BEAM DYNAMICS IN NS-FFAG EMMA WITH DYNAMICAL MAPS *

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Abstract

The Non-Scaling Fixed Field Alternating Gradient accelerator EMMA has a compact linear lattice, in which the effects of magnet fringe fields need to be modelled carefully. A numerical magnetic field map can be generated from magnet measurements or magnet design software. We have developed a technique that produces from the numerical field map, a dynamical map for a particle travelling in a full EMMA cell, for a given reference energy, without acceleration. Since the beam dynamics change with energy, a set of maps have been produced with various reference energies between 10 MeV and 20 MeV. For each reference energy, the simulated tune and time of flight have been compared with results in Zgoubi - tracking directly through numerical field map. The range of validity of a single map has been investigated by tracking particles with large energy deviation: the results can be used to implement a model of acceleration based on dynamical maps.

INTRODUCTION

In tracking studies, the behaviour of a single particle can be defined by six dynamical variables: the horizontal (vertical) position X (Y) and momemtum P_X (P_Y), the longitudinal position with respect to a reference particle Z, and the energy deviation δ. Tracking a particle through a sequence of magnetic elements in a beamline consists of computing the values of these variables at different locations or steps. If the numerical values of the magnetic field components are known on grid points throughout the magnetic element, then the equations of motion for the dynamical variables may be integrated numerically for given initial conditions, to find the values of the variables at the exit of the magnetic element. However, this method requires tracking through the beamline each time one wants a characterization of the beam behaviour. When tracking many particles through many steps, the process can be highly demanding in terms of computing time and memory.

In EMMA [1], a highly compact doublet cell is achieved using short quadrupole magnets. A large aperture requirement then leads to potentially significant fringe fields. Accurate simulations of the beam dynamics in EMMA require a dense description of the magnetic field, and numerous integration steps. Solving Maxwell's equations in an EMMA cell (by a Finite Element code, OPERA [2]) we have generated a 3D magnetic field map that can be used for numerical tracking in EMMA with PyZgoubi [3, 4, 5]. In most cases, PyZgoubi routines are fast and reliable. However, an alternative approach based on dynamical maps could provide some benefits, particularly where speed is important; for example, when tracking many particles through many cells. Dynamical maps also provide the possibility of reading significant quantities (such as tunes and chromaticities) directly from the map, giving an insight into the dynamics that is not provided directly by purely numerical methods.

To generate a dynamical map, one propagates a variable through the cell as a function instead of a numerical value. The magnetic field must be expressed in analytical form: an appropriate form can be obtained from a numerical field map by fitting an appropriate three-dimensional mode expansion [6]. Then, we use a symplectic integrator implemented in the differential algebra (DA) code COSY [7], to propagate a vector of six power series (one series for each of the six dynamical variables) through the field.

DYNAMICAL MAP DESCRIPTION

The DA integration routine outputs the dynamical map in explicit form as shown in Table 1. The first column gives the name of each coefficient following the TRANSPORT code nomenclature. The final six columns indicate, as exponents for the six dynamical variables, the term in the map to which the coefficient in second column refers. Thus, each variable is expressed as a power series in the values of the dynamical variables at the entrance of the cell. For instance the expression for the horizontal position is:

$$\begin{aligned} X_1 \;=\; & 1.0344\,X_0 + 0.2683\,P_{X0} - 0.0103\,\delta_0 + \\ & 4.6261\,X_0^2 + 1.7204\,X_0 P_{X0} \cdots + 0.0096\,\delta_0^2. \end{aligned}$$

Therefore, once the dynamical map has been obtained, tracking particles in the EMMA cell simply involves calculating the ouput values for a given set of input values. The power series is truncated at a certain order (in the example in Table 1, at 2nd order). Even though the integration

Table 1: Selected terms from the 2nd order dynamical map for the X variable, for one EMMA cell at 15 MeV reference energy.

