

**ICFSP**  
**2019**

**2019 5th International Conference on  
FRONTIERS OF SIGNAL PROCESSING**

**Marseille, France | September 17-21, 2019**



# A Simple Method for Increasing the Equal-amplitude Non-uniform Linear Thinned Array Directivity

Oleg V. Chernoyarov

International Laboratory of Statistics of Stochastic  
Processes and Quantitative Finance  
National Research Tomsk State University  
Tomsk, Russia  
Dept. of Higher Mathematics and System Analysis  
Maikop State Technological University  
Maikop, Russia  
e-mail: chernoyarovov@mpei.ru

Alexandra V. Salnikova

International Laboratory of Statistics of Stochastic  
Processes and Quantitative Finance  
National Research Tomsk State University  
Tomsk, Russia  
Dept. of Building Engineering and Urban Planning  
University of Zilina  
Zilina, Slovakia  
e-mail: amicus.lat@yandex.ru

Inna A. Kirpicheva, Aleksandr V. Ostankov

Dept. of Radio Engineering  
Voronezh State Technical University  
Voronezh, Russia  
e-mail: avostankov@mail.ru

**Abstract**—Calculating the distances between the elements of the non-uniform array is usually carried out through its parametric optimization using the computational algorithms. In this paper, a simplified method is introduced and tested for determining the distances between the neighboring elements of the equal-amplitude non-uniform linear array that provides a significant increase in the array directivity while preserving the array thinning. The developed method allows both choosing the element spacing on the non-uniform linear array aperture and ensuring an increase in the array directive gain without applying the computational algorithms.

**Keywords**—non-uniform linear array; directive gain; aperture geometry; array optimization

## I. INTRODUCTION

The arrays are widely applied in modern information and radio systems [1]. They are structurally implemented in a variety of ways, for example, as the periodic grating arrays excited by a surface wave of a dielectric waveguide at the millimeter wavelengths [2]. The reflecting arrays with a quasi-optical supply are also used operating in the mode of feed wave re-reflection or focusing [3].

In the millimeter, submillimeter and optical ranges, the distance between neighboring array elements may exceed the wavelength due to design constraints, for example, because of the need to place a phase shifter in the phased array in the vicinity of the millimeter array element. It must be emphasized that the pattern of such thinned arrays contains the additional diffraction lobes that leads to a significant decrease in both the directional properties and the directive gain. In order to improve the directional properties, the space tapering of the radiating elements is applied. But searching

the distances between multiple array elements providing the maximization of directional array properties is a very difficult task [4, 5].

The aim of the paper is to propose a relatively simple method for calculating the distances between the neighboring elements of the equal-amplitude linear array, which will maximize its directive gain while preserving the thinning.

## II. TASK SOLVING METHODOLOGY

Let the linear array elements are positioned along the X-axis. The distance between  $k$ -th and  $(k+1)$ -th elements is designated as  $d_k$  (Fig. 1). The total number of the elements composing the array is equal to  $N$ . The emission wavelength  $\lambda$  is set so that the propagation constant in free space is equal to  $k_0 = 2\pi/\lambda$ . The uniform amplitude and phase distributions imply the same amplitudes ( $A_k = \text{const}$ ) and initial phases ( $\Phi_k = \text{const}$ ) of the currents (fields) on all array elements. The direction angle to the observation point ( $\Theta$ ) is measured from the vertical Z-axis in accordance with Fig. 1.

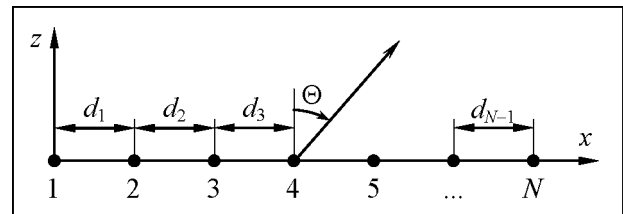


Figure 1. Model geometry of the optimized linear array

The amplitude array pattern is calculated assuming that the elements are isotropic by the classical expression [1]:

$$F_N(\Theta) = \left| \sum_{k=1}^N A_k \exp(j\Phi_k) \exp(-jk_0 x_k \sin\Theta) \right|,$$

where  $x_k = \sum_{i=1}^{k-1} d_i$ ,  $k = \overline{1, N}$ .

