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NUMERICAL INVESTIGATION OF HEAT TRANSFER IN A FORCED FLOW OF HE II

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ABSTRACT

In this paper, we use the complete two-fluid model to simulate transient heat transfer for a forced flow of He II at high Reynolds number following the setup of the experiments performed by Fuzier, S. and Van Sciver, S., *"Experimental measurements and modeling of transient heat transfer in forced flow of He II at high velocities,"* Cryogenics, 48(3–4), pp. 130 – 137, (2008). A particular attention has been paid to the heat increase due to forced flow without external warming. The simulation are performed using *HellFOAM*, the helium superfluid simulator based on the OpenFOAM® technology. Simulations results are then compared to the experimental data.

KEY WORDS: superfluid helium ; two-fluid model ; simulation ; forced flow ; Super-PISO ; heat-transfer

1. INTRODUCTION

The use of superfluid helium in forced flow has received an increasing attention these last decades (Srinivasan and Hofmann (1985a,b), Kashani et al. (1989), Rousset et al. (1994), Rao et al. (1996), Fuzier et al. (2001), Fuzier and Van Sciver (2008)). Indeed, the particular heat transfer properties of He II are of great importance for the cooling of large superconducting magnets and a better understanding of heat transfer in forced flow of superfluid helium could lead to significant enhancements in the design of new devices cooled by He II.

Transient heat transfer experimental studies in forced flow of superfluid helium at high velocities have shown that a pressure drop along the flow results in a very significant increase of the He II temperature (Fuzier et al. (2001); Fuzier and Van Sciver (2008)). In their work, the authors measured temperature profiles along a forced flow of superfluid helium in a smooth tube of $0.86\,\mathrm{m}$ length and $9.8\,\mathrm{mm}$ inside diameter. The liquid was pushed from a bellows pump to reach a velocity up to $22\,\mathrm{m/s}$. They showed that, at steady state, even without heater, the forced convection of He II leads to a temperature increase. Authors suggest that the temperature gradient along the tube they obtained is related to Joule-Thomson expansion (Huang (1986), Walstrom (1988)). However, as we will point out in the next sections, this explanation may be not sufficient to explain the entire temperature increase of the experimental data. A more complete solution of a superfluid He II flow model would provide also more insight on this fundamental aspect.

In a previous paper (Soulaine et al. (2014)) we have introduced *HellFOAM*, the multi-dimensional superfluid

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helium simulator we have developed to solve the complete two-fluid model using the Super-PISO algorithm. In this paper, we attempt to numerically reproduce Fuzier and Van Sciver experimental results with *HellFOAM* and propose to take into account an additional mechanism which contributes to the heat increase observed without external heating in the case of forced flow of He II at high velocities.

The paper is organized as follows. In Section 1, we present the superfluid equations, the main assumptions and the numerical implementation of the model. Then in Section 2, we solve numerically the superfluid equations in the configuration of the Fuzier and Van Sciver experiments, which allows us to discuss the physical interpretation of these results.

2. MATHEMATICAL MODEL

2.1 The two-fluid model

In this study, we use the Landau's two fluid model to simulate forced flow of superfluid helium (Landau (1941), Khalatnikov (1965)). In this model, He II is seen as though it was a mixture of different liquids. One of these is a normal viscous fluid (denoted by subscript n), the other is a superfluid (with subscript s) that moves with zero viscosity. The actual density of He II, ρ, is defined as the sum of the normal and superfluid mass density, ρ_n and ρ_s respectively,

$$\rho = \rho_n + \rho_s. \tag{1}$$

These quantities depend on the temperature: ρ_s vanishes at the so-called λ-point where the fluid becomes fully normal and ρ_n is null at absolute zero.

The mass conservation for the whole fluid reads

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s) = 0. \tag{2}$$

The momentum balance equation for normal component reads (gravity and diffusion energy effects neglected),

$$\frac{\partial \rho_n \mathbf{v}_n}{\partial t} + \nabla \cdot (\rho_n \mathbf{v}_n \mathbf{v}_n) = -\frac{\rho_n}{\rho} \nabla p - \rho_s s \nabla T + \nabla \cdot (\mu_n \nabla \mathbf{v}_n) - A \rho_n \rho_s |\mathbf{v}_n - \mathbf{v}_s|^2 (\mathbf{v}_n - \mathbf{v}_s) + \frac{\Gamma}{2} (\mathbf{v}_s + \mathbf{v}_n), \tag{3}$$

and for superfluid component reads,

$$\frac{\partial \rho_s \mathbf{v}_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{v}_s \mathbf{v}_s) = -\frac{\rho_s}{\rho} \nabla p + \rho_s s \nabla T + A \rho_n \rho_s |\mathbf{v}_n - \mathbf{v}_s|^2 (\mathbf{v}_n - \mathbf{v}_s) - \frac{\Gamma}{2} (\mathbf{v}_s + \mathbf{v}_n). \tag{4}$$

