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BALTIC SEA WAVES ANALYSIS BY USING CHAOS THEORY TOOLS

The motivation for this paper was to assess the applicability of the novel approach derived from chaos theory to the description and analysis of dynamics of the free sea surface, in particular to the phase space reconstruction of the dynamical system from the observed time series. The free sea surface elevation data sets were sampled at the Baltic Coastal Research Station Lubiatowo in Poland. After proper processing the experimental data, it was found that the sea surface elevations can be described as a result of a four-dimensional process, which appears to be weakly chaotic, characterized by a positive largest Lyapunov exponent and a short prediction horizon. It was confirmed that using chaos theory tools may be very promising for diagnosing certain properties of the sea waves. Moreover, in the paper, some new technique for evaluation of the average mutual information is introduced.

1. INTRODUCTION

Chaotic behavior is regarded as an ubiquitous feature of dynamical systems prevailing in nature (Schuster [19], Abarbanel [1]). *Chaocity*, defined as irregularity of motion and exponential sensitivity in response to minute variations in initial conditions, is likely imposed by, even a relatively simple, set of ordinary (and deterministic!) differential equations governing the system. However, the property of chaocity, viewed once almost solely as a confounding "noise", can serve as a source of better understanding of the dynamics observed, provided that adequate analysis is applied.

This paper reports on the so-called phase space reconstruction of the dynamics generating the free surface elevation time series observed at Lubiatowo Coastal Research Station, being dominated by gravity wind waves. The inspiration for the utilization of the novel approach emanated from the fact that - due to the complexity of the sea--atmosphere interactions - the universal description of sea waves is still

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lacking. For that reason the description of the sea surface motion is frequently accomplished by means of purely probabilistic theory.

The cornerstone for the employed methodology are the studies of Abarbanel [1] and [2], though the present work refers also to alternative techniques of analysis (Sato et al. [17], Cao [5], Rosenstein et al. [16], Lega et al. [13]). The concept of the phase space reconstruction of the dynamical system from the observed time series is founded on the theory of Takens, with the method of delays being the most appreciated. It should be strongly emphasized here that the only reported attempt at such treatment of wind waves is summarized by a paper of Frison and Abarbanel [9]. They investigated waves measured offshore in California and concluded their chaotical behavior. The temporal scale of Lubiatowo waves is different and so far this kind of waves have not been subjected to the presented method of analysis. The main objective is to cover this gap. The particular goals are:

- application and evaluation of routines for the phase space reconstruction of sea wave dynamics,
- attempt at addressing the intriguing question of the possible chaocity of the Lubiatowo surface wave system as well as the estimation of its predictability,
- assessment of utility of reconstructed phase space as a domain of description and diagnosis of sea wave dynamics in reconstructed phase space.

This paper is organized as follows. In Section 2 data sets from Lubiatowo are introduced. Section 3 gives theoretical basis for the analysis, including description of tools for phase space reconstruction, in particular Lyapunov exponents. In Section 4 the proposed technique of evaluation of the average mutual information is introduced and applied. Section 5 shows and discusses the results for sea waves from Lubiatowo. Finally, Section 6 contains a short summary.

2. SEA WAVE DATA

The Coastal Research Station (CRS) Lubiatowo (54.º 8N, 17.8.º E) is located some 70 km northwest of Gdansk. As tidal phenomena are very weakly expressed in this part of the Baltic, station records other types of wave motion: short-period wind waves and swell, infragravity and long waves, wave transformation processes, wave energy dissipation as well as sediment transport and the shoreline variability [11].

Data sampling was conducted by sensors mounted at three permanent multi-pile measuring towers, emplaced at the distance of 122, 200 and 278 m offshore, correspondingly. Free surface elevation was sampled at 10 Hz, and records of length 12000 (equivalent to 20 minutes) are collected every an hour, simultaneously at 3 distant towers. These are short data sets from the point of view of nonlinear analysis but they are can be considered quaite long as oceanographic measurements.

