FINDING MULTIPLE SUN-EARTH SADDLE-POINT FLYBYS FOR LISA PATHFINDER

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More than 70 years after its existence was postulated for the first time in order to explain the observed rotation curves of galaxies[1], the nature of Dark Matter remains a complete mystery. After several decades of research, no particles have been detected to support this theory. Thus, other theories have been developed to explain Dark Matter problem. Rather than postulating the existence of a new matter, they tend to explain the observations by modifying the gravitational laws. TeVeS and its non-relativistic limit MOND[2] is one of these theories. To date, proof to confirm it has not been provided either, but could be in a near future, thanks to ESA mission LISA Pathfinder.

LISA Pathfinder is a mission due to be launched in the next few years. It carries on-board an extremely sensitive gradiometer which would be able to measure deviations from Newtonian gravity, hence demonstrating MOND theory. Doing so, however, requires that LISA Pathfinder spacecraft reaches a specific point in the solar system, called the Sun-Earth Saddle Point (SP). The SP is the point located between the Sun and the Earth where the gravity of the Sun exactly equals the gravity of the Earth. This point is very singular because of its very low gravity gradient, which recent studies have demonstrated would make MONDian effects measurable[3].

However, LISA Pathfinder spacecraft is to be injected in a halo orbit around the first Sun-Earth Lagrangian point (L1), at more than one million kilometres from the Saddle Point. Therefore, it has been suggested to fly the satellite by the SP in an extension to its nominal mission. The challenge issued by this additional trajectory lies in the ΔV budget. While a total ΔV of approximately 3 km/s will be used to reach L1 from a LEO orbit, a budget of only 4 to 5 m/s is supposed to be remaining at the end of the nominal mission.

Despite this harsh constraint, this study shows that reaching the SP from a given L1 halo orbit is feasible. Furthermore, as it has been emphasized that flying by the SP more than once would be very profitable for the experiment's reliability, trajectories reaching twice the SP have been created. Nevertheless, these trajectories have not been designed as coming from a given halo orbit around L1, as it would be necessary once the exact orbit known during the nominal mission. On the contrary the solutions found, although respecting the specifications on LISA Pathfinder mission trajectory, are not independent of the halo orbit part of the trajectory. Until now, it has not been possible to find orbits reaching twice the SP from a given halo orbit.

Therefore, the final aim of this study is to assess the possibility of designing a trajectory flying twice by the Sun-Earth SP, once the actual orbit of LISA Pathfinder spacecraft is known. To do so, orbits like the ones designed by ESA/ESOC for the nominal mission are used. Conditions under which such double SP flybys could happen are evaluated, and methods to design interesting orbits are defined.

I. LISA PATHFINDER MISSION.

I.I. What is the reason to launch LISA Pathfinder?

Lisa Pathfinder (LPF) is an ESA mission which should be launched in 2015. Its aim is to test the concept of low frequency gravitational waves, with an accuracy never reached today. Those tests will enable astrophysicists to better understand events having an impact on the fabric of space-time, as for example the nature of binary black holes. The effects caused by those events are not measurable on Earth, because of our planet's own gravity field. Hence, the only way to measure them is to reach a place where the Earth gravity is less important: space.

I.II. LPF mission

In order to realize the experiments LPF is designed for, the spacecraft must be in a very stable environment. Thus, the orbit which has been chosen for the mission is a halo orbit around the Sun-Earth Lagrangian point L1. Halo orbits are orbits around a Lagrangian point, having the particularity to be closed (i.e. passing by the same point every period) in a frame keeping constant the position of the Earth and the direction of the Sun[4]. Their name comes from the fact that, viewed in three dimensions in this rotating frame, they actually look like a halo. This kind of orbits offers many advantages. First, it allows the spacecraft to remain at a great and quasi-constant distance from the Earth, thus minimizing the gravity encountered by the spacecraft. At the same time, this choice offers a constant communication possibility, because the Earth, the spacecraft and the Sun are never aligned and therefore the Sun's emissions do not interfere with the communications. A constant illumination by the Sun is another interesting characteristic offered by such an orbit, as there is consequently no need for very capacitive batteries.

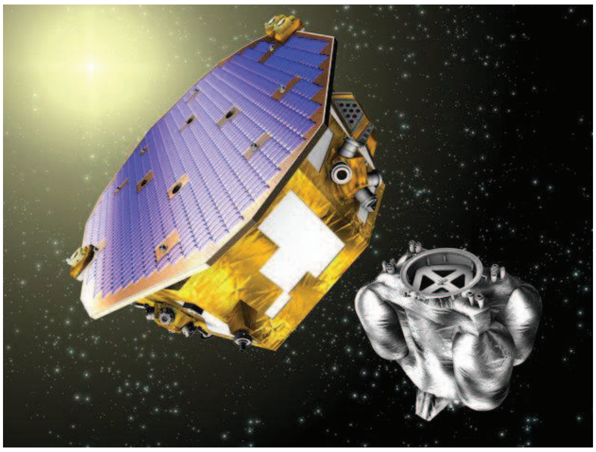

Fig I: Artist view of LPF spacecraft (credits ESA). One can see the propulsion module (bottom-right) after being jettisoned from the science module.

