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A bipolar consensus approach for group decision making problems

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This paper addresses the collaborative group decision making problems considering a consensus processes to achieve a common legitimate solution. The proposed resolution model is based on individual bipolar assessment. Each decision maker evaluates alternatives through selectability and rejectability measures which respectively represent the positive and negative aspects of alternatives considering objectives achievement. The impact of human behavior (influence, individualism, fear, caution, etc.) on decisional capacity has been taken into account. The influence degrees exerted mutually by decision makers are modeled through concordance and discordance measures. The individualistic nature of decision makers has been taken into account from the individualism degree. In order to achieve a common solution(s), models of consensus building are proposed based on the satisficing game theory formalism for collective decision problems. An application example is given to illustrate the proposed concepts.

1. Introduction

Nowadays, the increasing complexity of the socio-economic, engineering and environmental management make less possible decision by a single decision maker considering all aspects of a problem (Yue, 2011a). Therefore, the majority of decision problems are considered currently in the group decision process. This process is generally characterized by the existence of two or more persons (i) who have different perceptions, attitudes, motivations and personality, (ii) who recognize the existence of a common problem, and (iii) attempt to reach a collective decision (Herrera, Herrera-Viedma, & Verdegay, 1996b).

Solving a group decision making (GDM) problem often goes through the following phases: elicitation phase where different characteristics of the problem are defined (objectives, alternatives, attributes, etc.), evaluation phase and a selection and recommendation phase. In the evaluation phase, the way information is managed can lead to two families of aggregation approaches, we speak of input and output aggregation (Leyva Lopez, 2010) or common value tree for all decision makers and a value tree for each decision maker (De Brucker & Macharis, 2010). In the first case, aggregation is performed at the input when the decision group is invited to agree on a common set of attributes, weights and other parameters, which amounts to solving a problem as a single

decision making problem. In the second case, the individual evaluations are represented by individual value trees solved using standard process of decision support. The output aggregation is performed at the end. The present paper focuses on this second type of problem dealing with group decision making problem based on individual assessments.

Decision makers' evaluations can be represented by preference order (where the alternatives are ranked from best to worst), a utility function (where the alternatives are represented by real value – physical or monetary value–), or a frequently used preference relation (where alternatives are evaluated by pairwise comparison) (Herrera, Herrera-Viedma, & Chiclana, 2001). Depending on the nature of the data, the certainty of decision-makers, these preferences can be modeled by absolute evaluations when information is known or fuzzy evaluation based on the theory of fuzzy set, introduced by Zadeh (1965) in case of uncertainty in order to manage human subjectivity, imprecision and vagueness. The fuzzy evaluation is used in many areas due to the pressure, lack of knowledge and/or time.

Decision-makers' evaluations are then integrated into decision resolution procedures to reach an agreement on the selection of the best solutions. Traditionally, GDM problems have been solved by applying an alternative selection process in which the preferences of each decision maker over the alternatives are gathered and the best alternative or subset of alternatives is chosen (Roubens, 1997). However, as a group decision members usually come from different horizons with different specialty areas and different levels of knowledge, each group member has distinct

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information and sharing in general a part of the objectives with other decision members (Xu & Wu, 2011). This implies that individual assessments rarely meet (Roselló, Prats, Agell, & Sánchez, 2010; Ben-Arieh, Easton, & Evans, 2009) and the divergence of opinions can generate conflict (disagreement) and/or agreement within the group decision making. The recommendation phase in this case usually requires the establishment of a “consensus” building process in order to lead actors to a common decision (Khorshid, 2010).

To achieve a common accord, a variety of consensus reaching processes have been proposed in recent years (Eklund, Rusinowska, & de Swart, 2008; Gong, Forrest, & Yang, 2013; Herrera-Viedma, Cabrerizo, Kacprzyk, & Pedrycz, 2014). These approaches going from mechanist models of operational research to more sophisticated and soft computing oriented models that attempt to integrate human attitude (emotion, affect, fear, egoism, altruism, selfishness, etc.). The soft computing oriented models are used increasingly due to their ability to tolerate imprecision, uncertainty and partial truth in order to simulate human behavior with low cost (Pal & Ghosh, 2004), they allow to take into account the ambiguity in human thinking and uncertainty of the real world (Ko, Tiwari, & Mehnen, 2010).

2. Consensus building processes

Basically, group decision making aims at obtaining the consent, not necessarily the agreement of the participants by accommodating views of all parties involved to attain a decision that will yield what will be beneficial to the entire group (Herrera-Viedma et al., 2014). This is why the group consensus is usually considered as a total and final agreement between the decision members (Leyva Lopez, 2010). To reach a consensus, the researchers first proposed consensus approaches with the objective of reaching a full degree of agreement in the group, i.e. unanimity (Kline, 1972). The earliest approaches proposed to use group consensus functions that aggregate decision maker evaluations in a unique value representing the common opinion. Several aggregation methods have been proposed in the literature, simple average (Wheeler, Hora, Cramond, & Unwin, 1989), geometric mean (Cook & Kress, 1985), Bayesian aggregation (Bonano & Apostolakis, 1991), aggregation using the analytic hierarchy process (AHP) (see eg (Bard & Souk, 1990; Korpela & Tuominen, 1997; Lai, Wozniak, & Cheung, 2002; Tavana, Kennedy, & Joglekar, 1996)), fuzzy set theory (Hsu & Chen, 1996; Kacprzyk, Fedrizzi, & Nurmi, 1992; Moon & Kang, 1999; Yue & Jia, 2012), multi-criteria decision analysis methods (Hatami-Marbini & Tavana, 2011), etc.

In the best possible way, consensus should refer to unanimity of individuals because the selected solution(s) will be best representative for the entire group. Traditionally way to reach a consensus propose to model process by using matrix calculus or Markov chains to model the time evolution of changes of opinions toward consensus (Coch & French, 1948; French, 1956; Harary, 1959). However, unanimity may be difficult to attain, in particular in large and diversified groups of individuals as is the case in real world settings (Herrera-Viedma et al., 2014).

This resolution scheme does not take into account the agreement level between decision makers and some actors may not accept the final decision because their individual preferences are not taken into account sufficiently (Butler & Rothstein, 1987; Kacprzyk & Fedrizzi, 1988). For this reason, consensus reaching processes based on agreement levels were introduced as an additional phase in the resolution of GDM problems (Saint, 1994), as cited in (Palomares, Estrella, Martínez, & Herrera, 2014).

In this context, the concept of a soft consensus was introduced by Kacprzyk and Fedrizzi (1988) where some researchers assume

that unanimity is not required in the real decision problem and employed milder definitions of consensus (Butler & Rothstein, 1987; Verma, 2009) which consider for example a unanimity minus number of persons whose don't support the decision, percentage of actors (%) accepting decision, etc.

Generally, the soft consensus reached process is based on multistage setting where the individuals assumed collaborative, change their opinions until some consensus is reached.

The individual evaluation settings can be realized in discussion phases where the analyst or moderator –who is responsible for running the consensus reaching session– intervenes to guide stakeholders towards a common output solution, recommending them, based on rational arguments, to settle their preferences, and keeping the process within a period of time considered (Butler & Rothstein, 1987).

In some case, the individual settings are modeled by integrating the dynamic discussion phase in the reach consensus process and thus substituting the role of the analyst. (see for example (Choudhury, Shankar, & Tiwari, 2006)). Although the latter method is becoming increasingly popular in recent years, the method where moderator running a consensus reaching process is usually more effective and efficient (Herrera-Viedma et al., 2014).

In the present paper, a new adaptive consensus reached process based on semi-automated feedback mechanism –where analyst can intervenes– is developed. Considering a bipolar framework, initial decision makers' preferences are represented by bipolar measures that express the degree of supportability and rejectability of alternatives avoiding compensations.

To more realistic model, human behavior aspects (positive affect, negative affect, selfish, prudence, etc.) are integrated in the evaluation and recommendation phases of proposed approach. By considering social ties, the mutual influence of positive and negative interactions of the group members are integrated through individualism degree of each one. The weight of the decision makers which was the subject of several studies (Yue, 2011b; Yue, 2012a; Yue, 2012b; Yue & Jia, 2012) is also treated in this paper.

To our knowledge and as underlined in (Herrera-Viedma et al., 2014), although some authors introduce the decision maker importance degrees in the aggregation phase of actors' opinions (Herrera, Herrera-Viedma, & Verdegay, 1996a; Herrera, Herrera-Viedma, & Verdegay, 1997a; Lee, 2002), no one considers them in the recommendation phase when advising to the decision actors how to change their preferences to increase the consensus level. To remedy this, the present contribution integrates importance degrees of actors in proximity and bipolar consensus measures to adjust the actors' preference depending on his/her own knowledge level about the problem.

Considering that local preferences can be represented by a set of satisficing and non-dominated alternatives, developed consensus processes are defined considering two cases: when the local preferences of actors cannot be modified and in the case when modifications are possible. In the first case, consensus achievement is based on setting caution index and consensus measures are used in the second.

The remainder of the paper is organized as follow. Section 3 traces the evolution of consensus approaches in group decision making (GDM) and give a general classification of these process. Section 4 describe global framework of bipolar modeling of decision group problems when considering members interaction. The satisficing game theory used as aggregation tool is briefly described in this section. Section 4 presents proposed consensus and selection processes, an application example is given in Section 5. Eventually, Section 6 provides a conclusion and some perspectives.

