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Sensitivity Analysis of Likelihood Ratio Test in K Distributed and/or Gaussian Noise

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Abstract—**In a recent letter we addressed the problem of detecting a fluctuating target in K distributed noise using multiple coherent processing intervals. It was shown through simulations that the performance of the likelihood ratio test is dominated by the snapshot which corresponds to the minimal value of the texture. However, for this particular snapshot the clutter to thermal noise ratio is not large and hence thermal noise cannot be neglected. In the present letter, we continue our investigation with a view to consider detection in a mixture of K distributed and Gaussian noise. Towards this end we study the sensitivity of our previously derived detectors. First, we provide stochastic representations that allow to evaluate their performance in K distributed noise only or Gaussian noise only. Then, their robustness to a mixture is assessed.**

Index Terms—**Detector sensitivity, K distributed noise, likelihood ratio, radar processing.**

I. INTRODUCTION AND PROBLEM STATEMENT

D ETECTION of a target in competition with strong clutter and thermal noise is a predominant problem in most radar systems. The two sources of disturbance emanate from completely different physical phenomena and result in different power levels and statistical distributions. While the Gaussian assumption prevails for thermal noise, in a number of applications, such as low grazing angles or with sea clutter, the heavy-tail nature of clutter has been observed experimentally [1]–[5]. The compound-Gaussian model [6], [7] has been thoroughly used [8]–[14]. It consists in modeling the received signal as $\boldsymbol{c}_t = \sqrt{\tau_t}\boldsymbol{g}_t$ where $\boldsymbol{g}_t \sim \mathbb{C}\mathcal{N}(\boldsymbol{0}, \boldsymbol{R_c})$ follows a complex Gaussian distribution with zero mean and covariance matrix $\boldsymbol{R}_c$, and the texture τ_t is a positive random variable. The most well-known detector within this framework is the normalized matched filter [10], which is derived by considering τ_t deterministic. Most often, mainly because the clutter to noise ratio is generally high, it is assumed that thermal noise can be neglected.

In [15], we considered detection of a fluctuating target in fluctuating K-distributed clutter, a problem formalized as

$$
\begin{aligned}
H_0 &: \boldsymbol{x}_t = \sqrt{\tau_t}\boldsymbol{R}^{1/2}\boldsymbol{n}_t \\
H_1 &: \boldsymbol{x}_t = \boldsymbol{a}s_t + \sqrt{\tau_t}\boldsymbol{R}^{1/2}\boldsymbol{n}_t
\end{aligned}
\tag{1}
$$

where $\boldsymbol{x}_t$, $t = 1, \cdots, T$, are a set of T independent and identically distributed (i.i.d.) radar returns, and where $\boldsymbol{a}$ and s_t stand for the target signature and amplitude, respectively. In [15], we showed that, in contrast to what is commonly admitted, thermal Gaussian noise cannot be ignored, even if clutter to noise ratio is high. Indeed, it was shown that, for heavy-tailed distributions, the detection performance is in fact dominated by a single observation within the T i.i.d. observations available, namely the snapshot corresponding to the minimal texture value. To give an order of magnitude, for $T = 8$ and the shape parameter of the K distribution $\nu = 0.2$ [respectively $\nu = 0.1$] the average power of the minimal over T texture values is 25 dB [respectively 38 dB] below the ensemble power of the K distributed clutter. This shows that, with multiple i.i.d. observations, internal white Gaussian noise has to be considered even for practically observed input clutter to noise ratios.

Therefore, one should consider the problem of detecting a signal of interest in compound-Gaussian clutter and thermal Gaussian noise ($t = 1, \cdots, T$):

$$
\begin{aligned}
H_0 &: \boldsymbol{x}_t = \sqrt{\tau_t}\boldsymbol{R}_c^{1/2}\boldsymbol{n}_t + \sigma_w\boldsymbol{w}_t \\
H_1 &: \boldsymbol{x}_t = \boldsymbol{a}s_t + \sqrt{\tau_t}\boldsymbol{R}_c^{1/2}\boldsymbol{n}_t + \sigma_w\boldsymbol{w}_t
\end{aligned}
\tag{2}
$$

where $\boldsymbol{R}_c$ stands for the clutter covariance matrix, $\boldsymbol{n}_t \sim \mathbb{C}\mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$ and the additive white noise is assumed to be Gaussian, i.e., $\boldsymbol{w}_t \sim \mathbb{C}\mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$. The disturbance covariance matrix is given by

$$
\boldsymbol{R} = \mathcal{E}\left\{\boldsymbol{x}_t\boldsymbol{x}_t^H|H_0\right\} = \mathcal{E}\left\{\tau_t\right\}\boldsymbol{R}_c + \sigma_w^2\boldsymbol{I}.
\tag{3}
$$

