described the ones involving puzzles and sequences (e.g., Fibonacci numbers) as their favourites. Julie believes that the positive attitude towards mathematics, along with the improvement in using and applying mathematics, is a result of the work taking place in the pull-out group. She therefore believes that the implementation of this programme for the first year is successful. The only thing that she worries about is the "funding", because it is difficult for a small school to pay for an extra teacher like Claire.

5.5 Summary

This chapter presented and briefly discussed the findings from four case studies, which was the main study of the research. I described how I selected four teachers and twenty pupils and how I conducted my case studies. I have presented a snapshot of what was happening in their lessons, along with their perceptions and attitudes expressed in their interviews. I have also provided my analytical comments on what I discovered and the issues that emerged from each case study. These issues will be grouped together and discussed further, in more detail, along with my findings from the first phase of the research, in Chapter Six.

6 Chapter Six: Discussion

This thesis explored the range of strategies for educational provision for gifted and talented children in general, and specifically in mathematics, through a study of international theory and research, and by carrying out an empirical study in primary schools in England. The former were presented in Chapter Two. In that chapter, it was also highlighted that the concept of mathematical giftedness is complex, and a range of views regarding definition and identification were presented. However, despite the complexity of the issue, there is an agreement that mathematical giftedness can be developed within schools through the appropriate educational programmes and well-trained teachers of the gifted. The review of the literature provided a framework which helped to conduct the research, reported in this thesis, in English primary schools in London.

It was said in previous chapters that in the UK, there is comprehensive literature provided by the government, through the national strategies, relating to the education of gifted and talented children. In contrast, there is no national training programme for coordinators and teachers for provision specifically for mathematically gifted children (NCETM, 2009; Williams, 2008). The training courses in teaching mathematically able children, which some teachers claimed to have completed, were independent training courses occasionally organised by some schools or LEAs; courses organised by independent organisations such as the Brunel Able Children's Education (BACE) Centre at Brunel University (where I attended such a course); or were part of teachers' initial training in another country such as New Zealand (in the case of Emma). It was also mentioned earlier that there is little research in primary schools regarding provision for mathematically gifted children.

The purpose of this study was to find out what is happening in schools and to gain insights. The research was carried out in two stages seeking answers to the following research questions:

1. What strategies are schools using, if any, regarding the education of gifted and talented children in general and specifically in mathematics?
2. What are the teachers' perceptions of and attitudes towards mathematically gifted children, their education and the methods used by their schools?

3. How are the needs of mathematically gifted children met within classrooms in everyday practice?
4. What is the impact of the schools' strategies on pupils' achievement and attitudes?

The first stage, which was the preliminary phase, gave me the first insight into how maintained primary schools in a large area of Greater London addressed the needs of mathematically gifted children, as well as useful data about how teachers perceived gifted mathematicians, and the level of their own confidence in teaching these children. The second stage, which was the main study, involved four case studies in mathematics lessons, and explored in depth how specific methods of provision were used in practice as well as their impact on pupils' achievement, perceptions and attitudes. Each case study class was from a school that made provision for mathematically able children, but using a different method: (i) a pull-out group of Year 5 able mathematicians doing extra lessons in addition to the regular mathematics in School A; (ii) a higher-ability group of Year 2 pupils working within the regular classroom in School B; (iii) a group of Year 5 able pupils working in a top mathematics set in School C, and; (iv) a pull-out group of Year 6 gifted mathematicians working with a mentor, a secondary school teacher in School D.

All the data gathered and analysed from both stages of the research have contributed to answering the research questions as well as to highlighting aspects that may be of interest to audiences in the UK and in other countries, on aspects of specific provision for the education of mathematically gifted children. The findings were presented in Chapters Four and Five. In this chapter, these findings are discussed with reference to the theory and previous research presented in Chapter Two, along with my own commentary, which includes implications for practice. The discussion is structured under four main themes, which are the four research questions, so that it can be seen how the questions have been answered. Each research question is separated into subthemes which emerged from the thematic analysis of the findings presented in previous chapters.

6.1 First research question

"What strategies are schools using, if any, regarding the education of gifted and talented children in general and specifically in mathematics?"

6.1.1 Existence of a policy for identification and provision

Having a policy for the identification of gifted and talented children in different domains, and maintaining a register for them, is required for all schools as set up by the UK government (DCSF, 2008a). The National Strategy (DCSF, 2008b) provides guidance for all schools to organise effective provision for their identified gifted and talented children in all domains, such as in mathematics, within and also outside of the classrooms. However, it seems that schools have mainly conformed to the first requirement concerning identification rather than the second one regarding provision. For example, this was the finding which emerged from the present research, which found that almost all participating schools (43 out of 44) claimed to have a policy for the identification of their gifted and talented children and to keep a gifted and talented register (13 of these schools maintain a separate register for gifted mathematicians). The percentage of their registered gifted and talented pupils ranges from one to twenty percent, most commonly being between seven and ten percent for the general register, and four to six percent for the separate one (for gifted mathematicians). These differences in the percentages of identified gifted pupils are now accepted by the National Gifted & Talented Strategy (DCSF, 2008a), which allows schools to select any number of gifted and talented children — instead of the five to ten percent that was required in earlier years (DfEE, 1999a) — for their register in order to make adequate provision.

On the other hand, not all of these schools seemed to make provision for their identified children (34 out of 44). This may be because teachers felt more at ease with identification rather than with provision, especially when the former is based on standardised tests (something that appears to be the usual practice in the schools studied, but this is explained later). Making provision for gifted children seemed to be considered by teachers as a more difficult and demanding issue which required extra work.

However, beyond the question of whether or not schools have a policy of identification and provision, there is another important question: Does the existence of a policy ensure that the needs of mathematically gifted children are effectively addressed within schools?

Findings reported in previous government-commissioned reports and surveys suggest a "no". For instance, when the *Excellence in Cities* (DfEE, 1999a) initiative was launched, the schools in inner city were required to identify their 'gifted' pupils and provide them with a distinct teaching and learning programme. However, Ofsted inspections (Ofsted, 2004) in schools that took part in the EiC initiative — and, thus, in schools that had implemented a new policy with specific guidance — revealed that gifted pupils were not sufficiently catered for, and that one in five schools in the inner city areas did not identify or systematically measure the achievements of bright pupils.

The research presented in this thesis also suggests a "no". Looking, for instance, at the teachers' responses to the questionnaire, we can see that there is one teacher, at least, who responded that he or she was not sure if the G&T register that they maintain, according to the National Strategy (DCSF, 2008a) requirements, consisted of truly gifted children and, therefore, about whether provision was being made for the right pupils. This teacher pointed out: "The LEA has asked us to have a specific quota for G&T pupils. I worry this is artificial and would doubt that all those on the register are truly gifted" (A mathematics coordinator, School 42). The case studies also showed that teachers were not always doing what their school policy said. School B, for instance, appeared (through the questionnaire) to use a range of different methods for the identification of gifted and talented children, including parental nominations and discussions with children (see Table 3-1, which presents the participating schools, in Chapter Three), but, in practice, was found to use only classroom teacher assessments and children's results in achievement tests.

Another example that supports the above argument that simply having a policy does not solve the problem, is the case of the teacher, responsible for mathematics, in School C (third case study), who did not know (according to her responses to the questionnaire) that her school had a general policy of provision for gifted and talented children and a responsible coordinator for this, since 2006.

However, do all the aforementioned mean that no good practice exists regarding the education of gifted and talented and specifically of mathematically gifted children? It was in fact found that most of the schools that made specific provision for particular subjects appeared to do this for mathematics (29 out of 34). This is an encouraging finding because, as it was explained in Chapter Two, mathematically gifted children have special abilities, such as a ‘logical-mathematical intelligence’ (Gardner, 1983) or a ‘mathematical cast of mind’ (Krutetskii, 1976) that form mathematical giftedness; but, in order for this to be developed within schools, specific provision is needed (Gagne, 2004b, 2007; Gardner, 2006; Renzulli & Reis, 2003; Sternberg, 2003a) — ‘subject-specific’ provision, according to Koshy (2001) or ‘content-specific’ provision, according to VanTassel-Baska (1992). This finding showed that over half of the reviewed schools made specific provision for mathematically gifted children (many of them maintaining a separate register for them). Therefore, although this itself, as mentioned earlier, is not enough to prove that the needs of mathematically gifted children are effectively met in those schools, it does show that there is an awareness that these children need to be identified and, consequently, catered for within primary schools. However, the issue of whether and the extent to which the needs of these children are met within classrooms is discussed in the following sections.

However, the fact that there are many schools that do not have a specialist in mathematics to organise and teach lessons in higher levels of mathematics — as the Williams (2008) report suggests — and schools that do not make provision for any domain, or teachers who do not know their school policy, suggests that more effort is needed from policy makers, LEAs, or the heads of the schools, to raise the awareness of teachers on this matter. These schools may need some help and support to form a policy for provision from specialists and/or funds to employ a teacher with a background in higher mathematics in order to help them to plan mathematics lessons at higher levels suitable for gifted children. In addition, any kind of support offered for schools should be combined with a programme for teachers’ professional development; but this will be discussed later in more detail.

The following sections discuss the methods that the schools use for the identification of and provision for mathematically gifted children. This is based both on what the teachers said that they did (through the questionnaire) and on what was observed by the researcher through the case studies. It seems that there are some mismatches between what the teachers said that they did and what they were actually doing, as earlier presented, and this can also be seen in the following sections.

6.1.2 Methods used for the identification of mathematically gifted children

Although the focus of this study was on provision strategies for mathematically gifted children, identification issues were studied and are discussed here because identification is considered as part of an integrated provision for gifted and talented children (DCSF, 2008b).

Teachers' responses to the questionnaire showed that the methods that they mainly use to identify their gifted mathematicians are a range of tests, teacher assessments, the Attainment Level Descriptors provided by the National Curriculum (QCDA, 2010b) and teacher nominations. Achievement tests seem to be their main criterion for the identification of mathematically gifted children.

Achievement tests

Using the Attainment Level Descriptors provided by the National Curriculum (QCDA, 2010b) and criterion-referenced achievement tests (e.g., SATs and QCA tests), teachers seem to have found a practical way to split their classes into ability levels (e.g., School B from the second case study) and also to allocate pupils in sets (e.g., School C) or pull-out groups (e.g., School A and School D) for more able or gifted mathematicians. All twenty pupils observed through the four case studies were identified through achievement tests which take place at the end of each school term. In some cases, teacher assessments and nominations, based on what was observed in the classroom, were taken into account (e.g., School B, School C and School D). Even then, the results from standardised achievement tests were used as a criterion by two of the case study teachers in order to support their decisions for the allocation of children to a particular ability group/set, or their movement between groups/sets against parents' complaints. It seems that this quantitative evaluation of children's progress, based on standards such as the Attainment Level Descriptors (QCDA, 2010b), gives confidence and a sense of security to teachers to make decisions about the place of each child in class.

However, this heavy reliance on achievement tests for the identification of gifted mathematicians did not always succeed in identifying gifted mathematicians who could cope with more advanced topics that a special programme for the gifted involves. It was found, for instance, that case study pupils in special groups (School A, School C and School D) who were chosen because of their higher grades in standardised achievement tests, and who admitted during the interviews that they had systematic preparation on similar tests at home and continuous support from parents by doing mathematics from commercial books on topics that they were going to do in school, seemed to struggle with the advanced work required in the lessons observed. This finding reflects McClure's (2001) argument that there are cases in which high performance in mathematics tests does not prove mathematical giftedness, as, for instance, the exceptional ability in numeracy that a child displays at an early stage because of parental involvement.

Other studies have also found that achievement tests cannot identify some categories of gifted children, such as gifted children with learning disorders (Karolyi, Ramos-Ford & Gardner, 2003), with language problems (Koshy, 2001), the gifted minority and low-income students (VanTassel-Baska, Feng & Evans, 2007; Naglieri & Ford, 2003), because they require children to answer verbal and quantitative questions to receive high scores, while many gifted minority children, children from families with low educational backgrounds or children with learning disorders lack such reading and writing skills.

For these reasons, it is now widely accepted that there is not a single and unique method to identify mathematical giftedness (Eyre, 2001; Koshy, 2001; McClure, 2001) and that the best way for the identification of gifted and talented pupils is a continuous 'whole-school process' (DCSF, 2008b), which should draw on a range of information by utilising different sources through a teacher who is well-trained with regard to recognising giftedness (Eyre, 2001). The different sources that were used in the schools that took part in this research are now discussed.

IQ, cognitive and ability tests

Only a few schools were found, through the questionnaire, to use IQ tests (three schools), cognitive tests (four schools) and ability tests produced by NFER Nelson (recently changed to GL Assessment) (four schools) in order to identify their gifted mathematicians. IQ, cognitive and ability tests are useful for the identification of gifted children. IQ tests, for instance, are useful for measuring the intellectual domain of intelligence (Gagne, 2004a, 2004b). However, IQ tests are limited in one dimension, which relates to verbal and analytic skills (Renzulli, 2004) and, thus, like achievement tests, cannot measure other types of intelligence such as creativity, or 'creative-productive giftedness', as Renzulli suggests (1999).

A test that schools can use to measure aspects of mathematical ability without needing verbal skills and which is, therefore, appropriate for gifted pupils with language problems was not used in any school that took part in this research — the Naglieri non-verbal ability tests (Naglieri & Ford, 2003), which focus on the ability observed through spatial or logical organisation by utilising shapes or geometric designs for example, but not on the answers given on verbal and quantitative questions which normally characterise the other type of tests. In this test, children are not required to read, write, or speak. This kind of test on shape and space, if used by teachers, could help them to identify creative and spatial ability, both of which are associated with mathematical giftedness (Gagne, 1985, 2003; Gardner, 1983, 1999; Renzulli, 1978, 1998; Sternberg, 1997, 1999). There are, of course, other non-verbal tests that teachers can use for identifying creativity, spatial ability and mathematical potential. Some of these options were used by many schools, as was found by this research. These are discussed below.

In-class/teacher assessment and teacher nomination

In-class/teacher assessment and teacher nomination are discussed together here because the analysis of the questionnaire responses showed that both nominations and assessments are based on what teachers observe in their classrooms (e.g., pupils' performance and behaviour) as well as on some informal tests.

It was found that many of the schools surveyed use teacher assessment and teacher nomination in combination with tests (24 out of 44 schools) for the identification of their gifted mathematicians. Also, three out of four case study schools, as mentioned earlier, use teacher assessment, and teacher nomination along with achievement tests to support their decision to either allocate pupils in specific ability groups/sets, or to move them between groups (e.g., School B, School C and School D).

Assessment that is based on teachers' observations is recommended by Gardner and his colleagues (Gardner, 2006; Karolyi, Ramos-Ford & Gardner, 2003) as 'intelligence fair' assessment, when it involves observations of children's performance, behaviour and working styles under circumstances that encourage gifted behaviour to be displayed, when it is ongoing and when it does not confound intelligences. The issue of giving suitable opportunities that help giftedness to be displayed is very important, because mathematical potential is not easily observable (Koshy, Ernest & Casey, 2009) and because there may be cases in which mathematical ability is disguised because of certain fears, such as fear of having to do 'extra' work (Koshy, 2001) and fear of being labelled as a 'nerd' or 'geek' (Sheffield, 2003).

Nominations from current but also from previous teachers, are also believed to support the identification of gifted mathematicians (DCSF, 2008b), but on condition that the teachers have been trained in what to search for (Freeman, 1998). This is because an untrained teacher may confuse gifted children with tidy, neat and conforming children, which may result in 'inaccurate and dangerous' assessment (Eyre, 2001). This kind of assessment may also be subject to teachers' biases and limited to what is only observable, as described earlier in relation to teacher assessment, and, therefore, it should be combined with nominations from other sources, such as from parents or peers, as suggested by the National Strategy (DCSF, 2008b).

Nominations from parents and peers

Parent nominations

This research has found from the responses to the questionnaire that a small number of schools use parent or carer nominations (six schools out of forty-four). However, the case study in one of these schools (School B) raised questions about whether this method is actually used, or whether the teachers who replied to the question simply described their school's written policy for the identification of gifted and talented children. In School B, for instance, which appeared to involve parents and carers in the identification of gifted and talented children, as well as in provision, it was found that this does not actually happen, at least in relation to mathematics. On the contrary, it seemed that parents' and carers' involvement was found to be a problem by both the deputy headteacher, who was also the G&T coordinator of the school at that time, and the case study teacher (Sarah). The former, for instance, described parental involvement in the identification and provision as the number one problem, because it was said that almost all of the parents believe that their children are gifted.

The latter view is not new, as previous research (Davis & Rimm, 1985) also found this. But, parent nominations are described as helpful for identifying mathematical ability at early stages, because parents may notice children's abilities in mathematics before they start school (Straker, 1983). They can also give information about the aptitudes and abilities of their children, which may reveal indicators of mathematical ability (e.g., a fascination for numbers, an ability to spot number patterns, or make sophisticated constructions) (Koshy, 2001). Therefore, it would be wise for schools to use this source of information as much as possible. Teachers could evaluate parent nominations for possible bias. This could also be part of their training.

Peer nominations

Only two of the surveyed schools appeared to use peer nominations for the identification of gifted mathematicians, despite the suggestions for the use of this (DCSF, 2008b; Gagne, 2004a, 2004b; Koshy, 2001) and the views of some teachers, like Sarah from School B, who believes that children always "know who is good at maths". Furthermore, the case studies showed that peer nominations could be a reliable source for identifying able mathematicians. There were, for example, children in the interviews, like Alvin, who named good mathematicians with whom they would like to work as a team, and children, like Amardeep, who were recognised by the others when they all tried to solve a problem on the whiteboard. Based on what was experienced in the classrooms and expert views cited earlier, it would make sense for schools to use peer nomination as one of the ways of identifying mathematically gifted pupils.

