Errors-In-Variables-Based Approach for the Identification of AR Time-Varying Fading Channels

Ali Jamoos, Eric Grivel, William Bobillet, and Roberto Guidorzi

***Abstract*—This letter deals with the identification of time-varying Rayleigh fading channels using a training sequence-based approach. When the fading channel is approximated by an autoregressive (AR) process, it can be estimated by means of Kalman filtering, for instance. However, this method requires the estimations of both the AR parameters and the noise variances in the state–space representation of the system. For this purpose, the existing noise compensated approaches could be considered, but they usually require a long observation window and do not necessarily provide reliable estimates when the signal-to-noise ratio is low. Therefore, we propose to view the channel identification as an errors-in-variables (EIV) issue. The method consists in searching the noise variances that enable specific noise compensated autocorrelation matrices of observations to be positive semidefinite. In addition, the AR parameters can be estimated from the null spaces of these matrices. Simulation results confirm the effectiveness of this approach, especially in presence of a high amount of noise.**

***Index Terms*—Autoregressive processes, errors-in-variables, Rayleigh fading channels.**

I. Introduction

IN current mobile communication systems, estimating time-varying fading channels plays a key role for coherent symbol detection at the receiver.

According to the Jakes model, the theoretical power spectrum density (PSD) associated with either the real or the imaginary part of the time-varying fading channel is U-shaped and band-limited. Moreover, it exhibits twin peaks at $\pm f_d$, where f_d denotes the maximum Doppler frequency. The corresponding discrete-time autocorrelation function $R_{hh}(n)$ is a zero-order Bessel function of the first kind

$$R_{hh}(n) = J_0\left(2\pi f_d T|n|\right) \tag{1}$$

where f_dT is the Doppler rate, and T the symbol period.

In recent papers [1]–[3], the channel has been modeled as a pth-order autoregressive process, denoted by $\mathrm{AR}(p)$ and defined as follows:

$$h(n) = -\sum_{i=1}^{p} a_i h(n-i) + u(n) \tag{2}$$

where $\{a_i\}_{i=1,\ldots,p}$ are the AR model parameters, and $u(n)$ denotes the zero-mean complex white Gaussian driving process with variance σ_u^2.

Using a low order AR model for the channel is *a priori* debatable. On the one hand, some authors (e.g., [2], [3]) motivate this approximation arguing for the model simplicity, especially for first- or second-order AR processes, and its usefulness for channel prediction. On the other hand, from a theoretical point of view, according to the Kolmogoroff–Szego formula[1] [4], a deterministic model should be used for the channel due to the band-limited nature of its PSD. In between, solutions have been also studied. First, a sub-sampled ARMA process followed by a multistage interpolator has been considered for channel simulation [5]. Indeed, when down-sampling, the normalized maximal Doppler frequency moves towards 1/2. Nevertheless, only a very high down-sampling factor can lead to a PSD that is never equal to 0. Second, Baddour *et al.* [1] have suggested using high order AR processes for channel simulation. To allow the estimation of the corresponding AR parameters, they "slightly" modify the properties of the channel by considering the sum of the theoretical fading process and a zero-mean white process whose variance ϵ is very small (e.g., $\epsilon = 10^{-7}$ for $f_dT = 0.01$). At that stage, the AR parameters are estimated with the Yule–Walker (YW) equations based on the modified autocorrelation function $R_{hh}^{mod}(n) = J_0(2\pi f_d T|n|) + \epsilon\delta(n)$.

Taking into account the above results, an AR model whose order is high enough will be considered in this letter to approximate the channel fading process. Then, our purpose is to develop a training sequence-based method that makes it possible to estimate the channel AR parameters, without any *a priori* information about f_d. In that case, when considering a linearly modulated signal propagating through a time-varying frequency-flat Rayleigh fading channel, the received discrete-time signal can be written as follows:

$$r(n) = h(n)d(n) + w(n) \tag{3}$$

where $d(n)$ is the nth transmitted data symbol belonging to a constellation with unity radius, $h(n)$ is the fading process, and $w(n)$ is assumed to be a complex additive white Gaussian noise process with zero-mean and variance σ_w^2. In addition, $h(n)$ and $w(n)$ are assumed to be statistically independent.