Unfortunately, one immediately faces the problem that the distribution of the overall noise is unknown and, at this stage, it seems impossible to resort to a plain generalized likelihood ratio (GLR) approach.

As a preliminary attempt to cope with this problem, we adopt a common practice which consists in evaluating the sensitivity of known detectors to a mismatch or to uncertainties, see [16], [17] for example. More precisely, we consider the detector derived from problem (1) where we assumed that disturbance is composed of clutter only and thermal noise was neglected, and

we consider the "mirror" case where the disturbance is Gaussian and the problem is formulated as

$$H_0 : \boldsymbol{x}_t = \boldsymbol{R}^{1/2}\boldsymbol{n}_t$$
$$H_1 : \boldsymbol{x}_t = \boldsymbol{a}s_t + \boldsymbol{R}^{1/2}\boldsymbol{n}_t. \qquad (4)$$

In both problems (1) and (4) we assume that $\boldsymbol{R}$ is known: these frameworks can be viewed as the "extreme" cases of K only and Gaussian only disturbance while (2) is a mixture of the two kinds of noise. Whatever the case, the second order statistics of the noise are known and the difference lies in their different distributions. We tackle the problem by first investigating the robustness of the detectors derived under one hypothesis when applied to the other statistical model. This enables one to give a hint about the ability of these detectors to perform well under a mixture of the two sources of noise. Then we evaluate, through numerical simulations, their performance when (2) is in force.

II. Robustness to Each Type of Noise

In the sequel, and similarly to [15], we assume a Gamma distribution for τ_t, namely $\tau_t \sim \mathcal{G}(\nu, \beta)$, so that, under H_0, $\boldsymbol{x}_t$ follows a K distribution. Let $\boldsymbol{X} = [\boldsymbol{x}_1\ \boldsymbol{x}_2\ \dots\ \boldsymbol{x}_T]$. Under the assumption of known $\boldsymbol{R}$, an approximate log-likelihood ratio (LLR) test was derived in [15], given by

$$aLLR_K(\boldsymbol{X}) = -\sum_{t=1}^{T} \log\left[1 - t(\boldsymbol{x}_t)\right] \qquad (5)$$

where

$$t(\boldsymbol{x}_t) = \frac{\left|\boldsymbol{a}^H\boldsymbol{R}^{-1}\boldsymbol{x}_t\right|^2}{\left(\boldsymbol{x}_t^H\boldsymbol{R}^{-1}\boldsymbol{x}_t\right)\left(\boldsymbol{a}^H\boldsymbol{R}^{-1}\boldsymbol{a}\right)}. \qquad (6)$$

Interestingly enough, (5) coincides with the GLR obtained assuming that the variables τ_t are deterministic and unknowns [11], while in [15] it was obtained from a large $M - \nu$ approximation of the modified Bessel function. On the other hand, for the classical problem in (4) the GLR is the matched filter, i.e.

$$LLR_G(\boldsymbol{X}) = \sum_{t=1}^{T} \frac{\left|\boldsymbol{a}^H\boldsymbol{R}^{-1}\boldsymbol{x}_t\right|^2}{\boldsymbol{a}^H\boldsymbol{R}^{-1}\boldsymbol{a}}. \qquad (7)$$

The normalized matched filter (actually the non coherent integration of the NMF for a single snapshot) is given by

$$NMF(\boldsymbol{X}) = \sum_{t=1}^{T} t(\boldsymbol{x}_t). \qquad (8)$$

Finally, in [15], we discovered that the test statistic in (5) is mostly influenced by the term corresponding to the minimal value of τ_t. Let $\boldsymbol{x}_{\tau_{\min}}$ be this snapshot and let us consider the *hypothetical* test statistic

$$aLLR_K(\boldsymbol{x}_{\tau_{\min}}) = -\log\left[1 - t(\boldsymbol{x}_{\tau_{\min}})\right]. \qquad (9)$$