Other suggested methods for the identification of gifted mathematicians

The following methods were not found in any of the schools that took part in this research. They are discussed in this section because they are practical methods which can be easily used by classroom teachers and which, furthermore, do not demand changes in the curriculum and everyday programme nor any extra cost.

Pupils' portfolios

The use of portfolios can support teachers in their identification of able mathematicians, as they can collect evidence of high levels of pupil performance through their portfolios (Eyre, 2001; Feldhusen, 2001; Gardner, 1992; Karolyi, Ramos-Ford & Gardner, 2003; Koshy, 2001; Renzulli & Reis, 1985).

Characteristics checklists

The use of characteristics checklists, such as those suggested by Krutetskii (1976) and Sheffield (2003), can also help teachers' observations become more effective as they can help them to better understand the attributes of mathematically able or promising children (Eyre, 2001; Feldhusen, 2001; Koshy, 2001; Freeman, 1998). Furthermore, there is evidence from previous research that teachers who used a checklist of characteristics of mathematical gifted behaviours felt more confident to recognise mathematically able children, and understood that they cannot rely entirely on the results from achievement tests, which could be affected by external factors (Koshy & Casey, 2005).

Identification through provision

As explained earlier, the assessment of a specific type of giftedness, like mathematical giftedness, should be 'intelligence fair' and ongoing through an environment that will provide opportunities for the specific type of giftedness to emerge (Gardner, 2006; Karolyi et al., 2003) and be identified. This process of identification has been described as identification through provision (Freeman, 1998; Koshy, 2001; Koshy & Casey, 1997a). The idea is based on views, such as those of Krutetskii (1976), that we do not know how far the mathematical ability may go unless it is continuously challenged. This means that identification should not always come first, but should sometimes follow the appropriate provision. However, in order for this to succeed, schools must use a flexible programme of identification and provision, which will follow a circular process: identification–provision–identification (Koshy, 2001) or what Freeman (1998) calls 'Sports Approach'. According to Freeman's (1998) approach, children are offered a range of opportunities of progressively increased difficulty, aiming to help hidden abilities to be revealed and identified. In practical terms, this also means that schools that make provision for their gifted pupils through special grouping arrangements, must, at the same time, give opportunities for pupils in the regular classrooms to display possible hidden abilities. Additionally, pupils in the special groups should be continuously challenged (‘Sports Approach’), looking for exceptional ability that may need something even more special, such as one-to-one lessons (Sheffield, 1999) or acceleration.

6.1.3 Strategies of provision for mathematically gifted children

This section discusses organisational strategies for provision that schools use for their gifted mathematicians. Strategies for learning mathematics at higher cognitive levels along with their impact on pupils’ progress and attitudes are discussed later on in sections 6.3 and 6.4. It was found, through the questionnaire, that most of the thirty-four schools which claimed to make provision for mathematically gifted children use within-classroom provision (eighteen schools). However, there are many schools that also use special grouping programmes beyond the regular classroom for children identified as more able or gifted, such as setting (thirteen schools) and pull-out programmes (three schools). Some of these schools also combine ability grouping with mentoring (five schools) and strategies for acceleration (five schools), but only in a form that allows gifted pupils to bypass certain classes and attend more advanced lessons (e.g., in special groups for gifted mathematicians or in higher-year classes). Acceleration in a form that allows gifted pupils’ early entrance to or exit from primary school, or rapid movement through it, was not found in any of the schools that took part in this research.

Furthermore, eleven of the schools that make provision for mathematically gifted children appeared to combine within-class ability grouping with setting (nine schools), pull-out grouping (one school) or setting and pull-out grouping (one school). These combinations raise the number of schools that use within-class ability grouping to twenty-nine, making this type of grouping the most commonly used type amongst schools that make specific provision for mathematically gifted children.

The latter finding, about the use of ability grouping, revealed through the first phase of the research (the questionnaire survey), shows that schools that make specific provision for mathematically gifted children follow recommendations about using a variety of grouping approaches in classrooms for each subject (DCSF, 2008b). Ability grouping, in particular, for a specific area of curriculum — even for a part of their schooling day (within or outside of the regular classroom) — has been suggested by previous research (Kulik, 1992; Kulik & Kulik, 1991, 1992) as being beneficial for gifted children in specific domains as it produces substantial academic benefits in achievement, improves pupils' attitudes towards school and strengthens their self-efficacy.

However, despite the abovementioned benefits of grouping by ability for gifted pupils, there are some questions that need to be raised regarding whether a grouping method itself can ensure differentiated learning and, with respect to mathematics, learning at higher cognitive levels. For instance, many educational researchers with expertise in studying methods of teaching mathematics to more able or gifted children suggest that the idea of grouping by ability itself cannot meet the needs of mathematically gifted children. They argue, therefore, that teachers must be prepared to engage their pupils in higher-level cognitive activities (Casey 1999, 2002; Koshy 2001; Sheffield 1999, 2003). Gifted children may work in a higher ability group, but the work that they do in the group may be repetitive work relating to what they have already learnt, or just more difficult work that sometimes even gifted children cannot do by themselves. To avoid this happening, a grouping programme must be part of a well-organised programme for differentiation of the curriculum. Such a programme, according to VanTassel-Baska (2007), should first identify appropriate goals and outcomes, what is important for these students to learn and at what stages of development they will be able to do this; and then, should provide these students with experiences adequately different from the norm and differentiated instructions through flexible grouping and trained teachers of the gifted.

These requirements are now discussed below in relation to the main organisational strategies for provision that the schools were found to use. These were provision within the regular classroom, provision through setting, and provision through pull-out grouping. Issues of enrichment and learning mathematics at higher cognitive levels are discussed in section 6.3.

Within-classroom provision

As shown through teachers' responses to the questionnaire, grouping by ability is the practice that most schools use — schools with a policy of provision (twenty-nine) or without (eight) — to address the different levels of ability that exist in a regular classroom. Most teachers from these schools responded that they combine ability grouping with mixed-ability (twenty-three schools) in their classrooms. Only one school was found to make provision for more able mathematicians by giving them differentiated work in a mixed-ability setting. All teachers that use grouping arrangements within their classrooms appeared to rearrange their groupings throughout the year, either from one ability group to another, depending on pupils' progress (fifteen teachers), or from lesson to lesson according to the topic and the activity (fourteen teachers). Such movements between groupings are considered useful and it has been suggested that they should be involved in a differentiated curriculum (VanTassel-Baska, 2007). However, the issue of movement between groups, especially between ability groups, is not simple and is, therefore, discussed in a separate section below.

Previous research into the effects of different grouping arrangements carried out in UK primary schools has shown that although within-class ability grouping is considered a flexible grouping practice that allows opportunities for the whole class, there was no evidence of benefits for able or gifted pupils, but mostly for those who were considered weaker (Boaler, Wiliam & Brown, 2000). In order for this type of grouping to be successful for the whole class, it must be supported by appropriate teaching materials and a pace that is suited to the needs of the children (Harlen & Malcolm, 1999). Additionally, in order to successfully meet the needs of more able children, it must be combined, in every lesson, with differentiated work and teaching materials that should go beyond the single textbook, as well as with differentiated instruction (Tomlinson, 1995; VanTassel-Baska, 2007).

This research has found that most of the participating schools (thirty-three) use differentiation practices that involve grouping arrangements and differentiated work for more able pupils independently of whether or not they have a policy of provision for gifted and talented children. However, there is a difference in the frequency when grouping is organised by teachers and the frequency when different and extra work is given to more able pupils. Most teachers (thirty teachers) organised their groupings in every lesson, but only few teachers use extra support materials and differentiated work for their pupils in every lesson (nine teachers). The latter does not agree with the aforementioned recommendations for suitable materials and differentiated work in every lesson and, therefore, is an issue requiring further development in many schools.

Furthermore, the case study in one of the schools that implements within-classroom provision through grouping by ability showed that the percentage of schools that do not systematically use differentiated work may be bigger than it appears in teachers' responses to the questionnaire. For instance, in the lessons observed, Sarah from School B, who appeared (through her responses in both the questionnaire and the interview) to always prepare different work which was suitable for each ability group (lower, middle and higher), was found to mainly use the same teaching materials for the whole class. These materials, which were from higher levels of the syllabus and for older children, with the appropriate instructions and questioning from Sarah, worked well for the able pupils, but caused the other children to keep silent most of the time. Sarah gave different work once and this was only for the group of able pupils. However, this different work was given, not because it was selected as most suitable for higher ability students, thus, challenging them and extending their ability, but — as she explained — to enable her to work with those of middle and lower ability who had difficulties on some basic mathematics. In other words, this was done to keep the able children occupied while she supported the others. This practice has been criticised by Fielker (1997) as the weakness of within-class ability grouping that often serves the administrative convenience of the teachers rather than the different needs of the children. Sarah also admitted during the interview that this is something that she always does when she wants to help pupils of average or lower ability. She gives easier work to the able ones instead of a truly challenging one because she wants to be sure that they can work without help.

The aforementioned examples show that provision for more able pupils within the regular classroom is not an easy task, because apart from splitting the class into ability groups, teachers need to ensure that differentiated and suitable work should always be provided for every group, along with the appropriate instructions and attention.

Setting

Teachers' responses to the questionnaire showed that thirteen schools use setting in mathematics as grouping strategy for provision. This strategy has been recommended by the National Numeracy Strategy (DfEE, 2000b) for teaching gifted children. Setting is also recommended by educational researchers as a method of provision for able children in mathematics because it narrows the gap between those who cannot cope with their work and those who can learn faster and, therefore, helps to ensure homogeneity in ability. This homogeneity, on the one hand, may help teachers to prepare and provide differentiated work more easily through a class that is especially created for able pupils and, on the other hand, may benefit able pupils by engaging them in more challenging activities suited to their abilities (Koshy, 2001).

However, it seems that the abovementioned advantages of setting were not used by all schools that appeared, through the questionnaire, to use this method. For instance, with regard to the case study in School C, which was one of the thirteen schools that claimed to use setting in mathematics, it was found that its setting programme was restricted to one year group only (e.g., Year 5), which was separated into three ability 'sets' (lower, middle and top). The case study teacher (Kate) of Year 5 top set consequently did not have a homogenous group to teach. Instead, she had a large-sized class with different levels of ability — thirty-one pupils separated into in-class ability groups (lower, middle and higher) — and, therefore faced problems similar to those that teachers face in the regular classrooms. The higher-level work, which Kate chose for the whole set, combined with the lack of confidence that Kate felt to teach more able mathematicians (something that she admitted in her interview) increased these problems, leaving the group of more able pupils working by themselves, unable to complete their tasks. More about the role of teachers' confidence, as well as the work and its impact on pupils' performance and behaviour is discussed in sections 6.2 and 6.3 respectively.

The type of grouping, therefore, that was found to be used in School C was a variation of grouping by ability that worked on a full-time basis but which was not a real setting, as suggested by the literature. The literature suggests that setting reduces the gap between those who have difficulties in learning mathematics and those who can learn more, because the same-ability pupils are set together independently of their age (Koshy, 2001). In other words, each set may have pupils from different year groups having as priority the homogeneity of ability rather than age.

Teachers from School D also appeared to confuse setting with pull-out grouping, because their groups (apart from Year 6 pull-out group) consisted of able pupils from more than one year group. These examples show that teachers may not have thought about issues of setting in the traditional sense, therefore emphasising the need for appropriate and continuous training.

Pull-out groups

Only a few schools (three schools) from those participating in the research were found to organise pull-out grouping programmes for their more able or gifted pupils in mathematics. Two of these schools have recently started this programme and took part in the case study. Therefore, despite the small number of schools that use pull-out grouping, the fact that two of the three have just started and wanted to contribute to this study, shows an increasing interest in this method.

Pull-out groups have been suggested as more appropriate for meeting the needs of gifted or exceptionally able children whose needs are difficult to meet in a regular classroom or by the regular teacher (Sheffield, 1999). Renzulli (1987) also advocated that gifted pupils' interests are not taken into consideration within a regular classroom and, therefore, a pull-out group can engage them with more challenging activities.

Both pull-out groups in the case studies were taught by teachers with higher levels of confidence and subject expertise. Emma, a mathematics coordinator in School A, had initial training in New Zealand with an emphasis on teaching mathematics to able children, and in-service training specifically in identification of and provision for gifted and talented children through some short courses organised by a Local Educational Authority in London. Claire, a secondary school mathematics teacher in School D, had initial training for Key Stage 3 mathematics and, thus, in higher level mathematics. They both had no problems, within their small-sized classes, with continuously monitoring children's work, mediating, challenging and extending their abilities through questioning, as is discussed later (section 6.3) in more detail.

Mentoring through pull-out grouping

Mentoring was found, through the first phase of the research, to be used by five schools. One of them was School D, which employed a Key Stage 3 mathematics teacher (Claire) to work with identified gifted mathematicians in Year 6. The case study in Claire's class showed that the children accepted with enthusiasm the idea of being taught by a secondary school teacher. In their interviews, all five children said that they do more interesting lessons with Claire than with the other teacher in the regular classroom, and that they keenly awaited these lessons (e.g., "I look forward to Fridays and Mondays" (Lily).) Therefore, it seems that the idea works according to what is suggested by experts; that a teacher with a background in higher mathematics can provide promising students with more challenges (Sheffield, 1999), and that a mentor with subject expertise can act as a role model and bring forth enthusiasm for the subject and inspire the pupils (Koshy, 2001).

The choice to use a secondary school mathematics teacher as a mentor was made, as the teacher in charge explained in the interview, because they wanted to introduce the gifted children from Year 6 to the KS-3 mathematics curriculum. However, this generates some questions, such as: Is this part of a plan for accelerating these children? Will these children repeat the same content when they join secondary school? Is there any collaboration between primary and secondary schools in relation to the transition of these children? How does a programme demanding extra and different lessons work in Year 6, which usually emphasises preparation for SATs examinations? Is a secondary teacher without experience in primary education suitable to teach primary school pupils even if they are gifted or talented?

There were no plans for acceleration for these pupils or any collaboration with a secondary school. There was only an intention expressed by Julie, the teacher responsible for the programme, for future collaboration with secondary schools with respect to pupils' transition. With regard to the third question, the intent, as it was expressed by the teachers, was to introduce children to more advanced topics. In practice, however, this was moderated by an interest in the SATs examinations that were going to take place at the end of the year, about which both teachers, Claire and Julie, seemed to be very much concerned. This interest required pupils to be tested in past papers and to sometimes do repetitive work in order to secure higher grades in SATs. Here, there is another issue with which schools should deal — finding a balance between the need for preparation for the examinations and the need for enrichment through learning in depth more and advanced mathematics.

With regard to the fourth question, there is no doubt that Claire had the subject knowledge and the enthusiasm, both of which are required to act as a role model and inspire the pupils, according to Koshy (2001). However, the question needs to be raised about the lack of experience in teaching younger pupils. In the lessons observed, there were cases in which Claire, apart from prompting and praising the children like the other teachers, made negative comments to those who looked confused or made mistakes. This may have negative effects on those with a low level of confidence, making them feel more stressed with every new topic, such as Lily (who, it should be noted, was systematically one of the highest achievers in formal achievement tests). There were also cases in which Claire, seeming to become impatient when pupils were not able to answer immediately, gave the answers herself. Therefore, this seems to be an issue for consideration. Schools may need to organise some training courses for mentors to familiarise themselves with teaching styles in primary schools before they are sent to teach primary children. It is possible that a teacher with knowledge of higher mathematics but who may not have the suitable pedagogical approach to negatively affect motivation and attitudes which are important in the learning process, as this is described by Ernest's (1985) 'success cycle' of learning.

The integration of pull-out programmes with the curriculum

One of the main arguments against the pull-out programmes, which take place outside of the regular classroom, is the lack of communication and articulation that often exists between the pull-out class and the regular classroom (VanTassel-Baska, 1987).

The two pull-out programmes that were studied worked differently. Emma's pupils were pulled out not from the regular mathematics class, but from another subject (different every time) and that was only for 45 minutes every two out of three weeks. Claire's pupils were pulled out from the regular mathematics class twice a week for the whole lesson (60 minutes). In the first place, there was no case of anything missing from the regular mathematics curriculum, as whatever the selected children did in the pull-out group was additional. The question here might be what the children are missing from the other subjects from which they are pulled out. However, this question was not investigated by this study.

In the case of Claire's pull-out group, there was the danger of losing communication with the normal mathematics curriculum, because the selected children were skipping regular lessons and doing advanced topics from the KS-3 mathematics curriculum. However, there was good collaboration between the regular classroom teacher and Claire regarding the basic subjects that the children should know, so that no gaps in their knowledge occurred. There was also continuous assessment — with a focus on SATs tests, as mentioned earlier — and pupils who were found not to have secured the basics returned to the regular classroom. However, this practice of focusing on preparation for SATs examinations, worked in the opposite way from the targets of the pull-out programme (e.g., to continuously provide the able pupils with new and more advanced mathematics, as Claire explained) mentioned earlier.

Another practice that helped in having good communication and articulation between the pull-out group and the regular classroom was the use of starter lessons and homework (in Claire's case). Both teachers used the starter lessons to connect the present with the previous lessons and to reinforce previous knowledge — this also helped to connect the lessons taking place in the pull-out groups, as these were not on an everyday basis. Furthermore, Claire used activities from what the other teacher was doing in the regular classroom as starters or as homework, thereby helping the children to be connected with the main curriculum. This idea seems to work well as it helped selected pupils to do more advanced subjects outside of the regular class, without losing basic parts of the regular mathematics (as their results in formal assessments showed). Therefore, this could be an answer to the criticism that has been voiced (e.g., VanTassel-Baska (1987)) against pull-out programmes regarding the level of communication and articulation with the regular classrooms.