3. Consensus approaches in GDM

As mentioned in Herrera-Viedma et al. (2014), the first mathematical approaches of consensus reaching processes started with the pioneering works by French and his collaborators in the late 1940s and early 1950s (Coch & French, 1948; French, 1956). Authors employed matrix calculus to model the time evolution and reaching of the consensus process. They also describe the impact of involving people in changes that affect them through the introduction of a simple model of how a network of interpersonal influence enters into the process of opinion formation. Drawing on the algebra of a Markov chain process, the consensus theory was developed in a more general form by Degroot (1974); French (1981) and Harary (1959). These initial researches describes the formation of group consensus, but do not provide an adequate account of settled patterns of disagreement.

Later, many models of consensus reaching (formation) have been proposed, particularly in the realm of so called rational consensus where authors Lehrer and Wagner (1981) considered that a consensus reaching procedure is not just a pooling or aggregation but changes of individual preferences occur and are rationally motivated.

In fuzzy environment, the first work which addresses the problem of consensus reaching modeling in a fuzzy environment (Ragade, 1976) examined some applications of fuzzy set theory in the area of communications and information systems, and was followed by an increasing number of researches (see for example, (Yager, 1988; Kacprzyk, 1986; Szmidt & Kacprzyk, 2003; Fu & Yang, 2012)).

Considering that the unanimous agreement is not necessary to lead a consensus, Loewer and Laddaga (1985) introduced the first approach for a soft consensus was introduced where a consensus degree, or proximity to the "ideal" consensus were defined. According to Loewer and Laddaga, other authors (Kacprzyk & Fedrizzi, 1988; Kacprzyk & Fedrizzi, 1989; Kacprzyk & Fedrizzi, 1986) introduced the concept of a fuzzy majority using Zadeh's fuzzy linguistic quantifier to define soft consensus measures.

A new consensus model for GDM problems based on fuzzy linguistic preference relations was then defined in an ordinal fuzzy linguistic approach (Herrera, Alonso, Chiclana, & Herrera-Viedma, 2009; Herrera & Herrera-Viedma, 2000). As main novelty authors define two types of soft consensus measures, consensus degrees and proximity measures. Both measures are computed in the three representation levels of a preference relation: level of preference, level of alternative, and level of preference relation. The consensus degrees indicate how far the set of actors is from the maximum consensus, and the proximity measures indicate how far each individual is from current consensus labels over the preferences. In such a way, this proposal provides moderator a complete consensus instrument to control the consensus reaching process (Herrera-Viedma et al., 2014).

The consistency measures were introduced by the same authors in (Herrera et al., 1997a) to allow analyst to guide the decision process towards more consistent solution.

In order to consider decision situations in which the authors can use different representation formats to express their preferences, Herrera-Viedma, Herrera, and Chiclana (2002) proposed a new consensus model based on consensus measures computed by comparison between actor's solutions and not between actors' preferences. A feedback mechanism based on rules was then proposed to make settings to actors' preferences. This proposition can lead to an automated consensus process which does not require the intervention of an analyst who may be considered subjective.

Another automatic control system to guide the consensus process was proposed by Herrera-Viedma, Martinez, Mata, and

Chiclana (2005) if decision makers could use different linguistic domains to express their opinions. This consensus model uses the consensus degrees to decide when the consensus process should finish and the proximity measures to define a recommendation system that recommends actors about the preferences that they should change in the next consensus rounds.

In presence of incomplete information or missing values in GDM problems, Herrera-Viedma, Alonso, Chiclana, and Herrera (2007) propose other seminal automatic consensus model contribution based on three kind of measures: consensus measures, consistency measures and incompleteness measures.

The automated consensus models act generally in similar way during all consensus stages although the conditions of GDM problem change (Cabrerizo, Pérez, & Herrera-Viedma, 2010; Cabrerizo et al., 2010; Chiclana, Mata, Martinez, Herrera-Viedma, & Alonso, 2008), however adapted consensus model considering agreement degree may be interesting and more effective. In this context, an adaptive consensus model has been proposed by Mata et al. in (Chiclana et al., 2008) in which authors propose to adapt the number of changes required to the actors in each round of consensus with regard to the agreement degree. In the present paper, a consensus model is adaptive and instructions follow the agreement degrees in bipolar context.

3.1. Consensus approaches classification

The soft consensus process intervening in recommendation phase, can be divided on the following mains steps:

1. Evaluation of consensus measures: from individual preferences obtained in the evaluation phase.
2. Consensus control: where consensus measures are compared to fixed threshold level of agreement. If the consensus level is achieved, the group moves onto the final recommendation and selects solution(s), otherwise, the preference setting is necessary and another iteration is to do.
3. Preference setting or consensus progress: when the threshold level of agreement is not respected by actors, oriented instructions for preferences setting are proposed to divergent actors in order to obtain sufficient agreement level. This actions can be realized by moderator or automated feedback mechanism.
4. Final recommendation: once a sufficient level of agreement is reached, the selection of the final solution can be done.

According to defined soft consensus processes, consensus approaches defined in the literature can be, generally, distinguished according to the following criteria (Herrera-Viedma et al., 2014; Palomares, Martinez, & Herrera, 2014):

- *Type of preference structures*: the individual results of each alternative can be obtained using tools as preference relations, preference orderings, utility vectors (Herrera-Viedma et al., 2002; Yu & Lai, 2011), etc., or, aggregation operator based on distances to the positive and negative ideal solutions by considering weight of attributes and/or decision-makers (Li, 2007; Park, Park, Kwun, & Tan, 2011; Yue, 2011b; Yue & Jia, 2012), or information domains (e.g. numerical or linguistic information (Bordogna, Fedrizzi, & Pasi, 1997; Parreiras, Ekel, & Bernardes, 2012)) used by actors to express their preferences over alternatives, amongst others.

Additionally, some models are focused on multiple criteria GDM problems (MCGDM) (Fu & Yang, 2010; Parreiras et al., 2012), in which information fusion approaches are often utilized to combine preferences evaluated according to several criteria, whilst other models have been defined to deal with a

particular type of real-life decision problems (Choudhury et al., 2006; Eklund et al., 2008).

- *Reference domain*: considering reference domain, the proposed consensus approaches can be divided into two categories:
 - Approaches based on the actors set (Carlsson et al., 1992; Fedrizzi, Kacprzyk, & Zadrozny, 1988; Kacprzyk & Fedrizzi, 1988): in this case, consensus degrees of decision actors are obtained from an aggregation of an agreement degree of pair of individuals as to their opinions between all the pairs of options.
 - Approaches based on the alternative set (Cabrerizo, Alonso, & Herrera-Viedma, 2009; Herrera et al., 1996a; Herrera-Viedma et al., 2007): in this case, the proposed approach allows analyst to guide decision actors to change their preferences during the discussion process considering three levels of preference relations: level of preference (which indicates the consensus degree existing among all the preference values attributed by the actors to a specific preference), level of alternative (which allows to measure the consensus existing over all the alternative pairs where a given alternative is present), level of preference relation (which evaluates the social consensus, that is, the current consensus existing among all the actors about all the preferences).
- *Concept of coincidence*: which means observing the existing coincidence among actors' opinions, is used to compute soft consensus measures that can be valued in $[0, 1]$ (where 0 indicates no consensus and 1 the unanimity) (Bryson, 1996; Herrera-Viedma et al., 2007; Kacprzyk et al., 1992; Wu & Xu, 2012; Xu, Li, & Wang, 2013) or using linguistic labels (Herrera et al., 1997a; Herrera, Herrera-Viedma, & Verdegay, 1997b; Pérez, Cabrerizo, & Herrera-Viedma, 2011). Three different approaches can be distinguished:
 - Strict coincidence among preferences: The coincidence is obtained by computing distances between actors preferences, accepting only the total coincidence or null coincidence cases (Herrera et al., 1997a; Kacprzyk, 1987).
 - Soft coincidence among preferences: very extended in GDM and specially applied in contexts of GDM under preference relations, this approach considers also different partial coincidence degrees to obtain consensus. (Bordogna et al., 1997; Fedrizzi et al., 1988; Kacprzyk & Fedrizzi, 1988).
 - Coincidence among solutions: In this case, the coincidence is also a gradual concept assessed in $[0,1]$ (Ben-Arieh & Chen, 2006; Chiclana, Herrera-Viedma, Herrera, & Chiclana, 2002). Coincidence approach provides here more realistic consensus measure among actors thanks to the consideration of the position of individual and the collective solutions. This approach is used in decision situations under different formats of preference representation.
- *Generation method of recommendation*: in the recommendation phase, some instructions are given to actors in order to improve the consensus status and lead group decision to common legitimate solution. Two recommendation modes are possible.
 - based on analyst guidance (feedback mechanism): in these consensus approaches (Bordogna et al., 1997; Kacprzyk, 1987; Kacprzyk & Fedrizzi, 1988; Kacprzyk et al., 1992; Mata, Martinez, & Herrera-Viedma, 2009), the analyst's goal in each stage is to address the consensus reaching process towards success by achieving the maximum possible agreement degree and reducing the number of actors outside of the consensus.
 - based on automatically process (Cabrerizo et al., 2010; Chiclana et al., 2002; Herrera-Viedma et al., 2007; Tapia

García, Del Moral, MartíNez, & Herrera-Viedma, 2012): to avoid possible subjectivity of analyst, some authors propose to make the automated consensus process by introducing feedback mechanisms. In these approaches, feedback mechanism is based on the evaluation of the distance between individual preference measures and the collective one using proximity measures. One actors with less contribution to reach high consensus identified, some instructions are transmitted for them to obtain a solution with better consensus degree. The automated feedback mechanisms know actually a growing success specially when consensus processes are developed in crowded social environments, such as Web 2.0 (Alonso, Herrera-Viedma, Chiclana, & Herrera, 2010; Alonso & Pérez, 2013).

