Multiple Sun–Earth Saddle Point flybys for LISA Pathfinder

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Abstract

LISA Pathfinder is an ESA mission due to be launched in the next two years. The gravity gradiometer onboard has the sensitivity required to test predictions by gravitational theories proposed as alternatives to Dark Matter such as TeVeS. Within the Solar System measurable effects are predicted only in the vicinity of gravitational saddle points (SP). For this reason it has been proposed to fly LPF by the Earth–Sun SP, at some 259,000 km from Earth. This could be done in an extension to the nominal mission which uses a Lissajous orbit about the Earth–Sun L1 point. The responsibility for LPF mission design lies with ESA/ESOC, who have designed the transfer trajectories, orbits about L1, and station keeping strategies. This article describes an analysis performed by Astrium to support a suggestion for a possible mission extension to a saddle point crossing. With only very limited fuel availability, reaching the saddle point is a significant challenge. In this article, we present recent advances in the work on trajectory design. It is demonstrated that reaching the SP is feasible once the LPF mission is completed. Furthermore, in a significant enhancement, it is demonstrated that trajectories including more than one SP flyby are possible, thus improving the science return for this proposed mission extension.

Keywords: LISA Pathfinder; MOND; Saddle-point; Flyby; Trajectory

1. Introduction and motivation

The nature of Dark Matter remains a complete mystery, more than 70 years after its existence was first postulated in order to explain the observed rotation curves of galaxies (Zwicky, 1937). Despite concerted searches over many decades, to date no Dark Matter candidate particles have been detected. An alternative view of the Dark Matter problem is that the observed rotation curves are in fact a result of modified gravitational laws, rather than due to the existence of invisible matter. The fact that standard gravitation (General Relativity and Newtonian gravity as its non-relativistic limit) agrees extremely well with experimental observations on all accessible scales, means that this view has had only limited followers.

Nevertheless, a simple phenomenological formula that predicts deviations from Newtonian gravity in regions of extremely weak acceleration was proposed in order to account for galactic rotation curves (Milgrom, 1983). In a nutshell, the idea is that if internal and external accelerations of a system are below a threshold value of $a_0 = 10^{-10}$ m/s^2, Newtonian dynamics within that system are modified. In the original proposal, Newton's second law was re-written as

$$F = m \cdot \mu\left(\frac{a}{a_0}\right) \cdot a \tag{1}$$

With $\mu(\frac{a}{a_0})$ a transition function connecting the large and the low acceleration regimes, with appropriate limits. The simple prescription proposed by Milgrom, known as MOdified Newtonian Dynamics (MOND), had remarkable success in describing galactic rotation curves, and indeed was used to make predictions for various types of galaxies that were later confirmed.

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Eq. (1) naturally lends itself to the interpretation that inertia is modified, depending on the environment. However, given that the anomalous behaviour is only observed in systems dominated by gravitation, the observations themselves can be equally well described by an equivalent modification of gravity – Newton's first law.

The two interpretations (modified inertia vs modified gravity) are conceptually very different. In more recent years, the emphasis has been firmly on modified gravity, with the development of various relativistic theories reproducing the above MONDian behaviour in the non-relativistic regime. In fact, MOND was not taken seriously until much later, when the first relativistic theory was developed with MOND in the non-relativistic limit – the Tensor–Vector–Scalar (TeVeS) theory (Bekenstein, 2004). Other theories have been developed since; the main commonality is the non-relativistic regime, which reproduces the MONDian behaviour observed in galaxies.

Regardless of the particular theory, the fact remains that deviations from General Relativity are predicted only in regions of very low gravitational background acceleration, of order a_0, and therefore virtually inaccessible to direct experimentation. For example, Earth is exposed to a background acceleration of around 6×10^{-3} m/s^2 as it orbits the Sun – more than seven orders of magnitude larger than a_0. It was pointed out in 2006, however, that gravitational saddle points (SPs) offer regions of low gravitational acceleration even deep inside the solar system (Bekenstein and Magueijo, 2006). The order of magnitude of the anomalous gravity gradients predicted by TeVeS was estimated inside an ellipsoid of semi-axes 766 km and 383 km around the Earth–Sun saddle point. It was concluded that these could potentially be measureable by a gradiometer such as the one flying on LISA Pathfinder (LPF) – however the nominal LPF mission meant that the spacecraft, once operational, would remain at a distance of order 10^6 km from the SP, and at such a distance the effect would be unmeasureably small. Indeed, the SP of interest here is the one resulting from the equality of the Sun's and the Earth's gravitation, not taking into accounts the centrifugal forces. Thus, it is located approximately at 259,000 km from the Earth toward the direction of the Sun (see Fig. 1).

This was the state of affairs until it was suggested that, despite the limitations of its micropropulsion system, LPF could perhaps be made to fly past the Sun–Earth SP once its nominal mission was completed. The responsibility for the mission design for LPF lies with ESA/ESOC and the overriding goal of the mission is the testing of the main scientific payloads. However one possible mission extension that could allow the testing of the above mentioned theories has been suggested. It was also noticed at the same time that the natural speed of the spacecraft, of order 1 km/s, could potentially combine with the region size of order 1,000 km to produce a temporal signature of anomalous gravity gradients in the mHz frequency range – ideally suited to the sensitivity of the gradiometer on-board LPF (Trenkel and Kemble, 2009).

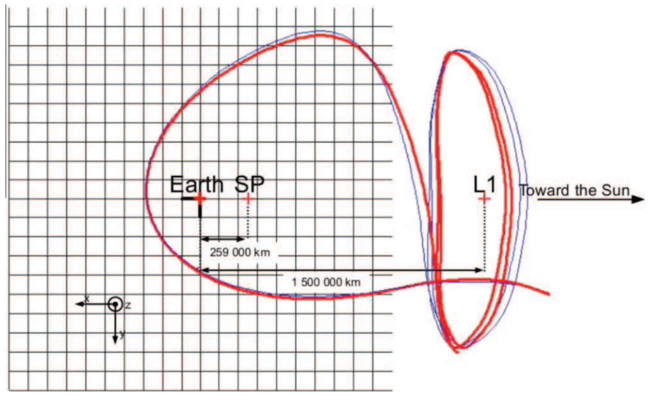


Fig. 1. Geometry of the problem – the reference frame is rotating, in order to keep the Earth–Sun direction constant. Grid is 1 million km from centre to edge. Red line represents the orbit seen in 3 dimensions from a point located along the z axis. Blue line shows the projection of the orbit on the Ecliptic plan. This representation is used for every orbit presented in this paper, unless another reference frame is precised. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Substantial progress on this idea has been made since then:

- numerical methods have been developed to produce robust estimates of the signals predicted by TeVeS, and also for other theories (Bevis et al., 2010; Magueijo and Mozaffari, 2012; Galianni et al., 2012)
- the LPF gradiometer instrument noise estimate has been consolidated as more and more of the actual flight hardware has been tested in preparation for the mission (Antonucci et al., 2011)
- the problem of finding suitable trajectories for LISA Pathfinder, including a preliminary assessment of the navigation challenge, has been progressed significantly (Toullec et al., 2010)

A general overview of the main issues associated with using LPF itself to conduct a direct test of MOND as derived by TeVeS can be found in Trenkel et al. (2012). An excellent description of the LISA Pathfinder mission, the spacecraft and the payload on-board, can be found in ESA-SCI (2007), in addition to Antonucci et al. (2011).