Movement between ability groups

This research, like the previous Davies, Hallam and Ireson (2003) study, showed that although all teachers who use grouping arrangements in their mixed ability classrooms — including Sarah, and some of those who organise and teach sets, like Kate — value the need for flexibility in grouping and the movement between sets, in practice, movement from a higher to a lower group or set seems to rarely take place. Moreover, it was found that decisions for such a movement are correlated with the level of monitoring of children’s assessment (e.g., the existence of a tracking system of their progress) and the level of confidence of the responsible teacher.

For instance, it was found that moving a child to a lower level sometimes caused difficulties such as parents’ complaints. In both the questionnaire survey and case studies, teachers avoided moving a child to a lower group or taking him/her out of a pull-out group and sending him/her back to the regular classroom. Emma, for instance, explained that this happens even if a child cannot follow the lessons in the pull-out group and that she is not saying, “You are not coming to the group anymore”, but is giving him/her time “to settle in”.

On the contrary, the other three teachers who took part in the main study (i.e., Sarah, Kate and Claire) appeared certain that children should be moved to either a higher or a lower group/set — or moved back to the regular classroom, in the case of Claire’s pull-out group — depending on their progress and also on their ability to collaborate effectively with the other members of the group. These three teachers did make such movements, at least once, either in the school year in which the research took place (Kate and Claire), or in a previous year (Sarah). These three teachers were also confident that pupils are ready to accept such changes because they all know where their places are in the class (Sarah), or because they are ready to accept their assessments at the end of every half term (Kate) or the decision of the teachers (Claire). Sarah, Kate and Claire were also confident that they could explain this to the parents and help them to accept a movement to a lower ability group if necessary (“[Parents] usually accept that some children achieve more at the top of the middle rather than the bottom of the top and that boosts their confidence”, Kate said). In the case of Kate, who did not show similar confidence in teaching mathematically able children, it seems that the Target Tracker programme and the whole school assessment record that they use to monitor children’s progress, as well as the collaboration with teachers from other sets and the mathematics coordinator before they decide who needs to change sets, play an important role in this confidence.

6.1.4 Summary

Almost all schools reviewed (forty-three out of forty-four) have a policy for the identification of mathematically gifted children, as per government requirement

(DCSF, 2008b), but not all these schools have a policy of provision for these children. The case studies, furthermore, showed that their main identification methods, in the first place, involved standardised achievement tests (e.g., SATs and QCA tests), and in the second place, teachers' assessments (e.g., observation of pupils' performance and informal tests) using, in both cases, the Attainment Level Descriptors provided by the National Curriculum (QCDA, 2010b). Most of the schools that make provision for mathematically gifted children use strategies for differentiation and enrichment through grouping by ability, mainly within the regular classroom. From schools that organise special grouping arrangements, setting appeared to be the favourite practice, while some schools had recently started implementing pull-out programmes. However, the case studies in a small sample of these schools showed that issues of differentiation through differentiated work and differentiated instructions are more easily addressed within the small-sized pull-out groups rather than in large-sized classes, in which a range of different ability levels exists, even when these classes are part of a setting structure.

6.2 Second research question

"What are the teachers' perceptions of and attitudes towards mathematically gifted children, their education and the methods used by their schools?"

6.2.1 How teachers describe mathematically gifted children

Most of the teachers, including the case study teachers, describe mathematically able children as those who display ability in solving problems, those who quickly understand new concepts and respond to and complete tasks and those who are able to explain their thinking pattern/logic and methods. In other words, they display attributes that are easily observed in classrooms through everyday mathematics lessons. These attributes described by the most teachers are usually the main criteria used to nominate a child as a gifted mathematician, and therefore, play an important role in the identification of these children in many schools in which, as discussed earlier, teachers' assessments and nominations are taken into account.

Most teachers prefer the term 'able' with variations, such as 'very able', 'more able' or 'exceptionally able', to describe children with higher mathematical ability than their peers. Others make a distinction between the terms 'able' and 'gifted', considering that the first refers to those who are above the average of the class, and the latter to those who go even higher or work at least "one level n/c [national curriculum] above class", according to a mathematics consultant/coordinator (School 13). For those latter children, they also suggest the term 'exceptionally able'. The latter descriptions show that the National Curriculum (QCDA, 2010b) Attainment Level Descriptors have influenced the way in which teachers define mathematically gifted children. This was also confirmed during the case studies when, for instance, Julie, from School D, described the children who were on their gifted and talented register saying: "They are not just children who are able. They are gifted... children who are already at Level 5".

However, these definitions are limited to those achieving higher grades in achievement tests and to enthusiastic learners who, as Karolyi, Ramos-Ford and Gardner (2003) contend, are those who usually display their abilities in classrooms, but not to those with hidden mathematical ability or potential, as discussed earlier. The question, therefore, is: How can these teachers identify children of the latter category if they have only the former children in mind?

Therefore, teachers need to become aware of these children who, as mentioned earlier, only show their abilities under the appropriate circumstances which encourage gifted behaviour to be displayed (Gardner, 2006; Karolyi et al., 2003). For this, suitable training in recognising giftedness and, in particular, mathematical giftedness could help.

6.2.2 Teachers' attitudes towards teaching mathematically gifted children

This study found that many teachers (sixteen, including Kate from the main study) feel that their work as teachers becomes more difficult when there are gifted mathematicians in their class, because they have to work harder to find the right materials and plan differentiated and challenging lessons for these children. Also, they find it hard because when there are very different levels of ability in a class, it is difficult to teach each of the different levels effectively. However, such views seem to be correlated with teachers' training background and their subject expertise. Looking, for instance, at what the case study teachers said about this issue during their interviews, we can see that only Kate, who had no specific training in gifted education or in teaching mathematics to able children (only general training in mathematics) and had no confidence about her subject knowledge (as she admitted), expressed similar concerns. In contrast, the other three teachers, who had specific training in gifted education and in mathematics (Emma and Sarah) or initial training in higher mathematics (Claire), expressed no concerns about this issue. Additionally, they proved both their self-confidence and knowledge in their classrooms later on during the observed lessons.

The views about finding the right materials for more able mathematicians, however, agree with the views expressed by most teachers, in the questionnaire survey and the interviews, that mathematically gifted children need differentiated work, which is not just different from what the other children do, but challenging work that, as Sarah said, "requires them to think". Nevertheless, despite this awareness from teachers on the differentiated work that gifted mathematicians need, the issue of finding available resources and choosing the most suitable work for these children seems to be a problem for many teachers, who did not appear confident in finding the right materials. This finding agrees with the Williams (2008) report and previous research carried out in London schools (Koshy & Casey, 2005), both of which highlight teachers' lack of knowledge of what is available for teaching mathematics to gifted children. This research, at the same time, shows that the situation in relation to this issue has not changed much during the last decade. Consequently, the need for teachers' training and support (e.g., by appointing a 'mathematics specialist', as the Williams (2008) report recommends) is once again highlighted.

The case studies found that there are teachers, like Kate, who believe that more able pupils in mathematics can cope with their work without help, and that it is only the less able children who need continuous support. Such views, however, have been mentioned as the main reasons that mathematically gifted children were neglected for a long time (Cockcroft, 1982; Johnson, 2000; NCTM, 1980). It is interesting that Kate, who was chosen to teach a top mathematics set — the main purpose of which, according to the literature (DfEE, 1997, 2000b; Koshy, 2001), is to better address the needs of more able pupils — had such views. The question, therefore, is: How likely is it for a teacher who believes that more able children do not need support, to give opportunities for these children to learn mathematics within their ZPD (Vygotsky, 1978), since this requires monitoring of their work and continuous support through a scaffolding approach?

Some answers were found through the observations of lessons, in which Kate showed in practice what she had previously said in the interview. Namely, she remained attentive towards the lower and middle ability groups, leaving the able children to work by themselves in all three lessons observed.

Teachers' confidence level and training background

This research found, through both the survey and the case studies, that there are teachers who feel anxious with the presence of one or more gifted children in their classrooms and not confident (e.g., Kate) to teach them mathematics. It seems that teachers' training background affects their confidence level in teaching mathematics to gifted children. It was found, through the teachers' responses to the questionnaire, and confirmed through the case studies by observing the four teachers (Emma, Kate, Sarah and Claire) in their classrooms, that training in higher level mathematics, in recognising and teaching high ability pupils, and, especially, specific training in teaching mathematics to able children, give more confidence to teachers facing the challenge of teaching gifted mathematicians. It was also found that many teachers are aware that the quality and effectiveness of provision that a school makes are affected by teachers' training background and confidence level. These findings, like those reported from other studies and reviews (Koshy & Casey, 2005; Williams, 2008), highlight the need for specific training, both initial teacher training and ongoing in-service training, in recognising mathematical giftedness and teaching mathematically gifted children.

6.2.3 Teachers' views about provision and perceived needs for making effective provision for mathematically gifted children

How teachers evaluate the effectiveness of provision that their schools implement

Teachers' responses to the questionnaire showed that most of them feel that the needs of mathematically able or gifted children are met "well" in their schools, and the provision that their school makes for them are "moderately effective". However, there are some issues raised through the responses of a small number of teachers that may be of interest, such as those that show that there are some teachers who feel that provision does not have the same impact on all children; that the effectiveness and quality of provision is mainly influenced by teachers' personal background, qualifications and confidence level, by the size of the school, its budgeting and the resources that it may provide to the teachers, as well as the stability and quality of the staff. Finally, there were a few teachers who replied that provision for mathematically able or gifted children is an area that requires development.

Teachers' perceived needs for making effective provision for mathematically gifted children

Most teachers who participated in both stages of the research (thirty-one, including three of the four case study teachers) responded that they felt that they needed more support or training in order to effectively address the needs of mathematically gifted children. The main areas in which teachers appear to ask for more support, are in finding suitable teaching materials, as mentioned earlier, and in in-classroom provision. Other areas mentioned were the identification of gifted mathematicians, monitoring pupils' progress, out-of-classroom provision, and working with parents.

For in-classroom provision, in particular, Kate and Sarah, two of the case study teachers who teach large classes in which there are different levels of ability, stated that additional adults in their classrooms could help them to offer more focused attention to able children. Also, although she taught small pull-out groups, Emma agreed that additional adults were needed in the classrooms to monitor children's work and to help them to extend their abilities. However, when voiced by classroom teachers, such claims may be heard by policy makers or others as excuses or attempts for them to do less work. But, here is the crucial point for schools and policy makers, to accept; that we are talking about exceptional students who have special needs and, as Sheffield (1999) contends, for whom the classroom teacher alone is not able to serve their needs effectively. Instead, the collaboration of a highly trained teacher of the gifted (not just a teaching assistant) is needed. This expert can help by working either within the regular classroom in collaboration with the other teacher or outside the classroom with only one or two exceptionally able students.

6.2.4 Summary

It seems that teachers' perceptions of mathematically gifted children and their attitudes towards making provision and teaching these children are influenced by their training background and level of subject knowledge. Furthermore, the way in which teachers perceive mathematical giftedness and mathematically gifted children seems to be influenced by the National Curriculum (QCDA, 2010b) Attainment Level Descriptors as they often describe a gifted child referring to a specific Attainment Level (e.g., a child "at Level 5", as Julie said, to describe a gifted pupil in Year 6). Teachers' main anxiety seems to be the gap that exists in mixed-ability classrooms between more able and less able children. Because of this, many teachers, independent of their level of confidence and training background, expressed their wish to have extra adults as teaching assistants in their classrooms to help them to better handle differentiation. Many teachers also expressed their wish to have more support with teaching materials suitable for more able pupils, in monitoring pupils' progress, in out-of-class provision and working with parents, as well as to have more training and support in identification of and provision for gifted mathematicians.

6.3 Third research question

"How are the needs of mathematically gifted children met within classrooms in everyday practice?"

In previous sections, the issues of differentiation through organisational strategies, along with teachers' perceptions and attitudes were discussed. It was mentioned that most teachers, in their classrooms, used differentiated and extra tasks for their able children (occasionally mainly) in order to achieve differentiation and enrichment in their lessons, independently of whether or not their schools had a policy of provision and whether or not they organised their classes into groupings. This section discusses issues of mathematics enrichment, through everyday lessons, taking place in different settings, such as a mixed-ability (regular) classroom, a top set, and pull-out groups. How teachers plan their mathematics lessons, the resources that they use, the kind of work that they select and the methods that they use to teach mathematics are discussed; and it explores whether or not they use higher cognitive levels of teaching as suggested by the literature for mathematics enrichment (Casey, 1999; Ernest, 1998; Koshy, 2001; Schoenfeld, 1992; Sheffield, 1999).

6.3.1 Teaching resources

The questionnaire survey, in the first phase of the research, showed that teachers who use extra support materials usually find these materials through the National Strategy publications (DCSF/DfES/DfEE and QCA) or online databases available through the Internet — commercial programmes (e.g., NRICH and Testbase) or the National Strategy website (i.e., Curriculum Online/National Curriculum).

The case studies confirmed the extended use of resources referenced by the National Strategy. Apart from Claire — the secondary school teacher, who uses commercial books for KS-3 students from her personal collection — the other three teachers (Emma, Sarah and Kate) mainly used the National Strategy resources, online sources or books specifically for mathematically able children (e.g., *Mathematical Challenges for Able Pupils in Key Stages 1 and 2* (DfEE, 2000a)), combined with commercial ICT programmes or publications. Additionally, Kate used a complete educational package from *Hamilton Plans* (Hamilton Trust, 2009) to plan her lessons in the top mathematics set. The case studies also showed that teachers with training background and subject knowledge (e.g., Emma, Sarah and Claire), or teachers who had the help of a mathematics coordinator with subject knowledge (e.g., Kate), did not have problems finding resources and materials for more able pupils in mathematics.

The next question that was investigated in more depth through the case studies was what kind of activities the teachers selected from the resources they used, and how the activities and teaching approach added depth or complexity in mathematics and challenged pupils' higher-order thinking.

6.3.2 Activities for enrichment

This research found through the case studies that the nature of activities that were used to address the needs of mathematically able children, were correlated with the method of provision that the school implements, the facilities and support provided by the school, and with teachers' knowledge and expertise. For instance, Sarah and Kate, teachers of large-sized classes in fully-equipped classrooms, used computers and interactive whiteboards in their classrooms for a major part of the lesson. Most of the computerised activities involved exercises or games for the starter lessons and, in some cases, exercises for revision or examples relating to the new learning objective.

Emma and Claire, teachers of pull-out groups, who worked outside a fully-equipped classroom (e.g., in a corner inside or outside the library), used photocopies from books and materials created by themselves (e.g., worksheets and cards) to enrich their lessons. In one case only, Emma used the electronic equipment in another classroom to show a five-minute video to her pupils. Claire, the secondary school teacher who worked part-time offering mentoring for the group of Year 6 gifted mathematicians, used her own resources and materials (e.g., jigsaw puzzles with algebra and a math domino game made by herself, as well as photocopies from books from her personal collection).

Unlike Davis and Rimm's (1985) conclusions that pull-out programmes often emphasise fun and games rather than theory-based training, this study found that both teachers (Emma and Claire) in pull-out groups emphasised learning mathematical facts and algorithms through enriched lessons. For example, Emma used activities which, although not so difficult, involved investigations, real-life and open-ended problems, suitable for challenging able mathematicians' higher-order thinking, as suggested by the literature for mathematics enrichment (Casey, 1999; Koshy, 2001; Sheffield, 1999). Claire used materials for older children (from KS-3 mathematics curriculum) to teach more advanced topics — something that is recommended by VanTassel-Baska (1998) for more able pupils who have already mastered the regular lessons — but she also made sure that these activities would involve investigations with numbers, number patterns and formulas, as suggested for mathematics enrichment (Casey, 1999; Koshy, 2001; Sheffield, 1999). In addition, the activities used in pull-out groups were from a variety of resources and beyond single textbooks, as recommended for differentiation (Tomlinson, 2004; VanTassel-Baska, 2007). They included self-designed worksheets and other support materials, such as commercial leaflets, pin boards and rubber bands (Emma), as well as self-made cards for jigsaw puzzles and math domino (Claire).

Claire, in particular, systematically used activities, created by herself, either for the starter lesson or, more often, as an extension for those who finished early. These activities involved exercises on the whiteboard (for reinforcement and connecting the previous with the present lesson) or math games (e.g., jigsaw puzzles and domino cards). However, all these games were puzzles on advanced mathematics for those who finished early (e.g. an algebra jigsaw puzzle) or for reinforcement in the starter lesson (e.g., a domino game with negative number calculations). There was only one case in which a game that was not connected with lesson objectives was given. This was a decoding game created by Claire. This game, however, like the other puzzles, is amongst those activities considered suitable to challenge children's higher-order thinking (Koshy, 2001).

In the other two cases, both teachers (Sarah and Kate), like Claire, used activities from higher levels and for older children, trying at the same time to combine the high level of difficulty with elements of enrichment; but this produced different results. For example, Sarah achieved this combination by choosing activities which, although they were for older children, involved problem-solving, open-ended questions and investigations. Kate also used extra support materials for enrichment (e.g., a 'ratio' exercise with real sweets), but she appeared to rely too much on computerised games, which, according to the literature, add no value in mathematics lessons (DfEE, 1999b). The latter brings out important questions related to the effective use of computers in lessons, namely when and how computers should be used by teachers and what kind of activities they should involve.

