Contribution of a micromechanics-based approach for reliability assessment

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The paper intends to widely develop the use of a new methodology for the design and optimization of composite materials and structures. Based on the coupling of reliability methods and homogenization techniques, such approach allows the integration of uncertainties at different scales in the problem analysis (i.e. from the microscopic scale of the composite material components up to the macroscopic scale of the structure) and the investigation of their consequences in the failure prediction. The principles and implementation steps of such original method have already been described in details (Int. J. Mech. Sci. 53 (2011) 935-945; Eng. Fail. Analysis 18 (2011) 988-998). This work focuses on its application and significant progress allowed in the design phase for engineering composite materials. Illustrations are presented on the case of a civil engineering structure, namely the Laroin footbridge (France) with carbon epoxy stay cables, and highlight reliability-based innovations and exploitations regarding the structure optimization.

Keywords: Reliability, composite materials, multi-scale analysis, design, optimization

1 Introduction

Uncertainties on constituent properties, fibre distribution, structural geometry, manufacturing process parameters or loading conditions clearly affect the design and optimization of composites structures. The related variability of their mechanical performances requires large safety factors in deterministic structural calculations and induces a significant loss in the weight/resistance ratio. In this context, reliability-based analyses have been developed for a few years to provide a consistent and rational estimation of the risk by introducing the random character of design parameters (materials properties, loads, geometry, ..). The quantitative evaluation of reliability indicators (such as the failure probability) obtained by these methods leads then to an enriched representation of the composites behaviour that significantly helps for both the design and maintenance of structures, and more generally for the development of composite materials [5, 10].

Most of reliability-based approaches are primarily designed to establish the reliability of a structure as function of the applied load and macroscopic parameters such as geometry and material strengths (for instance [1, 6, 11, 13, 14]). An original approach that simultaneously considers reliability and micromechanics has been recently proposed by the authors to improve this representation [4, 16]. For heterogeneous materials such as composites, homogenization techniques allow indeed the derivation of their overall behaviour from microscopic features (components behavior and morphology). At the same time, local stress or strain within components can also be obtained, which is particularly relevant for the consideration of a local failure criteria (see [8, 17]). The association of reliability methods with micromechanics offers then a better insight into the modelling of uncertainties affecting composites, including especially the fluctuations at various scales (microscopic to macroscopic) and also a physical motivation for the definition of the material reliability. These two points are important issues to understand the propagation of variability from micro to larger scales [15] but, above all, to derive a robust framework for reliability evaluation [3].

Interest of the coupled approach has been previously illustrated for material engineering through the identification and validation of micromechanical models [16]. The ability to derive multi-scale analyses has also been demonstrated [4]. The present work intends to focus on the question of design and optimization, through an application case in civil engineering. The objective is here to highlight the innovative tools provided by the methodology in the context of reliability–based design, for instance to get the best structure under a reliability constraint or to improve the manufacturing process.

After a general recall of the methodology and description of the case study, we present in what follows the design interest for an engineering material optimization, based on the integration of uncertainties related to the components properties and composite morphology.

2 General background and case study

The originality of the approach suggested by the authors lies in the association of two classical approaches for the study of composite materials:

- Reliability methods that introduce in the mechanical modelling the uncertain character of design variables and allow the study of the related consequences on the structure response,
- Homogenization techniques which aim at determining from the microstructure of a Representative Volume Element (RVE) of the material its overall (effective) mechanical behaviour and the local response within components for a given macroscopic load.

Theoretical developments and implementation details can be found in [4, 16]. We intend here to recall the key points of such coupling and assumptions used for the case study.