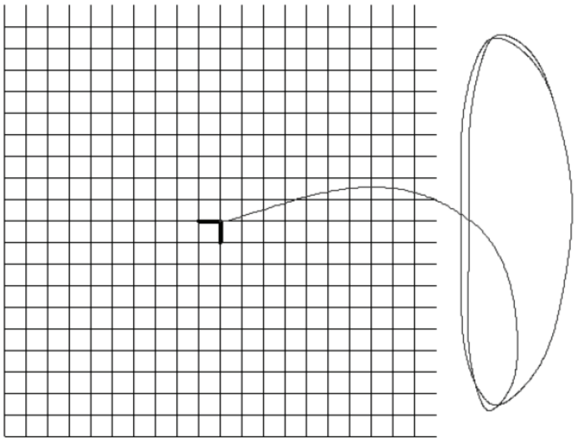

Fig II: Free injection into halo orbit from an Earth elliptic orbit. The frame, and positions of different key points are presented in figure III.

To reach the halo orbit designed for the mission, LPF spacecraft will first be put in a slightly elliptic orbit around the Earth by a launcher. The launcher chosen by ESA is VEGA which is fired from Kourou, in French Guyana. Once in orbit around the Earth, successive ΔV operated by the propulsion module (see figure I) will increasingly modify the apogee of the orbit, and finally enable a free injection in halo orbit, as it can be seen in figure II.

Figure III presents LISA Pathfinder nominal mission. If nothing is done, at the end of the mission the spacecraft will probably escape from its halo orbit, as it is showed on the figure. The figure also presents the geometry of the problem. Every orbit presented hereafter is drawn in the same frame. The frame is rotating, keeping always the Earth at the origin, and the Sun to the right. The plan of the graph is the ecliptic plan.

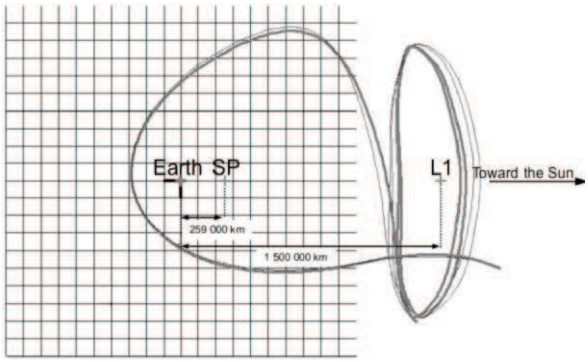

Fig III: Nominal orbit of LPF mission and geometry of the problem. The red thick line is a 3D view of the orbit, from a point above the ecliptic plane. The blue thin line is the projection of the trajectory on the ecliptic plane. The frame is rotating: the Sun lies to the right, the Earth at the centre. Each square represents 100000 km.

I.III. LPF design

As it can be seen in figure I, the spacecraft is composed of two main parts. The first one is the science module. This module contains the LISA Technology Package (LTP), which will realize the main experiment of the mission. It is fully equipped to carry out the mission, once injected on its nominal orbit. The other part of the spacecraft is the propulsion module. It will provide the thrust needed to increase the apogee of the spacecraft, once in orbit around the Earth. As soon as the nominal halo orbit is reached, the propulsion module will separate from the rest of the spacecraft, as it will not be useful anymore.

I.IV. LPF instruments

Because of the magnitude of the effects LPF is meant to measure, the spacecraft will carry on-board instruments having an accuracy greater than every instrument created up to now. The LISA Technology Package will include two gold masses in a near-perfect gravitational free-fall, which relative position will be measured by a picometric-precise laser interferometer. Moreover, the two test masses will be put under “drag free” conditions, which means that they will only be influenced by gravity, and no other force. In order to counter the effects of the solar radiation pressure, the spacecraft is equipped with a micro-propulsion system. This system enables to keep the spacecraft itself precisely positioned around one of the test mass, while the other is controlled by electromagnetic forces when needed. Once the propulsion module of the spacecraft is jettisoned, the micro-propulsion system will be the only possibility existing to manoeuvre.

II. EXTENDING LPF MISSION TO TEST NEW THEORIES

II.I. Theories alternative to Dark Matter

Although nearly adopted as official theory today, Dark Matter still resists the researchers who are looking for it. Indeed, whilst its existence has been postulated in 1937[1], no proof has been provided up to now which establishes the existence of Dark Matter and no Dark Matter candidate particle has been found.

Therefore, many theories have been developed, which do not require a new matter to exist. These theories are rather based on the modification of the gravitation laws. This explains why they have not encountered a great success until now: the gravitational laws seem perfect, at every scale experiments have ever been conducted.