- *Guiding measures*: used to guide decision makers toward a common solution through the analyst and/or feedback mechanisms. It is represented by:
 - consensus measures: as mentioned below, in the classical consensus process the consensus measures are used to evaluate the current consensus stage.
 - others kind of measures: avoids misleading solutions, in some consensus approaches, others measures are added to guide actors in the process, as a consistency measures (Herrera-Viedma et al., 2007; Cabrerizo et al., 2010), or index of comparability used to identify actors that have faced difficulties in expressing their preferences (Parreiras et al., 2012), or compatibility measure between actors' assessment (Fu & Yang, 2010; Jiang, Xu, & Yu, 2013), etc.

Considering group decision making problem, this paper present a new adaptive bipolar soft consensus building process based on flexible feedback mechanism where instructions follow the agreement degrees. According to previous definitions, the proposed model can be considered as an approaches based on the alternative set where coincidence is given among solutions.

4. Bipolar group decision making framework modeling under interactions

This section discusses the structuring and evaluation procedure based on bipolar analysis approach. Considering Multi-Attributes Multi-Objectives Group Decision Making problem (MAMOGDM problem), the general framework of the bipolar approach is presented and the satisficing game theory used as aggregation tool in individual evaluations is briefly described. The objective of proposed model is to provide a realistic framework to achieve a consensus, taking into account the potential impact of (positive/negative) influence among decision makers and their individualism degrees in different phases of problem resolution.

4.1. Proposed bipolar structuration of MAMOGDM problem

Let us consider a MAMOGDM problems characterized by the set of decision makers $D = \{d_1, d_2, \dots, d_m\}$ where each actor evaluate individually a common, finite and static set of alternatives noted $A = \{a_1, a_2, \dots, a_n\}$ to achieve fixed objectives $O = \{o_1, o_2, \dots, o_p\}$. The alternative evaluation is done using a set of attributes (or criteria) noted $C^{o_i} = \{c_1, c_2, \dots, c_q\}$ for each objective o_i . In the bipolar framework, alternatives are evaluated through bipolar 'a priori' measures. These measures noted $\mu_{s_0}^i(a_i)/\mu_{r_0}^i(a_i)$, represent respectively the 'supportability' and the 'rejectability' degree accorded by the decision maker d_j for an alternative a_i without considering potential influence of other decision makers.

These bipolar measures can be obtained in structured framework using BOCR analysis (Saaty, 2001; Saaty & Özdemir, 2005)

that brings structured evaluation considering in comprehensive way benefit, opportunity, cost and risk aspects. The opportunities in the BOCR analysis include usually positive expectations, future profits and income of the positive developments, while benefits are current revenues or profits of the positive developments relatively certain. Similarly, risks allow identifying the expected consequences of future negative developments, while costs are losses (in progress) where the consequences of the downturn are relatively certain.

Each decision maker can evaluate alternatives through b, o, c, r factors. The BOCR analysis can be associated with analytic hierarchy process to obtain individual preferences. In this case, criteria can be hierarchically distributed in clusters going from general to operational levels and to evaluate alternatives by pairwise comparisons (Bouzarour-Amokrane, Tchangani, & Pérès, 2012; Tchangani, Bouzarour-Amokrane, & Pérès, 2012).

We denoted by $\mathcal{B}_{o_i}^j(a_i)$, $\mathcal{O}_{o_i}^j(a_i)$, $\mathcal{C}_{o_i}^j(a_i)$, $\mathcal{R}_{o_i}^j(a_i)$ respectively the evaluations of (b), (o), (c), (r) factors given by the decision maker d_j for the alternative a_i with regard to achievement of the objective o_i . These evaluations are aggregated for each decision maker over all objectives to represent each alternative a_i with (b), (o), (c), (r) values noted $\mathcal{B}^j(a_i)$, $\mathcal{O}^j(a_i)$, $\mathcal{C}^j(a_i)$, $\mathcal{R}^j(a_i)$.

Synthesis methods of BOCR factors are usually based on multiplicative or additive expressions (Saaty, 2001; Saaty & Özdemir, 2005). However, due to the incommensurability of priority synthesis of the four factors, except in special cases, one cannot be certain that multiplicative or additive synthesis expression produces a correct order of alternatives (Wijnmalen, 2007).

Considering bipolar context, we propose to aggregate the benefit (b) and opportunity (o) factors in the 'selectability' measure and cost (c) and risk (r) factors in the 'rejectability' measure using the satisficing game theory introduced below.

4.2. Satisficing game theory

The satisficing game theory is based on the fact that decision makers in solving real problems do not necessarily seek the optimum solution –often costly in terms of time and money– but a satisfactory solution whose capabilities are estimated fairly good regarding to objectives achievement (Tchangani, 2009). The satisficing game theory is based on this observation and provides adequate tools for the selection of satisficing alternatives and reaching consensus. The concept of being good enough is suitable for our approach where an alternative can be considered good enough when its supporting contribution exceeds the rejecting one. The satisficing game theory is characterized by the triplet (A, μ_s, μ_r) where:

A : is a set of alternatives (discrete set).

$\mu_s(a_i)/\mu_r(a_i)$: are mass functions defined in A on the interval where $\mu_s(a_i), \mu_r(a_i)$ measure the degree to which a_i works towards success in achieving the decision maker's goal and costs associated with this alternative respectively (Tchangani, Bouzarour-Amokrane, & Pérès, 2011).

Thus, in the considered BOCR analysis framework, the bipolar measures using satisficing game theory are obtained with following equation.

$$\mu_s^j(a_i) = \frac{\sigma^j \mathcal{B}^j(a_i) + (1 - \sigma^j) \mathcal{O}^j(a_i)}{\sum_i \sigma^j \mathcal{B}^j(a_i) + (1 - \sigma^j) \mathcal{O}^j(a_i)} \quad (1)$$

$$\mu_r^j(a_i) = \frac{(1 - \sigma^j) \mathcal{C}^j(a_i) + \sigma^j \mathcal{R}^j(a_i)}{\sum_i (1 - \sigma^j) \mathcal{C}^j(a_i) + \sigma^j \mathcal{R}^j(a_i)} \quad (2)$$

where $0 \leq \sigma^j \leq 1$: is risk aversion index of decision maker d_j . More σ^j is close to 1, greater is risk aversion of decision maker who, pessimistic, will tend to give more importance to risk than cost in rejectability measure (Eq. (2)) and penalize opportunity in favor of

benefit in selectability measure (Eq. (1)). Inversely, when the risk aversion index tends to 0, the decision maker is optimist. He will focus on opportunity to benefit in the selectability measure, and will overlook risk against cost in the rejectability measure.

The satisficing game theory is characterized by the several sets that can be used on the recommendation phase. The satisficing set $S_{q_j} \subseteq A$ (at a boldness or caution index q_j of decision maker d_j) is the set of alternatives defined as following.

$$S_{q_j} = \{i \in A : \mu_s^j(a_i) \geq q_j \mu_r^j(a_i)\} \quad (3)$$

where q_j is the caution index of decision maker d_j that can be used to adjust the aspiration level: increase q_j allows reducing a satisficing set S_{q_j} (if too many alternatives are declared satisficing), on the contrary, decrease q_j allows increasing satisficing set (if S_{q_j} is empty for instance), see Fig. 1. To identify non-dominated alternatives which presenting a higher selectability measure and a lower rejectability measure an equilibrium set ε_j is defined by decision maker d_j as follow.

$$\varepsilon_j = \{a_i \in A : \mathcal{D}_j(a_i) = \emptyset\} \quad (4)$$

where $\mathcal{D}(a_i)$ is the set of alternatives that are strictly better than a_i (see Fig. 2). The set $\mathcal{D}_j(a_i)$ is defined with Eq. (5)

$$\mathcal{D}_j(a_i) = D_S(a_i) \cup D_R(a_i) \quad (5)$$

where $D_S(a_i)$ and $D_R(a_i)$ are defined by Eqs. (6) and (7) below

$$D_S(a_i) = \{a_f \in A : \mu_r(a_f) < \mu_r(a_i) \text{ and } \mu_s(a_f) \geq \mu_s(a_i)\} \quad (6)$$

$$D_R(a_i) = \{a_f \in A : \mu_r(a_f) \leq \mu_r(a_i) \text{ and } \mu_s(a_f) > \mu_s(a_i)\} \quad (7)$$

Thus, the satisficing equilibrium set $\varepsilon_q^{S_j}$ is given as follow

$$\varepsilon_q^{S_j} = \varepsilon_j \cap S_{q_j} \quad (8)$$

Notice that the set $\varepsilon_q^{S_j}$ constitutes a Pareto-equilibria set with an incomparability between a pair of alternatives and a trade-off process can be necessary for final choice in case of indecision. These alternatives are those laying on the portion of broken line green curve (above which there is no alternatives meaning the Pareto equilibrium) that is above the straight red line (separating satisficing and non-satisficing alternatives) of the following Fig. 3.

To provide a framework for integrating human factors evaluation, the following section proposes to model the influence between decision makers through concordance and discordance

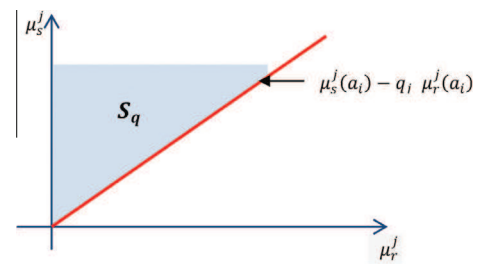


Fig. 1. Graphical representation of satisficing set.

