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# Codesign mechanics / attitude control for a simplified AOCS preliminary synthesis

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## ABSTRACT

*This paper aims to present the advantages of a multibody modeling approach, adapted to all kinds of satellites. This approach gives not only a linearized satellite model at a nominal parametric configuration but also a linearized parameterized model available on a parametric range. Resulting dynamic models are representative of the couplings between different axes and the impact of flexibility. The two contributions of the paper concern the ability of these dynamic models to simulation purposes and the parameterization of the overall satellite model in terms of the inclination of a flexible appendage, useful for a parametric sensitivity study.*

## 1. INTRODUCTION

Achieving as soon as possible mechanical design parameters, coherently chosen according to control capabilities, is a crucial issue during the preliminary design phase of a spacecraft. For instance it is useful to identify quickly which parameters (frequencies or dampings of flexible modes, inertia gaps between satellite axes, sizing of complex joints...) can limit the performance of AOCS (Attitude and Orbit Control Subsystem) or lead to instability. But common modeling approaches lead to complete nonlinear black-box-type models, whose accuracy is only appropriate to validation of control laws. Relevant mechanical parameters or parametric uncertainties cannot be explicitly isolated in such models, in order to be optimized or constrained.

In this context this paper exploits a suitable generic modeling method which directly gives structured linearized dynamic models of any multi-body flexible satellite. This approach was first explained in [1] where a linear dynamic model of a satellite around a nominal configuration is set up. The key idea consists in using Euler's equations applied to each subsystem, before connecting them to each other. The method has the great advantage to be generic, when all flexible appendages of a satellite are connected to its rigid main body.

This decomposition subsystem per subsystem also leads to linear synthesis models which highlight interesting physical transfer functions for the synthesis step : for example, in [2], constrained performance requirements are fulfilled with structured  $H_\infty$  controllers performed on such synthesis models.

Moreover such dynamic models analytically express the impact of dimensioning parameters for the attitude control. That is the reason why they can be easily translated into the LFT (Linear Fractional Transformation) framework such that parametric uncertainties can be taken into account. Resulting models are sufficiently accurate, of medium complexity and adapted to robustness analysis tools, as demonstrated in [3].

The satellite benchmark (described in section 2) studied here allows to prove the ability of the resulting models to AOCS validation and simulation purposes. Indeed an one-axis step SADM (Solar Array Driven Mechanism) is connected to the linearized dynamics model : as seen in section 3, all couplings between the flexible modes and varying axes are recovered. Besides in order to optimize both the structure and the performance of the AOCS, the modelling step can be easily adapted to parameterize the overall satellite model in terms of conception parameters, such as the inclination angle of the flexible appendage (section 4)

## 2. NOTATIONS

The main notations are here listed.

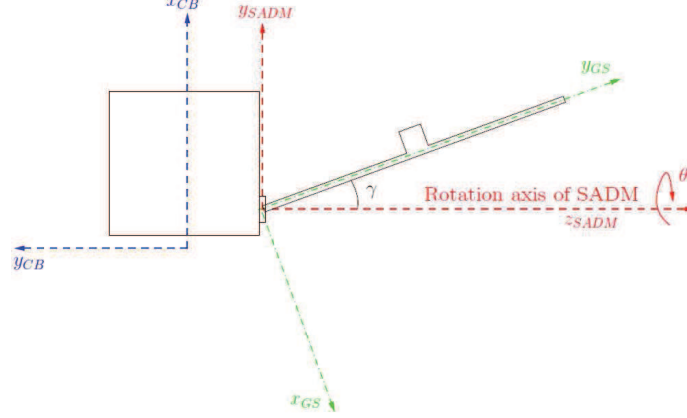
### A. MATHEMATICAL NOTATIONS

$0_{m \times n}$	:	$m \times n$ zero matrix
$I_{n \times n}$	:	$n \times n$ identity matrix
$T$	:	transpose operator

$\frac{d}{dt}$  : differential operator with respect to the variable  $t$

## B. FRAME DEFINITIONS

The application concerns a satellite composed of a rigid central body and a flexible solar array driven by an one-axis step SADM (Solar Array Driven Mechanism) (see Fig. 1). The solar array is tilted with an angle  $\gamma$  and there is a pivot joint between the central body and the flexible appendage.