2.1 Coupling of reliability methods and micromechanics

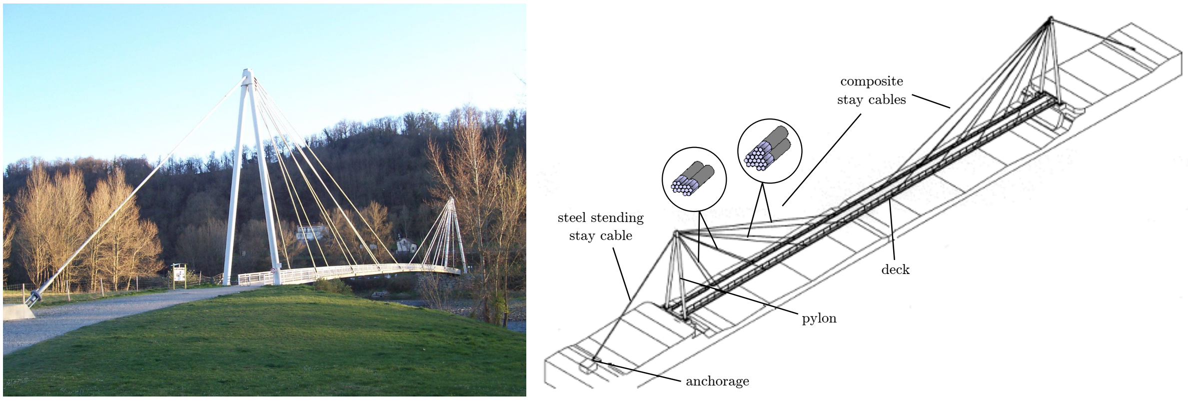
In the coupled approach, major stages encountered during any reliability study take into account the specific aspects of micromechanics:

1. First the selection of random variables $\{X_i\}_{i=1,N}$ in order to capture most of inherent fluctuations involved in the problem, and at the same time to offer at this starting point a broad analysis of design parameters; for heterogeneous materials such as composites, microstructural parameters clearly influence the material effective behaviour and should therefore be considered as random variables; important requirements are the availability of the statistical representation of these random parameters and the limit size of the problem (number N) that should be compatible with reasonable calculation time;
2. The choice of the failure scenario defined by a mathematical function G: this step includes both an adequate representation of the structure mechanical behaviour and a physical definition of the limit state (according either to strength achievement or serviceability); the ability provided by micromechanics to derive local stresses or strains induced by a macroscopic load is a major asset for a consistent definition of G in relation with the physical mechanisms involved ;
3. The estimation of probabilistic indicators (e.g. probability of failure P_f or reliability index β): in the context of composite structures investigation, one often resorts to numerical approximation methods in view of their suitability with finite element simulations and their design interest through sensitivity analyses on random variables.

Generally speaking, note that this approach should be conducted several times in order to select the significant random parameters of the problem considered and to ensure therefore the computational efficiency of reliability calculations.

2.2 Application to the Laroin footbridge

The demonstration of the potential of such approach for composites design and optimization is practically supported by a case study in civil engineering. The considered structure is a pedestrian footbridge located in Laroin (Pyrénées Atlantiques, France, Fig. 1(a)). Built in 2002, this is the first structure in this country conceived with composite stay cables [7]. It is composed of a steel deck, two steel reversed V-shaped pylons of 20.60 m height,



(a) (b)

Fig. 1: Laroin footbridge: Picture (a) and structure (b).

one high-strength steel stending cable for each pylon and its single span of 110 m length is maintained at each side by eight composite stay cables. Each stay cable is composed of two or three strands of seven unidirectional cylindrical composite rods (Fig. 1(b)).

Composite rods are made of Torayca high-strength carbon fibres T700SC-12K and Bostik Findley epoxy resin Eponal 401. They have been manufactured by pultrusion by the Toray Carbon Fibers Europe company (mean fibre volume fraction f_f of 67%). The mechanical behaviour of the composite material is described by means of the Mori-Tanaka formulation [2, 12] with a slight adjustment to reflect more effectively the manufacturing process. Based on the experience of Toray Carbon Fibers Europe, it seems indeed that only a fraction of fibres in the composite actually contributes to the composite response (due for instance to fibres misalignment, impregnation defects, etc.). Accordingly, the fibre volume fraction f_f appearing in the classical expressions of the Mori-Tanaka constitutive law has been changed in this way:

