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Fluctuating Target Detection in Fluctuating K-Distributed Clutter

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***Abstract*—This letter deals with the problem of fluctuating target detection in heavy-tailed K-distributed clutter over a number T of independent coherent intervals, e.g., in the case of a long observation interval ("stare mode"), or that of independent (range) resolution cells as per the track before detect techniques. The generalized likelihood ratio test for the problem at hand is derived, as well as an approximation of it, whose distribution under the null hypothesis is derived. We also show some significant differences as compared to the usual Gaussian case, in particular the influence of T and of the shape parameter of the K distribution.**

***Index Terms*—Adaptive detection, generalized likelihood ratio test, K distributed noise.**

I. Introduction and Problem Statement

IN THE last few decades, spurred by analysis of experimental radar data [1], [2], [3], [4], [5], considerable attention has been focused on target detection in heavy-tailed clutter environment, see e.g., [6], [7], [8], [9] and references therein. Most studies deal with detection of non-fluctuating over coherent integration time (CIT) target return in a clutter that can be presented as a compound-Gaussian process [10], [11] or, more generally, is assumed to follow an elliptically contoured distribution [12]. Traditionally the detection problem is formulated from a single resolution cell. Herein, we consider long observation interval ("stare mode") that significantly exceeds the target and clutter fluctuations spectra Nyquist rates, which can be recast as the following hypotheses testing problem ($t = 1, \cdots, T$) [9]:

$$\begin{aligned} H_0 &: \boldsymbol{x}_t = \sqrt{\tau_t}\boldsymbol{n}_t \\ H_1 &: \boldsymbol{x}_t = \boldsymbol{a}s_t + \sqrt{\tau_t}\boldsymbol{n}_t \end{aligned} \tag{1}$$

where $\boldsymbol{n}_t \sim \mathbb{C}\mathcal{N}(\boldsymbol{0}, \boldsymbol{R})$ follows a complex Gaussian distribution with zero mean and covariance matrix $\boldsymbol{R}$, and the texture τ_t is a positive random variable whose distribution will be specified shortly. The problem in (1) is also relevant in the so-called track before detect framework where the decision on target presence has to be made based on potential target presence in T independent range resolution cells. For Gaussian target and clutter models, (1) leads to the classical non-coherent integration of the coherent processing outputs, and the main focus of this study is to investigate the type of non-coherent inter-CIT processing for heavy-tailed distributions. Note, in this regard, that for the traditional single CIT problem, it was the data-dependent threshold arrangement [13] that made all the difference with respect to the Gaussian case. For multiple independent and identically distributed (i.i.d.) CIT, one can expect similarly changed nature of inter-CIT non coherent processing. Additionally, we wish to examine how the number of CIT T and the shape of the texture distribution impact the probability of detection. As such, the present letter is a follow-up of our recent submission [14], where we considered maximum likelihood (ML) direction of arrival estimation in K-distributed noise. It was shown there that the rate of convergence of the ML estimates is significantly faster than in the Gaussian case and increases as the shape parameter ν of the K distribution decreases. The aim of this letter is to investigate whether a similar behavior occurs in detection.

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II. Detection

As said before we consider the problem in (1) where we assume that τ_t follows a Gamma distribution with shape parameter ν and scale parameter β, i.e., its probability density function (p.d.f.) is given by

$$p(\tau_t) = \frac{\beta^{-\nu}}{\Gamma(\nu)}\tau_t^{\nu-1}e^{-\beta^{-1}\tau_t} \tag{2}$$

which we denote as $\tau_t \sim \mathcal{G}(\nu, \beta)$. The signal unknown random amplitudes s_t that fluctuates from CIT to CIT are considered as deterministic and unknown. The noise component $\sqrt{\tau_t}\boldsymbol{n}_t$ is known to follow a K distribution, and the probability density function of the data matrix $\boldsymbol{X} = [\,\boldsymbol{x}_1 \quad \boldsymbol{x}_2 \quad \ldots \quad \boldsymbol{x}_T\,]$ is given by

$$p_0(\boldsymbol{X}|\boldsymbol{R}) \propto |\boldsymbol{R}|^{-T}\prod_{t=1}^{T} g\left(\boldsymbol{x}_t^H\boldsymbol{R}^{-1}\boldsymbol{x}_t\right) \tag{3a}$$

