STRATEGIC INTERACTION IN THE PRISONER'S DILEMMA: A GAME-THEORETIC DIMENSION OF CONFLICT RESEARCH

submitted by

Louis Joshua Marinoff

for the Degree of Ph.D. in the Philosophy of Science
at University College London

ABSTRACT

This four-part enquiry treats selected theoretical and empirical developments in the Prisoner's Dilemma. The enquiry is oriented within the sphere of game-theoretic conflict research, and addresses methodological and philosophical problems embedded in the model under consideration.

In Part One, relevant taxonomic criteria of the von Neumann-Morgenstern theory of games are reviewed, and controversies associated with both the utility function and game-theoretic rationality are introduced. In Part Two, salient contributions by Rapoport and others to the Prisoner's Dilemma are enlisted to illustrate the model's conceptual richness and problematic wealth. Conflicting principles of choice, divergent concepts of rational choice, and attempted resolutions of the dilemma are evaluated in the static mode. In Part Three, empirical interaction among strategies is examined in the iterated mode. A computer-simulated tournament of competing families of strategies is conducted, as both a complement to and continuation of Axelrod's previous tournaments. Combinatoric sub-tournaments are exhaustively analyzed, and an eliminatory ecological scenario is generated. In Part Four, the performance of the maximization family of strategies is subjected to deeper analysis, which reveals critical strengths and weaknesses latent in its decision-making process.

On the whole, an inter-modal continuity obtains, which suggests that the maximization of expected utility, weighted toward probabilistic co-operation, is a relatively effective strategic embodiment of Rapoport's ethic of collective rationality.

TABLE OF CONTENTS

LIST OF GAMES, GRAPHS, TABLES, AND SAMPLE PROGRAMS

Games:

Graphs:

Tables:

<u>Sample Programs:</u>

INTRODUCTION

John von Neumann, who made several outstanding contributions to scientific endeavour, founded—together with Oskar Morgenstern—an entirely new branch of mathematics.[1] Their formal *Theory of Games* was developed between 1928-1943 and, in the five decades since its first appearance, it has been adapted, applied and extended to a broad range of philosophical, mathematical, and social scientific interests. This enquiry addresses itself to one of the formative problems that emerged from the theory of games; namely, the Prisoner's Dilemma.

This problem itself has developed into a panoply of multidisciplinary concerns, to the extent that it would require no mean feat of research even to classify the existing body of literature on the subject. Anatol Rapoport presented a graph of the number of scholarly papers on the Prisoner's Dilemma for each year of the decade 1960-69. He found 28 papers in 1960, and a peak of 100 papers in 1967.[2] In the mid-seventies, Shubik listed an eclectic bibliography containing hundreds of scholarly articles on the Prisoner's Dilemma.[3] For that whole decade (1970-79), Axelrod counted more than 350 citations on the Prisoner's Dilemma in *Psychological Abstracts* alone, which prompted his remark "The iterated Prisoner's Dilemma has become the *E. coli* of social psychology".[4]

The substantial and growing body of literature on the subject extant serves notice that the Prisoner's Dilemma is a model rich in implications and ramifications, both theoretical and empirical, to and for researchers in many disciplines. This enquiry examines the

---

[1] J. von Neumann & O. Morgenstern (1944), *Theory of Games and Economic Behaviour*, John Wiley & Sons Inc., N.Y., sixth edition, 1955.

[2] A. Rapoport (*ed.*), *Game Theory as a Theory of Conflict Resolution*, D. Reidel Publishing Co., Dordrecht, 1974, p.20.

[3] M. Shubik, *The Uses and Methods of Gaming*, Elsevier Scientific Publishing Company, N.Y., 1975.

[4] R. Axelrod, 'Effective Choice in the Prisoner's Dilemma', *Journal of Conflict Resolution*, 24, 1980a, pp.3-25.

two-person Prisoner's Dilemma within the sphere of game-theoretic conflict research. Through a consideraton of methodological and philosophical problems embedded in the model, the enquiry studies conditional resolutions in the static mode, and develops a parametric approach to strategic robustness in the iterated mode.

The enquiry consists of four principal parts. Parts One and Two are theoretical in nature; Parts Three and Four, empirical.

Part One recapitulates certain fundamental precepts of and difficulties latent in the theory of games, in so far as these pertain to the "classical" formulation of the Prisoner's Dilemma. A suitable frame of reference and appropriate terminology are thereby introduced, which in turn allow the problem itself to be set out both succinctly and unambiguously.

Part Two examines the static case of the Prisoner's Dilemma, and elucidates the fundamental conflict between two principles of choice (*dominance* versus *maximization of expected utility*). Two proposed "resolutions" of this conflict are considered: a decision-theoretic reformulation of Newcomb's paradox, and a stable meta-game-theoretic matrix, both of which favour mutual co-operation as a result of the maximization of expected utility. However, an argument is rehearsed which asserts that, notwithstanding the validity of these resolutions, the dilemma persists nonetheless.

Part Three examines the iterated case of the Prisoner's Dilemma, in which static principles of choice are replaced by dynamic strategies. The cogent outcomes of Axelrod's two computer-conducted tournaments are summarized,[5] and the results of a third tournament are analyzed and discussed in some depth. This third tournament (inspired by Axelrod's former two) features competition not only among individual strategies, but also among "families" of related strategies. In the computer-simulated environment of the third tournament, the family of strategies which maximizes expected utility proves relatively effective. But (in similarity to the static case)

---

[5] Axelrod, 1980*a*, & idem., 'More Effective Choice in the Prisoner's Dilemma', *Journal of Conflict Resolution*, **24**, 1980*b*, pp.379-403.

it is argued that no single strategy (or family of strategies) can claim absolute superiority in the iterated Prisoner's Dilemma.

Part Four examines the performance characteristics of the maximization family under a higher power of analytical resolution. The examination reveals some interesting and unexpected properties of this strategic family, and subsequent analysis is devoted to an account of how and why these properties emerge. The enquiry's perspective and main findings are then summarized, and some pertinent conclusions are drawn.

The Appendices offer the following supplementary information and/or data.

Appendix One provides a glossary of strategic families, acronyms and summarized decision rules, intended for rapid reference.

Appendix Two gives the complete table of raw scores for the main tournament involving twenty strategies. Each strategy competes against the others, and against its twin. A 20 x 20 matrix of raw scores results.

Appendix Three contains efficiency tables for the combinatoric sub-tournaments, which are employed in the evaluation of strategic robustness. (The generation and usage of this data are explained in Chapter Eight.)

Appendix Four affords documented samples of the computer programs used in the experiment and in subsequent data analysis. Ten tournament programs are listed, each of which simulates a competition between two different strategies. Thus each of the twenty strategic algorithms appears once in sample form. The main analytical programs, and some relevant supplementary routines, are also listed.

To a large extent, this study is inspired and motivated by invaluable works of Professors Anatol Rapoport and Robert Axelrod (among other game-theorists). Its intent is both to develop a context which permits juxtaposition of their significant contributions, and also to contribute a modest sum of findings to the great wealth of their tradition.

PART ONE:

GAME-THEORETIC BACKGROUND

Chapter One

Taxonomic Criteria

While the theory of games embraces concepts subject to divergent interpretation (such as *rationality* and *utility*), there is little dispute over the theory's ability to classify games effectively. The useful taxonomic criteria set out by von Neumann and Morgenstern have been adopted, virtually without dissent, as the *definientia* to date.

In game-theoretic terms, then, the basic Prisoner's Dilemma is classified as a two-person, non-zero-sum, non-co-operative game. A brief clarification of this terminology may serve to explain not only what kind of game the Prisoner's Dilemma is (and is not), but also why it holds such fascination for game theorists of many stripes.

Most generally, a degree of knowledge about any game is conferred by the very act of classifying it (or examining its prior classification, as the case may be). Just as fundamental properties of an element are revealed by its position in the periodic table, and similarly as common properties of flora and fauna are attributed by Linnaean nomenclature, so are the important properties of games spelled out by the respective method at hand. But a deeper purpose resides in the classification of games, in addition to their logical ordering as conceptual objects: once a game is correctly classified, one knows whether the theory is prescriptive, or merely descriptive, of its play. Thus the taxonomic structure of game theory allows the identification of those constituents over which the theory has normative power, and therein lies its usefulness. Examples will be cited to illustrate this point.

To begin with, however, one may justly ask: what is meant by a *game*? In reply, it seems reasonable to quote the authors of the theory:

> "The *game* is simply the totality of the rules which describe it. Every particular instance at which the game is played—in a particular way—from beginning to end, is a *play*. The game consists of a series of moves, and the play of a sequence of choices."[1]

---

[1] Neumann & Morgenstern, 1955, p.49.

Viewed in this way, virtually any activity or pursuit can be treated as a game, so long as it can be defined or otherwise described by some set of rules.

Thus the theory of games is not restricted to pastimes of the "parlour game" variety. It can be applied to a range of competitions, conflicts of interest, and situations of decision-making under risk. Most generally, then, from a game-theoretic perspective, bridge can be viewed as a game of cards defined by the rules of Hoyle; roulette, a game of chance governed by the rules of probability; mathematics, a game of symbolic association developed according to the rules of consistency; boxing, a game of pugilism ritualized by the rules of Queensberry; driving a motor vehicle, a game of transportation described by the rules of the road; banking, a game of monetary transaction affected by the rules of economy; running for public office, a game of politics influenced by the rules of expediency; diplomacy, a game of international relations mediated by the rules of policy.

The theory of games can scarcely be termed modest, at least in taxonomic scope. It can classify a staggering range of activities according to an elegant but limited set of criteria which are quantitative and/or Boolean in character, and which do not take into account the correspondingly broad set of qualitative purposes that may underlie such activities, from diversion to stimulation, from profit to ambition, from savagery to statesmanship. The theory, however, pays a fair price for its universality: although it can classify a great number of activities, its normative power turns out to be quite constrained. The theory thus describes the play of many games, but prescribes the play for relatively few.

Specifically, the principal taxonomic criteria that pertain to the Prisoner's Dilemma can be described as follows:

(i) <u>Number of Players</u>

In general, a game can be played by M persons, where $M \geq 1$ (is greater than or equal to unity).

Single-person games, with one player, take place against some state of nature, be it organic or synthetic. A solitary card game, for example, is played against a given state of the deck.

Two-person games form the core of game theory, whose axioms, postulates and theorems are extended, where possible, from the two-player case to cases involving more than two players.

By convention, "*N*-person" games refers to games involving three or more players. Not surprisingly, the complexity of game-theoretic analysis tends to increase as a function of the number of players. (The situation is loosely analogous to dynamical problems in physics involving two bodies, three bodies, and many bodies.) *N*-person Prisoner's Dilemmas lie beyond the scope of this study, which confines itself to the two-person game. However, multiple pairs are involved in the iterated mode, where the situation is analogous to a chess tournament. (Chess remains a two-person game, although multiple pairs of players can compete in iterated competitions.)

(ii) Constancy or Non-Constancy of Sum

With each game is associated a set of *payoffs*. These are the gains or forfeitures of each player, which result from the play. A *constant-sum game* is a game in which the algebraic sum of payoffs is constant. The constant itself may be less than zero, zero, or greater than zero. In tournament chess, for example, the winning player receives one point; the losing player, zero points; and in the event of a stalemate or draw, each player receives a half-point. Tournament chess is thus a constant-sum game whose sum equals unity.

A *zero-sum game* forms a special class of constant-sum games, in which the algebraic sum of payoffs equals zero. In poker, for instance, the total sum of monies (or matchsticks) won by the winning players equals the total sum of monies (or matchsticks) lost by the losing players. This remains vacuously true if all players "break even"; i.e. if no-one wins or loses. Poker is thus a zero-sum game.

A *non-constant-sum game* is a game whose sum of payoffs is not constant. In cribbage, for instance, each player accumulates points until one and only one player wins by surpassing one hundred and twenty points. The algebraic sum of all players' points is non-zero, and can assume a range of values up to and including $121 + 120(N-1)$, for an *N*-player game. Cribbage is thus a non-constant-sum game, with respect to points scored.

Every game is either constant-sum and non-zero—sum, zero-sum, or non-constant-sum, with respect to a particular set of payoffs. A game may have more than one set of payoffs. War, for instance, is a negative non-constant-sum game with respect to lives lost in battle; it is also a zero-sum game with respect to territory that changes hands as a result of battle.

It should be noted that any constant-sum game can be represented as a zero-sum game (by means of adjusting the payoffs in its matrix).[2] Consequently, "Every constant-sum game is strategically equivalent to a zero-sum game."[3] In the broadest sense, then, it is most convenient to refer to a game as either zero-sum or non-zero-sum.

(iii) Co-operation

A game is said to be *co-operative* (or *negotiable*) if the players can communicate their respective intentions prior to a move, or agree upon co-ordinated strategies, and thereby influence the play. Arbitration, negotiation, collusion, and conciliation, among other processes, reflect possible aspects of co-operation. The sphere of economics, for instance, admits of a host of co-operative games,[4] as do numerous social interactions in daily life.

A game is said to be non-co-operative (or non-negotiable) if "absolutely no preplay communication is permitted between the players".[5] Conceivably, the rapidity or automation of play itself can weigh heavily against co-operation, if such play outpaces the speed

---

[2] A proof can be found in R. Jeffrey, *The Logic of Decision*, McGraw-Hill Book Company, New York, 1965, pp.14-30.

[3] Neumann & Morgenstern, 1955, p.348.

[4] A pioneer of negotiable games is Nash. E.g. see J. Nash, `The Bargaining Problem', *Econometrica*, 18, 1950, pp.155-162. For a perspective on negotiated games, see e.g. A. Rapoport, *Two-Person Game Theory*, The University of Michigan Press, Ann Arbor, 1966, pp.94-122. For a study of co-operative games in terms of economic cybernetics, see e.g. Vorob'ev, N., *Game Theory, Lectures for Economists and Systems Scientists*, s.v. S. Kotz, Springer-Verlag, N.Y., 1977.

[5] D. Luce & H. Raiffa, *Games and Decisions*, John Wiley & Sons Inc., N.Y., 1957, p.89.

of communication. To cite a most drastic example, the Cold War has been non-co-operative in the sense that a nuclear war could be triggered accidentally, without adequate time for human intervention.[6] The installation of the so-called "Hot Line" between Washington and Moscow represented an early attempt, in game-theoretic terms, to offset cybernetic non-co-operativeness by introducing an element of human communication at the highest echelon of decision-making.

In general, a game may be co-operative, non-co-operative, or partly co-operative, with respect to the players' choices and their respective payoffs.

(iv) Strictness of Determination

A zero-sum game is said to be *strictly determined* if and only if a saddle point exists in its normal matrix representation. This property proceeds from the fundamental theorem of the functional calculus of two-person zero-sum games, which states the necessary and sufficient condition for the existence of a saddle point. From subsequent commentary in game-theoretic literature, it is evident that two foci of contention (utility and rationality) originate from the postulates leading to the statement of this theorem. An outline of the theorem follows.

A game matrix is constructed according to the following convention: suppose two players, *A* and *B*, have respective choices

$$\{a_1, a_2 \ldots, a_n\} \text{ and } \{b_1, b_2, \ldots, b_m\}$$

for a given move in a zero-sum game. Then an *n*-by-*m* matrix of mutual choices obtains:

[6] E.g. see A. Grinyer & P. Smoker, `It Couldn't Happen - Could It? An Assessment of the Probability of Accidental Nuclear War', *Richardson Institute for Conflict and Peace Research*, University of Lancaster, 1986; and D. Frei, *Risks of Unintentional Nuclear War*, Published in Cooperation with the United Nations Institute for Disarmament Research, Allanheld, Osmun & Co. Inc., Totowa, N.J., 1983.

Game 1.1 - Generalized Matrix of Choices

		B		
	(a_1, b_1)	(a_1, b_2)	. . .	(a_1, b_m)
	(a_2, b_1)	(a_2, b_2)	. . .	(a_2, b_m)
A	.	.		.
	.	.		.
	.	.		.
	(a_n, b_1)	(a_n, b_2)	. . .	(a_n, b_m)

Every possible joint choice of the two players—and thus every hypothetical game-state for that move—is uniquely represented by some entry in the matrix. But Game 1.1 is unplayable, since the players can neither express preference among possible choices, nor implement principles of choice, without first knowing the payoffs for each possible outcome of their joint choosing. Once the payoffs are stipulated, they must be value-ordered according to the preferences of the players. And so arises the necessity of transforming each outcome (or payoff) into its respective *value* to each player.

For the time being, let the existence of such a transformation be assumed. Von Neumann and Morgenstern call it Φ, the "utility function".[7] The function is mathematically acceptable, but game-theoretically controversial. It maps the preference for each game-state into the utility of that game-state, U, to each player. The utility itself is a real number. So, for the xth choice of player A, and the yth choice of player B,

$$U_{xy} = \Phi(a_x, b_y)$$

For convenience, let (a_x, b_y) be written as simply as (x,y). Then

$$U_{xy} = \Phi(x,y)$$

where, by convention, U_{xy} is the utility of the joint play (x,y) to

7 Neumann & Morgenstern, 1955, pp.88-123.

Player *A*. Since the game in question is zero-sum, the utility of the joint play (x,y) to player *B* is simply $-U_{xy}$. Applying Φ to all (x,y) in Game 1.1 results in a playable game:

Game 1.2 - Generalized Matrix of Utilities

			B	
	$\Phi(1,1)$	$\Phi(1,2)$	. . .	$\Phi(1,m)$
	$\Phi(2,1)$	$\Phi(2,2)$	. . .	$\Phi(2,m)$
A	.	.		.
	.	.		.
	.	.		.
	$\Phi(n,1)$	$\Phi(n,2)$	. . .	$\Phi(n,m)$

By virtue of the utility function, the players can assess the values of all possible outcomes for that move, and each player can then exercise his individual preference accordingly.

At this juncture, von Neumann and Morgenstern introduce the *Max* and *Min* operators.[8] $Max_x\Phi(x,y)$ is the maximum value of $\Phi(x,y)$ in column *y*, and $Min_y\Phi(x,y)$ is the minimum value of $\Phi(x,y)$ in row *x*. Then $Max_y Max_x\Phi(x,y)$ is the maximum of column maxima; $Min_x Min_y\Phi(x,y)$, the minimum of row minima.

It can be shown that the operators $[Max_x, Max_y]$ and $[Min_x, Min_y]$ commute. In other words, the maximum of column maxima is congruent with the maximum of row maxima, and the minimum of row minima is congruent with the minimum of column minima. But there is no generalization as to the commutativity or non-commutativity of $[Max_x, Min_y]$. Two examples illustrate the point:

Game 1.3 - A Case in Which All Operators Commute

		B	
	1,-1	2,-2	3,-3
A	4,-4	5,-5	6,-6
	7,-7	8,-8	9,-9

In Game 1.3, with respect to player *A*, the column maxima are seven, eight and nine respectively; the maximum of column maxima is

8 Ibid.

therefore nine. The row maxima are three, six and nine respectively; the maximum of row maxima is therefore nine. Similarly, the minimum of row minima is congruent with the minimum of column minima (at one).

And in this case, the minimum of column maxima happens to be congruent with the maximum of row minima (at seven). But consider another case:

<u>Game 1.4 - A Case in Which Not All Operators Commute</u>

		B	
	9,-9	2,-2	3,-3
A	4,-4	5,-5	6,-6
	7,-7	8,-8	1,-1

In Game 1.4, again with respect to player *A*, the maximum of column maxima is congruent with the maximum of row maxima (at nine); and the minimum of row minima is congruent with the minimum of column minima (at one).

But in this case, the minimum of column maxima is six, whereas the maximum of row minima is four. The two are not congruent.

Although Games 1.3 and 1.4 have been viewed from the perspective of player *A*, the assertions concerning column and row operators are symmetrically consistent for player *B*. If viewed from player *B*'s perspective, what was a column (to player *A*) is now a row, and vice-versa.

Mutatis mutandis, for player *B* in Game 1.3, the minimum of column minima is congruent with the minimum of row minima (at minus nine); the maximum of column maxima is congruent with the maximum of row maxima (at minus one); and the minimum of column maxima happens to be congruent with the maximum of row minima (at minus seven).

Similarly, for player *B* in Game 1.4, congruency obtains for the minima of column and row minima (at minus nine), and for the maxima of row and column maxima (at minus one). However, the minimum of column maxima (at minus four) is not congruent with the maximum of row minima (at minus six).

Thus the operators are symmetrically consistent for both players.