6.3.3 Using computers

As mentioned earlier, Sarah and Kate use the computers for a major part of each lesson, mainly for the starter lessons. However, the activities that they chose served different needs in the lessons observed. Sarah's computerised activities involved exercises from past SATs papers from the Testbase (2008) and exercises on number operations from the Mathematics Resource Library (DCSF, 2009). The first aimed at preparing pupils for the SATs examinations at the end of the year. The second was reinforcement in adding three one-digit numbers, and worked as an introduction for what Sarah did later on with pupils who had difficulties in that subject. This agrees with recommendations for using the computers to meet the objectives of the lesson (DfEE, 1999b). Additionally, based on what was observed, these activities worked very well in Sarah's class, as they seemed to stimulate able pupils' interest, made them think of different ways to present their solutions on symmetry or different ways to add three numbers, and engaged them in discussions about their own solutions or the solutions suggested by others. All these, of course, were encouraged by Sarah's questioning, in which she appeared very confident and skilled.

Also, in some instances, Kate used the computer in this way, namely to teach particular lessons (e.g., ratios and proportions). However, in Kate's class, most of the time was spent on computerised activities that involved fun games from the Fiery Ideas (2008) ICT programme. These looked like commercial electronic programmes made for video games. The children confirmed (in the interview) the systematic use of this programme and referred to some particular games (e.g., 'Space Invaders'). Such use of computers has been criticised as offering only aimless explorations or repetitive practice on calculations already mastered and, so, does not add any value in teaching and learning mathematics (DfEE, 1999b). In addition, the six children from Kate's class, albeit appearing to enjoy the games, expressed their wish to have more difficult mathematics. Only one child (Bridget) said that she would like to have more work with the computer, while there was another (Almirah) who said that she would prefer to play mathematics games without the computer, and against other people.

6.3.4 What the children say about the work that they do

Almost all children in the two pull-out groups appeared happy with the work that they were given. The same did not apply with regard to the children from the other classes. For instance, many pupils from Kate's set appeared unhappy or disappointed by the 'easy' work that they were given, whilst almost all pupils from Sarah's class complained that the work that they do was too difficult.

The most positive comments for the work that they do were expressed by children from Claire's Year 6 pull-out group; these concerned the work on new topics and the work that involved investigations in number sequences (e.g., Fibonacci numbers). Work that is relatively new or unfamiliar has been suggested by Sternberg's (1985) 'experiential sub-theory' for helping intelligence to be better expressed, and work that involves investigations has been suggested by Sheffield's (1999) 'three-dimensional model' for adding depth and complexity to the mathematics curriculum and also by Koshy, Ernest and Casey (2009) for enhancing gifted pupils' motivation to learn more mathematics from higher levels.

In contrast, the strongest complaints against the work that they do were made by Amardeep and Abdullah, two of the higher achievers in Kate's Year 5 top mathematics set, who appeared disappointed by the easy work that they were given.

A very interesting point is also the case of five able children in Sarah's class. They all seemed stressed when they were talking about the hard work that they do; especially Jake, who gave an example that made him feel "sad" when he was not able to cope with the higher-level work that he was given. It seems, therefore, that teachers who use work for older children in younger ages, like Sarah, need to be more careful about the level of difficulty of the work that they select so that this will suit pupils' abilities. In addition, they need to be aware that when they give something more difficult to able pupils, so that they will work within their ZPD (Vygotsky, 1978), they have to be prepared to offer pupils 'scaffolding' (Vygotsky, 1962, 1978) help and support. On the other hand, teachers who emphasise fun computerised games, like Kate, or who make too much use of computers, should be aware that, as the US framework Principles and Standards for School Mathematics (NCTM, 2004b) advocates, the use of computers cannot replace the teacher in mathematics and the teacher must decide when and how to use them, ensuring that this will help to enhance pupils' mathematical thinking. If this does not happen, there will be able children, like Amardeep and Abdullah, who could be left unsatisfied and disappointed.

6.3.5 Activities that motivate children to learn more mathematics

The observation of the lessons showed that most pupils worked with interest and zest on activities that involved investigations (e.g., Sarah's investigations in 3D shapes and 'data handling', and Claire's investigations on Pascal's Triangle and the decoding game); problems connected to real life (e.g., Emma's 'numeracy trail' and 'money & prices' exercises, and Kate's 'Smartie Party'); and teamwork (e.g., Claire's maths-domino game). The interviews with children, furthermore, found that there are children who wish to do more activities related to real life (or to their hobbies, as Daurama said), inside or outside the classroom.

Therefore, by using activities that encourage pupils to be engaged in investigations and teamwork, as well as activities connected to real life, teachers can enhance pupils' motivation in mathematics. In addition, if they make sure that these activities also provide opportunities for pupils to engage in higher cognitive levels, they should find a successful way to enrich the lessons for mathematically able children.

However, it seems that finding the 'golden mean' between the 'too difficult' and 'in-depth' activities while, at the same time, avoiding doing 'aimless' work, is not easy and remains an issue for further and continuous research.

6.3.6 Opportunities for learning mathematics involving higher cognitive levels

Using Bloom's (1956) *Taxonomy of Educational Objectives* as a guide, we can select suitable activities and plan mathematics lessons at higher cognitive levels (Koshy, 2001). Having Koshy's (2001) framework for planning and teaching mathematics to able children according to Bloom's Taxonomy as a guide, this study analysed and assessed teaching practices that were observed in primary classrooms to determine whether and to what extent teaching at higher cognitive levels is provided within mathematics lessons.

The case studies showed that although none of the teachers observed followed any specific framework for teaching mathematics to able children, they all (more or less) offered opportunities for knowledge, comprehension, application, analysis, synthesis and evaluation. The latter three are being considered higher-order educational objectives, according to Bloom's (1956) Taxonomy. How much higher in Bloom's Taxonomy or how much emphasis was given by teachers at every level of Taxonomy, depended on the size of the class, the level of homogeneity in each ability group and the professional skills of each teacher.

For example, Sarah and Emma, the two mathematics coordinators who had specific training backgrounds in identification of and provision for gifted children, appeared more aware that the able children need to do not only different work from their same-age peers, but work that will be challenging and work that will make them think. They were also aware that these children need continuous support from the teacher or another adult (e.g., a teaching assistant) in order to extend themselves to work at more advanced levels and teaching and learning within pupils' ZPD (Vygotsky, 1978).

Both Emma and Sarah — who also had training in teaching mathematics to able children and co-ordinating mathematics, respectively — and also Claire who, as a secondary school mathematics teacher, had subject knowledge, appeared confident and very effective in challenging pupils' higher-order thinking through discussions in which pupils had the opportunity to display their knowledge and skills of analysis and synthesis. Furthermore, Emma and Sarah enriched their lessons by adding depth and complexity — independently of how difficult or easy the activities that they used — through the higher-order questions that they asked (e.g., “Why?”, “Is there another way?”, “What if…?”, “Imagine if you were… how…?”). These questions gave the opportunity to more able children to display skills of reasoning, hypothesising and generalising, which involve *analysis* and *synthesis*, as well as skills for *evaluation* of others’ work, and *self-evaluation*, all of which are considered higher-order thinking skills and are at the top of Bloom’s Taxonomy.

However, in practice, it also seemed that teachers’ effectiveness in challenging pupils’ higher-order thinking and teaching within their ZPD, is affected by the size of the class and the level of homogeneity in pupils’ ability. For instance, Emma’s small pull-out group gave her the opportunity to continuously monitor all pupils’ work, mediate when someone got stuck and provide help and support using ‘scaffolding’ methods (Vygotsky, 1962, 1978). Claire, also in a small pull-out group, did not have problems in monitoring pupils’ work and challenging them with higher-order questions.

On the contrary, Sarah’s large-sized class and the different ability groups that existed within it made her choose particular groups on which to focus — in which she was monitoring their learning and offers support — and left the rest to work alone on easier activities. When she was working with the group of able pupils, she was mediating by using questions that made them think further, conjecture, try alternative ways and attempt generalisations. In addition, more able children (e.g. Alvin and Jason) were encouraged to be engaged in discussions, either with Sarah or among themselves, during the problem-solving process. In other words, the higher-ability children really did have the opportunities for learning mathematics at higher cognitive levels and the teacher’s support for work within their ZPD. In contrast, when able children were left to work unaided on a problem-solving activity, they were not able to find all the solutions correctly; they were not engaged in problem-solving procedures — apart from Alvin, who displayed his personal problem-solving skills through individual work — nor in discussions about the problem. Furthermore, some of them stopped trying with the first difficulty and started playing instead. There were, however, children who showed persistence with the problem or ‘task commitment’, according to

Renzulli's (1978) *Three Ring* model of giftedness, a characteristic of behaviour that the teacher can take into account when identifying the gifted mathematicians. Similar problems, caused by the lack of monitoring of able pupils' work, also appeared in Kate's large-sized set at an even greater level, as these pupils always worked unaided.

6.3.7 Summary

This section discussed the findings gathered mainly through the case studies. The study of four different mathematics classes showed that the teachers who had the responsibility to organise and teach mathematics classes did not use any specific framework of provision for gifted and talented children. However, all case study teachers applied (more or less) strategies for learning mathematics at higher cognitive levels, offering opportunities for 'analysis', 'synthesis, and 'evaluation', which are considered higher educational objectives in Bloom's (1956) Taxonomy. How effectively they did this seemed to depend on their subject expertise; their confidence and questioning skills to challenge able pupils' higher-order thinking; the amount of attention focused on these children; and the nature of activities used. Here, therefore, some issues requiring further development were raised. The issue of teachers' professional development through the subject-specific training; the choice of the right work between too easy or too difficult ones; the effective use of computers for mathematics enrichment by adding depth and complexity rather than for funny games; and the lack of focused attention for more able pupils in large-sized classes. The observed lessons showed that able children worked with more interest on activities that involved investigations, real-life and open-ended problems, and on activities that involved new topics. It was also observed that extension for more able pupils happened only when the teacher mediated using higher-order questions, holding discussions and offering support; in other words, when a constructivist approach to learning was used.

6.4 Fourth research question

"What is the impact of the schools' strategies on pupils' achievement and attitudes?"

6.4.1 Children's achievement

All able children in the pull-out groups and also in the within-classroom ability group progressed, according to their results in their assessments. But, the type of setting, as implemented by School C, did not seem to benefit all the children who were nominated as more able mathematicians, as the achievement of two children remained at the same level after two assessments, while the achievement of another child moved to a lower level. However, it should be noted at this point that children from the special groups with the highest scores in their assessments (e.g., Thanh, Amardeep and Lily) appeared, according to what they said in the interviews, to do extra work at home with their parents, and, therefore, this should be taken into account before any attempt is made to assess the effectiveness of each programme on pupils' progress. The issue of parental involvement is discussed in a later section (6.4.4).

6.4.2 Children's perceptions of mathematics and attitudes towards the grouping arrangements

In terms of the impact on children's attitudes, it was found that the pull-out programmes had positive effects on pupils' attitudes towards the grouping structure and the lessons that they had. In the set group (for Year 5 students), although all pupils appeared happy being in a top mathematics set and seemed motivated to learn more mathematics, some of them, in particular the most able ones, appeared unhappy about their progress and the lessons that they had. Additionally, the case study in the regular classroom found no impact of the in-class ability grouping on pupils' attitudes towards the grouping practice and the lessons.

In terms of the impact on pupils' perceptions, it seems that the work within the pull-out groups and with the mentor — both of which were involved in more focused instruction in higher-level cognitive activities and more time for thoughtful work without distractions — helped some pupils to see mathematics not just as numbers and number operations, as the majority of the children who took part in this study, but as lessons that involved "logic" and "thinking" (Matthew, from Claire's pull-out group) and a lesson that is useful in real "life" (Daurama, from Emma's pull-out group; Matthew and Daisy, from Claire's group).

However, it should be noted that teachers' expertise and the methodology used along with the size of the class, as discussed earlier, seem to play an important role in the effectiveness of each method and, thus, on pupils' progress, perceptions and attitudes.

6.4.3 Other organisational strategies that affect children's attitudes and motivation

Peer tutoring

It was revealed in the questionnaire in the first phase of the research and also in the case studies, that there were some teachers (six, including Emma and Kate, two of the case study teachers) who used more able pupils for peer tutoring in group work. They did this in order to make their own teaching work 'easy', or because they believed that by rearranging the groupings throughout the year and using more able children as peer tutors, all pupils were learning from each other. The first view considers peer tutoring from a teacher's perspective as an instrument that helps them to face administrative difficulties in their mixed-ability classrooms and, therefore takes teachers' needs into account rather than those of able pupils. The latter view — that through peer tutoring, all children learn from each other — can be true for the lower-ability pupils, if these children have suitable support from their more able peers to tackle a difficult exercise and, therefore, learn within their ZPDs (Vygotsky, 1978). But this is not always true for more able pupils. This is because able pupils' learning through peer tutoring is mainly related to their motivation rather than the knowledge gained from the exercise, which they have probably already mastered. Able mathematicians learn more mathematics when they develop positive attitudes and motivation towards the lesson (Koshy, Ernest & Casey, 2009). Therefore, peer tutoring can benefit more able children only if it positively affects their attitudes and motivation.

The case studies showed that able pupils who had had such experience of acting as a peer tutor for one or more lower-ability peers, helping them to finish their tasks, did not seem to be motivated by this. Amardeep (from Kate's class), for instance, appeared unhappy with this practice, complaining that he did not like to just help others to do a simple task. However, Amardeep viewed differently the idea of peer tutoring in a lower set where, as he said, he enjoyed explaining his methods to the whole class and acting like a teacher. The latter is a practice that is systematically implemented in School C, in collaboration with the teachers of the other two sets (the middle and the lower set); once a week, one child from the group of more able children in the top set goes to a lower set and offers peer tutoring. Therefore, teachers should be aware that peer tutoring does not motivate able children when it simply involves helping others to complete their exercises, because able children do not consider this a challenge. Teachers should give able pupils the opportunity to present their methods or solutions to new topics or difficult tasks.

Teamwork

Almost all children appeared to value teamwork within a group because they could hear what other people, good at mathematics, thought and because there were more people around who could help, if someone got stuck — or, as Matthew said, “more minds thinking on the...problem”. However, in many cases, children’s attitudes towards teamwork are affected by the kind of work that they do in the group or the ability of the other members of the group. For instance, many children from the upper years (Year 5-6) explained that they prefer to work alone when the task is easy because they can work quietly and without distractions, while a child from the highest achievers in Year 5 (Amardeep) said that he likes working only with pupils who are “clever”. Similarly, two of the highest achievers in Year 2 (Alvin and Jason) said that even though they prefer working alone, they would like to work with peers who are good at mathematics, or friends of theirs. Only one child (Emel from Year 5) made clear that she wishes to always work alone because she does not want to get distracted by others, adding that this is the main reason that she likes working in a pull-out group.

However, all the aforementioned, which show that almost all children favour teamwork, are conclusions from the interviews and, thus, based on what the children said. The observations of the lessons showed a different conclusion that is closer to Amardeep’s words, mentioned above. For instance, more children appeared to avoid working with peers with lower abilities in mathematics and to choose work partners of the same or higher ability compared to theirs. There were also cases in which pupils from the higher ability group (in Kate’s top maths set) hid their work from other children of a different ability group. These findings support the arguments that the allocation of pupils with the same intellectual peers is a criterion for the success of any special programme for gifted children (Belcastro, 1987).

In addition, most of the work which took place in four different classes was found to be mainly individual work rather than collaborative work, even when the children were grouped around some tables. True on-task discussion within the groups happened only when the teachers mediated by asking questions.

An important element that seemed to influence children's attitudes towards teamwork or make them want to work only with same-ability peers seems to be the nature of the work and the opportunities that it gives for all the members of the team to contribute to the solution. For instance, Emel and Alvin, two of the children who prefer working alone, explained that they do not like to work in a group in which the other children wait for them to find the answers or just copy them. In addition, Cathy (from Claire's pull-out group) suggested that mathematics lessons would become more interesting if they had more difficult problems, but which were still appropriate for teamwork ("that we can work in pairs"). Therefore, here there are some other issues that teachers have to take into account when they plan differentiated work that children should do as a group. They should choose work that is suitable for teamwork, place pupils with peers of the same intellectual level, monitor the collaboration of the children and encourage group discussion by mediating and asking questions.

6.4.4 Parental involvement as a factor that influences children's progress and motivation

According to Gardner (1983), parents' role, along with teachers' work in schools, is one of the factors that influence the development of mathematical intelligence. This study found that parents play an important role in both children's progress in mathematics, as well as in developing positive attitudes towards mathematics.

Most of the children who participated in special ability groupings (i.e., top mathematics set and pull-out groups) appeared (based on what they said in their interviews) to have systematic support at home from parents, which they believe is the main reason for being at the top of the class in mathematics and, therefore, in the special groups for able or gifted mathematicians. Some of them also appeared to be influenced by their parents for the practical use of mathematics in real life (e.g., to get better jobs) and, therefore, for developing positive attitudes towards the subject.

Children's interviews also revealed that parental support involved preparation on forthcoming topics, on tests similar to those provided by the national curriculum or on more advanced mathematics. However, in some cases, that kind of help, although it was enough for children to gain higher grades in formal tests, was not enough to make them feel comfortable in a pull-out group in which more advanced lessons took place, as was found by studying Lily’s case (School D). Lily had such kind of support that enabled her to achieve top grades in all formal achievement tests, but she displayed low performance during the lessons observed in the pull-out group as well as lack of confidence when tackling new topics (something that she also admitted in her interview). Also, the latter case of Lily also reinforces the argument that higher grades in tests by themselves do not prove the existence of mathematical giftedness (Eyre, 2001; Johnson, 2000; Koshy & Casey, 2005). However, it must be emphasised that parental impact was not investigated in depth by this study and, therefore, it can be an issue for future research.