$$p_1(\boldsymbol{X}|\boldsymbol{s}, \boldsymbol{R}) \propto |\boldsymbol{R}|^{-T}\prod_{t=1}^{T} g\left(\boldsymbol{z}_t^H\boldsymbol{R}^{-1}\boldsymbol{z}_t\right) \tag{3b}$$

where $g(z) = z^{\frac{\nu-M}{2}}K_{M-\nu}(2\sqrt{z/\beta})$ and $\boldsymbol{z}_t = \boldsymbol{x}_t - \boldsymbol{a}s_t$. We assume herein that $\boldsymbol{R}$ is known, so that the generalized likelihood ratio (GLR) for the problem at hand is given by

$$GLR(\boldsymbol{X}) = \frac{\max_{\boldsymbol{s}} p_1(\boldsymbol{X}|\boldsymbol{s}, \boldsymbol{R})}{p_0(\boldsymbol{X}|\boldsymbol{R})}. \tag{4}$$

Since $g(z)$ is monotonically decreasing, and observing that

$$[\boldsymbol{x}_t - \boldsymbol{a}s_t]^H \boldsymbol{R}^{-1} [\boldsymbol{x}_t - \boldsymbol{a}s_t] = \left[\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{a}\right] \left| s_t - \frac{\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{x}_t}{\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{a}} \right|^2 + \boldsymbol{x}_t^H \boldsymbol{R}^{-1}\boldsymbol{x}_t - \frac{\left|\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{x}_t\right|^2}{\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{a}} \quad (5)$$

it follows that $p_1(\boldsymbol{X}|\boldsymbol{s}, \boldsymbol{R})$ is maximized for

$$s_t = \frac{\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{x}_t}{\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{a}}. \quad (6)$$

Consequently, the GLR can be rewritten as

$$GLR(\boldsymbol{X}) = \frac{\prod_{t=1}^{T} g\left(\boldsymbol{x}_t^H \boldsymbol{R}^{-1}\boldsymbol{x}_t - \frac{\left|\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{x}_t\right|^2}{\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{a}}\right)}{\prod_{t=1}^{T} g\left(\boldsymbol{x}_t^H \boldsymbol{R}^{-1}\boldsymbol{x}_t\right)}. \quad (7)$$

In a view to simplify the detector, let us make use of an approximation of the modified Bessel function, which holds for large $(M - \nu)$ [15], and write

$$\begin{aligned} g(z) &= z^{\frac{\nu-M}{2}} K_{M-\nu}(2\sqrt{z/\beta}) \\ &\simeq z^{\frac{\nu-M}{2}} \times \sqrt{\frac{\pi}{2(M-\nu)}} \left(\frac{e\sqrt{z/\beta}}{(M-\nu)}\right)^{-(M-\nu)} \\ &= \text{const.} \times z^{\nu-M}. \end{aligned} \quad (8)$$

Our experience is that this approximation yields almost no loss compared to using the Bessel function [14]. The previous equation gives rise to an approximate GLR (aGLR)

$$\begin{aligned} aGLR(\boldsymbol{X}) &= \prod_{t=1}^{T} \left[\frac{\boldsymbol{x}_t^H \boldsymbol{R}^{-1}\boldsymbol{x}_t}{\boldsymbol{x}_t^H \boldsymbol{R}^{-1}\boldsymbol{x}_t - \frac{\left|\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{x}_t\right|^2}{\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{a}}}\right]^{M-\nu} \\ &= \prod_{t=1}^{T} \left[\frac{\boldsymbol{x}_t^H \boldsymbol{R}^{-1}\boldsymbol{x}_t}{\boldsymbol{x}_t^H \boldsymbol{R}^{-1/2} \boldsymbol{P}^{\perp}_{\boldsymbol{R}^{-1/2}\boldsymbol{a}} \boldsymbol{R}^{-1/2}\boldsymbol{x}_t}\right]^{M-\nu} \end{aligned} \quad (9)$$

where $\boldsymbol{P}^{\perp}_{\boldsymbol{R}^{-1/2}\boldsymbol{a}}$ is the projection onto the orthogonal complement of $\boldsymbol{R}^{-1/2}\boldsymbol{a}$. Taking the logarithm of (9) and scaling by $1/(M-\nu)$, we end up with an approximate log-likelihood ratio (LLR) given by

