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Topology optimization for robust damage localization using aggregated FRFs statistical criteria

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Abstract. This work focuses on the development of a damage localization tool using Topology Optimization (TO) as a solver for the inverse problem of localization. This approach is based on the correlation of a local stiffness loss and the change in frequencies due to damages. We use the loss in stiffness for updating undamaged numerical models towards similar models with embedded damages. This work is an extension of past work and aims at increasing the detectability of the method using using aggregated Frequency Response Functions (FRFs) statistical criteria. Good results have finally been achieved for the localization of close damages by the Topology Optimization method.

Introduction

The use of fibre composite materials in aeronautical structures has vastly increased during the last decade. Due to the specific tensile strength, composites are used for a variety of lightweight structures. Unfortunately, composite structures show a very complex mechanical behavior concerning dynamic loads and a variety of damage mechanisms, that are hard to predict. Some of these damages are fibre or matrix cracking, fibre matrix debonding and ply delaminations. Structural health monitoring (SHM) aims at analyzing the creation and evolution of damages in composites (damage detection), to identify affected regions (damage localization) and to evaluate their influence on the structure as a whole (damage classification and quantification) [1].

A structure can be considered as a dynamic system with stiffness, mass and damping. Once some damages emerge in the structures, the structural parameters will change, and the frequency response functions and modal parameters of the structural system will also change.

Change in natural frequencies is the most common parameter used in the identification of damaged regions [2,3,4]. Correlating damages with a degradation of structural stiffness and/or mass in numerical simulations has been proved [1]. Adapting this idea, a new approach, using Topology Optimization design variables for localizing damages, has been published first by Lee et al [5] and extended by our research team [6,7,8]. The key of this approach is that, due to the character of Topology Optimization, a search for defects is performed globally over the entire structure, which distinguishes this method from earlier proposed methods and offers interesting prospects. In this paper, the new proposed SHM method can be applied "online" (embedded sensors, operational analysis) in comparison to Non Destructive Tests (NDT) such as C-scan method which needs to disassembly the structure.

The original goal of topology optimization was to find, within a defined discretized solution domain, a structure of minimal compliance (highest rigidity) by changing the topology. This approach had first been proposed in [9] based on a homogenization of micro-structural elements with rectangular holes whose size are defined by the design variable. The dimensions of the holes could vary from 1 (*void*) to 0 (*solid*), normalized to the element's size, and also from element to element. By homogenization of the discrete elements, the material could be transformed in an equivalent homogeneous material. Since this method can deliver hazy results for the optimized structure, a slightly different approach has been presented in [9], which is often referred to as the *Power Law approach*. Here, the design variables are assumed to be an additional element property that can be understood as a relative density of the element. By reducing the stiffness and mass of certain elements with their newly assigned density fraction property, a local change of structural stiffness and mass can be obtained.

This work is an extension of previous works [6,7]. This method has already been validated using a numerical/experimental correlation on composites structures using a single FRF comparison. In the here presented we propose a simple statistical method which permits to distinguish close damages. As the result of the identification is dependent of the chosen FRF (dependency between one fix input and one moving output), the idea is to use the "local" content of the available measurements by choosing randomly few FRFs to include in the localization process. By averaging the results, an homogeneised "image" of the damage can be produced, leading to a best localization of close damages.

Modal Analysis

In this section, the theoretical background for the numerical approach used in this work is briefly presented. Basic equations for modal analysis and the calculation of frequency response functions for a discretized system are given. Also, the optimization problems statement for the presented damage localization method is developed.

Eq. 1 shows the set of equations of motion in matrix notation for a random discretized structure. Hereby, the system is considered to be discretized by Finite Elements and **M**, **C** and **K** are the system's mass, damping and stiffness matrices, respectively. The vector (f) is a time-dependent load vector.

$$\mathbf{M}\ddot{u} + \mathbf{C}\dot{u} + \mathbf{K}u = \mathbf{f}(t) \tag{1}$$

A modal analysis is performed to determine the principal dynamic characteristics of the system, like natural frequencies damping ratios and mode shapes. For a certain degree of freedom k, the frequency response function due to an excitation force at the l-th degree of freedom can than be calculated by eq. 2.

$$H_{kl}(\Omega) = \sum_{i=1}^{n} \frac{\psi_{ik}\psi_{il}}{m_i\left(\omega_i^2 - \Omega^2 + 2j\zeta_i\omega_i\Omega\right)} \tag{2}$$

If the natural frequencies and damping ratios are known, this equation can be evaluated for discrete excitation frequencies.