The work carried out to date appears to indicate that all the challenges can be overcome, and that such a direct test, with a considerable Signal-to-Noise-Ratio (SNR) for gravitational gradients derived from TeVeS, is feasible. It is currently planned to submit a mission extension proposal to ESA to fly LPF past the SP and to carry out this test. LPF presents us with a unique opportunity to conduct such a test in the foreseeable future at relatively low cost.

In this article, we present recent advances in the work on trajectories that we have been conducting, with a particular focus on identifying trajectories including more than one SP flyby.

Previous work identified solutions with just one SP flyby. The large predicted SNR, together with the fact that the time at which the anomalous signal should be detected is known precisely (namely when the spacecraft crosses the region near the SP), should result in an extremely low false alarm rate. Nevertheless, the signals predicted within other theoretical frameworks are considerably smaller, and in any event even the confidence in a positive result would increase dramatically if it could be confirmed in a repeat experiment.

This paper firstly presents, in Section 2, the nominal LPF mission, in terms of its trajectory and key spacecraft systems. This is followed in Section 3 by an overview of work carried out in previous studies on this problem and how these have been refined by the inclusion of additional manoeuvres. In Section 4 the objectives of the extended mission are expanded to include two SP crossings. A modified method using multiple shooting and local optimisation was then used to generate suitable trajectories.

2. Description of the nominal LPF mission

In this section we review, briefly, the elements of the LPF mission design that are relevant to the search for attractive trajectories from the nominal orbit to the Sun–Earth saddle point. For all other elements of the mission, the reader is referred to ESA-SCI (2007); Kemble et al. (2004); Landgraf et al. (2005).

LISA Pathfinder is a spacecraft currently under construction, which has been designed as a technology demonstrator for the LISA mission. The LPF spacecraft is composed of two different parts, as can be seen in Fig. 2. The first one is the propulsion module, containing approximately 1,300 kg of fuel. This module will enable LPF to reach its nominal orbit, after having been injected initially into a low Earth orbit. Once its operational orbit is reached, the propulsion module will separate from the spacecraft. The second module is carrying the LISA Technology Package. In order to control the attitude and the orbit of this science module once the propulsion module is separated from the spacecraft, a system of micro propulsion is being developed. It will enable LPF to stay on its nominal orbit and will provide the so-called "drag-free" environment that the instrument requires to achieve its performance.

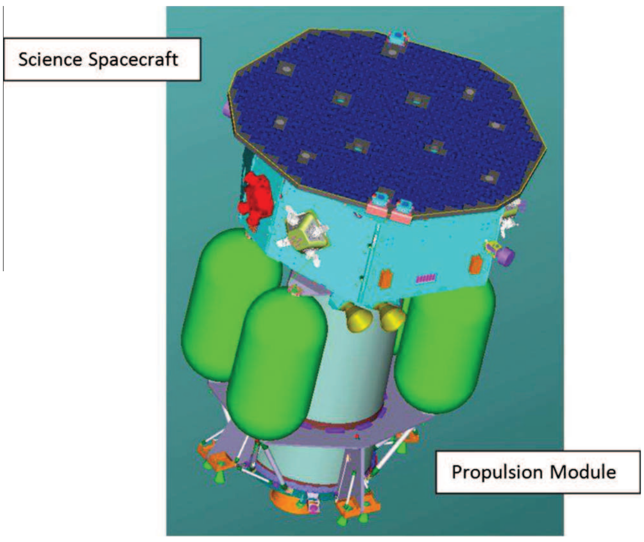


Fig. 2. LISA Pathfinder launch composite module.

The performance of the micropropulsion system constrain the search for potential trajectories which is the main subject of this paper, and the main characteristics are therefore described in some more detail.

The micropropulsion system on-board LPF is based on cold gas micro-thrusters and uses pressurized nitrogen as propellant. For the purposes of the work here, two key performance parameters are of relevance. The first one is the maximum net thrust that can be applied to the spacecraft. This is more than 500 μN. This maximum thrust will be available even in the case of a single failure. Given a spacecraft mass of approximately 450 kg, applying a 1 m/s manoeuvre will take around 9×10^5 s, or 10 days. Any manoeuvres of this magnitude will be applied only at points in time when the spacecraft is moving slowly (around L1 initially, and at subsequent apogees).

The second key parameter is the total amount of propellant available to any mission extension. A detailed propellant budget, based on conservative assumptions about specific impulse, operational modes, and disturbance levels, has been generated. According to this budget, and providing there have been no critical failures, the amount of cold gas at the end of the nominal mission could allow to apply a total ΔV of, typically, between 4 and 5 m/s to the spacecraft.

The nominal operational orbit is a Lissajous orbit about the Earth–Sun L1 point. The characteristics of such an orbit can be found in Gómez et al. (2002) or Gómez et al. (2001).

This has been chosen to ensure a very stable environment around the spacecraft. Indeed, due to the precision the instruments carried by LPF have to reach, any possible noise has to be carefully controlled. Furthermore, non-gravitational perturbations must be minimised.

The properties of the orbit chosen by ESOC will make it possible for LPF to perform all of the required orbit maintenance manoeuvres, even with the very small thrust available from its propulsion system. Perturbations at the Lagrange point are relatively low and therefore a Lissajous orbit becomes a near-stable orbit around a Lagrangian point, maintained by small correction manoeuvres. Its motion takes place not only in the ecliptic plane, but also in the plane perpendicular to it, with slightly different periods of motion in each direction. Halo orbits are a special case of a Lissajous orbit where the two periods are identical, with implications for the relationship between in and out of ecliptic amplitudes. The Lissajous orbit targeted for LPF is chosen to restrict the difference in amplitudes between in and out of ecliptic motions, to facilitate the communications solution.