Once it has been established that like operators always commute, while unlike operators do not always commute, then a *saddle point* can be defined:

> "Let $\Phi(x,y)$ be any two-variable function. Then (x_0,y_0) is a saddle point of Φ if at the same time $\Phi(x,y_0)$ assumes its maximum at $x=x_0$ and $\Phi(x_0,y)$ assumes its minimum at $y=y_0$."[9]

Now, the fundamental theorem under consideration states that $Max_x Min_y \Phi(x,y) = Min_y Max_x \Phi(x,y)$ if, and only if, there exists a saddle point (x_0,y_0). For a given game, there is no *a priori* guarantee of the existence of such a point. But if a saddle point exists, then that game is said to be *strictly determined*.

The property of strict determination is crucial both to the rationalization of a game, and to its play. Games which have this property are rationalized, and played, in a critically different way from those which lack it. To appreciate the difference in play, let games 1.3 and 1.4 be set out side-by-side, and re-interpreted (as games 1.5 and 1.6 respectively) in terms of this property.

In Game 1.5, Player *A* stands to gain no matter which outcome obtains. If *A* chooses the first row, he can gain no less than one; if the second row, no less than four; if the third row, no less than seven. A payoff of seven is the best of the worst possible outcomes for player *A*. Thus *A* should choose the row containing this payoff, since such a choice would *maximize his minimum* gain (hence the term "maximin").

[9] Ibid., p.95.

Game 1.5 – A Saddle Point			
		B	
	1,-1	2,-2	3,-3
A	4,-4	5,-5	6,-6
	7,-7	8,-8	9,-9

Game 1.6 – No Saddle Point			
		B	
	9,-9	2,-2	3,-3
A	4,-4	5,-5	6,-6
	7,-7	8,-8	1,-1

Similarly, in Game 1.5, player *B* stands to forfeit no matter which outcome obtains. If *B* chooses the first column, he can forfeit no more than seven; if the second column, no more than eight; if the third column, no more than nine. A payoff of minus seven is the best of the worst possible outcomes for player *B*. Thus *B* should choose the column containing this payoff, since such a choice would *minimize his maximum* forfeiture (hence the term "minimax").

Clearly, the existence of a saddle point at (7,-7) is prescriptive to both players. Each player fares best in choosing his maximin (or minimax, respectively), regardless of the other player's choice.

In Game 1.6, however, no saddle point exists. Player *A*'s maximin is four; player *B*'s minimax is minus six.

Knowing this, Player *A* might reason "*B* should choose the third column, since it contains his minimax. Therefore I should choose the second row, in order to gain six."

Knowing that, player *B* might reason "If *A* chooses the second row, then I should choose the first column, in order to forfeit only four."

Knowing this, player *A* might reason "If *B* chooses the first column, then I should choose the first row, in order to gain nine."

Knowing that, player *B* might reason "If *A* chooses the first row, then I should choose the second column, in order to forfeit only two."

Knowing this, player *A* might reason "If *B* chooses the second column, then I should choose the third row, in order to gain eight."

Knowing that, player *B* might reason "If *A* chooses the third row, then I should choose the third column, in order to forfeit only one."

Knowing this, player *A* might reason "If *B* chooses the third column, then I should choose the second row, in order to gain six."

Thus the players find themselves in a strategic infinite regress.[10] Clearly, in games without a saddle point, each player does not have a choice that is unconditionally "best", independent of what the other player chooses.

The difference between the play of zero-sum games with and without saddle points is readily appreciable. Even so, a questionable assumption was unavoidably smuggled into the argument; namely, that both players wish to maximize their gains and minimize their losses, respectively. If one assumes, for the time being, that to be "rational" is to play maximin or minimax (if the game has a saddle point), then an important feature of strictly determined games comes to light.

In Game 1.5, suppose that player *A* is rational, and player *B* is irrational. Then, according to the assumption about rationality, *A* would choose the third row, which contains his maximin. But player *B*, being irrational, would not choose the first column, which contains his minimax. In that case, *A* would gain eight or nine (as opposed to seven), and *B* would forfeit eight or nine (as opposed to seven). Generally stated, it amounts to this: in a strictly determined game, a rational player can fare no better than by playing maximin (or minimax) if his opponent is rational; and can fare no worse than by playing maximin (or minimax) if his opponent is irrational.

Von Neumann and Morgenstern bring the point home: for a rational player in a strictly determined game, it makes no difference whether his opponent is rational or irrational, and thus "the rationality of the opponent can be assumed, because the irrationality of his opponent can never harm a [rational] player."[11]

Once again, this conclusion is based upon a prior—and not necessarily justifiable—assumption about the meaning of rationality. So, one can assume the rationality of an opponent only in strictly

---

[10] The same point was made, using a different example, in A. Rapoport & A. Chammah, *Prisoner's Dilemma*, University of Michigan Press, Ann Arbor, 1965, p.23.

[11] Neumann & Morgenstern, 1955, p.128.

determined zero-sum games, and only if one has previously assumed a possible meaning of rationality itself. But in the universe of games, relatively few are strictly determined; so the prescriptive aspect of the theory wields absolute power over a fairly thinly-populated realm.

It would seem that the two-person, zero-sum game with a saddle point constitutes "the limit of applicability of game theory as a normative (or prescriptive) theory."[12] Nevertheless, as a descriptive theory, its power of classification appears virtually limitless. Although the theory is not prescriptive for the majority of games, it remains a triumph in taxonomy, and conduces to a better understanding of those games which it can only describe.

Despite the non-zero-sum status of the Prisoner's Dilemma, the saddle-point criterion can exert an inimical influence upon its play. Since the Prisoner's Dilemma is a two-person, non-zero-sum game, the theory cannot prescribe an unconditional resolution.[13] However, it does describe a multitude of conditional resolutions. And therein lies the dilemma's appeal, which devolves about elucidating variegated conditions under which resolutions can be achieved. Before examining the Prisoner's Dilemma, one must complete the game-theoretic background sketch, by addressing two questionable assumptions made in the development of the taxonomy; namely, the existence of the utility function, and the meaning of rationality.

---

[12] Rapoport & Chammah, 1965, p.23.

[13] Von Neumann and Morgenstern showed that any N-person, non-zero-sum game can be re-interpreted as an $(N+1)$-person, zero-sum game. However, if the two-person Prisoner's Dilemma were re-interpreted as a three-person, zero-sum game, novel problems of coalition formation would arise. E.g see A. Rapoport, *Fights, Games, and Debates*, The University of Michigan Press, Ann Arbor, 1960, pp.195-196.

Chapter Two
The Utility Function

The utility function can be regarded as both necessary to yet insufficient for the theory of games. In the absence of a utility measure, the players cannot value-order payoffs of different game-states (or outcomes) contingent upon their (the players') possible choices; in the absence of value-ordering, the players cannot express their preferences; and in the absence of expression of preference, no moves are made, and the game cannot be played. In the presence of a utility measure, however, game theory inherits problems already embedded, at the axiomatic level, in utility theory itself. As Luce and Raiffa point out:

> ". . . utility theory is not a part of game theory. It is true that it was created as a pillar for game theory, but it can stand apart and has applicability in other contexts."[1]

And while the edifice of game theory does not lack support from said pillar, its architecture is definitely constrained by weaknesses in the nature of the support.

The two chief assumptions in von Neumann's and Morgenstern's utility theory are well-summarized, by Luce and Raiffa, as follows:

> (1) "That, given two alternatives, a person either prefers one to the other or is indifferent between them."
> (2) "That there are certain well-defined chance events having probabilities attached to them which are manipulated according to the rules of probability calculus."[2]

But, as Luce and Raiffa indicate, both assumptions are subject to criticism.[3] Critical examples follow.

For the first assumption, it is understood that the utility function, U, quantifies the preferences of the players. To accomplish this, the utility function must have two minimally necessary properties: transitivity, and linear transformability.

---

[1] Luce & Raiffa, 1957, p.12.

[2] Ibid, pp.371-373.

[3] Ibid.

(i) Transitivity: given any outcomes v and w, if player P prefers v to w, then $U(v) > U(w)$. In other words, if P prefers an apple to an orange, then the utility of an apple must be greater than the utility of an orange, with respect to player P.

(ii) Linear transformability: if the probabilities that v and w obtain are p and $(1-p)$ respectively, then $U[p(v) + (1-p)(w)] = pU(v) + (1-p)U(w)$. In other words, the utility of the sum of the respective probabilities that P obtains an apple, and an orange, is equal to the sum of the products of the probability that each fruit obtains and the utility of that fruit, with respect to player P.

But the utility-theoretic assumption (1), concerning preference, breaks down in the following example: suppose P prefers an apple to an orange, an orange to a pear, a pear to a banana, and a banana to an apple. Let these preferences be represented by v, w, x and y respectively. By the property of transitivity, $U(v) > U(w)$, $U(w) > U(x)$, $U(x) > U(y)$, and $U(y) > U(v)$. Now suppose P is offered a choice between either an apple and an orange, or a pear and a banana. In this case the utility function cannot value-order P's preferences, since it cannot determine whether $[U(v) + U(w)]$ is greater than, less than, or equal to $[U(x) + U(y)]$.[4]

This breakdown stems from the circularity of P's preferences which, though conceivable, is not orderable by the relation of transitivity. The problem belongs to the same class as Arrow's "voter's paradox".[5]

Another breakdown of the assumption of preference occurs in the next example. Empirically,

> ". . .it was found that certain people preferred <u>any</u> bet in which they obtained one of two amounts of money with probability 1/2, to a bet in which the probabilities are

---

[4] Neumann & Morgenstern were well aware of this shortcoming. They termed it "the relationship of incomparability"; 1955, p.630.

[5] If P_1 prefers candidate $X > Y > Z$, P_2 prefers $Z > X > Y$, and P_3 prefers $Y > Z > X$, then two of three people prefer $X > Y$, two of three prefer $Y > Z$, and two of three prefer $Z > X$. see K. Arrow, *Social Choice and Individual Values*, Yale University Press, New Haven, 1970, p.33.

> 1/4 and 3/4, providing the average value obtained was the same."[6]

But modelling this result with the utility function leads swiftly to a contradiction.

Let the utility of £x = $U(x)$. Now consider these initial wagers:

Bet #1: $p(£150) = p(£100) = 1/2$; average gain of £125

Bet #2: $p(£200) = 1/4$, $p(£100) = 3/4$; average gain of £125

Since the first wager was found to be empirically preferable,

$$(1/2)U(100) + (1/2)U(150) > (1/4)U(200) + (3/4)U(100)$$

or

$$2U(150) > U(100) + U(200) \qquad (1)$$

If the amounts in Bet #1 are changed to £50 and £200, while those in Bet #2 are changed to £50 and £150, then

$$(1/2)U(50) + (1/2)U(200) > (1/4)U(50) + (3/4)U(150) \qquad (2)$$

If the amounts in Bet #1 are changed to £50 and £100, while those in Bet #2 remain at £50 and £150, then

$$(1/2)U(50) + (1/2)U(100) > (3/4)U(50) + (1/4)U(150) \qquad (3)$$

Adding inequalities (2) and (3), then multiplying by two gives

$U(100) + U(200) > 2U(150)$, which contradicts inequality (1).

Thus Morton concludes, "No utility function of the type we have been considering can possibly describe such preferences."[7]

Mathematically speaking, relation (1) should be an equation, not an inequality. It should be an equation because the probabilistically-averaged gains are equal for both initial wagers. The inequality relation was employed to express a *psychological* preference, but was then manipulated as though it were purely mathematical. The contradiction does not arise from a *reductio ad absurdum*; rather, it inheres in the initial employment of the utility function in mutually inconsistent senses: the logical and the psychological. The argument was constructed from a faulty implicit premise; namely, that x can be simultaneously equal to y, and greater than y. Nonetheless, the psychological preference is empirical and permissible, and one cannot

---

[6] M. Davis, *Game Theory*, Basic Books Inc., New York, 1970, pp.63–64.

[7] Ibid.

discount it in order salvage consistency. So the utility function, as constituted, remains inadequate for its purpose.

In order to generalize certain problems, one may appeal to Rapoport's exposition on the *scaling* of utilities.[8] There are three different scales on which utilities can be measured: the ordinal, the interval, and the ratio.

The ordinal scale is the weakest of the three. It employs the relation of transitivity, but does not assign differences of magnitude. The ordinal scale can specify only that $A > B > C$. It is invariant with respect to positive monotone transformations; i.e. if $U(A) > U(B)$, then $U(Ax + y) > U(Bx + y)$ (where $x > 0$).

The interval scale is stronger than the ordinal, but weaker than the ratio. It can specify both transitivity and difference of magnitude; i.e. $A > B > C$, and $(A-B)$ >,<, or = $(B-C)$. The interval scale is invariant with respect to linear transformations; i.e. if $U(A-B) > U(B-C)$, then $U[(A-B)x + y] > U[(B-C)x + y]$ (where $x > 0$).

The ratio scale is the strongest of the three. It can specify transitivity and the actual ratios of magnitude; i.e. $A > B > C$, and $A/B = y$, $B/C = z$, $C/A = 1/yz$. The ratio scale is invariant with respect to similarity transformations; i.e. if $U(A/B) > U(B/C)$, then $U[(A/B)x] > U[(B/C)x]$ (where $x > 0$).

By means of these scales, one may appreciate why any constant-sum game can be represented as a zero-sum game. Consider the following example:

<u>Game 2.1 - A Constant-Sum Game</u>

	B	
A	5,5	8,2
	7,3	-4,14

<u>Game 2.2 - A Zero-Sum Game</u>

	B	
A	0,0	3,-3
	2,-2	-9,9

The payoffs in Game 2.2 were obtained by subtracting five from each payoff in Game 2.1. Similarly, for any constant-sum game, some linear transformation exists which maps it to a zero-sum game. (And

[8] Rapoport, 1966, pp.24-28.

if either player has a winning strategy in the constant-sum game, then the same player has an identical winning strategy in its zero-sum representation.[9]) Note that any such linear transformation satisfies the requirements of the interval scale, but not those of the ratio scale. With respect to either player, the ratio of differences of payoffs remains constant for both games, while the ratio of magnitudes does not.

Returning to the problem of the fruit, one can see that if the preferences are defined on the interval scale instead of the ordinal scale, then the circularity in preference does not proscribe a solution. For instance, suppose that, in addition to *P*'s transitive preferences (apple > orange > pear > banana > apple), *P*'s interval preference for an apple over a banana is greater than his interval preference for an orange over a pear. In the notation of that problem, $U(y)-U(v) > U(w)-U(x)$. Then, if offered a choice between an apple and an orange or a pear and a banana, it follows immediately that $U(x) + U(y) > U(v) + U(w)$; so *P* prefers a pear and a banana to an apple and an orange.

Note that recourse to the ratio scale is not necessary for the solution of the above problem. Indeed, according to Rapoport,[10] the measure of utilities on the interval scale is sufficient for solving game-theoretic problems (where solutions exist). In practice, however, it might be difficult to establish such a measure. While most people can articulate preferences on the ordinal scale, it is not an accustomed practice to do so on the interval scale.

Note also that the second problem, which conflates logical with psychological properties, is insensible to a change of scale. Even if the ratio of the initial preferences were specifiable, the inbuilt inconsistency in relation (1) would remain. The probabilistically-averaged ratio of $2U(150) : U(100) + U(200)$ is 1:1, while the psychologically-preferred ratio is x:1, where $x > 1$. The subsequent mathematical manipulations would yield the reciprocal ratio, 1:x, and the

---

[9] Neumann & Morgenstern call this relation "the isomorphism of strategic equivalence"; 1955, p.504.

[10] Rapoport, 1966, p.28.

contradiction would remain.

Thus far, one has vindicated Luce and Raiffa's criticism of the first of two assumptions leading to Neumann and Morgenstern's utility theory, by having illustrated shortcomings of the theory with respect to its two necessary properties (transitivity and linear transformability). These illustrations have taken place in the mode of *intra-personal* comparison of utilities. One must also consider the mode of *inter-personal* comparison of utilities, which is not less beset by difficulties.

The question of the utility of money is classic problem, which manifests itself in both modes. Intra-personally, it has been generally assumed that the utility of money is a decreasing, non-linear function of the amount. When Daniel Bernoulli pondered the question in 1738, he concluded that

> "utility resulting from any small increase in wealth will be inversely proportional to the quantity of goods previously possessed."[11]

Empirical justifications for this assumption abound. For example:

> "H. Markowitz asked a group of middle-class people whether they would prefer to have a smaller amount of money with certainty or an even chance of getting ten times that much. The answers he received depended on the amount of money involved. When only a dollar was offered, all of them gambled for ten, but most settled for a thousand dollars rather than try for ten thousand, and all opted for a sure million dollars."[12]

While the assumption has been generalized in economics as "the law of diminishing marginal utility",[13] the actual function to be employed remains quite arbitrary. Bernoulli supposed that the value of money is proportional to its natural logarithm, and von Neumann and Morgenstern partially endorsed his supposition.[14]

---

[11] D. Bernoulli, `Exposition of a New Theory on the Measurement of Risk', s.v. L. Sommer, *Econometrica*, **22**, 1954, pp.23-37.

[12] Cited by Davis, 1970, p.51.

[13] I.e. see L. Savage, *The Foundation of Statistics*, John Wiley & Sons Inc., New York, 1954, p.94.

[14] Neumann & Morgenstern, 1955, p.629.

An example by Rapoport (which he left unsolved) can be used to illustrate the practical difference between treating the utility of money as a linear versus a non-linear function.[15] A man with £1000 to gamble is offered odds of 100 to 1 on roulette. What amount should he wager in order to maximize the utility of the gamble?

Answers to this question hinge on the utility of money. If the man wishes to maximize his utility, then he could adopt the classical but questionable "principle of mathematical expectation" (that the gamble with the highest expected winnings is best),[16] and wager the entire £1000. Then he would have a 36/37 chance of losing £1000, and a 1/37 chance of winning £100,000.

To maximize his utility in this linear case, he must solve the equation

$$(36/37)(1000-x) + (1/37)(1000 + 100x) = 1000$$

where x is the amount to be wagered. The solution is $x = 0$. So if the utility of money is linear, the man's optimal wager is no wager at all. He stands to gain nothing, and to forfeit nothing.

In the non-linear case, if the man adopts Bernoulli's suggestion, then his optimal wager is found by solving the equation

$$(36/37)\,ln(1000-x) + (1/37)\,ln(1000 + 100x) = ln(1000)$$

where x is the amount to be wagered and ln is the natural logarithm. The approximate solution is $x = 47.37$. The man then stands to gain £4,737, and to lose £47.37.

Thus, depending upon which utility rule he follows, the gambler may wager all, none, or part of his money. Rules can be devised that prescribe the wager of any fraction thereof. And, given that the gambler exercises some degree of freedom of choice, he may adopt any of the above rules, or invent his own. One cannot presume to say which monetary utility function seems to be the "best".

---

[15] Rapoport, 1960, pp.119–120.

[16] E.g. see Savage, 1954, p.91.

The problem becomes even more open-ended in the mode of inter-personal comparison of utilities. In this mode, one seeks a function that stipulates the different values, to different players, of a given payoff.

For example, suppose that two players, *A* and *B*, are competing for an apple. For the inter-personal comparison of utilities, the weakest possible scale (the ordinal) demands that the utility function stipulate whether the apple's value to *A* is greater than, less than, or the same as its value to *B*. One might assess the case in point, and attempt to make an evaluation. If *A* owns an apple orchard while *B* does not, one might argue that a single apple holds greater value for *B*. Then again, *B* may have recently consumed several apples, while *A* may be extremely hungry, in which case the apple holds greater value for *A*, his orchard notwithstanding. Or, if both *A* and *B* are severely allergic to apples, then the apple holds equally negative value to both, unless one of them keeps a horse. Any number of heuristic arguments, and counter-arguments, can be made; but such argumentation is a far cry from the articulation of a well-defined mathematical function.

Now, to complicate matters: suppose that both *A* and *B* prefer apples to oranges. The game-theoretic interval scale demands that the utility function stipulate whether *A*'s preference is greater than, less than, or the same as *B*'s preference. As Rapoport points out,

> ". . .the interval scale does not permit interpersonal comparison of utilities, because both the zero point and the unit of this scale remain arbitrary."[17]

Again, in the case of money, one may attempt to fix this scale acccording to some rule. Suppose that players *A* and *B* are competing for £100. Explicitly, if one asks whether this prize has greater value for *A* than for *B*, then the respective utilities of £100 seem to require measurement on the ordinal scale. But if one is implicitly asking whether this prize respresents a greater increase in wealth for *A* than for *B*, the respective utilities require measurement on the stronger interval scale. Now suppose that *A* is wealthy, while *B* is

---

[17] A. Rapoport, `Interpersonal Comparison of Utilities', *Lecture Notes in Economics and Mathematical Systems*, **123**, 1975*a*, pp.17–43.

impecunious. Then, arguably, £100 has greater value for *B*. In general, a fixed sum of money may have different utilities for different players.[18] Then one is confronted by a previous, unresolved problem; namely, the necessity of first fixing the intra-personal interval scale, in order to permit inter-personal comparisons.