We first study the behavior of the detectors in (5)-(9) under H_0. As was shown in [15] (except for NMF which was not considered there), both for K-distributed noise or Gaussian dis-

tributed noise, the distributions of $aLLR_K(\boldsymbol{X})$, $NMF(\boldsymbol{X})$ and $aLLR_K(\boldsymbol{x}_{\tau_{\min}})$ are the same and given by

$$aLLR_K(\boldsymbol{X})|K - G \sim \mathcal{G}\left(T, (M-1)^{-1}\right) \qquad (10)$$

$$aLLR(\boldsymbol{x}_{\tau_{\min}})|K - G \sim \mathcal{G}\left(1, (M-1)^{-1}\right) \qquad (11)$$

$$NMF(\boldsymbol{X})|K - G \overset{d}{=} \sum_{t=1}^{T} \mathbb{CB}_{1,M-1(t)}. \qquad (12)$$

Only $LLR_G(\boldsymbol{X})$ has a different distribution under each assumption, namely

$$LLR_G(\boldsymbol{X})|G \overset{d}{=} \sum_{t=1}^{T} \boldsymbol{n}_t^H \boldsymbol{P}_{\boldsymbol{R}^{-1/2}\boldsymbol{a}} \boldsymbol{n}_t \overset{d}{=} \sum_{t=1}^{T} \mathbb{C}\chi_T^2 \qquad (13)$$

$$LLR_G(\boldsymbol{X})|K \overset{d}{=} \sum_{t=1}^{T} \tau_t \boldsymbol{n}_t^H \boldsymbol{P}_{\boldsymbol{R}^{-1/2}\boldsymbol{a}} \boldsymbol{n}_t \overset{d}{=} \sum_{t=1}^{T} \tau_t \mathbb{C}\chi_{1(t)}^2 \qquad (14)$$

where $\boldsymbol{P}_{\boldsymbol{R}^{-1/2}\boldsymbol{a}}$ is the projection onto $\boldsymbol{R}^{-1/2}\boldsymbol{a}$. For $aLLR_K(\boldsymbol{X})$, $NMF(\boldsymbol{X})$ and $aLLR_K(\boldsymbol{x}_{\tau_{\min}})$, the same threshold can be set to ensure a desired probability of false alarm P_{fa} for both noise distributions. Only $LLR_G(\boldsymbol{X})$ can incur a variation of P_{fa} due to distribution mismatch. Indeed,

$$P_{fa}(LLR_G(\boldsymbol{X})|K) = \Pr\left[\sum_{t=1}^{T} \tau_t \mathbb{C}\chi_{1-t}^2 > \eta_{G|G}\right] \qquad (15)$$

where $\eta_{G|G}$ corresponds to the threshold of $LLR_G(\boldsymbol{X})$ to obtain the desired P_{fa} under Gaussian noise. Conditioned on $\mathbb{C}\chi_{1-t}^2$, the random variable $\sum_{t=1}^{T} \tau_t \mathbb{C}\chi_{1-t}^2$ is a sum of $\mathcal{G}(\nu, \beta\mathbb{C}\chi_{1-t}^2)$ distributed random variables. Its distribution can be obtained but its expression is rather involved [18], [19]. Moreover, in our case, the scale parameters are random and then one needs a further marginalization, which appears a formidable task. Some insights can be gained for instance by considering the case $T = 1$. Indeed, one has

$$\Pr\left[\tau_t \mathbb{C}\chi_{1-t}^2 > \eta_{G|G}\right]$$
$$= \int_{\eta_{G|G}}^{\infty} \frac{2\beta^{-(\nu+1)/2}}{\Gamma(\nu)} x^{(\nu-1)/2} K_{1-\nu}(2\sqrt{x/\beta})dx$$
$$= \frac{2}{\Gamma(\nu)}\left(\frac{\eta_{G|G}}{\beta}\right)^{\nu/2} K_\nu\left(\sqrt{4\eta_{G|G}\beta^{-1}}\right) \qquad (16)$$