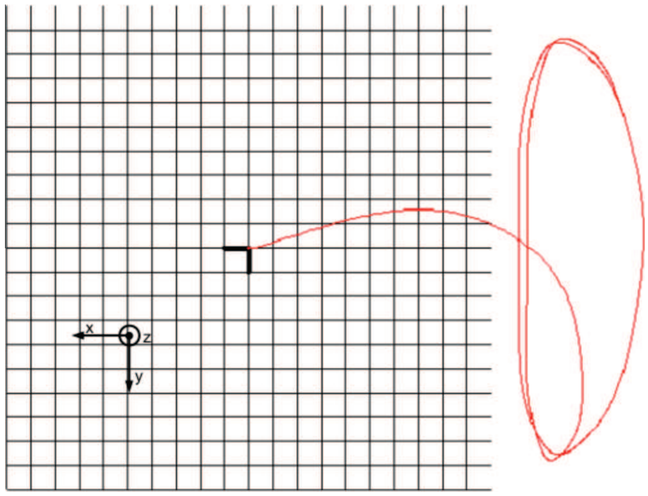


Fig. 3. Transfer and operational orbit achieved with free injection; View from ecliptic north in the rotating reference frame – grid is 1 million km from centre to edge. Sun lies to the right (see Fig. 1).

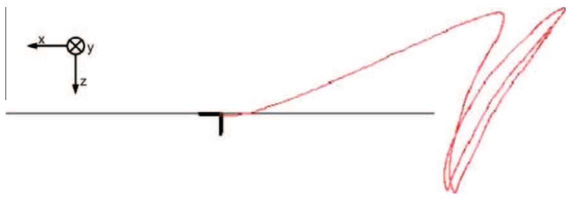


Fig. 4. Transfer and operational orbit; View from in Ecliptic showing motion out of ecliptic in the rotating reference frame – grid is 1 million km from centre to edge. Sun lies to the right (see Fig. 1).

The strategy used to reach the operational orbit is one of "free injection", presented in Figs. 3 and 4. The stable manifold of a Lissajous orbit can start from a point close to the Earth, from where the spacecraft can drift from its initially highly elliptical orbit into a Lissajous orbit, without manoeuvre (Kemble, 2006). In order to efficiently utilise the launcher, LPF is initially injected into a low Earth orbit, with apogee in the region of 900 to 1,300 km, depending on choice of launcher. Two launcher options are retained, Rockot and VEGA. The spacecrafts propulsion module is then used to deliver a ΔV of approximately 3 km/s to raise its apogee to a value in the range 1.3 to 1.5 million km, as presented in Fig. 5. From this highly elliptical orbit free injection can be achieved.

The characteristics of the free injection Lissajous orbit (in and out of ecliptic amplitudes and phasing) can be chosen by selecting appropriate combinations of apogee altitude, inclination, right ascension of ascending node and argument of perigee for the initially high elliptical orbit.

Maximising launcher injection mass means in general restricting inclination to a value close to the latitude of the launch site – allowing an Eastward launch. This lies close to 63° for a Rockot launch and 6° for VEGA. Choice of injection orbit Right Ascension of Ascending Node (RAAN) is free (depending on launch time) and significant freedom also exists in argument of perigee. The apogee raising strategy does not in general aim to modify the other orbit elements as this can be expensive in ΔV. Therefore the choice of launch vehicle primarily effects the inclination of the high elliptical orbit. This in turn influences the achievable out of ecliptic amplitude of the free injection Lissajous orbit. Low inclinations can only achieve low out of ecliptic amplitudes. The in-eclipic amplitude can be chosen with some freedom (providing it lies above the minimum achievable with free injection, circa 750,000 km). Typically it will lie in the range 800,000 to 900,000 km.

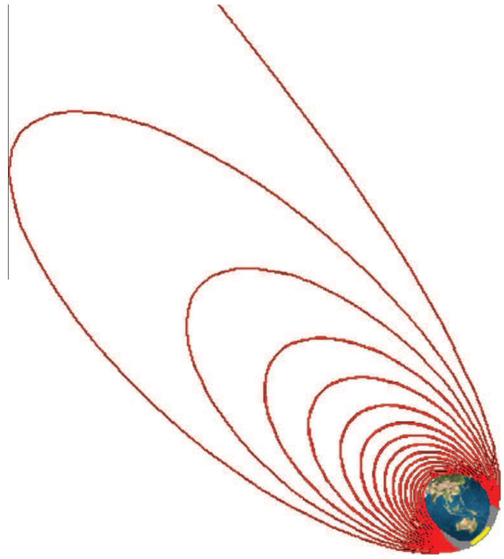


Fig. 5. Example of an Apogee raising sequence – starting from LEO. Earth centred inertial frame used.

A wide launch window for LPF is retained by ESOC, with launch available on most days of the year. The result is that due to the 23° plane difference between Earth's equator and the ecliptic, the out of ecliptic amplitude reached with the VEGA launch varies throughout the launch year, whilst with Rockot a near constant value of circa 500,000 km can be reached year round. Therefore the operational orbit used by LPF will differ with each launch day, and also differ between Rockot and VEGA launches.

3. Reaching the SP from a given libration orbit

3.1. Constraints and assumptions

An important characteristic of Lissajous orbits is that with only a small velocity increment, it is possible to step onto an unstable manifold. This is a trajectory that exponentially diverges from the nominal Lissajous orbit. Therefore in order to find a way to reach the SP with LPF once its mission completed, the most natural idea is to start simulations from an orbit typical of LPF at the end of its mission, but subject to small velocity perturbations. Then simulations can be run in order to test whether reaching the SP is possible. Some assumptions have to be made before running the simulations and these are now described.

In order to stay compliant with LPF propulsion capabilities, in terms of both ΔV and thrust authority, only very small manoeuvres are considered (less than 1.5 m/s in total). These manoeuvres are currently modelled as impulsive, although finite, low thrust effects could be simulated. The limited manoeuvre size and acceleration available via the propulsion system means that qualitatively similar solutions will be expected whether the manoeuvre model employed is impulsive or low thrust. The position and size of the MOND bubble around the SP is considered constant in time. Its centre is taken constant on the Sun–Earth axis, at 259,000 km from the Earth in the direction of the Sun. This is an approximation as in reality its location is dynamic due to the eccentricity of Earth's orbit and perturbations from other bodies. Variations of a few thousand km with respect to the above fixed rotating location result.

This investigation can only be performed after the end of the nominal LPF mission, assuming that the spacecraft remains in good health.