Presented approach requires formulation of assumptions and spatial and temporal constraints of the analysis. First of all, an assumption of the ergodicity of measured dynamics is taken. It is also assumed that any slow (i.e. occuring over time scales longer than those of the measurement) is either captured by the observation or does not affect it at all on the scales of data series (is “frozen”). This implies separate treatment of measurements taken at different times (i.e. non-continous data samples cannot be merged in one data series). The another question is spatial separation of the measurements (simultaneous sampling at three towers located at different distances off the shore). Hence, there is no a priori justification of the treatment of data sets obtained concurrently at detached towers as a manifestation of some common dynamics and each of the coincident data sets was regarded as a representative for disctinct dynamical system. One should bear in mind, however, that results obtained for those individual, spatially separated dynamical systems could possibly provide some information about wave-transformation processes taking place near sea shore.

Accordingly, in the presented work there are six dynamical systems represented by three wave time series sampled at three towers on two different days. Data was collected in autumn, that is the season of strongest storm activity. Each data series (and therefore each dynamical system by which it was generated) is attributed a code, date and starting time of measurement as well as a tower ID (ascending with a distance off-shore).

Table.1. Code table for data series of sea surface elevation.

Code	**Lub01**	**Lub02**	**Lub03**	**Lub51**	**Lub52**	**Lub53**
Date	27 Sept	27 Sept	27 Sept	3 Oct	3 Oct	3 Oct
Starting time	15:14:40	15:14:40	15:14:40	8:14:40	8:14:40	8:14:40
Tower ID	1	2	3	1	2	3

Following the indications given in [1], data series (free of any extreme values, so-called spikes), hasn't been filtered as to avoid the possible changes in structure of the system or loss of variability that might be caused by filtering process.

The investigation of a time trace and Fourier spectrum is a first step of the analysis scheme, that enables to detect *irregularity* and ensuing *nonlinearity*, the fundamental reasons for the need of utilising other, than linear, methods of analysis. Fig. 1a presents first 100 seconds of time trace of the sea waves generated by Lub0, including Lub01 (Tower 1) – up, Lub02 (Tower 2) – down, and Lub03 (Tower 3) – in middle.
Fig. 1b shows an analogous trace generated by Lub5. It may be observed that the signs of irregularity are evident in all cases.

Fig. 1c and Fig. 1d present power spectra for two selected data sets in logarithmic scale, after mean detrending. Shapes of displayed power density functions are similar to those typical of nonlinear and of chaotic systems [1].

