Upper and Lower Bounding Procedures for the Optimal Management of Water Pumping and Desalination Processes

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Plan

Introduction

Context

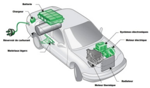
Energy considerations are becoming paramount in the resolution of real-world applications.

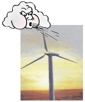
Objective

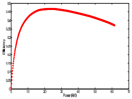
Address the (combinatorial) optimization challenge of integrating energy constraints in deterministic (scheduling) models with constraints related to their physical, technological and performance characteristics.

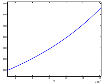
Challenges

Non-linearities come from energy efficiency functions









Introduction

PGMO project OREM (Combinatorial optimization with multiple resources and energy constraints)

- Previous studies involve multiple energy sources and general non-linear efficiency functions, but no scheduling.
- All our previous work on scheduling under energy constraint considered linear (and even identical) energy efficiency functions, which oversimplifies the problem.
- We want to solve explicitely and in an integrated fashion energy resource allocation problems and energy-consuming activity scheduling problems with non linear energy efficiency functions.
- This work (phase 1) proposes a proof of concept by studying a single-source scheduling problem involving realistic non-linear efficiency functions provided by the researchers in Electrical Engineering from LAPLACE.

System

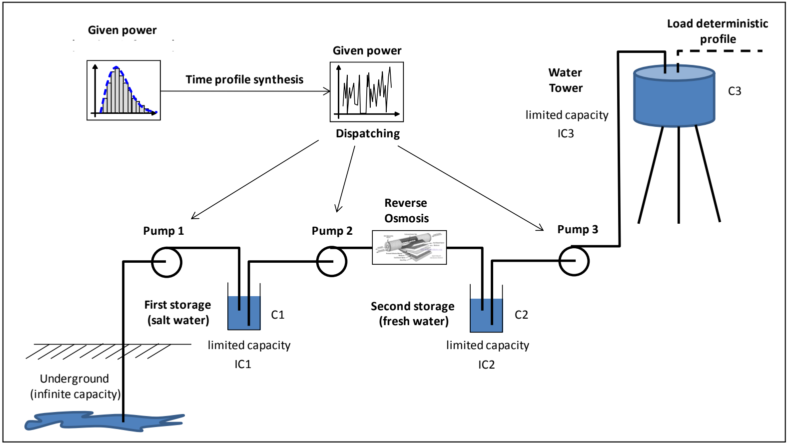


Figure : Source : (Sareni et al., 2012)

Mechanic-hydraulic-electric models

Electrical model

- V_m, I_m : electrical tension, courant
- T_m : motor electromag. torque
- Ω : rotation speed
- k_Φ : torque equivalent coefficient
- r : stator resistance

Electric motor equations (inertia neglected) :

$$V_m = rI_m + k_\Phi \Omega \quad (1)$$

$$T_m = \Phi_m I_m \quad (2)$$

Electrical power needed : $P_e = V_m I_m$.

Mechanical-Hydraulic conv.

- P_p : output pressure
- q : debit of water
- a, b : non linear girator coefs
- c : hydraulic friction
- p_0 : suction pressure
- $f_p + f_m$: mechanical losses

Static equations of the motor-pump (mechanical inertia neglected) :

$$P_p = (a\Omega + bq)\Omega - (cq^2 + p_0) \quad (3)$$

$$T_m = (a\Omega + bq)q + (f_m + f_p)\Omega \quad (4)$$

Pressure drop in the pipe

- $\Delta\mathrm{Pipe}$: pressure drop
- h : height of water pumping
- ρ : water density

Static+Dynamic pressure

$$\Delta\mathrm{Pipe} = kq^2 + \rho g h \quad (5)$$

Reverse Osmosis module model

Reverse Osmosis module

- $R\mathrm{Mod}$: losses in the pipes of the RO module
- $R\mathrm{Valve}$: variable restriction
- $R\mathrm{Mem}$: losses in the RO membrane
- q_c : debit of rejected water (concentrate)
- q_p : debit of fresh water (permeate)