3.2. Simulation aspects

The simulation used in these cases considers only the gravitational influences of Earth, Sun and Moon and is therefore an approximation to the full set of influences acting on the system. However it is adequate for demonstration of the feasibility of orbit designs that can cross the SP, which is the objective here. The orbits of the Earth about the Sun and of the Moon about the Earth are obtained from mathematical models described in Astronomical Almanac (2012) and therefore reflect the eccentricity of the both orbits and the inclination of the Moon's orbit. Hence, approximations such as use of the circular restricted three body problem are not employed here. Instead, the equations of motion under the influence of the three gravity fields are numerically integrated using a standard Runge–Kutta technique and Cowell's method. A more detailed design should subsequently include other gravitational perturbations from other major bodies in the Solar System and also the influence of Solar Radiation Pressure.

Therefore whilst it is true that the details of solutions to this problem will differ with the details of the mathematical models used, the nature of the solutions to the problem can be established with the above described approximations. This paper aims to describe a methodology, which if applied would allow similar solutions to be obtained using off the shelf orbit propagation packages. In this analysis multi-purpose, in-house mission design tools are used. Only deterministic manoeuvres are considered here, navigation aspects have not yet been assessed in detail. Similarly the initial Lissajous orbit properties are described in generic terms, as solutions to this problem can be found for a wide range of Lissajous orbits. For LPF the Lissajous orbit properties are different for each launch day, over a possible year of launches.

3.3. Previous study

A first study of this problem was performed previously (Toullec et al., 2010). As described in this paper, the goal had been to reach the SP from a Lissajous orbit with characteristics typical of those achieved after a Rockot or VEGA launch. In order to reach it, a manoeuvre was applied during the Lissajous orbit phase, followed by an optional second one, at a subsequent apogee. Only very restricted manoeuvres were considered. The most efficient direction to apply such an initial manoeuvre in order to leave the Lissajous orbit is in the escape direction (Hechler et al., 2002) lying at 28.5° from the Earth–Sun direction and parallel to the ecliptic. Many types of solutions exist, with an appropriate combination of first manoeuvre epoch and manoeuvre size. Trajectories that return towards Earth or even escape into a heliocentric orbit can be found. Search techniques based on these two variables are effective in finding solutions with the required properties. In order to cross the SP, solutions usually require several revolutions with high apogee around the Earth. In some cases solutions flying close to the Moon can make use of Lunar gravity to modify the orbit and approach the SP. In the case of a Rockot launch the initial orbit considered has an in-ecliptic amplitude of approximately 800,000 km and out of ecliptic amplitude of 500,000 km. In the case of a VEGA launch the out of ecliptic amplitude is typically 100,000 to 300,000 km, depending on launch date. Solutions can be further tuned by employing manoeuvres at subsequent apogees after leaving the Lissajous orbit. Where such manoeuvres are used they are nominally assumed to be tangential with the velocity direction. Encouraging results were found, the best ones reaching a flyby distance around 100 km far from the SP, for a journey time of roughly one year after a start time considered to be the end of the nominal mission.

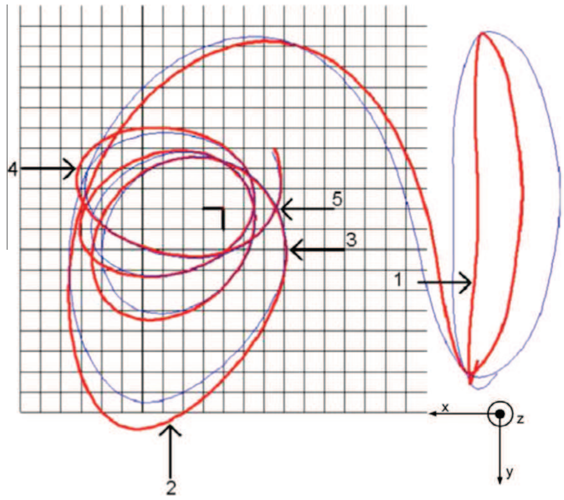
3.4. Improvement

The first part of the analysis described in this paper has been to improve the previous results. In order to reduce the flyby distance from the SP, more manoeuvres were added at key points during the orbit (before a possible lunar flyby or at apogees for example). Results were found for which the saddle point was exactly reached (the flyby distance being zero). Subsequent manoeuvres considered are now tangential to the velocity vector. Epoch of manoeuvre and manoeuvre size can be selected appropriately. Typically the final manoeuvre now occurs at the apogee preceeding the SP approach, from where miss distance can be efficiently reduced. The example shown in Table 1 presents one of these cases.

These results indicate that, once the Lissajous orbit is precisely characterised, it will be possible to reach the saddle point with a low global ΔV budget.

Lunar gravity assists are used in cases such as the one presented here. Although useful because of the limited

Table 1
Case reaching exactly the SP.

Lissajous orbit type: following Rockot launch (determines the out-of-ecliptic component) 1: first manoeuvre (0.86 m/s), used to escape from the Lissajous orbit. 2: 147 days after escaping from L1. Second manoeuvre (1 m/s), used to control the lunar flyby occurring at 3, 20 days later. 3: 167 days after escaping from L1. Lunar flyby. 4: 265 days after escaping from L1. Last apogee before reaching the SP. Manoeuvre (0.4 m/s) used to precisely control the flyby distance. 5: 285 days after escaping from L1. Reaching the SP. Total $\Delta V < 2.3$ m/s	 This plot shows the orbit as seen from a viewpoint removed perpendicularly from the ecliptic plane. See Fig. 1 for the geometry.

ΔV availability, they must nevertheless be handled with care: despite the possibility to use them to modify efficiently the orbit, it may be difficult to control them. The LPF current design uses a micro-Newton propulsion system; it is therefore less manoeuvrable than a satellite using a standard chemical propulsion system with higher thrust. Correction of dispersions occurring during such flybys can be relatively expensive to realise. Accurately targeting the flyby condition so as to minimise these dispersions is made more difficult by the low acceleration available from the propulsion system. In general only a limited time window exists within which such corrections can efficiently be executed. These aspects have not been assessed in this study.

4. Finding solutions with multiple SP flybys

4.1. Finding several flybys

The constraints and assumptions used are exactly the same as the ones described in part 3.1.

From the first part of the study, it seems that reaching the saddle point from a Lissajous orbit is feasible. Thus it should be possible to satisfy the basic goals of the mission extension. But as described in part 1, it would be really better if the experiment could be performed twice, or even more. Finding trajectories with multiple SP crossings is not an easy task. The difficulty of propagating long term trajectories must be considered in the context of their sensitivity to initial conditions.

Consequently, in order to find an orbit passing through the saddle point several times two different methods were studied. The first one consists of selecting a Lissajous orbit resulting from a typical Rockot or Vega launch and applying ΔV to reach the SP exactly; then extra manoeuvres are added after the first time the spacecraft reaches the SP. The second method consists of studying directly orbits passing twice through the SP, and checking if they can be reached from the nominal mission orbit.