One of the first of those theories has been published in 1983 by Milgrom[2] and is called MOND, for MOdified Newtonian Dynamics. Its principle is that if internal and external accelerations of a system are bellow a threshold value of $a_0 = 10^{-10} m/s^2$ Newton's law are modified. Indeed, the original publication suggested re-writing Newton's second law as follows:

$$F = m\,\mu\left(\frac{a}{a_0}\right)a$$

With $\mu\left(\frac{a}{a_0}\right)$ a function linking the low and large acceleration regime, and

$$\lim_{a \gg a_0} \mu\left(\frac{a}{a_0}\right) = 1$$

Though MOND was accurate enough to successfully predict galactic rotation curves for several types of galaxies, it was not completely recognized until 2004, when Bekenstein published the Tensor Vector Scalar theory[5] (TeVeS). Indeed, this theory includes MOND as non-relativistic limit.

Whichever the particular theory considered, the one developed to avoid the creation of a new mysterious matter all have non-relativistic behaviour similar to MOND.

II.II. A way to test those new theories

Because they differ from General Relativity only if the surrounding gravity is of order a_0, it has never been possible to test any theory evocated in part II.I. The Sun's gravity around the Earth is indeed approximately equal to $6 \cdot 10^{-3} m/s^2$, which is seven orders of magnitude greater than a_0. In order to be far enough from the Sun to be able to measure the effect of MOND theory, a spacecraft would have to be at approximately 7700 astronomical unit from the Sun, which is more than 60 times the distance of Voyager 1, the human crafted object the farthest away from us.

Fortunately, it was showed in 2006[3] that places exist inside the Solar System, were the low gravitational acceleration would enable experiments to be realized: gravitational saddle points (SPs). The Sun Earth saddle point, for example, is the point where the gravity of the Sun equals the gravity of the Earth. It is located on the Sun Earth axis, between the both celestial bodies, at approximately 259000 km from the Earth.

II.III. Possibility for LPF to measure their effects

It has been computed that around the Sun Earth SP, the effects of MOND theory could be measured in an ellipsoid of semi-axes 766 km and 383 km, called the MOND bubble[3].

Moreover, many studies have showed that the gradiometer on-board LISA Pathfinder would be able to realize those measures, if LPF spacecraft could fly by the SP. It has indeed been proven[6] that because of the natural speed LPF spacecraft would have at the SP and of the size of the MOND bubble, the effect of MONDian theories would result in an anomalous gravity gradient signal ideally suited to the sensitivity of LPF gradiometer: the mHz frequency range. Several studies[7,8,9] have even estimated the signal which could be measured.

Therefore, LPF mission provides a unique opportunity to test these theories.

In this context, a mission extension proposal will shortly be submitted to ESA, in order to fly LPF spacecraft by the SP.

II.IV. Challenge issued: very small ΔV

Finding a way for LPF to fly by the SP represents a challenge greater that one may think at first. The main reason for its difficulty is that at the end of the mission the only thrusters available to apply a ΔV to the spacecraft will be the one of the science module. Indeed, the propulsion module is to be jettisoned as soon as its mission is finished, i.e. as soon as the spacecraft reaches its nominal halo orbit. Therefore, the thrust power will be quite low. Moreover, the thrusters of the science module are designed to realize small manoeuvres and keep the spacecraft on its nominal orbit during the six months of the mission. It is estimated that only a ΔV of 4 to 5 m/s will remain at the end of the mission. In comparison, the ΔV used by the spacecraft to reach its halo orbit from its initial low Earth orbit is approximately 3 km/s.

The challenge raised is then to find a way for LPF spacecraft to escape from the vicinity of L1 in the direction of the Earth, and fly by the SP in a reasonable time, with less than 4 m/s of ΔV in total.

III. STUDY ALREADY REALIZED

A first study of the problem was conducted in 2010. The following first describes it, and then explains the reasons why the researches are continued.

III.I. Assumptions

The assumptions made during the first study in order to design trajectories are listed hereafter.

- The position of the SP is supposed to be constant in time, at 259000 km from the Earth on the Sun-Earth axis, between the Earth and the Sun. The real position varies in fact around the one used, because of the eccentricity of the Earth's orbit and of the perturbations from other celestial bodies (e.g. the Moon, Jupiter).
- The celestial bodies considered are the Sun, the Earth and the Moon. This approximation is acceptable for the purpose of these studies, which is to prove the existence of interesting trajectories. The influence of other bodies would have to be taken into account if the aim was to design a real trajectory. The orbits of the Earth around the Sun and the one of the Moon around the Earth are obtained from mathematical models described in the Astronomical Almanac[10]. They integrate the eccentricity of the Earth's orbit, the eccentricity and inclination of the Moon's orbit. The equations of motion are numerically integrated, using a Runge-Kutta technique and Cowell's method.
- The trajectories computed are "drag free": this simplification has been made because it will really be the case of LPF mission nominal orbit. As explained previously, Solar Radiation Pressure should be taken into account if the aim was to design the real mission extension trajectory, but this assumption is legitimate considering the purpose of this study.
- The manoeuvres are modelled as impulsive manoeuvres. This could be surprising at first, because of the low thrust availably on-board. But this hypothesis is considered to be representative even with the micro-propulsion system carried by LPF spacecraft, because of the very small manoeuvres planed (of order 1 m/s). Indeed, qualitatively similar solutions will be expected whether the manoeuvre model employed is impulsive or low thrust.