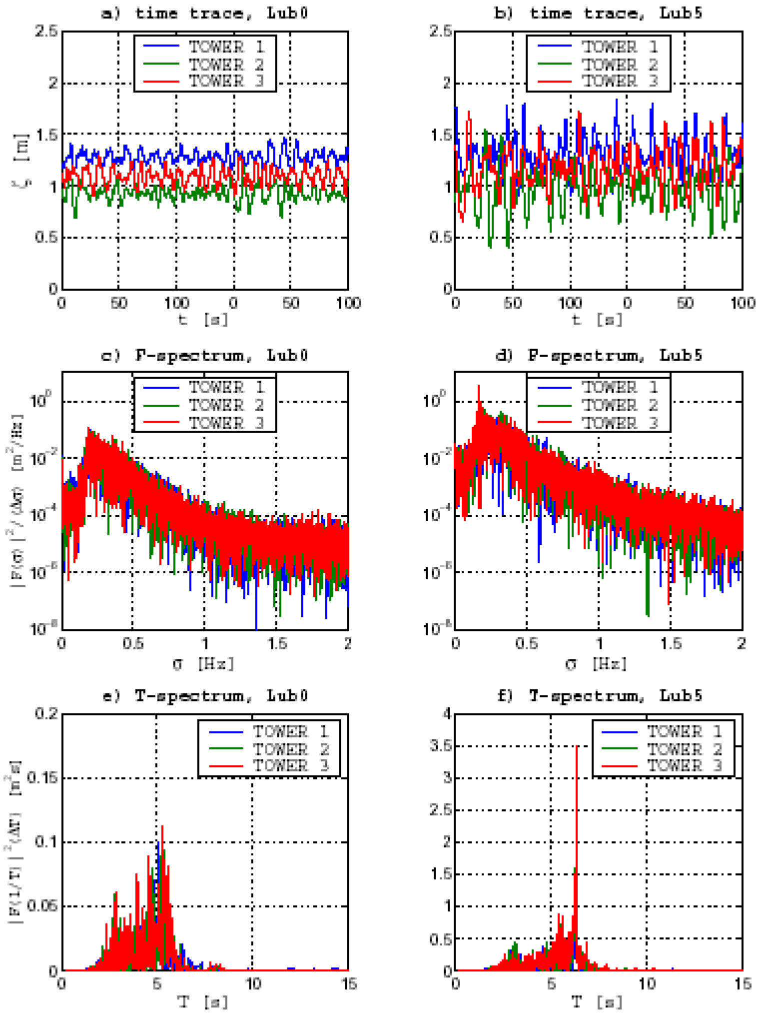

Fig. 1.Time trace of sea surface elevation measured at Lubiatowo, systems Lub0 (a) and Lub5 (b), first 100 seconds; power spectra against frequency (F-spectra) in logarithmic scale (c and d), as well as in relation to period (T-spectra) and in linear scale (e and f), the range being enlarged for the system Lub5. Calculations of spectra were carried out for N_0=8192 elements of the time series.

Energy is distributed at broad range of frequencies, with low frequency processes acting more vigorously - spectra are "red". The energy cascade towards high frequencies is clearly visible. These spectrum shapes are not yet the direct implication of the chaocity in the system, they are necessary but not sufficient feature of the dynamics. Still, spectra of similar shapes were recorded for chaotic wind waves from California, as reported by Frison and Abarbanel [9].

Thus, there is an clear evidence that sea wave time series studied are characterized by irregularity of temporal evolution and broadband spectra. These features might be indicative of chaotic nature of the processes observed and justify the application of the nonlinear time series analysis.

3. PHASE SPACE RECONSTRUCTION

The strategy scheme for the multidimensional phase space reconstruction is grounded on the embedding theorem attributed to Takens [20] and Mane [19]. Let $s(n)$ denote a discrete scalar signal, with n being the index of time series. According to our assumptions, this signal is generated by deterministic dynamical system governed by some unknown set of differential equations. The multidimensional geometric structure (trajectory), a *strange attractor* in case of a system with chaos, might be unfolded in the space of *time delay vector* $\boldsymbol{y}(n)$:

$$\boldsymbol{y}(n)=[\, s(n), s(n+\tau), s(n+2\tau),..., s(n+(d_E-1)\tau)\,] \tag{1}$$

The coordinates of this vector are simply the sequence of time series elements, $s(n)$, and time series obtained from $s(n)$ by time-lagging multiplies of a time delay τ, whereas d_E, the *embedding dimension* of the reconstructed space of motion, indicates the minimum necessary number of coordinates to unfold the trajectories from self overlaps emanating from projection of the original orbit to a one-dimensional measurement space, where the dynamics is represented by sea elevation data.

The orbit pictured with the vector $\boldsymbol{y}(n)$ is a proxy of unknown "original" dynamics, in the sense that the characteristics of the motion, the largest Lyapunov exponent as an example, obtained from the analysis in the reconstructed phase space of $\boldsymbol{y}(n)$ are believed to remain invariant under the transformation - are the same as for “original” dynamics. Each component of $\boldsymbol{y}(n)$, variables $s(n)$ and $s(n+k\tau)$, $k=1,..., d_E-1$ constitute some nonlinear combination of the original variables.

The phase space reconstruction is a two-step process, as τ and d_E are to be determined, then the evaluation of the dynamical invariants (Lyapunov exponents) proceeds. For a detailed comparison of different techniques of the estimation of those parameters see [12] in which the most reliable method for each of them are indicated.

3.1. TIME DELAY (τ)

<u>Requirements:</u> It should be a multiple of the sampling time, sufficiently long to assure mutual independence of components of the vector (1) and adequately short to avoid

the measurements $s(n+k\,\tau)$ being random with respect to each other since in case of the chaotic dynamics the memory of previous states is rather brief.