In the time interval allocated to the transmission of the training sequence, the data modulation can be wiped out by multiplying the signal samples with the complex conjugate of the training symbols, as follows:

$$y(n) = d^*(n)r(n) = h(n) + b(n) \tag{4}$$

[1] $\sigma_u^2 = \exp((1/2\pi)\int_{-\pi}^{\pi} \ln \Psi_{hh}(\omega)d\omega)$, where $\Psi_{hh}(\omega)$ is the PSD of the AR process that fits the theoretical Jakes spectrum.

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where the noise $b(n)$ is zero-mean white Gaussian with variance $\sigma_b^2 = \sigma_w^2$.

Among the existing estimation methods, Tsatsanis *et al.* [6] suggest estimating the AR parameters from the channel covariance estimates by means of a YW estimator. However, the method results in biased estimates. Alternative methods initially developed in other fields than wireless communications can be considered. Among them, a bias-correction least-square technique has been presented by Zheng [7], while Davila [8] has proposed to solve the so-called noise-compensated YW equations using a subspace-based method. Nevertheless, these methods may have convergence problems and are outperformed by the recursive instrumental variable technique based on two-cross-coupled Kalman filters proposed in [9]. One Kalman filter is used to estimate the AR process, while the second one makes it possible to estimate the corresponding AR parameters from the estimated AR process. It should be noted that we have analyzed the relevance of this approach to estimate multi-carrier direct-sequence code division multiple access (MC-DS-CDMA) fading channels in [10]. However, the variance of the additive noise is assumed to be known.

Here, we propose another approach, which views the estimation of the channel AR parameters as an errors-in-variables (EIV) issue [11]. As suggested by some of the authors in the framework of control [12], [13] and recently in the field of speech enhancement [14], the formulation of an EIV estimation problem consists in determining, on only the basis of noisy observations $y(n)$ and given (2), the set of κ-tuple $\{a_i\}_{i=1,\cdots,\kappa\geq p}$ that satisfies the condition

$$[h(n-\kappa) \quad \cdots \quad h(n-1) \quad (h(n)-u(n))] \times [a_\kappa \quad \cdots \quad a_1 \quad 1]^T = 0 \quad (5)$$

or equivalently

$$\bar{\mathbf{R}}_{\mathbf{h}}^{\kappa+1}[a_\kappa \quad \cdots \quad a_1 \quad 1]^T = 0 \quad (6)$$

where $\bar{\mathbf{R}}_{\mathbf{h}}^{\kappa+1}$ denotes the $(\kappa+1)\times(\kappa+1)$ positive semidefinite correlation matrix of $[h(n-\kappa) \cdots h(n-1)\ (h(n)-u(n))]$.

Therefore, due to (6), the estimations of the AR parameters and the variances of both the measurement noise and the driving process consists in estimating $\bar{\mathbf{R}}_{\mathbf{h}}^{\kappa+1}$ and its kernel, for specific values of κ.

The remainder of this letter is organized as follows. The parametric identification of the fading channel is presented in Section II. Simulation results are reported in Section III.

II. Identification of the Channel AR Parameters

Let N observation samples $\{y(n)\}_{n=1,\ldots,N}$ be obtained during the transmission of a training sequence and κ be set to p. In addition, let us define the following four $(p+1)\times 1$ vectors:

$$\boldsymbol{\theta}_{p+1} = [a_p \quad \cdots \quad a_1 \quad 1]^T \quad (7)$$
$$\mathbf{y}(n) = [y(n-p) \quad \cdots \quad y(n-1) \quad y(n)]^T \quad (8)$$
$$\mathbf{h}(n) = [h(n-p) \quad \cdots \quad h(n-1) \quad h(n)]^T \quad (9)$$
$$\mathbf{b}(n) = [b(n-p) \quad \cdots \quad b(n-1) \quad b(n)]^T. \quad (10)$$

Given (4), (8), (9), and (10), the observation autocorrelation matrix satisfies:

$$\mathbf{R}_{\mathbf{y}}^{p+1} = \mathrm{E}\left[\mathbf{y}(n)\mathbf{y}^H(n)\right] = \mathbf{R}_{\mathbf{h}}^{p+1} + \sigma_b^2\mathbf{I}_{p+1} \quad (11)$$

where $\mathbf{R}_{\mathbf{h}}^{p+1} = \mathrm{E}[\mathbf{h}(n)\mathbf{h}^H(n)]$, and $\mathbf{I}_{p+1}$ is the identity matrix.