III.II. First study: Single flyby

The first study of the problem has been realized in 2010 by Toullec[11]. Its aim was to prove that it is possible for LPF spacecraft to reach the SP with only small manoeuvres during the halo orbit, after the end of the nominal mission. At the time of the study, the launcher had not been chosen yet, and ESA was considering using either VEGA (launched from Kourou) or Rockot (launched from Plesetsk).

The study took as starting point halo orbits typical of the one which will be designed by ESOC for LPF. Because of the two launchers possibilities, the halo orbits considered had different characteristics. Indeed, Plesetsk being located at a far greater latitude than Kourou, the inclination of the Low Earth Orbit, and hence the shape of the nominal halo orbit, were not the same in both cases.

The results found were quite encouraging. Indeed, with only very small manoeuvres realized during the halo orbit, trajectories passing close to the SP were found. The smallest distances found between the spacecraft and the SP (along the respective trajectories) were of a few hundred kilometres. The time needed to reach the SP was approximately one year, after the end of the nominal mission.

This work also showed that the dependence of the trajectory from the manoeuvre realized is highly chaotic: with a tiny variation of the manoeuvre taken into account, the following trajectory can completely change. This is on the one hand problematic, because if not taken into account it could prevent from finding a suitable trajectory on which the spacecraft could remain. On the other hand, this chaotic dependence is profitable to find interesting trajectories for a possible LPF mission extension, because of the great number of trajectories reachable with only a very small amount of propellant remaining in the tanks.

III.III. Second study: double flyby

The first part of this second study was realised in 2011 by the author and was published in 2013[12]. The assumption made were the same as the ones of the first study, presented in part III.I. It first concentrated on improving the results obtained previously. By adding manoeuvres, it showed that trajectories flying by the SP and passing exactly through it could be found, even with a total ΔV bellow 4 m/s. To do so, the same orbits typical for LPF were taken as starting points. Interesting results were found using a single manoeuvre planned during the halo phase of the orbit. Then, other manoeuvres were added, at key points during the trajectory. For example, efficient manoeuvres had been found at apogees, and before lunar flybys.

Once the proof had been shown that trajectories suitable for a LPF mission extension could be designed, even under the harsh constraint of the very low thrust available, the study could have ended. But then, the reliability of the results supposed to be obtained while crossing the MONDian bubble was put into question. Although the probability for a false detection should be very low, it was pointed out that flying twice by the SP would annihilate any doubt possible. The decision was thus taken to search for trajectories enabling LPF spacecraft to fly twice by the SP.

The search for double flybys revealed itself more complicated than it could have been thought at the beginning. Several ideas were tested before finally obtaining satisfying results.

The first idea had simply been to select the trajectories found flying once by the SP, and try to control the orbit after the SP flyby. This, however, was

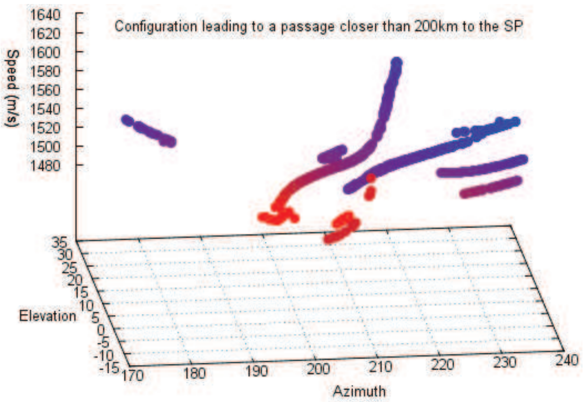

Fig. IV: Relationship between the manoeuvre velocity modulus and direction, for the different families found. The date of the first flyby is the 21/08/2016. The azimuth is the angle between the projection of the ΔV on the ecliptic plane and the Sun. The elevation is the angle between the manoeuvre ΔV and the ecliptic plane.

not satisfactory. Each trajectory designed to fly by the SP once was indeed unique, with precisely determined manoeuvres. Therefore, the velocity vector at the SP was fixed, and could not be modified even by modifying the manoeuvres realized, because otherwise the flyby would have been missed. But then, in order to reach a second time the SP, subsequent manoeuvres more powerful than the one possible for LPF would have been necessary. This idea was consequently abandoned.

Another idea, more successful, was based on the following reasoning: if trajectories flying twice by the SP were to be found, then the spacecraft following one of them would, at some time, be exactly at the SP. So the simplest was to start simulations from the SP, and find the trajectories flying by the SP a second time. This first step was quite successful, and families of trajectories were found and characterized. It was even possible to find for each family relationships between the velocity modulus and the direction of the speed at the SP, ensuring a following flyby with a distance equal to 0 km between the spacecraft and the SP. An example of these families of manoeuvre for a first SP flyby on the 21/08/2016 is presented in figure IV.