Method: **the first minimum of the average mutual information**

the criterion of nonlinear mutual dependence, defined by means of Shannon theory of information and expressed by (2) (e.g. Shapiro [17], Frasser and Swinney [10]):

$$I(k)=\sum s(n),s(n+k\Delta t)\ P(s(n),s(n+k\Delta t))\ \log_2[P(s(n),s(n+k\Delta t))/P(s(n))\ P(s(n+k\Delta t))] \quad (2)$$

where $s(n)$ is the measured time series $s(n+k\Delta t)$ – the series time-lagged by k multiple of the sample period Δt. The two time series are treated as random variables with probabilities $P(s(n)$, $P(n+k\Delta t)$ and $P(s(n),s(n+k\Delta t))$ being their joint probability. The value of $k\,\Delta t$ indicating the first minimum is considered to be the desired value for the time delay populated solely by the "noise" (in our case of instrumental origin or capillary waves), hampering the analysis. Dimensionality of reconstructed space is considered to be dependent on quality of data utilized, sampling time and time delay.

Method: **Cao method**. It examines the change in the mutual distance between the h number of adjacent points in the space of subsequently increased dimensionality compared to the distance in lower dimension. The term of variability of the distance indicates the unfolding process being successful [5]. The technique is also claimed to provide a criterion for discrimination between deterministic and stochastic systems.

Once reconstructed, the “proxy” phase space becomes a domain for further analysis, i.e. calculation of the two important quantities described in the next subsections.

3.2. THE LARGEST LYAPUNOV EXPONENT λ_L

Meaning: it is a measure of divergence of the nearby orbits in the phase space and indice of the chaotic behavior. For the deterministic and ergodic dynamical system, the positive value of this parameter implies that the sufficient condition for chaos is fulfilled (Schuster [18], Abarbanel [1], Rosestein at al. [16]).

Method: **Sato Method** [17] with the modifications of the original technique following the suggestion of other authors (e.g. Lega et. al [13]). If the initial separation $\varepsilon_i^j(0)\}$ between the point j and its neighbor i in the reconstructed phase space increases exponentially during evolution to ε^j (t_D) after divergence time t_D (multiple of the sampling period Δt) the largest Lyapunov exponent is given by (3):

$$\lambda_L\,(t_D\ h)={1\over h}{1\over t_D }{1\over (N_O-{t_D \over \Delta t})}\sum_{j=1}^{ N_O -(t_D \over \Delta t})\sum_{i=1}^{h} \ln {\varepsilon_i^j(t_D)\over \varepsilon_i^j(0) } \quad (3)$$

where N_O is a number of elements of the data series, corresponding to N_O points in the phase space. This method involves determining of a "plateau" in the shape of the curve λ_L (t_D) with respect to length of the divergence time of adjacent trajectories and it was found to serve well for Lubiatowo data, with well pronounced "plateau" [12].

3.3. PREDICTION HORIZON T_H

Meaning: based on the values of the largest Lyapunov exponent, prediction horizon allows to estimate a time limit of possible prediction, exerted by intrinsic chaocity of the dynamics (expressed in sensitivity to the minute change in initial conditions). The prediction horizon T_H of the dynamical behavior refers to the time interval necessary to pass in order to magnify the uncertainty of the determination of the system state up to the half of the "radius of the attractor" R_A (equal to root mean square error of a given data set).
Method: Provided the initial state of the system is limited by the accuracy of the measuring device given by ε_M, the prediction horizon can be expressed by (4) (Frison and Abarbanel [9], Schuster [18]):

$$T_H = {1 \over \lambda_L} \ln \left({0.5 R_A \over \varepsilon_M} \right) \qquad (4)$$

4. RESULTS & DISCUSSION

4.1 TIME DELAY FOR LUBIATOWO SEA WAVE DATA

Figure 2. presents estimated average information function for the wave systems Lub01-03 and Lub51-53. The corresponding time delay values (the first minimuma) are listed in table 2. From our extensive survey on the effect of time series length and partition depth , we concluded that in case of Lubiatowo data, $N_O = 12000$ and $Pd=100$ and the new method introduced was found to to be robust than traditional one (Koszalka [12]).