$$\begin{aligned} aLLR(\boldsymbol{X}) &= -\sum_{t=1}^{T} \log\left[1 - \frac{\left|\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{x}_t\right|^2}{\left(\boldsymbol{x}_t^H \boldsymbol{R}^{-1}\boldsymbol{x}_t\right)\left(\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{a}\right)}\right] \\ &= -\sum_{t=1}^{T} \log\left[\frac{\boldsymbol{x}_t^H \boldsymbol{R}^{-1/2} \boldsymbol{P}^{\perp}_{\boldsymbol{R}^{-1/2}\boldsymbol{a}} \boldsymbol{R}^{-1/2}\boldsymbol{x}_t}{\boldsymbol{x}_t^H \boldsymbol{R}^{-1}\boldsymbol{x}_t}\right]. \end{aligned} \quad (10)$$

For comparison, the LLR for Gaussian distributed noise is given by

$$aLLR_G(\boldsymbol{X}) = \sum_{t=1}^{T} \frac{\left|\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{x}_t\right|^2}{\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{a}}. \quad (11)$$

Let us investigate the properties of the test statistic under H_0 first. Since $\boldsymbol{R}^{-1/2}\boldsymbol{x}_t = \sqrt{\tau_t}\boldsymbol{w}_t$, where $\boldsymbol{w}_t \sim \mathbb{C}\mathcal{N}(\boldsymbol{0}, \boldsymbol{I}_M)$, one has

$$\begin{aligned} t(\boldsymbol{x}_t) &= \frac{\left|\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{x}_t\right|^2}{\left(\boldsymbol{x}_t^H \boldsymbol{R}^{-1}\boldsymbol{x}_t\right)\left(\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{a}\right)} \\ &= \frac{\boldsymbol{w}_t^H \boldsymbol{P}_{\boldsymbol{R}^{-1/2}\boldsymbol{a}} \boldsymbol{w}_t}{\boldsymbol{w}_t^H \boldsymbol{w}_t} \overset{H_0}{\sim} \frac{\mathbb{C}\chi_1^2}{\mathbb{C}\chi_1^2 + \mathbb{C}\chi_{M-1}^2}. \end{aligned} \quad (12)$$

It follows that the p.d.f. of $t(\boldsymbol{x}_t)$ does not depend on the actual value of τ_t, and hence all snapshots are treated equally. Furthermore, $1 - t(\boldsymbol{x}_t) \overset{H_0}{\sim} \frac{\mathbb{C}\chi_{M-1}^2}{\mathbb{C}\chi_1^2 + \mathbb{C}\chi_{M-1}^2}$ which implies that

$$-\log\left[(1 - t(\boldsymbol{x}_t)\right] \overset{H_0}{\sim} \mathcal{G}\left(1, (M-1)^{-1}\right). \quad (13)$$

Consequently, the p.d.f. of the approximate LLR under H_0 is given by

$$aLLR(\boldsymbol{X}) = -\sum_{t=1}^{T} \log\left[(1 - t(\boldsymbol{x}_t)\right] \overset{H_0}{\sim} \mathcal{G}\left(T, (M-1)^{-1}\right). \quad (14)$$

The probability of false alarm, for a given threshold η, is thus

$$\begin{aligned} P_{fa} &= \int_{\eta}^{\infty} \frac{(M-1)^T}{\Gamma(T)} u^{T-1} \exp\left\{-(M-1)u\right\} du \\ &= 1 - \gamma((M-1)\eta, T) \end{aligned} \quad (15)$$

where $\gamma(a, z) = \int_0^z t^{a-1} e^{-t} dt$ is the incomplete Gamma function [15]. Observe that the distribution of $aLLR(\boldsymbol{X})$ is *independent of* ν under H_0, and is thus the same as if the data were Gaussian. However, in the latter case, the Gaussian likelihood ratio, as given by (11), follows a $\mathbb{C}\chi_T^2$ distribution, which is different from (14).