6.4.5 Summary

It seems that the pull-out programmes in mathematics have most positive effects on both children’s progress and attitudes towards the subject, the grouping strategy and the lessons that they do. However, most of the benefits seem to be correlated with the small size of the class as well as the teacher’s professional qualifications. What increases the possibilities for these programmes to be more successful, compared to the other organisational strategies, seems to be that schools appoint the best-trained teachers from their staff, or sometimes choose to employ a new person with higher subject expertise (e.g., School D) to teach their pull-out groups. However, the interviews with children, as well as the observation of their working style and behaviour, revealed more factors that influence their attitudes, their motivation and their progress. These factors are the allocation according to pupils’ ability for teamwork, and the use of able pupils as peer tutors, not just helping their peers, but also presenting methods or solutions on new subjects or difficult tasks. The interviews, in particular, revealed that parental involvement, through extra help and support at home, plays an important role in children’s progress, motivation and their attitudes towards mathematics.

7 Chapter Seven: Conclusions

This study drew on a range of expert theories of giftedness and talent and aspects of provision which were reviewed in Chapter Two. These views and positions informed the empirical work. In particular, Gardner’s (1983) theory of Multiple Intelligences and his ideas about the existence of a specific mathematical ability (‘logical-mathematical intelligence’), Krutetskii’s (1976) ‘mathematical cast of mind’ and Vygotsky’s (1978) Zone of Proximal Development (ZPD). It also drew on theories for learning mathematics, such as the ‘success cycle’ (Ernest, 1985) and for learning at higher cognitive levels, such as Bloom’s Taxonomy (1956).

With these ideas in mind, this study aimed:

- To explore the strategies used by teachers for educational provision for gifted and talented children, particularly in mathematics. This also involved a study of international theory and research into the teaching of gifted mathematicians.
- To carry out a questionnaire survey, interviews and classroom observations in English primary schools to explore how the needs of mathematically gifted children are addressed in practice.
- To make an assessment of what is happening in primary schools — where there is specific attention given to gifted and talented children — by comparing the findings of aims 1 and 2 against recommendations for specific provision and effective teaching for mathematically gifted children.

In order to achieve the aims of the study, mixed methods for collecting information were used. These methods involved the use of a questionnaire, interviews, classroom observations, school documentation and pupils’ written work. The different methods that were employed and the different data sets that were used made it possible to triangulate the findings gathered and also enhance the credibility and validity of the research.

Personal learning

The review of international theory and research, such as the study of prominent theories and models of giftedness and talent (e.g., Gagne, 1985; Gardner, 1983; Renzulli, 1978; Sternberg, 1985) and pioneering empirical studies relating to specific mathematical ability (e.g., Krutetskii 1976) provided me with a robust understanding of issues relating to the multifaceted aspects of giftedness, the nature of specific mathematical giftedness, its assessment and its development. Additionally, the review of specific frameworks for teaching mathematics to able children (e.g., Casey, 1999; Koshy, 2001; Sheffield, 1999) and current educational policies in the UK and also internationally provided me with practical knowledge relating to organisational methods for provision and strategies for teaching mathematically gifted children in primary education. The empirical study, carried out in a small sample of primary schools in England, on methods for provision for mathematically gifted children, helped me to identify aspects that could be improved or changed in the UK, where the research took place. This study also enables me to make recommendations about the development of mathematical giftedness within primary schools that may be of interest not only to audiences in the UK, but also in my country, Greece, and elsewhere, as the need for the development of mathematical giftedness for the benefit of society has no boundaries. It is an aim that should be sought by educationalists and educational authorities everywhere. This was also the main aim relating to the contribution of this study, namely, to raise awareness for making specific provision for mathematically gifted children within primary schools, and to raise understanding of the strategies of educational provision for the development of mathematical giftedness. In addition, the study of research methods and methodologies and their implementation into practice provided me with valuable experience in collecting, organising, analysing and interpreting data. This will be a great advantage in carrying out future research.

Contribution to knowledge

There has not been much research on strategies for educating mathematically gifted children and empirical findings from real classroom work in the UK or internationally. In the UK, for instance, although there has been extensive literature from the Government, through the National Strategy, to guide practitioners to support gifted and talented education, none of the training courses offered has addressed the teaching of mathematics to able mathematicians. There has been little effort to support practitioners in teaching mathematically gifted pupils. This thesis, therefore, aims to fill this gap in knowledge, through making the findings widely available.

This study adds to knowledge and contributes to the literature by illuminating aspects relating to:

- The teachers' perceptions and attitudes towards mathematically gifted children and their education.
- The pupils' perceptions of mathematics and their attitudes towards the work that they do in classrooms as well as the setting in which the lessons take place.
- The effectiveness of particular organisational strategies for provision, such as in-class ability grouping, setting, and pull-out grouping, highlighting problems in handling differentiation within large-sized classes.
- Teaching strategies that provide opportunities for enrichment and extension for more able pupils, illustrating useful examples for teaching mathematics at higher cognitive levels observed in classrooms through the case studies.
- Activities that motivate pupils to learn more mathematics.
- The use of computers in mathematics lessons along with pupils' views and attitudes towards computer-based activities.
- The role of focused attention and 'scaffolding' of pupils' learning.
- The role of training in both the self-confidence and effectiveness of teachers to teach mathematics to gifted children.

An article based on the first case study (Dimitriadis, in press) has already been accepted for publication by a peer-reviewed educational journal making a first step to international contribution. The findings from the other case studies will become the material for other publications. Profiles of children who were studied would also make illustrative case studies of what works in practice.

This study adds to UK educational research as it reports findings that may contribute to a better understanding of the real situation in everyday classrooms. Although the 'gifted and talented' policy of the UK Government was launched in 1999, there has been only one reported research study (Koshy, Ernest & Casey, 2009) published. There have been no evaluation or reviews commissioned by the government of how mathematically able pupils have been provided for. This study, therefore, along with the aforementioned, illuminates:

- Whether and how policy for the identification of and provision for mathematically gifted children is implemented by schools.
- Experiences of teachers on the implementation of methods for identification and strategies of provision, as well as their views about the practicality of identification programmes and the effectiveness of provision.
- Experiences of pupils from their participation in special grouping arrangements, such as in-class higher ability groups, top ability sets, and pull-out groups.
- Teachers' perceived needs for making effective provision for gifted mathematicians, highlighting the areas in which teachers feel that they need support.
- The views of more able pupils about how mathematics lessons could become more interesting.

Finally, through publishing this study, I hope to raise awareness of the education of gifted children in mathematics (and in other academic domains) in my country, Greece, and to inspire educationalists and policy makers to start thinking about the development of a national policy for the education of these children. To increase the possibility of this happening, I intend to translate this study into Greek, publish some papers in Greek educational journals, and present the findings of this research at educational seminars.

Conclusions from the study

The review of the current literature from the field of both psychology and education showed that there is agreement that mathematical giftedness is associated with a specific ability that can be developed through experiences, instruction, training and continuous challenge (Gagne, 1985; Gardner, 1983; Krutetskii, 1976; Renzulli, 1978; Sternberg, 1985) and, therefore, through appropriate educational programmes. Educational systems in many countries of Europe and North America as well as in Australia and New Zealand have recognised the need to identify gifted mathematicians at early stages and effectively nurture their mathematical ability within schools. In England and Wales, in particular, during the last decade, especially since the publication of the 2005 white paper, *Higher Standard, Better Schools for All* (DfES 2005), and the 2006 renewed Primary National Strategy for Literacy and Mathematics

(DfES 2006), there has been a growing interest in addressing the needs of children gifted in mathematics within schools and a continuous development of new policy frameworks which provide schools with guidance for identification (DCSF, 2008a) and provision (DCSF, 2008b) as well as support through an electronic version of curriculum (QCDA, 2010a). This creates an optimistic picture in the history of gifted education in England and Wales, where there have been no active policies to meet the needs of the most able in the schools.

Gifted and talented policy in schools

This research found that almost all schools reviewed have complied with the recommendations about the identification of gifted and talented children in any academic domain and, therefore, also in mathematics, and about maintaining a register for them. However, not all the schools have a policy of provision for gifted children or coordinators specifically appointed to plan and run particular programmes for gifted children in particular areas, such as in mathematics.

Identification

With regard to the identification of mathematically gifted children, it was found that there is heavy reliance on achievement tests using the Attainment Level Descriptors provided by the National Curriculum (QCDA, 2010b). The latter seems to have influenced the way in which teachers describe a child gifted or exceptionally able in mathematics (e.g., a child "at Level 5" for a Year 6 student). This method of identification relies on a narrow definition of the nature of mathematical promise. Teachers should be encouraged to use other methods of identification, such as characteristics checklists, pupils' portfolios, as well as nominations from parents and peers.

Provision

Most teachers from schools that have a policy of provision, appeared to use within-classroom provision with differentiation and enrichment as the most preferable organisational strategies compared to acceleration. This study found that the effectiveness of differentiation and enrichment depends not only on the existence of a policy, resources and modern equipment, but mainly on the quality of the selected work and teaching methodology; in other words, on the professional quality of the teacher. The case studies showed that teachers, based either on their own knowledge or on the help of a mathematics coordinator, have the appropriate teaching resources or equipment (e.g., computers and interactive whiteboards), but they may use them differently, depending on their professional skills. There were, for instance, examples of use of mathematics materials from higher levels of the mathematics syllabus, in which more able pupils engaged in higher-order thinking because they had the teacher mediating during the problem-solving process, asking them higher-order questions and raising discussions; and examples of use of materials from higher-level mathematics, in which more able children were left to work alone, unable to successfully complete their tasks. There were also examples of computer use for activities relating to the learning objective and activities that gave opportunities, through teachers' questioning again, for higher-order thinking and extension (e.g., to find alternative solutions); and examples of use of computers for games that seemed to emphasise mainly fun rather than knowledge. Differentiation and enrichment for adding depth and complexity in mathematics for more able mathematicians were found to be better achieved within pull-out groups, because even in a top ability set (part of a setting programme), a range of different ability levels exists.

Teachers' expertise

Teacher training and ongoing professional development support in both mathematics and gifted education seem to be factors that influence the quality of provision offered to gifted children. Lack of knowledge about organisational strategies for provision, for instance, caused misunderstandings in the application of setting, as it was found that some teachers confused setting with either pull-out grouping (e.g., pull-out groups consisted of children of different ages) or with ability grouping within a Year (e.g., a Year 5 separated in ability groups on a full-time basis). Lack of subject knowledge was found to affect teachers' confidence to organise differentiated and challenging lessons to teach mathematics to gifted children. Teachers' perceptions of mathematically gifted children and their attitudes towards their education were also found to be influenced by teachers' training background and their confidence level. It was found, through teachers' responses to the questionnaire and the interviews, as well as through the observation of the lessons, that teachers' specific training in recognising and educating gifted and talented children, and especially specific training in teaching mathematics to gifted children, enhances their confidence to teach these children and helps teachers to understand their needs, thus, offering the right opportunities in class through both teaching materials and methodology. Teachers themselves were found to be aware of the role that their professional development plays in the effectiveness and quality of provision that the gifted children may experience in mathematics. However, their responses to the questionnaire revealed that there are many teachers who, independently of their training background, would like to have more support in teaching materials and in in-classroom provision. The case studies confirmed that the main concern of teachers in the case of in-classroom provision is associated with the gap that exists in mixed-ability classrooms between the children who can do more difficult mathematics, and children who have difficulties understanding the basics. Because of this, three of the four case study teachers, independently of their level of confidence and training background, highlighted the need for extra adults in large-sized classes, who may help them to monitor all pupils' work and to better handle differentiation.

Children's progress, perceptions and attitudes

Factors that influence children's progress, perceptions of mathematics and attitudes towards their lessons were found, through their interviews and observed lessons, to be the following:

The setting in which the lessons take place

Pull-out groups, as mentioned earlier, seemed to allow more opportunities for differentiated lessons and differentiated instructions than other types of grouping. This was found to have positive effects on pulled-out children's progress as well as their perceptions of and attitudes towards mathematics. Children in pull-out groups were taught by well-trained and experienced teachers, and received more focused instruction on higher-level cognitive activities as well as lessons at their own pace. The small size of the groups also ensured continuous monitoring of each pupil's work, plenty of time for interaction with the teacher, and time for thoughtful work without distractions, something that seemed to keep all children satisfied. Continuous collaboration between the regular classroom teacher and the pull-out teacher seemed to ensure good communication and articulation between the pull-out group and the regular classroom, as well as the progress of pulled-out pupils with no breaks from the regular mathematics.

Teamwork and peer tutoring

Practices of pairing or grouping pupils in small mixed-ability groups for teamwork and peer tutoring within large-sized mixed-ability classrooms did not always seem to have positive effects on more able pupils' attitudes and motivation to learn, as many teachers appeared to believe. For instance, even though they appeared through the interviews to value group work, the majority of able children were found in practice to prefer working individually or with peers of the same or higher ability. Additionally, more able pupils who had experience from peer tutoring did not appear happy and motivated when this was limited to helping others to just complete a simple task, but only when they presented to the class or a specific group methods or solutions related to new topics or difficult tasks.

The kind of work that children do

More able children were found to work with an interest and to enjoy activities that involved investigations, open-ended problems and problems connected to real life, as well as activities that involved new and novel topics and activities that required teamwork with peers of the same ability. The interviews, furthermore, revealed that more able children wished to have had more activities related to real life and their hobbies, inside or outside of the classroom. Children who had experienced such activities were found to perceive mathematics as a lesson that involves real-life problems which help people in everyday life. Children who had had experience from systematic use of computerised activities expressed different views about computerised maths games. More children appeared to find them too easy, asking for more difficult work, while there were others who seemed happy with the computerised games, and still others who wished that they had more maths games but with people and not with a computer. Therefore, children's views, expressed in the interviews, along with their working styles and behaviour displayed in observed lessons, highlight the role of the teacher in the way that they perceive mathematics and in their attitudes towards the lessons.

Teachers' focused attention, questioning and 'scaffolding' skills

The case studies showed that able children made progress when they worked in classes in which they had not only activities such as the aforementioned, but also when they had their teachers' attention and continuous support, something which highlights, again, the role of the teacher in the progress of the children. More specifically, the observation of the lessons showed that extension for more able children happened only when the teacher mediated during the problem-solving process using higher-order questions, and raised discussions about the problem as well as about the solutions suggested by children. In other words, there was more evidence of learning when the children had 'scaffolding' support every time they faced difficulties with an exercise.

Parental involvement

Most children appeared to be influenced by their parents on views relating to the usefulness of mathematics in life, in that they can help in finding better jobs. Parents' help and support was also found to play an important role in children's progress and motivation to learn mathematics.

Implications for practice

After careful consideration of the findings of the present research, and what was presented in the literature review, this research suggests that mathematically gifted children have special needs that ought to be addressed within every school through a subject-specific programme of provision. This programme should involve grouping by ability, which can be pull-out groups, setting or within-classroom ability grouping, depending on the size of the school. However, in any case, schools should consider the use of special pull-out groups, particularly for exceptionally able mathematicians whose needs cannot be met within the regular class or even within setting or pull-out groups that are often used for pupils who simply achieve higher grades than their same-age peers.

The work that the able or gifted pupils do in the class (regular or special group) should not just be 'more' work of the same that they have already mastered, or more difficult work, but work different from the standard work for the average child, which should challenge their higher-order thinking skills.

Even gifted mathematicians need teachers' attention and continuous support, in order to be able to work within their Zone of Proximal Development (ZPD) (Vygotsky, 1978). Likewise, teachers of the gifted also need continuous support in order to be able to offer suitable differentiated work and instruction to gifted children within the normal curriculum.

Continuous support, in practice, may involve cooperative planning between teachers who, with the guidance of an expert — a highly trained teacher in both gifted education and higher-level mathematics, member of staff or an external consultant, depending on the size of the school and its financial background — can plan effective differentiation for more able pupils and organise the lessons. An expert teacher may provide practical help by modelling examples of teaching mathematics at higher levels, encouraging teachers with low self-confidence or even teaching exceptionally able pupils outside the classroom in small pull-out groups or in one-to-one lessons, offering mentoring and more focused support. Taking into account that it is very possible for a mathematics class, even in a top mathematics set, to have one or more pupils who stand at more than one attainment level higher than the others and whose exceptional abilities are not easily addressed in a class with the other pupils or by their regular teacher, mentoring outside the classroom may be a strategy that is very useful to both classroom teachers and exceptionally able mathematicians.

Continuous support needs to be practical and moulded into the needs expressed by teachers. It also needs to be easily integrated into the everyday programme (e.g., reasonable and manageable in terms of teachers' time and energy) so that teachers will not feel that they need to do extra and very hard work, which was found to be one of the teachers' anxieties when they were asked to express their opinion about making provision for mathematically gifted children.

Limitations of the study

- This study was based mainly on a qualitative research approach which, as discussed in Chapter Three, involves a risk of being biased, subjective and selective. I have tried to avoid this as much as possible.
- It is possible that the questionnaire responses were from people who were responding to the government policy, which may have introduced some bias.

- Although the sample chosen for the case studies was schools from different LEAs and from areas that represented a variety of socio-economic backgrounds, they were all within a small geographical area (Greater London), because of constraints of time, finance and accessibility.
- The findings are based on a relatively small sample and, therefore, despite the triangulation that occurred through the use of different methods of data collection, generalisations based on this study can only be tentative. But, I hope that the findings will be useful for both policy makers and practitioners. I hope to publish the case studies which should illuminate the significant issues and hope to generate useful discussion and debate.
- Although the intent was to find participating teachers of both genders, I was unable to find any male teachers for the case studies as most of the teachers in those schools were women. The participation of male teachers was only through the questionnaire in the first phase of the research and the majority were headteachers or deputy headteachers. This seems to be a pattern in English primary schools.
- The time plan for the case studies (e.g., for the observations and interviews with teachers and children) was dependent on each school's schedule, which was not so flexible. Therefore, the case studies in each school had to be completed within a limited time period.
- The research was carried out between 2008 and 2009, and, therefore, should be regarded as a snapshot of methods and strategies of provision applied by schools for mathematically gifted children at that time.