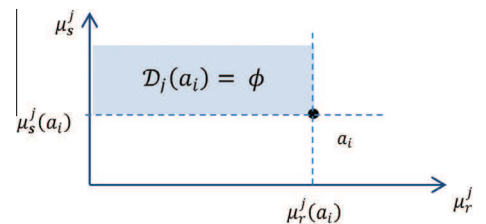


Fig. 2. Graphic representation of non-dominated alternatives.

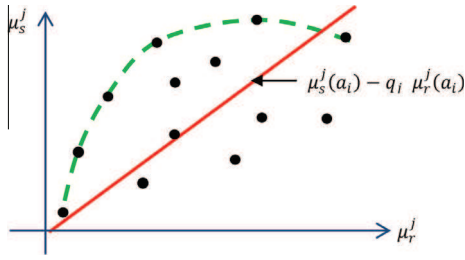


Fig. 3. Positions of alternatives in the plan (μ_r, μ_s) .

measures. The degree of individualism and the weight of decision makers will also be defined and integrated into the resolution model.

4.3. Influence modeling procedure

A group decision is usually composed of persons whose perceptions, attitudes, motivations and personality are different. In the GDM problems with an important number of jugged suitable and unsuitable actors to solve decision process, it is difficult to achieve acceptable consensus.

To consider the importance of each actor in the decisional process, some researches introduce the decision maker importance degrees in the aggregation phase of actors' opinions (Herrera et al., 1996a; Herrera et al., 1997a; Lee, 2002), but without consider them in the recommendation phase. In its recent research Alonso and Pérez (2013) propose to integrate the actors importance in consensus process but considering only subgroup of actors considered more important and final opinions given by this subgroup is used to obtain the final solution.

In present contribution, a confidence degree is introduced in the context of collaborative group decision where all decision makers are brought to cooperate in the evaluation phase. Each actor has a confidence degree obtained considering the positive or negative opinions that other members on the group have of him.

Depending on the personality of the actors, positive and/or negative influences can be exercised; some persons in the decision group are exclusively individualistic and take into account only their point of view, while other persons say 'holistic' prefer to take into account the opinion of other actors according to the importance degree assigned to them.

To model the influence related to each decision maker's opinion, this part proposes to define concordance and discordance degrees awarded by each decision maker to actors he considers influential. These measures allow decision makers to express their level of agreement or disagreement overlooked these actors. Indices of concordance and discordance are defined following (Bouzarour-Amokrane, Tchangani, & Pérès, 2013; Tchangani, 2013).

Definition 1. let $V(j)$ be the vicinity of decision maker d_j , which represents a set of m_j decision makers whose opinion matters (influence) the decision maker d_j in a positive or negative way. For each decision maker d_j belonging to a vicinity of de d_j , we define respectively $\omega_{jj}^c, \omega_{jj}^d$ as relative concordance and discordance degrees accorded by decision maker d_j to opinion of decision maker d_j compared to other members of the vicinity, where $\sum_{d_j \in V(j)} \omega_{jj}^c = 1, \sum_{d_j \in V(j)} \omega_{jj}^d = 1$.

The selection of vicinity $V(j)$ can be obtained for each decision maker d_j in an instinctive way in the case of a small group of decision where a pairwise comparison can be done for example to select a favorite actors considering a weight threshold. Considering a large decision group, one can adapt recent models proposed in

the literature to identify the trust network and deduce the subgroup of decision makers (Alonso & Pérez, 2013), or, detect non cooperative behaviors and dealing with them used fuzzy clustering as proposed in Palomares et al. (2014).

Once vicinity sets are defined, several methods can be used to obtain concordance and discordance degrees, the analytic hierarchy process (AHP) can be proposed for a relatively quick estimate. The AHP method is based on pairwise comparison by answering question of the form "what is the concordance (resp. discordance) degree accorded by the decision maker d_j to the opinion of d_j compared to the opinion of $d_{j'}$ " (where $d_j, d_{j'} \in V(j)$).

Based on this definition, we can consider that a decision maker is even more important in the community if other decision makers give him a good confidence. Thus, the degree of importance of decision maker d_j can be defined as follows.

Definition 2. let $\omega_{jj}^c, \omega_{jj}^d$ be respectively the relative concordance and discordance degrees accorded by the decision maker d_j to an opinion of d_j . The importance degree of d_j can be defined by Eq. (9)

$$\Theta_j = \frac{\sum_j \max(0, \omega_{jj}^c - \omega_{jj}^d)}{\sum_{p=1}^m \left\{ \sum_l \max(0, \omega_{lp}^c - \omega_{lp}^d) \right\}} \quad (9)$$

Or, Eq. (10) which allows decision makers to have non-zero degrees of importance, unlike formulation given by Eq. (9) in which zero importance is possible. Expression (10) may be useful in the case of a holistic and collaborative group consists of decision makers sensitive to opinion of their neighborhood.

$$\Theta_j = \frac{\sum_j \left(\frac{1}{1 + \left(\exp \left(-\alpha \left(\omega_{jj}^c - \omega_{jj}^d \right) \right) \right)} \right)}{\sum_{p=1}^m \left(\sum_k \left(\frac{1}{1 + \left(\exp \left(-\alpha \left(\omega_{kp}^c - \omega_{kp}^d \right) \right) \right)} \right) \right)} \quad (10)$$

where α is a parameter setting.

Considering the relative importance degree, the relative bipolar measures are then defined to include neighborhood effects in the final bipolar measures.

In the BOCR analysis where decision makers evaluations are expressed by $\mathcal{B}^j(a_i), \mathcal{O}^j(a_i), \mathcal{C}^j(a_i), \mathcal{R}^j(a_i)$. The relative bipolar evaluations in this case can be expressed using Eq. (11) and (12).

$$\mu_s^{j/V(j)}(a_i) = \frac{\sum_{j' \in V(j)} \Theta_{j'} (\sigma^{j'} \mathcal{B}^{j'}(a_i) + (1 - \sigma^{j'}) \mathcal{O}^{j'}(a_i))}{\sum_i \sum_{j''} \left(\sum_{j' \in V(j)} \Theta_{j'} (\sigma^{j'} \mathcal{B}^{j'}(a_i) + (1 - \sigma^{j'}) \mathcal{O}^{j'}(a_i)) \right)} \quad (11)$$

$$\mu_r^{j/V(j)}(a_i) = \frac{\sum_{j' \in V(j)} \Theta_{j'} ((1 - \sigma^{j'}) \mathcal{C}^{j'}(a_i) + \sigma^{j'} \mathcal{R}^{j'}(a_i))}{\sum_i \sum_{j''} \left(\sum_{j' \in V(j)} \Theta_{j''} ((1 - \sigma^{j''}) \mathcal{C}^{j''}(a_i) + \sigma^{j''} \mathcal{R}^{j''}(a_i)) \right)} \quad (12)$$

where

$\sigma^{j'}$: risk aversion degree of decision maker $d_{k'} \in V(d_k)$,

$\Theta_{j'}$: relative importance degree of decision maker $d_{k'} \in V(d_k)$.

These equations mean that relative bipolar measures of decision maker d_j considers only the opinion or evaluation of his vicinity according to the importance of each one. The final bipolar measures (selectability and rejectability measures) are then obtained by Eqs. (13) and (14).

$$\mu_s^j(a_i) = \delta^j \mu_{s_0}^j(a_i) + (1 - \delta^j) \mu_s^{j/V(j)}(a_i) \quad (13)$$

$$\mu_r^j(a_i) = \delta^j \mu_{r_0}^j(a_i) + (1 - \delta^j) \mu_r^{j/V(j)}(a_i) \quad (14)$$

where $\mu_{s_0}^j(a_i)/\mu_{r_0}^j(a_i)$ represent a priori measures of alternative a_i obtained by aggregating the corresponding factors.

$0 \leq \delta^j \leq 1$, is the individualism degree of decision maker d_j . While δ^j tends to 0, the decision maker is considered as 'holistic' (altruist) and gives a more importance to the global opinion represented by his vicinity. Inversely, if δ^j tends to 1, the decision maker is 'individualist' and considers his opinion above his vicinity.

When decision makers present their evaluations directly with 'a priori' bipolar measures $\mu_{s_0}^i(a_i)/\mu_{r_0}^i(a_i)$, the relative bipolar measures can be obtained using the following equations.

$$\mu_s^{j/V(d_j)}(a_i) = \frac{\sum_{j \in V(d_j)} (\omega_{jj}^c \mu_{s_0}^j(a_i) + \omega_{jj}^d \mu_{r_0}^j(a_i))}{\sum_i (\sum_{j \in V(d_j)} (\omega_{jj}^c \mu_{s_0}^j(a_i) + \omega_{jj}^d \mu_{r_0}^j(a_i)))} \quad (15)$$

$$\mu_r^{j/V(d_j)}(a_i) = \frac{\sum_{j \in V(d_j)} (\omega_{jj}^c \mu_{r_0}^j(a_i) + \omega_{jj}^d \mu_{s_0}^j(a_i))}{\sum_i (\sum_{j \in V(d_j)} (\omega_{jj}^c \mu_{r_0}^j(a_i) + \omega_{jj}^d \mu_{s_0}^j(a_i)))} \quad (16)$$

For decision maker d_j , these relative bipolar measures take into account the 'a priori' bipolar measures of his vicinity. In the selectability measure, the decision maker considers also the 'a priori' rejectability measure of his vicinity $V(j)$ according to their discordance degree. Inversely in relative rejectability measure, decision maker integers 'a priori' selectability measures of his vicinity $V(j)$. The finale bipolar measures can then obtained by the Eqs. (13) and (14).