$$f_f \to f'_f = p_{act} \times f_f \tag{1}$$

with $p_{act} \leq 100\%$ the amount of fibres really active from the mechanical point of view. This parameter, which characterizes the quality of the manufacturing process, has been identified on the pultrusion production line. In this study, reliability assessment corresponds to the mechanical strength achievement of these rods under monotonic tensile load. The failure scenario is defined to account for the microstructural origin of the failure, namely by the fibres brittle behaviour through the following limit state function:

$$G = \sigma_f^F(\mathbf{n}) - \langle \sigma_I \rangle_{(f)} \tag{2}$$

where $\sigma_f^F(\mathbf{n})$ denotes the fibre axial failure strength (axial direction denoted by unit vector $\mathbf{n}$) and $\langle \sigma_I \rangle_{(f)}$ the maximum principal value of the average local stress over the fibre phase. If the first term is a material data provided by manufacturers, the second is derived through the micromechanical model.

Reliability calculations have been carried out with the probabilistic code FERUM (Finite Element Reliability Using Matlab [9]) by means of a direct coupling with the micromechanical model. FORM (First-Order Reliability Method [5, 10]) approximation method has been used since it exhibits a remarkable computational efficiency and provides valuable data for design issues (such as elasticities, see section 3.1).

3 Micromechanical-based reliability analysis

The present work aims at studying the reliability-based design of composite rods used for the stay cables of the Laroin footbridge, subjected to monotonic longitudinal tension.

3.1 Reference point

Based on the micromechanical model developed in [4, 16], the five random variables $\{X_i\}_{i=1,5}$ taken into account in this study concerns both micro and macro scale data of the problem:

- components mechanical properties : fibre yield strength $\sigma_f^F(\mathbf{n})$,
- manufacturing process parameters : fibre volume fraction f_f, active fibre fraction p_{act}, rod diameter ϕ,
- load condition : axial load F,

since their variability clearly affects the composite reliability. It has been shown in [16] that components elastic properties (fibre and resin) could be chosen as deterministic due to their weak influence. Regarding the considered case study, manufacturers (of fibre materials and composite materials) have provided the statistical distribution of the first four parameters that follow a Normal law (mean value and standard deviation are respectively denoted by $\overline{X}_i$ and S_{X_i}). We assume also a Normal distribution for the load and the study is centred around the operating point $\overline{F} = 76\,\text{kN}$ that corresponds to reliability index $\beta = 3.0514$ (such security level is approved in civil engineering structures for which β should be greater than 3 or, equivalently, failure probability P_f should be greater than 10^{-3}). All these data are detailed in Table 1 and will be considered as the *reference point* in what follows.

Random variables		ref. mean value $\overline{X}_i^{\text{ref}}$	ref. standard deviation $S_{X_i}^{\text{ref}}$
Fibre yield strength (MPa)	$\sigma_f^F(\mathbf{n})$	4870	162
Fibre volume fraction (%)	f_f	67	0.333
Active fibre fraction (%)	p_{act}	95	0.667
Rod diameter (mm)	ϕ	6	0.03
Axial load (kN)	F	76	2.533

Tab. 1: Random variables – *Reference point.*

The sensitivity analysis of the FORM method provides elasticities $\{e_{r_i}\}_{i=1,N}$ according to the distribution parameters of each random variable (here the mean value $r_i = \overline{X}_i$ or the standard deviation $r_i = S_{X_i}$) defined by:

$$e_{r_i} = \frac{r_i}{\beta}\frac{\partial\beta}{\partial r_i}(P^*), \quad \forall i \in [1..N] \tag{3}$$