	Coefficient	Order	Exponents
R11	1.0344	1	1 0 0 0 0 0
R12	0.2683	1	0 1 0 0 0 0
R16	-0.0103	1	0 0 0 0 0 1
T111	4.6261	2	2 0 0 0 0 0
T112	1.7204	2	1 1 0 0 0 0
⋮	⋮	⋮	⋮
T166	0.0096	2	0 0 0 0 0 2

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routine is symplectic, the truncation results in a symplectic error. A symplectic transformation satisfies $J^T \cdot S \cdot J = S$, where S is a block diagonal matrix constructed from the 'unit' 2×2 antisymmetric matrix, and J is the Jacobian of the transformation. A symplectic error in a map may be significant if the map is applied iteratively many times, or if small non-symplectic physical effects are being investigated. If the map is computed using a symplectic integrator (as is the case for the results presented here), then the symplectic error depends on the order of truncation, rather than integration step size.

The symplectic integrator that we use requires the paraxial approximation. This requires some care, since in an FFAG, beam excursions can be large (of the order of a few cm). However, when one computes a dynamical map, it is necessary to make a choice of reference trajectory. Since energy and transverse position are correlated, a sensible choice is to look for closed orbits for various reference energies over the full energy range (in EMMA, from 10 MeV to 20 MeV), and use these closed orbits as reference trajectories. For small energy deviations, particle trajectories should then remain close to the reference trajectory, and the paraxial approximation should be valid.

In practice, we do not use exactly the closed orbit as the reference trajectory at a given reference energy. For simplicity in the integration, we use instead a straight line starting (and ending) at the middle of a long drift, where the field is close to zero; the position of the straight line is chosen to minimize the excursion of the closed orbit with respect to this reference trajectory. Since there are 42 cells in total in EMMA, concatenating the dynamical map around the straight reference trajectory with a rotation through $2\pi/42$ (about a vertical axis) produces a map for one periodic section of the EMMA lattice.

MAPS WITH VARIOUS REFERENCE ENERGIES

We first compare the results of the dynamical map with the results of numerical tracking in Zgoubi. Eleven dynamical maps were calculated around the closed orbits for reference energies from 10 MeV to 20 MeV, in steps of 1 MeV. The tunes (phase advances per cell) can be obtained from the eigenvalues λ of the linear part of a given map:

$$\lambda = e^{\pm 2\pi i \nu},$$

where ν is the tune.

The zeroth-order term in the map for the fifth variable (longitudinal coordinate, Z) represents the difference in path length of the closed orbit with respect to the reference trajectory.

The comparisons of these features with numerical tracking through the magnetic field map with PyZgoubi are plotted in Figs. 1 and 2. The two codes show good agreement for the horizontal tune and path length, although a slight discrepancy occurs in the path length for high energy. This might be due to a truncation in the relativistic conversion

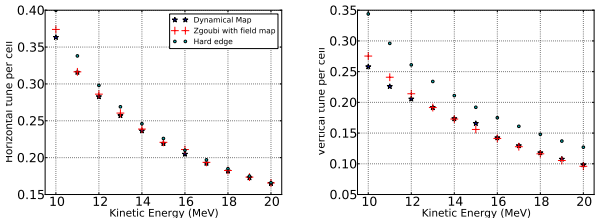

Figure 1: Horizontal (left) and vertical (right) tune versus kinetic energy using multiple dynamical maps.

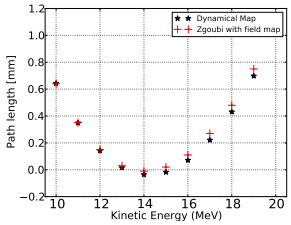

Figure 2: Path length versus kinetic energy from dynamical map (blue) and PyZgoubi (red).

from time to flight to path length and has to be studied in more detail. A hard-edge model of the magnets in Zgoubi is also shown (in green) on the plot, and indicates the impact of the fringe field on the vertical tune: a discrepancy of about 10% is found at 10 MeV.

Betatron motion may be studied in more detail by applying the dynamical maps to particles with some initial transverse offset with respect to the reference trajectory. Fig. 3 shows the horizontal phase space for reference energies from 10 MeV to 20 MeV, constructed by applying the appropriate dynamical map iteratively to particles with 1 mm initial transverse offset (with respect to the reference trajectory). We notice that there is some non-physical growth in the amplitude over time for 10 MeV and 11 MeV: this is a consequence of the truncation of the dynamical map to 2nd order. The effect disappears if terms up to 4th order are retained.