From the individual evaluation results, the selection of a solution or set of solutions approved by the group decision generally requires a consensus building process. The following section provides a process achieved consensus from local preferences represented by different sets obtained from the satisficing game theory. The scheme of proposed consensus model can be given as follow, Fig. 4.

5. Consensus and selection processes

The individual bipolar measures are used by decision makers to select the best alternative or the set of the best alternatives for each one. According to the knowledge and perceptions of actors, the decision-makers choices can be contradictory. To find a common satisficing alternative(s), a consensus processes is necessary.

The reach consensus can be done in two ways: by aggregating final bipolar measures of decision makers into collective bipolar measures using Eqs. (17) and (18). Then selected alternative(s) can be obtained from satisficing equilibrium set according to proposed selection criteria. The second way to reach a consensus can be done using local preferences. The local preferences are defined as the selected alternatives by each decision maker according to his bipolar measures. The local preferences can be expressed by selection of unique alternative considered as 'the best' one, or by the equilibrium satisficing set defined in the satisficing game theory (Section 4.2). If selected alternative(s) is(are) the same for all decision makers, the common alternative can be chose easily,

however, if selected alternatives are different a consensus process is necessary. In the following, we propose a consensus model for each case of unique selected alternative or selected alternatives set.

$$\mu_s^c(a_i) = \frac{\sum_j \Theta_j \mu_s^j(a_i)}{\sum_j \Theta_j} \quad (17)$$

$$\mu_r^c(a_i) = \frac{\sum_j \Theta_j \mu_r^j(a_i)}{\sum_j \Theta_j} \quad (18)$$

where $\mu_s^c(a_i)$ and $\mu_r^c(a_i)$ represent respectively, the collective selectability and rejectability measures.

5.1. Case of unique local preference (unique selected alternative)

In this case, each decision maker d_j selects his 'optimal' or 'more satisficing' alternative noted a_j^* (among a set of equilibrium satisficing for example). The selected alternative can be obtained by the following.

$$a_j^* = \arg \max_{a \in \mathcal{E}_j} (\pi(a_i)) \quad (19)$$

where $\pi(a_i) = f(\mu_s^j(a_i), \mu_r^j(a_i))$ is value function that can be take particular form depending on the decision goal of decision maker d_j , for example :

$$\pi(a_i) = \mu_s^j(a_i) - q \mu_r^j(a_i) \quad (20)$$

that gives the priority to alternatives with large difference between the selectability measure and the rejectability measure given the index of caution, or

$$\pi(a_i) = \mu_s^j(a_i) / \mu_r^j(a_i) \quad (21)$$

that considers alternatives with the largest index of caution, or

$$\pi(a_i) = \mu_s^j(a_i) \left(\text{respct. } \pi(a_i) = \frac{1}{\mu_r^j(a_i)} \right) \quad (22)$$

that gives priority to alternatives with the largest selectability (respect. lowest rejectability); this later case is suitable when one of the measure is uniformly distributed over alternatives.

If the selected alternative a_j^* is the same for all decision makers, the final solution can be deduced easily. Conversely, if selected alternative a_j^* varies from one person to other a reach consensus processes based on selection criteria is proposed to find a common acceptable alternative.

According to the group nature, context and society where the decision problem is considered, the decisions rules can be changed. In his paper, Urfalino (2007) addresses a question of decision rules choice according to periods and societies. These parameters will favor unanimity, consensus or majority voting in order to achieve legitimate collective decision. The group nature and social ties are also determinant parameters in the decision rules choice. These

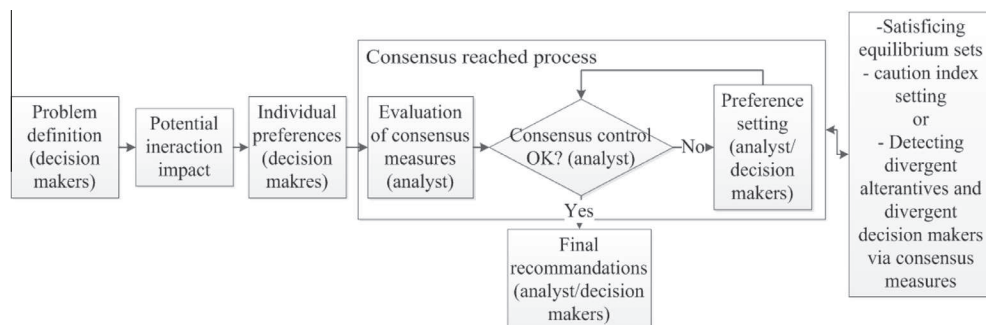


Fig. 4. General scheme of proposed consensus models.

parameters can be introduced with the ‘individualism’ and ‘holism’ notions that play important role in the rules selection. Considering these notions, we propose some possible selection criteria in the following.

1. Select alternative chosen by decision maker who presents the more important importance degree according by the group.
2. Select alternative that presents the more important selectability $(a^* = \arg \max_{a_i \in \mathcal{E}_q^{S_j}} (\sum_j \Theta_j \mu_s^i(a_i)))$.
3. Select alternative that presents the less important rejectability $(a^* = \arg \min_{a_i \in \mathcal{E}_q^{S_j}} (\sum_j \Theta_j \mu_r^i(a_i)))$.
4. Select the alternative that has the maximum benefit for all decision makers $\max_{a_i \in \mathcal{E}_q^{S_j}} (\sum_j \Theta_j (\mu_s^i(a_i) - \mu_r^i(a_i)))$.
5. Selecting a qualified majority: in some situations, the use of qualified majority decision is recommended in decision making. In this case, only the responses from a subset of makers are considered. We define ‘Qualified Majority’ as follows.

Definition 3. a ‘Qualified Majority’ (QM) is a set of decision makers who present $\alpha\%$ of the group decision members and $\beta\%$ of total importance.

$$QM = \{d_1, d_2, \dots, d_t\} \text{ where } t = \alpha m \text{ and } \sum_{j=1}^t \Theta_j \geq \beta, \sum_{j=1}^P \Theta_j = 1 \quad (23)$$

where α, β , are respectively possible rates that define ‘majority members’ and ‘global importance of the majority’, these rate can be defined by group decision or analyst.

Consider the ‘qualified majority’ (QM), the selection of the final decision can be done using the following steps.

Step1. If decision makers have a common alternative (obtained from the previous selection criteria) then this alternative is selected.

Step2. Otherwise, make adjustments to the caution index q_j of decision makers who show great caution successively in descending order until a common alternative.

Note that the latter definition ensures to reach a final solution on a partial consensus scheme; the notion of consensus in this case should be considered with regards to the agreement about the parameters α and β by the members of the group.

5.2. Case of the set of local preferences (expressed by satisficing equilibrium set)

In this part, local preferences are expressed according to the equilibrium satisficing set defined on satisficing game theory (see Section 4.2). Considering that each decision maker d_j identified his equilibrium satisficing set $\mathcal{E}_{q_j}^S$, if $\bigcap_{j=1}^m \mathcal{E}_{q_j}^S \neq \emptyset$, then the selection of common alternative can be done for example with the following.

$$a_j^* = \arg \max_{a_i \in \bigcap_{j=1}^m \mathcal{E}_{q_j}^S} (\pi(a_i)) \quad (24)$$

where $\pi(a_i) = \prod_{j=1}^m \left(\frac{\mu_s^j(a_i)}{\mu_r^j(a_i)} \right)$ for example.

Conversely, if $\bigcap_{j=1}^m \mathcal{E}_{q_j}^S = \emptyset$, a reached consensus process is necessary. To deal with this, we propose two possible reaching consensus process; fist process is based on varying caution index of decision makers when second proposed process is based on the distance measures between individual preferences. These measures are commonly called ‘proximity measures’ when it comes to comparing individual decision maker opinions to global or collective opinion on the ‘soft’ consensus process.

5.2.1. Consensus model based on caution index

One possible way to reach a consensus is to vary caution index of some decision makers to enlarge their satisficing equilibrium set and tend towards a non-empty intersection of these sets. To select decision makers who must modify their caution index, some selection criteria can be proposed as, importance of decision maker, satisficing equilibrium set dimensions $\mu_s^i(a_i)/\mu_r^i(a_i)$ rapport, etc.

We propose to decrease caution index of decision makers according to

- The importance degree: decrease caution index of decision makers who presents the less important degree one by one until reach consensus $(\bigcap_{j=1}^m \mathcal{E}_{q_j}^{S_j} \neq \emptyset)$.

- The qualified majority (QM): if the qualified majority of decision maker present one or more common alternatives, according to the group nature relations and decisional context (unanimity or not, confidence or not, complementarity or not), select a more satisficing alternative among QM common alternatives or ask to the rest of group to enlarge their equilibrium satisficing set to achieve a consensus using importance degree as regulation factor for example.

- Satisficing equilibrium set dimension: decrease caution index of decision makers who present the smallest set of satisficing equilibrium, one by one until reach a consensus $(\bigcap_{j=1}^m \mathcal{E}_{q_j}^{S_j} \neq \emptyset)$.

- Proximity limits: decision makers who present a large number of alternatives with $\mu_s^i(a_i)/\mu_r^i(a_i)$ tend to $q_j^{max} = \max_{a_i \in A} \left(\frac{\mu_s^i(a_i)}{\mu_r^i(a_i)} \right)$

are brought to reduce their index caution. In other words, decision makers consider alternative solutions with selectability measures much higher rejectability measures are brought to expand their range of solutions by reducing their caution indices one by one until reaching a consensus. This allows the decision makers to include in their satisficing equilibrium sets alternatives with better spreads.