Due to (5) and (11), it follows that

$$\begin{aligned}\bar{\mathbf{R}}_{\mathbf{h}}^{p+1} &= \mathbf{R}_{\mathbf{h}}^{p+1} - \mathrm{diag}\left[\underbrace{0 \ \cdots \ 0}_{p} \ \ \sigma_u^2\right] \\ &= \mathbf{R}_{\mathbf{y}}^{p+1} - \mathrm{diag}\left[\sigma_b^2\mathbf{I}_p, \sigma_s^2\right] \quad (12)\end{aligned}$$

where

$$\sigma_s^2 = \sigma_b^2 + \sigma_u^2. \quad (13)$$

By referring to the EIV-based methods [12], [13], the actual point $P^a = (\sigma_s^2, \sigma_b^2)$ belongs to the set of solutions $P = (\alpha, \beta)$, making the following matrices positive semidefinite:

$$\bar{\mathbf{R}}_{\mathbf{h}}^{p+1}(P) = \mathbf{R}_{\mathbf{y}}^{p+1} - \mathrm{diag}[\beta\mathbf{I}_p, \alpha] \geq 0. \quad (14)$$

The set of solutions $P = (\alpha, \beta)$ is the convex curve $S(\mathbf{R}_{\mathbf{y}}^{p+1})$ belonging to the first quadrant of the $(\alpha\beta)$-plane[2] and whose concavity faces the origin (see Fig. 1). In addition, every point P can be associated with the parameter vector $\boldsymbol{\theta}_{p+1}(P)$ satisfying (6) as follows:

$$\bar{\mathbf{R}}_{\mathbf{h}}^{p+1}(P)\boldsymbol{\theta}_{p+1}(P) = 0. \quad (15)$$

However, this procedure provides a family of solutions. When carrying out the EIV method with $\kappa = p+1$, i.e., considering a $(p+1)$th-order AR process, another set of solutions is obtained. The true solution can hence be extracted since it belongs to both sets. By extending the right-hand side of the vectors in (8)–(10) with a new value at time $n+1$, one obtains the $(p+2)\times 1$ vectors $\mathbf{y}(n+1)$, $\mathbf{h}(n+1)$, and $\mathbf{b}(n+1)$. It follows that

$$\bar{\mathbf{R}}_{\mathbf{h}}^{p+2}(P) = \mathbf{R}_{\mathbf{y}}^{p+2} - \mathrm{diag}[\beta\mathbf{I}_{p+1}, \alpha] \geq 0 \quad (16)$$

and

$$\bar{\mathbf{R}}_{\mathbf{h}}^{p+2}(P)\boldsymbol{\theta}_{p+2}(P) = 0. \quad (17)$$

When $P = P^a$, one has more particularly

$$\bar{\mathbf{R}}_{\mathbf{h}}^{p+2}(P^a)\begin{bmatrix} 0 \\ \boldsymbol{\theta}_{p+1}(P^a)\end{bmatrix} = 0. \quad (18)$$

Given (16) and (18), the actual point P^a belongs to both $S(\mathbf{R}_{\mathbf{y}}^{p+1})$ and $S(\mathbf{R}_{\mathbf{y}}^{p+2})$ convex curves (see Fig. 1). So, once the point P^a is determined, the parameter vector $\boldsymbol{\theta}_{p+1}$ can be extracted from the null space of $\bar{\mathbf{R}}_{\mathbf{h}}^{p+1}(P^a)$ using (15).

However, in all practical cases that rely on limited sequences of data, the EIV assumptions do not hold. Hence, no point P^a belongs to both $S(\mathbf{R}_{\mathbf{y}}^{p+1})$ and $S(\mathbf{R}_{\mathbf{y}}^{p+2})$. Therefore, several criteria (see, e.g., [13]) have been proposed to apply the EIV scheme in these cases. The algorithm [13] that will be used in the following is based on the shift-invariant property of the dynamic systems described by (18).