4.2. THE EMBEDDING DIMENSION FOR LUBIATOWO SEA WAVES

The results of estimation of d_E for wave systems Lub01-03 and Lub51-53, obtained by means of the Cao method [5], are listed in Table 2. The resulting dimensionality of the reconstructed phase space was found more likely to be four in case of the dynamical systems studied (Koszalka [12]). Due to extensive calculational time requined for the analysis, the initial data sets were cropped to N=2000 elements. The investigation of influence of such an operation lead to the conclusion that this operation do not alter the final outcome [12]. Rather, it was the value of time delay and, most importantly, of the number of nearest neighbors h that seem to influence final results. This finding clearly revokes the statement of the author of a method (Cao [5]) that the only parameter needed to be imposed externally is τ - it turns out that the number of nearest neighbors covered in computations may impact the interpretation. After examination of different values of the amount of adjacent points included in calculations, the number $h \approx 30$ was decided as most adequate for Lubiatowo wave dynamics [12]. In some cases the determination of d_E was troublesome (in Table 2 other considered possibilities are also enclosed) regardless the change in value of parameters τ and h.

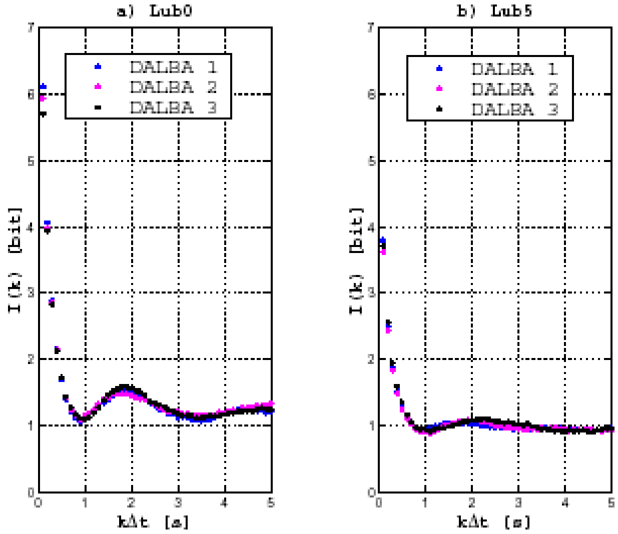

Fig. 2. Average mutual information for dynamical systems Lub1 (a) and Lub5 (b). Results for wave dynamics sampled at distinct towers are juxtaposed together. Estimation carried by means of the new technique introduced in (Koszalka [12]). Calculations were conducted for N_O =12000.

Table.2. Time delay from the first minimum of mutual average information (τ). Embedidng dimension (d_E) from the Cao method, more likely values marked with a bold font, obtained for calculations with h *30* and N_O =2000. The largest Lyapunov exponent from Sato method (λ) and corresponding prediction horizon (T_H) for wave systems measured at Lubiatowo Station.

Code Parameter	Lub01	Lub02	Lub03	Lub51	Lub52	Lub53
T	0.9	0.8	0.8	0.9	0.9	0.9
d_E	**4**	**4**(5)	**4**(6)	**4**	**4** (7)	**4**
λ	0.52	0.47	0.42	0.30	0.45	0.44
T_H	2.1	2.6	3.2	3.9	5.2	5.4

4.3. THE LARGEST LYAPUNOV EXPONENT FOR LUBIATOWO SEA WAVES

Out of two techniques for estimation of this quantity examined in [12], the Sato method [17] has been used. The possible influence of the variable amount of nearest neighbors was investigated (as suggested by (Lega et al. [13]). This number is not known a priori; if it is too high, the near orbits considered are not in fact neighboring. However, it cannot be too low, to assure a significant result upon averaging. The Sato method proved to give an estimate quite insensible to the number of neighboring orbits and to other parameters used in calculations (dimensionality of the phase space, data set lengthts). The results - curves illustrating the relation of the value of λ to the "divergence time" (t_D) of the adjacent trajectories are shown in Fig. 3. Computations were held in the phase space of d_E = 4 - 6 dimensions and time delay given by averaged mutual information criterion (Table.2).

In case of sea waves the detection of the "plateau" (reported troublesome for some dynamical systems) was not difficult, probably due to the strong oscillatory character of the motion, reflected in the shape of the curve. The position of "plateau" is clearly discernible and depicted by two "minima" of a corresponding curve. The location of the first minimum corresponds to the value of τ used for phase space reconstruction, whereas the position of the latter can be associated with the value of τ doubled. The minima correspond to the length of time span, for which the divergence of nearby trajectories is small. The high values of the exponent for short time spans, on the other hand, imply strong initial divergence of the neighboring orbits. The values of λ averaged over the "plateau" region, attained for N_O =2000, h=27, d_E =4 are listed in Tab. 2.

