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Ruzinazar Beshimov
National University of Uzbekistan, rbeshimov@mail.ru

Dilnora Safarova
National University of Uzbekistan, safarova.dilnora87@mail.ru

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GENERALIZED METRIC SPACES AND HYPERSPACES

Beshimov R.B., Safarova D.T.
National University of Uzbekistan, Tashkent, Uzbekistan
e-mail: rbeshimov@mail.ru, safarova.dilnora87@mail.ru

Abstract

In this paper, we investigate the heredity of some kind of generalized metric spaces to $\exp_c X$ and $\exp_n X$. We will study the connection between a σ-space, Σ-space, a stratifiable space, $\aleph$-space, $\aleph_0$-space and its hyperspace.

Keywords: *Hyperspace, σ-space, Σ-space, stratifiable space, $\aleph$-space, semistratifiable space, $\aleph_0$-space.*

Mathematics Subject Classification (2010): *54B20, 54E18, 54E20, 54D50.*

Introduction

Exponential spaces were introduced in the work [1]. Michael [2] studied various topologies on the collection of nonempty closed subsets of a topological space. In the works [3, 4], they studied some categorical and topological properties of the functors exp, $\exp_c$, $\exp_\omega$ and $\exp_n$. In that work [5], categorical and cardinal properties of hyperspaces with a finite number of components were investigated and it was proved that this functor is not a normal functor. They proved that the functor $C_n : Comp \to Comp$ is not normal, i.e., it does not preserve epimorphisms of continuous mappings. They also discussed the density, the caliber, and the Shanin number of the space C_nX. This space is of interest since it contains the hyperspaces $\exp_n X$ of closed sets with cardinalities not greater than n elements. T.Mizokami [6] studied the hereditary property to the hyperspaces $\exp_c X$ and $\exp_\omega X$ when X is a generalized metric space around Moore space. For example, as it is well known, Moore space, stratifiable space, metrizable and σ-space are inherited to $\exp_\omega X$ [7]. As for as the reciprocal relationship of topological properties between X and $\exp_c X$ and $\exp_n X$, there has been a natural problem as follows: Let C be a class of spaces with some property. if $X \in C$, does then $\exp_c X$ or $\exp_n X$ belong to C? In this paper, we survey this problem when C is restricted to class of generalized metric spaces known already. As candidates for C, we consider the σ-spaces, paracompact Σ-spaces, stratifiable spaces, $\aleph$-spaces, semistratifiable spaces and $\aleph_0$-spaces classes. It is shown that the functor $\exp_n X$ preserves the class of Σ-spaces, paracompact Σ-spaces, stratifiable spaces, $\aleph$-spaces, semistratifiable spaces and $\aleph_0$-spaces.

In [8], V.V. Fedorchuk stated the following general problems in the theory of covariant functors that defined a new direction of investigations in the given field of the topology: Let P be some geometrical property and F some covariant functor. If topological space X has a property P, then $F(X)$ has the same property P? Or, conversely, i.e. for what functors F, if $F(X)$ has property P would imply that X has the same property? In our case $F = exp_nX$ and $X \in T_1$.

In [5], categorical and cardinal properties of hyperspaces with a finite number of components were investigated and it was proved that this functor is not a normal functor. It was proved in [9] that the Radon functor satisfies all the normality conditions. In [10], the topological properties of topological groups were studied.

Let X be a topological T_1-space. Denote by $\exp X$ the set of all nonempty closed subsets of the space X. The family of all sets in the form of $O\langle U_1, ..., U_n\rangle = \{F : F \in \exp X, F \subset \bigcup_{i=1}^{n} U_i, F \bigcap U_i \neq \emptyset, i = 1, 2, ..., n\}$ where $U_1, ..., U_n$ is a sequence of open subsets of X, generates the topology on the set $\exp X$. This topology is called the Vietoris topology. The set $\exp X$ with the Vietoris topology is called the exponential space or the hyperspace of the space X.