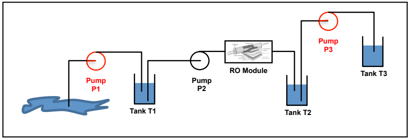
Quasi static model of the RO module
(storage effect neglected) :

$$P_p - \Delta\mathrm{Pipe} = (R_{\mathrm{Mod}} + R_{\mathrm{Valve}}){q_c}^2 \quad (8)$$

$$q_p = \frac{P_p - \Delta\mathrm{Pipe}}{R_{\mathrm{Me}}} \quad (9)$$

$$q = q_p + q_c \quad (10)$$

Efficiency functions of pumps 1 & 3



The electric power required is expressed in function of the water level of the intake tank **h** and the water debit **q**.

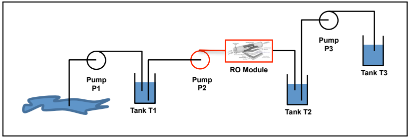
power required =

$$r * \mathcal{K}(\mathbf{h},\mathbf{q}) + ((f_m + f_p) * \Omega(\mathbf{h},\mathbf{q}) + \mathbf{q} * (a * \Omega(\mathbf{h},\mathbf{q}) + (b * \mathbf{q}))) * \Omega(\mathbf{h},\mathbf{q})$$

where

$$\begin{cases} \Omega(\mathbf{h},\mathbf{q}) = \frac{-(b*\mathbf{q}) + \sqrt{(b*\mathbf{q})^2 - 4*a*(-(p_0 + \rho_g*(\mathbf{h} - l_{\mathrm{out}}) + (k+c)*\mathbf{q}^2))}}{2*a} \\ \mathcal{K}(\mathbf{h},\mathbf{q}) = (((f_m + f_p) * \Omega(\mathbf{h},\mathbf{q}) + \mathbf{q} * (a * \Omega(\mathbf{h},\mathbf{q}) + (b * \mathbf{q})))/k_\phi)^2 \end{cases}$$

Efficiency function of pump 2 + RO



The subsystem resulting from the combinaison of pump 2 and the Reverse Osmosis module is modeled with equation :

power required =

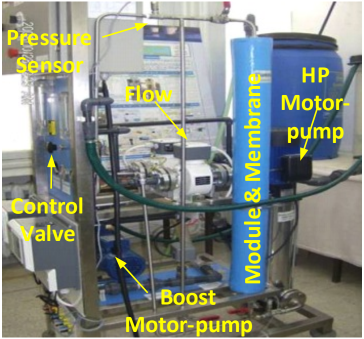
$$r * \mathcal{K}(\mathbf{q_c}, \mathbf{h}) + ((f_m + f_p) * \Omega(\mathbf{q_c}, \mathbf{h}) + (\mathbf{q_c} + \mathcal{F}(\mathbf{q_c})/R_{\mathrm{Me}}) * \mathcal{M}(\mathbf{q_c}, \mathbf{h})) * \Omega(\mathbf{q_c}, \mathbf{h})$$