1) Compute the estimates of $\mathbf{R}_{\mathbf{y}}^{p+1}$ and $\mathbf{R}_{\mathbf{y}}^{p+2}$.
2) Start from generic points $P_1 = (\alpha_1, \beta_1)$ on $S(\hat{\mathbf{R}}_{\mathbf{y}}^{p+1})$ and $P_2 = (\alpha_2, \beta_2)$ on $S(\hat{\mathbf{R}}_{\mathbf{y}}^{p+2})$ such as $\beta_1/\alpha_1 = \beta_2/\alpha_2$ (see Fig. 1). Note that this is an easily solvable generalized eigenvalue problem [15].

[2] α corresponds to σ_s^2 and stands for the abscissa, while β corresponds to σ_b^2 and stands for the ordinate.

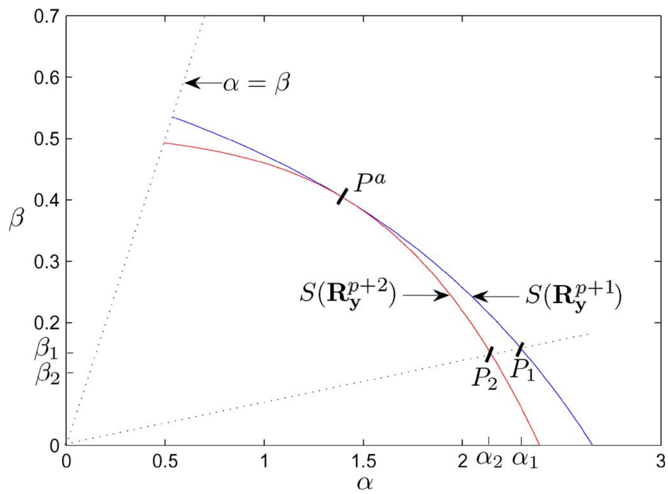

Fig. 1. Typical shapes of $S(\mathbf{R}_{\mathbf{y}}^{p+1})$ and $S(\mathbf{R}_{\mathbf{y}}^{p+2})$.

3) Compute $\bar{\mathbf{R}}_{\mathbf{h}}^{p+1}(P_1)$, $\bar{\mathbf{R}}_{\mathbf{h}}^{p+2}(P_2)$, and $\boldsymbol{\theta}_{p+1}(P_1)$ by using relations (14), (16), and (15), respectively,
4) Compute the following cost function:

$$J(P_1,P_2)=\begin{bmatrix}0\\ \boldsymbol{\theta}_{p+1}(P_1)\end{bmatrix}^H \bar{\mathbf{R}}_{\mathbf{h}}^{p+2}(P_2)\begin{bmatrix}0\\ \boldsymbol{\theta}_{p+1}(P_1)\end{bmatrix}. \quad (19)$$

5) Search on $S(\hat{\mathbf{R}}_{\mathbf{y}}^{p+1})$ for the noise variances that minimize $J(P_1,P_2)$.

III. Simulation Results

In this section, we carry out a comparative simulation study on channel identification between the following three methods:
1) proposed EIV-based approach;
2) two-cross-coupled Kalman filters approach [10];
3) Yule–Walker estimator, used, for instance, in [6].

Despite a computational cost higher than [6] and [10], the proposed approach has various advantages. To illustrate them, two simulation experiments are carried out.

In the first one, the fading process $h(n)$ is generated according to a pth-order autoregressive model-based simulator with a given Doppler rate f_dT. Thus, when p is set to 2, according to Table I, the proposed approach yields better AR parameter estimates than the two other approaches, especially for low SNR. In addition, it has the advantage of providing accurate estimates of the measurement noise variance. When increasing the model order, the proposed approach still results in better AR PSD estimates than the other approaches. See Figs. 2 and 3. As pointed out in [1], increasing the model order allows a better fit between the PSD of the resulting process and the PSD of the realistic Jakes channel. It should be noted that for every method, when the model order is getting higher (for instance, $p = 50$), the accurate estimation of all the AR parameters becomes difficult, even at high SNR. This is due to the U-shaped low-pass band-limited nature of the channel spectrum or, equivalently, to the positions of the corresponding AR poles,[3] which are close to the unit circle in the z-plane.