Issues for further research

During this study, the following issues emerged as worthy of further investigation:

- The use of computers in classrooms as tools for teaching and learning mathematics
- The use of grouping (mixed or ability grouping), teamwork and peer tutoring within the regular classrooms and their influence on pupils' progress and attitudes

- The relationship between identification of and provision for gifted mathematicians, as well as the level of monitoring of pupils' progress throughout the year
- The level of communication between pull-out group and the regular classroom teachers
- The level of association between school provision and outside-school life, such as the work at home or that in special mathematics clubs
- The role of parents in promoting and supporting mathematical giftedness
- Collaboration amongst the teachers within the school for planning and implementing identification and provision for gifted mathematicians
- Collaboration between primary and secondary schools on the transition of the gifted children
- The impact of teachers' subject expertise and professional development on the quality of provision for mathematically gifted pupils

Final thoughts

"The education of young gifted mathematicians in the UK is at a critical crossroad" (Koshy, Ernest & Casey, 2009, p. 226). After many years of neglect, there is some progress, an opportunity to develop systems for identification using multiple sources, frameworks for planning, evaluating the relative merits of grouping and organisational structures (Koshy et al., 2009). Targeted Professional Development for mathematics teachers remains an issue. As the William Report (2008) is asking for more subject knowledge, whether this will address the pedagogical issue relating to provision for mathematically gifted children remains unknown.

This study has been an exploratory journey for me, the researcher, constructing an account of provision for mathematically gifted pupils. This journey will continue.

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Appendices

List of Appendices

Appendix 1: Guidance for developing a portfolio in mathematics

Developing a Portfolio of Mathematical Achievement

Guidance Sheet for Pupils

This special file is designed to help you to keep a record of your mathematical achievement. You are invited to select examples of your 'best' work to be included in your portfolio. The following notes will guide you in the development of this very special file.

- Select a variety of pieces of work. For example, a completed investigation (this may have taken you several hours of work), a completed mathematical puzzle or a mathematical project.
- When you record your work, be clear and systematic. This will help other people to see how you developed your work, your particular strengths and where you may need help.
- Make sure you complete a Selection Sheet and attach it to each of the pieces of work included.
- Your portfolio will be of interest to your class teacher, your parents and for teachers in your secondary school.
- Don't be ashamed of crossings out and rough workings out. Remember that being a good mathematician involves thinking hard, making mistakes and being persistent.
- You can include photocopies of any special work you have done in school or at home. Any certificates you have obtained for mathematical achievement can also be photocopied and included in this file.
- If you take on other challenges such as constructing a glossary of mathematical words, a fact sheet or write your own version of a text book page to teach a mathematical topic, you should include these in your portfolio.
- You may refer to the following words and phrases describing the special processes which help you to be a real mathematician:
  *enjoying the challenge *recording ideas clearly *reasoning
  *estimating *organising information systematically
  *trying out different ideas *checking for sensible results
  *learning new words *using new techniques
  *breaking down a problem into manageable parts
  *making conjectures *searching for proof to convince others
  *spotting ideas learnt before *posing new questions to extend ideas
  *being persistent

Source: "Mathematics Enrichment Project" (Casey & Koshy, 2002, 2003)

Appendix 2: The Questionnaire Survey

The Cover letter

EDUCATING MATHEMATICALLY ABLE CHILDREN IN FIVE LONDON LOCAL EDUCATIONAL AUTHORITIES IN THE UK PRIMARY SCHOOLS

Dear Headteacher

I am writing to ask for your help with an investigation I am carrying out on how primary schools in England face the challenge of educating able mathematicians. The investigation is part of a doctoral degree in education at Brunel University of London. Could you, please, ask a full-time teacher from the upper Years or a mathematics co-ordinator to complete this questionnaire?

I am a primary school teacher with 21-years teaching experience in Greek state schools. I have noticed that teachers face a challenge in meeting the different needs of mathematically able children within the class. Since 2000, I have been living, working and studying in London. In 2005, I obtained my Masters Degree at Brunel University, which helped me broaden my ideas about the concept of ‘mathematical giftedness’, raised my interest in the education of able children and provided a starting point for my doctoral studies and this particular investigation.

The aim of this investigation is to explore how primary schools address the needs of mathematically able children in everyday practice. The information, which I hope you will provide, will be very important for me to fulfil my doctoral degree but it may also be more useful when the findings of the study are published and your thoughts reach policy makers in education. Furthermore, all the collected data may eventually help the development of a new model of provision for mathematically able children, which should be useful for teachers who face the challenge of effectively educating mathematically able children.

Please, do not feel overwhelmed by the seeming length of the questionnaire. It would take only 20 minutes (approximately) to be completed, as most of the questions only need a tick. If you require further information about the questionnaire, please do not hesitate to contact me at xxxxxxxxxxxx@brunel.ac.uk.

This questionnaire is strictly confidential; so all answers will remain anonymous. I also intend to carry out some case studies in schools willing to cooperate, in order to obtain a clear picture of educational provision for able mathematicians in practice. Please, fill in the appropriate space at the end of the questionnaire, if you wish to participate in the case study.

Thank you in advance

Christos Dimitriadis
Research student

Professor Valsa Koshy
Supervisor of the study

The Questionnaire

Please tick a box for the following questions apart from those that indicate otherwise or/and invite you to write in the spaces provided.

Section 1: About Yourself

1.1 Please, write your current position in the school (e.g. mathematics co-ordinator).

……………………………………………………………………………………………

1.2 Do you have a specific responsibility for any aspect of gifted and talented children?

Yes………… ☐
No………….. ☐
If yes, please provide details:
……………………………………………………………………………………………………
……………………………………………………………………………………………………

1.3 Have you (personally) ever received any training (apart from the Initial Teacher Training) in identification of and provision for gifted and talented children?

Yes………… ☐
No………….. ☐
If yes, please provide details (e.g. when, what programme, sponsor, etc.):
……………………………………………………………………………………………………
……………………………………………………………………………………………………

1.4 Have you (personally) ever received any training (apart from the Initial Teacher Training) in teaching mathematics?

Yes………… ☐
No………….. ☐
If **yes**, did this concern? (You may choose more than one)

- General teaching practices ………………………………………...……… ☐
- Specific strategies for teaching able (or gifted and talented) children…. ☐
- Specific strategies for teaching less able or underachiever children.…. ☐
- Other (please specify below) ……………………………………………… ☐

………………………………………………………………………………..

Section 2: About Your School

2.1 Local Authority of your school: …………………………………………………………..

2.2 School type (e.g. Community, Foundation, Voluntary, etc): ………………………….

2.3 Number of pupils enrolled: ……….

2.4 Number of pupils in your mathematics class (classroom teachers only): ……….

2.5 Number of teaching staff: ………..

2.6 Number of support staff: …………

2.7 Do you have a co-ordinator for planning and running particular programmes at your school for gifted and talented children?

Yes…………… ☐

No……………. ☐

Don't know….. ☐

If **yes**, does this involve? (You may choose more than one)

Sports…………………………………..…. ☐

Music…………………………………..…. ☐

Arts……………………………………..… ☐

English………………………………….... ☐

Mathematics…………………………..…. ☐

Science………………………..………… ☐

Other (please specify below) …………….. ☐

………………………………………………………………………………………….

Section 3: About your school's policy in relation to identification of and provision for gifted and talented children

<u>Identification</u>

3.1 Does your school **identify** Gifted and Talented children?

Yes…………. ☐

No………….. ☐ *Go to question **3.7***

Don't know….. ☐ *Go to question **3.7***

3.2 Does your school maintain a register for Gifted and Talented children?

Yes………… ☐
No………….. ☐
Don't know….. ☐

If yes, what is the percentage (approximately) of Gifted and Talented children registered in your class? ……………

3.3 Does your school keep a separate register for **mathematically gifted children**?

Yes………… ☐

No………….. ☐

Don't know….. ☐

If yes, what is the percentage (approximately) of gifted mathematicians in your class? ………

3.4 Do you review and reconsider the register of **mathematically gifted children**?

Yes………… ☐

No………….. ☐

Don't know…. ☐

If yes, how often? ……………………………………………………………………

3.5 Which of the following do you use to identify **gifted mathematicians**?

a. Test results?

Yes………… ☐

No………….. ☐

Don't know….☐

If **yes**, please indicate which of the following test you use (you may choose more than one):

IQ tests……………………………………………………………………….. ☐

Cognitive tests………………………………………………………….. ☐

Achievement tests provided by the National Curriculum (i.e. SATs)… ☐

Other (please specify below)…………………………………………….. ☐

………………………………………………………………………………………

b. Nominations?

Yes………… ☐

No………….. ☐

Don't know….☐

If **yes**, please tick the appropriate box or boxes below:

Nominations from parents or carers ………………… ☐

Nominations from teachers ………………………… ☐

Nominations from peers …………………………… ☐

Other (please specify below)……………………….. ☐

……………………………………………………………………………

c. Teacher assessments?

Yes………… ☐

No………….. ☐

Don't know….☐

If **yes**, please explain what methods you use in the following space:

3.6 How would you evaluate the identification process of **gifted mathematicians** in your school?

In regards to its practical use:

Very simple…………………… ☐

Relatively simple……………… ☐

Neither simple nor difficult…… ☐

Relatively difficult………….. ☐

Very difficult…………………. ☐

In regards to its reliability:

Very reliable…………………... ☐

Relatively reliable…………….. ☐

Neither reliable nor unreliable…. ☐

Relatively unreliable………..…. ☐

Very unreliable………………... ☐

Provision

3.7 Does your school have a policy for **provision** for addressing the needs of Gifted and Talented children?

Yes………… ☐

No………….. ☐ *Go to question **4.1***

Don't know….. ☐ *Go to question **4.1***

3.8 Do you make any special provision for gifted and talented children **in mathematics**?

Yes………… ☐

No………….. ☐

Don't know…. ☐

If **yes**, does this involve any of the following? (You may choose more than one)

Skipping years ……………………… ☐

Bypassing certain classes ………... ☐

Early entry to the school.………….. ☐

Early exit from the school ………… ☐

Differentiated tasks for children within classroom….. ☐

Outside classroom activities (e.g. in break time)……. ☐

After school activities (e.g. 'Master Classes')……….. ☐

Grouping children by ability within class ….…………………………… ☐

Grouping children by ability independently of their age (*setting*)…….. ☐

Working with a mentor.…………………………………………………… ☐

Other (please specify): ____ ☐

3.9 How effective do you think is your provision for **gifted children in mathematics**?

Very effective…………… ☐

Moderately effective……. ☐

No impact……………….. ☐

Moderately ineffective….. ☐

Very ineffective………… ☐

You may want to provide further information below:

…………………………………………………………………………………………………
…………………………………………………………………………………………………
…………………………………………………………………………………………………
…………………………………………………………………………………………………

Section 4: About classroom practice in mathematics from your experience

4.1 Do you (personally) use any **extra support teaching materials** beyond what is usually used in your mathematics lessons?

Yes………… ☐

No………….. ☐ *Go to question **4.4***

If **yes**, please specify the name(s) of the resource(s) (e.g. DfES/DCSF, QCA, NACE, Curriculum Online, NRICH): ……………………………………………………

4.2 How often do you use extra support material?

In every lesson……… ☐

Occasionally……….. ☐

4.3 When you select the extra support materials, which children do you give them to?

(Please specify)

……………………………………………………………………………………………

……………………………………………………………………………………………

4.4 If you (personally) **do not** use any extra support teaching materials, is this because?

There is no time for this during everyday lessons…………… ☐

I don't know where I can find appropriate resources…………. ☐

Other (please write in) ……………………………………………………………….

4.5 Do you (personally) use any **special grouping arrangements** in your mathematics class?

Yes… ☐

No… ☐ Go to question 5.1

4.6 How often do you group your students?

In every lesson……… ☐

Occasionally………… ☐

4.7 How do you organise the groups?

By ability ……….……………. ☐

In mixed ability groups ……… ☐

Other (please write in) ……………………………………………………………….

4.8 Do you change children's groupings?

Yes………….. ☐

No………….... ☐

If **yes**, please explain why in the following space:

Section 5: About your own thoughts on mathematically able children

5.1 How would you describe mathematically able children? (Please write in the following space)

5.2 Which of the following terms do you think describes children with higher ability in mathematics than their age peers?

- Gifted ………………………………………………………………………… ☐
- Talented …………………………………………………………………….. ☐
- Gifted and talented ………………………………………………………… ☐
- Able and variations such as 'very able', 'exceptionally able' ……………... ☐
- Promising ……………………………………………………………………… ☐
- Other (please indicate) ________________________________ ☐

5.3 Do you feel that having some 'gifted' or 'able' or 'promising' mathematicians in your class makes your role as a teacher easy?

- Very easy……. ☐
- Easy………..… ☐
- Neutral………. ☐
- Difficult……… ☐
- Very difficult… ☐

Please explain why

…………………………………………………………………………………………………
…………………………………………………………………………………………………

5.4 How comfortable do you feel in teaching mathematics to gifted children?

Very comfortable ……………………………. ☐

Moderately comfortable …………………… ☐

Neither comfortable nor uncomfortable … ☐

Moderately uncomfortable ……………….. ☐

Very uncomfortable ……………………….. ☐

5.5 How well do you think your school addresses the needs of able mathematicians?

Very well………..☐

Well………………☐

Adequately…….. ☐

Poorly…………… ☐

Very poorly…….. ☐

5.6 Do you think you need more support or training to address the challenge of educating gifted mathematicians?

Yes…………….. ☐

No……………… ☐

Don't know……. ☐

If yes, in which of the following areas do you think you need more support or training? (You may choose more than one)

Identification……………………….. ☐

Provision in the classroom………. ☐

Teaching materials……………….. ☐

Provision outside classroom…….. ☐

Monitoring children's progress….. ☐

Other (please write below) ……… ☐

…………………………………………………………………………………………..

5.7 I attempted to make this questionnaire as comprehensive as possible but you may feel that there are things I missed out. Please write in the following space any other thoughts about the education of mathematically able children, using an extra page if necessary.

Please indicate whether you would like to participate in a case study, which will help us to broaden our ideas, as it will possibly bring out aspects from everyday practice that might not have been noticed yet.

Yes…… ☐

No…… ☐

If you indicated **yes**, or you would like to receive a copy of the report with the findings and any future publications, please complete the following:

Name of contact: ……………………………………………………………………………………

School Name and address: ……………………………………………………………………

……………………………………………………………………………………………………..
……………………………………………………………………………………………………..

Telephone number: …………………………..   E-mail: ……………………………………..

Appendix 3: A Sample of Analysis of the Questionnaire with 'Excel'

How teachers explain the level of easiness or difficulty that they feel when there are able mathematicians in their class (Question 5.3)

Very easy	Easy	Neutral	Difficult	Very difficult	Details	School Code
0	1	0	0	0	"Age related expectations are easier for them to reach. New concepts/methods they can adopt too much quicker."	01
0	0	1	0	0	"It depends the range of the rest of the group and how many able children are promising/able/gifted"	02
0	1	0	0	0	"I use them for peer mentoring across the groups in the Year group or in the lower school."	04
0	1	0	0	0	"Can depend on task. Is a challenge to teacher to be able to motivate and inspire..."	07
0	0	1	0	0	"It is a part (no more/no less) of teaching a mixed-ability class and trying to enable all children to achieve their learning potential"	09
0	0	0	1	0	"Very conscious of ensuring appropriate provision & 'moving' pupil… they are often 'grasshoppers'. Such pupils can make good peer mentors. Able children are often 'grasshoppers' & do have to learn the rudimentary skills of presenting working-out methodically in a test situation."	10
0	1	0	0	0	"Because we have provision in place"	13
0	0	0	1	0	"Because you need to plan in order to truly extend them not just give them harder number sentences. But part of the job, so I endeavour to do my best and meeting their needs"	14
0	0	0	1	0	"Need to think how to extend them and make each learning task relevant"	15
0	0	1	0	0	"Easy: They act as mentors in group work; they raise the bar for the rest, especially rest of top group. Difficult: Time consuming to find the right challenges for them. We are discouraged from going to standard textbooks of year above. Specific resources may be too expensive"	16
0	0	0	1	0	"Difficult activities to extend their thinking - their work rate is so much quicker than the rest in class"	17
0	0	1	0	0	"Able to teach a range of abilities"	18
0	0	0	1	0	"Consistently seeking ways to motivate and interest children to prevent boredom"	19
0	0	0	1	0	"Hard trying to cover both ends of spectrum L2B=L5!!!"	20
0	1	0	0	0	"They enjoy the challenge - Can differentiate work easily to meet the needs of all learners in the class"	21
0	0	1	0	0	"All children are taught to their own level."	23
0	0	1	0	0	"All children have individual needs so 'gifted' just extension of normal groupings"	24

Appendix 4: Tools used in observations

4.1. Observation Form

School:………………………………… **Class:………..…**
Date:…………………………………... **No of pupils:…. /Teacher:…./Staff:….**
Duration of lesson:………
Starting time:…… **End time:………**
Topic of the lesson:…………………………………………………………………..

Time	Observations

4.2. Checklist

School:…………………………………   Class:………..…

Date:…………………………………...   No of pupils:…. /Teacher:…./Staff:….

Duration of lesson:………

Starting time:……   End time:………

Topic of the lesson:………………………………………………………………….