The only part missing was the trajectory between the nominal halo orbit of the mission and the trajectories found between the two SP flybys. But this part could not be found, as it was simply not possible to reach the SP with the desired speed and direction from a given halo orbit around L1. Moreover, this time the date of the flyby was fixed because taken as starting date by the previous simulations between the two SP flybys. So even a satisfying velocity vector would not have been enough, if the date when the spacecraft reached the SP was not the one taken as starting date by the simulation between the two SP flybys.

This issue was solved by using reverse-time simulations. Indeed, this enabled to take as a starting date the same starting date as the simulation between the two SP flybys. As good starting conditions (i.e. velocity modulus and speed direction) had already been found, the challenge consisted in finding good trajectories involving manoeuvres realizable by LPF spacecraft. Starting from the conditions found, a small manoeuvre was added at the beginning of the simulation, and the position of the spacecraft was propagated backward in time. Because of the low perigee of many trajectories found by the first step, the small manoeuvre was enough to efficiently control the trajectory in the new simulations. As a result, many interesting trajectories were found, coming from a halo orbit prior to flying twice by the SP.

To sum up, the work done enabled to find trajectories coming from a halo orbit around L1, and flying twice by the SP. All what remained to do was to check whether the halo orbit was suitable for LPF mission, which was the case for many trajectories.

III.IV. Why continuing the study?

We have just seen that the last study realized on the subject enabled to find trajectories satisfying the needs of LPF mission, and at the same time offering the possibility for a double SP flyby after the end of the nominal mission. Why then trying to improve once more the results found?

The study realized in 2011 proved that trajectories linking a halo orbit typical of LPF to a double SP flyby trajectory exist. However, the method used to find them implied that these trajectories cannot be found once the halo orbit around L1 has been fixed, if the halo orbit has not been specially chosen for it to be possible. Therefore there was no way, up to now, to find a trajectory which flies twice by the SP, once in orbit around L1.

IV. RESULTS: WAY TO FIND DOUBLE FLYBYS

The following presents the latest results found in the research for double SP flyby trajectories. Its aim is to define methods enabling to find double SP flybys trajectories, starting from a given halo orbit. It is based, as the previous studies, on a typical LPF halo orbit. The same hypothesizes than the one used for the other studies are adopted here.

IV.I. What makes small manoeuvres efficient to control the flyby distance?

One could be surprised that with ΔVs as small as 1 or 2 m/s, it is possible to efficiently control the trajectory after the manoeuvre. There are several reasons which can explain this.

To begin with, one must not forget that the position of the spacecraft is propagated during a very long time in the simulations. Indeed, there can be up to 1.5 year between the manoeuvre and the first flyby of the SP by the spacecraft. Therefore, the trajectory can diverge, although slowly, from the trajectory without manoeuvre.

Moreover, the evolution of the trajectories in regard to the manoeuvre changes can be chaotic because of the many events that can have an impact on them. For example, if the manoeuvre modifies the time the spacecraft spends in halo orbit around L1, even by just a few days, the way the spacecraft escapes from the halo orbit can be completely modified. Then, the following trajectory will be completely different from the one without manoeuvre. Figure V gives two examples of trajectories obtained with a manoeuvre bellow 2 m/s, which are completely different.

The Moon has also a great impact on every trajectory between the halo orbit considered and the SP. Indeed, the SP is located between the Moon orbit and the Earth. So LPF will necessarily cross it, while trying to reach the SP. If by any chance the Moon is located close enough, its gravity will greatly modify the trajectory. For example, the second trajectory presented in figure V realizes two lunar flybys.

The presence of the Moon will have a very important impact on LPF trajectories during a possible mission extension. Indeed, as ESA decided that the launcher will be VEGA, the launch will take place from Kourou (French Guyana), at a latitude of approximately 5 degrees north. Therefore, the initial Low Earth Orbit of LPF spacecraft will have a relatively low inclination, compared to what would have been the case with a Rockot launch from Plesetsk (which has a latitude of approximately 63 degrees north). As a result, its halo orbit will have a lower out of ecliptic component, and it will also be the case for the following trajectory. This will increase the chance for the spacecraft to be close to the Moon while approaching the Earth.