5.2.2. Consensus model based on proximity measures

Supporting that a final agreement between decision makers is not necessary to resolve group decision problems, this section proposes to reach a common solution through a soft consensus process (Herrera et al., 1996a; Herrera-Viedma et al., 2002) based on proximity and bipolar consensus measures defined from final bipolar measures.

The proximity measures are used to assess the gap between decision makers’ evaluation on each alternative while bipolar consensus measures allow evaluating the gap between decision maker d_j and the rest of the group considering bipolar measures of alternatives. These measures are part of a feedback mechanism proposed to guide decision makers in adjusting their evaluations to tend towards a single solution and reach a consensus.

Unlike soft consensus process proposed in the literature (Herrera-Viedma et al., 2007; Khorshid, 2010), the model presented here does not aim to converge all decision makers assessments but focuses on alternatives presenting initially the same evaluation trend (convergent). The targeted recommendations are given to decision-makers to correct their differences and to reach the common solution from selected alternatives (Bouzarour-Amokrane et al., 2013).

In other hands, as mentioned bellow, some literature proposes to take into account the importance of decision makers when aggregating the actor’s opinions (Herrera et al., 1996a; Herrera et al., 1997a; Lee, 2002) but not when advising to the actors how to change their preferences to increase the consensus level. Proposed feedback mechanism consider this point and propose to

use weighted consensus measures to integrate decision maker's importance in the recommendation phase.

Considering that the intersection of the equilibrium satisficing sets of decision makers is empty $\bigcap_{j=1}^m E_q^{S_j} = \emptyset$, reaching a consensus amounts to establishing an iterative process allowing to have several phases of consultation until an agreement. The objective is to identify alternatives on which decision-makers have convergent opinions and support them to adjust their evaluation in order to reach a final common solution based on initial convergent opinion. In each iteration, considering importance degree of decision makers, the proximity measures are calculated to determine convergent alternatives (that present the smallest distance compared to others), when bipolar consensus measures are measured to identify decision makers who present a large gap of supportability and/or rejectability evaluations for the preselected alternatives compared to the rest of the group.

The main problem in this situation is the difficulty to determining the way to convergences individual preferences (Tapia García et al., 2012). To achieve this convergence, boundary conditions (threshold) are fixed for each level (for proximity and bipolar consensus measures). This process is carried out in a discussion session led by a feedback mechanism that can guide decision makers in changing their opinions. The alternatives with confused evaluations will be readjusted by the decision makers who present a strong divergence. A discussion session is led by a feedback mechanism to lead decision makers towards a common legitimate solution. The parameters of the proposed reached consensus process are developed below.

5.2.2.1. Proximity and bipolar consensus measures. The proximity and bipolar consensus measures are defined as follow (Bouzarour-Amokrane et al., 2013).

Definition 4. proximity measure d_i is used to calculate the average distance between decision maker evaluations considering the alternative a_i . It is obtained as follows, Eq. (25).

$$d_i = \frac{\sum_j \sum_{j' \neq j} \left[\left(d_{s_i}^{jj'} \right)^2 + \left(d_{r_i}^{jj'} \right)^2 \right]^{\frac{1}{2}}}{\binom{m}{2}} \quad (25)$$

where

$d_{s_i}^{jj'} = \Theta_j \mu_{s_i}^j(a_i) - \Theta_{j'} \mu_{s_i}^{j'}(a_i)$, distance between the supportability measure of decision maker d_j and $d_{j'}$ for the alternative a_i , with regard to the importance of each decision maker.

$d_{r_i}^{jj'} = \Theta_j \mu_r^j(a_i) - \Theta_{j'} \mu_r^{j'}(a_i)$, distance between the rejectability measure of decision maker d_j and $d_{j'}$ for the alternative a_i , with regard to the importance of each decision maker.

$\binom{m}{2} = \frac{m!}{2!(m-2)!}$, binomial coefficient takes into account combinations with avoiding redundancy distances (for example: $d_{s_i}^{12} = d_{s_i}^{21}$).

Definition 5. bipolar consensus measures are used to calculate the distance between a decision maker d_j and the rest of the group considering supportability and rejectability evaluation for the alternative a_i . The bipolar consensus measures are given by Eq. (26) and (27) respectively.

$$d_{s_i}^j = \frac{\sum_{j' \neq j} \left(d_{s_i}^{jj'} \right)}{m-1} \quad (26)$$

$$d_{r_i}^j = \frac{\sum_{j' \neq j} \left(d_{r_i}^{jj'} \right)}{m-1} \quad (27)$$

where $d_{s_i}^j, d_{r_i}^j$ represent respectively selectability and rejectability consensus measures.

5.2.2.2. feedback mechanism. The feedback mechanism is able to assist the moderator in the process of reaching consensus or even replace it allowing actors to change their preferences in order to achieve a tolerated proximity degree. The main issues of this mechanism are: how to ensure that individual positions converge? And how to support a decision maker in obtaining and accepting a given solution? (Tapia García et al., 2012; Chiclana et al., 2002).

Literature generally represents the feedback process by identification and a recommendation phases. According to this structure, the following section presents a feedback process proposed in connection with the bipolar approach.

Identification phase. The identification phase evaluates the closeness of decision maker assessments for each alternative and compares them to tolerances threshold fixed by the decision group or by the moderator. The alternatives which represent a convergent opinion among group decision members (manifested by a smallest proximity measure) considered more interesting, are selected. Then, bipolar consensus measures are used to identify decision makers who present a significant deviation compared the rest of the group according selected alternatives. From these measurements, evaluations of alternatives can be modified using the following steps.

(a) Identification of alternatives whose proximity measure d_i respects the condition (1) $d_i \leq \omega$, where ω is the tolerance threshold representing the permissible difference between alternatives (average distances on the set of alternatives can be considered as tolerance, for example). This allows excluding alternatives that may create conflicts and focus on alternatives already having a certain convergence.

(b) For each alternative a_i that respect condition (1), identification of decision makers that don't respect one or both following conditions: (2) $d_{s_i}^k \leq \omega_s$, (3) $d_{r_i}^k \leq \omega_r$, where ω_s/ω_r are a tolerance thresholds of selectability/rejectability measure respectively. These conditions allow identifying decision makers who present a divergent opinion regarding a rest of group. The average of all alternatives can be proposed as threshold.

Recommendation phase.

In this phase, recommendations are given to decision makers do not fulfill the conditions (2) and (3) in order to change their assessments concerning alternatives that meet the condition (1). The recommendations are based on the following rules:

- for $d_{s_i}^j > \omega_s$, the decision maker d_j has a large gap related to its selectability measure which means that the selectability measure given by d_j concerning alternative a_i , moves away from the evaluations given by other actors. To know the divergence direction and thus know if the considered alternative a_i , has more important selectability measure (positive divergence) or a low measure (negative divergence) compared to other actors, the following equation is proposed.

$$div_{s_i}^j = \frac{\sum_{j' \neq j} \left(d_{s_i}^{jj'} \right)}{m-1} \quad (28)$$

If $div_{s_i}^j > 0$, the alternative a_i has a good selectability measure, no change is required. Otherwise ($div_{s_i}^j < 0$), the selectability measure is smaller than the measures average, an increase selectability measure is recommended for considered alternative a_i .

The same recommendations are implemented for rejectability measure.

- for $d_{r_i}^j \geq \omega_r$, the decision maker d_j has a strong divergence from the rest of the group. The divergence direction indicates whether the alternative has a low rejectability (positive divergence) or a significant rejectability (negative divergence). The divergence direction of rejectability is given by Eq. (29).

$$div_{r_i}^j = \frac{\sum_{j' \neq j} (d_{r_i}^{j'})}{m-1} \quad (29)$$

If $div_{r_i}^j > 0$, the alternative a_i has a low rejectability measure, no change is required. Otherwise ($div_{r_i}^j < 0$), the rejectability measure is greater than the measures average, reduction of rejectability measure is recommended for considered alternative a_i .

Once modifications realized, the equilibrium satisficing sets for each decision maker are reconstructed. The iterative process is stopped when $\bigcap_{j=1}^m e_q^{S_j} \neq \emptyset$. If obtained solution satisfies group decision maker, the process can be stopped. Else, other iteration can be proposed. The next section presents an application example to illustrate proposed approach.

6. Application example

In this section, we use proposed group decision making bipolar analysis approach to resolve real size wind farm implantation problem adapted from Lee, Chen, and Kang (2009).

Install a wind farm involves various actors in society such as; wind specialist, local administration and public authority. In France for example, the selection of potential implantation sites (selection of alternatives) returns generally to the local administration, which taking into account previous studies, will retain potential installation areas approved by the prefect. A more sophisticated study is then performed to select the site chosen for the implementation, bipolar collective approach presented in this chapter can be used for analyze study. To illustrate the developed bipolar model, this section adapts original problem data of the of installing a wind farm (Lee et al., 2009), considering a decision committee composed of three entities: wind specialist, local administration and public authority denoted respectively d_1, d_2, d_3 . The goal is to select a location for a wind farm installation to achieve a set of objectives related to socio-economic, performance and operability notions. Five potential sites are under consideration.

Table 1
Evaluation results of BOCR analysis.