In these equation, $\rho_s s \nabla T$ represents the thermo-mechanical force which occurs when a temperature gradient exists. It is responsible for creating the counterflow in which the normal fluid moves down the temperature gradient from the heat source to the bath, while the superfluid flows toward the heat source. The term $A \rho_n \rho_s |\mathbf{v}_n - \mathbf{v}_s|^2 (\mathbf{v}_n - \mathbf{v}_s)$ represents the Gorter-Mellink mutual friction term as introduced by Gorter and Mellink (1949), Gorter et al. (1950) to estimate the interaction between the two components when the superfluid velocity reaches a certain critical value. A is a coefficient determined empirically (Srinivasan and Hofmann (1985a)). The last term in the right hand side of Eqs (3)-(4) represents the momentum transfer between the two fluids. More exactly, Γ is the rate at which the superfluid particles become normal during the heating process. It can be computed uncoupling normal continuity equation and superfluid continuity equation by: $\Gamma = \frac{1}{2} \left(\frac{\partial (\rho_n - \rho_s)}{\partial t} + \nabla \cdot (\rho_n \mathbf{v}_n - \rho_s \mathbf{v}_s) \right).$

Beside these two momentum conservation equations, we should consider the energy conservation which is only transported by normal fluid. We have,

$$\frac{\partial \rho s}{\partial t} + \nabla \cdot (\rho s \mathbf{v}_n) = \nabla \cdot \left(\frac{k_n}{T} \nabla T \right) + \frac{A \rho_n \rho_s |\mathbf{v}_n - \mathbf{v}_s|^4}{T}, \tag{5}$$

where k_n represents the heat conductivity of the normal fluid while the last term on the right-hand side expresses the energy dissipation based on the mutual friction between the two components. As we will see in the next section, this term may behave as a source term in case of flow motion.

We note that besides the mutual friction term, the equations Eqs (1)-(5) are strongly coupled by the presence of a temperature gradient in the momentum equations. Moreover, the physical variables s, ρ_n, ρ_s, k_n and A vary according to temperature, which also contribute to the coupling of the equations. We assume that all these quantities are independent of the pressure. Their values are provided by polynomial interpolations from the HePak thermodynamic package by Cryodata Inc.

The superfluid equations must be completed with boundary conditions. They are defined as follows (Landau and Lifshitz (1969)). Firstly, the normal component of the total mass flux must vanish at any surface at rest:

$$\mathbf{n} \cdot (\rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s) = 0 \tag{6}$$

Moreover, the tangential component of the normal velocity $\mathbf{v}_n$ must be zero at a solid surface,

$$\mathbf{v}_n - \mathbf{nn} \cdot \mathbf{v}_n = 0 \tag{7}$$

In the absence of heat transfer between the solid surface and the fluid, we write

$$\mathbf{n} \cdot (\rho s T \mathbf{v}_n - k_n \nabla T) = 0. \tag{8}$$

Neglecting the diffusive term $k_n \nabla T$ the component of $\mathbf{v}_n$ perpendicular to the surface is also zero. Hence, for this case, we obtain a no-slip boundary condition for the normal velocity ($\mathbf{v}_n = \mathbf{0}$) and a slip boundary condition for $\mathbf{v}_s$.

At the heater, the energy condition boundary becomes,

$$\mathbf{n} \cdot (\rho s T \mathbf{v}_n - k_n \nabla T) = \mathbf{n} \cdot \mathbf{q}. \tag{9}$$

At the helium bath entrance, temperature and pressure are fixed values.