Owing to this kind of insuperable difficulty, inter-personal utility theory draws moderate to severe criticism:

> "The problem of trying to conceptualize and apply interpersonal comparisons of utility is still unsolved."[19]

> ". . .interpersonal comparison of utility has no meaning."[20]

There is a way in which these problematic issues can be sidestepped, and the preferential aspect of utility theory salvaged, at least for game-theoretic purposes. It consists in measuring payoffs in units of *pure utility*, or *utiles*. Unlike the utility, the utile is assumed to conserve the player's preferences. Unlike utilities, utiles can be compared, both intra-personally and inter-personally. The game-theoretic distinction between utility and utile is somewhat analogous to the physical distinction between weight and mass. The first is a relative measure; the second, absolute (in a Newtonian sense, at non-relativistic velocities). For the theory of games, the adoption of a pure utile measure restricts the range of empirical application, but greatly enhances the domain of theoretical development. And the theory, if sufficiently developed in terms of players' preferences, may yet discover ways of applying itself anew.

Now recall the second assumption of Neumann's and Morgenstern's utility theory, as summarized by Luce and Raiffa:

> (2) "That there are certain well-defined chance events having probabilities attached to them which are manipulated according to the rules of probability calculus."[21]

---

[18] This is also the view of Luce & Raiffa, 1957, p.43.

[19] R. Singleton & W. Tyndall, *Games and Programs*, W.H. Freeman & Co., San Francisco, 1974, p.39.

[20] Arrow, 1970, p.9.

[21] Luce & Raiffa, 1957, pp.371–3.

This assumption is not less subject to criticism than the first. One centre of contention lies in the plausibly-worded phrase "according to the rules of probability calculus", in which the definite article ("the") implies the existence of a singular or universally-accepted set of rules for the calculation of probabilities. This implication fails to acknowledge—or bridge—the enormous rift between two most general schools of probabilistic thought: the *a priori* (which includes classical and Bayesian systems), and the *a posteriori* (or frequentist interpretation);[22] each of which assesses probabilities according to a different set of rules.

While it lies beyond the scope of this enquiry to embody a disquisition on the philosophy of probability theory, it is minimally necessary to differentiate, in passing, between the two general schools. In so far as this enquiry has recourse to both probability paradigms, as occasion warrants, it seems prudent to draw a fundamental—if limited—distinction between them.

First, it must be said that the distinction itself is one of recognition rather than definition. One can equally well recognize four schools of probabilistic thought, or more.[23] And one can draw ever-finer distinctions between proponents of similar schools. But the two suffice for this purpose.

The distinction can be drawn quite readily. Suppose two players wish to shoot craps in a casino. The rules are as follows: the shooter wagers an amount of money, then rolls a pair of dice. If he obtains seven or eleven on that roll, he wins; if two, three or twelve, he loses. If he obtains any other number, then he must roll the dice repeatedly until: either that number appears again, in which case he wins, or seven appears, in which case he loses. Suppose that

---

[22] This distinction is drawn e.g. by T. Seidenfeld, *Philosophical Problems of Statistical Inference*, D. Reidel Publishing Company, Dordrecht, 1979. He classifies Laplace, De Morgan, Pearson, Keynes, Jeffreys, Carnap, Finetti, and Savage as Bayseians; Boole, Venn, Fisher, Neyman, von Mises, Reichenbach, Wald, Hacking, and Kyburg as frequentists: p1.

[23] E.g. see R. Weatherford, *Philosophical Foundations of Probability Theory*, Routledge & Kegan Paul, London, 1982, pp.6*ff*. Weatherford recognizes four types of theory: classical, *a priori*, relative frequency, and subjectivist.

one of the players is an *a priori* probabilist; the other, an *a posteriori* probabilist.

The *a priori* probabilist assumes that the dice are fair, and calculates the likelihood of different game-states occurring according to the rules of classical probability theory (by finding the ratio of equipossible cases to all possible cases, for each state).[24] He finds that a pair of dice rolled simultaneously can produce thirty-six possible game-states: rolls of two or twelve can occur in only one way each; three or eleven, in two ways each; four or ten, in three ways each; five or nine, in four ways each; six or eight, in five ways each; while seven can occur in six different ways. He then finds the associated probabilities of obtaining each number on a given roll: two or twelve, 1/36; three or eleven, 2/36; four or ten, 3/36; five or nine, 4/36; six or eight, 5/36; seven, 6/36. He then finds that his chances of winning on the first roll are 8/36; of losing, 4/36; of having to roll again, 24/36. But if he has to roll again, his chances of winning will vary from 2/36 to 5/36, while his chances of losing will remain constant at 6/36.

The *a posteriori* probabilist, however, makes no assumption whatsoever about the "fairness" of the dice. For him, the concept of equipossibility has no meaning.[25] The *a posteriori* probabilist makes a long series of observations of the game, recording the outcome of each roll of the dice. After a sufficiently large number of rolls have been observed, he calculates the relative frequency with which each outcome has occurred. If the dice are "fair", then as the number of observations increases, the frequency distribution will tend

---

[24] A rigorous justification for this method was developed by James Bernoulli in his *Ars Conjectandi*; see e.g. I. Todhunter, *A History of the Mathematical Theory of Probability*, Macmillan & Co., London, 1865, pp.70–73.

[25] The frequentist position was developed in order to avoid circular definitions and other inherent problems of classical theory. See R. von Mises, *Probability, Statistics and Truth*, Dover Publications Inc., New York, 1981 (translation of revised edition of 1951).

toward the classical probability values.[26] If the dice are not fair (i.e. are "loaded"), then the weight of the loading will be reflected in the given frequency distribution.

Thus, in an honestly-run game, both probabilists will agree on their chances of winning and losing. But in a dishonestly-run game, the *a priori* probabilist stands to be cheated, while the *a posteriori* probabilist will have fuller knowledge of the true odds. It is of interest that the *a posteriori* probabilist need know nothing of classical probability theory to make his assessment. The dice yield an empirical result; if loaded, they will not be presumed to have deviated from an *a priori* expectation. Thus the *a posteriori* probabilist not only cannot be cheated; he also avoids making moral presumptions upon the honesty, or dishonesty, of this type of game.

One more example is instructive. Suppose the probabilists are about to play the children's game of Rock, Scissors, Paper. In this game, two players each place one hand behind their backs, then simultaneously present their hands in one of three configurations: a fist (signifying rock), a Churchillian "V" (signifying scissors), or a palm (signifying paper). Rock defeats scissors (by virtue of smashing); scissors defeat paper (by virtue of cutting); paper defeats rock (by virtue of enveloping). The matrix is as follows:

Game 2.3 - Rock, Scissors, Paper

			B		
		R	*S*	*P*	
	R	0,0	1,-1	-1,1	*R* means Rock
A	*S*	-1,1	0,0	1,-1	*S* means Scissors
	P	1,-1	-1,1	0,0	*P* means Paper

This is a two-person, zero-sum, non-co-operative game that is not strictly determined. The matrix of Game 2.3 not only has no saddle point: it is completely symmetric with respect to payoffs.

[26] This consequence is explicit in James Bernoulli's Law of Large Numbers. Laplace also developed a method for acertaining how many trials are necessary to obtain a given result that lies within pre-assigned limits. See Todhunter, 1865, pp.548-54.

From either player's point of view, each row (and column) contains exactly one winning outcome, one losing outcome, and one drawn outcome. Furthermore, all non-zero payoffs are identical in absolute magnitude. Thus neither player can express a logical preference for any row or column.

Suppose that player *A* is an *a priori* probabilist, and that nothing whatever is known about player *B*. Player *A* must resort to the "principle of insufficient reason"; namely, that

> "alternatives are always to be judged equiprobable if we have no reason to expect or prefer one over another."[27]

So *A* assumes that player *B* will choose *R*, *S*, or *P* with probabilities of 1/3 each. *A*'s expected utility of choosing *R* is then

$$\begin{aligned} EU(R) &= (1/3)U(R,R) + (1/3)U(R,S) + (1/3)U(R,P) \\ &= (1/3)(0) + (1/3)(1) + (1/3)(-1) \\ &= 0 \end{aligned}$$

Similarly, *A*'s expected utilities of choosing *S*, and *P*, are identically zero.

In such a case, Von Neumann and Morgenstern also recommend that *A* play equiprobably:

> "Thus one important consideration for a player in such a game is to protect himself against having his intentions found out by his opponent. Playing several different stratgies at random, so that only their probabilities are determined, is a very effective way to achieve a degree of such protection: by this device the opponent cannot possibly find out what the player's strategy is going to be, since the player does not know it himself."[28]

This is a compelling argument, which holds as long as the opponent is also playing with uniform randomness. Indeed, if both *A* and *B* proceed to play as such, then over the course of many plays, they will each tend to win one third of the games, lose one third of the games, and draw one third of the games.

And in this case, the *a posteriori* probabilist, observing that the relative frequencies with which *B* chooses *R*, *S*, and *P* are ap-

---

[27] Weatherford, 1982, p.29; see also Luce & Raiffa, 1957, p.284.

[28] Neumann & Morgenstern, 1955, p.146.

proximately 1/3 each, adopts the same strategy of random play, and fares the same as the *a priori* probabilist. As long as *B* plays with uniform randomness, both probabilists achieve the same result.

But suppose that *B* plays with non-uniform randomness; i.e. that the *a priori* probabilities of his choices are weighted. Let the weights be such that *B* choses *R* with probability 1/2, and *S* and *P* with probabilities 1/4 each.

(One can posit any number of plausible reasons for the uneven weightings. For example, suppose that *B* intends to play with uniform randomness by rolling a die, and that he will choose *R* if he rolls one or two; *S* if he rolls three or four; and *P* if he rolls five or six. But unknown to *B*, he uses a die that is "loaded" to yield one and two with probabilities 1/4 each; and to yield three, four, five and six with probabilities 1/8 each. Then the above distribution would obtain.)

If *B* plays according to these weights, and *A* plays with *a priori*, uniform randomness, then the matrix of probabilities for each outcome is as follows:

Game 2.4 – Weighted Probability Matrix for Rock, Scissors, Paper

Player *A*: $p(R) = p(S) = p(P) = 1/3$

Player *B*: $p(R) = 1/2;\ p(S) = p(P) = 1/4$

		B		
		$p(R)$	$p(S)$	$p(P)$
	$p(R)$	1/6	1/12	1/12
A	$p(S)$	1/6	1/12	1/12
	$p(P)$	1/6	1/12	1/12

Unknown to player *A*—who is not recording the relative frequencies of *B*'s choices—his expected utilities for Game 2.4 are now

$$EU(R) = p(R,R)\,U(R,R) + p(R,S)\,U(R,S) + p(R,P)\,U(R,P)$$
$$= (1/6)(0) + (1/12)(1) + (1/12)(-1)$$
$$= 0$$

$$EU(S) = p(S,R)\,U(S,R) + p(S,S)\,U(S,S) + p(S,P)\,U(S,P)$$
$$= (1/6)(-1) + (1/12)(0) + (1/12)(1)$$
$$= -1/12$$

$EU(P) = p(P,R)U(P,R) + p(P,S)U(P,S) + p(P,P)U(P,P)$
$= (1/6)(1) + (1/12)(-1) + (1/12)(0)$
$= 1/12$

Although *A*'s set of expected utilities in Game 2.4 differs from that of Game 2.3, *A*'s average result is identical in both cases. After a large number of plays of Game 2.4, he will have won one third of the games, lost one third of the games, and drawn one third of the games, for a net average gain of zero utiles.

Now suppose that the *a posteriori* probabilist takes his turn as player *A*. He has been observing player *B*, and has recorded the relative frequencies of *B*'s choices. The *a posteriori* probabilist counters *B*'s weighted play with a weighting of his own. As player *A*, he makes random choices with weighted probabilities one-quarter for Rock and Scissors, and one-half for Paper. The new probability matrix is as follows:

Game 2.5 - Re-Weighted Probability Matrix for Rock, Scissors, Paper

Player *A*: $p(R) = p(S) = 1/4$; $p(P) = 1/2$

Player *B*: $p(R) = 1/2$; $p(S) = p(P) = 1/4$

		B		
		$p(R)$	$p(S)$	$p(P)$
	$p(R)$	1/8	1/16	1/16
A	$p(S)$	1/8	1/16	1/16
	$p(P)$	1/4	1/8	1/8

For Game 2.5, player *A*'s expected utilities are

$EU(R) = p(R,R)U(R,R) + p(R,S)U(R,S) + p(R,P)U(R,P)$
$= (1/8)(0) + (1/16)(1) + (1/16)(-1)$
$= 0$

$EU(S) = p(S,R)U(S,R) + p(S,S)U(S,S) + p(S,P)U(S,P)$
$= (1/8)(-1) + (1/16)(0) + (1/16)(1)$
$= -1/16$

$EU(P) = p(P,R)U(P,R) + p(P,S)U(P,S) + p(P,P)U(P,P)$
$= (1/4)(1) + (1/8)(-1) + (1/8)(0)$
$= 1/8$

This result certainly favours player *A*. On average, *A* wins three-eighths of the games, loses five-sixteenths of the games, and draws five-sixteenths of the games. The average quantity won exceeds the average quantity lost by one-sixteenth of a utile. After a large number of plays of Game 2.5, *A*'s net average gain will be one utile for every sixteen plays.

Again, the *a posteriori* probabilist need not impute any motives, whether logical or psychological, to account for and to counter player *B*'s weighted choices. Just as in the example of the dishonestly-run casino, *A*'s observation of the relative frequency of events is a value-neutral process.

The purpose of these two examples is most assuredly *not* to make a case for the relative superiority of one school of probabilistic thought over another; rather, it is to argue that the outcomes of certain games can be affected by a particular choice of probabilistic paradigm on the part of the player.

For games involving random (or pseudo-random) moves, a player must assign some probabilistic distribution to outcomes in order to calculate the expected utilities of different choices. It has been illustrated that, in some cases, the results of *a priori* and *a posteriori* probability assignments are convergent. When the two methods do not converge, it has been shown that the player who employs an *a posteriori* calculus may forfeit less, or gain more, than one who employs an *a priori* calculus.

The objection can be made that no example was given which explicitly favours the *a priori* over the *a posteriori* method. The latter method is not without potential shortcomings, one of which can be illustrated in the following way.

Suppose two players are "matching pennies". Player *A* first predicts either "even parity" (two heads or two tails) or "odd parity" (one head and one tail). Next, they each flip a penny and allow the coins to fall. If player *A* predicted the outcome correctly, he wins; if not, he loses. The matrix is as follows:

Game 2.6 - Matching Pennies

		B		
		E_o	O_o	*E* means "even parity"
	E_p	1,-1	-1,1	*O* means "odd parity"
A				*p*-subscript means "predicted"
	O_p	-1,1	1,-1	*o*-subscript means "occurred"

Suppose that player *A* is an *a priori* probabilist, and suppose also that both coins are fair. Then the probability of each outcome is 1/4. Player *A* constructs a probability matrix:

Game 2.7 - Probability Matrix for Matching Pennies

Player *A*: $p(H) = 1/2$, $p(T) = 1/2$
Player *B*: $p(H) = 1/2$, $p(T) = 1/2$

		B		
		$p(H)$	$p(T)$	
	$p(H)$	1/4	1/4	
A				$p(H)$ means "probability of heads"
	$p(T)$	1/4	1/4	$p(T)$ means "probability of tails"

Since *A* predicts even and odd parity with random probabilities of one-half each, his prediction percentage is approximately fifty percent correct over a large number of games. On net average, he neither wins nor loses.

Now suppose that player *A* is an *a posteriori* probabilist. Suppose also that both coins are fair, but that *A* does not make a sufficiently large number of observations. Let him make ten observations, in which he finds his coin to have landed "heads" four times, and "tails" six times; and in which he finds *B*'s coin to have landed "heads" seven times and "tails" three times. Now let *A* conclude from these observations that his coin is weighted 6:4 in favour of tails, while *B*'s coin is weighted 7:3 in favour of heads. *A* then constructs a (fallacious) relative frequency matrix, based upon his limited observations:

Game 2.8 - Fallacious Relative Frequency Matrix

Player *A*: $f(H) = 4/10$, $f(T) = 6/10$

Player *B*: $f(H) = 7/10$, $f(T) = 3/10$

		B	
		$f(H)$	$f(T)$
A	$f(H)$	28/100	12/100
	$f(T)$	42/100	18/100

$f(H)$ means "frequency of heads"
$f(T)$ means "frequency of tails"

From this matrix, *A* finds that the combined relative frequency of even parity is (28+18)/100 = 46/100, while that of odd parity is (42+12)/100 = 54/100. Thus *A* concludes that he should play randomly but not uniformly, weighting his predictions to favour even parity in forty-six of each one hundred subsequent games, and odd parity in fifty-four.

Consequently, after a large number of subsequent games, *A*'s prediction percentage is only about forty-two percent correct. On net average, he loses eight utiles per hundred games.

When interpreting *a posteriori* probabilities, then, it is of paramount importance to ensure that an observed relative frequency attains a limiting value.[29] This *A* failed to do, by observing an insufficient number of events.

Furthermore, it is not always feasible to employ the *a posteriori* method. In countless situations where one must take a decision under risk or conflict of interest, without the benefit of a sufficiently lengthy series of observations of outcomes in similar situa-

[29] If it is to meet the von Mises criterion of randomness, this value must be independent of any "selection rule" for the observed events. For instance, a "fair" coin will land "heads" in about fifty percent of trials—given a large enough number of trials—and this limiting value should be obtainable from any large sub-sequence of the observed trials, according to any rule of place-selection. For instance, the relative frequency of "heads" in even-numbered trials, odd numbered-trials, prime-numbered trials, i^{th} trials, etc., should have a limiting value of fifty percent, within confidence limits defined by appropriate statistical tests. See Mises, 1981, pp.87-9.

tions, the *a posteriori* calculus is inapplicable. Then one has recourse to classical, or to Bayesian, or to subjective interpretations. Not every calculus is available in every game-theoretic situation.

While the introduction of a pure utility measure allows one to evade unsolved problems of comparing utilities (at the cost of restricting the applicability of the theory), classes of games exist in which one cannot avoid the employment of a probability calculus. The difficulty then lies in selecting an appropriate calculus for the given situation, and is compounded by the fact that each school of probabilistic thought admits of particular strengths and weaknesses.

It can be seen that the von Neumann-Morgenstern utility function is formed by the concatenation of two problematic calculi: one of preferences, the other of probabilities. That both are subject to criticisms seems clear enough. The severest criticism, though, is not necessarily the most instructive. One has it from Savage that

> "The postulates leading to the von Neumann-Morgenstern concept of utility are arbitrary and gratuitous."[30]

That they are rich in controversy is apparently beyond dispute. Utility theory, however incompletely formulated, remains indispensable to game theory.

And one further concept, not less dispensable but perhaps more controversial, requires elaboration in this game-theoretic background; namely, the concept of rationality.

[30] Savage, 1954, p.99.

Chapter Three
Game-Theoretic Rationality

Before delving into the intricacies of the Prisoner's Dilemma, it is necessary to review the concept of *rationality* in a game-theoretic context. The concept is laden with difficulties, but must be addressed; for it is of central importance to both game theory in general and the Prisoner's Dilemma in particular.

An assumption about rational choice was ineluctably smuggled into the synopsis, in Chapter One, of the property of strict determinateness. It was implicitly assumed that a so-called "rational" player would select that row (or column) of a game matrix which contains a saddle point, if indeed such a point exists in the given game. Recall that the grounds for this assumption were that if the so-called "rational" player chooses the minimax (or maximin, as the case may be), then he can fare no worse in that game, regardless of whether his opponent plays "rationally" or "irrationally". The example was given in order to illustrate the importance of the saddle point; its corollary implication, however, was that a "rational" player always plays minimax (or maximin, as the case may be), while an "irrational" player may not always do so. The soundness of this implication must now be called into question.