The directive gain towards the peak array pattern (directivity) is determined by the formula

$$D_m = 2F^2(\Theta_m) / \int_{-\pi/2}^{\pi/2} F^2(\Theta) \cos\Theta d\Theta,$$

where  $\Theta_m$  is the angular direction of the peak array pattern.

It is presupposed that when the distances between the elements of the non-uniform array are found, its length always remains constant and equal to the length of the corresponding uniform array with the specified step  $d$  and the thinning coefficient  $\kappa = d/\lambda$ :

$$L = \kappa\lambda(N-1).$$

Let the position of the peripheral array elements is rigidly fixed and their coordinates are equal to  $x_1 = 0$  and  $x_N = L$ . In addition, it is useful to place the array elements symmetrically relative to the array center. Under this condition, the coordinates of the elements located to the right from the array center are related to the coordinates of the elements located to the left from the center as follows:

$$x_k = L - x_{N-k+1}.$$

One needs to find the coordinates  $x_k$  of the elements located to the left from the array center.

At the stage of development of the simplified method for determining the distances between the array elements, numerical optimization has been repeatedly carried out, with the values of the required distances calculated as close to optimal. It should be noted that the most important component of optimization is the use of a numerical algorithm for searching the global extremum of a function [6]. Genetic algorithms appear to make the achievement of the global extremum of the functions with the several arguments highly probable. In the present study, the modified genetic algorithm described in detail in [7] is applied. The software implementation of this algorithm is freely available on the Internet [8].

As an objective function, the maximum of which should be achieved in the parametric optimization, we take the functional dependence of the directive gain on the coordinates  $x_k$  of equal-amplitude linear array elements.

### III. THE RESULTS OF THE STUDY AND THEIR EXAMINATION

Below the selected results obtained in the numerical optimization are presented.

Let the number of array elements be equal to eight. The spacing of the initial equidistant array is  $1.1\lambda$ . In Fig. 2, by dashed line the normalized array pattern is shown. It can be seen that the pattern has two marked side lobes formed as a result of diffraction. The array directive gain is 7.8 dB. Here by the solid line the array pattern is drawn, for which the distances between the elements are obtained by optimization. From Fig. 2, it follows that the non-uniform array pattern has one main lobe only. Increase of the side lobes closest to the main one, their level being minus 4.8 dB (unlike the minus 12.8 dB for the uniform array), is obviously due to the level of diffraction pattern lobes decreasing. The directive gain after optimization is 10.7 dB, which is 2.9 dB more.

The geometry of the non-uniform array is shown in Fig. 3. And here the elements of the initial uniform array are also presented. While analyzing Fig. 3, one can conclude that the optimal non-uniform array is generally consistent with the some uniform array with the spacing of  $0.77\lambda$  and the 11th elements that does not have three central elements. In such array, the directive gain is 10.6 dB (less than the optimal value by 0.1 dB).

Let us increase the initial uniform array spacing up to  $1.6\lambda$ . For this case the normalized array pattern is drawn in Fig. 4. The directive gain of such array is 9.3 dB, while the width of the main lobe at the level of minus 3 dB is  $3.0^\circ$ . In the same Figure, by continuous line, the array pattern is shown for the case when the distances between the array elements are optimized. The directive gain of the optimized array is 10.9 dB. The width of the main lobe is  $3.1^\circ$ , while the maximum level of the side lobes is minus 2.1 dB.

The distribution of the elements on the non-uniform optimized array aperture is shown in Fig. 5. The elements of the initial uniform array are also marked there. It can be seen from this Figure that, by the position of the peripheral elements, the optimal non-uniform array, even without 7 central elements, is almost equivalent to the uniform array that has the spacing of  $0.80\lambda$  and consists of 15 elements.

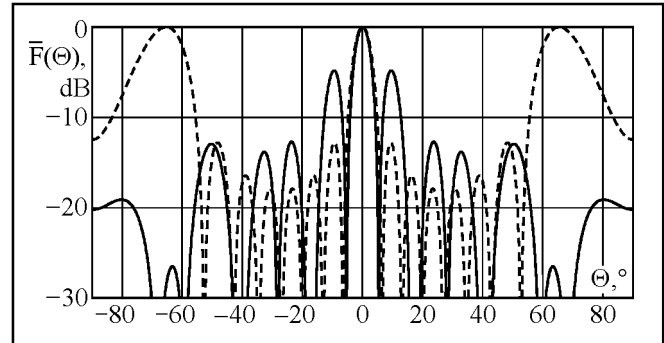


Figure 2. The pattern of the optimized non-uniform array with  $N = 8$  and  $\kappa = 1.1$

