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Международная конференция по геометрическому анализу в честь 90-летия академика Ю. Г. Решетняка

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Сборник содержит тезисы некоторых докладов, представленных на Международной конференции по геометрическому анализу, проводимой в честь 90-летия академика Ю. Г. Решетняка (22–28 сентября 2019 года). Темы докладов относятся к современным направлениям в геометрии, теории управления и анализе, а также к приложениям методов метрической геометрии и анализа к смежным областям математики и прикладным задачам.

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The digest contains abstracts of some of the talks presented on the International Conference on Geometric Analysis in honor of the 90th anniversary of academician Yu. G. Reshetnyak (22–28 of September, 2019). Topics of talks concern modern trends in geometry, control theory and analysis, as well as applications of the methods of the metric geometry and analysis to related fields of mathematics and applied problems.

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The mixed volumes of convex hull of distinct leaves can be obtained by the polarization of the polynomial $V(p)$. According to SP, the coefficient at $s^k t^{8-k}$ is equal to the multiplied by $\binom{8}{k}$ mixed volume of k bodies $\widehat{F}_{\varpi_1}$ and $8-k$ bodies $\widehat{F}_{\varpi_2}$, where ϖ_1, ϖ_2 are the fundamental weights (see [4, 5.1.26]).

Due to BKK-theory we know the explicit formula for Bézout's number (i.e. the number of solutions) for systems of equations $f_k = 0$ with matrix elements of finite dimensional representations of a complex reductive Lie group. It involves volumes and mixed volumes of the convex hulls of integral orbits of the coadjoint representations (see [5]). We are grateful to Boris Kazarnovskiĭ for making us aware of this fact and for useful comments. These results were extended and generalized in several directions but to the best of our knowledge the case of isoparametric foliations is not covered yet.

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Composition operators on Sobolev spaces and the spectrum of elliptic operators

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This work is devoted to applications of the geometric theory of composition operators on Sobolev spaces to spectral problems of the A-divergent form elliptic operators with the Neumann boundary condition:

$$L_A = -\operatorname{div}[A(z)\nabla f(z)], \quad z = (x, y) \in \Omega, \quad \left.\frac{\partial f}{\partial n}\right|_{\partial\Omega} = 0, \tag{1}$$

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in quasiconformal regular domains $\Omega \subset \mathbb{C}$ with $A \in M^{2\times 2}(\Omega)$. We denote, by $M^{2\times 2}(\Omega)$, the class of all 2×2 symmetric matrix functions $A(z) = \{a_{kl}(z)\}$, $\det A = 1$, with measurable entries satisfying the uniform ellipticity condition

$$\frac{1}{K}|\xi|^2 \leq \langle A(z)\xi, \xi\rangle \leq K|\xi|^2 \text{ a.e. in } \Omega, \tag{2}$$

for every $\xi \in \mathbb{C}$, where $1 \leq K < \infty$.

Recall that a simply connected domain $\Omega \subset \mathbb{C}$ is called an A-quasiconformal β-regular domain, $\beta > 1$, if

$$\iint\limits_{\mathbb{D}} |J(w, \varphi^{-1})|^{\beta} \, dudv < \infty,$$

where $\varphi : \Omega \to \mathbb{D}$ is an A-quasiconformal mapping [3]. Since (see, for example, [1]) A-quasiconformal mappings $\varphi : \Omega \to \mathbb{D}$ are defined up to conformal automorphisms of $\mathbb{D}$, this definition doesn't depend on a choice of φ and depends on the quasihyperbolic geometry of Ω only. A class of quasiconformal regular domains includes Lipschitz domains, Gehring domains and also some fractal domains like snowflakes.

The suggested method is based on the connection between composition operators on Sobolev spaces and quasiconformal mappings, that refines (in the case $n = 2$) results of [2]. As applications we obtain the solvability of the spectral problem (1) and lower estimates of the first non-trivial Neumann eigenvalues in A-quasiconformal β-regular domains [3]:

Theorem. *Let A be a matrix satisfies the uniform ellipticity condition (2) and Ω be an A-quasiconformal β-regular domain. Then the spectrum of the Neumann divergence form elliptic operator L_A in Ω is discrete, and can be written in the form of a non-decreasing sequence:*

$$0 = \mu_0(A,\Omega) < \mu_1(A,\Omega) \leq \mu_2(A,\Omega) \leq \ldots \leq \mu_n(A,\Omega) \leq \ldots,$$

and $$\frac{1}{\mu_1(A,\Omega)} \leq \frac{4}{\sqrt[\beta]{\pi}} \left(\frac{2\beta-1}{\beta-1}\right)^{\frac{2\beta-1}{\beta}} \left(\iint\limits_{\mathbb{D}} |J(w,\varphi^{-1})|^{\beta} \, dudv\right)^{\frac{1}{\beta}},$$

where $J(w, \varphi^{-1})$ is a Jacobian of the quasiconformal mapping $\varphi^{-1} : \mathbb{D} \to \Omega$.

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