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HOW DO DAMPED ACOUSTIC WAVES LOOK LIKE ON A MESOSCALE? A DISCONTINUOUS VARIATIONAL APPROACH

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The *discontinuous Lagrangian approach*, allowing for a variational description of irreversible phenomena in continuum theory such as viscosity and thermal conductivity, is utilised for the analysis of damped acoustic waves. Starting from a Lagrangian for general viscous flow theory, by linearisation of the resulting Euler-Lagrange equations and performing an ensemble average, a single wave equation for the density perturbations is obtained, being the one resulting from classical Navier-Stokes theory with an additional term due to thermodynamic non-equilibrium. By considering harmonic waves, the respective non-classical dispersion relation and its implications are analysed against the background of discontinuities occurring on a mesoscale. Related physical models are suggested.

Wave theories in general have their origins in equations stemming from underlying continuum theory; in the case of damped acoustic waves propagating in a liquid or gas these comprise the continuity and Navier-Stokes-Duhem equations together with an appropriate thermal conduction-convection equation. An alternative approach is starting from a variational principle. As is well known, the use of Hamilton's principle is ideally suited to, for example, the field of conservative Newtonian mechanics. Contrary to this, in continuum theories many open problems remain unsolved, typically in the field of viscous flow; since there are, in general, no obligatory construction rules for establishing variational principles, for certain problems a variety of suggestions have appeared from various authors based on different approaches. One has to distinguish between two major categories, namely between variational formulations based on a field description (Eulerian description), and a stochastic variational description based on a material description (Lagrangian description) and averaging particle motion. Based on a rigorous analysis of the fundamental symmetries the Lagrangian has to fulfil, with particular regard to Galilean invariance [1] and utilizing the complex field of thermal excitation proposed by Anthony [2], a Lagrangian for viscous flow considering both shear viscosity and volume viscosity has been formulated [3], provided with a discontinuity and containing an additional parameter ω_0 that can be interpreted as a relaxation rate toward thermodynamic equilibrium. By careful analysis it is proven that the dynamics resulting from Hamilton's principle can consistently be interpreted as a generalisation of the theory of viscous flow towards thermodynamic non-equilibrium, giving rise to recovery of the well-known Navier-Stokes equations and the balance of inner energy when applying the limit $\omega_0 \to \infty$ to the resulting equations of motion.

Sound waves can be obtained as solutions of the linearised or weekly nonlinear fluid equations of motion. This topic has been well researched [4]. If transmitted through a fluid medium over a long distance, damping due to dissipation may become relevant. Two competing mechanisms for damping exist, based on thermal conductivity and on viscosity. Damping due to viscosity is considered in [5], where also conceptual similarities of the discontinuous Lagrangian approach [3] with the stochastic variational approach [6] are exposed by interpreting the discontinuities occurring on a microscopic scale as fluctuations. Therefore, although originally motivated by previous research involving deterministic field theories, the use of the discontinuous Lagrangian seems to embrace aspects of both concepts and can therefore be considered as kind of 'in-between' or lying betwixt deterministic and statistic approaches, since equations of motions result which are the classical ones plus 'deterministic' fluctuations.

In the subsequent paper [7] a generalisation of the Lagrangian toward thermal conduction is provided and a wave equation for viscously damped sound waves influenced by thermodynamic non-equilibrium effects and its associated dispersion relation is derived after ensemble averaging of the discontinuous equations of motion. If the angular frequency ω of the wave is much smaller than ω_0, the dispersion relation well known from classical Navier-Stokes theory is discovered.

Despite the recovery of classical results on the macroscopic scale, less attention has been paid to the dynamics of the non-equilibrium fluctuations becoming manifest as discontinuities of the original (non-averaged) equations of motions resulting from Hamilton's principle. A detailed analysis of discontinuous slip-waves and shock waves occurring on a mesoscopic scale and the discussion of the results against the background of physical models is subject of the investigations in the present paper. Perspectives and forthcomings of the discontinuous Lagrangian approach are shown.

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