Let X be a topological T_1-space. Denote by $\exp_n X$ the set of all nonempty closed subsets of X of cardinality not greater than the cardinal number n, i.e. $\exp_n X = \{F \in \exp X : |F| \leq n\}$. Put $\exp_\omega X = \bigcup\{\exp_n X : n = 1, 2, ...\}$, $\exp_c X = \{F \in \exp X :\ F \text{ is compact in } X\}$. It is clear, that $\exp_n X \subset \exp_\omega X \subset \exp_c X \subset \exp X$ for any topological space X [11].

It is known that for a Hausdorff space X and a natural number n, the space $\exp_n X$ is closed in the space $\exp X$. It is easy to see that if X is a T_1- space, then the mapping $i : X \to \exp X$ corresponding to the point $x \in X$ to the one-point set $i(x) = \{x\}$, is an embedding, that is, considered as a mapping onto the set $\exp_1 X$, it is a homeomorphism. For a Hausdorff space X, we have a chain of embeddings: $\exp_1 X \subset \exp_2 X \subset ... \subset \exp X$. It is clear that if $n \geq m$ then the space $\exp_m X$ is closed in the space $\exp_n X$ [11].

Proposition 1. *[12]. Let X be a T_1-space. To each point $(x_1, x_2, ..., x_n) \in X^n$ we associate a point $\{x_1, x_2, ..., x_n\} \in \exp_n X$ with a point. Then we get a continuous surjective map:*

$$\pi_{n,X} = \pi_n : X^n \to \exp_n X.$$

Theorem 1. *[13]. If X is Hausdorff, then π_n is a closed continuous surjection.*

T. Ganea ([13, p. 306.]) proved that in general π_n is not an open mapping.

A continuous mapping $f : X \to Y$ is called a *perfect map* if X is a Hausdorff space, f is a closed, onto and $f^{-1}(y)$ are compact subsets of X. A continuous mapping $f : X \to Y$ is called a *quasi-perfect map* if X is a Hausdorff space, f is a closed, onto and $f^{-1}(y)$ are countable compact subsets of X.

Proposition 2. *[14]. If $f : X \to Y$ is a perfect mapping, then for any closed $A \subset X$ and any $B \subset Y$ the restrictions $f_A : A \to Y$ and $f_B : f^{-1}(B) \to B$ are perfect.*

1 Main Result

A topological space X is called a *Lasnev space* if there is an image of a metrizable space under a closed continuous mapping [15]. It is clear that the closed continuous image of each Lasnev space is Lasnev.

Proposition 3. *An arbitrary subset of the Lasnev space is also Lasnev.*

Proof. Let X be a Lasnev space and L is an arbitrary subset of X . Then there exists a metric space M and a closed continuous map f from M onto X. Since each subspace of a metric space is also a metric space, it follows that the subspace $f^{-1}(L)$ of M is so. It follows from [14] that the restriction $f_L : f^{-1}(l) \to L$ is a closed mapping. Consequently, the subset L is Lasnev space subspace of the space X. Proposition 3 is proved. □

The product of two Lasnev spaces does not preserve Frechet property, sequentiality and countable tightness. However, the finite Cartesian product of Lasnev spaces is not Lasnev space.

Example 1. *[7]. There exists a countable Lasnev space X such that $\exp_c X$ is not Lasnev space.*

Corollary 1. *If the space $\exp_n X$ is Lasnev, then X is also Lasnev.*

Proof. It is clear X is a subset of $\exp_n X$ and class of Lasnev spaces is hereditarily all subspaces. Therefore, X is a Lasnev space. Corollary 1 is proved. □

A family $\mathcal{N} = \{M_s\}_{s\in S}$ subsets of a topological space X is a *network* for X if for every point $x \in X$ and any neighbourhood U of x there exists an element $s \in S$, such that $x \in M_s \subset U$. A family $\{A_s\}_{s\in S}$ of subsets of a topological space X is *locally finite* if for every point $x \in X$ there exists a neighbourhood U such that the set $\{s \in S : U \cap A_s \neq \emptyset\}$ is finite. A family of subsets of a topological space is called *σ-locally finite* (*σ -discrete*), if it can be represented as a countable union of a locally finite (discrete) families [14].