Let one commence with the Von Neumann-Morgenstern caveat to their qualification of rationality:

> "The individual who attempts to obtain these respective maxima [maximin and minimax] is also said to act 'rationally'. But it may be safely stated that there exists, at present, no satisfactory treatment of the question of rational behaviour."[1]

Rapoport expresses the wish to modify the caveat itself, by denuding game theory of "psychological" overtones.[2] The term "behaviour" is connotative of psychology, and Rapoport argues that psychological

[1] Neumann & Morgenstern, 1955, p.9. See also Luce & Raiffa, 1957, p.5.

[2] Rapoport, 1966, p.103.

orientations of "rational players" should be irrelevant in a formal game-theoretic context.[3]

One reason for this viewpoint is as follows: in an idealized situation, just as one can compare utilities in units of pure utiles, so it would be convenient to define a "rational player" in a way that depends purely on his play. As the ideal unit of utility, the utile orders the values of preferences by mapping them to the real numbers. It transcends the intransitivity of circular preferences, and permits the interpersonal comparison of utilities. Similarly, the ideal definition of rationality would map each play to a Boolean statement, either "rational" or "irrational", in a way that transcends the psychological motives of the players. Can such a definition be articulated, even in the ideal case?

In the game of poker, it is often useful to employ the tactics of "bluffing" and "sandbagging", which entail, respectively, the occasional over-playing of weak hands, and under-playing of strong hands, in order to mislead one's opponents. These tactics are workable because poker is a game of imperfect information.[4] As such, the outcome of a given hand does not necessarily depend on the cards that the players are actually holding, and frequently depends rather upon the fictitious cards that they *believe* one another to be holding.

Suppose a player decides to bluff on a weak hand. He wagers increasingly large amounts of money on his cards, as though he held a strong hand. If his bluff is not "called", then the bluffer wins with a hand that would normally have lost; and the losing players, who did not pay to view his cards, might assume that he did indeed hold a winning hand.

But if his bluff is "called", the bluffer must reveal his weak hand to the players who have matched his wager. They immediately realize that he was attempting to bluff. The bluffer thus loses the hand in question, but sets a potentially lucrative precedent in the process. For when he next holds a very strong hand, he may again

---

[3] Ibid.

[4] Simply stated, a game of imperfect information is a game in which some moves are concealed. E.g. see Neumann & Morgenstern, 1955, pp.51–52 *ff*.

wager large amounts of money (perhaps feigning nervousness as he does so), in order to induce the other players into believing that he is once again attempting to bluff. They may match his wager and call what they suppose to be his "bluff", only to find that he has not been bluffing on this occasion.

Thus the astute poker player is willing to lose one or more hands quite deliberately, in order to potentiate a future situation in which he expects not only to recoup his previous losses, but also to realize a net gain.

If one defines poker-theoretic rationality as the wish to maximize one's overall winnings (or minimize one's overall losses), then it is also poker-theoretically rational to employ the tactic of bluffing from time to time (although game theory can prescribe neither the frequency nor the cost of the tactic). Then, if a player loses a given hand because his bluff has been called, he is not irrational, but perhaps ambitious. Suppose another player loses several hands in this fashion, but the game ends before he can recoup his losses. That player is not irrational, but perhaps unfortunate. Suppose another player wins the game without ever having bluffed. That player is not irrational, but perhaps fortunate. And suppose another player loses all his money, without ever having bluffed. That player is not irrational, but perhaps unskilled. Suppose a player is winning by a substantial amount, but wagers this entire amount on the final hand, and loses. That player is not irrational, but perhaps avaricious. Thus no poker player is irrational, if he wishes to win in the long run.

However, this definition of rationality is the antithesis of the sought-after "ideal" definition, because it depends not at all on the play and hinges solely upon the motives of the players.

Yet it does not seem at all sensible to alter the working definition of rationality with respect to poker, by claiming that it is rational to seek to win as much as possible, or to lose as little as possible, on each individual hand. While this new working definition would conform to the ideal, by assessing the play and discounting the motives of the players, it could prove paradoxical. Suppose the overall winner of a poker game turns out to be a player who bluffs quite frequently. The working definition labels him as "irrational", yet he fares better than the "rational" player. A definition of rational play that both urges a player to win by rational means and acknowledges the potential superiority of irrational means, is self-contradictory and therefore unsatisfactory.

At first blush, this state of affairs may seem to arise because poker is a game of imperfect information which is not strictly determined. (Recall that in a strictly determined game, a player who chooses the minimax fares even better if his opponent does not choose the maximin; and a player who chooses the maximin fares even better if his opponent does not choose the minimax.) In a game without a saddle point, the "rational" player has no inherent defense against an "irrational" player, if rationality means maximizing gains or minimizing losses on every play.

However, it can be demonstrated that the paradox is a consequence, not of imperfect information and the absence of strict determination, but of the attempt to articulate an ideal definition of rationality. Consider chess, which is a strictly determined game of perfect information.[5] A chess game is either won, lost, or drawn, according to the disposition of the pieces, which are always in plain view of the players. The tactic of bluffing would seem to have no relevance in this game.

In world championship chess, a match is the best of twenty-four games (in each of which a player receives one point for a win, no points for a loss, and one-half point for a draw). Thus, the first player to attain twelve-and-one-half points is the victor.[6] The working definition of rationality, which proved paradoxical in poker, prescribes that the rational player attempt to win as many chess games as possible, and lose as few as possible, in order to win the match.

If that seems reasonable, then consider what actually took place in the 1972 world championship match in Reykjavik, between

---

[5] Simply stated, a game of perfect information is a game in which no moves are concealed from any player.

[6] If the score is tied at twelve points each, then the incumbent champion retains the title.

Bobby Fischer and Boris Spassky.[7] Fischer, who had given his prior written assent to the presence of television cameras, found that their proximity interfered with his concentration. He therefore refused to play until the cameras were removed, and remained in his hotel room when the match officially commenced. Fischer's apparent "bluff" was called, and he proceeded to forfeit the first two games of the match. An accommodation was then reached, and Fischer played in subsequent games. Spassky held the initial lead of two games to none (a considerable advantage at this level of competition), but was unnerved by Fischer's cold-blooded forfeitures. Fischer eventually won the match with brilliant play, while Spassky made several blunders unworthy of a player of his stature.

According to the working definition of rationality, Fischer played irrationally in the first two games, by losing them deliberately. A "rational" player would have elected to play under conditions of slightly impaired concentration, because he could not have fared worse by playing, and might indeed have fared better. But in retrospect, Fischer's "irrational play" in the first two games was an ingredient of his eventual victory in the match.

One seems obliged to concede that, whether the game is one of perfect or imperfect information, and whether strictly determined or not, a certain number of losses may conduce to an overall win in the long run. In that case, one cannot demand, by definition, that a "rational" player seek to maximize his wins, and minimize his losses, at every opportunity. But then one cannot define rationality in terms of the play itself, and one is thrown back upon the undesirable necessity of gauging rational, or irrational play, in terms of the motives of the player.

At this juncture, one might argue that the problem stems not from the working definition of rationality as such, but from the

[7] E.g. see C. O'D. Alexander, *Fischer v. Spassky: Reykjavik 1972*, Penguin Books Ltd., Harmondsworth, 1972.

failure to draw a categorical distinction between games and meta-games.[8]

Poker is a meta-game, in that a "game" of poker consists of many hands. Each hand may be evaluated as a separate game, and the meta-game outcome is the algebraic sum of the outcomes of the hands. Similarly, a chess match is a meta-game consisting of many games of chess. The outcome of the match is the algebraic sum of the outcomes of the games. Given this distinction, is it possible to formulate an ideal definition of rationality which takes into account that deliberate losses of a game (or games) may still conduce to victory in the associated meta-game?

The distinction between games and meta-games necessitates a similar distinction between *move* and *strategy*, in the sense that a losing move in a given game may form part of a winning strategy in the associated meta-game. In that case, rationality is embodied not in the move itself, but rather in the strategy that gives rise to the move. Thus, one can attempt the following reformulation: the ideal definition of rationality would map each *strategy* (instead of *move*) to a Boolean statement, either "rational" or "irrational", in a way that transcends the psychological motives of the players. The question is, can one infer the rationality (or irrationality) of a player merely by observing his strategy? If so, then "rationality" is ideally defined.

Unfortunately, the answer to the question seems to be: not necessarily. Consider this example. Suppose a wealthy but eccentric sportsman sponsors a poker game according to the following rules: each player begins the game with £1000. There is a maximum bet of £5 and one raise per hand. The first player to *lose* £1000 wins a prize of £10,000. Now suppose a game-theorist, who is unaware of the meta-game situation, observes the play of several hands. Based on the strategies he observes, he may speedily conclude that the players are irrational (if not utterly mad). But if the game-theorist were informed that the first player to lose £1000 in the poker game wins

[8] Formal Meta-game theory was developed by N. Howard, *Paradoxes of Rationality: Theory of Metagames and Political Behaviour*, The M.I.T. Press, Cambridge, Massachusetts, 1971.

£10,000 in the meta-game, he could conclude from the same observations that the players are quite rational. Thus one cannot always infer a player's rationality (or irrationality) from his game strategy alone; one may also require knowledge of the rules of an associated meta-game, in order to draw such an inference.

It seems that an ideal definition of rationality cannot always be based solely upon the game strategy of a player; it must also take into account the rules of the associated meta-game. And, to further complicate matters, while many players may be involved in the same game, with each player may be associated a different meta-game. The game-theorist cannot infer a player's rationality unless he knows the rules of the particular meta-game associated with that player.

Consider, for instance, the hypothetical case of a wealthy poker player who loses money deliberately to his fellow poker-players, as an act of charity. He may, inadvertently, win a few hands in the process; but his meta-game rule is to maximize his long-term losses. Suppose the other players are playing "normally"; that is, they share the meta-game rule of attempting to maximize their long-term winnings. If the game-theorist observer is unaware of the charitable player's meta-game rule, he might infer, based on his observations of strategy, that the player is irrational. But if made aware of the charitable player's rule, he would infer from his observations that the charitable player is indeed rational.

Note that one does not need to know the actual motives of the player in order to draw such an inference. It is not necessary that the game-theorist be told that the player in question is motivated by charitability; he need only know whether the player's meta-game rule is the long-term maximization of winnings, or of losses. If that player's game-strategy is consistent with his meta-game rule, then that player may be called "rational"; if not, then "irrational."

Note also that the other players in this hypothetical game, though they share an identical meta-game rule (the maximization of long-term winnings), may do so for completely different reasons. One player may wish to purchase a gift for his wife; another may wish to make a donation to medical research; a fourth may wish to pay for music lessons for his child. Again, the game-theorist does not need to be told what motivates these players; he need only know that their meta-game rule is the maximization of long-term winnings. If a player's game strategy is consistent with his rule, then that player may be called "rational"; if not, then "irrational".

This dyadic definition of rationality, which assesses consistency between a game strategy and its associated meta-game rule, entails no moral judgement concerning intra-personal motives, nor does it attempt an inter-personal comparison of motives. It satisfies Rapoport's demand that the psychology of a player be excluded from consideration of his rationality.

From the foregoing example, it is clear that one cannot infer a player's meta-game rule simply by observing his game-strategy. While the losing strategy of the charitable poker-player is consistent with his meta-game rule of maximizing long-term losses, an identical losing strategy could also be adopted by an irrational player whose meta-game rule is to maximize his long-term winnings. The observer of these players would err by inferring the identity of their meta-game rules from the identity of their game strategies. Of course, if the observer were told that one of the players is rational, and the other irrational, he could then infer that their meta-game rules are different. But he could not identify the rational (or the irrational) player without knowing which meta-game rule a particular player obeys.

In order to ascertain whether a given player is rational or not, the observer can construct a meta-matrix for that player, as follows:

Game 3.1 - Observer's Meta-Matrix for Player *A*

		A's Meta-Game Rule	
		Maximize Winnings	Maximize Losses
A's Game Strategy	Winning Strategy	player *A* is rational	player *A* is irrational
	Losing Strategy	player *A* is irrational	player *A* is rational

Thus far, it has been possible for the observer to discern between winning and losing game strategies, independently of his knowledge (or lack thereof) about a player's meta-game rule. It is possible to conceive of a worse case, however, in which the observer cannot discern between winning and losing strategies purely from the context of the game. Would such a case preclude the construction of a meta-matrix, and thus prevent him from assessing a player's rationality?

Reconsider, for example, the game of Rock, Scissors, Paper. It has been established that, if both players are *a priori* probabilists, they should both adopt a mixed strategy of uniform random play. Then, over the course of a large number of games, both players' net scores will tend toward zero. In the prior consideration of this game, it was tacitly assumed that both players obeyed a meta-game rule of maximizing their long-term winnings (or, equivalently in this class of game, of minimizing their long-term losses).

But suppose both players now obey a meta-game rule of minimizing their long-term winnings (or, equivalently, of maximizing their long-term losses). Instead of attempting to win as often as possible, both players are now (for some plausible reason) attempting to lose as often as possible. What strategies should they adopt?

If player *A* wishes to lose and player *B* wishes to win, player *A* would choose a pure strategy of either Rock, Scissors, or Paper. Suppose he chooses Rock. Player *B* would soon respond with a pure strategy of Paper. Player *A* would lose, and player *B* would win, every game thereafter. Thus each would satisfy his respective meta-game rule (and both would be rational to the game-theoretic observer).

However, if both players wish to lose, then *A* cannot adopt a pure strategy. (If he did so, again choosing Rock, then *B* would soon respond with a pure strategy of Scissors, and *A* would win every game thereafter.) If both players wish to lose, then they must each adopt a mixed strategy of uniform random play. This strategy is therefore degenerate: it is best both for mutually-desired long-term wins and for mutually-desired long-term losses. Thus the game-theoretic observer cannot ascertain, purely from the context of the play, whether both players obey a meta-game rule that maximizes wins, or losses. Nevertheless, the observer can readily construct a meta-matrix for either player, as follows:

Game 3.2 - Observer's Meta-Matrix for Player *A*

(with degenerate winning/losing stragegy)

		A's Meta-game Rule	
		Maximize Winnings	Maximize Losses
A's Game Strategy	Mixed, Uniform Random Strategy	player *A* is rational	player *A* is rational
	Pure or Non-Uniform Strategy	player *A* is irrational	player *A* is irrational

Strategic degeneracy does not affect the observer's ability to assess the rationality of either player, according to the working definition of rationality under consideration.

Do situations arise which demand more of this concept of rationality than it can afford? Apparently, they do. The working definition becomes less workable in the following examples.

Suppose player *A* is both very fond of strawberries and mildly allergic to them. He derives considerable pleasure from eating strawberries, but suffers a temporarily uncomfortable though otherwise harmless allergic reaction after eating them. If *A* is offered strawberries and declines them, is he rational? Certainly, if his meta-game rule prescribes the avoidance of discomfort whenever possible. But if *A* is offered strawberries and accepts them, is *A* irrational? Certainly not, if his meta-game rule permits the indulgence of a gustatory pleasure with the consequence of a mild discomfort.

Now suppose that *A* is offered strawberries at consecutive meals; he declines them at breakfast, but accepts them at lunch. What can a game-theoretic observer infer about *A*'s rationality? He can infer that, if *A*'s meta-game rule was the same for both meals, then *A* was rational at one meal and irrational at the other. He can also infer that, if A's meta-game rule was not the same for both meals, then *A* was either rational at both meals, or was irrational at both.

This example is one in which the meta-game rule does not necessarily remain fixed or constant throughout the duration of the meta-game itself, but is subject to change according to the shifting preferences of the player. Meta-game theorist Howard puts forward a definition of rationality based squarely upon this premise:

> "We say that *rational* behaviour consists in choosing the alternative one prefers."[9]

The working definition under consideration here is consistent with Howard's. A meta-game rule orders a player's preferences, while a game strategy chooses that alternative which reflects the ordering (if the player is rational), or which does not reflect it (if the player is irrational).

The problem is that the player's rationality can be assessed only if the observer is informed of every shift in the player's preference.

Now suppose the observer is player *B* in a game of imperfect information without a saddle point, in which changes in the players' preferences are mutually concealed. In that case, the rationality or irrationality of each player is indeterminate with respect to the other. This situation is worse than that of a game of perfect information without a saddle point, in which player *B* can be harmed by player *A*'s irrationality (and vice-versa). In a game of imperfect information without a saddle point, player *B* can be harmed not only by *A*'s irrationality, but also by *B*'s possible mistaking of *A*'s actual rationality for apparent irrationality (and vice-versa).

The first example of the indeterminacy of rationality takes place in an inter-personal context; that is, each player behaves either rationally or irrationally ("behaves" in Howard's sense, by choosing the alternative he prefers), but neither player can infer the rationality or irrationality of the other.

A second example, the indeterminacy of whose expectations is well-known to game theorists, takes place intra-personally. It

---

[9] Howard, 1971, p.xx.

involves Pascal's question of whether to subscribe, or not to subscribe, to Roman Catholic theology. Pascal's matrix is as follows:[10]

Game 3.3 - Pascal's Question

		State of Nature	
		God exists	God does not exist
Pascal's Decision	Practice Catholicism	eternal reward	pious life only
	Not Practice Catholicism	eternal punishment	impious life only

In games against a state of nature, the player does not know which state actually obtains, but assumes the actual state to be causally independent of his beliefs. In order to calculate his expected utilities, the player normally assigns a probability distribution to the states of nature.[11] The result of Game 3.3 is indeterminate in terms of expected utilities, owing to the infinite positive and negative payoffs associated with eternal reward and punishment, respectively, and the ensuing transfinite arithmetic.

But the concern here is not with the utility of Pascal's decision; rather, with its rationality. If Pascal decides to believe in the Catholic deity's existence (as a meta-game rule), then he would be rational to practice Catholicism (as a game strategy).

But would Pascal be irrational to believe in such a deity's existence and not practice Catholicism? Not necessarily. If Pascal is a fatalistic theist, he might believe that his decision is preordained. But if Pascal's decision is causally pre-determined by a deity, then it is not solely Pascal's decision. And if Pascal cannot make a free choice, then the meaning of the rationality or irrationa-

[10] Variants of the matrix can be found e.g. in Jeffrey, 1965, p.12; and Howard, 1971, p.7.

[11] Pascal, for instance, assigned a subjective probability of .00001 to the state in which God exists; see Jeffrey, 1965, p.12.

lity of his choice alters drastically. And if the deity in which Pascal believes allows him a choice, then Pascal is still not necessarily irrational not to practice Catholicism. For Pascal may prefer to sin now, and to seek absolution or redemption later.

On the other hand, if Pascal decides not to believe in the Catholic deity's existence (as a meta-game rule), then he would be rational not to practice Catholicism (as a game strategy).

But would Pascal be irrational not to believe in the deity's existence and to practice Catholicism anyway? Again, not necessarily. Given that his belief need not be absolute, Pascal may simply doubt the existence of such a deity, while practicing Catholicism in order to "hedge his bet". Or, Pascal's disbelief may be absolute, and he remains in a state of atheism, but practices Catholicism publicly to protect himself in the event of an Inquisition.

The example of Pascal is meant to illustrate that, no matter what the player's beliefs in a game against nature, arguments can be found which support the "rationality" of any personal decision he takes. But if a distinction cannot be drawn between rationality and irrationality, then the game-theoretic concepts are indeterminate in this context.

Thus the working definition of rationality (consistency between a player's meta-game rule and his game strategy), which satisfies Rapoport's game-theoretic criterion of independence from psychology and Howard's meta-game-theoretic criterion of choosing the alternative that one prefers, is not universally applicable. Classes of inter-personal games exist in which neither player can ascertain the other's rationality, or irrationality; and classes of intra-personal games exist in which the player cannot discern between rational and irrational choice.

But the problems of game-theoretic rationality hardly end there. As will be seen next, one dimension of the Prisoner's Dilemma —and arguably the most significant dimension with respect to conflict resolution—entails divergent meanings of rational choice. In games considered thus far, rationality and irrationality have been associated with (and, where definable, defined in terms of) the individual player. But with reference to the Prisoner's Dilemma, Rapoport suggests

> "that the concept of *rationality* should be re-examined, perhaps split into two concepts, individual rationality and collective rationality."[12]

A re-examination of the Prisoner's Dilemma will certainly bear out the cogency of Rapoport's suggestion.

Sufficient essentials of game theory have been reviewed to enable such a re-examination. With these basic necessities in hand (an understanding of principal taxonomic criteria, and an appreciation of the range of difficulties latent in utility theory and game-theoretic rationality), one is minimally equipped to consider some of the complexities in the Prisoner's Dilemma.

---

[12] Rapoport (*ed.*), 1974, p.4.