Several specimens have been fabricated and have undergone vibration testing in the undamaged and damaged state [4]. Resin-containing carbon-fiber/epoxy prepregs of T300/914 are used to fabricate the test specimens. The specimens have a thickness of $3mm$ and consist of 24 plies of stacking sequence $[(0/90/45/-45)_3]_s$.

The vibration tests are carried out with two steel masses attached at the ends using the Oberst beam test-rig.

The composite beams are analyzed in the three different states: An undamaged state (UD), a first damage state due to four impacts (D1) and a second damage state after eight impacts (D2). The locations of the impacts are also shown in fig. 1. Vibration tests are carried out at each of these three states. A simple case is studied where the impact points are chosen as such as the damage is symmetrical on both sides of the two axes of symmetry

After the estimation of the set of FRFs, the modal parameters can be determined using the PolyMAX method. Both modal parameters or FRFs can be compared to numerical results.

Using Nastran software, Harmonic modal analysis are then performed to obtain the modal parameters of the undamaged and the damaged case. The experimentally tested beams have been modeled by shell elements. The connecting piece and the end masses that were glued to the specimens for the vibrations tests had to be explicitly modeled by three-dimensional solid elements. A distributed load

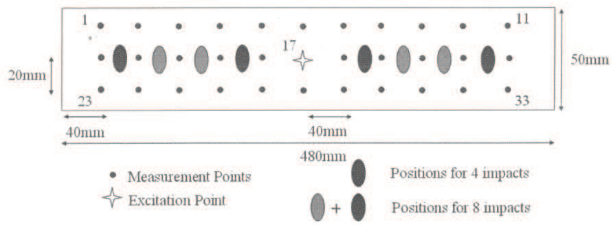

Fig. 1: Location of impacts and measurement points on composite beams [4]

with a combined amplitude of $1N$ has been applied to the bottom of the connecting piece to simulate the excitation force of the shaker.

Optimization Problem Formulation for Damage Localization

The principle objective of the performed optimizations is the matching of a set of modal parameters, any kind of matching function (e.g. least square formulations) can be considered. We reformulate the problem as the exemplary pseudo objective function of eq. 3, where f is the original objective function, and P a penalty function consisting of a set of inequality or equality constraints, which are **g** and **h**, respectively. The parameter r_p is a further penalization coefficient that can be applied to the constraints.

$$\min : \Phi(\boldsymbol{\chi}) = f(\boldsymbol{\chi}) + P(r_p, \mathbf{h}(\boldsymbol{\chi}), \mathbf{g}(\boldsymbol{\chi})) \tag{3}$$

The vector of design variables $\boldsymbol{\chi}$ consists of the Topology design variables corresponding to the *Power Law approach* [9].These design variables are principally an additional element property that can be understood as a relative density of each element e as stated in eq. 4.

$$\chi_e = \frac{\rho_e}{\rho_0} \quad \text{subject to} \quad 0 \leq \chi_{min} \leq \chi \leq 1 \tag{4}$$

Since the variable is normalized by the original densities, it can only assume values between 0 and 1, as stated in the side constraints. The design variable is also penalized, which is supposed to help in getting a clearer *solid-void* solution by making intermediate design variables more "costly". The values of the penalization exponents p and q are problem-dependent, but a penalization factor of about 3 is generally proposed in common literature [8].

In the following, the constraints consisting of modal parameters are defined, where the primed value ($\overline{\omega}$) always denotes the reference data of the damaged structure, and the plain value (ω) corresponds to the current data of the optimized model. Since the goal is to minimize the difference between these values, a proximity ϵ is usually defined.