Definition 1. *[16]. A topological space X is called a σ-space if it has a σ-locally finite network.*

The class of σ-spaces is very well behaved in terms of various topological operations. It is easy to check that this class is hereditary and countably productive. It is also true that the countable product of paracompact σ-spaces is again a paracompact σ-space [17].

Proposition 4. *A space X is a σ-space if and only if $\exp_n X$ is a σ-space.*

Proof. Necessity. Let X be a σ-space. Then by [17] the countable product space X is a σ-space, that is, X^n is a σ-space. It follows from [17] that the class of σ-space is preserved under a closed map. Since the mapping π_n is closed and $\pi_n(X^n) = \exp_n X$, it follows that the spaces $\exp_n X$ is a σ-space.

Sufficiency. Obviously, X is a subspace of $\exp_n X$ and the class of the σ-spaces is hereditary for all subspaces. Therefore, the space X is a σ-space. Proposition 4 is proved. □

Borges [18] showed the following property by which a stratifiable space, paracompact σ-space, and σ-space are not hereditary to $\exp_c X$, though they are so to $\exp_\omega X$.

Corollary 2. *A Hausdorff space X is a paracompact σ-space if and only if* $\exp_n X$ *is a paracompact σ-space.*

Corollary 3. *[19]. Let X be a locally compact paracompact and Y a paracompact. Then $X \times Y$ is a paracompact space.*

Lemma 1. *Let the family of open sets $\mu = \{ V_i^\alpha : \alpha \in A_i,\ i = 1, 2, ..., n \}$ is a refinement of the family $\{G_1, G_2, ..., G_n\}$ in open subsets of a topological space X. Then the family $\mu_1 = \{ O\langle V_1^{\alpha_1}, V_2^{\alpha_2}, ..., V_n^{\alpha_n}\rangle : \alpha_i \in A_i,\ i = 1, 2, ..., n \}$ is a refinement of the family $\{O\langle G_1, G_2, ..., G_n\rangle\}$ in the hyperspace* $\exp X$*, where $\{ V_i^\alpha \in \mu : \alpha \in A_i,\ i = 1, 2, ..., n \}$*

Theorem 2. *A locally compact space X is paracompact if and only if* $\exp_c X$ *is paracompact.*

Proof. Necessity. Let X be a locally compact paracompact space and consider its arbitrary open cover $\mu = \{O\langle U_1^\alpha, U_2^\alpha, ..., U_n^\alpha\rangle : \alpha \in A\}$ in $\exp_c X$. Consider the trace of the family μ in the space X, i.e. $\mu_1 = \{ U_i^\alpha : \alpha \in A,\ i = 1, 2, ..., n \}$. It is clear that μ_1 is an open cover of X. Since X is paracompact, there exists a locally finite open cover of $\nu = \{V^\beta : \beta \in B\}$, which is a refinement of μ_1.

Consider all possible finite combinations of the cover ν and put $\nu_1 = \left\{O\left\langle V_1^\beta, V_2^\beta, ..., V_k^\beta\right\rangle : \beta \in B\right\}$. It is clear that ν_1 is a cover of the space $\exp_c X$ and ν_1 is inscribed in the cover μ_1 by virtue of Lemma 1. Let us show that the system ν_1 is locally finite. Let $F \in \exp_c X$ be an arbitrary element, then F is compact and $F \subset X$. Since the cover of $\nu = \{V^\beta : \beta \in B\}$ is locally finite, every point $x \in F$ has a neighborhood $O(x)$ such that $\{\beta \in B : O(x) \cap V^\beta \neq \emptyset\}$ is finite. Let the point x run along the compact set F. Since F is compact, there exists $O(x_1), O(x_2), ..., O(x_k)$, that $F \subset \bigcup_{i=1}^{k} O(x_i)$ and $\{\beta \in B : O(x_i) \cap V^\beta \neq \emptyset\}$ are finite for every $i = 1, 2, ..., k$. Then the set $O\langle O(x_1), O(x_2), ..., O(x_k)\rangle$ is a neighborhood of the compact set $F \in \exp_c X$ and $\left\{\beta \in B : O\langle O(x_1), O(x_2), ..., O(x_k)\rangle \cap O\left\langle V_1^\beta, V_2^\beta, ..., V_k^\beta\right\rangle \neq \emptyset\right\}$ is finite.