The most important and striking result is that for all the systems studied here the largest Lyapunov exponent was found positive. Assuming ergodicity and deterministic nature of the examined dynamics, it implies that chaotic behavior has been detected. In Tab. 2 are also shown the corresponding values of the prediction horizon T_H, defined by (4) and obtained based on the value of the exponent from Sato method, with the assumption that the initial state was known with the accuracy ε_M =1 cm (4). For all the dynamic systems studied the prediction horizon is very short and mounts up to few seconds.

The fluctuational character of the presented curves may imply a strongly oscillatory (wavy) behavior of the system generating sea wave, the observation that is rather to be expected. System trajectories scan the bounded region of the phase space, in the way determined partly this inherent oscillatory character. Nearby trajectories spread away in the first term of evolution, depicted by the values of λ for "divergence time" $t_D < \tau$. Provided the "divergence time" equal to τ is chosen, the nearby trajectories are close to each other again (do not diverge exponentially) and the corresponding value of λ is lower. However, the phenomenon surveyed is not purely periodic (which was shown already by frequency spectra), since the increasing "divergence time" results in the decline of aforementioned tendency. Most likely after several cycles around attractor the nonlinear effects suppress the effect of oscillations.

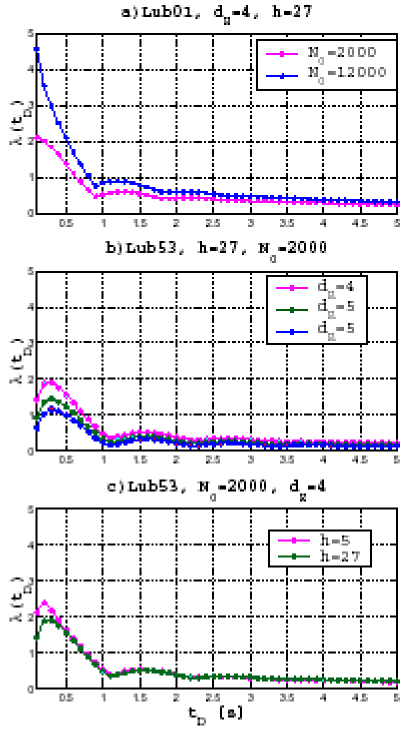

Fig. 3. Estimation of the largest Lyapunov exponent. Curves illustrating the relation of the value of λ in relation to the time span (t_D) "divergence time" of adjacent orbits for Sato method. (a) The effect of the time series length N_O , for Lub01. The amount of nearest neighbors included in calculation h=27, (b) the effect of the dimensionality of the phase space for Lub53. The amount of nearest neighbors included in calculation h=27, (c) The effect of the number of nearest neighbors h for Lub53 (d_E =4, N_O =2000).

5. BASIC PROBLEMS IN ANALYSIS

The main goal of the paper was to assess the applicability of the phase space reconstruction derived from the chaos theory to the sea wave motion measured at Lubiatowo Research Station.

In author opinion, it seems to be important to encounter and address the following basic problems associated with the introduced approach:

Time series length. The requisite condition for the phase space reconstruction is the ergodicity of the dynamics and representativeness of the time series for the original dynamical system as a whole. Is this demand still fulfilled for short data sets? The work presented has been founded on the hypothesis that even a short sample of temporal evolution of a variable, which constitutes a nonlinear combination of all the variables of the mother system, contains sufficient information about the global dynamics. This information is accessible only after application of the suitable method of extraction. This concept is a cornerstone for the development of new techniques and improvement of the existing chaos theory tools for the scrutiny of small observed data sets being proxies for the underlying dynamics. The results described in this work suggest that all the procedures utilized here are to some degree affected by *N*. The computations of time delay estimators, as quite fast, were carried out for the whole data sets untouched. As for the multidimensional calculations, generally the outcomes obtained for =2000 compared to N_O =12000, were found to be reliable.