We used set of constraint equations ($\mathbf{g}^H$) requires that the magnitude value of the nodal FRF (FRF of an FE node) at a certain excitation frequency Ω_j is within the proximity of that of the damaged structure at the same frequency (5) or just modal frequencies depending on the application.

$$g_j^H = \left(|H_{kl}(\Omega_j)| - |\overline{H}_{kl}(\Omega_j)|\right)^2 < \epsilon_H \quad \text{for} \quad j = 1, 2, ..., n_H \tag{5}$$

Results and Discussion

In the following, some Topology Optimization results are presented for the modal parameters of the beam that was impacted with an energy of $8J$. Thereby, two sets of damage state modal parameter data existed for the beams (D1 and D2). Thus, Topology Optimization was performed with the goal to either localize the first four impact sites or all eight. The results for the both cases using only resonance frequencies as constraints are shown in fig. 2 and 3 with the corresponding ultra-sound scan. For the first two impacts the right locations have effectively been found with similar sizes as in ultra-sonic images that have been produced after vibration testing (fig. 3; the outer two impact damage sites have been introduced at first (D1), the two inner impacts later (D2, see fig. 1); ultra-sonic testing could only be performed after all other testing had been completed.)

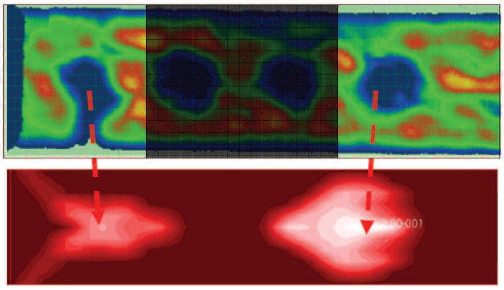

Fig. 2: Comparison between C-SCAN results (top) and Damage Localization by TO (bottom) for the damage state D1 - The two middle points are blackened as they correspond to the damage state D2. The applied algorithm is able to easily detect this kind of damage (2 isolated impacts) with high accuracy.

The next example will show the past limitation of our method and it highlights the classical non uniqueness of the solution in inverse problem. Direct parameters i.e. FRFs and resonance frequencies show a relatively good match with the damaged cases (see tab. 1) whereas the results of the inverse problem i.e. damage localizations are not in perfect match (see fig. 3).

Table 1: Resonance frequencies of tested CFRP beams after 4 and 8 impacts and of the corresponding model with optimized density distribution. Especially the higher frequencies show a relatively good match.

Mode	Frequencies / Hz			
	4 Impacts		8 Impacts	
	Target	Optimization	Target	Optimization
1 (1st bending)	37	30	36	32
2 (2nd bending)	157	148	155	156
3 (3rd bending)	687	679	678	679
4 (4th bending)	1363	1351	1326	1326

For the more damaged beam with four impact sites only a rather uncertain region of damaged material with lower densities could be located (fig. 3), but not the discrete impact points as shown in the ultra-sound picture, since the damage is very closely spaced and is difficult to differentiate. To solve this inverse problem, we need to add data containing information. This is done here by adding much FRFs in the problem to ensure a good localization.

This choice of design constraints is not very intuitive. Since higher resonance frequencies show usually a higher shift due to introduced damages, it seems obvious to use modal parameters of a large spectrum. But with higher frequencies the necessary modal identification becomes more difficult as higher modes are more susceptible to noise and participate less and less in the actual dynamic response.

Due to the fact that the results presented in the experimental validation section are not very satisfactory and punctuated, a further approach was tried to obtain better results for the damage localization.

With each single FRF used in each optimization run, more or less different results were obtained. In order to reduce uncertainties on the results we try to average element densities from the achieved results with different nodal FRF data.This average can be calculated empirically by eq. 6 for n optimization results.

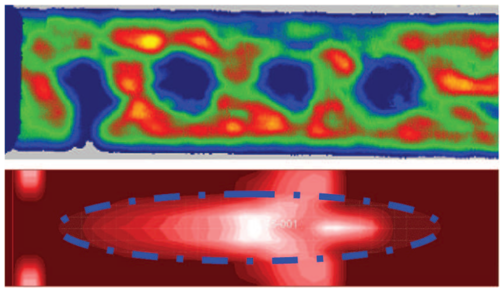

Fig. 3: Comparison between C-SCAN results (top) and Damage Localization by TO (bottom) for damage state D2. The developed method highlights here a real limitation: Damage can be detected but it cannot be distinguished between each of the damage sites. The applied algorithm is only capable to show a damage zone delimiting the 4 impacts.

$$\chi_{avg} = \sum_{i=1}^{n} \frac{a_i \cdot \chi_i}{a_i} \tag{6}$$

Here a_i is a weighting factor that is calculated by eq. 7. In practice n is the number of "interesting" sensors ("interesting" means sensors with optimal placement for damage localization).

$$\frac{1}{a_i} = \sum_{j=1}^{n_\omega} |\omega_j - \overline{\omega_j}| \tag{7}$$

The results of this approach for the CFRP half beam with four impacts are shown in fig. 4. The displayed pictures show a slightly better correlation with the corresponding ultra-sound image in (fig. 3) as before.