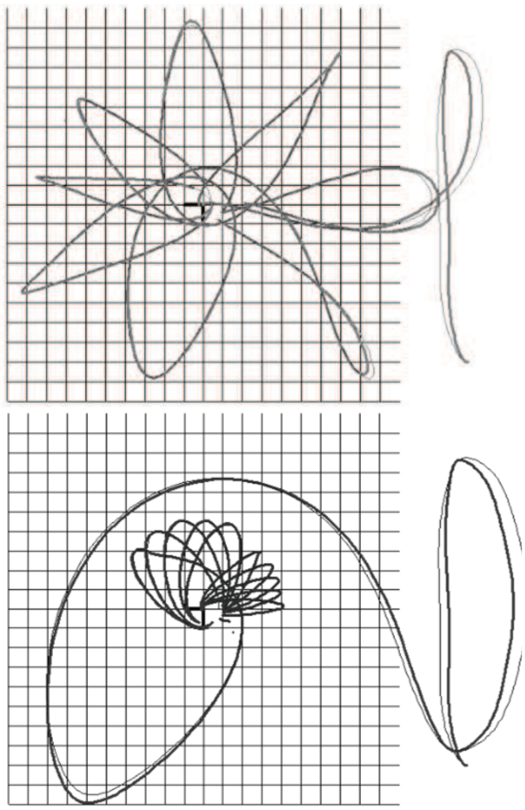

Fig. V: Examples of different orbits found, resulting from different very small manoeuvres (bellow 2 m/s) realised during the halo orbit. See figure III for frame definition

IV.II. First method: reaching one the orbit families previously characterized

The first method tested in this new study consisted in starting from a given halo orbit, and trying to reach one of the families found in 2011, and presented in figure IV for a first SP flyby on the 21/08/2016. It is known that such families exist for every date considered, so they also exist for the date when the spacecraft would reach the SP for the first time.

This method did not give any satisfying results however, because it encountered the same problem as before: as the ΔV possibility for LISA pathfinder are very small, it is not possible to modify the velocity vector at the SP enough to reach a velocity known to belong to one of the interesting orbit families.

IV.III. Second method: direct double flyby optimization

The following method consists in starting simulations from a given halo orbit, and optimizing directly the double flybys. To do so, a first simulation is launched, to analyse the result of a ΔV in every direction possible. If configurations exist for which the spacecraft passes in the vicinity of the SP twice after the manoeuvre, they are optimized. This means that only one manoeuvre is realized during the whole trajectory.

IV.III.I. Optimization algorithm

In order to optimize a trajectory, a criterion must be chosen and measured, which will be the optimized parameter. As this method aims to directly optimize the two SP flybys with only one manoeuvre, the parameter chosen is the sum of the two smallest distances between the SP and the spacecraft, along the trajectory.

Because of the chaotic dependence between the manoeuvres and the flyby distance, standard techniques as for example Newton's method did not appear to be very efficient to minimize the flyby distance. Therefore, other algorithms were created, which were less likely to stay blocked in a local minimum.

Indeed, it was chosen to adapt a method of conjugate gradient to the problem. The parameters optimized in order to minimize the total flyby distance are the velocity modulus and the direction of the manoeuvre ΔV. As it is done in the conjugate gradient method, each step of the optimization process minimizes the total flyby distance in regard to one of the two parameters.

In our case, the first step minimizes, with a constant manoeuvre direction, the total flyby distance in regard to the manoeuvre ΔV. It is done by computing the total flyby distance for 11 ΔVs regularly spaced within a range predefined $[\Delta V_{min}\ \Delta V_{max}]$. The ΔV corresponding to the minimum distance (ΔV_{opti}) is then taken as reference, and the range for the following ΔV optimization step is defined as:

$$\left[\Delta V_{opti} - \frac{\Delta V_{max} - \Delta V_{min}}{10};\ \Delta V_{opti} + \frac{\Delta V_{max} - \Delta V_{min}}{10}\right].$$

The second step works on the direction of the manoeuvre. The same process is applied than for the ΔV. The only difference is that both elevation and azimuth of the manoeuvre are optimized at the same time. To do so, the sum of the two flyby distances is computed for 121 directions regularly spaced within $[azi_{min}\ azi_{max}] \times [ele_{min}\ ele_{max}]$, where azi is the angle between the projection of the ΔV on the ecliptic plane and the Sun (the azimuth), and ele is the angle between the manoeuvre ΔV and the ecliptic plane (the elevation).

The direction corresponding to the minimum distance is then taken as reference, and the range for the following direction optimization step is defined as:

$$\left[azi_{opti} - \frac{azi_{max} - azi_{min}}{10};\ azi_{opti} + \frac{azi_{max} - azi_{min}}{10}\right] \times \left[ele_{opti} - \frac{ele_{max} - ele_{min}}{10};\ ele_{opti} + \frac{ele_{max} - ele_{min}}{10}\right].$$

At the end of those two steps, they are performed again, until it is decided that the optimization could not find much better.

To decide whether to stop the simulation, the criterion used is the following: if the ratio $\frac{\text{last total distance} - \text{new total distance}}{\text{last total distance}}$ is greater than 2%, then the simulation continues. If it is not the case, the simulation stops.

IV.III.II. Results

This process enables to find manoeuvres leading to quite satisfying results. Figure V gives some examples of the cases which might be encountered. One can notice that some cases use a lunar flyby, as the second one of figure V for example. This modifies efficiently the subsequent trajectory, but should be used with care, as it could be quite difficult to control with a small thrust capacity.