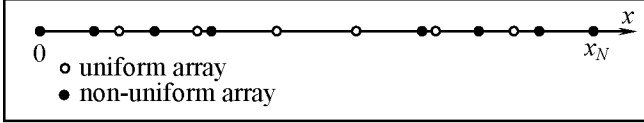


Figure 3. The geometry of the initial and optimized arrays with  $N = 8$  and  $\kappa = 1.1$

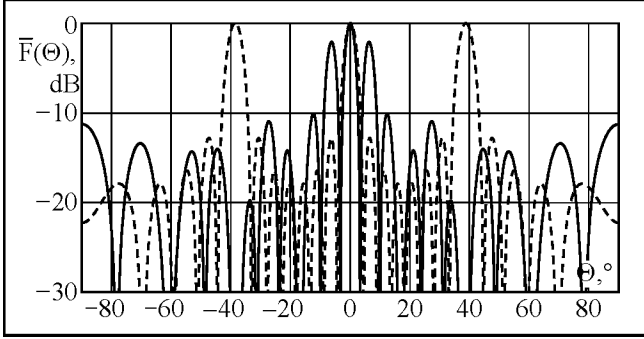


Figure 4. The pattern of the optimized non-uniform array with  $N = 8$  and  $\kappa = 1.6$

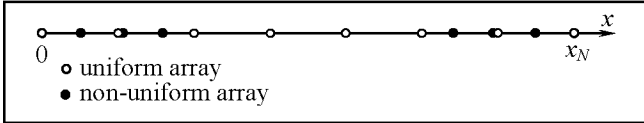


Figure 5. The geometry of initial and optimized arrays with  $N = 8$  and  $\kappa = 1.6$

Let the spacing of the initial uniform array is equal to  $2.1\lambda$ . Then the normalized antenna pattern on a logarithmic scale has the form shown by the dashed line in Fig. 6. The directive gain of such array is 8.5 dB, and the width of the main lobe at the level of minus 3 dB is  $3.0^\circ$ . In the same Figure, by continuous line the array pattern is shown in case when the distances between the array elements are optimized. The directive gain of the optimized array is 10.8 dB that is by 2.3 dB greater. The width of the main lobe is  $2.25^\circ$ , while the maximum level of the side lobes is minus 0.94 dB.

The geometry of the non-uniform array is shown in Fig. 7. And the elements of the initial uniform array are also shown here.

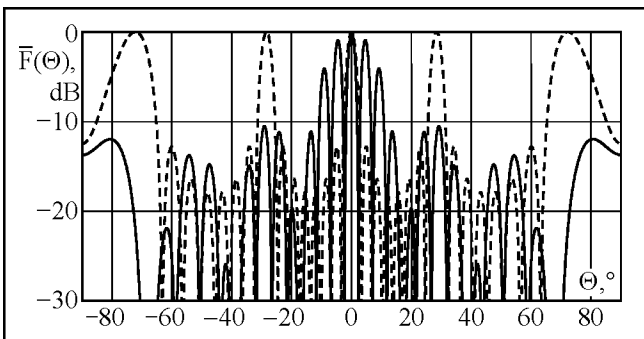


Figure 6. The pattern of the optimized non-uniform array with  $N = 8$  and  $\kappa = 2.1$

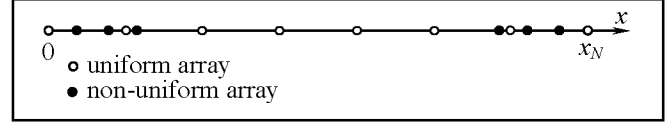


Figure 7. The geometry of initial and optimized arrays with  $N = 8$  and  $\kappa = 2.1$

From Fig. 7 it follows that the optimal non-uniform array practically coincides by the position of the elements with the uniform array with the spacing of  $0.817\lambda$  and 19th elements, while 11 central elements are removed. It should be noted that the calculated directive gain of this uniform array is 10.7 dB (that is, less by 0.1 dB only).

Similar calculations are performed for the other values of the array thinning. The results of the parametric optimization for different values of the number of elements and the sparseness of the array are presented in the Table I. The fourth column of the Table specifies the average distance between the peripheral elements of the quasi-uniform array obtained during the optimization by the criterion of maximum directive gain.