4.1.1. Adding a manoeuvre after reaching the SP for the first time

In this methodology, the best cases found in the first part of the study were taken as initial conditions. For an orbit reaching the SP after a libration orbit escape, manoeuvres were added after flying by the SP. The aim of these manoeuvres was to reach the SP a second time. The results obtained were not completely successful. For the case presented in Table 1 for example, even adding two successive different manoeuvres (with ΔV constraints already discussed) does not reduce the flyby distance below a few 1000 kms. This method could however be studied further, as it may be a good technique to find a trajectory that reaches the SP twice, once the final characteristics of the actual mission orbit are known. More manoeuvres can in practice be considered than were used in this preliminary investigation, to potentially enable further classes of solution to be obtained.

4.1.2. Starting from the SP

This part of the article deals with another way to tackle the design of the LPF post-mission trajectory. The aim is to find orbits passing twice through the SP, that are also suitable for the LPF main mission.

The focus of the method presented here is to start simulations from the SP. Indeed, if the satellite has to pass twice through the SP, then the subsequent trajectory needs to pass just once through the SP. In this case, it is easy to find directions and speeds that the satellite must have at the SP in order to return to it at a later time. Then, it only remains to check whether this trajectory linking the two SP flybys

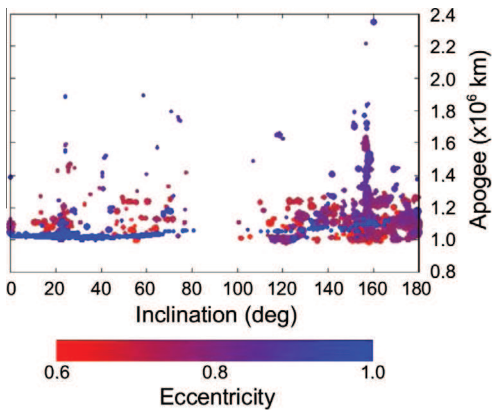


Fig. 6. Initial orbital elements leading to several SP flybys. The colour (from red to blue) gives the eccentricity (from 0.6 to 1). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

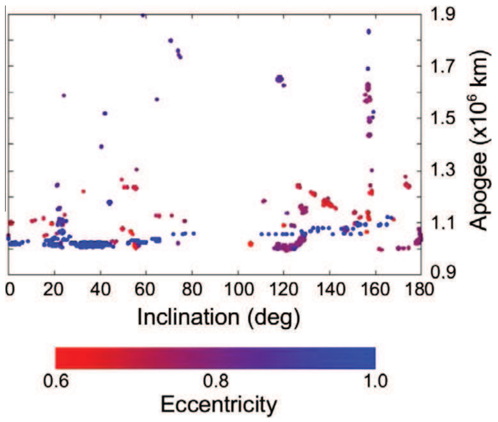


Fig. 7. Initial orbital elements leading to unstable double SP flybys. The colour (from red to blue) gives the eccentricity (from 0.6 to 1). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

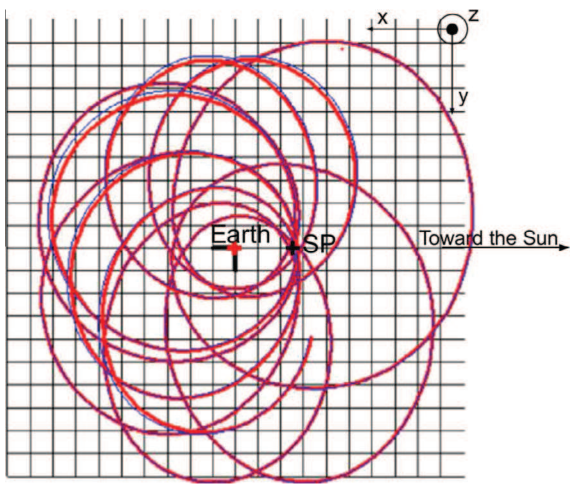


Fig. 8. Example of an orbit considered as too stable – viewed in rotating reference frame, grid is 1 million km from centre to edge. The orbit passes 6 times in the vicinity of the SP.

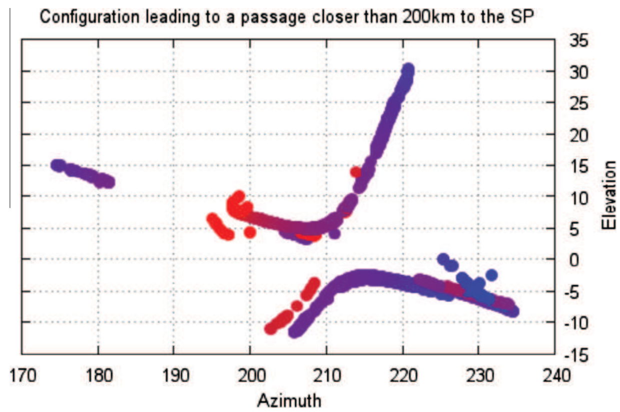


Fig. 9. Variables relationships for a SP flyby closer than 200 km – the colour (from red to blue) represents the speed (from 1480 m/s to 1640 m/s). The angles are linked to the rotating reference frame, i.e. fixed relatively to the Earth–Sun direction. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

can be reached by the LPF spacecraft from its orbit about the Lagrange point. This multiple shooting based method reduces the sensitivity with respect to initial conditions and therefore simplifies the solution process. In summary, the problem is divided into two smaller ones: can a trajectory be propagated forwards returning to the SP and can this same trajectory, starting at the SP, be reached by a spacecraft coming from a libration orbit? Finally, is this libration orbit suitable for the LISA Pathfinder mission?

The three parts of the process are presented in the following subsections.