Let us now investigate what happens under H_1. We now have $\boldsymbol{R}^{-1/2}\boldsymbol{x}_t = \boldsymbol{R}^{-1/2}\boldsymbol{a}s_t + \sqrt{\tau_t}\boldsymbol{w}_t$, so that

$$\begin{aligned} 1 - t(\boldsymbol{x}_t) &= \frac{\boldsymbol{x}_t^H \boldsymbol{R}^{-1/2} \boldsymbol{P}^{\perp}_{\boldsymbol{R}^{-1/2}\boldsymbol{a}} \boldsymbol{R}^{-1/2}\boldsymbol{x}_t}{\boldsymbol{x}_t^H \boldsymbol{R}^{-1}\boldsymbol{x}_t} \\ &= \frac{\tau_t \boldsymbol{w}_t^H \boldsymbol{P}^{\perp}_{\boldsymbol{R}^{-1/2}\boldsymbol{a}} \boldsymbol{w}_t}{\left\|\boldsymbol{R}^{-1/2}\boldsymbol{a}s_t + \sqrt{\tau_t}\boldsymbol{w}_t\right\|^2}. \end{aligned} \quad (16)$$

As illustrated in [14], the logarithm operation in (10) will tend to emphasize the snapshots for which τ_t is very small. Indeed, for $\tau_t \ll 1$,

$$1 - t(\boldsymbol{x}_t) \simeq \frac{\tau_t \boldsymbol{w}_t^H \boldsymbol{P}^{\perp}_{\boldsymbol{R}^{-1/2}\boldsymbol{a}} \boldsymbol{w}_t}{|s_t|^2 \left(\boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{a}\right)} \quad (17)$$

while, for $\tau_t \boldsymbol{w}_t^H \boldsymbol{w}_t \gg |s_t|^2 \boldsymbol{a}^H \boldsymbol{R}^{-1}\boldsymbol{a}$,

$$1 - t(\boldsymbol{x}_t) \simeq \frac{\boldsymbol{w}_t^H \boldsymbol{P}^{\perp}_{\boldsymbol{R}^{-1/2}\boldsymbol{a}} \boldsymbol{w}_t}{\boldsymbol{w}_t^H \boldsymbol{w}_t} \quad (18)$$

whose average value is $(M-1)/M$ an τ_t and thus close to 1. Moreover, there is a very high probability that some of the τ_t take very small values [14]. Hence, one can surmise that, for very heavy-tailed noise, i.e., for very small ν, the approximate log likelihood ratio will be dominated by the term involving the snapshot with minimal T_t. In order to confirm this conjecture, let $\boldsymbol{x}_{t_{\min}}$ be this snapshot and let us define $u_T = \min_{1 \leq t \leq T} \tau_t$. Let us now consider an hypothetical detector that would use $\boldsymbol{x}_{t_{\min}}$ only, i.e.,

$$\begin{aligned} aLLR(\boldsymbol{x}_{t_{\min}}) &= -\log\left[1 - t(\boldsymbol{x}_{t_{\min}})\right] \\ &\simeq -\log\left[u_T \frac{\boldsymbol{w}_{t_{\min}}^H \boldsymbol{P}_{\boldsymbol{R}^{-1/2}\boldsymbol{a}}^{\perp} \boldsymbol{w}_{t_{\min}}}{|s_{t_{\min}}|^2 \left(\boldsymbol{a}^H \boldsymbol{R}^{-1/2} \boldsymbol{a}\right)}\right]. \quad (19) \end{aligned}$$

Observe that, under H_0, $aLLR(\boldsymbol{x}_{t_{\min}}) \overset{H_0}{\sim} \mathcal{G}(1, (M-1)^{-1})$ and, hence, the threshold $\eta_{\min}$ of this detector is related to P_{fa} as

$$P_{fa} = 1 - \gamma((M-1)\eta_{\min}, 1). \quad (20)$$

The test statistic $aLLR(\boldsymbol{x}_{t_{\min}})$ depends mostly on u_T which is the minimum value of a set of T independent Gamma distributed random variables. In order to analyze its behavior, one must consider statistics of extreme values [16], for which only asymptotic (as $T \to \infty$) results are available. More precisely, we showed in [14] that, for small ν and large T,

$$\Pr\left[T^{1/\nu} u_T \geq x\right] \simeq \exp\left\{-\frac{\beta^{-\nu} x^{\nu}}{\nu \Gamma(\nu)}\right\}. \quad (21)$$