Decision makers	Factors	a_1	a_2	a_3	a_4	a_5
d_1	B	0.1892	0.2284	0.1498	0.2250	0.2076
	O	0.1965	0.1990	0.1980	0.2020	0.2045
	C	0.2072	0.1814	0.2132	0.2298	0.1684
	R	0.1976	0.2064	0.1986	0.2058	0.1915
d_2	B	0.1924	0.1900	0.2347	0.1581	0.2249
	O	0.1964	0.1980	0.1898	0.2103	0.2055
	C	0.1618	0.2172	0.1507	0.2216	0.2487
	R	0.2000	0.2043	0.2045	0.1894	0.2018
d_3	B	0.2137	0.1519	0.1957	0.2313	0.2073
	O	0.1981	0.1945	0.2026	0.2005	0.2043
	C	0.1902	0.1829	0.1891	0.1792	0.2587
	R	0.2078	0.1955	0.2115	0.1994	0.1857

Table 2
'A priori' bipolar measures.

Decision makers	Bipolar measures	a_1	a_2	a_3	a_4	a_5
d_1	Selectability measure ($\mu_{s_0}^1$)	0.1943	0.2078	0.1835	0.2089	0.2055
	Rejectability measure ($\mu_{r_0}^1$)	0.2043	0.1889	0.2088	0.2226	0.1753
d_2	Selectability measure ($\mu_{s_0}^2$)	0.1944	0.1940	0.2122	0.1842	0.2152
	Rejectability measure ($\mu_{r_0}^2$)	0.1809	0.2108	0.1776	0.2055	0.2252
d_3	Selectability measure ($\mu_{s_0}^3$)	0.2090	0.1647	0.1978	0.2221	0.2064
	Rejectability measure ($\mu_{r_0}^3$)	0.2025	0.1917	0.2048	0.1933	0.2076

The importance degrees of socio-economic, performance and operability objectives are given by decision makers d_1, d_2, d_3 respectively as follows: $\omega^{1,O} = [0.1 \ 0.8 \ 0.1]$, $\omega^{2,O} = [0.8 \ 0.1 \ 0.1]$, $\omega^{3,O} = [0.1 \ 0.1 \ 0.8]$. It is assumed that specialists wind (entity d_1) give more importance to the performance objective, their first goal is to establish a production site with a good yield. To manage the budget and preserve the municipal property, local administration (d_2) promotes socio-economics aspects while public authority (d_3) are supposed to pay more attention to the operability of the future site.

The bipolar evaluation of alternatives is carried out through BOCR analysis that consists in evaluating alternatives over benefit, opportunity, cost and risk factors through a distribution of attributes on these factors (Bouzarour-Amokrane et al., 2012). The results are summarized in Tables 1 and 2, representing respectively the evaluation results of the BOCR analysis and 'a priori' bipolar measures obtained for a risk aversion given by the vector $\sigma^j = [0.3 \ 0.5 \ 0.7]$. It is assumed in this case that the wind specialists (d_1) have a low risk aversion against a medium risk aversion for local administration (d_2) and an important caution about the public authority (d_3).

To represent the social link and the potential influences between decision makers, the proposed approach suggests modeling the interactions between decision makers through concordance and discordance measures notes respectively $\omega_{jj}^c / \omega_{jj}^d$ and represented by the following matrices.

$$\omega_{jj}^c = \begin{bmatrix} - & 0.1 & 0.9 \\ 0.7 & - & 0.3 \\ 0.2 & 0.8 & - \end{bmatrix} \quad \omega_{jj}^d = \begin{bmatrix} - & 0.7 & 0.3 \\ 0.2 & - & 0.8 \\ 0.6 & 0.4 & - \end{bmatrix}$$

These matrices show for example that decision maker d_2 gives good confidence in decision maker d_1 through a good degree of concordance and a low degree of discordance. Considering the concordance and discordance given above, the importance degree Θ_j of each decision maker d_j is deduced from Eq. (10) is given by the following vector.

$$\Theta_j = [0.3334 \ 0.3285 \ 0.3381] \quad (30)$$

The relative measures taking into account the vicinity can be obtained using Eqs. (11) and (12) and bipolar final measures from Eqs. (13) and (14). Considering that the individualism degree is medium ($\delta^k = [0.5 \ 0.5 \ 0.5]$), the results are respectively presented in Tables 3 and 4.

The graphical representation of the evaluation results (Table 4) in the plane (μ_r, μ_s) is given by the following Fig. 5. The index of caution q_j is assumed to be 1 for each decision maker d_j . The satisficing equilibrium sets $(\varepsilon_q^{S,j})$ of actors are as follows: $\varepsilon_1^{S,1} = \{a_5, a_2, a_4\}$, $\varepsilon_1^{S,2} = \{a_3\}$ and $\varepsilon_1^{S,3} = \{a_1, a_4\}$. In this case there is no common solution ($\bigcap_{j=1}^3 \varepsilon_1^{S,j} = \emptyset$), we propose then to seek a consensus using the proposed models. By varying the caution index q_j first, satisficing equilibrium sets of decision makers will be resized to make their non-empty intersection ($\bigcap_{j=1}^3 \varepsilon_q^{S,j} = \emptyset$). The process reached consensus based on proximity and consensus measures, is proposed in a second time.

7. Consensus building process based on caution index variation

The proposed model achieved with this consensus approach allows decision makers to avoid changing their evaluations. Only a concession on the caution index is required based on a selection criteria proposed in section 3.2.1 In this example, we note that the equilibrium satisficing sets of qualified majority of 67% composed of decision makers d_1, d_3 which consider the alternative a_4 as a final solution ($\varepsilon_1^{S,1} \cap \varepsilon_1^{S,3} = \{a_4\}$). Depending on the relationships nature and group decision-making context, the decision maker d_2 can accept selecting the alternative a_4 unsatisfactory for him without requiring modification. Otherwise, decision maker d_2 will have to change its caution index to expand its satisficing equilibrium set until include alternative a_4 . The caution index variation may also be according to the importance degree of decision makers. In this case, the decision makers classification deduced from Eq. (26), $(d_3 d_1 d_2)$ indicates that the decision maker d_2 must reduce its caution index first. If the intersection is still empty after changing, the decision maker d_1 changes its index of caution, and so on until a common solution is obtained.

Table 3
Relative bipolar measures.

Decision makers	Relative bipolar measures	a_1	a_2	a_3	a_4	a_5
d_1	$\mu_s^{1/V(d_1)}$	0.1976	0.1861	0.1926	0.2102	0.2135
	$\mu_r^{1/V(d_1)}$	0.1996	0.1893	0.2050	0.1951	0.2109
d_2	$\mu_s^{2/V(d_2)}$	0.2008	0.1929	0.1968	0.2060	0.2035
	$\mu_r^{2/V(d_2)}$	0.2049	0.1815	0.2013	0.2166	0.1957
d_3	$\mu_s^{3/V(d_3)}$	0.1947	0.1972	0.2014	0.2026	0.2041
	$\mu_r^{3/V(d_3)}$	0.1900	0.2043	0.1894	0.2040	0.2122

Table 4
Final bipolar measures.

Decision makers	Final bipolar measures	a_1	a_2	a_3	a_4	a_5
d_1	Selectability measure (μ_s^1)	0.1960	0.1969	0.1881	0.2095	0.2095
	Rejectability measure (μ_r^1)	0.2020	0.1891	0.2069	0.2089	0.1931
d_2	Selectability measure (μ_s^2)	0.1976	0.1935	0.2045	0.1951	0.2093
	Rejectability measure (μ_r^2)	0.1929	0.1961	0.1894	0.2111	0.2105
d_3	Selectability measure (μ_s^3)	0.2019	0.1809	0.1996	0.2123	0.2053
	Rejectability measure (μ_r^3)	0.1963	0.1980	0.1971	0.1987	0.2099

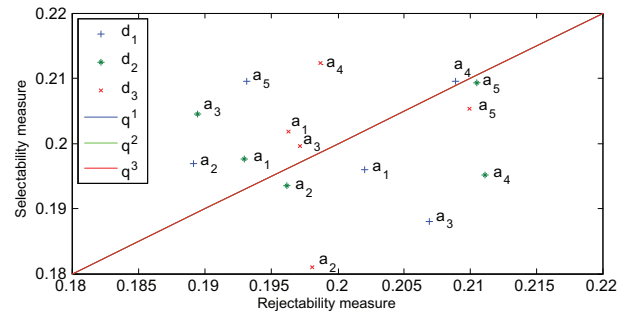


Fig. 5. Graphic representation of final bipolar measures for each decision maker.

Considering that the caution index of decision maker d_2 is given by $q^2 = 0.95$ this modification allows to enlarge its satisficing equilibrium set to $\varepsilon_{0.95}^{S,2} = \{a_3, a_5\}$, however the intersection $\bigcap_{j=1}^3 \varepsilon_{0.95}^{S,j}$ remains empty. The decision maker d_1 must change its caution index, considering that caution index becomes $q^1 = 0.95$, the satisficing equilibrium set of d_1 remains identical $\varepsilon_{0.95}^{S,1} = \{a_5, a_2, a_4\}$, the alternative a_1 is satisficing but dominated and then excluded from satisficing equilibrium set $\varepsilon_{0.95}^{S,1}$. The intersection at this level is still empty. It then asks the decision maker to modify its caution index. Considering that $q_3 = 0.95$, the satisficing equilibrium set of d_3 is given by $\varepsilon_{0.95}^{S,3} = \{a_1, a_4\}$, the alternative a_5 is satisficing but dominated by a_4 . In this case, a new iteration is realized, decision maker d_2 must reduce its caution index more. Considering that $q^2 = 0.91$, the satisficing equilibrium set is equal to $\varepsilon_{0.91}^{S,2} = \{a_5, a_3\}$, where the alternative a_4 is satisficing but dominated. It is noted that all the remaining alternatives can be satisfactory, but are still dominated. In this case, since the adjustments of assessments are not accepted, a possible solution amounts to consider the intersection of the satisficing equilibrium set of the most important decision maker with the satisficing sets of the rest of the group regardless of dominance for the rest of the group. This involves the selection of a qualified 'more satisfactory' solution by the largest maker. The solution in this case is the result of the following intersection $\varepsilon_1^{S,3} \cap \varepsilon_{0.95}^{S,1} \cap \varepsilon_{0.91}^{S,2} = \{a_4\}$.