Sufficiency. Let $\exp_c X$ be paracompact. Let $\mu = \{U^\alpha : \alpha \in A\}$ be an arbitrary open cover of the space X. Consider all possible finite combinations of the cover μ and put $\mu_1 = \{O\langle U_1^\alpha, U_2^\alpha, ..., U_n^\alpha\rangle : \alpha \in A,\ U_i^\alpha \in \mu,\ i = 1, 2, ..., n\}$. It is clear that μ is an open cover of the space $\exp_c X$. Since $\exp_c X$ is paracompact, there exists a locally finite open cover $\nu = \left\{O\left\langle V_1^\beta, V_2^\beta, ..., V_s^\beta\right\rangle : \beta \in B\right\}$ which is refinement of the cover μ_1.

Consider the trace ν_1 of a family in the space X. Let us show that the trace $\nu_1 = \left\{ V_i^\beta : \beta \in B,\ i = 1, 2, ..., n \right\}$ is a locally finite open cover of X. Let x be an arbitrary point of X, then $\{x\} \in \exp_c X$. Since $\exp_c X$ is paracompact and ν is a locally finite open cover of X, then there exists a neighborhood $O\langle G\rangle$ of $\{x\}$, that $\left\{\beta \in B : O\langle G\rangle \cap O\left\langle V_1^\beta, V_2^\beta, ..., V_s^\beta\right\rangle \neq \emptyset\right\}$ is finite. This means that $x \in G$ and $G \cap V_i^\beta \neq \emptyset$, $i = 1, 2, ..., s$ and for finite $\beta \in B$. Hence we have that the system ν_1 is locally finite. Theorem 2 is proved. □

Corollary 4. *A locally compact space X is paracompact if and only if the space $\exp_n X$ is paracompact.*

Remark 1. *In Theorem 2.1, the condition of locally compactness is essential. There exists a paracompact space X^* such that $\exp_n X^*$ is not paracompact space.*

Indeed, consider the space "One arrow" of P.S.Alexandroff $X^* = [0, 1)$, the base of which is formed by subsets of the form $(\alpha,\ \beta)$, where $0 \leq \alpha < \beta \leq 1$. It was proved in [14] that X^* is a hereditarily paracompact space. By virtue of Theorem 5.1.5 [14], every paracompact Hausdorff space is normal. If $X^* * X^*$ is normal. We get a contradiction. In this case a space $\exp_n X^*$ is not a paracompact space.

K. Nagami gave the definition of Σ-spaces that is more useful to use here than the original one in [20].

Definition 2. *[20]. A space is a (strong) Σ-space if there exists a pair $\{\mathcal{J},\ \mathcal{C}\}$ of families satisfying the following conditions:*

1. *$\mathcal{J}$ is a σ-discrete family of subsets of X;*
2. *$\mathcal{C}$ is a cover of X by closed countably compact (respectively compact) subsets of X;*
3. *If $C \in \mathcal{C}$ and U is an open subset of X such that $C \subset U$, then $C \subset F \subset U$ for some $F \in \mathcal{J}$.*

K. Nagami [20] showed that the class of paracompact Σ-spaces is preserved under a quasi-perfect map and a countable product.

More strictly, T.Mizokami constructed an example that there exists a paracompact Σ-space such that is not a Σ-space. But, If we consider class of paracompact Σ-spaces, then we obtained following result.