Number of the nearest neighbors *h*. The determination of the dimensionality of the reconstructed space d_E, as well as the largest Lyapunov exponent, requires the examination of interrelations between the points called "the nearest neighbors" in the phase space. The choice of the apt number of the points for averaging procedures is essential, since:

(i) too small *h* included during the averaging is likely to result in "unstable", incidental estimate - particularly in case of observed dynamical systems, inevitably contaminated by a definite amount of noise,

(ii) too large *h* involved in the averaging process might distort the results due to the fact that some of them are likely not to be nearest neighbors at all.

The universal, reliable criterion for the proper value of this parameter has not yet been developed. It seems that every single dynamical system should be treated separately. This is a serious impediment of the phase space reconstruction process, requiring from analysts an intuition and most likely iteration of calculation for various amount of neighrest neighbors and subsequently, the comparison of the results. The multiple test - calculations discussed in this work were conducted for two selected data sets generated by Lub01 and Lub53, with the results being a basis for most appropriate parameter choice applied to all examined data sets. The number of h≈ 30 seems to be the most adequate for Lubiatowo waves.

Signal contamination. The noise in nonlinear analysis is defined as a high-dimensional component of the dynamical system. It might come from the deterministic prescription of the dynamics of phenomena studied (capillary waves superimposed on those wind-generated) or be a kind of instrumental noise. The Lubiatowo sea wave systems has been embedded in four-dimensional space, but some results (namely, outcomes from Cao method, not shown here, see Koszalka [12] and Cao [5]) suggest that large amount of "noise" is likely to be existent in observed dynamics. However, the methods of chaos theory applied here are not mind for quantification of the "noise" level.

Lack of measures of reliability of estimation. The main drawback of the analysis based on the theory of chaos is lack of the methods of determination of the estimation error. We have attempted to face the problem by creation of "reference base" for interpretation, composed of the results obtained by means of methods applied for exemplary, numerically generated (e.g. Lorenz model, Hénon map), dynamical systems, widely recognized in the field of chaos theory (see [12]). Reports on other observed dynamical systems also helped e.g. [1], [2], [5]. One should be aware, however, that those syntheticaly generated, “pure” time series are very distinct from the observed sea waves, therefore a question of formulation of the adequate theory of reliability of estimation is still open and calls for futher studies.

5.2. PHASE SPACE RECONSTRUCTION FOR LUBIATOWO WAVES

Once the dynamics of waves measured at Lubiatowo station had been embedded in four-dimensional space, this “proxy” phase space served as a domain of analysis of the system dynamics. The conclusions presented here refers to the paper of Frison and Abarbanel [9], where the results of similar treatment of wind waves from California coast are reported.

The postulations placed prior to the analysis can be briefly summed up in the following way: a nonlinear combination of variable from the unknown set of differential equations governing the motion observed permits the embedding of the dynamics in d_E - dimensional space. The number of "original" variables might be very high (representing the interaction processes between the sea and the atmosphere as well as an array of wave dissipation and transformation processes in the coastal zone), but the number of coordinates in the reconstructed space can be lower. The ergodicity of the dynamics measured and an appropiate choice of the sampling rate have been likewise assumed. The slow variability has been "frozen". Ruminative on these assumptions, the following characteristics have been allocated to the wave motion studied:

- Existence of well-pronounced minima of the average information allows for estimation of the delay time. The absence of a minimum would imply that the too long sample period hinders the acquirement of information about the system state from the time-lagged measurement (Abarbanel et al. [2]), for Lubiatowo waves $\tau \approx 1$ (s), while embedding dimension is $d_E = 4$. These values are smaller in comparison to the corresponding values reported in [9] for the tide-filtered, long period wind waves from the coast of California (1997). However, the character of dynamical systems studied in their work and the time series parameters were different: data sets of N=8192 elements were sampled for 3 hours with 1 (s) sampling period. The Californian measurements spanned longer range of scales and processes involved. It is reflected in the dimensionality found: the embedding dimension value of d_E =6-7 was concluded by aforementioned authors for the case of California waves.
- The fundamental assumption of the analysis was that of deterministic origin of the observed motion. Some of the results presented here, namely uncertainty of the output of Cao method, might contradict the deterministic nature of the