4.1.2.1. Double flyby trajectories. The first part of the process consists of finding trajectories linking two SP flybys. Starting simulations from the SP itself must be associated with a simulation starting date, as this determines the initial position in an inertially oriented frame. This initial selection of date means that the results obtained represent only a fraction of the possible results. However the results obtained for any given date are expected to be similar to

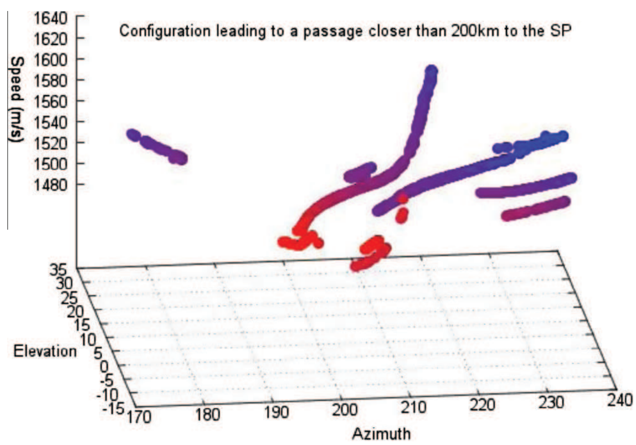
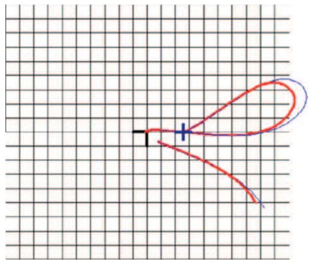
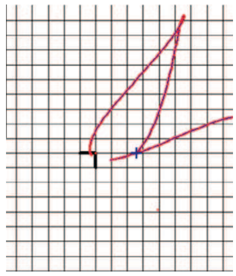
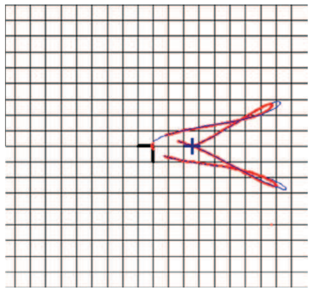
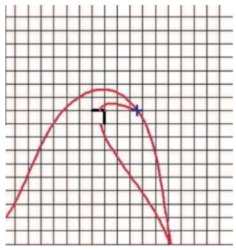
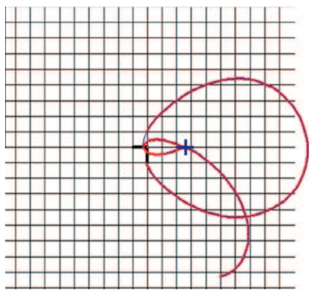


Fig. 10. Variables relationships for a SP flyby less than 200 km; 3D-view.

Table 2
Examples of different orbit families leading to SP flyby of less than 200 km. All orbits are presented in the rotating frame – grid is 1 million km from centre to edge. The position of the SP is given by the blue cross. The geometry is presented in Fig. 1.

Initial velocity toward the Sun	The spacecraft reaches the SP for the second time before a complete revolution around the Earth.	
	The spacecraft reaches the SP for the second time after a revolution around the Earth.	
	The spacecraft makes a complete revolution around the Earth and half of the following one before reaching the SP for the second time.	
Initial velocity toward the Earth	The spacecraft reaches the SP for the second time before a complete revolution around the Earth.	
	The spacecraft reaches the SP for the second time after a revolution around the Earth.	

those for other dates, if initial states are expressed in a rotating reference frame.

Once the simulation is initialised with a date, the parameters corresponding to trajectories flying by the SP twice can be found. In order to find them, simulations are run in which the spacecraft has an initial speed in every possible direction. Of course, in order to keep a reasonable computing time to explore the solutions, only speeds around the typical values obtained in the first part of the study are considered (around 1.5 km/s). The closest distance to the SP is measured for each trajectory, and a minimisation of this flyby distance is performed by exploring the direction of the initial velocity. The result is a list of combination of variables (speed, azimuth, elevation) leading to a SP flyby with the smallest flyby distance possible.

The results of the optimisation are initial conditions leading to a second SP flyby. There are a very large number of solutions, but each one cannot be considered as satisfactory. Many of the orbits found are indeed very stable: most are simply orbits around the Earth in the ecliptic plane. Therefore, those cases are very unlikely to be reached by LPF, using only low thrust and a small ΔV whilst approaching from the Lissajous orbit. In order to simplify the following steps of the process, those cases are discarded.

Figs. 6 and 7 present an example of the orbital elements of different solutions, before and after removing the ones considered as too stable. The colour (from red to blue) gives the eccentricity (from 0.6 to 1). The apogee is in km and the inclination in degrees.

The orbit considered as too stable in this study are the one passing more than twice in the vicinity of the SP (some passing even more than ten times, like the orbit presented in Fig. 8). Discarding those cases could lead to missing interesting trajectories, but the large number of remaining cases justifies the use of this criterion. It is interesting to notice that there are far more stable retrograde orbits than stable prograde orbits. This may be explained by the fact that the prograde orbits are more influenced by the moon, as they stay longer in its vicinity. The explanation lying behind the concept of "too stable" is that orbits with, for example, low apogee, tend to be only weakly perturbed by the Sun and so tend to remain in a similar orbit for extended periods. Whilst presenting a valid solution for a double SP crossing, they are unlikely to experience sufficient perturbation when propagated backwards from the SP to match high apogee approaching orbits that are typical of orbits departing from Lissajous orbits. They are therefore "too stable" i.e. not consistent with the orbital variations required for an end to end mission design here. Such orbits would only be usable if a larger ΔV was available.

The optimisation of the second flyby distance reveals particularly promising ranges of directions, for which the second flyby distance could be minimised. Those ranges, presented in Fig. 9 and Fig. 10, can be seen as continuously spread families of variables leading to a successful second flyby.

The results presented here have a starting date on the 21. August 2014. The colour (from red to blue) represents the speed (from 1480 m/s to 1640 m/s). All the points presented correspond to directions leading to double flybys for which the first one is exactly at the SP, and the second one below 200 km.

When viewed in 3 dimensions, as presented on Fig. 10, structures appear, corresponding to the different families of trajectories. Some of those different families are presented on Table 2.

For all the results presented here, the maximum time between two SP flybys has been limited to be below 80 days.

4.1.2.2. Trajectories arriving from a libration point orbit. The previous part described the method used to find double SP flybys. The result is a list of initial conditions (initial speed, azimuth, elevation in the Sun rotating reference frame) corresponding to orbit leading to a double flyby.

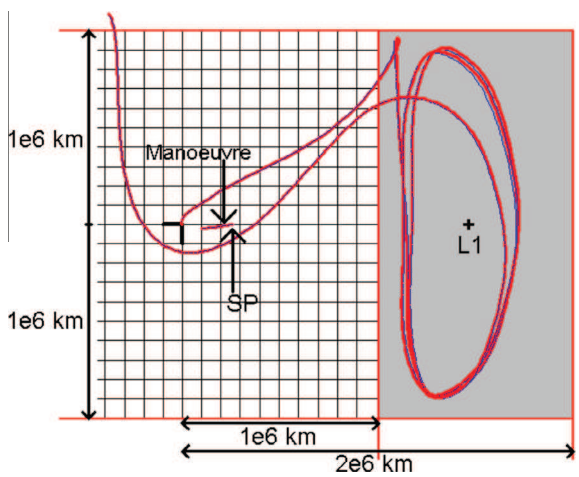


Fig. 11. Range used to characterise the libration orbit.