Theorem 3. *If X is a paracompact Σ-space, then $\exp_n X$ is also a paracompact Σ-space.*

Proof. Let X be a paracompact Σ-space. Then by [20] the class of paracompact Σ-spaces preserves the countable product, it follows that X^n is a paracompact Σ-space. Obviously, the class of Σ-spaces is preserved under a quasi-perfect mapping, in particular, a perfect mapping. In addition, paracompactness is preserved under a perfect mapping. Since the mapping π_n is perfect and $\pi_n(X^n) = \exp_n X$, it follows that the space $\exp_n X$ is paracompact Σ-space. Theorem 3 is proved. □

Definition 3. *[21]. A topological space X is called a srtatifiable space if X is T_1-space and to each open $U \subset X$, one can assign a sequence $\{U_n\}_{n=1}^{\infty}$ of open subsets of X such that*

1. *$[U_n] \subset U$;*
2. *$\bigcup\limits_{n=1}^{\infty} U_n \subset U$;*

3. $U_n \subset V_n$, whenever $U \in V$ for each $n \in N$.

It is easy to check that each stratifiable space is regular. We give another properties of this space. A stratifiable space is hereditarily and countably productive. The stratifiable space is preserved under a closed continuous mapping [21].

Corollary 5. *A space is srtatifiable if and only if is a srtatifiable space.*

Proof. Necessity. Let X be a srtatifiable space. Then by [17] the countable product space X is a srtatifiable space, that is, X^n is a srtatifiable space. It follows from [21] that the class of a stratifiable space is preserved under a closed mapping. Since the mapping π_n is closed and $\pi_n(X^n) = \exp_n X$, it follows that the spaces $\exp_n X$ is a stratifiable space.

Sufficiency. Obviously, X is a subspace of $\exp_n X$ and the class of the stratifiable space is hereditary for all subspaces. Therefore, the space X is a stratifiable space. Corollary 5 is proved. □

Definition 4. *[22]. A collection $\mathcal{F}$ of subsets of a space X is a k-network if whenever K is a compact subset of an open set U, there exists a finite $\mathcal{F}' \subset \mathcal{F}$ such that $K \subset \bigcup \mathcal{F}' \subset U$. A regular space with σ-locally finite (countable) k-network is called an $\aleph$-space ($\aleph_0$-space).*

Proposition 5. *[23]. Let $f : X \to Y$ be a perfect mapping of a topological space X onto a topological space Y. If X has a k-network of cardinality $\tau \geq \aleph_0$, then Y has a k-network of cardinality $\leq \tau$.*

Theorem 4. *[17]. The classes of $\aleph$-spaces, $\aleph_0$-spaces and paracompact $\aleph$-spaces are hereditary and countable productive.*

Theorem 5. *A space X is an $\aleph$-space if and only if $\exp_n X$ is an $\aleph$-space.*

Proof. Let X be an $\aleph$-space. By [17], the space X^n is an $\aleph$-space. Since the cover of $\nu = \left\{V^\beta : \beta \in B\right\}$ is locally finite, every point $x \in X^n$ has a neighborhood $O(x)$ such that $\left\{\beta \in B : \ O(x) \cap V^\beta \neq \emptyset\right\}$ is finite. Let a family $\mu = \bigcup\limits_{w=1}^{\infty} \nu_w$ (where a family $\nu_w = \left\{V_w^\beta : \beta \in B \ w = 1, 2, ...\right\}$ is a locally finite) k-network and σ-locally finite in X^n. It is obviously that k-network is preserved under the perfect mapping [23]. Consider all possible finite combinations of the cover ν and put $\nu_1 = \left\{O\left\langle V_1^\beta, V_2^\beta, ..., V_k^\beta\right\rangle : \beta \in B\right\}$. It suffices to show that the family $\nu_1 = \left\{O\left\langle V_1^\beta, V_2^\beta, ..., V_k^\beta\right\rangle : \beta \in B\right\}$ is locally finite in $\exp_n X$. Let us show that the system $\nu_1 = \left\{O\left\langle V_1^\beta, V_2^\beta, ..., V_k^\beta\right\rangle : \beta \in B\right\}$ is locally finite. Let the point x run along the set F. Since $F = \{x_1, x_2, ..., x_n\}$, then there exists $O(x_1), O(x_2), ..., O(x_k)$, that $F \subset \bigcup\limits_{i=1}^{k} O(x_i)$ and $\left\{\beta \in B : \ O(x_i) \cap V^\beta \neq \emptyset\right\}$ are finite for every $i = 1, 2, ..., k$. Then the set $O\langle O(x_1), O(x_2), ..., O(x_k)\rangle$ is a neighborhood of the set $F \in \exp_n X$ and $\left\{\beta \in B : O\langle O(x_1), O(x_2), ..., O(x_k)\rangle \cap O\left\langle V_1^\beta, V_2^\beta, ..., V_k^\beta\right\rangle \neq \emptyset\right\}$ is finite.