PART TWO:

THE STATIC PRISONER'S DILEMMA

Chapter Four

Conflicting Choices and Rationalities

Game-theoretic literature attributes the original Prisoner's Dilemma to A.W. Tucker.[1] As to its early development, Rapoport narrates:

> "To my knowledge, the earliest experiments with Prisoner's Dilemma were performed by Flood in 1952. . .and do not seem to have attracted much attention at the time. . . The `paradox' was discussed by several of the Fellows at the Centre for Advanced Study in the Behavioural Sciences in Palo Alto during the first year of its operation (1954-55). . .Possibly a decisive impetus to experimental work was given by a paper by Schelling, published in 1958. At any rate, it seems that the first experiment since Flood's was performed by Deutch in 1958. Thereafter the number of experimental papers on Prisoner's Dilemma increased very rapidly."[2]

Both theoretical and experimental interest in the Prisoner's Dilemma are stimulated by the model's structural properties. As a non-zero-sum, non-co-operative game, the Prisoner's Dilemma resists absolute theoretical prescriptions as to the "best" line of play. In consequence, a limitless range of experiments can be conducted, whose results may correlate with a wide variety of factors, from differing characteristics of the players to variants of the game itself.

The Prisoner's Dilemma can be played in both the static and the iterated modes. Logically and chronologically, the former gives rise to the latter, so it is mete to commence with the former. As von Neumann and Morgenstern declared in their general theory of two-person, zero-sum games:

---

[1] E.g. Luce & Raiffa, 1957, p.94; Rapoport & Chammah, 1965, p.24; Singleton & Tyndall, 1974, p.101.

[2] Rapoport (*ed.*), 1974, pp.19-20. The papers to which Rapoport refers are: M. Flood, `Some Experimental Games', *Research Memorandum RM*-789, The Rand Corporation, Santa Monica, 1952; T. Schelling, `The Strategy of Conflict: Prospectus for the Reorientation of Game Theory', *Journal of Conflict Resolution*, 2, 1958, pp.203-64; M. Deutch, `Trust and Suspicion', *Journal of Conflict Resolution*, 2, 1958, pp.267-79.

> "We make no concessions. Our viewpoint is static and we are analyzing only a single play."[3]

Indeed, numerous aspects of game-theory were subsequently developed, maintaining theoretical pace with the empirical transition from static to iterated non-zero-sum games. An understanding of the static Prisoner's Dilemma is a prerequisite for an appreciation of the increased complexities of iterated Prisoner's Dilemmas.

The static Prisoner's Dilemma arises from a particular type of scenario, many versions of which are rehearsed in the literature. Though the model has been embellished in a variety of ways, variations in the narrative details do not alter the problem itself, which inheres in specific properties of the game-matrix.

One version, then, is as follows: suppose two suspects are arrested, held incommunicado, and interrogated. Call them prisoner *A* and prisoner *B*. Each prisoner faces an identical choice: he can either divulge evidence against his fellow-prisoner, or refuse to do so. Since each prisoner must make a choice, the prisoners will thus generate a joint outcome, but without collusion. Both prisoners are made aware of the payoffs of each possible outcome, which are:

(1) If both *A* and *B* refuse to divulge evidence against one another, they will both be set free.

(2) If *A* divulges evidence against *B* and *B* does not divulge evidence against *A*, then *A* will be given a bribe and set free, while *B* will serve a heavy sentence.

(3) If *B* divulges evidence against *A* and *A* does not divulge evidence against *B*, then *B* will be given a bribe and set free, while *A* will serve a heavy sentence.

(4) If both *A* and *B* divulge evidence against one another, they will both serve light sentences.

In the conventional terminology of the Dilemma, each prisoner must choose between *co-operating* and *defecting*, with respect to his fellow prisoner. To *defect* means to divulge evidence; to *co-operate* means to refuse to divulge evidence. The game matrix can be constructed as follows:

---

[3] Neumann & Morgenstern, 1955, p.147.

Game 4.1 – The Prisoner's Dilemma

		B	
		c	*d*
A	*C*	*R,R*	*S,T*
	D	*T,S*	*P,P*

where $T > R > P > S$

for prisoner *A*: *C* means *co-operate*, *D* means *defect*

for prisoner *B*: *c* means *co-operate*, *d* means *defect*

The numerical values of the payoffs may fluctuate in a given Prisoner's Dilemma, but their transitive ordering does not change. *T* stands for the temptation to defect; *R*, for the reward of mutual co-operation; *P*, for the punishment of mutual defection; *S*, for the so-called "sucker's payoff".[4]

As will be seen throughout Part Two, the dilemma admits of several facets of interpretation.

The initial dilemma can be viewed as arising from the breakdown of the fundamental property of strictly-determined zero-sum games, when applied to certain non-zero-sum games; namely, the minimax criterion.[5] In game 4.1, a generalized Prisoner's Dilemma, the (*P,P*) outcome resulting from mutual defection is, in effect, a saddle point of the matrix.

Recall that, in a two-person zero-sum game with a saddle point, a player who seeks to maximize his payoff fares best by choosing that row (or column) which contains the saddle point, regardless of the other player's choice. In the Prisoner's Dilemma, however, this

[4] This conventional notation is used e.g. by Rapoport & Chammah, 1965, pp.33–4 *et passim*; by R. Axelrod, 'The Emergence of Cooperation Among Egoists', *The American Political Science Review*, **75**, 1981, pp.306–18; by R. Axelrod & W. Hamilton, 'The Evolution of Cooperation', *Science*, **211**, 1981, pp.1390–6; among others.

[5] A number of zero-sum game properties are violated in non-zero-sum games; e.g. see Luce & Raiffa, 1957, pp.90–94.

property no longer holds. For, in game 4.1, if both players choose that row (and column) containing the saddle point, they attain the outcome (*P*,*P*), and thereby fail to realize the more mutually favourable outcome (*R*,*R*). This situation cannot obtain in a zero-sum game with a saddle point. The non-zero-sum game differs critically, in that a player who seeks to maximize his own payoff is obliged to take the other player's possible choice into account (saddle points notwithstanding).

In game 4.1, it can still be argued that player *A* fares better by defecting, in terms of possible payoffs to himself alone, regardless of player *B*'s choice. But if player *B* reasons similarly, then the resultant outcome is not the most mutually favourable outcome. Then again, if player *A* risks co-operation, then he stands either to gain relatively less, or else to lose relatively more, than through defection, depending upon player *B*'s choice. Thus each player must run the risk of incurring the most detrimental individual payoff if he wishes to achieve the most beneficial collective payoff.

It is desirable to describe this situation in more formal terms. A useful way in which to do so is to represent the initial dilemma as a conflict between two principles of choice: *dominance* versus *maximization of expected utility*.[6]

The dominance principle operates as follows: choice *X* *strongly* dominates choice *Y* if and only if, for each game-state (joint outcome), *A* prefers the consequences of *X* to those of *Y*. Choice *X* *weakly* dominates choice *Y* if: for each game-state, *A* either prefers the consequences of *X* to those of *Y* or is indifferent between them; and for some game-state or states, *X* prefers the consequence(s) of *X* to that (those) of *Y*. Two simple examples illustrate this principle.

---

[6] These principles are common to game theory and decision theory; e.g. see R. Nozick, 'Newcomb's Paradox and Two Principles of Choice', in N. Rescher (*ed.*), *Essays in Honour of Carl G. Hempel*, D. Reidel, Dordrecht, 1969, pp.114-146.

Game 4.2 – Strong Dominance

		B x	*B* y
A	*X*	4,1	9,5
A	*Y*	3,2	8,6

Game 4.3 – Weak Dominance

		B x	*B* y
A	*X*	4,1	9,5
A	*Y*	4,2	8,2

In game 4.2, choice *X* strongly dominates choice *Y* for *A* (since $4 > 3$ and $9 > 8$), while choice *y* strongly dominates choice *x* for *B* (since $5 > 1$ and $6 > 2$). In Game 4.3, choice *X* weakly dominates choice *Y* for *A* (since $4 = 4$ and $9 > 8$), while choice *y* weakly dominates choice *x* for *B* (since $5 > 1$ and $2 = 2$).

In Game 4.1, the Prisoner's Dilemma, defection is strongly dominant for both *A* and *B* (since, for both prisoners, $T > R$ and $P > S$). Hence the dominance principle dictates that each prisoner should defect. But if both prisoners defect, the outcome (P,P) is mutually detrimental.

The principle of maximization of expected utility was encountered in Chapter Two. To re-iterate: the expected utility of a given row (or column) is the sum of the products of the utility of each game-state in that row (or column) and the respective probability with which that game-state obtains. Most generally, if a given row (or column) contains n states, and the utility of the i^{th} state is U_i, and the i^{th} state obtains with probability p_i, then the expected utility of that row (or column) is

$$EU = \sum_{i=1}^{n} (U_i)(p_i)$$

To maximize expected utility, then, one chooses that row (or column) for which the EU is greatest.

In Game 4.1, the respective utilities of each game-state are ordered (on the ordinal scale), but the probability that each game-state obtains has yet to be assigned. How are probabilities to be distributed among the game-states of the static Prisoner's Dilemma?

In the iterated case, it will be seen that probabilistic and causal dependencies arise which favour the frequency and likelihood of symmetric game states, either (R,R) or (P,P). But since the static case is being treated as logically prior to the iterated case, it seems inappropriate to admit iterated criteria at this juncture. Since the static case is an isolated case, *a posteriori* probabilities (frequency distributions) are presumably unavailable to the prisoners. Thus the prisoners would be obliged to assign probabilities on some *a priori* basis.

For example, were player A to apply the principle of insufficient reason, then he would assume that player B will co-operate or defect with equal probability (1/2). In that case, his expected utility of co-operation would be

$$EU(C) = (1/2)R + (1/2)S$$

while his expected utility of defection would be

$$EU(D) = (1/2)T + (1/2)P$$

Since $T > R$ and $P > S$, maximization of expected utility *via* the principle of insufficient reason suggests mutual defection.

However, an argument can be made that a player should not apply said principle. By definition, the principle of insufficient reason states that

> "alternatives are always to be judged equiprobable if we have no reason to expect or prefer one over another."[7]

While a prisoner may have no reason to *expect* one joint outcome over another, he certainly has valid reason to *prefer* one joint outcome to another. Each prisoner will order the joint outcomes according to the payoffs they contain for him; e.g., for prisoner A: $(T,S) > (R,R) > (P,P) > (S,T)$. Given the expressibility of preferences, the principle of insufficient reason seems to rule itself out.

The prisoners have recourse to a more interesting—and arguably more appropriate—*a priori* probability distribution, which follows

---

[7] Weatherford, 1982, p.29.

from particular properties of the game matrix. The matrix of game 4.1 has an *equilibrium outcome* at (P,P). An equilibrium outcome is one

> ". . .from which neither player can shift without impairing his payoff, assuming that the other player does not shift."[8]

Tha matrix also has a *Pareto-optimal* outcome at (R,R):

> "An outcome of a game is called Pareto-optimal if there is no other outcome in which both players get a larger payoff."[9]

The existence of equilibrium and Pareto-optimal outcomes may justify an *a priori* assumption of their probabilistic dependence. In other words, each prisoner may deem it likely that their joint decision will result in either an equilibrium or a Pareto-optimal outcome.

In that case, each prisoner would weight the probabilities such that $p(R,R) > p(S,T)$ and $p(P,P) > p(T,S)$. In terms of individual choice, prisoner A would weight $p(c/C) > p(d/C)$ and $p(d/D) > p(c/D)$, where $p(c/C)$ means "the probability that prisoner B co-operates (c), conditional on the assumption that prisoner A co-operates (C)", and so forth.[10] Similarly, prisoner B would weight $p(C/c) > p(D/c)$ and $p(D/d) > p(C/d)$.

Now prisoner A finds his expected utilities to be

$$EU(C) = p(c/C)(R) + (1-p)(d/C)(S)$$
$$EU(D) = (1-p)(c/D)(T) + p(d/D)(P)$$

where $p > 1/2$

If prisoner A assumes *complete* probabilistic dependence, then

---

[8] A. Rapoport, M. Guyer, D. Gordon, *The 2x2 Game*, The University of Michigan Press, Ann Arbor, 1976, p.18. See also R. Weber, 'Noncooperative Games', *Proceedings of Symposia in Applied Mathematics*, **24**, 1981, pp.83–125.

[9] Rapoport *et al*, 1976, pp.18–19.

[10] This notation is from R. Campbell, 'Background for the Uninitiated', in R. Campbell & L. Sowden (*eds.*), *Paradoxes of Rationality and Cooperation*, The University of British Columbia Press, Vancouver, 1985, p.18*ff*.

$p(c/C) = p(d/D) = 1$ and $(1-p)(d/C) = (1-p)(c/D) = 0$. Explicitly, then, A's expected utilities are

$$EU(C) = R$$
$$EU(D) = P$$

Since $R > P$, maximization of expected utility prescribes co-operation. The argument is symmetric for prisoner B. Thus both prisoners co-operate, to their mutual benefit.

Of course, if a prisoner assumes *partial* probabilistic dependence, then the general result is indeterminate. For instance, if for some reason prisoner A assumes $p(c/C) = p(d/D) = x$ and $p(d/C) = p(c/D) = (1-x)$, then his expected utilities are

$$EU(C) = xR + (1-x)S$$
$$EU(D) = (1-x)T + xP$$

Prisoner A co-operates if $EU(C) > EU(D)$. For this to be the case,

$$xR + (1-x)S > xP + (1-x)T$$

or

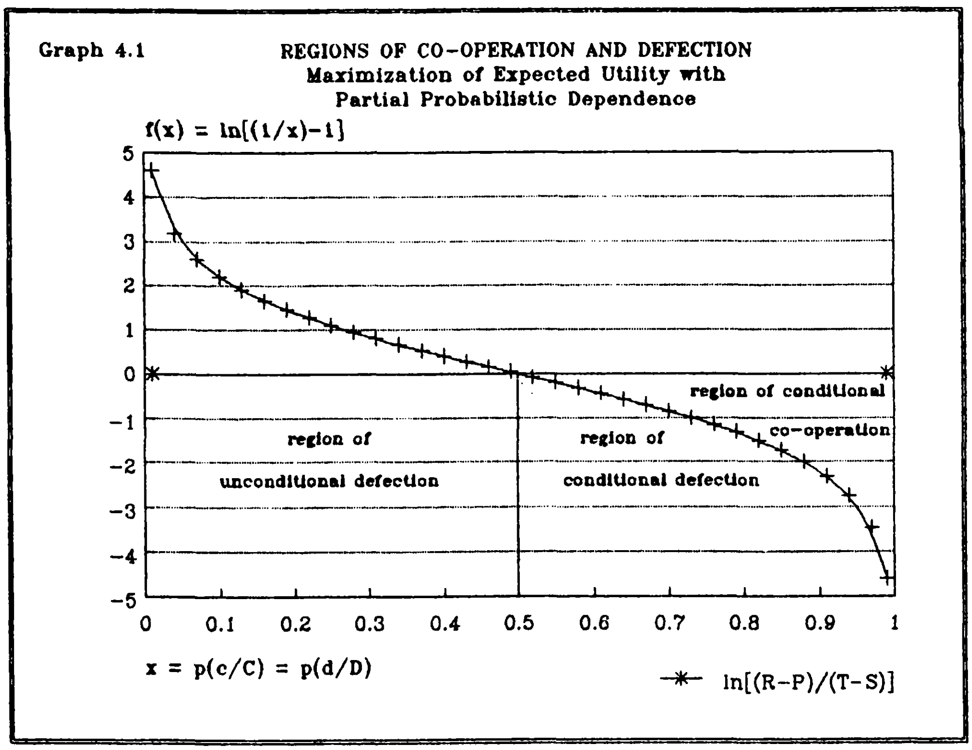
$$(R-P)/(T-S) > (1/x)-1 \qquad (4.1)$$

Consider the left hand side of inequality (4.1). Since $T > R$ and $S < P$, the denominator is always larger than the numerator. Thus the left hand side of this inequality must always be smaller than unity. It approaches (but never reaches) the value of unity as an upper limit, in cases where R is almost as large as T and S is almost as large as P.

Now consider the right hand side of inequality (4.1). It can take on a range of values for the permitted domain of x $(0 < x \leq 1)$. As x approaches zero, the right hand side blows up; as x approaches unity, it tends toward zero. At $x = 1/2$, the right hand side equals unity, which is the upper limit of the left hand side.

So, for inequality (4.1) to be satisfiable, the value of x must exceed one-half. When it does so, the right hand side is less than unity, and the inequality can be satisfied by appropriate values of *T, R, P,* and *S*. Thus, if the maximization of expected utilities is to prescribe co-operation *via* the rule of partial probabilistic dependence, the conditional probabilities of mutual co-operation, $p(c/C)$, and of mutual defection, $p(d/D)$, must exceed the critical value of 1/2. When they do so, the principle of maximizing expected utilities may prescribe co-operation, depending on the particular payoffs. But when the conditional probabilities do not exceed the critical value of one-half, the principle always prescribes defection, regardless of the payoffs.

These considerations can be illustrated graphically, where the natural logarithms of both sides of inequality (4.1) are plotted against the permitted domain of *x*.



Graph 4.1 delineates *regions* of co-operation, and of defection.

The *region of unconditional defection* is bounded above by the *x*-axis for $0 < x \leq 1/2$. The graph depicts a previous algebraic result, that inequality (4.1) cannot be satisfied in this domain. In other words, maximizing expected utility with partial probabilistic dependence of less than one-half prescribes defection for all values of T, R, P and S (such that $T > R > P > S$).

The *region of conditional defection* is bounded above by the curve $f(x) = ln[(1/x)-1]$, for $1/2 < x < 1$. In this domain, maximizing expected utility prescribes defection if inequality (4.1) is not satisfied, i.e. if $(R-P)/(T-S) < (1/x)-1$. For any partial probability x, in this domain, the result depends upon the particular payoffs of the given game.

The *region of conditional co-operation* is bounded above by the x-axis, and below by the curve $f(x) = ln[(1/x)-1]$, for $1/2 < x < 1$. Maximization of expected utility prescribes co-operation if inequality (4.1) is satisfied, i.e. if $(R-P)/(T-S) > (1/x)-1$. Note that the area of the region of conditional co-operation increases as x approaches unity. This area is proportional to the number of possible values of T, R, P and S for which inequality (4.1) is satisfied.

At $x = 1$, $f(x)$ is undefined, since unity is that value of x for which partial probabilistic dependence becomes complete probabilistic dependence. The area in this region increases without bound as x gets very close to unity, and the graph depicts a previous algebraic result: that in the case of complete probabilistic dependence, maximization of expected utility prescribes unconditional co-operation, for all values of T, R, P and S (such that $T > R > P > S$).

In so far as the dilemma confronting the prisoners arises from diverging dictates of two decision-theoretic principles of choice, the situation can be summarized as follows.

For each prisoner, defection strongly dominates co-operation. The dominance principle dictates that each prisoner fares better by defecting than by co-operating, no matter what the other prisoner does. But if both prisoners defect, they achieve an equilibrium outcome, which is mutually detrimental.

On the other hand, the existence of equilibrium and Pareto-optimal outcomes in the matrix may incline each prisoner to maximize his expected utility. If both adopt the rule of complete probabilistic dependence, then both co-operate, and they achieve a Pareto-optimal outcome, which is mutually beneficial. If both adopt the rule of partial probabilistic dependence, then the joint outcome depends upon their respective probability weights and the given payoffs of the game.

The inner workings of this dilemma are not trivial, even in the static case under consideration. Although the two decision-theoretic principles may prescribe conflicting choices, they do not do so unequivocally. An inner problem is embedded in the calculus of each principle, which prevents a rational player from adopting either unreservedly. Briefly stated, these problems are:

(1) If the dominance principle is rational for each prisoner, why does its mutual adoption result in a detrimental joint outcome?

(2) If the principle of maximizing expected utility is rational for each prisoner, is the associated rule of probabilistic dependence to be complete, or partial? And, if the principle is adopted with partial probabilistic dependence, how does a rational prisoner assign the corresponding probability weights?

But in asking these two questions, one begs a third:

(3) What, if anything, constitutes "rational" choice in the Prisoner's Dilemma? The current working definition, that it is rational to choose the alternative one prefers, can lead to any of the four joint outcomes. In that case, the prisoners may as well flip coins as apply decision theory. Since the working definition of game-theoretic rationality cannot distinguish between individually and mutually beneficial, or detrimental, outcomes, one might posit a criterion of game-theoretic meta-rationality: to be "meta-rational" is to be aware of the deficiency of the game-theoretic concept of rationality as it stands.