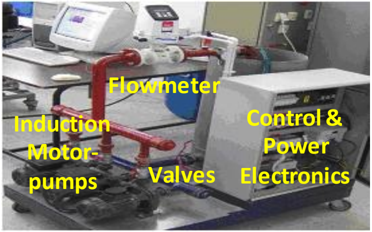
where

$$\begin{cases} \mathcal{F}(\mathbf{q_c}) = (R_{\mathrm{Mod}} + R_{\mathrm{Valve}}) * \mathbf{q_c}^2 \\ \mathcal{G}(\mathbf{q_c}) = (b * (\mathbf{q_c} + \mathcal{F}(\mathbf{q_c})/R_{\mathrm{Me}})) \\ \mathcal{M}(\mathbf{q_c}, \mathbf{h}) = a * \Omega(\mathbf{q_c}, \mathbf{h}) + \mathcal{G}(\mathbf{q_c}) \\ \Omega(\mathbf{q_c}, \mathbf{h}) = \frac{-\mathcal{G}(\mathbf{q_c}) + \sqrt{\mathcal{G}(\mathbf{q_c})^2 - 4a*(-(p_0 + \rho g*(\mathbf{h} - l_{\mathrm{out}}) + (k+c)*((\mathbf{q_c} + \mathcal{F}(\mathbf{q_c})/R_{\mathrm{Me}})^2) + \mathcal{F}(\mathbf{q_c})))}}{2*a} \\ \mathcal{K}(\mathbf{q_c}, \mathbf{h}) = (((f_m + f_p) * \Omega(\mathbf{q_c}, \mathbf{h}) + (\mathbf{q_c} + \mathcal{F}(\mathbf{q_c})/R_{\mathrm{Me}}) * (a * \Omega(\mathbf{q_c}, \mathbf{h}) + \mathcal{G}(\mathbf{q_c})))/k_\phi)^2 \end{cases}$$

Literature review

Roboam X., Sareni B., Nguyen D. T., and Belhadj J. . *Optimal system management of a water pumping and desalination process supplied with intermittent renewable sources*. In Proceedings of the 8th Power Plant and Power System Control Symposium, vol 8, p. 369-374, 2012.





Literature review

Mathematical programming-based resolution methods on similar problems

Camponogara E., De Castro M. P. and Plucenio, A. *Compressor scheduling in oil fields : A piecewise-linear formulation*. IEEE International Conference on Automation Science and Engineering, p. 436 - 441, 2007.

Borghetti A., D'Ambrosio C., Lodi A. and Martello S., *An milp approach for short-term hydro scheduling and unit commitment with head-dependent reservoir*. IEEE Transactions on Power Systems, 23(3), p. 1115 - 1124, 2008.

Generic MINLP resolution methods

Grossmann I.E.. *Review of nonlinear mixed-integer and disjunctive programming techniques*. Optimization and Engineering, 3, p. 227 - 252, 2002.

Hybrid algorithms and frameworks

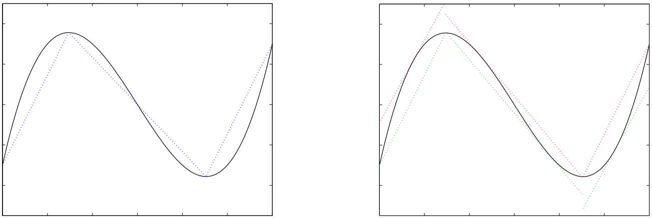
Polisetty P.K. and Gatzke E.P.. *A decomposition-based minlp solution method using piecewise linear relaxations*. Technical report, Univ. of South Carolina, 2006.

Bonami P., Biegler L.T., Conn A.R., Cornuéjols G., Grossmann I.E., Laird C.D., Lee J., Lodi A., Margot F., Sawaya N., and Wächter A. *An algorithmic framework for convex mixed integer nonlinear programs.* Discrete Optimization, 5(2), p.186 - 204, 2008.

Floudas C.A. and Gounaris C.E.. *A review of recent advances in global optimization*. Journal of Global Optimization, 45, p. 3 - 389, 2009.

Resolution method

Step 1 : Piecewise linear bounding of the nonlinear energy transfer/efficiency functions



(a) Linear approximation (b) Piecewise bounding

Step 2 : Reformulation of the problem into two mixed integer problems (MILP)

- the problem is originally a MINLP
- using the pair of bounding functions previously defined

Piecewise bounding

Principle

Piecewise bounding a function f of m variables within a tolerance value ϵ consists in identifying two piecewise linear functions $(\overline{f}^{\epsilon}, \underline{f}^{\epsilon})$ that verify :

$$\underline{f}^{\epsilon}(x) \leq f(x) \leq \overline{f}^{\epsilon}(x), \quad \forall x \in \mathbb{R}^m \tag{11}$$

$$f(x) - \underline{f}^{\epsilon}(x) \leq \epsilon f(x), \quad \forall x \in \mathbb{R}^m \tag{12}$$

$$\overline{f}^{\epsilon}(x) - f(x) \leq \epsilon f(x), \quad \forall x \in \mathbb{R}^m \tag{13}$$