where we made use of [20, 6.592.12]. Since for $T = 1$, $P_{fa}(LLR_G(\boldsymbol{X})|G)_{T=1} = \exp\{-\eta_{G|G}\}$, one obtains a direct relation between the actual $P_{fa}(LLR_G(\boldsymbol{X})|K)_{T=1}$ and the designed P_{fa}. In the general case $T > 1$, one needs to resort to numerical evaluations but, as will be shown shortly, the ratio between $P_{fa}(LLR_G(\boldsymbol{X})|K)_{T=1}$ and $P_{fa}(LLR_G(\boldsymbol{X})|G)_{T=1}$ is approximately constant over T. As an illustration, Fig. 1 displays the actual P_{fa} of $LLR_G(\boldsymbol{X})$ under K-distributed noise. Clearly, this detector is not at all robust and a threshold cannot be set which guarantees approximately the same P_{fa} in K distributed noise or in Gaussian noise.

Let us turn now to what happens under H_1. Let us address first the situation where noise is K-distributed. In this case, since $\boldsymbol{R}^{-1/2}\boldsymbol{x}_t|\tau_t \sim \mathbb{CN}(\boldsymbol{R}^{-1/2}\boldsymbol{a}s_t, \tau_t\boldsymbol{I})$, one has

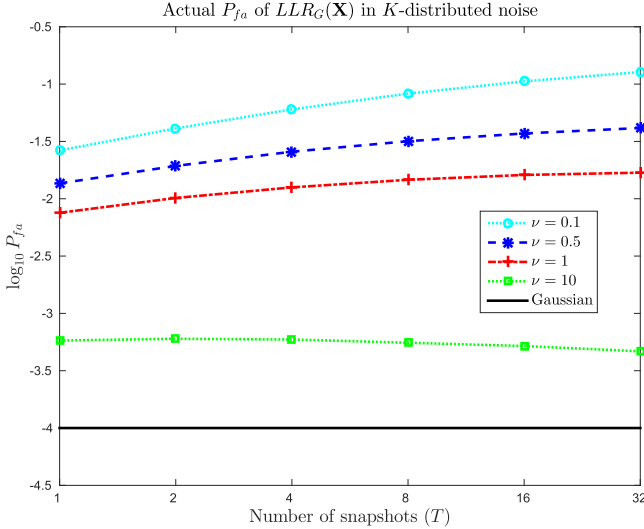|	K noise	Gaussian noise
$aLLR_K(\boldsymbol{X})$	$\sum_{t=1}^{T} \log\left[1 + \mathbb{C}\mathcal{F}_{1,M-1}\left(\tau_t^{-1}\lambda_t\right)\right]$	$\sum_{t=1}^{T} \log\left[1 + \mathbb{C}\mathcal{F}_{1,M-1}\left(\lambda_t\right)\right]$
$LLR_G(\boldsymbol{X})$	$\sum_{t=1}^{T} \tau_t \times \mathbb{C}\chi_1^2\left(\tau_t^{-1}\lambda_t\right)$	$\mathbb{C}\chi_T^2\left(\sum_{t=1}^{T}\lambda_t\right)$
$NMF(\boldsymbol{X})$	$\sum_{t=1}^{T} \mathbb{C}\mathcal{B}_{1,M-1}\left(\tau_t^{-1}\lambda_t\right)$	$\sum_{t=1}^{T} \mathbb{C}\mathcal{B}_{1,M-1}\left(\lambda_t\right)$

Fig. 1. Probability of false alarm of $LLR_G(\boldsymbol{X})$ in K distributed noise. Varying T and ν. $P_{fa} = 10^{-4}$ in Gaussian noise.