2.2 Numerical implementation

For the simulations, we use *HellFOAM*, the superfluid code we have developed (Soulaine et al. (2014)) using the OpenFOAM® technology (Jasak (1996), Weller et al. (1998)). In this program, superfluid equations Eqs (1)-(5) are first discretized according to the volume of fluid method and then solved with the segregated algorithm called Super-PISO (Soulaine et al. (2014)). This latter is an extension to two-fluid model of the Pressure Implicit Operator Splitting (PISO) algorithm designed by Issa (1985) to solve transient Navier-Stokes equations. With Super-PISO, a pressure equation is directly derived from the total mass balance equation and both momentum equations. As for the energy equation, it is recast into a temperature equation, and the heated area is considered adding a volume heat source distributed in some cells of the computational domain. This trick allows the heat flux to be relaxed in several cells, which improves the stability of the simulations. More details regarding the Super-PISO algorithm and its implementation in OpenFOAM® can be found in Soulaine et al. (2014).

Thanks to the OpenFOAM® technology, *HellFOAM* can handle 3D simulations by default. It also benefits of its parallel computation features without any additional modifications. The solver has been successfully

validated by comparison of the numerical results with the analytical solutions of superfluid flow in a capillary containing He II in both Landau and Gorter-Mellink flow regimes (Soulaine et al. (2014)).

3. RESULTS AND DISCUSSION

In this section, we simulate transient heat transfer in forced flow of He II at high Reynolds number following the setup of the experiments performed by Fuzier and Van Sciver (2008). The objective is not a complete accurate description of these experiments since *HellFoam* model is a direct model not suited for modeling developed turbulence, which would appear at very high Reynolds numbers. Instead, our objective is to investigate the impact of the various terms in the two-fluid model and discuss potential explanations for the increase in temperature observed in the experiments when no external heating is applied to the device. To achieve this goal, we performed 2D simulations on a tube geometry similar to Fuzier et al. (2001) and Fuzier and Van Sciver (2008) experimental setup. The considered computational domain is depicted in Fig. 1. It consists in a $0.86\,\mathrm{m}$ long, $4.9\,\mathrm{mm}$ thick 2D half tube with symmetry conditions. The grid is made of 500×10 hexaedral cells. The heater is placed between 0.30 and $0.31\,\mathrm{m}$ from the left-hand side. Liquid helium enters into the domain from the left-hand side at $\mathbf{v}_{in}$ and flows out at the right end of the tube. Initially, we assume that the superfluid and normal velocities are equal to $\mathbf{v}_{in}$ within the tube and the temperature is T_b. Several probes are placed along the tube axis. They correspond to the temperature measurements T3 to T8 of Fuzier and Van Sciver (2008). Simulations are performed with the full two-fluid model involving Gorter-Mellink mutual friction terms.
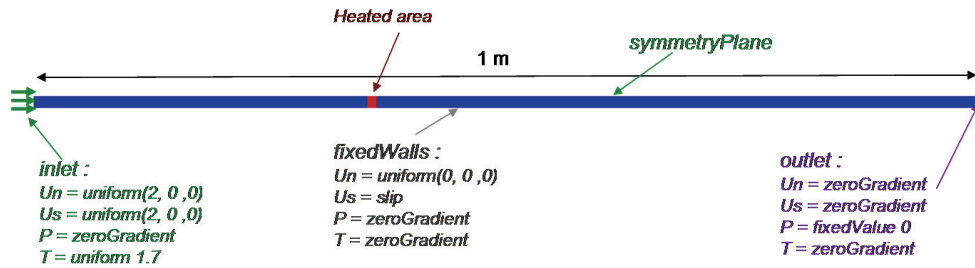

Fig. 1 Geometry and boundary conditions of the domain.

In a first stage, we let the flow to be established without external warming. To remain consistent with the experimental conditions of Fuzier et al. (2001), we have $T_b = 1.675\,\mathrm{K}$. The resultant temperature profiles at steady-state for $\mathbf{v}_{in}$ in the range $[0.5; 16]\,m/s$ are plotted in Fig. 2. We clearly notice that the temperature increases along the pipe and that this phenomenon is more pronounced with increasing flow velocity. Such a temperature increase was also observed experimentally. This phenomenon was related to the Joule-Thomson effect. However, in our mathematical model, the Joule-Thomson expansion has not been introduced! This temperature increase is in fact related to the mutual friction term. Since, as we shall see, this temperature increase associated to the Gorter-Mellink mutual friction term has a magnitude similar to the Joule-Thomson effect estimated by Fuzier and Van Sciver (2008), it is important to evaluate and compare more quantitatively the various contributions. This is the purpose of the discussion in this section.