According to this hypothetical criterion, Rapoport is highly meta-rational. In his view:

> "Either the concept of rationality is not well-defined in the context of the non-negotiable non-zero-sum game; or if the definition of rationality in the context of the zero-sum game is applied to the "solution" of some non-zero-sum games, the results are paradoxical."[11]

In the case of the Prisoner's Dilemma, it seems that Rapoport's disjunction is actually a conjunction. The "paradox", in this case,

[11] Rapoport, 1966, p.142.

arises because each prisoner has a strongly dominant choice (defection) that leads to a mutually-detrimental result. Is the soundness of the dominance principle suspect? Not necessarily. The core problem, which Rapoport identifies, lies in the application of zero-sum rationality to a non-zero-sum game.

Rapoport's insight provides answers to the first two questions by addressing itself to the third.

In applying the dominance principle, prisoner *A* chooses the set of outcomes that is best for him. In a two-person zero-sum game, the best set of outcomes for *A* is also the worst set of outcomes for *B*, since *B* must always forfeit exactly what *A* what gains (and vice-versa). If *A* prefers to maximize his payoff, and if one choice dominates another, then *A* is rational to make the dominant choice.

<u>Game 4.4 - The Prisoner's Non-Dilemma</u>

		B: *c*	*B*: *d*
A	*C*	$R,-R$	$-T,T$
	D	$T,-T$	$P,-P$

where $T > R > P$

Game 4.4 represents an attempt to impose a zero-sum condition upon the Prisoner's Dilemma. For prisoner *A*, defection is strongly dominant, since $T > R$ and $P > -T$. For prisoner *B*, defection is also strongly dominant, since $T > -R$ and $-P > -T$. Thus both prisoners defect. But in this case, the outcome $(P,-P)$ is not mutually detrimental; rather, mutually optimal. Why? Because $(P,-P)$ is a saddle point of the matrix. If *A* defects, he gains at least *P* utiles; if *B* defects, he loses at most *P* utiles. Since mutual defection leads to minimax, a zero-sum Prisoner's Dilemma presents no dilemma to the prisoners.

In the non-zero-sum Prisoner's Dilemma, however, mutual defection leads to an outcome that is not mutually optimal. Why? Because the zero-sum criterion of rationality prescribes defection to each prisoner. According to this criterion, if prisoner *A* prefers to maximize his gains and minimize his losses, and if a dominant choice exists, then he should make that choice. Hence he is rational to defect. And so with *B*. But this criterion originates in the context of a zero-sum game, in which, by definition, the algebraic sum of the joint payoffs of any outcome is always zero. In a non-zero-sum game, however, this constraint vanishes; differences between algebraic sums of joint payoffs now exist. These differences must be taken into account, since they can generate a Pareto-optimal outcome.

Game 4.4 (the Prisoner's non-Dilemma) has an equilibrium outcome at (*P*,−*P*). Neither prisoner can shift from it without impairing his payoff, assuming that the other player does not shift. If either prisoner prefers the equilibrium outcome, he applies the dominance principle, and obtains, at worst, his preference. Dominance is effective in zero-sum games because the criterion of rationality is workable in zero-sum games. The criterion in turn is workable because, in zero-sum games, every outcome is Pareto-optimal.[12]

Game 4.1 (the Prisoner's Dilemma) also has an equilibrium outcome at (*P*,*P*). But the criterion of zero-sum rationality, which demands only that a player choose the alternative *he* prefers, fails to guarantee Pareto-optimality in this non-zero-sum case, because the equilibrium outcome (*P*,*P*) is no longer Pareto-optimal.

One can now appreciate the cogency of Rapoport's differentiation between *individual* and *collective* rationality.[13] Individual rationality is applicable in zero-sum games. But in non-zero-sum games, collective rationality must be applied, in order that the players do not pre-empt a Pareto-optimal outcome by exercising individually rational choices. A working definition of collective rationality demands that a player attempt to achieve a Pareto-optimal outcome, if one exists. At the same time, a player who is collectively rational must be able to protect himself—in so far as a given

---

[12] In any zero-sum game, every outcome satisfies the condition of Pareto-optimality; namely, that no other outcome contains larger payoffs for both players. This condition, being universally true in zero-sum games, retains little significance in them.

[13] Rapoport (*ed.*), 1974, p.4.

game allows—from a player who is individually rational, or otherwise irrational.

This enquiry does not attempt to formulate a definition of collective rationality that is workable across the broad spectrum of non-zero-sum games. It does, however, attempt to realize a more limited objective; namely, an implementation of Rapoport's concept of collective rationality in the context of the Prisoner's Dilemma. Thus far, the attempt provides an answer to question (1) above: the dominance principle leads to a mutually-detrimental outcome because, although individually rational, it is not collectively rational.

Next, one seeks answers to questions (2) and (3). It is possible to formulate a working definition of collective rationality that answers these questions simultaneously. Suppose that prisoner *A* is collectively rational if

(i) he elects to maximize his expected utility, and

(ii) he adopts the rule of either complete or partial probabilistic dependence, assigning to $p(c/C)$ and $p(d/D)$ *the probability that prisoner B is collectively rational.*

It can be immediately contested that condition (ii) of this proposed definition is impredicative; nonetheless, given that the type of rationality under consideration is <u>not</u> of the individual kind, it may be permissible in these unusual circumstances to define the collective aspect in terms of the collective itself. One may put this objection on one side, and see whether the definition can counter it in operation.

Let prisoner *A* be collectively rational, according to conditions (i) and (ii). Now suppose the probability of *B*'s collective rationality is unity. In that event, *A* maximizes his expected utility with $p(c/C) = p(d/D) = 1$, which is the case of complete probabilistic dependence. Consequently, *A* co-operates. But *B* is also collectively rational, and the probability of *A*'s collective rationality is also unity. So, according to conditions (i) and (ii), *B* maximizes his expected utility with $p(C/c) = p(D/d) = 1$, and consequently *B* too co-operates. So mutual collective rationality, on these terms, leads to the desired outcome of Pareto-optimality.

Now let prisoner *A* be collectively rational, again according to conditions (i) and (ii), and suppose the probability of *B*'s collective rationality is zero. *A* maximizes his expected utility with $p(c/C) = p(d/D) = 0$, which lies in the region of unconditional defection. So *A* defects. But *B* is not collectively rational, and may defect upon a whim. In that case, *A* protects himself against *B*'s individual rationality, and against any other form of irrationality that leads *B* to defect. If *B*'s irrationality leads him to co-operate, for a bizarre or capricious reason, then *A* fares even better by defecting.

Now let prisoner *A* be collectively rational, again according to conditions (i) and (ii), and suppose the probability of *B*'s collective rationality lies between zero and unity. If said probability is less than or equal to one-half, then *A* defects unconditionally. If it is greater than one-half, then *A*'s maximization of expected utility lies in the region of conditional co-operation or defection. *A* either co-operates or defects, depending on the actual payoffs involved. In general, the greater the probability that prisoner *B* is collectively rational, the greater the number of cases in which collectively rational prisoner *A* will choose co-operation.

This working definition of collective rationality answers questions (2) and (3), and seems to overrule the objection of impredicativity. It allows two collectively rational prisoners to achieve a Pareto-optimal outcome, and also affords a measure of protection to a collectively rational prisoner whose fellow-prisoner is not collectively rational.

Unfortunately, the static Prisoner's Dilemma is not so handily resolved. The proposed definition of collective rationality, while quite workable in theory, encounters a formidable barrier in practice. There is simply no analytic method, in the static mode, by which one prisoner can ascertain the probability of the other's collective rationality. A prisoner who wishes to do so falls into one of two broad streams of probabilistic thought: the *a priori*, and the *a posteriori*. The *a posteriori* probabilistic prisoner cannot be in possession of a frequency distribution of the other prisoner's previous choices (made in other dilemmas), since the static model represents a single, isolated case. Nor can the *a priori* probabilistic prisoner be made aware of the other prisoner's current deliberations or intentions since, according to the ground rules of the model, the prisoners are held incommunicado.

It would seem that one prisoner's evaluation of the probability of the other prisoner's collective rationality is a matter of guesswork. As such, a prisoner may make a grossly inaccurate assessment, with disastrous results for either himself or his fellow-prisoner. And if both prisoners are collectively rational, but both incorrectly assess the other's probability of being such as less than one-half, then both prisoners defect, to their mutual detriment. Unless the collectively rational prisoner is able to find a reliable way to ascertain the probability of the other's collective rationality, then his own collective rationality amounts to no more than a beneficial intention. While a beneficial intention may be an estimable factor in the resolution of conflict generally, it is plainly susceptible to misdirection in the static Prisoner's Dilemma, where it can prove as inimical, to either prisoner, as a hostile predisposition. Again, the Prisoner's Dilemma resists an infallible resolution.

Game-theorists who are unwilling to be confounded by the dilemma have brought no small ingenuity to bear upon the problem. Two significant proposed resolutions are examined in the next two chapters. The model, however, exhibits a disquieting, Hydra-like property: the resolution of one dilemma seems to engender the appearance of another.

For example, Rapoport's notion of collective rationality has drawn the following criticism: Davis argues that if each prisoner is concerned with the joint outcome, then the Prisoner's Dilemma ceases to be a dilemma.[14] Suppose one alters the ground rules, and permits the prisoners to signal or even to discuss their intentions. In other words, one changes the model from a non-co-operative to a co-operative game. If the prisoners collude, and make a pact not to defect, then the dilemma appears to vanish.

---

[14] Davis, 1970, pp.101-102.

Not so, according to Rapoport and Chammah, who anticipated and countered the criticism:

> "It is clear, however, that if the pact is not enforce-able, a new dilemma arises. For now each of the prisoners faces a decision of keeping the pact or breaking it. This choice induces another game exactly like the Prisoner's Dilemma, because it is in the interest of each to break the pact regardless of whether the other keeps it."[15]

It is clear that the conflict within the Prisoner's Dilemma may be transposed from one set of issues to another. In this chapter, a transposition was effected from conflicting principles of choice to conflicting concepts of rationality. Similarly, Davis's criticism and Rapoport's reply effect a transposition from a conflict between dominance and utility to a conflict between temptation and integrity.

It is also clear, however, that the conflict is not resolved merely by virtue of being transposed.

---

[15] Rapoport & Chammah, 1965, p.25.

Chapter Five
A Resolution Via Newcomb's Paradox

Nozick's publication of Newcomb's Paradox, and his treatment of it, engenders ongoing debate in game-theoretic, decision-theoretic, and philosophical literature.[1] In some respects, Newcomb's Paradox and Prisoner's Dilemma pose similar problems; in other respects, quite different ones. Consideration of the similarities led both Brams and Lewis to revelations of relevance to this enquiry; namely, that the static Prisoner's Dilemma can be viewed as constituting a particular case of Newcomb's Paradox.[2] This view is relevant because Brams also gives an attempted resolution of the paradox. One can enquire whether the resolution seems sound and, *mutatis mutandis*, whether it perforce applies to the particular case of the dilemma as well.

To begin with, then, let Newcomb's demon be introduced:

> "Suppose a being in whose power to predict your choices you have enormous confidence. . .You know that this being has often correctly predicted your choices in the past (and has never, so far as you know, made an incorrect prediction about your choices), and furthermore you know that this being has often correctly predicted the choices of other people, many of whom are similar to you, in the particular situation to be described below. One might tell a longer story, but all this leads you to believe that almost certainly this being's prediction about your choice in the situation to be discussed will be correct."[3]

The player then finds himself in this situation. Two boxes, *B1* and *B2*, are placed in front of him. *B1* is transparent; *B2*, opaque. *B1* contains £1,000. *B2* contains either £1,000,000 or nothing, depending upon what Newcomb's demon predicts about the player's upcoming choice. The player must choose between taking either the contents of

---

[1] Nozick, 1969, pp.114–46.

[2] S. Brams, 'Newcomb's Problem and Prisoner's Dilemma', *Journal of Conflict Resolution*, 19, 1975, pp.596–612; and D. Lewis, 'Prisoner's Dilemma is a Newcomb Paradox', *Philosophy and Public Affairs*, 8, 1979, pp.235–240.

[3] Nozick, 1969, p.114.

both boxes, or the contents of $B2$ only. If the being predicts that the player will choose the contents of both boxes, it does not place £1,000,000 in $B2$. If the being predicts that the player will choose the contents of $B2$ only, it places £1,000,000 in $B2$.

The play unfolds in a strict sequence. First, the being makes its prediction. Second, according to its prediction, it places either nothing or £1,000,000 in $B2$. Third, the player makes his choice. The game matrix is as follows.

<u>Game 5.1 – Newcomb's Paradox</u>

		being	
		predicts $B2$ only	predicts $B1$ & $B2$
player	chooses $B2$ only	$\pounds M$	£0
	chooses $B1$ & $B2$	$\pounds M + \pounds T$	$\pounds T$

where $\pounds M$ = £1,000,000 and $\pounds T$ = £1,000

As in the Prisoner's Dilemma, one encounters a conflict between two principles of choice: dominance, and maximization of expected utility.

Choosing both boxes strongly dominates choosing $B2$ only, since $\pounds M + \pounds T > \pounds M$ and $\pounds T > \pounds 0$. [This remains true despite the arbitrariness of the utility of money, as long as said utility is taken to be any transitive function of the amount; i.e. if $X > Y$, then $U(\pounds X) > U(\pounds Y)$.] The dominance principle dictates that, no matter what the being predicts, the player fares better by choosing both boxes.

Then again, the player's expected utilities of choosing $B2$ only, and of choosing both $B1$ and $B2$ are, respectively,

$$EU(B2) = p(B2)U(\pounds M) + (1-p)(B1 \,\&\, B2)U(\pounds 0)$$
$$EU(B1 \,\&\, B2) = (1-p)(B2)U(\pounds M + \pounds T) + p(B1 \,\&\, B2)U(\pounds T)$$

where $p(B2)$ is the probability that, if the player chooses $B2$, the being has correctly predicted this choice; and so forth.

If one takes the utility of money to be proportional to the base ten logarithm of the amount, and if one assumes the utility of £0 to be nothing, then one has

$$EU(B2) = 6z$$
$$EU(B1 \ \& \ B2) = 6(1-z) + 3z$$

where z is the probability that the being has correctly predicted the player's choice. According to this utility assignment, $EU(B2) >$ $EU(B1 \ \& \ B2)$, if $z > 2/3$. Thus, the player should choose only box two if the being's predictive success rate exceeds two-thirds.

It should be noted that the selection of a monetary utility function has a pronounced effect upon the overall expected utilities. If, for example, one now takes the utility of money to be proportional to its actual amount, then one has

$$EU(B2) = 10^6 z$$
$$EU(B1 \ \& \ B2) = (10^6 + 10^3)(1-z) + 10^3 z$$

where z is once again the probability that the being has correctly predicted the player's choice. In this case, $EU(B2) > EU(B1 \ \& \ B2)$ if $z > 1001/2000$. Thus, the player should choose only box two if the being's predictive success rate exceeds one-half (by more than one two-thousandth). This substantial relaxation of the probabilistic demand results from the selection of a linear utility function.

Notwithstanding the range of probabilistic demands made possible by the arbitrariness of the utility of money, one can safely infer from Nozick's description that the being's predictive success rate is such that the expected utility of choosing only box two is much greater than that of choosing both boxes. Hence the principle of maximizing expected utility suggests that the player choose the contents of box two only.

It is also clear from Nozick's description that a player is able to make use of an *a posteriori* probability calculus, if he so wishes. Newcomb's demon apparently has unlimited funds to disburse if need be, and a player may avail himself of a long series of observations in order to ascertain the limiting frequency with which the being makes correct predictions. The player who elects to maximize his expected utility in this model has therefore a more objectively reliable method of assigning probabilities than in the static Prisoner's Dilemma.

But this advantage is negatively-compensated—if not reduced to irrelevancy—by another circumstance, peculiar to Newcomb's paradox. If one re-considers the strict temporal order of the moves (first, the being's prediction; second, the being's consequent placement or non-placement of *£M* in box two; third, the player's subsequent choice), one detects an implicit flaw in the argument for maximizing expected utility.

Suppose that the first two moves have been made; i.e. that the being has made its prediction, and has acted upon it. Now the player must choose either the contents of box two alone, or the contents of both boxes. It is most certainly the case that box two presently contains either *£M*, or nothing. The contents of box two cannot now be affected by the player's choice, and the player obtains the contents of box two regardless of his choice. If the player chooses both boxes, he is then guaranteed of obtaining no less than *£T*; whereas if he chooses box two only, and if the being has predicted incorrectly, then the player obtains nothing.

This is not simply a restatement of the dominance principle, for the following reason. An *a posteriori* probabilist may well object to the foregoing argument, on the ground that the being's observed frequency of predictions is, let one suppose, 99.9999% correct. If the player now chooses both boxes, the being will almost certainly have predicted his choice, and will have placed nothing in box two. But if the player now chooses box two only, the being, by the same token, will have predicted this choice with the same high degree of accuracy, and will have placed *£M* in box two. The player should therefore choose box two only.

The *a posteriori* probabilist's objection is countered by the assertion that, while the being's prediction and the player's choice are evidently probabilistically dependent (even to the extent that the partial dependence approaches complete dependence), there is absolutely *no causal dependence* between the two. The being's prediction has no causal influence over the player's choice; in consequence, the being's prediction can be incorrect. And neither can the player's choice have any causal influence over the being's prediction; for that would entail a violation of the temporal succession of events. In other words, if the being has predicted incorrectly, then

(i) if box two now contains nothing, then the player's choice of box two only cannot cause the being to place *£M* therein; and

(ii) if box two now contains *£M*, then the player's choice of both boxes cannot cause the being to remove the *£M* therefrom.

Bolstered by the assertion of causal independence, a player may be tempted to choose the contents of both boxes.

So Newcomb's paradox embodies a conflict not only between the principles of dominance and maximization of expected utility, but also between corollary arguments of complete causal independence and near-complete probabilistic dependence, respectively.

Nozick put Newcomb's problem to a great many people, and elicited their choices as hypothetical players. He found:

> "To almost everyone it is perfectly clear and obvious what should be done. The difficulty is that these people seem to divide almost evenly on the problem, with large numbers thinking that the opposing half is just being silly."[4]

Given such a response, Newcomb's problem may justly bear the mantle of a paradox. While opinion may divide as evenly in the Prisoner's Dilemma, either half can appreciate why the other chooses as it does, without necessarily accusing it of irrationality (or silliness). Conflict of choice in the Prisoner's Dilemma can be understood as a conflict between individual and collective rationality, and alternatively as uncertainty in a collectively rational prisoner's assessment of the other prisoner's rationality. Perhaps Newcomb's problem is paradoxical because, among other reasons, it brings the zero-sum concept of rationality into a non-zero-sum game (in which the being,

---

[4] Nozick, 1969, p.117.

having an infinite supply of funds, gains or loses nothing) without having to re-define rational play.

In consequence, every player can exercise individual rationality with impunity, and seek the maximum possible gain. There are no collective outcomes to be weighed; a player can neither exploit the being, nor be exploited by it. A player has nothing to lose, and stands to gain substantially. In these respects, Newcomb's problem differs patently from the Prisoner's Dilemma. If a multitude of players can be thus described, it seems paradoxical indeed that their choices should manifest the same divergence, according to the same principles, as in the Prisoner's Dilemma.

Given these critical differences between the two models, one seeks an explanation for the similarities between the dilemmas that the players face. As intimated earlier, the Prisoner's Dilemma can be regarded as a particular case of Newcomb's paradox. Explicitly, both Brams and Lewis have argued that the Prisoner's Dilemma can be viewed as two interacting Newcomb's paradoxes.[5]

To appreciate this perspective, one must first grant that the player in Newcomb's paradox is playing a game, in effect, against a state of nature.[6] This follows from the strict sequence of moves, which begins with the being's prediction, and continues with its placement, or non-placement, of *£M* in box two. When the player makes his choice, the possible outcomes are already halved, from four to two, by the being's previous moves. From the being's point of view, the player is facing a state of nature which has only one possible pair of outcomes, because the being has already predicted one of the player's two possible choices. Before the player actually makes his choice, the being then perceives the game in one of two ways, depending upon what it has predicted:

[5] Brams, 1975; & Lewis, 1979.

[6] This point, and its decision-theoretic ramifications, are spelled out in some detail by Nozick, 1969.

Game 5.2 - Being's View			being
			predicted B2 only
A	will choose B2 only		£M
	will choose B1 & B2		£M + £T

Game 5.3 - Being's Alternative View			being
			predicted B1 & B2
A	will choose B2 only		£0
	will choose B1 & B2		£T

The player (*A*) also realizes this, but the nature of the game prohibits him from knowing which pair of outcomes actually remains, until after he will have made his choice. As such, Newcomb's paradox is played against a state of nature.