Obviously, X is a subspace in $\exp_n X$, and the class of the $\aleph$-space is hereditary for all subspaces. Therefore, the space X is a $\aleph$-space. Theorem 5 is proved. □

Theorem 6. *Hausdorff space X is an $\aleph_0$-space if and only if $\exp_n X$ is an $\aleph_0$-space.*

The proof consists of repeating the arguments in the proof of Theorem 5.

Corollary 6. *Hausdorff space X is a paracompact $\aleph$-space if and only if $\exp_n X$ is paracompact $\aleph$-space.*

Definition 5. *[24]. A topological space X is a semistratifiable space if to each open set $U \subset X$ one can assign a sequence $\{U_n\}_{n=1}^{\infty}$ of closed subsets of X such that*

1. $\bigcup\limits_{n=1}^{\infty} U_n \subset U$;
2. *$U_n \subset V_n$, whenever $U \in V$ for each $n \in N$, where $\{V_n\}_{n=1}^{\infty}$ is the sequence assigned to V.*

It is obviously the following property. The class of a semistratifiable spaces is hereditary and countably productive [24].

Theorem 7. *A topological space X is semistratifiable if and only if the space $\exp_n X$ is semistratifiable.*

Proof. Necessity. Let X be a semisrtatifiable space. Then by [24] the countable product space X is a semisrtatifiable space, that is, X^n is a semisrtatifiable space, since the mapping π_n is closed and $\pi_n(X^n) = \exp_n X$. Take an open set $O\langle U_1, U_2, ..., U_n\rangle \subset \exp_n X$. Then the set $\pi_n^{-1}(O\langle U_1, U_2, ..., U_n\rangle)$ is open in X^n. There exists a sequence $\{F_m\}_{m=1}^{\infty}$ to a closed subset of X^n such that $\bigcup\limits_{m=1}^{\infty} F_m = \pi_n^{-1}(O\langle U_1, U_2, ..., U_n\rangle)$. It follows that $O\langle U_1, U_2, ..., U_n\rangle = \pi_n\left(\bigcup\limits_{m=1}^{\infty} F_m\right) = \bigcup\limits_{m=1}^{\infty} \pi_n(F_m)$. Since the set F_m is closed in X^n for every $n \in N$, the mapping π_n is closed. Therefore, the set $T_m = \pi_n(F_m)$ is closed in $\exp_n X$. Hence, $O\langle U_1, U_2, ..., U_n\rangle = \bigcup\limits_{m=1}^{\infty} T_m$. Take an arbitrary open subset of $O\langle V_1, V_2, ..., V_t\rangle \subset O\langle U_1, U_2, ..., U_n\rangle$. The inclusion $\pi_n^{-1}(O\langle V_1, V_2, ..., V_t\rangle) \subset \pi_n^{-1}(O\langle U_1, U_2, ..., U_n\rangle)$ implies that there exist closed subsets of $\{K_m\}_{m=1}^{\infty}$ such that $\pi_n^{-1}(O\langle V_1, V_2, ..., V_t\rangle) = \bigcup\limits_{m=1}^{\infty} K_m$. Hence we have $O\langle V_1, V_2, ..., V_t\rangle = \bigcup\limits_{m=1}^{\infty} \pi_n(K_m)$ and $K_m \subset F_m$ for each $m = 1, 2, \ldots$. It is clear that $\pi_n(K_m) \subset \pi_n(F_m)$ for each $m = 1, 2, ...$ Consequently, we obtained that the space $\exp_n X$ is semistratifiable.