DOCUMENTS	
School Policy:	
Planning:	
Records of Children's Assessment:	
Photocopies of Children's Work:	
Resources - Activities:	
Other:	

4.3. A sample of a ten-minute observation notes from the first lesson observed in Sarah's class

21-30	The teacher then says that they are going to use small cubes (plastic cubes for construction games) as "bricks", in order to build their own houses. She asks them to make as many different cuboids as they can and then find ways to count the bricks that they will use. She writes their task on the board and reads it loudly: *How many different cuboids can you and your partner make?* *How can you record them?* The teacher divides the children into groups by ability, keeps the five children of my focus group on the carpet with her and sends the rest with her assistant to their tables to start working. The teacher gives some directions to the five children and then she leaves them to sit together in a table in a front row. I am moving a little closer to them in order to be able to observe them and hear more clearly what they say, but not too close in order to avoid distracting them. The teacher and her assistant are circulating amongst the children and offer help. My focus children work mostly on their own. Sometimes they are speaking to the person who is sitting closest, such as Alvin with Nevil, Nevil with Jake and Jake with Jason, while Amy seems to work alone. Jason walks to another table and speaks with other children too. Alvin has done a cube. He shows it to the others and says: *I've done a cube!* He shows it to the teacher also and the teacher asks him to find how many "bricks" he has used. He says eight. The teacher asks him to try more and think about a pattern. Alvin suggests that numbers like eight, six and four (even numbers) make a cube. The teacher asks him to try it and goes to Jason. Jason shows his own construction to the teacher, which is a rectangular parallelepiped. The teacher asks him to count the "bricks" and Jason counted them correctly (twelve). She asks him then if he can transform it to a cube. Jason now starts adding more rows of bricks.

Appendix 5: Interviews

5.1. A Schedule of Interview Questions

TEACHER INTERVIEW

	Themes	Questions
i.	Planning for identification and provision	- How have the selected children been identified? - How do you assess your pupils? - How do you plan the lessons?
ii.	Resources	- What resources do you use?
iii.	Having able or gifted mathematicians in the class	These were mostly follow-up questions to their responses given in the questionnaire, asking for clarification and further explanations: - Why do you feel that having some very able mathematicians in your class makes your work *easy* (or *difficult*, depending on what the teacher had written)? - Why do you feel *comfortable* (or *uncomfortable*, depending on what the teacher had written) teaching mathematics to able children?
iv.	Organising and teaching the groups (pull-out groups, sets or in-classroom groups, depending on the case)	- How do you organise the groupings? - Have you experienced any problems in organising or reorganising these groups? (If "yes", can you give an example?)
v.	The effectiveness of provision offered (pull-out, setting, or in-classroom grouping programme)	- What do you think is the impact of your methods on pupils' achievements and attitudes? - What do you think is the most important aspect in making successful provision for mathematically able children?
vi.	Further support or training	- What kind of support would you find useful?

CHILD INTERVIEW

	Themes	Questions
i.	Mathematics as a subject	- What do you feel about mathematics? - What does mathematics mean to you?
ii.	Work done in the class or group (pull-out group, set, or abiity group within class, depending on the case)	- How would you describe all the work that you do in this class/group? - What kind of work do you like most? - What are your feelings about this group you are in?
iii.	Working habits and team work	- Do you prefer working alone or with other children as a group? Why?
iv.	Making mathematics lessons more interesting	- If you were asked to give some ideas to make mathematics lessons more interesting, what would they be?

5.2. A sample of a child interviewing

[..........]

INTERVIEWER: Can you tell me what you feel about mathematics?

JAKE: Fun

INTERVIEWER: Fun?

JAKE: Yes

INTERVIEWER: Why do you think it is fun?

JAKE: Because I like number work and I like doing learning objectives in maths. Sometimes I struggle with them; sometimes it's quite easy.

INTERVIEWER: When do you feel is like a struggle?

JAKE: I am not very sure.

INTERVIEWER: What do you think mathematics is?

JAKE: …

INTERVIEWER: What does mathematics mean to you?

JAKE: …

INTERVIEWER: For example, when you hear the word 'mathematics', what comes to your mind?

JAKE: Sometimes it's happy, sometimes it's sad.

INTERVIEWER: Can you tell me more about this?

JAKE: …

INTERVIEWER: When is it happy? When it is sad?

JAKE: One time I felt a little bit sad when Mrs Sarah gave me some other people's work that I thought that I might be able to do, but the work she gave me was quite hard.

INTERVIEWER: And what happened then?

JAKE: I had to have her coming to me all the time. I didn't like it too much.

[..........]

Appendix 6: Participant Information Sheets & Consent Forms

RESEARCH PARTICIPATION INFORMATION SHEET
(LETTER TO SCHOOL HEADTEACHER)

Dear Headteacher

Thank you for your interest in participating in my study.

Following a questionnaire I sent to your school before the end of the last school year, I would like to invite a teacher to participate in a research project, which is part of my doctoral degree in education at Brunel University London. The project is entitled *Educating mathematically able children in five London Local Educational Authorities in the UK primary schools*. I write to ask for your approval and assistance to conduct this research at your school. I have attached a copy of the teacher's Participant Information Sheet, a copy of the Letter to Parents/Guardians and a Consent form for you. If you agree to participate in my research project, please complete the form and return it to me in the enclosed s.a.e.

The purpose of this research study is to explore how primary schools address the needs of mathematically able children in everyday practice. It will investigate the:

- teachers' experiences in making provision for able children in mathematics;
- conditions that support teachers, or not, to address the needs of mathematically able children;
- impact of provision for mathematically able children on students' achievement, behaviour and beliefs.

I am requesting approval for visiting your school for four (4) days over a two-month time. During these visits I, the researcher, would like to observe a teacher in his/her mathematics class for four teaching hours (one per day) and collect a few samples of the children's work. I would also like to interview the teacher for 30 minutes about his/her experiences of teaching mathematics to able children as well as some of his/her students (in groups or individually but always in the presence of the teacher) to explore their views on mathematics and everyday practices. All the interviews will take place at school and will be audiotaped. The classroom observations will focus on teaching strategies, implemented by the teacher, in order to meet the needs of mathematically able children (e.g. differentiation, grouping), and on pupils' interactions.

The findings of this research study will be published in a PhD and possibly in educational journals (anonymously) so that your teacher's and students' views together with other ideas, arisen from practice, reach policy makers and maybe even influence them. The detailed data may also eventually help to the development of a new model of provision specifically for able mathematicians, which should be useful for teachers who face the challenge of effectively educating mathematically able children.

This research study has been reviewed by the Research Ethics Committee of Brunel University London. Please find attached to this letter the Participant Information Sheets for the teacher and children's parents/guardians. If you require any further information please do not hesitate to contact me at xxxxxxxxxxxx@brunel.ac.uk.

Thank you,

Christos Dimitriadis	Professor Valsa Koshy
Research Student	Supervisor of the study

CONSENT FORM

Please tick the appropriate box

	YES	NO
Have you read the Research Participation Information Sheet?	☐	☐
Have you had an opportunity to ask questions and discuss this study?	☐	☐
Have you received satisfactory answers to all your questions, if you had any?	☐	☐
Do you understand that neither your school nor anyone from participants will be referred to by name in any report concerning the study?	☐	☐
Do you understand that participants are free to withdraw from the study:		
• at any time	☐	☐
• without having to give a reason for withdrawing?	☐	☐
I/we approve the interviews with children and the use of an audio recorder.	☐	☐
I/we approve the observations of mathematics lessons taking place.	☐	☐
I/we agree to the use of samples from children’s work when the study is written or published.	☐	☐
I/we agree to the use of non-attributable direct quotes when the study is written or published.	☐	☐
I/we agree the research project to be conducted at our school.	☐	☐
Signature of Headteacher:		
Date:		
Name in capitals:		
If you would like to receive a copy of the report with the findings and any future publications, please complete a name of contact (if it is different from above), address and/or e-mail: …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… ……………………………………………………………………………………………		

PARTICIPANT INFORMATION SHEET
(For the teacher)

PROJECT TITLE: Educating Mathematically Able Children in five London Local Educational Authorities in the UK Primary Schools

THE RESEARCH STUDY

We are inviting you to participate in a research project, which is part of my doctoral degree in education at Brunel University London. As you showed interest to participate in my research case study, it is important for you to understand the purpose of the research and what it will involve, in more detail. Please take time to read the following information carefully and ask me at xxxxxxxxxxxx@brunel.ac.uk. if anything is unclear or if you would like more information. If you decide to take part in the research, please complete the attached consent form and return it to me in the enclosed s.a.e.

THE PURPOSE OF THE RESEARCH STUDY

The purpose of this research study is to investigate how primary schools address the needs of mathematically able children in everyday practice. The study will explore teachers' experiences in making provision for able children in mathematics, as well as the effects of provision on students' achievement and experiences.

METHOD AND DEMANDS ON PARTICIPANTS

If you agree to be included in the research, you will be asked to participate in an in-depth case study of your class. This will involve four (4) observations of your mathematics class (mixed ability class or class on a high ability set, if your school uses 'setting' as a grouping strategy) for one teaching hour each to see what strategies of provision for mathematically able children you are implementing in practice and how your students are responding. You will also be asked to participate in a 30-minute interview where you will have the opportunity to further explain the answers you have already given in the questionnaire. 30-Minute interviews will, also, be conducted with some of your able students in mathematics (in groups or individually but always in your presence) to explore their views on mathematics and everyday practices. All the interviews will be audio taped. A few samples of the children's work will also be collected.

If you agree with the aforementioned interviews and observations of your teaching, I cannot foresee any possible risks or discomfort for you. Your participation in the study is voluntary and you may withdraw freely at any time, withdrawing at the same time any data that you have given.

The results of this study will be published in a thesis and possibly in educational journals so that your views, together with other ideas, arisen from practice, reach policy makers and maybe even influence them. The detailed data may also eventually help the development of a new model of provision specifically for able mathematicians, which should be useful for teachers who face the challenge of effectively educating mathematically able children.

Your identity as a participant, the name of your school and your students will be kept confidential in any publication of this study's results. The information obtained during this research project will be kept confidential at the University until its final publication after which it will be destroyed. This research study has been reviewed by the Research Ethics Committee of the Brunel University London.

Thank you for your interest in participating in this study

Christos Dimitriadis Research Student	Professor Valsa Koshy Supervisor of the study

CONSENT FORM

Please tick the appropriate box

	YES	NO
Have you read the Participant Information Sheet?	☐	☐
Have you had an opportunity to ask questions and discuss this study?	☐	☐
Have you received satisfactory answers to all your questions, if you had any?	☐	☐
Do you understand that neither your school nor anyone from participants will be referred to by name in any report concerning the study?	☐	☐
Do you understand that you and the children are free to withdraw from the study:		
• at any time	☐	☐
• without having to give a reason for withdrawing?	☐	☐
I agree to my and students' interviews being recorded.	☐	☐
I agree to the use of samples from children's work when the study is written or published.	☐	☐
I agree to the use of non-attributable direct quotes when the study is written or published.	☐	☐
Do you agree to take part in this study?	☐	☐

Signature of Research Participant:
Date:
Name in capitals:
If you would like to receive a copy of the report with the findings and any future publications, please complete your address and/or e-mail: ……………………………………………………………………………………… ……………………………………………………………………………………… ………………………………………………………………………………………
Witness statement
I am satisfied that the above-named has given informed consent.
Witnessed by:
Date:
Name in capitals:

RESEARCH PARTICIPATION INFORMATION SHEET
(LETTER TO PARENTS/GUARDIANS)

Dear Parent/Guardian

Following your school joining a research project, your child will be invited to participate in a research project, which is part of my doctoral degree in education at Brunel University London. The project is entitled *Educating mathematically able children in five London Local Educational Authorities in the UK primary schools* and your school showed interest in taking part. I write to ask for your permission to conduct research in your child's class and involve him/her as a participant. Before you decide to approve this, it is important for you to understand the purpose of the research and what it will involve. Please take time to read the following information carefully and ask me at xxxxxxxxxxxx@brunel.ac.uk., if anything is unclear or if you need more information. If you approve your child's participation in the research, please complete the attached consent form and send it to your school.

The purpose of this research study is to investigate how primary schools address the needs of mathematically able children in everyday practice. The study will explore teachers' experiences in making provision for able children in mathematics as well as the effects of provision on students' achievement and experiences.

If you consent to your child being included, your child will be observed, without any intrusion, doing his/her regular mathematics lessons for four (4) teaching hours. A few samples of your child's work may also be collected. The observations will be carried out by me without the involvement of any third person. All the observations will take place in everyday classrooms and will not affect the daily practice. Therefore, there is no change needed for children who may continue to act as usual. Your child may be asked to participate in a 30-minute interview concerning his/her views about mathematics and everyday practices. The interview will be carried out in a group or individually in the presence of the teacher and will be audiotaped. Typical questions include: What do you feel about mathematics? What is your favourite mathematics lesson? Why? Do you prefer working alone or with other children in a group? Why? etc.

Your child's participation in the study is voluntary and he/she may withdraw freely at any time, withdrawing at the same time any data that he /she has given. As my presence in his/her class, the possible participation in a 30-minute interview (with the teacher nearby) and the collection of a few photocopies of his/her work will not discomfort your child, I cannot foresee any possible risks or discomfort for him/her.

The results of this study will be published in a thesis and possibly in educational journals so that your child's views together with practical issues reach policy makers and maybe even influence them. The detailed data may also eventually help the development of a new model of provision for able mathematicians, which should be useful for teachers who face the challenge of effectively educating mathematically able children within the whole class.

Your child will not be identified in any part of this research. The information obtained during this research project will be kept confidential at the University until its final publication after which it will be destroyed.

Thank you for your interest in this study

Christos Dimitriadis	Professor Valsa Koshy
Research Student	Supervisor of the study

CONSENT FORM

Please tick the appropriate box

	YES	NO
Have you read the Research Participation Information Sheet?	☐	☐
Have you had an opportunity to ask questions and discuss this study?	☐	☐
Have you received satisfactory answers to all your questions, if you had any?	☐	☐
Do you understand that your child will not be referred to by name in any report concerning the study?	☐	☐
Do you understand that your child is free to withdraw from the study:		
• at any time	☐	☐
• without having to give a reason for withdrawing?	☐	☐
I agree to my child's interview being recorded.	☐	☐
I agree to the use of samples from my child's work when the study is written or published.	☐	☐
I agree to the use of non-attributable direct quotes when the study is written or published.	☐	☐
Do you agree your child to take part in this study?	☐	☐

Signature of Parent/Guardian of Research Participant:
Date:
Name in capitals:
If you would like to receive a copy of the report with the findings and any future publications, please complete your address and/or e-mail:
Witness statement
I am satisfied that the above-named has given informed consent.
Witnessed by:
Date:
Name in capitals:

Appendix 7: Samples from Planning and Teaching Materials

7.1. Emma's 'Money & Prices' exercise (Day 2)

1a

POUNDS & pence

A world without money

Sometimes we can understand things better by imagining what we would do without them.

Imagine money had not been invented. Have a go at answering these questions.

1. How would you 'buy' things?

2. How would you 'sell' things?

3. How would you get 'paid' for working?

Think about what you can do with money to see how money works.

4. Make a list of the things you buy.

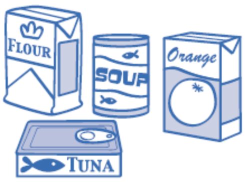
Things I buy often	Things I buy sometimes

5. Make a list of the things your parents or carers buy.

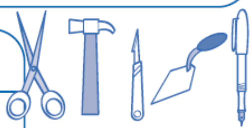
Things adults buy often	Things adults buy sometimes

6. Make a list of some ways that people can make money or earn money.

7. Choose some items from your lists and put them in the correct group.



Goods e.g. food	Services e.g. haircuts, repairs



POUNDS & pence

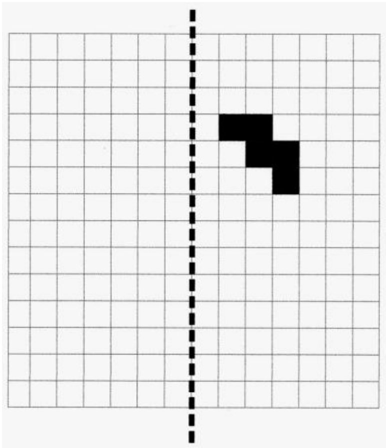
Can you think of anything else we can do with money?