Let $v = \frac{\boldsymbol{w}_{t_{\min}}^H \boldsymbol{P}_{\boldsymbol{R}^{-1/2}\boldsymbol{a}}^{\perp} \boldsymbol{w}_{t_{\min}}}{|s_{t_{\min}}|^2 (\boldsymbol{a}^H \boldsymbol{R}^{-1/2} \boldsymbol{a})} \overset{d}{=} \frac{\mathbb{C}\chi^2_{M-1}}{|s_{t_{\min}}|^2 (\boldsymbol{a}^H \boldsymbol{R}^{-1/2} \boldsymbol{a})}$. Then

$$\begin{aligned} &\Pr\left[aLLR(\boldsymbol{x}_{t_{\min}}) \geq \eta | v\right] = \Pr\left[-\log(u_T v) \geq \eta | v\right] \\ &= \Pr\left[u_T \leq v^{-1} e^{-\eta} | v\right] = \Pr\left[T^{1/\nu} u_T \leq v^{-1} e^{-\eta} T^{1/\nu} | v\right] \\ &\simeq 1 - \exp\left\{-\frac{\left[\beta v e^{\eta}\right]^{-\nu}}{\nu \Gamma(\nu)} T\right\}. \quad (22) \end{aligned}$$

In order to obtain the probability of detection, one must integrate over the p.d.f. of v. Although this marginalisation appears infeasible, (22) provides very interesting insights into the speed of convergence of the probability of detection as a function of T. Indeed, and this confirms what was observed for direction of arrival estimation in K-distributed noise, the probability of detection of a signal in K-distributed noise grows much faster with T than in the Gaussian case. Moreover, since the detector in (10) based on all snapshots is expected to perform better than the detector in (19) which uses only $\boldsymbol{x}_{t_{\min}}$, its probability of detection should be at least as fastly growing. This conjecture will be illustrated in the next section.

III. Numerical Simulations

We assume a set of $M = 16$ pulse repetition intervals and a moving target with Doppler frequency $f_d = 0.0868$ so that $\boldsymbol{a} = [1 \quad e^{i2\pi f_d} \quad \dots \quad e^{i2\pi(M-1)f_d}]^T$. The K-distributed clutter is assumed to have unit power, so that $\beta = \nu^{-1}$. The fluctuating amplitude s_t was generated from i.i.d. Gaussian variables with power P and the signal to noise ratio (SNR) is defined as $SNR = P(\boldsymbol{a}^H \boldsymbol{R}^{-1} \boldsymbol{a})$ where $\boldsymbol{R}$ is assumed to be known. The probability of false alarm is set to $P_{fa} = 10^{-5}$. Figs. 1 and 2 display the probability of detection P_d of the approximate LLR as given by (10) and the approximate LLR (19) based on the snapshot with minimum τ_t. These figures confirm the very