Purpose

- Two MILP ($\overline{\text{MILP}}$ and $\underline{\text{MILP}}$) are obtained
- Linearizations before the optimization allow :
  - the respect of the predefined tolerance value
  - the minimization of the number of sectors
    - min. of the number of additional integer variables in $\overline{\text{MILP}}$ and $\underline{\text{MILP}}$

Piecewise bounding

Proposition

$\exists \epsilon^*$ such that $\forall \underline{f}^\epsilon$, the optimal solution cost of the corresponding $\underline{\text{MILP}}$ is the global optimal solution cost of the original MINLP.

Proof outline

Based on two properties :

(i) The solution value of $\underline{\text{MILP}}$ does not decrease with the decrease of ϵ.

(ii) The theorem of (Duran and Grossman,1986) which is the basis of the OA algorithm states that if all feasible discrete variables are used as linearization points then the resulting MILPCP problem (denoted M-OA by Grossmann in 2002) has the same optimal solution than the original MINLP.

Piecewise bounding heuristics

Bounding the efficiency function of pump 1

Assumptions

The efficiency function is either convex or concave.

Objective

For $\underline{f^{\epsilon}}$ and $\overline{f^{\epsilon}}$, identify the minimum number of sectors n, and the parameters of each sector i : slope a_i, y-intercept b_i and limits $q_{\min_i}$ and $q_{\max_i}$.

Principle

Each sector i verifies : $p = a_i q + b_i$.
Consecutive sectors $i-1$ and i satisfy : $q_{\min_i} = q_{\max_{i-1}}$

Idea

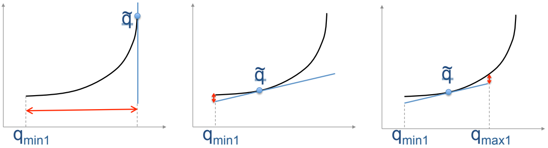
Use supporting linear functions tangent to f^1 at predefined points, to control where the max error will be located, to respect eq. (11)-(13).

Piecewise bounding heuristics

Bounding the efficiency function of pump 1

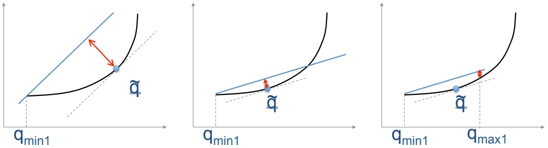
Lower bounding

For a potential tangent point $\tilde{q}$: $a_i = \frac{df^1}{dq}(\tilde{q})$ and $b_i = f^1(\tilde{q}) - \tilde{q}\frac{df^1}{dq}(\tilde{q})$.



Upper bounding

For a potential tangent point $\tilde{q}$: $a_i = \frac{df^1}{dq}(\tilde{q})$ and $b_i = f^1(q_{min_i}) - q_{min_i}\frac{df^1}{dq}(\tilde{q})$.



Piecewise bounding heuristics

Extension to the efficiency function of pump 3

Specificity

f^3 takes into account the water level h from its intake tank. It is a function of two variables (q, h) instead of one.

Solution chosen

Piecewise bounding functions in the form $p = aq + b - sh$ where s is a correction parameter.
Idea similar to (Borghetti et al., 2008) but here we ensure that eq. (11)-(13) remain verified.

Extension to the efficiency function of pump 2 and RO module

Solution chosen

Bounding of the global efficiency function of the subsystem "pump 2 + RO module" (instead of separated efficiency functions).