MAPS WITH ENERGY DEVIATION

The number of dynamical maps required to model the dynamics over the full energy range in EMMA will depend on the range of validity of each map with respect to vari-

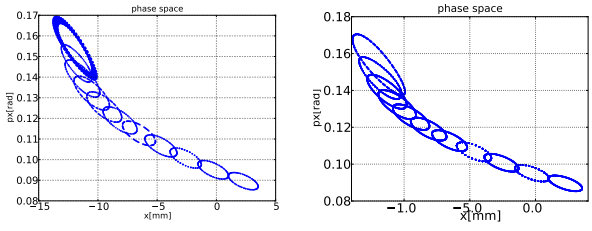

Figure 3: Betatron motion with multiple dynamical maps.

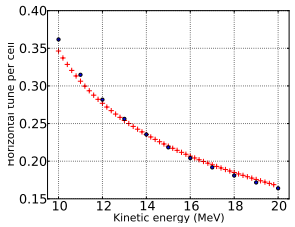

Figure 4: Betatron tunes computed with a set of dynamical maps at different reference energies (blue dots), and with a single dynamical map with different energy deviations (red crosses).

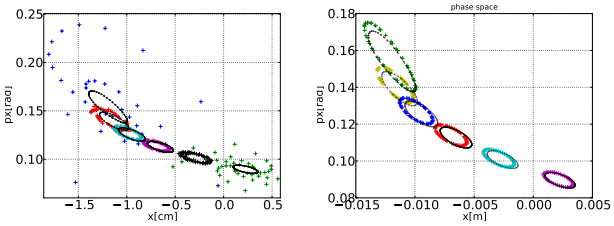

Figure 5: Betatron motion at different energies, simulated with dynamical map up 4th order (left), and 9th order (right).

ations in the energy deviation δ. Fig. 4 shows the tune as a function of energy obtained in two different ways: first, from different dynamical maps computed for different reference energies (blue dots); and second, from a single dynamical map at a single reference energy (15 MeV), but with different values for the energy deviation δ (red crosses). The good agreement between the two methods suggests that it may be possible to use a single map to describe the transverse dynamics with good accuracy, even for large energy deviations.

Fig. 5 shows the horizontal phase space for different energies, generated using maps with different reference energies (dots), and with fixed 15 MeV reference energy but different energy deviations (crosses). We see a strong effect from the symplectic error with a map truncated at 4th order; the effect of the error is greatly reduced by retaining terms in the map up to 9th order.

For modelling acceleration in EMMA, it is important that the dynamical maps also describe accurately the variation in path length with energy. Fig. 6 shows the length of the closed orbit computed in two different ways: first, from a set of dynamical maps at different reference energies (blue dots); and second, from a single dynamical map (15 MeV reference energy) with different values for the energy deviation (red crosses). We see there are large discrepancies (more than 10%) at large energy deviations when the maps are truncated at 2nd order; however, there is good agreement for the maps truncated at 4th order.

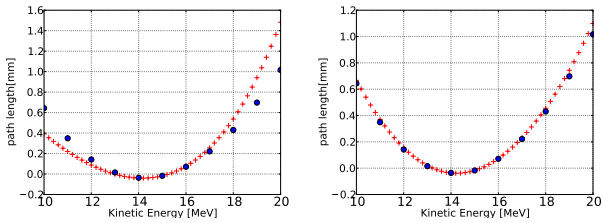

Figure 6: Closed orbit path length computed with a set of dynamical maps at different reference energies (blue dots), and with a single dynamical map with different energy deviations (red crosses). Left: dynamical map up to 2nd order. Right: dynamical map up to 4th order.

SUMMARY AND NEXT STEPS

The large range of transverse positions in an FFAG can be modelled using multiple dynamical maps with different reference trajectories. The phase advance per cell and the path length computed using a dynamical map show good agreement with the results obtained using a numerical tracking code, Zgoubi. Accurate description of the betatron motion requires the dynamical maps to be computed to at least the 4th order, at small energy deviation. If the energy deviation is large (for example, to cover the full energy range in EMMA from 10 MeV to 20 MeV in a single dynamical map), then a map up to 9th order may be required to model the horizontal motion.

Acceleration may be included in the dynamics by making an appropriate adjustment to the energy deviation at the end of each cell (representing the effect of an RF cavity in the cell). The results presented here suggest that it may be possible to achieve a reasonable description of the dynamics using a fixed reference energy at the mid point of the energy range. However, if a very accurate description is required, then it may be necessary to change to a different reference energy at one or more points during the acceleration.

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