[3] $H(z) = 1/(1+\sum_i a_i z^{-i}) = 1/\Pi_i(1-p_i z^{-1})$, where p_i is the so-called pole.

TABLE I
Average AR(2) Parameter and Measurement Noise Variance Estimates Based on 1000 Realizations. The True Values are $a_1 = -1.5513$, $a_2 = 0.9018$, $\sigma_u^2 = 0.0625$. $N = 200$ and $f_dT = 0.14$

SNR		5 dB $\sigma_b^2 = 0.316$	10 dB $\sigma_b^2 = 0.1$	15 dB $\sigma_b^2 = 0.0316$
Proposed	$\hat{a}_1$	-1.5304	-1.5418	-1.5440
	$\hat{a}_2$	0.8798	0.8917	0.8938
	$\hat{\sigma}_u^2$	0.0752	0.0641	0.0623
	$\hat{\sigma}_b^2$	0.3121	0.0986	0.0313
Two-cross-coupled Kalman filters [10]	$\hat{a}_1$	-1.1977	-1.3565	-1.4549
	$\hat{a}_2$	0.5979	0.7330	0.8174
	$\hat{\sigma}_u^2$	0.2545	0.1406	0.1046
Yule-Walker [6]	$\hat{a}_1$	-0.7059	-1.0726	-1.3248
	$\hat{a}_2$	0.1632	0.4626	0.6896
	$\hat{\sigma}_u^2$	0.7717	0.3798	0.1941

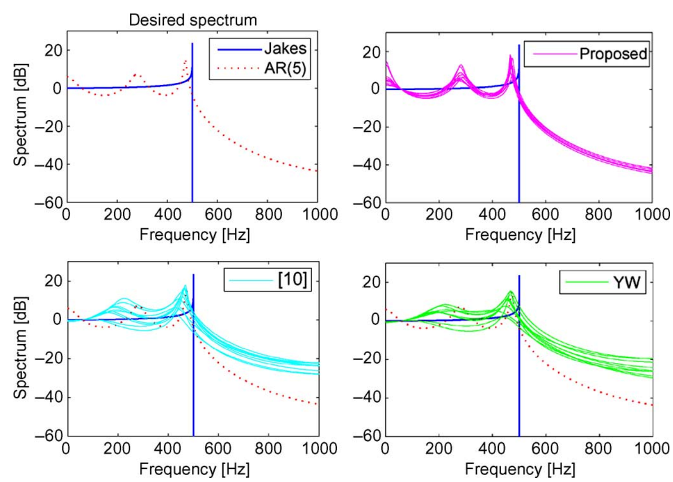

Fig. 2. Estimations of AR(5) PSD obtained by using three different approaches. $N = 200$, SNR $= 30$ dB, and $f_dT = 0.2$. Ten realizations are plotted. The channel is generated from AR(5) model-based simulator.

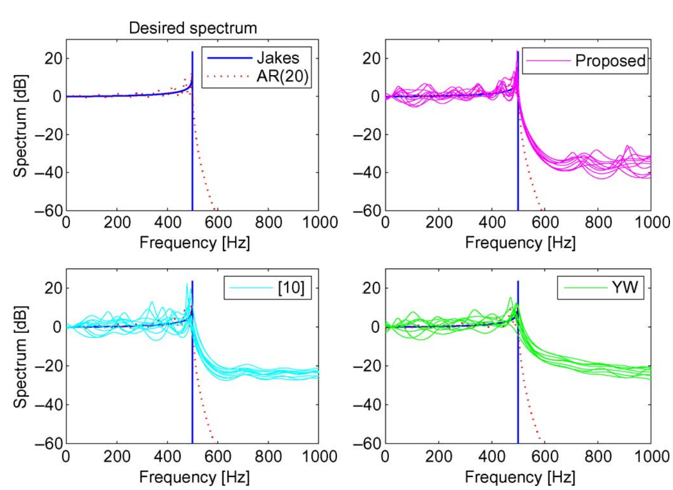

Fig. 3. Estimations of AR(20) PSD obtained by using three different approaches. $N = 200$, SNR $= 30$ dB, and $f_dT = 0.2$. Ten realizations are plotted. The channel is generated from AR(20) model-based simulator.

