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Uktam Farkhodovich Mamirov
Tashkent State Technical University Address: 2 Universitetskaya st., 100095, Tashkent city, Republic of Uzbekistan E-mail: uktammamirov@gmail.com, uktammamirov@gmail.com

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Mamirov Uktam Farkhodovich

Tashkent State Technical University
Address: 2 Universitetskaya st., 100095, Tashkent city, Republic of Uzbekistan
E-mail: uktammamirov@gmail.com

Abstract: *The questions of formation and construction of algorithms for identification of elements of the transition matrix of controlled objects in conditions of parametric uncertainty are considered. To identify elements of the transition matrix, we suggest using a custom model based on the type of Kalman filtration equations. Algorithms for stable calculation of a pseudo-inverse matrix using matrix decomposition are analyzed. Regular algorithms for solving argumentative problems of minimization of the identification quality criterion are given. These algorithms allow us to regularize the problem of adaptive identification of the transition matrix of an object based on regular methods for minimizing functionals under conditions of parametric uncertainty of a dynamic control object.*
Key words: *parametric uncertainty, controlled object, Kalman filter, transition matrix, identification, pseudoinverse matrix, regularization.*

Аннотация: *Параметрик ноаниқлик шароитида бошқарилувчи объектларнинг ўткинчи матрица элементларини идентификациялаш алгоритмларини шакллантириш ва қуриш масалалари кўрилган. Ўткинчи матрица элементларини идентификациялаш учун Калман фильтрлаш тенгламаларига ўхшаш созланувчи моделдан фойдаланиш таклиф этилган. Матрицани ажратишдан фойдаланиб, псевдотескари матрицани турғун ҳисоблаш алгоритмлари таҳлил қилинган. Идентификациялашнинг сифат мезонини аргументли минималлаштириш масаласини ечишнинг мунтазам алгоритмлари келтирилган. Келтирилган алгоритмлар динамик бошқариш объектининг параметрик ноаниқлик шароитида функционални минималлаштиришнинг мунтазам усуллари асосида объектнинг ўткинчи матрицасини адаптив идентификациялаш масаласини мунтазамлаштиришда фойдаланилади.*
Таянч сўзлар: *параметрик ноаниқлиқ, бошқарилувчи объект, Калман фильтри, ўткинчи матрица, идентификация, псевдотескари матрица, мунтазамлаштириш.*

Аннотация: *Рассмотрены вопросы формирования и построения алгоритмов идентификации элементов переходной матрицы управляемых объектов в условиях параметрической неопределенности. Для идентификации элементов переходной матрицы предлагается использовать настраиваемую модель по типу уравнений фильтрации Калмана. Проанализированы алгоритмы устойчивого вычисления псевдообратной матрицы с использованием разложения матриц. Приводятся регулярные алгоритмы решения аргументных задач минимизации критерия качества идентификации. Приведенные алгоритмы позволяют в условиях параметрической неопределенности динамического объекта управления регуляризовать задачу адаптивной идентификации переходной матрицы объекта на основе регулярных методов минимизации функционалов.*
Ключевые слова: *параметрическая неопределенность, управляемый объект, фильтр Калмана, переходная матрица, идентификация, псевдообратная матриц, регуляризация.*

Introduction

Modern stochastic control theory uses the mathematical apparatus for describing control systems in the state space, with the help of which the motion of the system is described by a system of ordinary differential equations of the first order [1-6]. These equations contain functions with the help of which the processes of change in time of disturbing and control actions are specified. In this case, the processes of time variation of the disturbing influences were approximated by centered white Gaussian noise. At the same time, the fundamental idea of optimality of the synthesis of a control system arose and was successfully developed – the control action generated by the system of optimal control should optimize a certain indicator of the quality of control, taking into account the constraints created by the mathematical model of the control system.