**Fig.1:** Reference frames of central body, SADM and flexible array.

Three reference frames must be considered, as highlighted in Fig.1 : the main body reference frame  $R_b = (x_{CB}, y_{CB}, z_{CB})$ , the appendage reference frame  $R_a = (x_{GS}, y_{GS}, z_{GS})$  and the SADM reference frame  $R_{SADM} = (x_{SADM}, y_{SADM}, z_{SADM})$ . The transformation matrix from  $R_{SADM}$  to  $R_b$  is denoted  $T_{SADM \rightarrow CB}$ .  $B$  is the center of mass of the main body and  $P$  denotes the anchorage point between the appendage and the main body.

## C. MATRIX EXPRESSIONS OF VECTORIAL FORMULAS

If  $[u_1, u_2, u_3]^T$  and  $[v_1, v_2, v_3]^T$  are the coordinate vectors of respectively the two vectors  $\vec{u}$  and  $\vec{v}$  in a specified frame  $R$ , the cross product  $\vec{u} \times \vec{v}$  is written in  $R$  as a matrix-vector product  $(*u)v$  where the skew-symmetric matrix  $(*u)$  associated to the vector  $u$  is defined as follows :

$$(*u) = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix}$$

This notation is useful to express the transport of a dynamic model from one point to another. Indeed the first-order approximation of the vector, composed of the absolute linear acceleration vector of a body at a point  $A_1$  ( $\vec{a}_{A_1}$ ) and of the absolute angular velocity vector of the body w.r.t. the inertial frame ( $\vec{\omega}$ ), can be rewritten in terms of the absolute accelerations vector of the body at another point  $A_2$  thanks to the relation :

$$\begin{bmatrix} \vec{a}_{A_1} \\ \vec{\omega} \end{bmatrix} = \underbrace{\begin{bmatrix} I_{3 \times 3} & (*A_1 A_2) \\ 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix}}_{\tau_{A_1 A_2}} \begin{bmatrix} \vec{a}_{A_2} \\ \vec{\omega} \end{bmatrix} \quad (1)$$

The transport of the forces vector  $\vec{F}$  and of the torques vector  $\vec{T}$  from  $A_1$  to  $A_2$  uses the same matrix :

$$\begin{bmatrix} \vec{F} \\ \vec{T}_{A_1} \end{bmatrix} = \tau_{A_2 A_1}^T \begin{bmatrix} \vec{F} \\ \vec{T}_{A_2} \end{bmatrix} \quad (2)$$

## D. LFT NOTIONS

A linear system submitted to parametric uncertainties can be described by a LFT (Linear Fractional Transformation). This representation consists in isolating the uncertainty matrix  $\Delta$  from the nominal model  $H(s)$  and in connecting them through a feedback as shown in Fig.2. The LFT framework thus allows to represent a continuum of models and over all, avoids launching the modeling procedure again for a new parametric configuration. This is the reason why a LFT model saves CPU time during control attitude validation process, as explained in part 4.

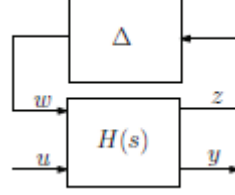


Fig.2: A common LFT model.

## 3. LINEAR MULTIBODY DYNAMICS EQUATIONS

This section explains briefly how to get a dynamical model of the satellite as physical as possible for simulations and robustness. The reader must refer to [1,3] for further details.

### A. LINEAR DYNAMIC MODEL OF THE RIGID MAIN BODY

Under assumptions of small displacements, the Newton's second law and the Euler's equations applied to the rigid main body give :

$$\begin{bmatrix} \vec{F}_{ext} - \vec{F}_{b/a} \\ \vec{T}_{ext,B} - \vec{T}_{b/a,B} \end{bmatrix} = \underbrace{\begin{bmatrix} m_b I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & I_{b,B} \end{bmatrix}}_{D_B^b} \begin{bmatrix} \vec{a}_B \\ \vec{\omega}_b \end{bmatrix} \quad (3)$$

where :

- $\vec{F}_{ext}, \vec{T}_{ext,B}$  are the external forces/torques (at  $B$ ) vectors, applied to the main body,
- $\vec{F}_{b/a}, \vec{T}_{b/a,B}$  are the forces/torques (at  $B$ ) vectors, applied by the main body to the appendage,
- $\vec{a}_B$  is the absolute linear acceleration vector of the main body at  $B$ ,
- $\vec{\omega}_b$  is the absolute angular velocity vector of  $R_b$  w.r.t. the inertial frame,
- $m_b$  is the mass of the main body,
- $I_{b,B}$  is the  $3 \times 3$  moment of inertia tensor of the main body at  $B$ .