Actually, if the introduction of the Joule-Thomson expansion effect is almost straightforward in *HellFOAM*, its impact will be underestimated since our code is not predictive for pressure losses at very high velocities, i.e., in turbulent regimes. Such improvements could be considered by the development of a turbulent model in the spirit of the Reynolds Averaged Navier-Stokes model used in classical computational fluid mechanics for flow at high flow rate. In consequence, this thermal gradient comes from a different origin. We suggest that the thermal gradients we obtain are a consequence of the friction term: because of the difference in the wall boundary condition, hydrodynamic generates a difference between the normal and the superfluid velocities that increases the value of the mutual friction term in the energy equation which finally acts as a bulk heat source term. In fact, as we can see in Fig.3, the Joule-Thomson effect is not sufficient to explain the entire heat

increase along the pipe. In this figure, we have plotted the temperature increase ($\triangle T$) experimentally obtained by Fuzier et al. (2001) (adapted from their Fig. 4), an estimation of the temperature increase, $\triangle T_{JT}$, due to the Joule-Thomson expansion and the heat increase, $\triangle T_{GM}$, related to the Gorter-Mellink mutual friction term that results on our simulations. $\triangle T_{JT}$ is evaluated on the basis that $\triangle T_{JT} = j_m \triangle P$ where the pressure drop is estimated using $\triangle P = f \frac{L \rho}{R} U^2$ with the friction factor $f = 0.004$ according to the value of Fuzier et al. (2001), and where the Joule-Thomson coefficient at $T = 1.7 \, \text{K}$ is about $j_m = \left(\frac{\partial T}{\partial P} \right)_H = -3.11 \times 10^{-6} \, \text{K/Pa}$ (value taken from HePak). Actually, this temperature elevation due to the Joule-Thomson expansion is overestimated since the absolute value of the Joule-Thomson coefficient is strongly temperature-dependent and decreases while the temperature increases (according to HePak, we have $j_m = -1.76 \times 10^{-6} \, \text{K/Pa}$ for $T = 1.9 \, \text{K}$). For this configuration, the temperature increase due to the mutual friction between normal and superfluid particles can be well correlated by $\triangle T_{GM} \approx 0.0011 U^{1.5}$. We notice that both $\triangle T_{JT}$ and $\triangle T_{GM}$ are of the same order of magnitude. Moreover, adding these two contributions, we recover the experimental $\triangle T$ obtained by Fuzier et al. (2001).
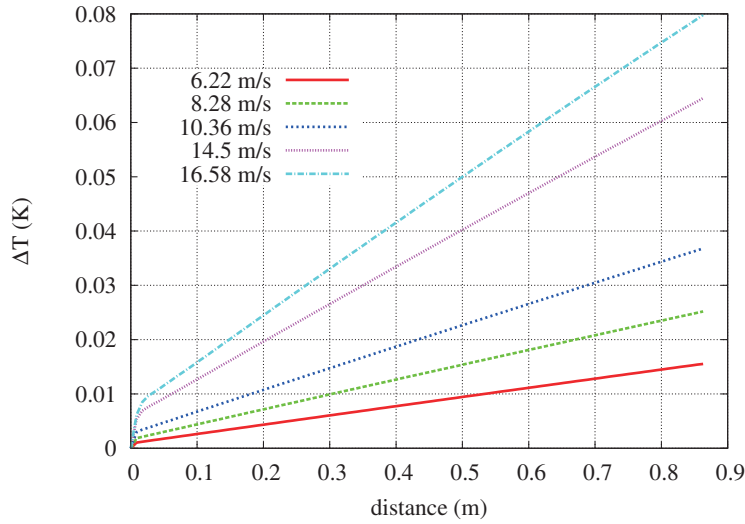

Fig. 2 Plot of the temperature increase at steady-state in case of forced flow of He II in a pipe without external warming.