MILP reformulation

Data

- $\mathbb{N}_I, \mathbb{N}_P, \mathbb{N}_T$: set of time intervals, pumps, set of tanks
- ts : scale of time, duration of the time intervals
- hq : section of the tanks, used to convert the debit into water level
- $P^i_{\min}, P^i_{\max}, \forall i \in \mathbb{N}_P$: pumping power limits of pump i
- $L^i_{\min}, L^i_{\max}, \forall i \in \mathbb{N}_P$: capacity limits of tank i
- $l^j_{\text{init}}, j \in \mathbb{N}_T, \geq 0$: initial water level of tank j
- $\text{Pin}_i, \forall i \in \mathbb{N}$: input power available at time interval i
- **piecewise functions data** computed with the heuristics : $\{n_{p_i}, a^j_i, b^j_i, s^j_i, \alpha^j, \beta^j, Q^{i,j}_{\min}, Q^{i,j}_{\max}\}, \forall i \in \mathbb{N}_P, \forall j \in 1..n_{p_i}$

Binary variables

- $r_i, \forall i \in \mathbb{N}_I$: equal to 0 iff all tanks are full at time interval i.
- $sect^{j,k}_i, \forall i \in \mathbb{N}_I, \forall j \in 1..n_{p_1}, \forall k \in 1..3$: equal to 1 iff pump k is used at the j^{th} section of its piecewise power function during time interval i.

Continuous variables

- $q^{j,k}_i, \forall i \in \mathbb{N}_I, \forall j \in 1..n_{p_1}, \geq 0$: equal to the flow of water pumped by pump k at time i if it is used at the j^{th} sector of the piecewise power function and 0 otherwise.
- $l^j_i, \forall i \in \mathbb{N}_I, \forall j \in \mathbb{N}_T, \geq 0$: equal to the level of water going in tank j at time interval i
- $v^{j,k}_i, \forall i \in \mathbb{N}_I, \forall j \in 1..n_{p_2}, \geq 0$: equal to the level of water in tank k if pump $k+1$ is used at the j^{th} section of the piecewise power function at time i and 0 otherwise.

MILP reformulation

$$\min \sum_{i \in \mathbb{N}} r_i * \text{ts} \tag{8}$$

subject to

$$r_0 = 1 \tag{9}$$

$$l_0^j = l_{\text{init}}^j, \qquad \forall j \in \mathbb{N}_T \tag{10}$$

$$r_i - r_{i-1} \leq 0, \qquad \forall i \in \mathbb{N}_I \tag{11}$$

$$\sum_{j \in \mathbb{N}_T} l_i^j + \Big(\sum_{j \in \mathbb{N}_T} L_{\max}^j\Big) r_i \geq \sum_{j \in \mathbb{N}_T} L_{\max}^j, \qquad \forall i \in \mathbb{N}_I \tag{12}$$

$$\sum_{k \in \mathbb{N}_P} \sum_{j \in 1..n_{p_k}} \big(a_k^j q_i^{j,k} + b_k^j sect_i^{j,k} - s_k^j v_i^{j,k}\big) \leq \text{Pin}_i, \qquad \forall i \in \mathbb{N}_I \tag{13}$$

$$l_i^1 - l_{i-1}^1 - \sum_{j \in 1..n_{p_1}} \text{hq} * \text{ts} * q_{j,1}^i + \sum_{j \in 1..n_{p_2}} \text{hq} * \text{ts} * \big(\alpha^j q_{j,2}^i + \beta^j sect_i^{j,2}\big) \leq 0, \qquad \forall i \in \mathbb{N}_I \tag{14}$$

$$l_i^2 - l_{i-1}^2 - \sum_{j \in 1..n_{p_2}} \text{hq} * \text{ts} * \big((\alpha^j - 1) q_{j,2}^i + \beta^j sect_i^{j,2}\big) + \sum_{j \in 1..n_{p_3}} \text{hq} * \text{ts} * q_{j,3}^i \leq 0, \qquad \forall i \in \mathbb{N}_I \tag{15}$$

$$l_i^3 - l_{i-1}^3 - \sum_{j \in 1..n_{p_3}} \text{hq} * \text{ts} * q_{j,3}^i \leq 0, \qquad \forall i \in \mathbb{N}_I \tag{16}$$

$$L_{\min}^k \leq l_i^k \leq L_{\max}^k, \qquad \forall i \in \mathbb{N}_I, k \in \mathbb{N}_P \tag{17}$$