8. Consensus building process based on distance measures

The second proposed process is based on distance measures that allow the analyst to identify alternatives and decision makers with strong differences. Using a feedback mechanism, decision makers have the ability to change their assessments based on recommendations made by the analyst during the discussion sessions, with the aim to converge to a common solution.

The first phase of the feedback mechanism uses proximity measures and bipolar consensus to identify respectively, divergent alternatives and decision makers with assessments that deviate from those of other decision makers. The second phase allows the analyst to make targeted recommendations to divergent decision makers.

Identification phase. The identification of the alternatives with strong differences consists in calculating proximity measures defined by Eq. (25) from the final bipolar measures (Table 4). The obtained results are given by the following vector $d_i = [0.0077 \ 0.0079 \ 0.0119 \ 0.0113 \ 0.0090]$. Assuming that the average distances on the set of alternatives is the tolerance threshold, the proximity distance must not exceed 0.0096 ($d_i \leq 0.0096$). We can deduce that the alternative a_3 and a_4 have widely divergent and should therefore be discarded. Assuming that the thresholds \hat{u}_s , \hat{u}_r were obtained from averages of bipolar distances on the set of alternatives, the following Table 5 shows the gaps observed at the actor level for each alternative.

Recommendation phase. Table 5 shows that decision maker d_1 presents a deviation from the average concerning selectability measure of alternative a_2 . The meaning of the divergence is positive ($div_{s_2}^1 = 0.003$), the selectability measure is important and cannot be modified.

- Decision maker d_2 presents a divergence regarding the rejectability measures of alternatives a_1 and a_5 . The divergence direction of the rejectability measures is given by $div_{r_1}^2 = 0.0035$ and $div_{r_5}^2 = -0.0015$. The negative divergence of alternative a_5 leads to a recommendation to reduce its rejectability measure. Alternative a_1 which has a low rejectability is spared.

- Decision maker d_3 presents a strong rejectability measure on alternative a_5 compared to the rest of the group. The negative divergence direction ($div_{r_5}^3 = -0.0042$) implies a recommendation of reducing this measure.

The reduction of rejectability measures of alternatives a_5 by decision makers d_2 and d_3 to $r_1^2(a_5) = 0.2005$ and $r_1^3(a_5) = 0.1980$ respectively allows for the following graphical representation (see Fig. 6). The satisficing equilibrium sets ($e_q^{s,j}$) of each decision maker d_k are deduced as follows; $e_1^{s,1} = \{a_5, a_2, a_4\}$, $e_1^{s,2} = \{a_3, a_5\}$ and $e_1^{s,3} = \{a_1, a_4, a_5\}$. The solution obtained by the intersection of the sets is the alternative a_5 ($\bigcap_{j=1}^3 e_1^{s,j} = a_5$).

In the example discussed here, the integration of positive and negative influences of decision makers in the model and the relatively small number of decision makers allowed reaching a

Table 5
Bipolar consensus measures.

D	$d_{s_i}^k$			$d_{r_i}^k$		
	d_1	d_2	d_3	d_1	d_2	d_3
a_1	0,0017	0,0019	0,0031	0,0025	0,0035	0,002
a_2	0,0033	0,0022	0,0034	0,0026	0,002	0,0032
a_3	0,0046	0,0024	0,0026	0,0045	0,0056	0,0034
a_4	0,0039	0,0067	0,0048	0,0014	0,0012	0,0023
a_5	0,0007	0,0009	0,0005	0,0057	0,0033	0,0042

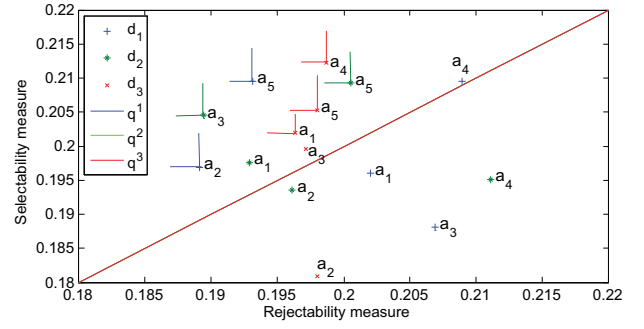


Fig. 6. Graphic representation of final bipolar measures (iteration1).

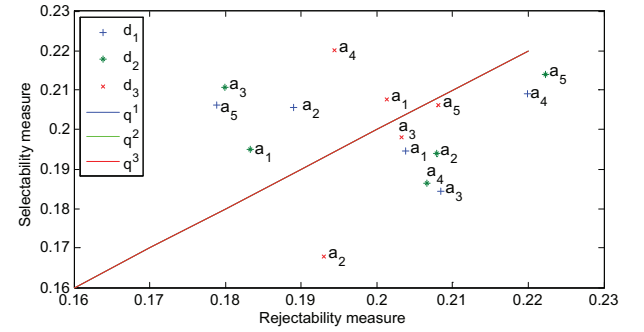


Fig. 7. Graphic representation of final bipolar measures for $\delta_j = [0.90.90.9]$.

consensus quickly after a single recommendation step. The individualism average rate considered for all decision makers allows also to nuance the individual assessments and reduce differences that a high degree of individualism could make appear as shown in Fig. 7 for individualism degrees δ^j equal to 0.9 for each decision maker. A sensitivity analysis can be performed by varying the caution index and/or individualism degree to test different possible scenarios and stability of recommended solutions.

9. Conclusion and perspectives

Considering collaborative decision problem based on individual evaluation, this paper proposes a new resolution approach to deal with group decision problems in multicriteria framework.

To more realistic model, human behavior aspects (positive and negative influences, selfish, prudence, etc.) are integrated in the evaluation and recommendation phases. Based on bipolar context, local preferences which considers the vicinity influence are expressed by selectability and rejectability measures, to avoid compensations.

A consensus building processes are proposed from final bipolar measures in order to guide decision makers toward a common solution. Based on the formalism of the satisficing game theory, common solutions were represented by the result of the intersection of satisficing equilibrium sets of decision makers. The first proposed process for achieving consensus is based on the caution index variation, considering that the adjustments and changes in individual assessments were not admitted. In this case, decision actors had to reduce their caution index degree based on a set of selection criteria. This readjustment has expanded the satisficing equilibrium sets and increase the chances of obtaining a non-empty sets intersection. When individual assessments adjustments are possible, an iterative process of reaching consensus based on proximity and bipolar consensus measures has also been proposed to find common solutions.

In the principle, our proposition can be considered similar to other consensus models proposed in the literature (Herrera et al., 1996a; Herrera-Viedma et al., 2002; Yeh, Kreng, & Lin, 2001) in the sense that the consensus seeking consists in adjusting initial assessment of some parameters obtained by actors but in practice the implementation of approaches may completely differ. For example regarding proposed model in where the principle is similar.

The approach developed in this paper goes from bipolar measures then obtaining a satisficing short list of alternatives by each decision makers; the adjustment is considered only with regards to these satisficing alternatives using divergence measures contrary to what is proposed in Yeh et al., 2001 for example, where a mathematical programming problem that seeks new assessments when minimizing a deviation measure is formulated and solved by a genetic algorithm.

Moreover, in contrast to soft consensus processes proposed in the literature ((Herrera-Viedma et al., 2007; Khorshid, 2010) for example), the model presented here does not aim to converge individual assessments on the set of all alternatives but focuses mainly on alternative with the same trend assessments initially (convergent). Targeted recommendations are given to divergent actors (with incoherent opinion compared to other members) in adaptive process. Another feature of the proposed model lies in the fact that the importance degree of actors is seen in evaluation and recommendation phases by integrating it in the consensus reached process unlike what is done to present literature, to our known.

Although the proposed model has some new features such as those just mentioned and flexibility and ease of immersive excursion, some weakness can be felt when applying the approach developed in this paper particularly due to the amount of parameters that have to be elicited and the interpretation issues by decision makers mainly when they are trained in different backgrounds. But if the process is conducted by an analyst step by step, these difficulties may be reduced.

To apply the approach developed in this paper, different materials must be gathered; among these materials, the notion of vicinity and related influence parameters elicitation may raise difficult issues such how different partners do understand this notion. Therefore, future researches must consider addressing scientifically this issue by formalizing this notion building on social network concept for instance. Human attitude modeling is another subject that we think need to be seriously considered in the future works as well as uncertainty consideration during selection process (fuzzy selectability and rejectability measures for instance).

On the other hand, consensus process assumed generally that decision makers accept to give modifications to their preferences to achieve a common solution. However, some actors can decide not to accept the recommendations (Herrera-Viedma et al., 2014). In this case, two alternatives can be studied, the first amounts to consider the refusal of these decision makers in the consensus processes though settings made according to the importance of refractory actors. Or, conversely, develop a support for the consensus process based on psychology concepts (social proof, authority, linking, scarcity, consistency, etc) to convince recalcitrant decision makers (Cialdini, 2001).

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