However, some cases having a total flyby distance as low as 10000 km, the results found are encouraging.

IV.III.III. Need to optimize also the date or the manoeuvre

There is a reason which might explain why the optimization is not able to find results flying twice exactly by the SP. Indeed, the time of the manoeuvre is not optimized in this process. It means that the

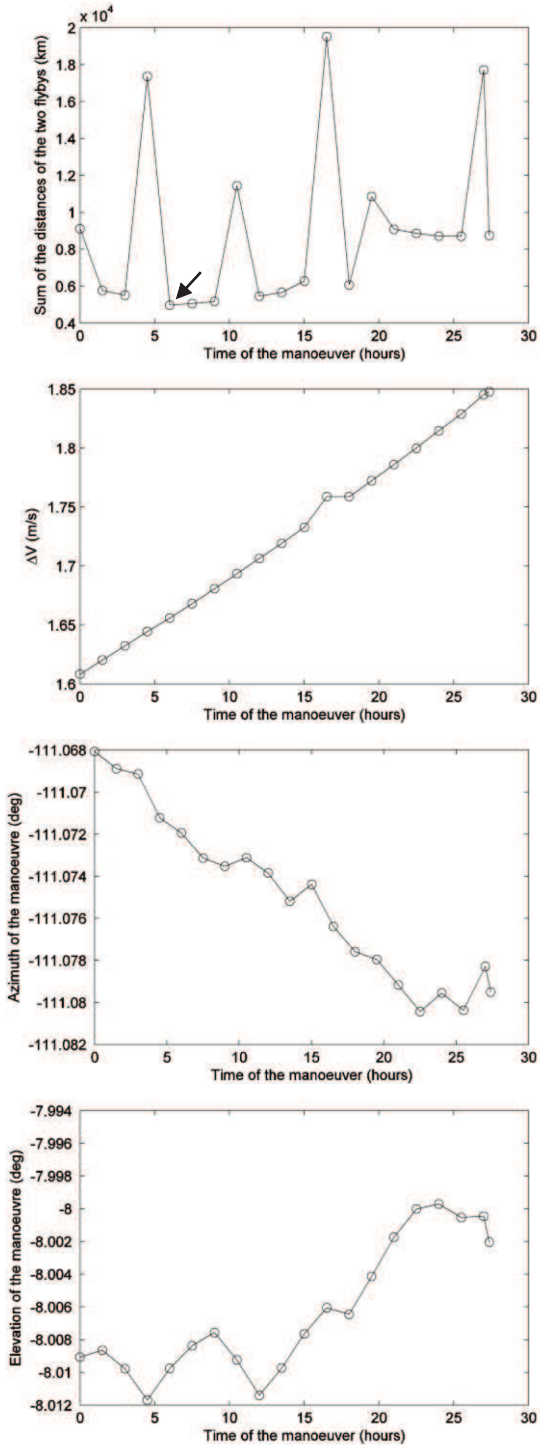

Fig. VI: Evolution of the manoeuvre and of the sum of the two flybys distances, when the manoeuvre time changes.
Graph 1 presents the total distance.
Graph 2 presents the manoeuvre ΔV.
Graph 3 presents the manoeuvre azimuth.
Graph 4 presents the manoeuvre elevation.

	Time (d)	ΔV (m/s)	Azimuth (deg)	Elevation (deg)	1st flyby dist. (km)	1st flyby time (d)	2nd flyby dist. (km)	2nd flyby time (d)	Total dist. (km)
Without 2nd manoeuvre					**3616.2**	**419.8**	**1363.2**	**570.9**	4979.4
With 2nd manoeuvre	360	1.0035	105.4	-41.19	**2510.8**	**419.8**	**0.03**	**570.7**	2510.8

Table I: Result of a second manoeuvre realised after the one marked by an arrow in the first graph of figure VI. One can see that the second flyby distance is equal to zero, while the first is still high.

algorithm tries to determine the best speed vector enabling to reach two different points, while a third one is also already fixed. In order to really optimize the double flyby, the time of the manoeuvre should also be variable and taken as parameter in the optimization process.

Rather than optimizing also the time, the decision has been taken to look for the evolution of the direction and ΔV optimums, when the date of manoeuvre changes. To do so, an algorithm has been created, which enables to follow these evolutions.

At first, a reference manoeuvre is optimized, and the direction and ΔV optimums are stored. An optimization is then started, with the same manoeuvre happening some time later. The new result is then kept in memory, and the process starts again.

This process is quite successful in improving the results obtained before: the sum of the two flyby distance can nearly be divided by four. Figure VI shows the evolution of the total flyby distance, and of the manoeuvre itself, when the manoeuvre time changes. As showed by the plots, this method enables to find the best date for the manoeuvre: one can simply chose the date when the total flyby distance is the smallest.