TABLE I. THE CHARACTERISTICS OF THE OPTIMIZED EQUAL-AMPLITUDE NON-UNIFORM ARRAYS BY THE CRITERION OF MAXIMUM DIRECTIVE GAIN

$N$	$\kappa$	$D_{m1}^{(1)}$ , dB	$D_{m2}^{(2)}$ , dB	$\xi_2^{(3)}$ , dB	$d_{\text{mean}}/\lambda^{(4)}$	$d_{\text{simple}}/\lambda^{(5)}$	$N_1^{(6)}$
8	1.1	7.8	10.7	-4.8	0.80	0.77	3
8	1.6	9.3	10.9	-2.1	0.85	0.80	7
8	2.1	8.5	10.8	-0.9	0.83	0.82	11
8	2.6	9.2	10.8	-0.6	0.82	0.82	15
12	1.1	9.6	13.0	-6.9	0.90	0.86	3
12	1.6	11.1	12.8	-2.1	0.86	0.88	9
12	2.1	10.1	12.6	-1.9	0.90	0.89	15
12	2.6	11.0	12.8	-0.6	0.86	0.89	21

<sup>1)</sup> the directive gain of the initial uniform array;

<sup>2)</sup> the directive gain of the optimized non-uniform array;

<sup>3)</sup> the maximum level of the side lobes of the optimized array;

<sup>4)</sup> the average distance between the peripheral elements of the quasi-uniform array;

<sup>5)</sup> the distance between the array elements obtained by the simplified method;

<sup>6)</sup> the number of the missing elements if the central part of the array.

Summarizing the numerous results obtained, there can be proposed the following simplified method for synthesizing the non-uniform linear thinned array with the uniform amplitude and phase distributions, even number of elements and directive gain close to maximum:

- for the specified array length  $L$ , such an odd number  $N_1$  is determined of the uniform array elements, that the distance between those elements has the value closest to  $(0.8 - 0.9)\lambda$  (see the 5th column of the Table);

- from the resulting uniform array aperture the central elements are removed, the odd number of which is

determined by the difference between  $N_1$  and  $N$  (see the 6th column of the Table).

This method has been tested and its efficiency confirmed. In order to illustrate the efficiency of the proposed method, there is considered the array with the number of elements  $N = 16$  and the thinning coefficient  $\kappa = 1.1$ .

In Fig. 8, by the dashed line the pattern is shown of the uniform array with the spacing of  $1.1\lambda$  (the directive gain is 10.8 dB), while by the solid line the pattern is drawn of the non-uniform one synthesized using the proposed method (the directive gain increases up to 14.0 dB).

The arrays geometry is shown in Fig. 9. During the numerical optimization, the non-uniform array is obtained which is also presented in Fig. 9. The array geometry generated by the computational algorithm provides 14.3 dB of the directive gain (that is greater by 0.3 dB). In Fig. 8, the pattern of such optimized array is plotted by points.

Let us decrease the number of the array elements down to 12, while the sparseness coefficient remains the same. In Fig. 10, by the dashed line, the pattern is shown of the uniform array with the spacing of  $1.1\lambda$  (the directive gain is 9.6 dB). In accordance with the introduced technique, the spacing of the quasi-uniform array with the elements shifted to the periphery is set equal to  $0.86\lambda$  (see Table). In the same Figure, by the solid line, the pattern is shown of the array synthesized by means of the simpler techniques.

It is estimated that the directive gain of the array has increased up to 12.7 dB, while the side lobe radiation level has decreased down to minus 6.0 dB. In addition, one can notice that, by positioning the array elements in accordance with the results of the numerical optimization by the criterion of the maximum directive gain, you can obtain almost identical pattern that is drawn by points in Fig. 10. The directive gain of such array is greater by 0.3 dB only, while the side lobe radiation level is less by 0.9 dB.