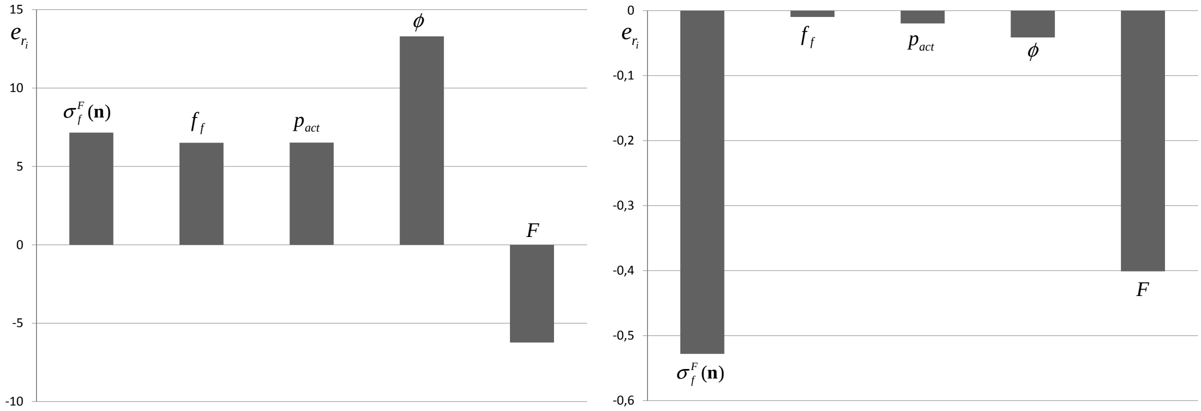
with P^* the design point (most probable failure point) [10]. Such analysis gives the global tendency regarding the origin of the scatter in the material response. This helps to discriminate variables that could be considered as deterministic (such as components elastic properties [16]) and to compare variables between them. As an illustration, figure 2 gives the reliability elasticities for the composite rod regarding respectively the variables mean value (fig. 2(a)) and their standard deviation (fig. 2(b)). We can distinguish *strength variables* with $e_{r_i} \geq 0$ (respectively *loading variables* with $e_{r_i} \leq 0$) for which an increase in their mean value improves (resp. alters) the structure reliability. Whatever the variable, an increase of the standard deviation obviously leads to an increase in the failure probability. Here, we note that the mean value of the geometric parameter ϕ and the deviation scatter on the fibre yield strength $\sigma_f^F(\mathbf{n})$ play the most crucial role in the structure reliability.

Yet, such analysis does not provide practical arguments for designers to take quantitative decisions regarding the composite structure. Our aim is then to develop in what follows the micromechanics-based analysis from another point of view in order to really quantify the influences detected before and help for design choices.

3.2 Evolution range

In this way, we need first to consider allowable evolutions of the distribution parameters of the different variables. In particular, these evolutions have to be chosen according to the potentialities of materials and manufacturing machines and should also be compatible with assumptions of the mechanical model. To illustrate the approach, we have considered in table 2 an acceptable evolution range for the mean value $\overline{X}_i$ of each random variable (range ratio relative to their reference value is indicated for clearness). Without specific data, an extended and uniform range ratio has been taken for the standard deviation S_{X_i} ($S_{X_i} = 0$ corresponding to the deterministic case).

Several simulations have then been carried out in which one distribution parameter of each random variable evolves independently within its interval defined in table 2, the other being fixed to their value given in table 1. For each calculation configuration, explicit formulation provided by the Mori-Tanaka scheme and the small number



(a) (b)

Fig. 2: Elasticities e_{r_i} according to the distribution parameters of random variables at the *reference point*: mean value $r_i = \overline{X}_i$ (a) and standard deviation $r_i = S_{X_i}$ (b).

Random variables		mean value $\overline{X}_i$	standard deviation S_{X_i} ($[0, 150\%]$ of $S_{X_i}^{\text{ref}}$)
Fibre yield strength (MPa)	$\sigma_f^F(\mathbf{n})$	$[4384, 5356]$ ($\pm 10\%$ of $\overline{X}_i^{\text{ref}}$)	$[0, 243.5]$
Fibre volume fraction (%)	f_f	$[64, 70]$ ($\pm 5\%$ of $\overline{X}_i^{\text{ref}}$)	$[0, 0.5]$
Active fibre fraction (%)	p_{act}	$[90, 100]$ ($\pm 5.2\%$ of $\overline{X}_i^{\text{ref}}$)	$[0, 1]$
Rod diameter (mm)	ϕ	$[5.8, 6.2]$ ($\pm 3\%$ of $\overline{X}_i^{\text{ref}}$)	$[0, 0.05]$
Axial load (kN)	F	$[64.6, 87.4]$ ($\pm 15\%$ of $\overline{X}_i^{\text{ref}}$)	$[0, 3.8]$

Tab. 2: Evolution range of the distribution parameters of random variables around the *reference point*.

of variables allow to derive a precise value of reliability index β with a reasonable number of calls to function G (around 50).