$$LLR_G(\boldsymbol{X})|K \overset{d}{=} \sum_{t=1}^{T} \tau_t \times \mathbb{C}\chi_1^2\left(\tau_t^{-1}\lambda_t\right) \qquad (17)$$

where $\lambda_t = |s_t|^2 \boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{a}$. Furthermore,

$$
\begin{aligned}
t(\boldsymbol{x}_t)|\tau_t &= \frac{\boldsymbol{x}_t^H \boldsymbol{R}^{-1/2} \boldsymbol{P}_{\boldsymbol{R}^{-1/2}\boldsymbol{a}} \boldsymbol{R}^{-1/2}\boldsymbol{x}_t}{\boldsymbol{x}_t^H \boldsymbol{R}^{-1}\boldsymbol{x}_t} \\
&\overset{d}{=} \frac{\mathbb{C}\chi_1^2\left(\tau_t^{-1}\lambda_t\right)}{\mathbb{C}\chi_1^2\left(\tau_t^{-1}\lambda_t\right) + \mathbb{C}\chi_{M-1}^2}
\end{aligned}
\qquad (18)
$$

from which we infer that

$$aLLR_K(\boldsymbol{X})|K \overset{d}{=} \sum_{t=1}^{T} \log\left[1 + \mathbb{C}\mathcal{F}_{1,M-1}\left(\tau_t^{-1}\lambda_t\right)\right] \qquad (19)$$

$$aLLR_K(\boldsymbol{x}_{\tau_{\min}})|K \overset{d}{=} \log\left[1 + \mathbb{C}\mathcal{F}_{1,M-1}\left(\tau_{\min}^{-1}\lambda_{t_{\min}}\right)\right] \quad (20)$$

$$NMF(\boldsymbol{X})|K \overset{d}{=} \sum_{t=1}^{T} \mathbb{C}\mathcal{B}_{1,M-1}\left(\tau_t^{-1}\lambda_t\right) \qquad (21)$$

where $\mathbb{C}\mathcal{F}_{p,q}(\lambda) \overset{d}{=} \mathbb{C}\chi_p^2(\lambda)/\mathbb{C}\chi_q^2$ and $\mathbb{C}\mathcal{B}_{p,q}(\lambda) \overset{d}{=} \mathbb{C}\chi_p^2(\lambda)/[\mathbb{C}\chi_p^2(\lambda) + \mathbb{C}\chi_q^2]$. These distributions are recapitulated in Table I. The case of Gaussian noise is obtained by simply setting $\tau_t = 1$ in the above equations. Note that any compound-Gaussian distribution can be handled by an appropriate choice of the distribution of τ_t, and hence these expressions are

not restricted to K-distributed noise. These novel stochastic representations have a double interest. First, even though they do not provide analytical formulas for the probability of detection, they allow for fast evaluation of P_d from well-known scalar distributions. Moreover, they provide insights onto the influence of τ_t. Indeed, for the scaled invariant detectors $aLLR_K(\boldsymbol{X})$ and $NMF(\boldsymbol{X})$, τ_t appears only in the non-centrality parameter of the corresponding distribution. Therefore, the smaller τ_t the larger this non-centrality parameter and hence the greater its influence. This explains why, for very heavy-tailed distributions (for very small ν), for which $\min \tau_t$ might be very small, the latter has a strong influence on the detection performance. On the other hand, for $LLR_G(\boldsymbol{X})$, τ_t appears both in the non-centrality parameter (through τ_t^{-1}) but also in the "weighting" of the distribution through τ_t: hence, there is a sort of balance, which means that the snapshot with minimal τ_t is not necessarily the most influential. In fact, we might expect that snapshots with large values of τ_t will have a strong impact.

III. PERFORMANCE IN A MIXTURE OF K DISTRIBUTED AND GAUSSIAN NOISE

We now evaluate the performance of the detectors when applied to a mixture of K-distributed and Gaussian noise. We consider the case of $M = 16$ pulses and a moving target with Doppler frequency $f_d = 0.09$ so that $\boldsymbol{a} = [1 e^{i2\pi f_d} \ldots e^{i2\pi(M-1)f_d}]^T$. The power of the K-distributed clutter is assumed to be one and hence $\beta = \nu^{-1}$. The clutter covariance matrix is $\boldsymbol{R}_c(k,\ell) = 0.99^{|k-\ell|}$ and the clutter to white noise ratio is defined as $CWNR = \mathrm{Tr}\{\boldsymbol{R}_c\}/\mathrm{Tr}\{\sigma_w^2\boldsymbol{I}\}$. Since the probability of false alarm of $LLR_G(\boldsymbol{X})$ cannot be controlled, while a single threshold can be set for the scale invariant detectors $aLLR_K(\boldsymbol{X})$, $aLLR_K(\boldsymbol{x}_{\tau_{\min}})$, $NMF(\boldsymbol{X})$ to ensure the same P_{fa} under either K distributed noise or Gaussian noise, we consider only the three latter detectors from now on. For each of them, the threshold is set to obtain a P_{fa} under K distributed noise only or Gaussian noise only equal to $P_{fa} = 10^{-4}$. In Fig. 2, we display the actual P_{fa} of all detectors (obtained from $1000/P_{fa}$ simulations) as a function of $CWNR$.