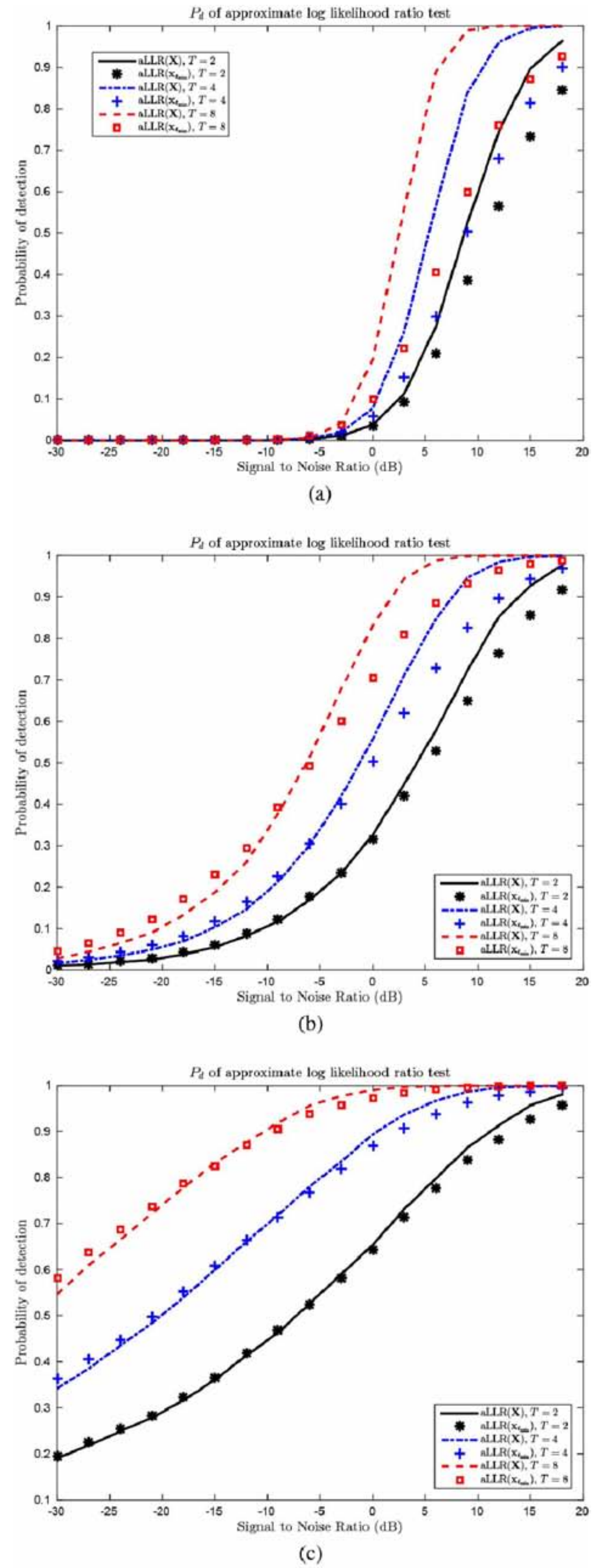

Fig. 1. Probability of detection of approximate log likelihood ratio test versus SNR. $M = 16$. (a) $\nu = 2$ (b) $\nu = 0.5$ (c) $\nu = 0.2$.

strong influence of the shape parameter ν on P_d. Indeed, for a fixed number of snapshots T, the probability of detection rises