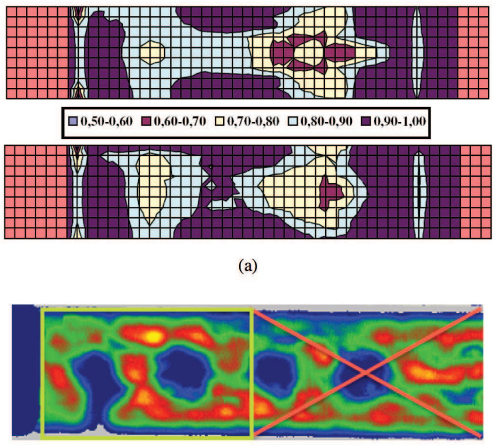

Fig. 4: Results of a statistical analysis for the half beam with 4 impacts taking into account the results of several optimization runs with data from different nodal FRFs: (a-top) Averages of element densities; (a-bottom) averages of element densities weighted by the correlation of resonance frequencies; (b) corresponding ultra-sound scan with interesting zone highlighted in the yellow square.

On fig.4 the topmost image is a simple average of the density distributions of all optimization runs with special weight on the run with only resonance frequencies as constraints. The lower image is a weighted average taking into account the match of the resonances of the optimized model with respect to the target frequencies.

Further studies using different optimization algorithms (stochastic or evolutionary methods, etc.) are encouraged to reach the good solution (global minimum), the principal problem in this damage localization approach is however still the inclusion of further design responses that are sensitive to damages in the fibre composites.

Conclusions

The presented damage localization approach tries to adapt an existing TO method by increasing the number of observation (using local FRFs) to lead the optimization problem to one good solution. The can also be divided into two sub-problems, the optimization routine using Topology Optimization and the experimental estimation of modal parameters with the associated correct numerical modeling. This work aims at increasing the detectability of the method using using aggregated FRFs statistical criteria. Good results have finally been achieved for the localization of close damages. Further prospects of this work could be to develop an electromechanical monitoring of a bonded assembly joint, for example a very used one in aeronautics: composites/composites.

References

[1] S. W. Doebling, C. R. Farrar, M. B. Prime, D. W. Shevitz, Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: A literature review, Research Report LA-13070-MS Los Alamos National Laboratory (1996).

[2] R. D. Adams, R. Cawley, The localization of defects in structures from measurements of natural frequencies, Journal of Strain Analysis 14 (1979) 49-57.

[3] M. H. Richardson, M. A. Mannan, Correlating minute structural faults with changes in modal parame- ters, Proceedings of Spie, International Societe of Optical Engineering 1923 (2) (1993) 893-898.

[4] A. Shahdin, J. Morlier, and Y. Gourinat, Correlating Low-energy Impact Damage with Changes in Modal Parameters: A Preliminary Study on Composite Beams. Structural Health Monitoring, 8(6) (2009), 523-536.

[5] J. S. Lee, J. E. Kim, Y. Y. Kim, Damage detection by the topology design formulation using modal parameters, International Journal for Numerical Methods in Engineering 69 (2007) 1480-1498.

[6] H. Niemann, J. Morlier, A. Shahdin, Y. Gourinat, Damage localization using experimental modal pa- rameters and topology optimization, Mechanical systems and signal processing 24 (3) (2010) 636-652.

[7] A. Shahdin, J. Morlier, H. Niemann, Y. Gourinat, Correlating low energy impact damage with changes in modal parameters: diagnosis tools and fe validation, Structural Health Monitoring 10 (2) (2011) 199-217.

[8] Z. Y. Mao, G. P. Chen, B. Q. Zhang, Tiny damage localization using topology optimization, Journal of Vibration and Shock 4 (2012) 006.

[9] M. P. Bendsoe, O. Sigmund, in: *Topology Optimization - Theory, Methods and Applications*, Springer Verlag, 2003.