4 Results and discussion

Figure 3 shows the evolution of index β according to the mean value of each random variable. An increase (respectively decrease) of β with $\overline{X}_i$ is obviously obtained for *strength variables* (resp. *loading variables*). For the present model and application case, we obtain a quasi linear evolution of β regarding mean values $\overline{X}_i$ of random variables. In agreement with the model correction (1) and elasticities described on figure 2(a), we find a similar behaviour for f_f and p_{act} variables and a major influence of ϕ through the steepest slope. For designers, such result allows to define either changes of parameters or the maximum load amplitude to respect a given reliability level. For instance, the reliability index $\beta = 3.0514$ at the *reference point* can be raised to $\beta = 3.5$ (corresponding to an increase of 15%) either by an increase of 2% of the fibre yield strength (that is with $\overline{\sigma}_f^F(\mathbf{n}) = 4970$ MPa) or with an increase of 1% of the rod diameter (that is with $\overline{\phi} = 6.07$ mm). On the other hand, we note that a reliability level of $\beta = 2$ remains satisfied with an increase of 6% of the mean load (that is with $\overline{F} = 80.6$ kN).

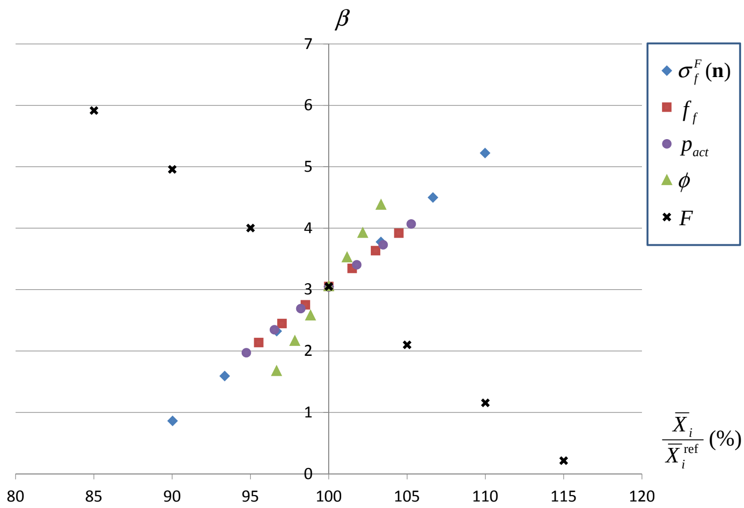


Fig. 3: Evolution of reliability index β with respect to the mean value $\overline{X}_i/\overline{X}_i^{\text{ref}}$ for each random variable X_i.

Regarding the effect of the standard deviation, figure 4 quantitatively highlights the influence on the reliability of the uncertainties on variables. As expected, more (resp. less) the variables are reproducible, more reliability increases (resp. decreases). The tendency observed on figure 2(b) is also reinforced with a weak influence of the scatter on manufacturing process parameters (f_f, p_{act} and ϕ). This indicates where to put the effort to get better reliable structures, namely on the fibre strength $\sigma_f^F(\mathbf{n})$. For the same mean value as the *reference point*, an highly reproducible $\sigma_f^F(\mathbf{n})$ (with $S_{X_i} \to 0$) can increase the reliability index up to $\beta = 4.4$ (increase of 44%). The variability of the load F plays also an important role on reliability, but the control over it is much more difficult, especially for civil engineering cases. Moreover, the evolution of β regarding the standard deviation of these two variables is of polynomial form with a crossing of the curves at the *reference point*: a better (resp. worse) reproducibility of $\sigma_f^F(\mathbf{n})$ induces a greater increase (decrease) in β than the load F. This confirms the main importance of the fibre strength in the composite performance.