Equation (3) is often written in  $R_b$ .

### B. LINEAR DYNAMIC MODEL OF THE APPENDAGE

If the flexible appendage is cantilevered on the main body at the interface point  $P$ , the dynamic model of the appendage describes the relationship between the 6 dof acceleration vector of the point  $P$  and the 6 dof forces/torques vector applied by the main body to the appendage at point  $P$  :

$$\begin{cases} \begin{bmatrix} \vec{F}_{b/a} \\ \vec{T}_{b/a,P} \end{bmatrix} = D_P^a \begin{bmatrix} \vec{a}_P \\ \vec{\omega}_b \end{bmatrix} + L_P \ddot{\eta} \\ \ddot{\eta} + \text{diag}(2\xi_i \omega_i) \dot{\eta} + \text{diag}(\omega_i^2) \eta = -L_P^T \begin{bmatrix} \vec{a}_P \\ \vec{\omega}_b \end{bmatrix} \end{cases} \quad (4)$$

where :

- $D_P^a$  is the  $6 \times 6$  mass/inertia model matrix of the appendage at point  $P$ ,
- $L_P$  is the  $6 \times N$  matrix of modal participation factors of the  $N$  flexible modes of the appendage at point  $P$ ,
- $\omega_i$  and  $\xi_i$  are the pulsation and the damping ratio of the  $i$ -th flexible mode.



The expression of  $\ddot{\theta}$  can be deduced from (5). To have a generic Matlab code, the best procedure consists in considering a state space representation  $(A_1, B_1, C_1, D_1)$  of  $M_P^a(s)|_{\mathcal{R}_{SADM}}$  where  $x_1$  denotes the state vector :

$$\ddot{\theta} = -\frac{C_1(6,:)}{D_1(6,6)}x_1 - \frac{D_1(6,:)}{D_1(6,6)} \begin{bmatrix} a_{P,x_{SADM}} \\ a_{P,y_{SADM}} \\ a_{P,z_{SADM}} \\ \dot{\omega}_{b,x_{SADM}} \\ \dot{\omega}_{b,y_{SADM}} \\ \dot{\omega}_{b,z_{SADM}} \end{bmatrix} + \frac{1}{D_1(6,6)}C_m \quad (6)$$

The dynamic model of the appendage is thus augmented with a new input  $C_m$  and a new output  $\ddot{\theta}$ . And as shown in Fig.4.

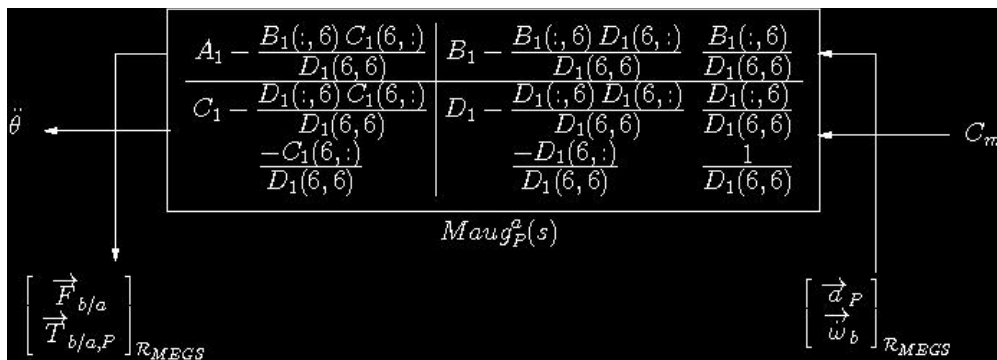


Fig.4: Augmented direct dynamic model of the appendage on a motorized pivot.

The final dynamic model of the overall system (central body + SADM + flexible array) counts~:

- 4 inputs : the 3 axes torques of attitude control and the torque applied by the SADM to the pivot joint,
- 4 outputs : the 3 angular accelerations of the central body and the angular acceleration of the pivot joint.

Fig.5 shows how a nonlinear model of SADM can be directly connected to the linear direct dynamic model of the appendage. Frame change ( $T_{CB \rightarrow SADM}$  in Fig.5) allows to restore correctly the dynamic couplings between main body and appendage for each satellite configuration.

Next section details a more precise modeling of a SADM, useful for the representativeness of the simulation.