This figure is worthy of some observations. First, it should be noted that the actual probability of false alarm in K+ Gaussian noise is less than the specified P_{fa} designed under the assumption of K distributed noise only or Gaussian noise only. It seems that the two extreme cases are the worst cases and constitute an upper bound. For $aLLR_K(\boldsymbol{X})$ and $NMF(\boldsymbol{X})$ the probability of false alarm first decreases, then can be 10 times lower than the P_{fa} for K only or Gaussian only noise, and then re-increases to meet the designed P_{fa}. For $aLLR_K(\boldsymbol{x}_{\tau_{\min}})$, the actual P_{fa} can
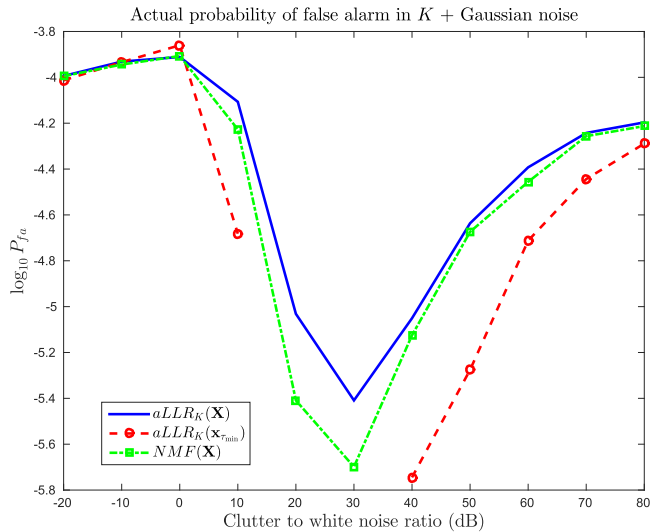
Fig. 2. Mixture of K-distributed clutter and Gaussian white noise. Probability of false alarm of $aLLR_K(\boldsymbol{X})$, $aLLR_K(\boldsymbol{x}_{\tau_{\min}})$ and $NMF(\boldsymbol{X})$ versus $CWNR$. Designed P_{fa} under K distributed noise only or Gaussian noise only is 10^{-4}.

go to zero, at least among the $1000/P_{fa}$ simulations, none of them exceeded the threshold.

We now study the probability of detection in Fig. 3. 10^4 simulations were run to estimate P_d. The fluctuating amplitude s_t was generated from i.i.d. Gaussian variables with power P and the signal to clutter and noise ratio is defined as $S/(C + N) = P(\boldsymbol{a}^H \boldsymbol{R}^{-1} \boldsymbol{a})$. As before, the thresholds of each detector are set to ensure a probability of false alarm under K distributed noise only or Gaussian noise only equal to $P_{fa} = 10^{-4}$. Therefore, as illustrated in Fig. 2 the detectors do not operate at the same P_{fa}. However, the P_d shown in Fig. 3 would be that obtained in practice where a threshold cannot be set based on the correct distribution assumption. This figure shows that $aLLR_K(\boldsymbol{X})$ and $NMF(\boldsymbol{X})$ perform nearly the same, with the former slightly better. The impact of including thermal noise is visible, especially on $aLLR_K(\boldsymbol{x}_{\tau_{\min}})$: the latter departs from $aLLR_K(\boldsymbol{X})$ when $CWNR$ is about 20 to 40 dB. This indicates that when thermal noise is present, the snapshot with minimal τ_t does no longer prevail over all other snapshots.