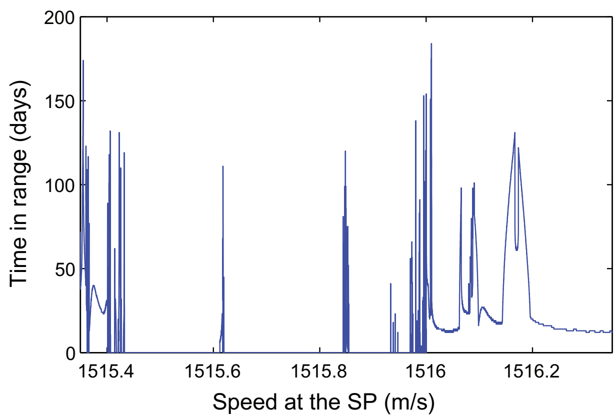


Fig. 12. Zoom on the time spent in the range shown in Fig. 11, as a function of speed at the SP.

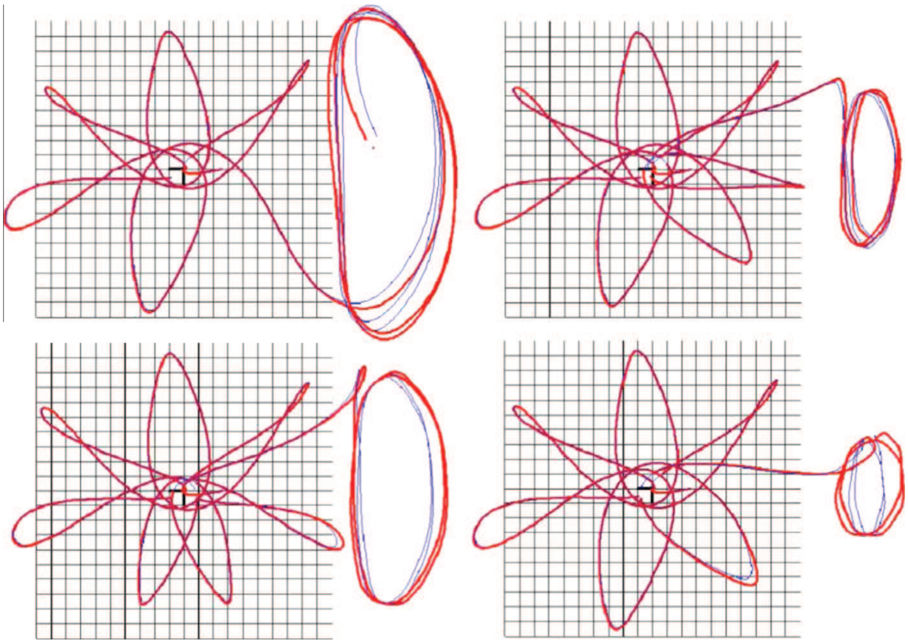


Fig. 13. Example of backward propagated transfers from the SP to Lissajous orbits about L1 with a speed very similar. Speed values, from top-left to bottom-right: 1516.180 m/s, 1516.059 m/s, 1515.664 m/s, 1516.154 m/s. Geometry is presented in Fig. 1.

The issue is then to try to reach those initial conditions with a spacecraft coming from a libration point orbit around L1. In order to do so, backward propagations are used. Therefore, instead of trying to start from a given initial libration orbit to reach the SP with a fixed velocity, simulations are run from the SP backward in time with the initial conditions found previously.

If the initial conditions (speed, azimuth, elevation obtained previously) are not changed, the cases leading to a libration orbit (backward in time) are very few. But many of the cases are orbits which might be interesting if slightly modified. For example, many have a low perigee when propagated backward. Thus, it is possible to slightly modify the speed immediately before the SP, in order to control the perigee, and thereby the orbit before it. Others have an apogee close to the distance between the Earth and the SP, which may lead to a free libration orbit.

To characterise the libration orbits, the time spent in the range define in Fig. 11 is used. The range is defined by the grey part of the figure. The out-of-ecliptic coordinate is not taken into account for its definition.

Having a low perigee makes an orbit highly sensitive to its speed before it. When studying closely the time spent in the range previously defined, this chaotic dependence is clearly noticeable. Fig. 12 is an example of this dependence.

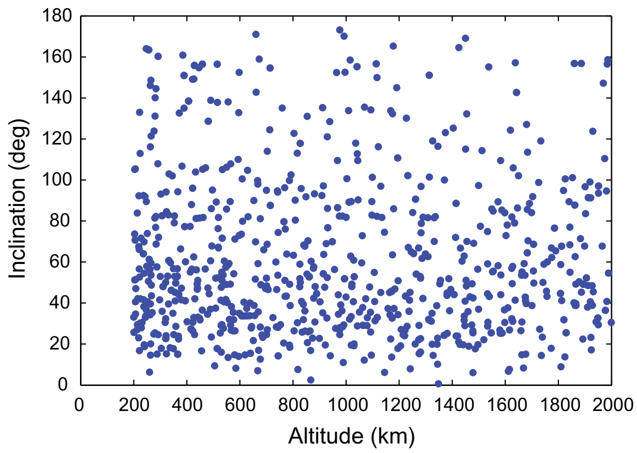


Fig. 14. Characteristics of the perigee of interesting cases on 22/09/2016 – altitude and inclination.

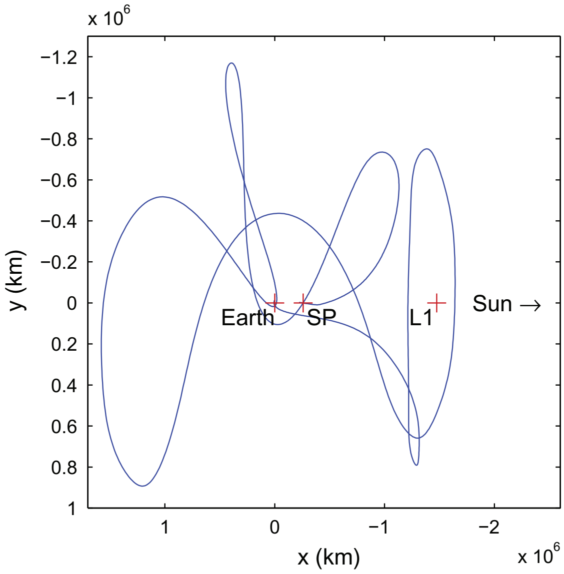


Fig. 15. Example of an end to end mission following a VEGA launch. Two SP crossings occur, separated by 65 days. Earth centred plot in rotating reference frame. Sun lies to the right of this figure.