In the Prisoner's Dilemma, the very absence of a strict sequence of moves provides a convenient justification for viewing the game as a dual Newcomb's paradox. Neither prisoner is concerned with the order in which the individual choices are made, because all joint outcomes are insensitive to temporal permutations of choice. Whether prisoner *A* makes his choice first, and *B* second, or vice-versa, has no bearing on the outcome. This circumstance arises because the choices in both models under consideration are causally independent.

For the sake of contrast, one may briefly consider a causally dependent game against a state of nature. Suppose two collectors wish to acquire a given work of art at an auction. Suppose also that collector *A* can afford a maximum bid of £1,000; collector *B*, £5,000. Each is willing to bid his respective maximum. With respect to the work of art, both collectors are playing a game against a state of nature. Though perhaps unknown to both collectors, the outcome is determined before the bidding commences: *A* will neither part with money nor acquire the work, and *B* will both part with money and acquire the work. But for *B*, the cost of the outcome is not predetermined; that depends upon who bids first.

Should *A* enter an opening bid of, say, £400, *B* might reply with £500. If bidding continues in this fashion, *A* will soon bid his maximum of £1,000, and *B* will acquire the work for, say, £1,100. On the other hand, should *B* enter an opening bid of, say, £2,000, he would unknowingly pre-empt *A*, and acquire the work at a higher cost. In general, when the players' choices exert mutual causal influence, the temporal sequence of play must be taken into account.[7]

Returning to the Prisoner's Dilemma, one can imagine that, from prisoner *A*'s point of view, prisoner *B* may be assumed to have conveyed his choice to the authorities. Then *A*'s situation is similar to that of the player in Newcomb's paradox: from the authorities' point of view, *A* is now playing against a state of nature, in which only one pair of outcomes is attainable. Although *A* realizes this, he is prohibited from knowing which of the original two pairs remains, until he will have made his choice. The situation is completely symmetric with respect to prisoner *B*, who may assume, without contradiction, that prisoner *A* has already conveyed his choice to the authorities, and so forth.

Obviously, from everyone's point of view, it is logically impossible for both prisoners to convey their choices first; they must do so either one after another, or with approximate simultaneity. But it is logically possible for each prisoner to assume that the other has done so first, in which case they find themselves in a dual Newcomb's paradox. In that original paradox, however, a player can fare no worse than no gain; whereas in this dual paradox, either prisoner can incur a loss of liberty.

It seems advantageous to regard the Prisoner's Dilemma as a particular case of Newcomb's paradox, if only because this view affords one explanation as to why the models exhibit both radical differences (in criteria of rationality) and pronounced similarities (in divergence of rational choice).

But this view offers a second advantage, of arguably greater moment than the first. Newcomb's paradox has been reformulated in a way that eliminates one of the two principles of choice from con-

---

[7] E.g. see also M. Bar-Hillel & A. Margolit, 'Newcomb's Paradox Revisited', *The British Journal for the Philosophy of Science*, **23**, 1972, pp.295–305.

sideration. If the reformulation of the paradox is sound, then it ought to be applicable to the dilemma as well.

Brams credits Ferejohn with the insight of reformulating Newcomb's paradox.[8] Essentially, the being's *desiderata* are transposed from the game-theoretic, to the decision-theoretic variety. The transposition is effected in the following way: instead of defining the game-states in terms of what the being predicts about the player's choice (i.e. that the player will choose either the contents of box two only, or the contents of both boxes), one defines them in terms of the eventual astuteness of the prediction itself (i.e. that the being's prediction proves either correct, or incorrect).

Game 5.4 - Newcomb's Paradox Reformulated

		being	
		predicted correctly	predicted incorrectly
player	will have chosen *B2* only	£*M*	£0
	will have chosen *B1* & *B2*	£*T*	£*M* + £*T*

The outcomes themselves remain unaltered. But the choice of both boxes no longer dominates the choice of box two only. The principle of dominance is thus eliminated from contention, while maximization of expected utility yields the same results as in Game 5.1.

But note the changes in tense. In Game 5.1, both the being's prediction and the player's choice are couched in the present tense (with the understanding that the prediction preceeds the choice). In Game 5.4, the astuteness of the being's prediction can be assessed only after the player will have made his choice. Thus, in order to construct the reformulated game matrix, which presumably aids him in making a choice, the player must imagine that he has already made a

[8] Brams, 1975, pp.599–600.

choice. Though he cannot attain an outcome before making a choice, he knows full well what the possible outcomes are, so is free to imagine the consequences of future choices, as if they were already made, and to juxtapose them with the subsequent correctness or incorrectness of the being's (previous) prediction.

It does not seem unsound to alter the temporal perspective from which the player views the game, provided that one does not tamper with the strict temporal sequence of the moves. (This point will shortly become relevant in another context also.)

One hastens to add that, although in this reformulation the principle of mazimizing expected utility prevails over dominance by default, the player is not obliged to adopt either principle. Whether playing Game 5.1 or 5.4, the player may arrive at his choice by flipping a coin, consulting an oracle, or by any other means he deems fit. But a player wishing to employ an analytical method of choice may be caught by the paradox in Game 5.1, and be convinced by the transposition in Game 5.4.

Brams is not slow to apply this reformulation to the Prisoner's Dilemma.[9] He leaps directly from the reformulated Newcomb's paradox to the reformulated dilemma, and takes as a premise that both prisoners will realize that the dominance principle now gives way to the maximization of expected utility. However, as Brams realizes, this is still a far cry from the elicitation of mutual co-operation. Brams is able to show that each prisoner co-operates only if the probability of the correctness of his prediction of the other player's choice is sufficiently high. This, in turn, invites comments from Rapoport on the manifest difficulties involved in evaluating Bayesian probabilities.[10]

It is not necessary to recapitulate Brams's argument in order to appreciate that the reformulation of the Prisoner's Dilemma, compelling though it is, does not resolve the conflict. An effective (though by no means unique) way to represent the reformulated dilemma

---

[9] Ibid., pp.603*ff*.

[10] A. Rapoport, 'Comment on Brams's Discussion of Newcomb's Paradox', *Journal of Conflict Resolution*, **19**, 1975*b*, pp.613–19.

was developed, in another guise, in the preceeding chapter. Suppose that both prisoner *A* and prisoner *B* are collectively rational. Then, in order to reformulate the game matrix, either prisoner need only ask "Can the other prisoner correctly predict my choice?".

Recall the criteria of collective rationality. Prisoner *A* is said to be colletively rational if

(i) he elects to maximize his expected utility, and

(ii) he adopts the rule of either complete or partial probabilistic dependence, assigning to $p(c/C)$ and $p(d/D)$ *the probability that prisoner B is collectively rational.*

Then, for example from *A*'s point of view, the following matrix obtains:

Game 5.5 – Prisoner's Dilemma Reformulated

		A's belief about *B*	
		B will correctly predict *A*'s choice	*B* will incorrectly predict *A*'s choice
A	*C*	*R,R*	*S,T*
	D	*P,P*	*T,S*

Defection no longer dominates. Since *B* is collectively rational, *A* knows that if *B* predicts *A* will co-operate, then *B* will co-operate; and if *B* predicts *A* will defect, then *B* will defect. If *A* assigns sufficiently high probability to *B*'s ability to predict correctly, then *A* co-operates; otherwise, he defects. The situation is symmetric for *B*. But the reformulation of the matrix leads to the same problem posed by the transposition in the preceeding chapter; namely, that no analytical method exists by which either prisoner can assign said probabilities in the static mode.

Thus far, one has not encountered any ambiguity about the actual mechanics of the calculus of maximizing expected utility; one has simply failed to find an objective method for assigning the probability distribution which forms a necessary component of that calculus. Rapoport's concept of collective rationality, and Brams's attempted resolution of the Prisoner's Dilemma *via* the reformulation of Newcomb's paradox, both prescribe the maximization of expected utility. That an *a priori* probabilist cannot readily implement the calculus in the static mode, does not negate the soundness of the prescription itself.

But Nozick's treatment of the maximization of expected utility, which is consistent with Rapoport's, Brams's, this enquiry's, and many others, is subjected to a vigorous attack by Levi.[11] Since the principle has theoretical value in the static Prisoner's Dilemma (and will be of demonstrable empirical value in the iterated Prisoner's Dilemma), Levi's objection ought to be examined. If Nozick's application of the principle to Newcomb's paradox is flawed, then the flaw should be exposed, lest it be incoporated into applications of the identical principle to the Prisoner's Dilemma.

Levi presents three different cases, in each of which Newcomb's demon has made at least 1,000,100 total predictions of the player's choice. (It is irrelevent to the problem whether 1,000,100 players have each played one game, or one player has played 1,000,100 games, and so forth.) The three different distributions of predictions and choices are as follows:[12]

<u>5.6 - Newcomb's Paradox, Levi's Case 1</u>

		being	
		predicts *B2*	predicts *B1* & *B2*
player	chooses *B2*	900,000	10
	chooses *B1* & *B2*	100,000	90

[11] I. Levi, `Newcomb's Many Problems', *Theory and Decision*, 6, 1975, pp.161-175.

[12] Ibid., p.165.

If the player wishes to maximize his expected utility, he asks two questions: "What is the frequency with which the being correctly predicted the player's choice of only box two?", and "What is the frequency with which the being correctly predicted the player's choice of both boxes?". The answer to the first question is: 900,000/1,000,000 or 9/10; the answer to the second is: 90/100 or 9/10. In this case, the *a posteriori* probability that the being predicts correctly is 9/10.

Now the player can calculate his expected utilities. The expected utility of choosing only box two is

$$EU(B2) = p(C_{B2}/P_{B2})\,U(\pounds M) + (1-p)(C_{B2}/P_{B1\&B2})\,U(\pounds 0)$$

where $p(C_{B2}/P_{B2})$ means "the probability that the player chooses only box two, if the being predicted his choice of only box two"; and $(1-p)(C_{B2}/P_{B1\&B2})$ means "the probability that the player chooses only box two, if the being predicted his choice of both boxes."

For simplicity, assume the utility of money to be proportional to the amount. Then

$$EU(B2) = (9/10)(\pounds M) = \pounds 900{,}000$$

Similarly, the player calculates his expected utility of choosing both boxes:

$$EU(B1\ \&\ B2) = p(C_{B1\&B2}/P_{B2})\,U(\pounds M+\pounds T) + (1-p)(C_{B1\&B2}/P_{B1\&B2})\,U(\pounds T)$$

where $p(C_{B1\&B2}/P_{B2})$ means "the probability that the player chooses both boxes, if the being predicted his choice of only box two"; and $(1-p)(C_{B1\&B2}/P_{B1\&B2})$ means "the probability that the player chooses both boxes, if the being predicted his choice of both boxes." Then

$$EU(B1\ \&\ B2) = (1/10)(\pounds M+\pounds T) + (9/10)(\pounds T) = \pounds 101{,}000$$

Hence, maximizing expected utility prescribes that the player choose box two only.

Levi's other cases are as follows:

Game 5.7 - Newcomb's Paradox, Levi's Case 2

		being	
		predicts *B2*	predicts *B1* & *B2*
player	chooses *B2*	495,045	55,005
	chooses *B1* & *B2*	55,005	495,045

Game 5.8 - Newcomb's Paradox, Levi's Case 3

		being	
		predicts *B2*	predicts *B1* & *B2*
player	chooses *B2*	90	100,000
	chooses *B1* & *B2*	10	900,000

In Games 5.7 and 5.8, the answer to the questions "What is the frequency with which the being correctly predicted the player's choice of only box two?", and "What is the frequency with which the being correctly predicted the player's choice of both boxes?", is identical to the answer in Game 5.6; namely, 9/10. It follows that the maximization of expected utility prescribes a choice of only box two, for Games 5.7 and 5.8 as well. All seems as it should be.

Levi's objection is based on a consideration of converse conditional probabilities, $p(P_{B2}/C_{B2})$ and $p(P_{B1\&B2}/C_{B1\&B2})$; in other words, "the probability that the being predicts a choice of only box two, if the player chose only box two", and "the probability that the being predicts a choice of both boxes, if the player chose both boxes", respectively.

Levi tables the conditional probabilities, and their converses, as follows:[13]

Table 5.1 – Newcomb's Paradox: Conditional Probabilities & Their Converses

	$p(C_{B2}/P_{B2})$	$p(C_{B1\&B2}/P_{B2})$	$p(P_{B2}/C_{B2})$	$p(P_{B1\&B2}/C_{B1\&B2})$
Case 1	0.9	0.9	0.9999888	0.0008991
Case 2	0.9	0.9	0.9	0.9
Case 3	0.9	0.9	0.0008991	0.9999888

And, cautions Levi,

> "Care should be taken, however, not to confuse the conditional probabilities $p(C_{B2}/P_{B2})$ and $p(C_{B1\&B2}/P_{B2})$ which are conditional probabilities of *X*'s [the player's] choosing an option given that the demon predicts it with the conditional probabilities $p(P_{B2}/C_{B2})$ and $p(P_{B1\&B2}/C_{B1\&B2})$ which are the conditional probabilities of the demon's predicting *X*'s choosing that option given that *X* chooses that option. Assuming that the first two conditional probabilities are high, it does *not* follow that the second two conditional probabilities are both high."[14]

The point is well taken. But Nozick stands accused of having made just this "fallacious inference" in his argument supporting the maximization of expected utility in Newcomb's paradox.[15] (Nozick eventually favours dominance, but first presents detailed arguments both for and against both principles of choice.) Based on his accusation that Nozick has unjustifiably assumed the converse conditional probabilities to be high, Levi concludes

> ". . .not only that Nozick's first argument is invalid but that, from the point of view of someone committed to using the principle of maximizing expected utility, no recommendation can be made concerning what *X* should do without filling in more details concerning *X*'s predicament than Nozick has done. . .
>
> Suppose *X* faces Newcomb's problem and does not himself know whether he is facing a variant of case 1, case 2, or

---

[13] Levi, 1975, p.165. To minimize confusion, Levi's notation has been replaced, throughout, by the notation adopted in this enquiry.

[14] Ibid, p.166.

[15] Ibid.

case 3. . .*X* cannot apply the principle of maximizing expected utilities."[16]

This is so because, in Levi's view, the maximization of expected utility, against a state of nature, is calculated by using the *converse* conditional probabilities. Thus, for instance, in Levi's opinion, maximizing expected utility in cases 1 and 3 prescribes a choice of both boxes, not box two only.

However, it would seem that Levi himself has made a fallacious inference, which in turn invalidates his argument against Nozick. In the context of Newcomb's Paradox, the converse conditional probabilities are inadmissible, because they permute the temporal order in which the moves are made. Compare the tensed meanings of the conditional and converse conditional probabilities, $p(C_{B2}/P_{B2})$ and $p(P_{B2}/C_{B2})$: "the probability that the player chooses only box two, if the demon predicted his choice of only box two", and "the probability that the demon predicts his choice of only box two, if the player chose only box two", respectively. If strict temporal succession of moves is to be preserved, then the converse conditional probability cannot be admitted, since it entails the being's prediction of the player's choice after the player has made the choice.

This does not refute Levi's general point, that one cannot reflexively equate the values of conditional and converse conditional probabilities. However, the set of games to which Levi's point applies does not and cannot include Newcomb's paradox, at least as Nozick presents it. And as Nozick's use of conditional probabilities in his calculation of expected utilities appears to be correct, his argument in favour of maximizing expected utilities seems quite sound.

But if one relaxes the temporal constraint, and allows the being to make its prediction after the player makes his choice, one arrives at a situation resembling the Prisonr's Dilemma, which is insensitive to permutations of temporal order of choice. In this set of games, one must indeed exercise care to adopt the appropriate probability distribution, depending on which player's utilities one seeks to maximize.

---

[16] Ibid, p.166 & pp.173-4.

Before concluding this chapter, it must be fairly said that Newcomb's paradox admits of many other avenues of investigation, which may be broader, longer, and also more winding than that explored herein. For this enquiry's purpose, it is sufficient to note salient respects in which the Prisoner's Dilemma both can, and cannot be considered as a kind of Newcomb's paradox.

It seems interesting and relevant that the reformulation of the paradox points to a similar theoretical resolution of the dilemma as does the implementation of collective rationality; namely, adoption of the principle of maximizing expected utility. Unfortunately, when the dilemma occurs on a single and isolated occasion, neither prisoner has recourse to *a posteriori* probability distributions, and both prisoners must rely on *a priori* probabilistic criteria. As such, there seems no reliable way for a given prisoner to assess either

(i) the probability that the other prisoner is collectively rational, or

(ii) the probability that the other prisoner can correctly predict the given prisoner's choice.

The appeals to collective rationality of choice (in the previous chapter) and to mutual predictability of choice (in this chapter) hold more in common than their respective theoretical but not-quite-realizable resolutions of the Prisoner's Dilemma; both make implicit use of meta-game theory in their respective processes.[17] The next chapter examines an explicit meta-game-theoretic resolution of the static Prisoner's Dilemma.

---

[17] See also Brams, 1975, p.608.

Chapter Six
A Meta-Game Resolution

A meta-game is defined by Howard in the following way:

> "If *G* is a game in normal form, and if *k* is a player in *G*, the (first-level) metagame *kG* . . .is the normal-form game that would exist if player *k* chose his strategy in *G* in knowledge of the other players' strategies (in *G*)."[1]

It is understood that the phrase "in knowledge of" is congruent with the epistemological concept of knowledge as "justified true belief". Hence, in the absence of factual knowledge, if player *k* assigns probability distributions to the other players' strategies, or otherwise hypothesizes about them, then player *k* is considered to be playing a meta-game.

This consideration endorses the claim that the implementation of collective rationality and the reformulation of the Prisoner's Dilemma matrix both entail (first-level) meta-games. In order to apply the working definition of collective rationality adopted herein, prisoner *A* must assign a probability value to the proposition that prisoner *B* is collectively rational. This involves *A*'s deliberation of prisoner *B*'s principle(s) of choice. Similarly, if prisoner *A* reformulates the game matrix, he must determine the probability with which prisoner *B* will correctly predict *A*'s choice. This also involves *A*'s deliberation of *B*'s principle(s) of choice. In this sense, *A* is playing a meta-game in both cases.

Howard defines a meta-game, not in terms of principles of choice, but in terms of strategies. Note the difference: a principle can be thought of as a decision rule which generates a single choice; a strategy, as a decision rule which generates a sequence of choices. In both meta-games and iterated games, the former gives way to the latter. At the same time, the meta-game still takes place in the static mode. Meta-game strategies are said to *expand* the matrix,[2] not

---

[1] N. Howard, `"General Metagames": An Extension of the Metagame Concept', in Rapoport (*ed.*), 1974, pp.260–83. See also idem, 1971.

[2] E.g. see A. Rapoport, `Escape From Paradox', *Scientific American*, July 1967, pp.50–56.

to iterate it. Nevertheless, meta-game decision rules do generate sequences of choices, and are therefore justly termed strategies.

At this stage, it is appropriate to mention that in any game (or meta-game), the number of possible strategies tends to preponderate greatly over the number of possible moves. Even in a relatively simple game, the preponderance can be numerically staggering. To illustrate the point, one can cite Rapoport's example of tic-tac-toe.[3] He computes the number of possible ways to make the first five moves as 9x8x7x6x5, or 15,120. But player *A*, who moves first, has 9 possible strategies on move one, then 7^8 possible strategies on move three (by associating any of his seven replies to any of *B*'s eight choices), then 5^6 possible strategies on move five (by associating any of his five replies to any of *B*'s six choices), for a total of $9 \times 7^8 \times 5^6$, or 8.1×10^{11} possible strategies. Player *A* has an average ratio of more than fifty-three million strategies per move.

As Rapoport mentions, considerations of symmetry would reduce these numbers by several orders of magnitude, but the ratio of strategies to moves would still remain exceedingly large.[4] If one applies this calculus to the relatively complex game of chess, the mind rapidly boggles at the number of possible strategies available to the players.[5]

The point applies to the Prisoner's Dilemma as well. In the static meta-game, each player faces only two possible moves, but has recourse to a multiplicity of possible strategies. The idea in Howard's meta-game resolution is to eliminate the dominance principle by introducing "conditional" strategies, which generate an expanded

---

[3] Idem., 1960, pp.146-7.

[4] Ibid.