Lubiatowo sea wave dynamics. This uncertainty for Lubiatowo waves may be brought by the significant contribution of noisy nature of data, likely of instrumental origin, introducing randomness of states in the phase space and suppressing the natural, deterministic tendencies in point distribution. Unfiltered capillary waves might also contribute. The other explanation feasible is that assumption of ergodicity or the adequacy of the "freezing" of long-scale variability might be fallacious. A hidden influence of some slow, unresolved system variable might also contribute and affect the distribution of the points in the phase space. We cannot still exclude the possibility of the whole, phase space reconstruction approach, being "fooled" by the observed dynamical system of sea wave, so distinct from "classical" dynamical systems widely studied (e.g. voltage fluctuations or numerically generated data, see e.g. Abarbanel [1]). This question certainly calls for further examination, which should be based on different sea wave data sets. The most reliable method of treatment of the problem, suggested here, would be a simulation of wind waves of suitable parameters in a wave tank, measurement, subsequent analysis by means of the methods applied here, and comparison of results with waves observed in nature, an experiment of this kind was conducted by the same Frison and Abarbanel [9]. Nevertheless, the whole analysis presented here has been completed as if for deterministic systems, since no sufficient foundation for the deny of the deterministic behavior was found.

- The values of the maximum Lyapunov exponent, obtained from investigation of the divergence of the nearby trajectories, suggest that Lubiatowo sea wave systems are weakly chaotic (according to the notion used by Lega et al. [13]), characterized $\lambda_L \approx 0.4$. The corresponding prediction horizon T_H mounts up to few seconds. This value can be considered as low in terms of the wave forecast. In that case probabilistic methods might prove more relevant for prediction purposes rather then modelling in the local phase space [1], [9]. For the waves observed in California the values of $\lambda_L \approx 0.2$and $T_H \approx 30$s were detected. However, the results reported here as well as those obtained by Frison and Abarbanel [9], might suggest that chaotic behavior is a commonplace in the dynamics of the free sea surface. Validation of this statement requires the examination of other wave systems.
- The presence of several peaks in power spectrum, the similarity of the values of the time lag obtained from the autocorrelation function and average mutual information criteria, the periodic fluctuations curve examined by Sato method depict the important role of oscillatory components of the whole observed, nonlinear wave dynamics.

5.3 THE SPATIAL VARIABILITY OF LUBIATOWO WAVES

Before the analysis, the motion sampled at different towers was regarded as generated by separate dynamical systems. The results reported enabled the embedding in four-dimensional space in all cases considered, which supports the hypothesis of the common origin of the observed systems. However, the variation of the Lyapunov exponent with a distance offshore (see Table.2), might be an indication of the wave trasformation processes in the coastal zone. The application of the phase space dynamics to the study of those transformation phenomena would be an attractive alternative to the traditional methods, mostly based on empirical formulae as the costistent physical description is still lacking (see e.g. Crapper [7] and Darlymple and Dean [8]). Consequently, we postulate that futher studies should be done in order to verify this particular, potential applicability of the chaos theory tools for sea wave systems.

6. SUMMARY

The presented work aims at the assessment of applicability of the chaos theory tools for the sea wave motion study, presents and discusses the results of various methods and creates new domain for dynamical investigation, being a phase space reconstructed solely from the observations themselves.

The conclusions might be summed up as follows:

- methods derived from studies of the observed chaotic systems, namely the phase space reconstruction, has been proved to be in general applicable to the dynamics of sea waves represented by the free surface elevation data collected at Lubiatowo Coastal Research Station. The results corresponds to those reported in references. However, due to certain drawbacks of the theory itself as well as the properties of data coming from in-situ measurements, the interpretation of the results is sometimes troublesome,
- results obtained by a new technique for estimation of time delay from averaged mutual information, recently introduced in (Koszalka [12]) has been presented,
- sea waves from Lubiatowo show distinct signs of four-dimensional dynamics,
- presented approach likely provides the possibility for extraction of the aspects of the sea wave dynamics that are imperceptible with linear methods, as the detection of the chaocity and assessment of limits of predictability of the observed wave motion,
- work reflects on the possibility for the description and diagnosis of the certain aspects of the spatially distant coastal wave dynamics in phase space reconstructed.

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