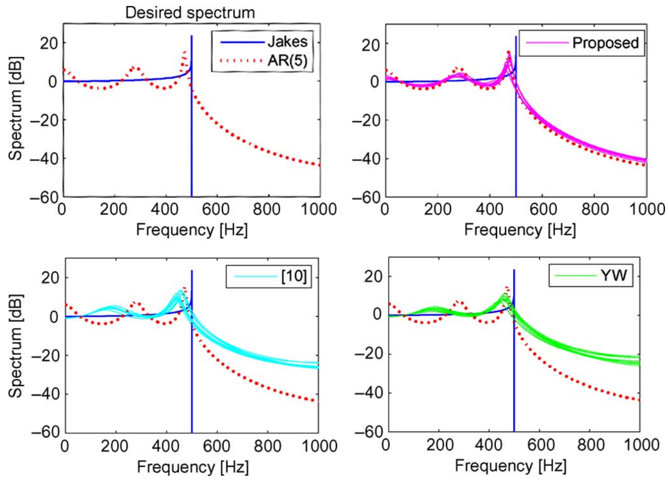

Fig. 4. Estimations of AR(5) PSD obtained by using three different approaches. $N = 200$, SNR $= 30$ dB, and $f_d T = 0.2$. Ten realizations are plotted. The channel is generated from Jakes sum-of-sinusoids simulator.

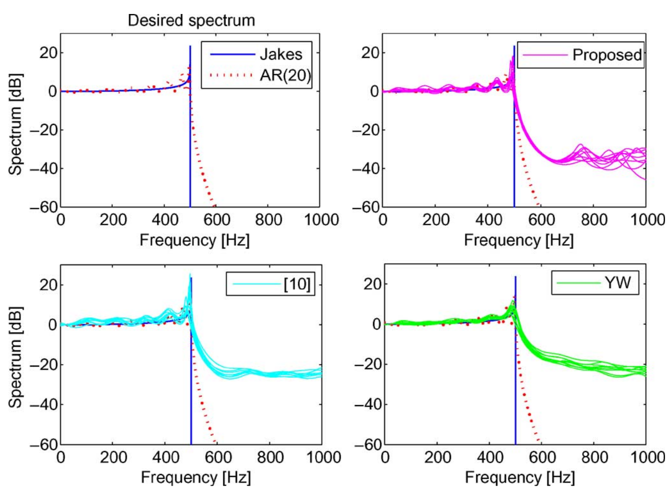

Fig. 5. Estimations of AR(20) PSD obtained by using three different approaches. $N = 200$, SNR $= 30$ dB, and $f_d T = 0.2$. Ten realizations are plotted. The channel is generated from Jakes sum-of-sinusoids simulator.

In the second experiment, the fading process $h(n)$ is generated according to the Jakes sum-of-sinusoids channel simulator with Doppler rate $f_d T = 0.2$. According to Figs. 4 and 5, which, respectively, show the estimation of the PSD of AR(5) and AR(20) processes modeling the Jakes spectrum, the proposed approach provides closer estimates to the true spectrum than the other approaches [6], [10].

Once the AR parameters are estimated, the fading process can be estimated by using Kalman filtering or an H_∞ algorithm [16]. Such an estimation can be included in the design of receivers such as proposed in [10] for an MC-DS-CDMA system. In that case, according to the simulation tests we carried out based on the Jakes sum-of-sinusoids channel simulator, the proposed approach yields slightly lower bit-error rate (BER) than the other approaches. In addition, increasing the AR model order beyond 5 will not improve much the BER performance of the system. Therefore, although an AR(20) model yields much better approximation to the Jakes spectrum than low-order models, an AR(5) process can reduce the computational cost of the estimation algorithm.

IV. Conclusion

In this letter, an EIV approach makes it possible to estimate the channel AR parameters and the variances of both the additive measurement noise and the driving process in a congruent manner. Unlike existing approaches in the field of mobile communications, the proposed approach provides reliable estimates even at low SNR (e.g., SNR $= 5$ dB) and/or when the number of samples is limited (e.g., $N = 200$). In addition, when dealing with channel estimation, an AR(5) process can provide a tradeoff between the accuracy of the model, the computational cost, and subsequent BER.

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