To create it, simulations have be run in the following configuration: a satellite is at the SP at time $t = 0$. Its velocity is characterised by $azimuth = -168.5°$ and $elevation = 8.8°$. The value of its speed is taken in the range [1515.35 m/s,1516.35 m/s]. The value plotted is the time the satellite spends in the range defined in Fig. 11, for each particular speed step. Each peak detected shows a configuration for which the satellite enters a libration orbit (backward in time). As the speed of the satellite is initially fixed equal to 1515.85 m/s (given as a result by the algorithm described in part 4.1.2.1), the manoeuvre that has to be implemented in order to reach a libration orbit is given by $\Delta V = 1515.85 - speed_{peak}$.

Thus, only the points where a vertical asymptote can be defined are interesting here. Indeed, the other local maxima correspond only to a passage in L1 area, not to a possible libration orbit, whereas the speeds where a vertical asymptote can be defined correspond to initial conditions very close to combinations leading to a libration orbit. In this case for example, if the precision is increased, it is possible to find all the orbit presented in Fig. 13.

Although the four cases presented each have a manoeuvre with a ΔV value below 1 m/s, the way they reach L1 (or conversely arrive from it in forward time propagation), and the characteristics of the libration orbits are completely different. This proves once more that the response to a manoeuvre is potentially highly chaotic. This is on the one hand problematic, as an orbit so designed can therefore be very difficult to achieve in practice, but on the other hand it enables the finding of many different interesting orbits and hence increases the number of satisfactory ones.

To sum up, the second step of this method is to find the orbits passing through the SP twice and also coming from a libration orbit. At this point of the study, the part still missing is the trajectory which has to be used to reach the libration orbit, from the orbit the satellite is injected into by the launcher. So the next step is to find whether the cases found previously could possibly have a low perigee before they reach L1, and whether the characteristics of this perigee match LPF requirements.

4.1.2.3. Finding trajectories that match the LISA Pathfinder injection requirement. By using a process of "fine tuning" of the speed at the SP and running backward simulations, it appears that it is possible to achieve at least a targeted minimum time to be spent around L1, and that is also consistent with a low perigee such as achieved by a Rockot or VEGA injection, without needing to apply an additional manoeuvre. The minimum time to be targeted is an input parameter to the analysis and can be updated to reflect the duration of the main LPF mission plus any extensions to that mission phase. This strategy is developed hereafter.

In order to find orbits which at the same time pass twice through the SP, stay at least a minimum length in a libration orbit, and previously arrive from a very low perigee, an optimisation program has been created. Its inputs are the different initial conditions obtained at the second step: leading to a double SP flyby and arriving from a libration point orbit. First, it optimises the speed in order to ensure that at least a minimum time (of 180 days for example) is spent in the spatial range defined in Fig. 11. Then, it minimises the perigee altitude prior to reaching the libration point orbit (subject to a mimimum altitude of 200 km).

Using this algorithm allows many different orbits to be found which fulfill the different conditions. The orbits are then characterised, in order to be able to compare their dimensions, perigee condition, etc with the requirements of LPF mission. Fig. 14 presents the perigee conditions of all those orbits, for a first SP flyby fixed on the 22. September 2016.

The results found are widely scattered: it is possible to find an acceptable perigee altitude (i.e. close to 200 km) with every possible inclination. Here, only the ones with altitude higher than 200 km and lower than 2000 km are represented. For the higher perigee solutions, the initial perigee could potentially be slightly higher, and compensated by an additional small manoeuvre, expanding the range of feasible solutions beyond those indicated on the plot. Because of the large number of trajectories having a low perigee, it is possible to find many trajectories whose perigee characteristics match the two possibilities for LISA Pathfinder launch: Rockot and Vega. Rockot is launched from Plesetsk (62.6°N) and Vega is launched from Kourou (5.2°N). Taking into account a possible variation of a few degrees, Fig. 14 shows many potential solutions.

4.1.2.4. End to end mission example. Using the multiple shooting technique has enabled the generation of many possible solutions, that differs qualitatively in certain features such as total mission duration, nature of the orbit about the Lagrange point. In the example presented Fig. 15 the mission is launched from VEGA into a low inclination injection orbit and follows a free injection transfer, reaching a Lissajous orbit after approximately 30 days. The Lissajous orbit has an in-plane amplitude of approximately 800,000 km and out of plane amplitude of 150,000 km. After making just over one revolution of the Lagrange point where the nominal LPF science occurs (a period of approximately 200 days) the spacecraft starts its extended journey towards the SP, reaching it for the first time 470 days after launch (or approximately 250 days after leaving the vicinity of the Lagrange point orbit. The SP is reached for the second time after a further 65 days.

5. Conclusions and future work

The method presented in this study enables trajectories for a complete LISA Pathfinder mission to be easily found. This method determines the trajectories for the transfer from launch towards L1, the libration point orbit, and a trajectory that reaches the SP twice.

Nevertheless, this method implies two main consequences. First, fixing the date of the first SP crossing makes the trajectory unique. Second, because of the use of backward propagation, the launch date is therefore restricted to a narrow window. However this is only an artefact of the analysis process. In order to generate an extended launch window a range of epochs for the first SP flyby must be considered. The result is that the inclusion of the twin SP crossing phase results in no change to the nominal LPF launch window which spans a year.

This study has demonstrated that it is feasible to achieve a trajectory that utilises a free injection transfer to a Lagrange point orbit, stay there for a given minimum period (at least 180 days, more if required) and then leave that orbit to pass twice through the Earth–Sun gravitational SP. As such it has fulfilled its objectives. Finding these solutions is facilitated by using a starting point at the first SP flyby and then considering forward and backward propagations to reach the respective goals. The use of multiple shooting trajectory propagation to satisfy the several goals of the mission simplifies the task of finding solutions.

The mission designs so obtained are consistent with both Rockot and VEGA injection. The deterministic ΔVs required are typically less than 2 m/s. In addition to this, small navigation manoeuvres are required. Analysis is ongoing to assess this navigation problem and to consequently confirm that the total manoeuvre requirement for the extended mission lies with the LPF capability.

In practice the final mission designs used for Lisa Pathfinder will likely employ a forward only propagation method so that the launch conditions and variants/dispersions can be more readily defined and subsequently compensated. ESA/ESOC's priority for mission design will be the opimisation of the nominal science mission, and the possibility of extended operations around L1 may also need to be considered. Whilst the features of a mission designed by a forward only propagation method will be similar to those derived here, an enhanced methodology will be required, representing a further challenge in this evolving trajectory design problem. The final mission design for LPF is the responsibility of ESA/ESOC.

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