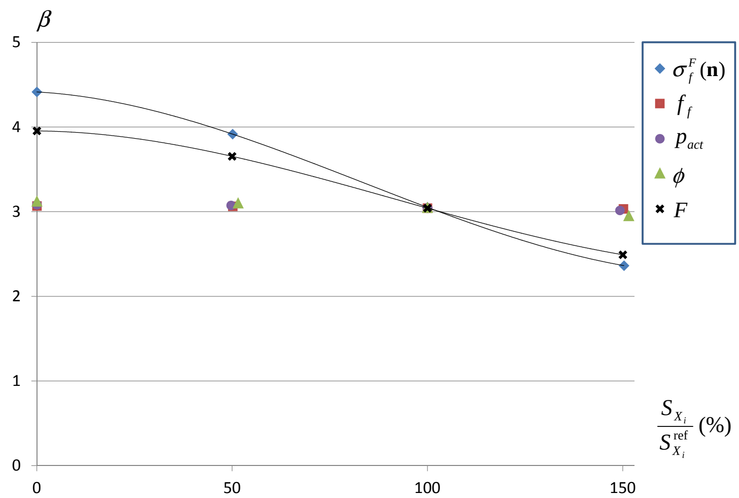


Fig. 4: Evolution of reliability index β with respect to the standard deviation $S_{X_i}/S_{X_i}^{\text{ref}}$ for each random variable X_i.

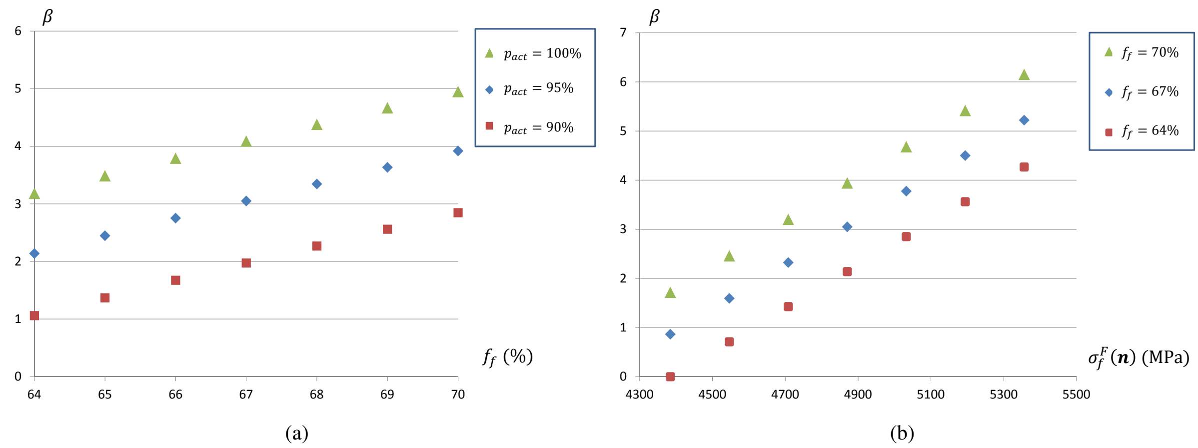


Fig. 5: Cross-analyses of the evolution of reliability index β with respect to the mean values of couple of parameters: (f_f, p_{act}) (a) and $(\sigma_f^F(\mathbf{n}), f_f)$ (b).

The micromechanical-based study allows also to implement quantitative cross-analyses between random variables. This provides to engineers various combined solutions for design changes. We illustrate on the figure 5 the coupled study of manufacturing process parameters (namely the fibre content and active fibre fraction) and material parameters (namely the fibre yield strength and fibre content). As shown of figure 5(a), the reliability index β can be improved to 3.5 either:

- by enhancing the fibre content (with $\overline{f}_f = 68.5\%$) while keeping the present manufacturing process,
- or by keeping the fibre content and improving the manufacturing process (with $\overline{p}_{act} = 100\%$).

Regarding material features investigated on figure 5(b), same reliability improvement can be obtained either :

- with higher strength fibre ($\overline{\sigma}_f^F(\mathbf{n}) = 5180$ MPa) and a reduced fibre content ($\overline{f}_f = 64\%$),
- or with higher fibre content ($\overline{f}_f = 70\%$) and lower strength fibre ($\overline{\sigma}_f^F(\mathbf{n}) = 4770$ MPa).