In a second stage, we focus our attention to the transient heat transfer in a forced flow of He II as described in Fuzier and Van Sciver (2008). In this case, $T_b = 1.7 \, \text{K}$ and $\mathbf{v}_{in} = 2 \, \text{m/s}$, $8 \, \text{m/s}$ or $16 \, \text{m/s}$. As previously, the simulations start without warming until they reach their steady-state. At this point, the different probes are set to zero. Then, after these preliminary simulations, the heater is activated at $q = 9.9 \, \text{W/cm}^2$ for $20 \, \text{ms}$ and the temperature evolution is recorded until $250 \, \text{ms}$ for the different probes as depicted in Fig. 4 for $\mathbf{v}_{in} = 2 \, \text{m/s}$. The results are in very good agreement with the experimental results shown in Fig. 5 of Fuzier and Van Sciver (2008): the temperature peak for each probe are comparable both in amplitude and in time. This comparison example is preliminary and semi-quantitative in the sense that some assumptions do not correspond exactly to the actual situation (for example: Joule-Thomson effect non considered, 2D geometry as opposed to a cylindrical geometry, no turbulence model). Moreover, the situation at the tube inlet is perhaps more complicated since it corresponds in reality to the end of a flow divergence. It is intended to show that the expected physics is captured by the proposed model and the implemented numerical tool, *i.e.*, correct evolution of physical variables. Further work is in progress towards a more comprehensive comparison, but this requires more detailed knowledge of the experimental setup.

Then, following the same procedure, transient heat transfer simulation of He II flowing at $8 \, \text{m/s}$ is performed. Temperature records are plotted in Fig.5. Once again, results are in very good agreement with Fuzier and Van Sciver experiments (see Fig. 6 of Fuzier and Van Sciver (2008)).
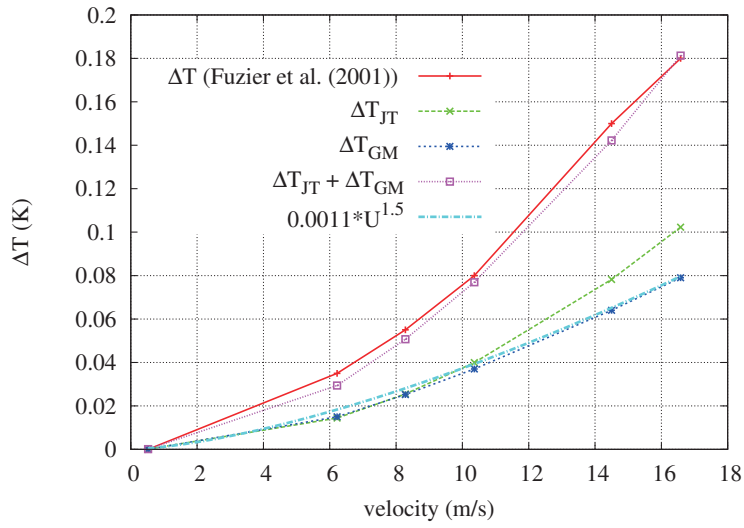
Fig. 3 Comparison of the different contributions of the temperature increase at steady-state for a forced flow of He II in a pipe without warming.
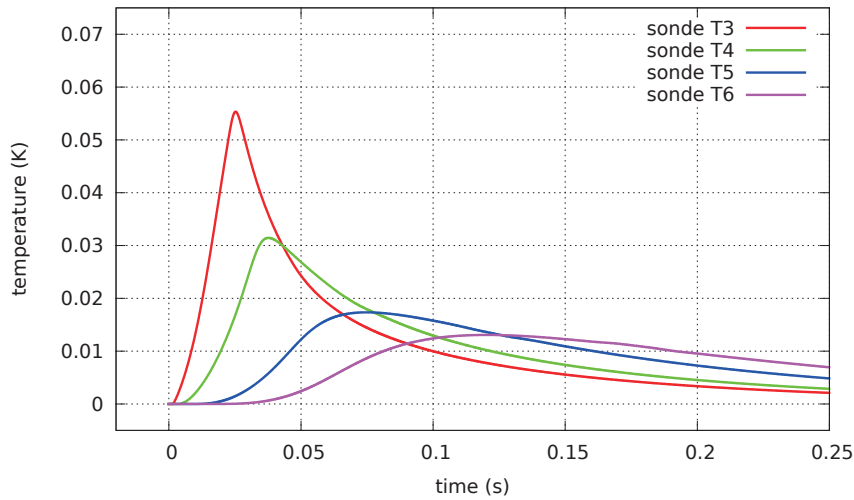

Fig. 4 Transient temperature profile recorded by several probes placed along the tube in case of a forced convection flow at $2\,\mathrm{m/s}$.

Finally, simulation results of forced flow of He II at $16\,\mathrm{m/s}$ are plotted in Fig.6. Results are clearly comparable to Fuzier and Van Sciver experiments (see Fuzier and Van Sciver (2008), Fig. 7).