Consider a linear continuous stochastic dynamical system, which in discrete time can be described by the equations

$$x_{i+1} = A_i x_i + \Gamma_i w_i, \tag{1}$$

$$y_i = Hx_i + v_i, \tag{2}$$

where x_i, y_i – vectors of state and observations of a system of dimension *n*, *m*, respectively; w_i and v_i – vectors of object noise and observation noise of dimensions *q* and *p*, respectively, which are a sequence of the form of Gaussian white noise with characteristics $E[w_i]=0$, $E\left[w_i w_k^T\right] = Q_i\delta_{ik}$, $E[v_i]=0$, $E\left[v_i v_k^T\right] = R_i\delta_{ik}$, $E\left[w_i v_k^T\right] = 0$; *A* – transition matrix of order *n*; *Γ* and *H* are matrices of corresponding dimensions. These sequences are also independent of the random initial state of the system x_0 with mathematical expectation $\bar{x}_0$ and covariance P_0.

Let the considered system (1), (2) be invariant to a shift in time (the matrices *A*, *Γ*, *Q*, *H*, *R* are constant), stable (the spectral radius of the matrix *A* is less than one: $\rho(A)<1$), not observable ($|A| \neq 0$) and quite observable:

$$U = \left[H^T (HA)^T \ldots \left(HA^{n-1}\right)^T\right]^T, \ rank U = n. \tag{3}$$

Optimal steady-state estimates $x_{t|i}$, $y_{t|i}$ position x_t and measurements y_t, and measurements that minimize the mean square errors of prediction and filtering are unique and obey the equations of the Kalman filter [2,4,7,8]:

$$x_{i+1|i} = Ax_{i|i-1} + G(y_i - Hx_{i|i-1}), \ \ i \geq 1, \tag{4}$$

$$x_{i|i} = Ax_{i-1|i-1} + K(y_i - HAx_{i-1|i-1}), \ \ i \geq 1, \tag{5}$$

$$x_{t|i} = A^{t-i} x_{i|i}, \ \ y_{t|i} = Hx_{t|i}, \ \ t > i, \tag{6}$$

$$G = AK, \ K = MH^T\left(HMH^T + R\right)^{-1} = PH^T R^{-1}, \tag{7}$$

$$M = APA^T + \Gamma Q \Gamma^T - AMH^T\left(HMH^T + R\right)^{-1} HMA^T, \tag{8}$$

$$\begin{aligned} P = (APA^T + \Gamma Q\Gamma^T) - (APA^T + \Gamma Q\Gamma^T)H^T \times \\ \times \left[H\left(APA^T + \Gamma Q\Gamma^T\right)H^T + R\right]^{-1} H(APA^T + \Gamma Q\Gamma^T). \end{aligned} \tag{9}$$

In fact, for a number of reasons, the parameters *A*, *Γ*, *Q*, *H*, *R* of model (1), (2) when solving equations (4) - (9) are not known exactly, which forces them to be determined from the measurement data y_i.

The optimality of Kalman-type estimation algorithms, as is known, largely depends on the reliability of knowledge of the matrices in equations (1), (2) describing the system. However, in most practical cases, knowledge of the numerical values of all or several elements of these matrices is incomplete. Although usually, based on physical considerations, it is possible to set approximate values and a range of unknown parameters, nevertheless, due to the lack of accurate data, the quality of the results obtained during the filtering process may turn out to be low. The required quality of the results is determined by the specific case of the algorithm application. In many cases, it turns out to be necessary to improve the a priori information by estimating essential parameters using measurement data obtained during the operation of the system. Due to this uncertainty, the initially formulated problem of assessing the state becomes more complicated. In many cases, it turns out to be necessary to evaluate not only the state, but also the parameters of the system.

There are known methods [7, 9, 10], which allow, under some very realistic assumptions about stationary systems, to identify the elements of the state transition and control matrices regardless of the errors that exist in the accepted values of the elements of the noise covariance matrices. In some more complicated cases, in [7,11] algorithms for solving nonlinear matrix equations for identifying the elements of the transition matrix of the system are given.

Let us present regular algorithms for constructing approximate solutions of the considered ill-posed problem of identification of the transition matrix of a dynamical system, which have the property of being stable to small changes in the initial data.