## 4. COMPUTATION OF A CONTINUUM OF PARAMETERIZED SATELLITE MODELS

### A. LFT FRAMEWORK FOR PARAMETERIZED SATELLITE MODELS

When a parameter whose nominal value is  $x_0$  is submitted to uncertainties, two kinds of description are available :

- either  $y = (x_0 + \alpha\delta)u$ . This is the "additive formulation" and the parameter value is then between  $x_0 - \alpha$  and  $x_0 + \alpha$ . The block-diagram in Fig.7 (a), which describes the uncertain static transfer function, is equivalent to the LFT model represented in Fig.7 (b).

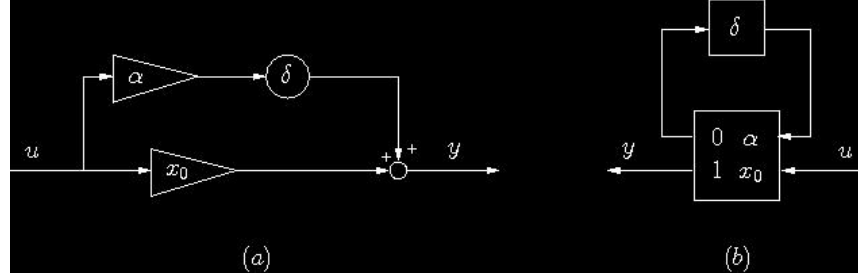


Fig. 7: Block-diagram in (a) and a LFT in (b) of  $y = (x_0 + \alpha\delta)u$ .

- or  $y = x_0(1 + \alpha\delta)u$ . It is the so called "multiplicative formulation". The parameter value is then between  $x_0(1 - \alpha)$  and  $x_0(1 + \alpha)$ . As above, it can be proved that a LFT model exactly describes this new uncertain static transfer function.

These elementary blocks are the basis of the construction of the uncertain complete satellite model. Indeed each uncertain parameter is defined by a characterizing relation (either additive or multiplicative formulation) thanks to a Matlab object (the standard *uss* in Matlab, or *lfr* in the LFR Toolbox [5]). This Matlab object can be used as the Matlab object *ss* : that is the reason why all connections (series, parallel, feedback...) systematically become a LFT model.

As a result, each sub-block in Fig.5 becomes a LFT, depending on the possible parametric uncertainties. The obtained overall linearized dynamic model is finally a LFT model, containing all necessary information to generate all satellite configurations inside the definition range of the uncertain parameters (masses, inertia terms, pulsations and damping ratios of the flexible modes, position of the anchorage points...).

This continuum of parameterized satellite models are useful because it avoids computing the new resulting linear dynamic model when parameter values must be changed.

### B. PARTICULAR CASE OF THE INCLINATION ANGLE

This satellite configuration can also be parameterized according to the inclination angle  $\gamma$  (see Fig.1) between the rotation axis of the SADM and the surface of the solar array. Indeed it can be interesting to analyze the impact of this angle value, when the solar array is excited by a step motor. Moreover the obtained LFT model can be parameterized according to the rotation angle  $\theta$  of the flexible appendage and can be used to validate an attitude control system over a complete revolution of the appendage. To avoid computing models for each  $\gamma$  (or  $\theta$ ) value, the trick consists in isolating, through a feedback, a parameter that allows to generate all models over a  $\gamma$  variation range (or during a complete appendage revolution), so does the LFT building step.

Only a parameterization according to  $\gamma$  will be here detailed.

Let us recall that the angle  $\gamma$  appears in the computation of the matrix  $Maug_p^a(s)$  (see Fig.1). More precisely the transformation matrix from  $R_{SADM}$  to  $R_a$  can be written as follows :

$$T_{SADM \rightarrow a} = \begin{bmatrix} -\cos \gamma & \sin \gamma & 0 \\ 0 & 0 & 1 \\ \sin \gamma & \cos \gamma & 0 \end{bmatrix} \quad (7)$$

If the value of the inclination angle  $\gamma$  can be between 0 deg and  $\gamma_{MAX}$  deg, this  $\gamma$  range can be described with a normalized uncertainty  $\delta_\gamma$  :



$$\gamma = \gamma_0 + \gamma_1 \delta_\gamma \quad \text{with } \gamma_0 = \gamma_1 = \frac{\gamma_{MAX}}{2} \text{ and } \delta_\gamma \in [-1,1] \quad (8)$$