$$\sum_{j \in 1..n_{p_k}} P_{\min}^k sect_i^{j,k} \leq \sum_{k \in N_p\{1\}} \sum_{j \in 1..n_{p_k}} \big(a_k^j q_i^{j,k} + b_k^j sect_i^{j,k} - s_k^j v_i^{j,k}\big) \leq \sum_{j \in 1..n_{p_k}} P_{\max}^k sect_i^{j,k}, \qquad \forall i \in \mathbb{N}_I, k \in \mathbb{N}_P \tag{18}$$

$$Q_{\min}^{i,j} sect_i^{j,k} \leq q_i^{j,k} \leq Q_{\max}^{i,j} sect_i^{j,k}, \; \forall i \in \mathbb{N}_I, k \in \mathbb{N}_P, j \in 1..n_{p_k} \tag{19}$$

$$\sum_{j \in 1..n_{p_k}} sect_i^{j,k} \leq 1, \qquad \forall i \in \mathbb{N}_I, k \in \mathbb{N}_P \tag{20}$$

$$v_i^{j,k} - l_i^j \leq 0, \; \forall k \in \mathbb{N}_P, i \in \mathbb{N}_I, j \in 1..n_{p_k} \tag{21}$$

$$v_i^{j,k} - L_{\max}^j sect_i^{j,k} \leq 0, \qquad \forall k \in \mathbb{N}_P, i \in \mathbb{N}_I, j \in 1..n_{p_k} \tag{22}$$

Figure : Source : (Ngueveu et al. 2014)

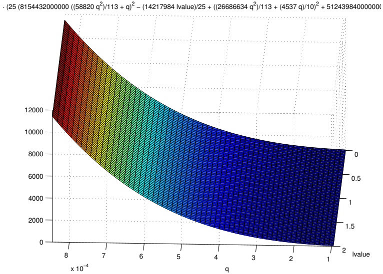
Implementation and data

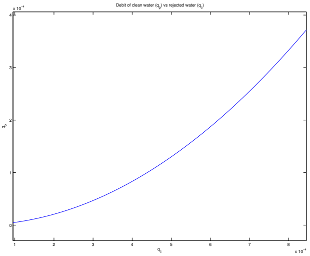
Implementation

- Matlab (for the efficiency functions)
+ GLPK (for transfer)
+ CPLEX 12.5 (for the MILP resolution)
- Intel Core 2Duo, 2.66 GHz 4GB of RAM

Data :

- Same pump characteristics as (Roboam X., Sareni B., Nguyen D. T., and Belhadj J. . 2012).[1]
- Input power profile deduced from the Guadeloupe wind site





Computational Evaluation

	Pump 1		(Pump 2+RO)		Pump 3	
ϵ	$\overline{n}_{p_1}$	$\underline{n}_{p_1}$	$\overline{n}_{p_2}$	$\underline{n}_{p_2}$	$\overline{n}_{p_3}$	$\underline{n}_{p_3}$
5%	2	2	11	21	3	2
1%	5	5	21	29	8	5
0.5%	8	7	35	62	13	7
0.3%	10	9	43	74	17	9

Table : Number of sectors per tolerance value

	$\overline{\text{MILP}}$		$\underline{\text{MILP}}$			Gap	opt
ϵ	UB	s	LB	s	UB*	%	
5%	20580	4	19740	15	-	4.25	no
1%	20100	15	19920	140	-	0.9	no
0.5%	20040	178	19980	117	-	0.3	no
0.3%	20040	64	19980	321	19980	0.3	yes

Table : Upper and Lower bounds values obtained

Conclusion

Done

- Groundwork for the integration of energy source characteristics in production scheduling problems.
- Resolution scheme is based on piecewise linear bounding and integer programming
- Bounding heuristics for convex and concave efficiency/transfer functions have been introduced.
- Good results on a water production optimization problem with non linear efficiency functions : global optimization problem solved to optimality on the given data sets.

Ongoing

- Address multiple energy sources with different characteristics and their resulting problems untractable for black-box solvers.