IV.III.IV Adding a second manoeuvre

This method is not limited to a manoeuvre taking place during the halo orbit. Indeed, once a first good manoeuvre has been found, it is possible to add more manoeuvres, in order to reduce the sum of the two flyby distances. This other manoeuvre can also be optimised to minimise the total flyby distance. Doing so enables to reduce once more the flyby distances. For example, a manoeuvre realised 60 days before the first SP flyby, in the case of the trajectory marked by an arrow in figure VI (first graph) enables to reduce the total distance from 4979 km to 2510 km in total for the two flybys. The result of this manoeuvre is presented in table I.

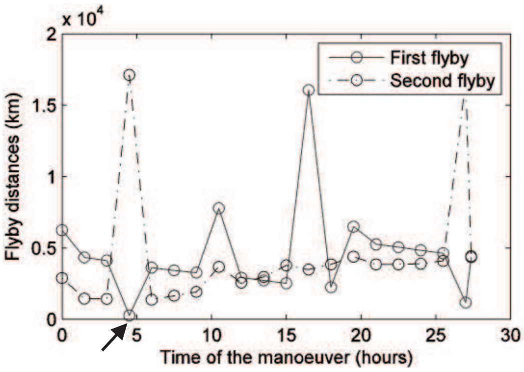

Fig. VII: First and second flyby distance, for the same manoeuvres than the ones presented in figure VI

IV.III.V. Limits of the method

For the different trajectories found during this study, the delay between the first and the second SP flybys is of order a hundred days. Therefore, the second flyby is much more affected by the manoeuvre realized during the orbit before the first flyby. So when the manoeuvre is optimized, the value weighing the most in the total SP flyby distance is the distance of the second passage. Therefore, minimums can be found for which the first flyby is not really optimized. It is the case for the result presented in table I. Indeed, although the second flyby distance is nullified by the computed manoeuvre, the first one stays higher. This is quite problematic. Indeed, adding a manoeuvre before the first flyby to reduce its distance would then dramatically increase the distance of the second one. Moreover, the later this manoeuvre is done, the higher the ΔV would need to be. However, there is a solution to this problem, which is presented in part IV.IV.

IV.IV. Adding a manoeuvre after the first flyby

Just as results have been found (part IV.III.) for which the distance of the first flyby was a few thousands kilometres and the distance of the second was near zero (see table I), it is possible to find results for which the first flyby distance is close to zero, and the second is of a few thousands kilometres. It is the case for the manoeuvre designed by the arrow in figure V for example. This is a very interesting opportunity. Indeed, if a single manoeuvre enables to reach exactly the SP and fly at a distance of a few thousand kilometres from the SP after the first flyby, adding a second manoeuvre after the first flyby should permit to fly exactly through the SP during the second flyby.

To find these trajectories, another algorithm has been developed. Its first step is the same than the previous algorithm: find interesting double flyby trajectories, with a single manoeuvre during the halo orbit phase of LPF trajectory. The interesting manoeuvres are then first optimized by direct optimization of the two flybys, in order to assess the total distance of the two flybys. If it is bellow a threshold of a few thousands of kilometres, the case is processed by a last algorithm.

This last algorithm optimizes the distance of the first flyby, using the same optimization process than the one described in part IV.III. Two values are then kept in memory: the distance of the first flyby on the one hand, and the distance of the second one on the other hand.

When interesting trajectories are found, a manoeuvre is added after the first flyby to reduce the second flyby distance. Therefore, the interest of optimizing as well as possible the first manoeuvre is to reduce the ΔV needed to realize the second manoeuvre.

It is not always possible to find a solution flying twice by the SP. Indeed, if the second flyby distance is too high, then LPF spacecraft's propulsion is not powerful enough to improve much the second flyby distance. For the case with an arrow in figure VII for example, the second flyby distance cannot be reduced bellow 16000 km with a ΔV maximum of 3 m/s.

However, some features have been characterised, which enable to find better trajectories. For example, passing in the vicinity of the Lagrangian points L1 or L2 between the two SP flybys offers the possibility to efficiently control the second one. Moreover, the longest the time between the two flybys, the more efficient the manoeuvre after the first one can be.

V. CONCLUSIONS

The first part of this study proved that double SP flyby trajectories suitable for LPF existed, but did not provide a way to find them once the spacecraft in nominal orbit. The study was therefore continued, and the challenge tackled in a more direct way. Indeed, the purpose of this paper was to demonstrate that it is possible to find double SP flyby trajectories once the actual mission orbit fixed.

To do so, several methods were tested, among which one revealed itself particularly promising: direct optimisation of the sum of two SP flyby distances. It enabled to find trajectories having a total flyby distance as low as 2500 km, with a total ΔV bellow 3 m/s. Moreover, the time between the end of the mission and the first flyby was of order 250 days, which is very satisfying.

Therefore it is believed that this method, combined with a manoeuvre between the two SP flybys, should enable the European Space Operations Centre to design a satisfying trajectory for a possible LISA Pathfinder mission extension once LPF spacecraft in halo orbit.

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