Building a Custom Model

By now, in the formulated conditions, the most frequently used methods are fictitious noise, nonlinear extended filtering, Bayesian estimation schemes, maximum likelihood, etc. [1-9]. These methods have a common drawback, which is that their practical use is accompanied by a bias in the parameter estimates. The indicated drawback can be eliminated only with the optimal choice of the measurement model y_i. The optimal model for systems (1), (2) is an adaptive filter that coincides in structure with the Kalman filter (4) - (6). This well-known result of the theory of optimal filtering turns out to be closely related to solving the general question of choosing an optimal tunable model in the problems of identifying parameters and estimating the state of a controlled object [10-12].

To identify the matrix A, it is proposed to use a tunable model similar to the Kalman filtering equations in the form:

$$\hat{g}_{i+1|i} = \Phi\hat{g}_{i|i-1} + B(z_i - C\hat{g}_{i|i-1}), \;\; i \geq 1, \tag{10}$$

$$\hat{z}_{t|i} = C\hat{g}_{t|i}, \; t > i, \tag{11}$$

$$B = \Phi D, \tag{12}$$

observing the conditions of nondegeneracy, stability and full observability in it:

$$|\Phi| \neq 0, \;\; \rho\big[(I - DC)\Phi\big] = \rho\big[\Phi(I - DC)\big] < 1,$$

$$\widehat{U} = \left[C^T \, (C\Phi)^T \; \ldots \; (C\Phi^{n-1})^T\right]^T, \;\; rank\widehat{U} = n,$$

since they are performed in the system (1), (2). Model (10) - (12) coincides with the optimal Kalman filter for the input signal z_i and for the dimension of the corresponding vectors and matrices, but according to the initial assumption, none of the matrices A, Γ, Q, H, R can be used to build a model.

Based on this assumption of complete a priori uncertainty of system (1), (2), it is required to form the residual of system (1), (2) and model (10) - (12), as well as the corresponding quality criterion, the minimum of which provides a necessary and sufficient condition for fulfillment of equality $\Phi = A$; determine the smallest requirements for the amount of a priori information regarding the type and values of matrices A, Γ, Q, H, R at which the method of a tunable model can be used.

The dependence of the process $\hat{g}_{i+1|i}$ on the parameters Φ, B, C will be denoted $\hat{g}_{i+1|i} = \hat{g}_{i+1|i}(\Phi, B, C)$.

For an optimal estimate that minimizes the criterion $\left\| g_{i+1} - \hat{g}_{i+1|i} \right\|^2$, we use the notation $g_{i+1|i}$. Then, following [8-10], for the magnitude of the residual, one can write the expression

$$\varepsilon_{j+1|j} = \Delta^+ (Z_{j+1} - \widehat{U}\hat{g}_{j+1|j}), \;\; j \geq 0, \tag{13}$$

in which

$$\Delta = \left[\theta^T \;\vdots\; (\theta\Psi)^T \;\vdots\; \cdots \;\vdots\; (\theta\Psi^{n-1})^T\right]^T, \;\; \text{rank}\, \Delta = n, \tag{14}$$

$$Z_{j+1} = [z_{j+1}^T \;\vdots\; z_{j+2}^T \;\vdots\; \cdots \;\vdots\; z_{j+n}^T]^T.$$

The efficiency of the identification quality criterion significantly depends on the accuracy of the matrix inversion (13). In a number of computational problems in linear algebra, it is required to find the Moore-Penrose pseudoinverse matrix Δ^+ with respect to the matrix Δ. An extensive literature is devoted to the discussion of the problems of numerical determination of pseudoinverse matrices [13-18].

Calculating the Pseudoinverse Matrix

We will consider the following decompositions of the matrix Δ [13-15]:

$$D\Delta = \left[R^T \;\vdots\; 0\right]^T, \tag{15}$$

$$\tilde{D}\Delta S = \left[\tilde{R}^T \;\vdots\; 0\right]^T. \tag{16}$$

In expressions (15), (16) $R_{p\times p}$ and $\tilde{R}_{p\times p}$ and are the upper triangular matrices, D, $\tilde{D}$ – are orthogonal matrices of the corresponding dimensions S – is the permutation matrix.