These results lead to two conclusions : *i)* they confirm the possibility of simulating such experiments with the use of a complete two-fluid model, *ii)* they are an additional point towards the validation of the *HellFOAM* code.

4. CONCLUSIONS AND PERSPECTIVES

We have presented in this document simulations of forced flow of helium superfluid in a pipe using *HellFOAM*, a simulator based on the Super-PISO algorithm and the OpenFOAM$^{\circledR}$ technology. The simulation results are in very good agreement with the experimental data by Fuzier et al. (2001); Fuzier and Van Sciver (2008). We have demonstrated the possibility to simulate such experiments with the use of a complete two-fluid model
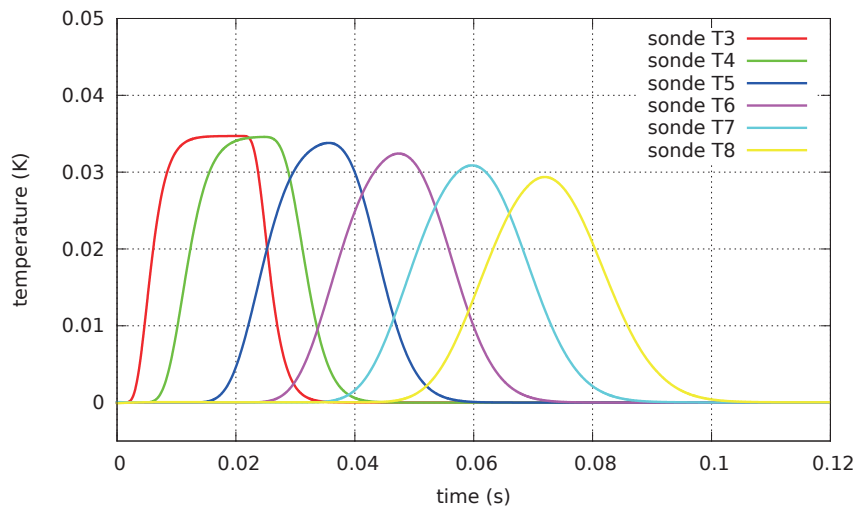
Fig. 5 Transient temperature profile recorded by several probes placed along the tube in case of a forced convection flow at $8\,\mathrm{m/s}$.
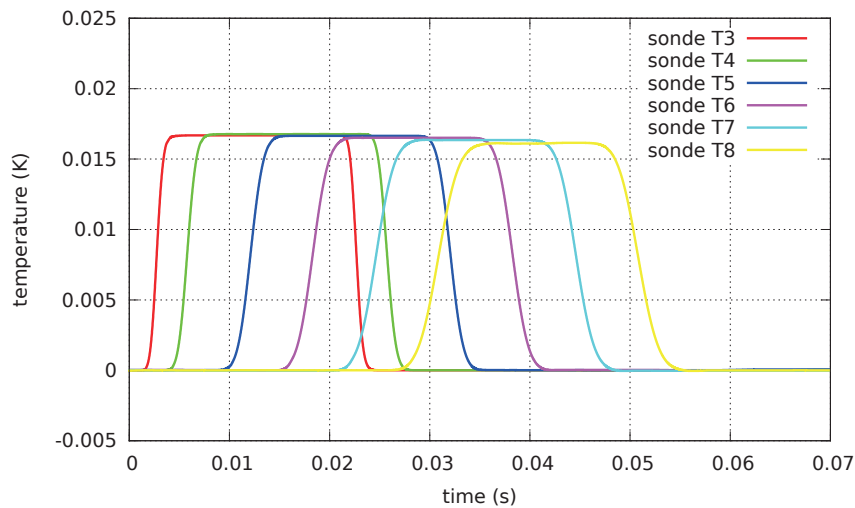

Fig. 6 Transient temperature profile recorded by several probes placed along the tube in case of a forced convection flow at $16\,\mathrm{m/s}$.

and have emphasized the role played by the mutual friction term in the energy equation to the temperature increase in case of forced flow of He II without external heat sources. This bring more thorough insight into the discussion about observed experimental temperature increases for forced flow of He II through tubes. The numerical results show that, at least for small pressure drops and at temperatures where the normal fluid concentration is significant, Gorter-Mellink friction terms provide a significant temperature increase, in the same order of magnitude as the temperature increase due to Joule-Thomson expansion. The combination of these two contributions can explain actual data.

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