Obviously, such kind of cross-analysis can be done also on standard deviations of variables or even between mean value and deviation of different parameters according to the needs and possible actions of designers.

5 Conclusion

The development of composite materials and composite structures requires a convenient consideration of the stochastic variability of the data entering their mechanical behaviour. The combination of reliability analysis and micromechanics allows to adress such issue. The use of classical and well-known frameworks (reliability methods and homogenization techniques) associated with the broad integration of uncertainties at different scales (micro to macro) and on various features (constituents properties, microstructural morphology, geometry, load) leads to a relevant tool for structural applications. This paper has illustrated some innovations provided by this approach for the reliability assessment of composite materials. Through various examples, we have investigated the consequences of design choices for composites in a probabilistic context. The quantitative description of different solutions at different scale or regarding different parameters such as manufacturing process, geometry, materials properties indicates which designs are qualified to achieve a given reliability level and helps for the optimization of mechanical systems.

From this basic results, further work will now be conducted in order to show others design applications provided by the approach, and especially to extend it at a structure scale through the global study of the Laroin footbridge.

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References

[1] C. Baxevanakis, D. Jeulin, B. Lebon, and Renard J. Fracture statistics modeling of laminate composites. *Int. J. Solids Structures*, 35:2505–2521, 1998.

[2] Y. Benveniste. A new approach to the application of Mori-Tanaka's theory in composite materials. *Mech. Mater.*, 6:147–157, 1987.

[3] M. Chiachio, J. Chiachio, and G. Rus. Reliability in composites - A selective review and survey of current development. *Comp.: Part B*, 43:902–913, 2012.

[4] H. Dehmous and H. Welemane. Multi-scale reliability analysis of composite structures - Application to the Laroin footbridge. *Eng. Fail. Anal.*, 18:988–998, 2011.

[5] O. Ditlevsen and H. Madsen. *Structural reliability analyses*. Wiley & Sons, Chichester, 1996.

[6] D.M. Frangopol and S. Recek. Reliability of fiber reinforced composite laminate plates. *Prob. Eng Mech.*, 18:119–137, 2003.

[7] R.L. Geffroy. La passerelle de Laroin. *Freyssinet Mag*, 214, 2002.

[8] P. Gilormini and Y. Bréchet. Syntheses: Mechanical properties of heterogeneous media: Which material for which model ? Which model for which material ? *Model. Simul. Mater. Sci. Eng*, 7:805–816, 1999.

[9] T. Haukaas and A. Der Kiureghian. *FERUM user's guide*. University of California, 2000.

[10] M. Lemaire, A. Chateauneuf, and J.-C. Mitteau. *Structural reliability*. ISTE Ltd, London, 2009.

[11] P.F. Liu and J.Y. Zheng. Strength reliability analysis of aluminium-carbon fiber/epoxy composite laminates. *Loss Prev. Proc. Ind.*, 23:421–427, 2010.

[12] T. Mori and K. Tanaka. Average stress in matrix and average elastic energy of materials with misfitting inclusions. *Acta Metal.*, 21:571–574, 1973.

[13] M.R. Motley and Y.L. Young. Influence of uncertainties on the response and reliability of self-adaptative composite rotors. *Comp. Struct.*, 94:114–120, 2011.

[14] T.P. Philippidis and D.J. Lekou. Probabilistic failure prediction for the FRP composites. *Comp. Sci. Tech.*, 58:1973–1982, 1998.

[15] A. Shaw, S. Sriramula, P.D. Gosling, and M.K. Chryssanthopoulos. A critical reliability evaluation of fibre reinforced composite materials based on probabilistic micro and macro-mechanical analysis. *Comp.: Part B*, 41:446–453, 2010.

[16] H. Welemane and H. Dehmous. Reliability analysis and micromechanics : a coupled approach for composite failure prediction. *Int. J. Mech. Sci.*, 53:935–945, 2011.

[17] A. Zaoui. Continuum micromechanics : survey. *J. Eng Mech.*, 128:808–816, 2002.