It can be shown [17-19] that for (15) and (16) at $rank\Delta = p$, respectively, the following relations are valid:

$$Z = (\Delta^T\Delta)^{-1} = R^{-1}(R^{-1})^T, \tag{17}$$

$$Z = (\Delta^T\Delta)^{-1} = P\tilde{R}^{-1}(\tilde{R}^{-1})^T P^T.$$

Then, when inverting the triangular matrix R in (17), we can use the expressions.

$$t_{ii} = r_{ii}^{-1}, \quad i = 1,\ldots,p,$$

$$t_{ij} = -t_{jj}\sum_{j=i}^{j-1} t_{ii}r_{ij}, \quad j = i+1,\ldots,p, \quad i = 1,\ldots,p-1.$$

For case (16), it is also necessary to take into account the operations of left and right multiplication by the permutation matrices S and S^T, respectively.

Identification Quality Criterion

For unknown matrices *A* and *H*, expression (14) is written in accordance with (3) by replacing the matrices *H*, *A* with some of their well-known analogs θ, Ψ, then we can write [10, 12]

$$\left\|\varepsilon_{j+1|j}\right\|^2 = \left\|e_{j+1|j}\right\|^2 + \delta^2, \tag{18}$$

in which $e_{j+1|j} = \Delta^{+}Ux_{j+1} - \Delta^{+}\hat{U}\hat{g}_{j+1|j}$, δ^2, is a constant value independent of the parameters *Φ, B, C* of the model (10) - (12).

We take the value (18) as a criterion for the quality of identification

$$J = J(\Phi, B, C) = \left\|\varepsilon_{j+1|j}\right\|^2.$$

Let us introduce the notation $(A_1, G_1, H_1) = \arg\min_{\Phi, L, C} J(\Phi, B, C)$, where $A_1 = SAS^{-1}$, $G_1 = SG$, $H_1 = HS^{-1}$.

In the general formulation of the problem, the parameters of the matrices *A*, *Γ*, *Q*, *R* of the model will be considered unknown and they are subject to estimation, and the matrix *H* is assumed to be known. Then, with the known *H* and unknowns *A*, *Γ*, *Q*, *R*, we can write the following expression: $(A_1, G_1) = \arg\min_{\Phi, B} J(\Phi, B, H)$, $\hat{g}_{i+1|i}(A_1, G_1, H) = g_{i+1|i} = Sx_{i+1|i}$, $H = HS$.

We will consider the minimization problem

$$(A_1, G_1) = \arg\min_{\Phi, B} J(\Theta) = \arg\min_{\Phi, B} J\left\|\varepsilon_{j+1|j}\right\|^2,$$

in this case, the sought vector $\Theta \in \Xi$ consists of elements of matrices *Φ*, *B*, its dimension is equal to $n(n+l)$.

In the practical implementation of this algorithm, computational difficulties arise due to the fact that the algorithms presented are related to argument minimization problems. Such problems under the conditions of an approximate assignment of the initial data refer to ill-posed ones, in which arbitrarily small changes in the initial data can correspond to arbitrarily large changes in the minimum value of the functional [20-22].

Regular Algorithms for Solving Argument Problems of Minimizing the Identification Quality Criterion

It is shown in [22,23] that one of the most popular and effective methods for minimizing functional is the Gauss - Newton method:

$$\Theta_{k+1} = \Theta_k - [J'^*(\Theta_k)J'(\Theta_k)]^{-1} J'^*(\Theta_k)J'(\Theta_k)$$

and modified iterative process

$$\Theta_{k+1} = \Theta_k - [J'^*(\Theta_k)J'(\Theta_k) + \alpha_k I]^{-1}[J'^*(\Theta_k)J'(\Theta_k) + \alpha_k(\Theta_k - \xi)], \qquad (19)$$

where ξ - some element of the Hilbert space H_1 and $\{\alpha_k\}$ – sequence of positive parameters

$$\lim_{k\to\infty} \alpha_k = 0, \quad \left|\frac{\alpha_k}{\alpha_{k+1}}\right| = O(1), \quad \lim_{k\to\infty}\sup\left|\frac{\alpha_k - \alpha_{k+1}}{\alpha_{k+1}}\right| \le \delta .$$

In [23], it was proved that the convergence of the process (19) can be investigated using a priori information on the asymptotic behavior of the points of minimum of the functional by A.N. Tikhonov.