Sufficiency. It is clear that X is a subspace of the space $\exp_n X$ and the class of semistratifiable spaces is hereditary. Therefore, the space X is a semistratifiable space. Theorem 7 is proved. □

References

[1] Vietoris A. Bereiche Zweiter Ordnung. Monatsh, fur math, und phys, Issue 32, 258–280 (1922).

[2] Michael E. Topologies on spaces of subsets. Trans. Amer. Math. Soc., Vol. 71, Issue 1, 152–172 (1951).

[3] Fedorchuk V.V., Todorcevic S. Cellularity of Covariant functors. Topology and its applications, Vol. 76, Issue 1, 125–150 (1997).

[4] Beshimov R.B. On some cardinal invariants of hyperspaces. Mathematychni Studii, Vol. 24, Issue 2, 197–202 (2005).

[5] Beshimov R.B., Mamadaliev N.K. and Eshtemirova Sh.Kh. Categorical and cardinal properties of hyperspaces with a finite number of components. Journal of Mathematical Sciences, Vol. 245, Issue 3, 390–397 (2020).

[6] Mizokami T. On hyperspaces of spaces around Moore spaces. Hauston Journal of Mathematics, Vol. 22, 297–306 (1996).

[7] Mizokami T. On hyperspaces of generalized metric spaces. Topology and its applications, Vol. 76, 169–173 (1997).

[8] Fedorchuk V.V. Covariant functors in the category of compacta absolute retracts and Q manifolds. UMN, Vol. 3, Issue 36, 177–195 (1981).

[9] Beshimov R.B., Mamadaliev N.K. Categorical and topological properties of the functor of Radon functionals. Topology and its Applications, Vol. 275, 1–11 (2020).

[10] Beshimov R.B., Zhuraev R.M. Some properties of a connected topological group. Mathematics and Statistics, Vol. 7, Issue 2, 45–49 (2019).

[11] Fedorchuk V.V., Filippov V.V. General topology. Basic constructions. Moscow: Fizmatlit, 314 (2006).

[12] Fedorchuk V.V. Infinite-dimensional bicompacta. Izvestia. USSR Academy of Sciences, ser. maths, Vol. 42, 1162–1178 (1978).

[13] Ganea T. Symmetrische Potenzen Topologischer Raume. Math. Nach., Vol. 11, 305–316 (1954).

[14] Engelking R. General topology. Revised and completed edition. Berlin. Helderman, 752 (1986).

[15] Banakh T. Topological spaces with an ω-base. Dissertations Mathemeticae, Warszawa, (2019).

[16] Nagata J. Modern General Topology. North-Holland Mathematical Library, 522 (1984).

[17] Gruenhage G. Generalized metric spaces. Handbook of Set Theoretic Topology, 423–502 (1983).

[18] Borges C.R. Normality of hyperspaces. Math. Japon, Vol. 25, 507–508 (1980).

[19] Morita K. On the dimension of product spaces. Amer Journal of mathematics, Vol. 75, 205–223 (1953).

[20] Nagami K. Σ-space. Fundamental Mathematic, Vol. 65, 169–192 (1969).

[21] Borges C.R. On stratifiable spaces. Pacific Journal of Mathematics, Vol. 17, 1–16 (1966).

[22] Lin F., Liu Ch. The k-spaces property of the free Abelian topological groups over non-metrizable Lasnev spaces. Topology and its Applications, Vol. 220, 31–42 (1917).

[23] Beshimov R.B., Mamadaliyev N.K. and Mukhamadiyev F.G. π-Irreducible Mappings and k-network of Infinite Compacts. British Journal of Mathematics Computer Science, Vol. 10, Issue 5, 1–7 (2015).

[24] Creede G.D. Concerning semi-stratifiable spaces. Pacific Journal of Mathematics, Vol. 32, Issue 1, 1–7 (1970).