ACTIVITY CARD

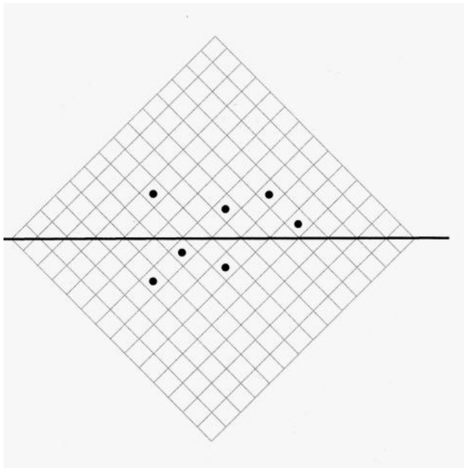
Resource: Bank of England Educational Resources (Bank of England, 2008b)

7.2. Sarah's computerised activities in starter lessons (A representation of activities presented on the interactive whiteboard)

7.2.1. Sarah's 'starter' activities, day 1



(a) First 'symmetry' activity



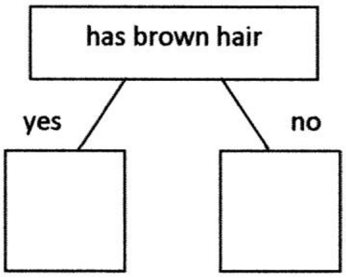
(b) Second 'symmetry' activity

Resource: Test Base website (Testbase, 2008)

7.2.2. Sarah's 'starter' activities, Day 3

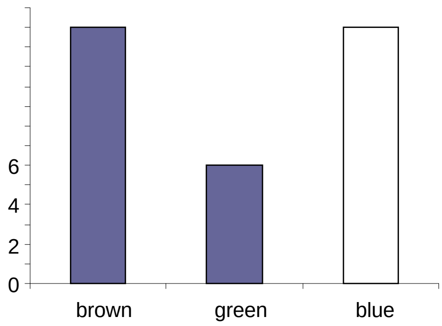
(a) First 'data handling' activity

name	has brown hair
Sara	yes
Lee	no
Anna	no
Carl	yes



Resource: Test Base website (Testbase, 2008)

(b) Second 'data handling' activity



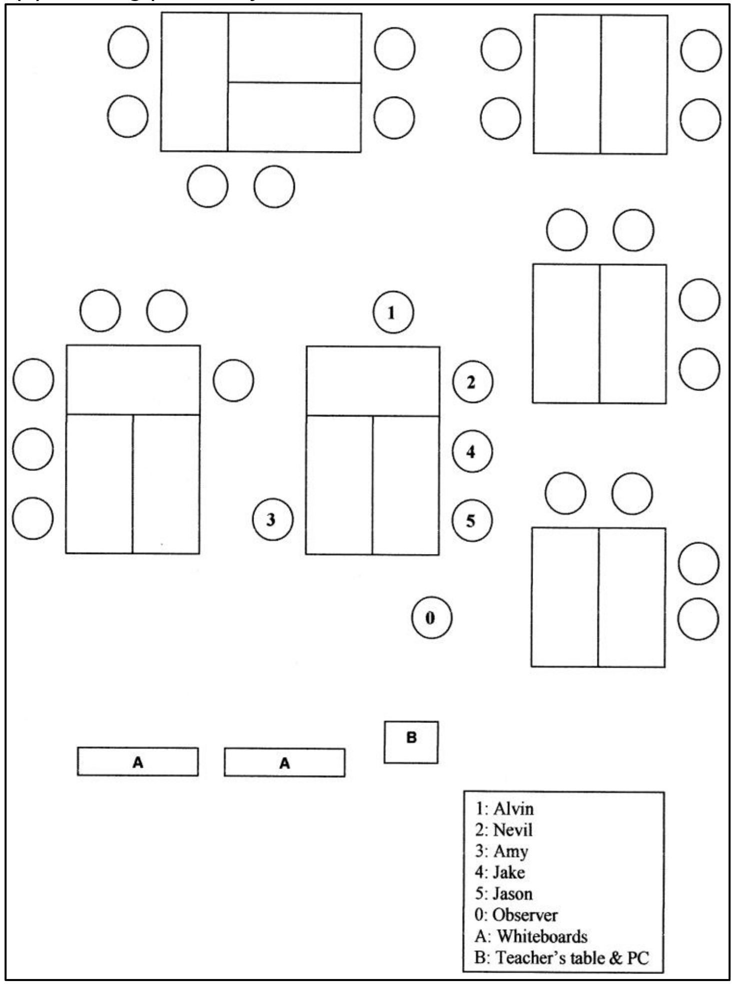
a) 5 children have blue eyes. Show this on the right bar.

b) More children have brown eyes than green eyes. How many?

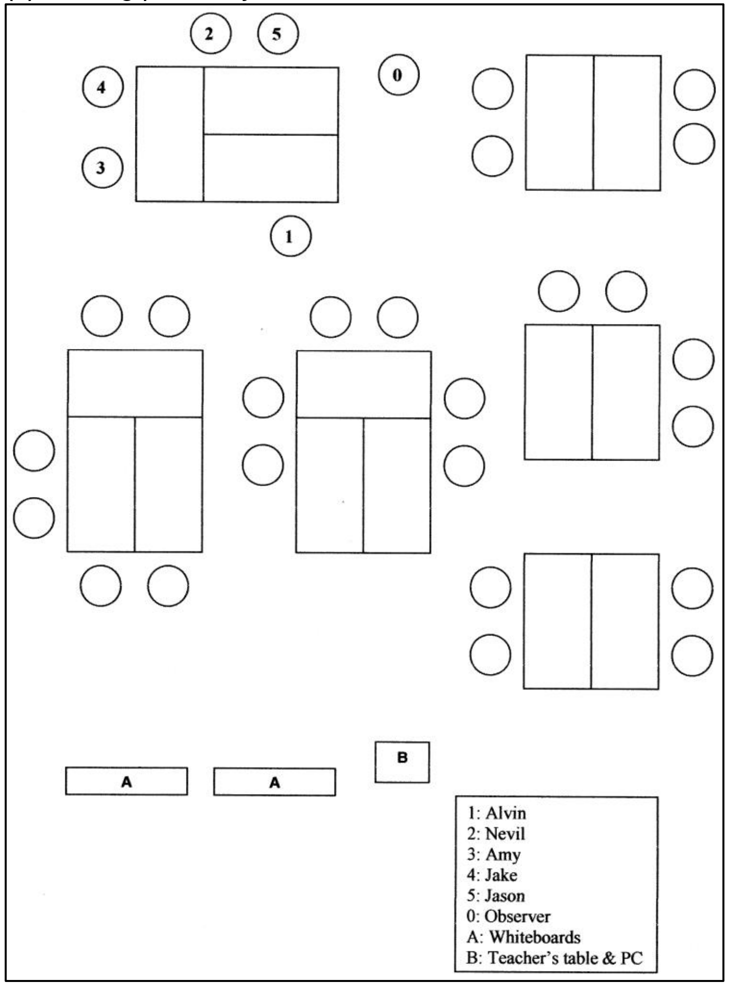
Resource: Test Base website (Testbase, 2008)

7.3. Sarah's seating plans

(a) Seating plan, Day 1



(b) Seating plan, Day 2



7.4. Sarah's 'Space &shape' problem-solving activity (Day 2)

Problem Solving Task Card

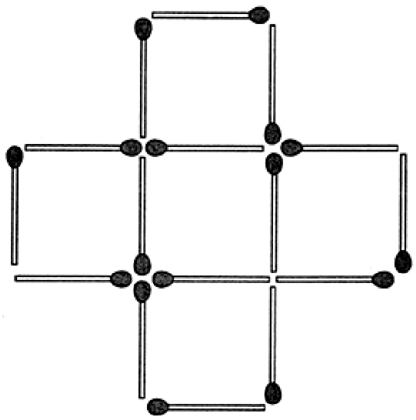
Using equipment and acting out **10**

PROBLEM 10 Shape & Space

Level 3

Matchstick squares

This pattern is made from matchsticks.



★ How many squares are in the pattern?

★ Make the pattern.

★ Move 3 matchsticks to make a different shape with 5 squares.

(Remember: all the squares must touch.)

★ Move 3 matchsticks to make a new shape with 5 squares.

★ How many different shapes with 5 squares can you make?

(Remember: you can only move 3 matchsticks at a time.)

Resource: Badger Maths Problem Solving: Years 1-2 (Nathan, 2007)

7.5. Kate's weekly plan: a sample

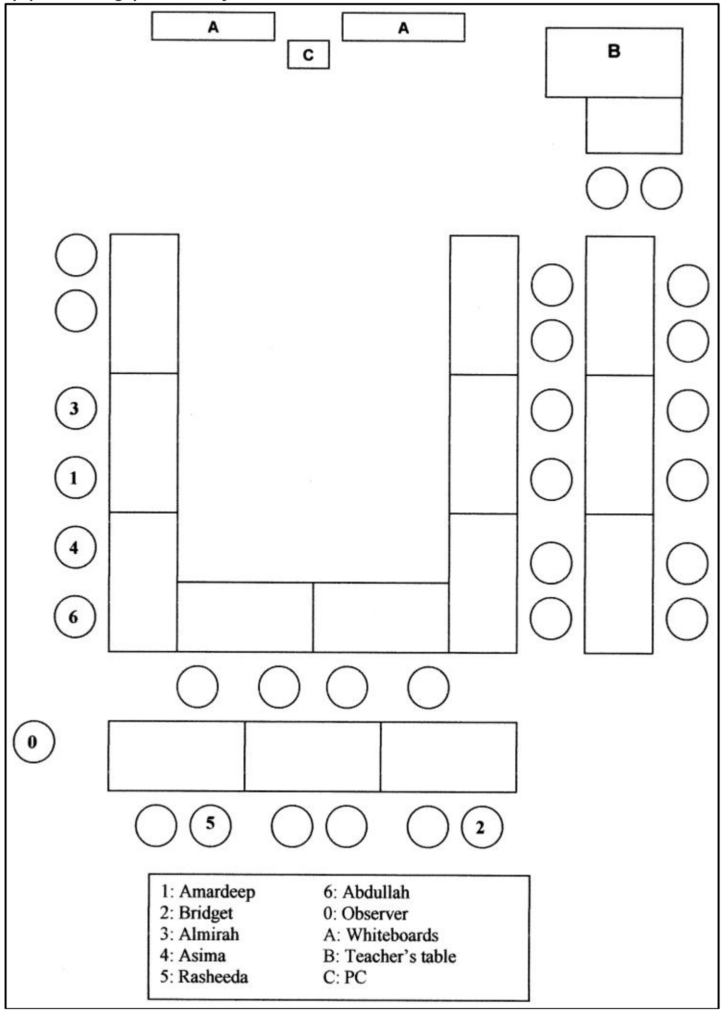
Maths Weekly Plan Year 5/6 **Spring Week 6**

	Mental/oral		Main Teaching Activity							Plenary
	Obj	**Activity**	**Obj**	**Strategies**	**Y5 Easy**	**Y5 Hard**	**Y6 Easy**	**Y6 Hard**	**Resources**	**Key points**
M	Calculate the area of a rectangle. Find the surface area of a box where the faces are all rectangles.	Explain that you are going to find the surface area of a box. On the f/c, write 'surface area = sum of area of faces'. Draw a 5 by 5 by 5 box. *What shape is this? Cube. How many faces? 6 faces all the same size. How can we calculate the area of one face? 5x5=25 square centimetres. What calculation must we do to find the surface area of the whole shape? 25x6=300 square centimetres.* Start Y6 tasks	Under-stand area measured in square centimetres. Under-stand & use the formula in words, 'length times breadth' for the area of a rectangle.	Year 5 Draw a rectangle on the f/c & label the length '17cm' & the breadth '10cm'. *How can we find the area? Multiply 17 by 10. We multiply the length by the breadth.* Ensure chn understand the term 'breadth'. Write 'length times breadth = area of a rectangle'. 17x10=170 square centimetres. Show chn how to write this using a small 2 after cm, i.e. cm^2. Next give each child a postcard & a ruler. Work together to find the area, first measuring the length to the nearest cm & then the breadth, then multiplying. Chn then write the answer in cm^2.	Chn work in pairs. They each draw a rectangle on cm squared paper, with sides less than 10cm, then work out its area & pass it to their partner for checking. Next they each draw a second rectangle with sides twice as long as the first & work out its area also. What do they notice? (Area is 4 times greater.) Needs adult	Draw a rectangle on the f/c. Label its breadth 7.5cm & its area 75cm. Ask chn to work out its length, i.e. 10cm. Ask them to explain what they did. Write 7.5 x __ = 75cm. Draw another rectangle & label its length 15cm & its area 129 square cm. Give chn calculators to find breadth, i.e. 8.6cm. Ensure chn record trial & improvement attempts. **TD**	Give chn a page of rectangles and ask them to calc their areas by multiplying, e.g. 4m by 7m or 20mm by 60mm. Ext: Ask chn to find the areas of given compound (L) shapes after dividing them up into two rectangles. Allow the use of calculators, if necessary. *(SDM Y6 p12)*	Give chn a page of cuboid shaped boxes and ask them to find their surface areas, e.g. 2 by 8 by 4 or 7 by 3 by 6. Ext: Give chn some cuboids with surface areas of three faces shown and challenge them to find the length, width & height of each box. *(SDM Y6 p14 nos1-8)*	Abacus T Cards Y6 M4 & Y5 M3, SDM Y6 pp12 & 14, centimetre squared paper, different sized cuboids.	*Understand that the formula for the area of a rectangle is 'length times breadth'.* *Know that the surface area of a box is the sum of the areas of all its faces.* Choose an object. Give an estimate of the area of one of its rectangular faces. Chn guess the object. Chn take turns to estimate areas of faces of objects, e.g. book cover, desk, door, ceiling, etc. Others guess the object.

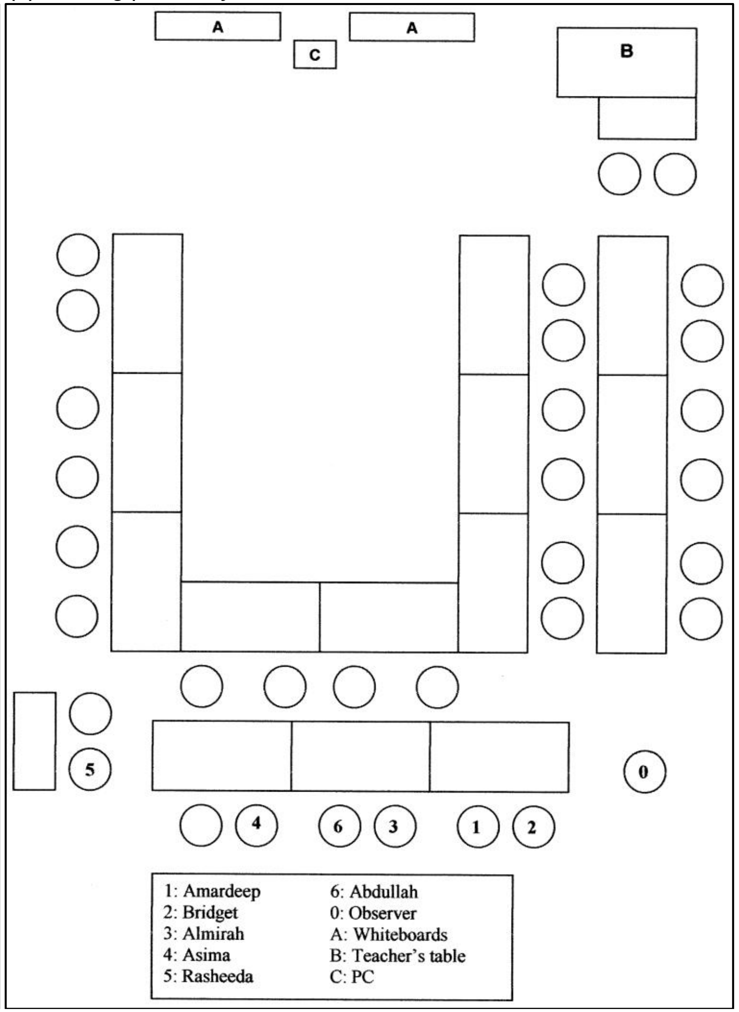
Y5/6-Spr – Week 6 - v2.0

7.6. Kate's seating plans

(a) Seating plan, Day 1



(b) Seating plan, Day 2



7.7: Kate's 'Smartie Party' ratio activity (Day 3)

Smartie Party

In order to eat the Smarties, you first have to complete the work on this sheet!

1. Count the number of smarties you have of each colour, and list them.
   eg. Red 1
   Green 3

2. Find the total number of smarties and write it down.

3. Write down the ratio of each colour, to every other colour.
   eg. Red:Green
   1:3
   Work in a methodical way and don't miss any.

4. Write down each colour as a proportion of the total.
   eg. Green 3 in 15

****

Extension: Now try to write down each colour as a proportion of the total
as a) a fraction eg. Green ${}^{3}/_{15}$ (or ${}^{1}/_{5}$)
b) a decimal eg. Green 0.2
c) a percentage eg. Green 20%
You may use a calculator to help you.

****

5. Eat them!

Source: Primary Resources website (2008)

7.8: Claire's 'Sum Fun' homework (Day 1)

ACTIVITY 3

NAME ______________________________

Sum Fun!

The Fibonacci sequence is 1, 1, 2, 3, 5, 8, and so one. Each number is the sum of the previous two numbers. You can represent all positive integers either as a Fibonacci number or as the sum of non-adjacent (meaning "not next to each other in the sequence") Fibonacci numbers. Show the following integers represented this way. Some have been done for you.

List the Fibonacci numbers in the right column. Use this list to create sums that equal the integers. Do not use two numbers next to each other!

Look for patterns!

			Fibonacci numbers
			1
1 = 1	21 = 21	41 =	**1**
2 = 2	22 =	42 =	**2**
3 = 3	23 =	43 =	**3**
4 = 3 + 1	24 =	44 =	**5**
5 = 5	25 =	45 =	______
6 =	26 =	46 = 34 + 8 + 3 + 1	______
7 = 5 + 2	27 =	47 =	______
8 = 8	28 =	48 =	______
9 =	29 =	49 =	______
10 =	30 =	50 =	______
11 =	31 =		______
12 =	32 = 21 + 8 + 3	60 =	______
13 = 13	33 =		______
14 =	34 = 34	70 =	______
15 =	35 =		______
16 =	36 =	80 =	______
17 =	37 =		______
18 = 13 + 5	38 =	100 =	
19 =	39 =		
20 =	40 = 34 + 5 + 1	1000 =	

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Source: "Fibonacci fun: fascinating activities with intriguing numbers" (Garland, 1997)

7.9: Claire's maths-domino game (Day 3)

7.9.1. Claire's maths-domino game, the beginning

I am 8 divided by -4	I am -2 plus -6	I am -8 divided by -2

7.9.2. Claire's maths-domino game completed