The Gauss-Newton method can be obtained on the basis of preliminary linearization of the residual function $\varphi(\beta) = \varepsilon_{j+1|j}$ in the vicinity of the current point β_c [24] of the iterative process of solving a nonlinear optimization problem:

$$\varphi(\beta) \approx \varphi(\beta_c) + \nabla^T_\beta \varphi(\beta_c)\Delta\beta, \quad \Delta\beta = \beta - \beta_c, \qquad (20)$$

where $\nabla^T_\beta \varphi(\beta_c)$ – transpose the Jacobian matrix.

Based on the least squares problem (20), we can write [24,25]:

$$\Delta\beta = \left(\nabla^T_\beta \varphi(\beta_c)\right)^+ \left(\nabla^T_\beta \varphi(\beta_c)\Delta\beta\right). \qquad (21)$$

Let us consider the situation when β_c is divided into two subvectors in an arbitrary way: β_c^1 and β_c^2, then algorithm (21) takes the form:

$$\begin{bmatrix} \beta_c^1 \\ \beta_c^2 \end{bmatrix} = \left[\nabla^T_{\beta^1}\varphi(\beta_c) \;\vdots\; \nabla^T_{\beta^2}\varphi(\beta_c)\right]^+ \left(\nabla^T_\beta \varphi(\beta_c)\Delta\beta\right).$$

Further implementation of algorithm (21) is related to the computation of the pseudoinverse matrix $(\nabla^T_\beta \varphi(\beta_c))^+$ using the Kline formula for pseudoinverse of block matrices [13,14,18,25]:

$$\left[\nabla^T_{\beta^1}\varphi(\beta_c) \;\vdots\; \nabla^T_{\beta^2}\varphi(\beta_c)\right]^+ = \begin{bmatrix} \left(\nabla^T_{\beta^1}\varphi(\beta_c)\right)\left(I - \left(\nabla^T_{\beta^2}\varphi(\beta_c)\right)L_c\right) \\ L_c \end{bmatrix},$$

where

$$L_c = C_c^+ + \left(I - C_c^+ C_c\right)K_c\left(\nabla^T_{\beta^2}\varphi(\beta_c)\right)\left(\nabla^T_{\beta^1}\varphi(\beta_c)\right)^+\left(\nabla^T_{\beta^1}\varphi(\beta_c)\right)^+\left[I - \left(\nabla^T_{\beta^2}\varphi(\beta_c)\right)C_c^+\right],$$

$$C_c = \left(I - \left(\nabla^T_{\beta^1}\varphi(\beta_c)\right)\left(\nabla^T_{\beta^1}\varphi(\beta_c)\right)^+\right)\left(\nabla^T_{\beta^2}\varphi(\beta_c)\right), \; K_c = \left(I + M_c^T M_c\right)^{-1},$$

$$M_c = \left(\nabla^T_{\beta^1}\varphi(\beta_c)\right)^+\left(\nabla^T_{\beta^2}\varphi(\beta_c)\right)\left(I - C_c^+ C_c\right).$$

Using these relations, (22) will be written in the form:

$$\begin{bmatrix} \beta_c^1 \\ \beta_c^2 \end{bmatrix} = \begin{bmatrix} \left(\nabla^T_{\beta^1}\varphi(\beta_c)\right)\left(I - \left(\nabla^T_{\beta^2}\varphi(\beta_c)\right)L_c\right) \\ L_c \end{bmatrix}\left(\nabla^T_\beta \varphi(\beta_c)\Delta\beta\right).$$

then

$$\beta_c^2 = L_c\left(\nabla_\beta^T \varphi(\beta_c)\Delta\beta\right),$$
$$\beta_c^1 = \left(\nabla_{\beta^1}^T \varphi(\beta_c)\right)^+ \left(\nabla_\beta^T \varphi(\beta_c)\Delta\beta - \left(\nabla_{\beta^2}^T \varphi(\beta_c)\right)\beta_c^2\right).$$

Conclusion

The algorithms above allow, under the conditions of parametric uncertainty of a dynamic control object, to regularize the problem of adaptive identification of the transition matrix of the object on the basis of regular methods of minimizing functionals.

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