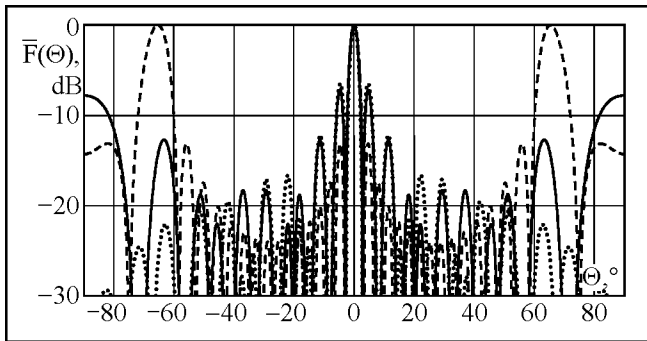


Figure 8. The pattern of the non-uniform array with  $N = 16$  and  $\kappa = 1.1$  obtained by the introduced method

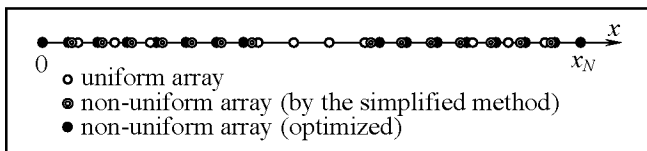


Figure 9. The geometry of the arrays with  $N = 16$  and  $\kappa = 1.1$ : initial, optimized and obtained by the introduced method

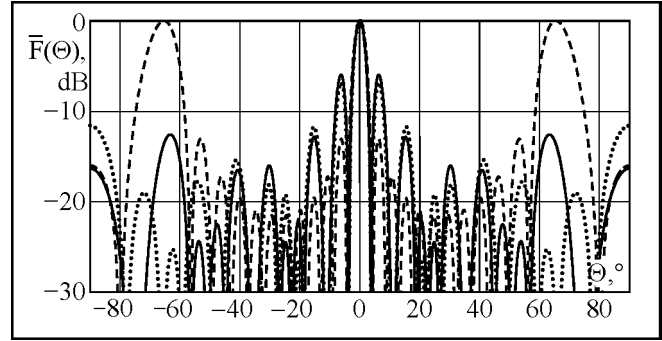


Figure 10. The pattern of the non-uniform array with  $N = 12$  and  $\kappa = 1.1$  obtained by the introduced method

#### IV. CONCLUSION

Thus, based on the numerical parametric optimization of the geometry of the linear equal-amplitude non-uniform array in terms of the directive gain maximum, a simple method for determining the aperture geometry has been proposed providing a significant improvement in the array directional properties. It is established that the optimal non-uniform array composed of  $N$  elements is consistent with the uniform array with the spacing  $(0.8 - 0.9)\lambda$  and  $N + k$  elements, where  $k$  central elements are removed from the aperture, by the directive gain magnitude.

#### ACKNOWLEDGMENT

This study was financially supported by the Russian Science Foundation (research project No. 17-11-01049).

#### REFERENCES

- [1] R. C. Hansen, Phased Array Antennas. New Jersey: Wiley, 2009.
- [2] A. V. Ostankov, S. A. Antipov and K. A. Razinkin, "Optimization of directional and energetic properties of diffraction antenna", Global Journal of Pure and Applied Mathematics, vol. 12, Apr. 2016, pp. 3845-3864.
- [3] A. V. Ostankov, S. A. Antipov and Y. E. Kalinin, Analysis and Synthesis of the Aperture of Diffraction Radiation Antennas Based on the Quasi-periodic Ridged Arrays (in Russian). Voronezh: Voronezh State Technical University, 2016.
- [4] A. V. Ostankov and I. A. Kirpicheva, "Optimization of the directional properties of linear nonequidistant antenna arrays" (in Russian), Bulletin of Voronezh State Technical University, vol. 9, Jun. 2013, pp. 8-11.
- [5] A. V. Ostankov, S. A. Antipov and Y. S. Sakharov, "Minimax level of side radiation of the nonequidistant antenna array with the uniform amplitude distribution" (in Russian), Bulletin of Voronezh State Technical University, vol. 9, Dec. 2013, pp. 10-12.
- [6] A. G. Trifonov, The Statement of the Optimization Problem and Numerical Methods for Its Solution (in Russian). Electronic Publication: [http://matlab.exponenta.ru/optimiz/book\\_2/index.php](http://matlab.exponenta.ru/optimiz/book_2/index.php).
- [7] V. R. Sabanin, N. I. Smirnov and A. I. Repin, "Parametric optimization and diagnostics using genetic algorithms" (in Russian), Industrial Automatic Control Systems and Controllers, Dec. 2004, pp. 27-31.
- [8] Electronic Publication: [http://twf.mpei.ac.ru/ochkov/Mathcad\\_12/3\\_31\\_genetic.mcd](http://twf.mpei.ac.ru/ochkov/Mathcad_12/3_31_genetic.mcd).