IV. CONCLUSIONS

In this letter, we pursued further our analysis of the detectors derived in [15] under the assumption of K distributed noise only. First, we analyzed their performance in K distributed noise only or Gaussian noise only. New stochastic representations for the test statistics under H_1 were obtained, which provided insights into the influence of the values of the textures τ_t. Then, we assessed the detectors under a mixture of K and Gaussian noise. It was shown that the probability of false alarm obtained with the mixture was inferior to the probability of false alarm under K only or Gaussian only noise. As for the probability of detection, it was shown that thermal noise has an impact even for moderate clutter to white noise ratio, and hence improvement of the detection schemes is worthy of further investigation. Furthermore, it was shown that, in the presence of thermal noise, the hypothetical detector that operates on the clairvoyantly selected snapshot with the minimal τ_t becomes inferior to the detector that uses all snapshots, after the number
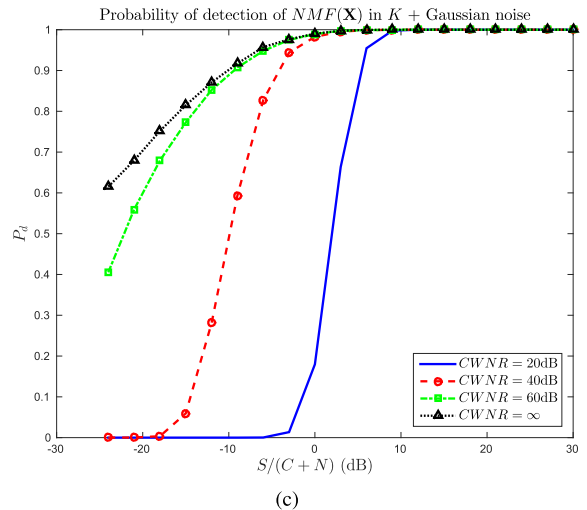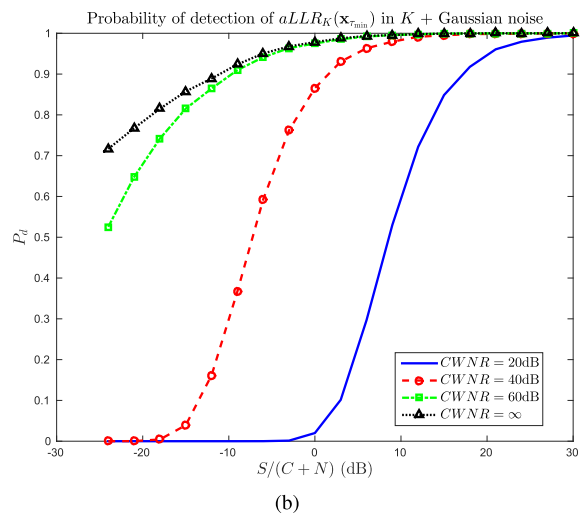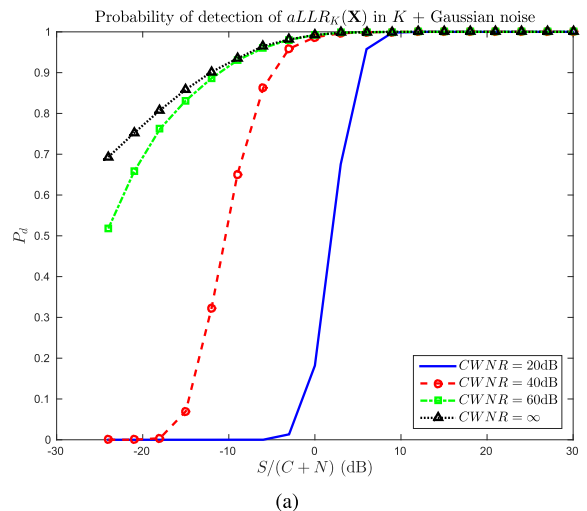

Fig. 3. Probability of detection of $aLLR_K(\boldsymbol{X})$, $aLLR_K(\boldsymbol{x}_{\tau_{\min}})$ and $NMF(\boldsymbol{X})$ versus SNR. $T = 8$, $\nu = 0.2$ and designed P_{fa} under K distributed noise only or Gaussian noise only is 10^{-4}.

T of independent observations grows above a certain value specified by the CWNR. It is expected that in this case, in order to be as efficient as the detector which uses all snapshots, one could resort to a detector that would somehow select only samples with τ_t values below WGN power.

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