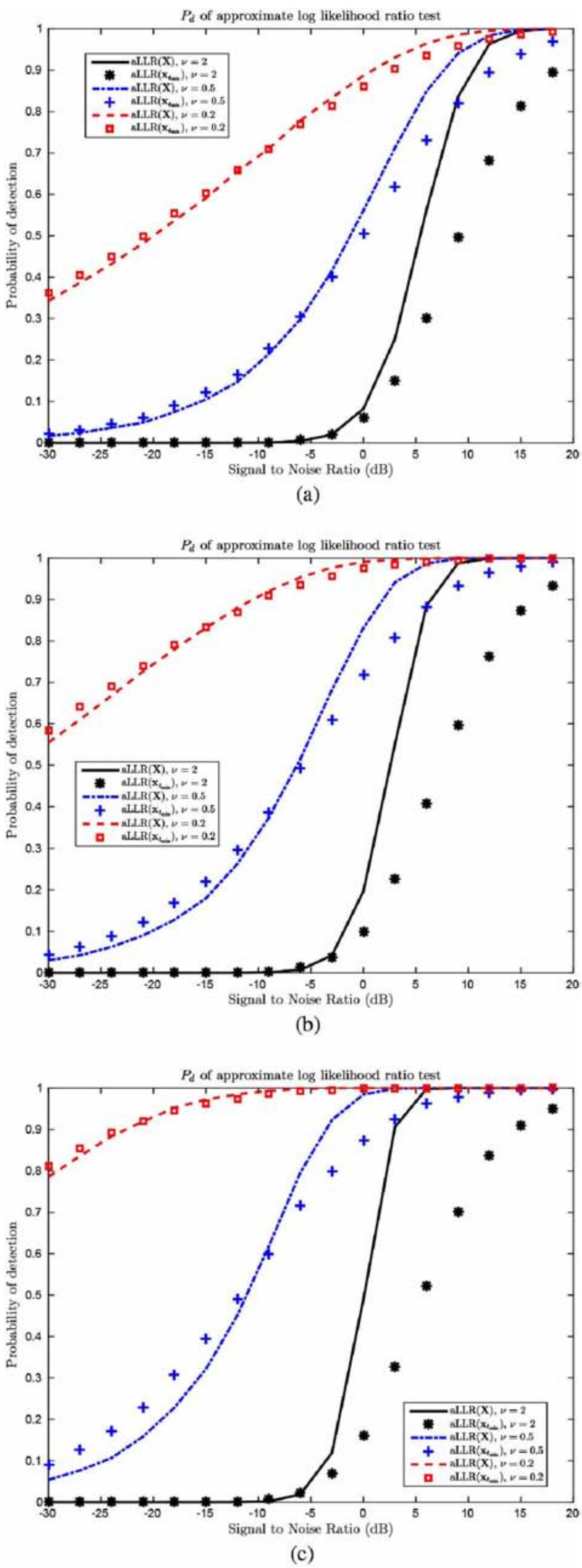

Fig. 2. Probability of detection of approximate log likelihood ratio test versus SNR. $M = 16$. (a) $T = 4$ (b) $T = 8$ (c) $T = 16$.

very fastly as ν decreases, see Fig. 2. Accordingly, the influence of T is more pronounced when ν decreases, see Fig. 1.

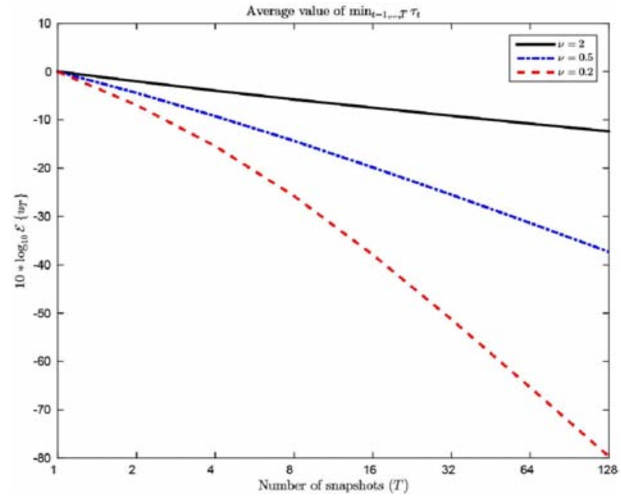

Fig. 3. Average value of $\min_{t=1,\dots,T} \tau_t$ versus T and ν.

Moreover, one can observe that for small ν, $aLLLR(\boldsymbol{X})$ and $aLLR(\boldsymbol{x}_{t_{\min}})$ perform nearly the same, which confirms the fact that, for heavy-tailed distributions, the detector is mostly influenced by the snapshot with minimum τ_t, i.e., with maximum signal to clutter ratio.

IV. Discussion

In this letter we addressed the problem of detecting a fluctuating target over multiple CIT, in the presence of K-distributed noise, whose covariance matrix is known. An approximate likelihood ratio test was derived, whose distribution under the null hypothesis was shown to be independent of the shape parameter ν of the Gamma texture. This yielded a simple and closed-form expression of the detector threshold as a function of the probability of false alarm. Qualitative insights were provided to gain comprehension of the probability of detection P_d. We showed that, for very small ν, the detector is mostly influenced by the snapshot corresponding to the minimum value of the texture. Furthermore, the detector exhibits a rate of convergence of P_d much faster than in the Gaussian case.

In this letter, and similarly to many studies, we considered the non-Gaussian clutter only and somehow ignored the additive thermal Gaussian noise. This assumption is usually justified by assumption of a limited clutter mitigation efficiency of the optimum filter, i.e., the clutter to white noise ratio at the output of this filter is still large. On the other hand, as indicated above, the detection performance is mainly driven by the snapshot with minimum τ_t. However, since we assume a constant input clutter total power ($\varepsilon\{\tau_t\} = 1$) for any ν, the clutter power in $\boldsymbol{x}_{t_{\min}}$ may be very small, as illustrated in Fig. 3. This figure provides an insight on when this commonly made assumption on ignored additive white noise remains valid for different T and ν. When this is no longer the case, one needs to consider both non-Gaussian clutter and white Gaussian noise, a problem which is rather tedious [17] as the p.d.f. of the total noise is not really tractable. This is a further line of research that needs to be investigated.

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