The matrix raises problems for the LFT building, because it is composed of trigonometric functions, which must verify, whatever the uncertainty value :

$$\cos^2(\gamma_0 + \gamma_1 \delta_\gamma) + \sin^2(\gamma_0 + \gamma_1 \delta_\gamma) = 1$$

As a complete revolution is not considered here, a solution, combining ease of the LFT building and size minimization of the uncertainty block, consists in taking the tangent of the half angle. Mathematically,,the notation

$$t = \tan\left(\frac{\gamma_1 \delta_\gamma}{2}\right) \in \left[-\frac{\gamma_1}{2}, \frac{\gamma_1}{2}\right]$$

leads to :  $\cos(\gamma_1 \delta_\gamma) = -1 + \frac{2}{1+t^2}$  and  $\sin(\gamma_1 \delta_\gamma) = \frac{2t}{1+t^2}$

If  $[u_1; u_2; u_3]$  and  $[y_1; y_2; y_3]$  denote respectively the inputs and the outputs of  $T_{SADM \rightarrow a}$ , it is easily to prove that :

$$y_1 = \frac{1-t^2}{1+t^2} (-\cos \gamma_0 u_1 + \sin \gamma_0 u_2) + \frac{2t}{1+t^2} (\sin \gamma_0 u_1 + \cos \gamma_0 u_2)$$

This equation is represented in Fig.8.

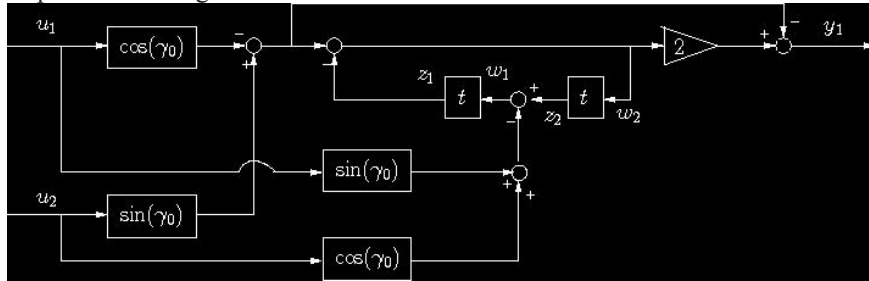


Fig. 8: Block-diagram of the transfer between  $u_1, u_2$  and  $y_1$ .

Same developments can be carried on for the expression of  $y_3$ . Fig.9 finally gives a minimal LFT realization of the matrix  $T_{SADM \rightarrow a}$ .

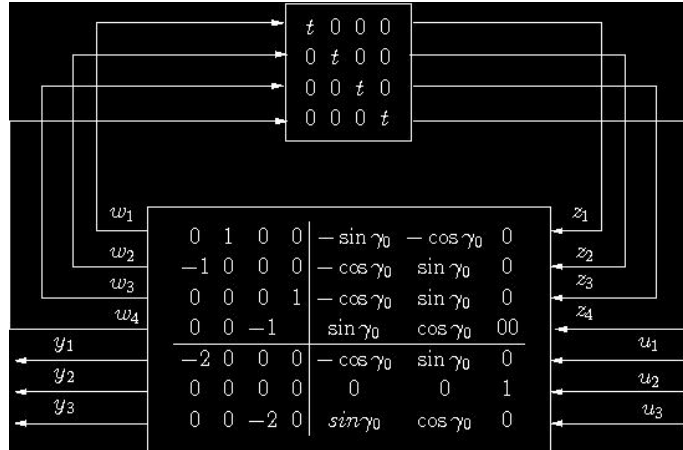


Fig. 9: LFT realization of the matrix  $T_{SADM \rightarrow a}$ .

Let us note that a variable change can normalize the uncertainty : instead of considering  $t$ , it is better to work with  $\tilde{t}$  defined by :

$$t = \tan\left(\frac{\gamma_1}{2}\right) \tilde{t} \quad \text{with} \quad \tilde{t} \in [-1,1]$$

As explained before this LFT modelling is a CPU time saving procedure, when the best compromise between the definition of the satellite structure and the AOCS performance must be fulfilled.

## E. CONCLUSION

This paper proposes an adapted linear modelling procedure for representativeness of the satellite dynamics for AOCS design, for simulation purposes and for all kinds of preliminary steps –namely, the codesign mechanics/attitude control and the robustness analysis of first AOCS. The two main advantages of this method is the genericity of physical models and the simplicity of physical parameter dependence. Extensions are today studied, so that many flexible subsystems can be connected together. This generalization will be useful for in orbit services.

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