[5] White has 20 possible opening moves; so does Black. White has no fewer than 20 possible second moves; so does Black. The number of possible ways in which the players can each make two moves is 20x20x20x20, or 160,000. But White has 20 possible strategies on his first move, and no fewer than 20^{20} possible strategies on his second move, or 2×10^{27} possible strategies for only two moves. Black has 20^{20} possible strategies on his first move, and no fewer than 20^{20} possible strategies on his second move, or 1×10^{52} possible strategies for only two moves.

matrix containing new equilibrium outcomes.

Suppose prisoner *A* has four conditional strategies:

A_1 : *A* co-operates regardless of *B*'s expected choice. This strategy always generates choice *C*.

A_2 : *A* chooses what he expects *B* to choose. This strategy generates choice β, which can be either *C* or *D*.

A_3 : *A* chooses what he expects *B* not to choose. This strategy generates choice β^*, which can be either *D* or *C*.

A_4 : *A* defects regardless of *B*'s expected choice. This strategy always generates choice *D*.

Let the payoffs to the prisoners be represented in utiles, as follows:

<u>Game 6.1 - Prisoner's Dilemma</u>

		A	
		C	*D*
B	*c*	1,1	-2,2
	d	2,-2	-1,-1

Now let prisoner *A*'s conditional strategies be applied to Game 6.1. The following meta-game matrix obtains:

<u>Game 6.2 - First-Level Meta-Game of Game 6.1</u>

		A			
		A_1	A_2	A_3	A_4
		(*C*)	(β)	(β^*)	(*D*)
B	*c*	1,1	1,1	-2,2	-2,2
	d	2,-2	-1,-1	2,-2	-1,-1

In game 6.2, prisoner *A*'s choices are generated by his respective conditional strategies. The choices themselves are parenthesized, in order to emphasise their dependence on the respective strategies.

In game 6.2, prisoner *B* no longer has a dominant choice. For *B*, defection dominates co-operation if *A* adopts A_1, A_3 or A_4, but co-operation dominates defection if *A* adopts A_2. If *B* were to maximize his expected utility, by assigning (via the principle of insufficient reason) an equiprobability of 1/4 to the likelihood that *A* adopts any strategy, then $EU(c) = -1/2$ and $EU(d) = 1/2$. With this probability distribution, maximizing expected utilities prescribes defection. But *B* has no assurance that *A*'s choice among strategies will be made equiprobabilistically. How, then, is *B* to choose?

B might examine the matrix from *A*'s point of view, and ask himself whether *A* has a dominant strategy. Then *B* would find that *A* stands to minimize his losses by choosing either A_2 or A_4, and to maximize his gains by choosing either A_3 or A_4. If *A* is individually rational, his best strategy is therefore A_4. But if *A* is collectively rational, and believes *B* to be collectively rational also, then *A* might risk choosing A_2. So *B* has narrowed the field of *A*'s likely choices, to A_2 and A_4. In effect, *B* is now looking at the following reduced game:

<u>Game 6.3 - Sub-Matrix of Game 6.2</u>

		A	
		A_2	A_4
		(*B*)	(*D*)
B	*c*	1,1	-2,2
	d	-1,-1	-1,-1

B now realizes that, for prisoner *A*, strategy A_4 weakly dominates A_2. Thus, if *B* has no extenuating reason to believe that *A* is collectively rational, *B* must defect in order to protect himself against *A*'s impending defection. In sum, game 6.2 offers no resolution to the Prisoner's Dilemma. Weak dominance lures *A*, and thus impels *B*, to the mutually detrimental outcome of (-1,-1).

But all is not yet lost. Howard defines the second-level metagame as follows:

> ". . .by recursion, the second-level metagame *jkG*, where *j* and *k* are players, is the game in which *j* chooses his strategy (in *kG*) in knowledge of the other's strategies (in *kG*); in terms of strategies in *G*, it is a game in which *j* reacts (a) to *k*'s reactions to the actions of the players other than *k*; (b) to the actions of the players other than *j* and *k*."[6]

Only condition (a) is applicable to the two-person Prisoner's Dilemma. In terms of condition (a), then, let prisoner *B* react to prisoner *A*'s strategies (which are *A*'s reactions to *B*'s possible actions) by taking first-level meta-game 6.2 to a second-level metagame.

B associates two choices (either *c* or *d*) with each of *A*'s four possible strategies, and so *B* generates 2^4, or sixteen, possible meta-strategies. The following meta-meta-game obtains:

<u>Game 6.4 - Second-Level Meta-Game of Game 6.1</u>

		A			
		A_1	A_2	A_3	A_4
	$A_1A_2A_3A_4$	(*C*)	(*B*)	(*B**)	(*D*)
	cccc	1,1	1,1	−2,2	−2,2
	cccd	1,1	1,1	−2,2	−1,−1
	ccdc	1,1	1,1	2,−2	−2,2
	cdcc	1,1	−1,−1	−2,2	−2,2
	dccc	2,−2	1,1	−2,2	−2,2
	ccdd	1,1	1,1	2,−2	−1,−1
	cdcd	1,1	−1,−1	−2,2	−1,−1
B	*dccd*	2,−2	1,1	−2,2	−1,−1
	cddc	1,1	−1,−1	2,−2	−2,2
	dcdc	2,−2	1,1	2,−2	−2,2
	ddcc	2,−2	−1,−1	−2,2	−2,2
	dddc	2,−2	−1,−1	2,−2	−2,2
	ddcd	2,−2	−1,−1	−2,2	−1,−1
	dcdd	2,−2	1,1	2,−2	−1,−1
	cddd	1,1	−1,−1	2,−2	−1,−1
	dddd	2,−2	−1,−1	2,−2	−1,−1

---

[6] Howard, in Rapoport (*ed.*), 1974, p.261.

In game 6.4, prisoner A has no dominant strategy. If A were to maximize his expected utility, by assigning (via the principle of insufficient reason) an equiprobability of 1/16 to the likelihood that B adopts any meta-strategy, then

$$EU(A_1) = -1/2$$
$$EU(A_2) = 0$$
$$EU(A_3) = 0$$
$$EU(A_4) = 1/2$$

Given such a probability distribution, maximization of expected utilities prescribes unconditional defection. But A has no assurance that B will choose his meta-strategy equiprobabilistically. How, then, is A to choose? A might examine the matrix from B's point of view, and ask himself what B would choose.

If B examines column one of game 6.4, he notices that, should A choose strategy A_1, then eight of B's sixteen meta-strategies result in a payoff (to B) of two utiles; the other eight, of only one utile. B defines the set of meta-strategies, whose members yield two utiles, as S_1. Explicitly,

$$S_1 = \{dccc,\ dccd,\ dcdc,\ ddcc,\ dddc,\ ddcd,\ dcdd,\ dddd\}$$

Similarly, should A choose strategies A_2, A_3, or A_4, B defines S_2, S_3, and S_4 as those respective sets of meta-strategies whose members yield a better payoff to B (of one as opposed to minus one utiles, two as opposed to minus two utiles, and minus one as opposed to minus two utiles; in columns two, three, and four respectively). Explicitly,

$$S_2 = \{cccc,\ cccd,\ ccdc,\ dccc,\ ccdd,\ dccd,\ dcdc,\ dcdd\}$$
$$S_3 = \{ccdc,\ ccdd,\ cddc,\ dcdc,\ dddc,\ dcdd,\ cddd,\ dddd\}$$
$$S_4 = \{cccd,\ ccdd,\ cdcd,\ dccd,\ ddcd,\ dcdd,\ cddd,\ dddd\}$$

Now B finds the intersection of these sets:

$$S_1 \cap S_2 \cap S_3 \cap S_4 = \{dcdd\}$$

In other words, the choice of meta-strategy *dcdd* guarantees that, no matter what strategy *A* chooses, *B* cannot fare better by choosing any other meta-strategy. *B*'s next-best meta-strategy is *ccdd*, which yields identical results except when *A* chooses A_1, in which case *ccdd* yields only one utile to *B* (as opposed to two utiles yielded by *dcdd*). Although *dcdd* is neither strongly nor weakly dominant, it is the sole meta-strategy that yields the best possible result to *B* irrespective of what *A* chooses. In consequence, this meta-strategy exhibits a novel property which may, with methodological justification, be termed *set-theoretic dominance*.[7]

If, on the other hand, *B* elects to maximize his expected utility, again assigning equiprobable values of 1/4 to each of A's four strategies, he finds

$$EU(dcdd) = 1$$

which is indeed the maximum expected utility for all meta-strategies. (Again, *ccdd* is next-best, with $EU(ccdd) = 3/4$.) Both set-theoretic dominance and maximization of expected utility prescribe meta-strategy *dcdd*. And set-theoretic dominance does so analytically, without recourse to the principle of insufficient reason or any other potentially objectionable *a priori* probabilistic rule.

To recapitulate: prisoner *A*'s examination of the matrix of meta-game 6.4 leads him to the realization that prisoner *B* possesses a set-theoretically dominant meta-strategy, *dcdd*. Now *A* reasons that, if *B* is individually rational, *B* will opt for set-theoretic dominance. In that case, if *A* is individually rational, *A* should choose strategy A_2, not A_4. Although strategy A_4 has a higher expected utility than A_2, those expected utilities were found by assigning a

---

[7] Reflection shows that if a strategy is strongly dominant, or weakly dominant, then it is also set-theoretically dominant. In other words, both strong and weak dominance imply set-theoretic dominance. But set-theoretic dominance implies neither strong nor weak dominance, and therefore comprises a weaker but logically distinct category of dominance.

uniform probability distribution to *B*'s choice of meta-strategies. But, given that *B* is individually rational, the existence of *B*'s set-theoretically dominant meta-strategy nullifies *A*'s application of the principle of insufficient reason, and its consequent assignment of *a priori* equiprobabilities.

If *B* is individually rational, *B* chooses the set-theoretically dominant meta-strategy *dcdd*. Now *A* examines the sub-matrix of the reduced game that obtains:

Game 6.5 - Sub-Matrix of Game 6.4

			A		
		A_1	A_2	A_3	A_4
B	*dcdd*	2,-2	1,1	2,-2	-1,-1

Now *A* has a strongly dominant strategy; namely, A_2. Thus if *A* is individually rational, *A* chooses strategy A_2. Now both prisoners realize that the individually rational joint choice is (A_2,*dcdd*), whose joint outcome is the mutually beneficial (1,1). Thus, individual rationality leads both prisoners to co-operate, and both prisoners gain one utile.

This is in marked constrast to the previous games in this chapter. In game 6.1 (the Prisoner's Dilemma), strong dominance leads to the mutually-detrimental equilibrium outcome (-1,-1). In game 6.2 (first-level meta-game of 6.1), weak dominance leads to the same result: (-1,-1). But in game 6.4 (second-level meta-game of 6.1), set-theoretic dominance leads to the mutually-beneficial, Pareto-optimal outcome (1,1). In the second-level meta-game, the dominance principle has finally "reversed" its prescription.

And at the same time, a significant change occurs with respect to rational choice. In game 6.4, individual rationality leads both prisoners to mutual co-operation. Now suppose *B* is collectively rational. What should he do? Clearly, *B* should still choose meta-strategy *dcdd*, for two reasons. First, it contains a Pareto-optimal outcome, thus enabling both prisoners to benefit by mutual co-operation if *A* is also collectively rational. Second, if *A* is not collectively rational, then meta-strategy *dcdd* remains set-theoretically dominant, since it is the only choice that yields the maximum possible payoff to *B* no matter what *A* chooses. So, whether *B* is an individual or a collective rationalist, he chooses meta-strategy *dcdd*.

Now suppose *A* is collectively rational. Then, by definition, he chooses strategy A_2 which, by definition, consists in choosing what *A* expects *B* to choose. So, whether *A* is an individual or a collective rationalist, he chooses strategy A_2.

This turn of events, in which either type of rationality (or indeed a mixture of types) leads to the same meta-strategic outcome, (A_2, *dcdd*), evinces a singular observation by Rapoport: that individual and collective rationality are reconciled in this second-level meta-game of the Prisoner's Dilemma.[8]

However, the undeniable elegance and ingenuity of Howard's meta-game-theoretic resolution do not dispel the problem that haunts Prisoner's Dilemmas; rather, they transform it into a meta-problem. The reconciliation of individual and collective rationalities does not, in and of itself, guarantee that the dilemma is resolved.

This point can be illustrated by a thought-experiment. Suppose three hundred pairs of people are to be placed in two-person Prisoner's Dilemmas, with payoffs as in game 6.1. With respect to the participants, the pairs are formed by blind random selection, so that no prisoner knows the identity of the other prisoner in his particular game. The three hundred pairs are then formed into three groups of one hundred pairs each, again by random selection. It is assumed that none of the six hundred participants has any prior knowledge of game theory, or of the Prisoner's Dilemma.

The three groups are then isolated from one another, and each group is given a preparatory lecture on the Prisoner's Dilemma. These lectures, however, are not identical.

Group One is informed about conflicting principles of choice and diverging concepts of rationality. The nature of the dilemma is

---

[8] Rapoport, 1967, pp.55-6.

made clear, but no meta-game resolution is presented.

Group Two is given the same lecture as Group One, and is additionally informed about the meta-game resolution of the dilemma.

Group Three is given the same lecture as Group One (with no information about the meta-game resolution).

Each pair of each group then plays one game of Prisoner's Dilemma. The players make their choices by a ballot-box method, and thus retain mutual anonymity. (The ballots are coded in such a way that only the experimentor, and not the players, can associate them pair-wise.)

Next, the groups are asked to play one more game each. Groups One and Two proceed as above, but Group Three is first given an additional preparatory lecture, in which the meta-game resolution is presented. Then Group Three proceeds as above.

The empirical question is, of course, whether an awareness of the meta-game resolution makes mutual co-operation more frequent. If it does, then Group Two should show a greater frequency of mutual co-operation than Group One. And if it does, then Group Three's frequency of mutual co-operation in its first hundred trials should correlate with that of Group One, and in its second hundred trials with that of Group Two.

It might be interesting to conduct this experiment, or some variation of it. (If such an experiment has already been conducted, may this enquiry's ignorance be excused.)

The point to be made is simply this: there is no *a priori* guarantee that an awareness of the meta-game resolution promotes unconditional mutual co-operation. Such awareness can have the opposite effect, in exactly the same way as consideration of collective rationality, and a reformulation of the game-matrix, can result in non-Pareto-optimal outcomes. If the meta-game resolution convinces prisoner *A* to co-operate, it may equally well convince prisoner *B* to exploit *A* by defecting (and vice-versa). Similarly, both may be tempted into defection, each hoping to exploit the other.

Another difficulty with the meta-game resolution is, as Rapoport points out, that it requires translation into a social context.[9] With respect to conflict resolution, one may infer that Rapoport wishes for a more significant translation than that which merely utilizes the resolution as the basis for an experiment in social psychology.

In any case, the Prisoner's Dilemma persists in the static mode, resisting attempted resolutions by re-definition of rationality, reformulation of game matrix, and expansion into meta-game decision space. These attempted resolutions are of undeniable theoretical value, as they acknowledge that a mutually beneficial outcome is most certainly attainable. And in practice, while these resolutions cannot compel the prisoners to co-operate, they can make a profound appeal for collective rationality to prevail. Indeed, the meta-game resolution demonstrates that, in the last analysis, any type of rationality (whether individual or collective) is preferable to irrationality.

The enquiry now turns to an examination of the Prisoner's Dilemma in the iterated mode, in order to investigate strategic interactions that obtain therein.

---

[9] Ibid., p.56.

PART THREE:
THE ITERATED PRISONER'S DILEMMA

Chapter Seven

A Tournament of Strategic Families

In 1966, Rapoport asserted that

> " 'What is the best way to play chess?' is not a game-theoretical question. On the other hand, 'Is there a best way to play chess?' is a game-theoretical question."[1]

Rapoport's assertion is a reflection of the taxonomic orientation of game-theory.

The theory answers Rapoport's second question in the affirmative. Chess is classified as a two-person, constant-sum game of perfect information that is strictly determined. As such, chess belongs to the same class of games as tic-tac-toe. Thus von Neumann and Morgenstern remark:

> "This shows that if the theory of Chess were really fully known there would be nothing left to play . . ."[2]

To appreciate why this is so, consider tic-tac-toe, which ceases to fascinate players as soon as they realize that it is always possible for either player to force a draw. Between two experienced players, a drawn outcome is a foregone conclusion. From a game-theoretic point of view, chess differs only in so far as the number of possible chess games is incomparably greater than the number of possible games of tic-tac-toe. So, even between two experienced chess players, a draw is not a foregone conclusion, since either or both players are likely to become entangled in a welter of possible combinations of moves, and thus fail to find the "best move" in a given situation. If all chess players continually found their best moves, all chess games would be drawn.

But the theory of games does not provide an answer to Rapoport's first question ('What is the best way to play chess?'). The theory of games merely asserts that a "best" chess move always exists, on every turn of every game, whether or not a player actually finds it. The theory does not purport to tell the player *how* to find it. The player who wishes to answer this question must consult the

---

[1] Rapoport, 1966, p.14.

[2] Neumann & Morgenstern, 1955, p.125.

copious literature on the theory of chess openings, middle-games, and end-games. The player who does so will soon discover that, although the literature abounds with sound recommendations on how *not* to play the game, the vast number of possible combinations of good moves precludes exhaustive deterministic analysis.

In 1980, Axelrod enquired "What is the most effective way to play the iterated Prisoner's Dilemma?"[3] At first blush, this question may seem not only non-game-theoretic, but also self-contradictory. It seems to assume that the game-theoretic question "Is there a most effective way to play the iterated Prisoner's Dilemma?" can be answered in the affirmative. But the Prisoner's Dilemma belongs to the class of non-zero-sum, non-co-operative games. So game-theory asserts that, in the static mode, there does not exist a "most effective way" to play the game. (And Part Two of this enquiry certainly corroborates the theory, if by "most effective way" one understands "the way that guarantees a Pareto-optimal outcome".) How then, one may ask, can Axelrod expect to find, in the iterated mode, something which the theory pronounces non-existent in the static mode?

To answer this, one recalls the von Neumann-Morgenstern disclaimer:

> "We make no concessions: Our viewpoint is static and we are analyzing only a single play."[4]

Since its formal articulation by von Neumann and Morgenstern, the theory of games has been extended into many new domains, including that of iterated games. Rapoport has made significant contributions to the theory of iterated Prisoner's Dilemmas,[5] and has thus helped the theory to keep pace with the profusion of iterated experiments.

Theories of the iterated Prisoner's Dilemma posit a tendency toward joint similar play (either mutual co-operation or mutual

---

[3] Axelrod, 1980*a* & 1980*b*, p.3 & p.379 respectively.

[4] Neumann & Morgenstern, 1955, p.147.

[5] E.g. for treatments of Markov chain models, equilibrium models, stochastic learning models, and classical dynamic models, see Rapoport & Chammah, 1965, pp.115-50. For an inductive theory of iterated Prisoner's Dilemmas, see Rapoport, 1966, pp.145-57.

defection).[6] Which outcome obtains depends upon initial conditions and the length of the game. Rapoport also finds agreement between theory and experiment:

> "The initial gross trend in repeated plays of Prisoner's Dilemma is toward more defection. After a while 'recovery' sets in, and the frequency of co-operative responses increases. This recovery is relatively quick and pronounced when the matrix is displayed but comes much later and is relatively weak when the matrix is not displayed. The steady decline of the unilateral states, i.e. the increasing predominance of *CC* and *DD* states, is evidently responsible for the fact that paired players become more and more like each other in repeated plays of Prisoner's Dilemma."[7]

Given these developments, it seems quite reasonable to assume that some ways of playing the iterated Prisoner's Dilemma are more effective than others (if by "more effective" one understands "more likely to lead to repeated mutual co-operation than to repeated mutual defection"). Armed with that assumption, Axelrod conducted two experiments aimed at discovering precisely which ways might prove more, and less, effective. The experiments were conducted as computer-run tournaments, in which competitors submitted strategies in the form of programs.

Before summarizing the results of Axelrod's tournaments, and prior to analyzing the results of a third tournament conducted as part of this enquiry, one must state Axelrod's overriding conclusion: there is no "best" strategy independent of environment.[8] This conclusion is consistent with the theory of the Prisoner's Dilemma. No strategy is unconditionally Pareto-optimal, in either the static or the iterated mode. But, in a given environment, certain strategies may prove more conducive to Pareto-optimality than others. Having defined a particular environment, Axelrod is able to identify such strategies.

---

[6] Ibid.

[7] Rapoport & Chammah, 1965, p.102. See also R. Clarke, *The Science of War and Peace*, Jonathan Cape, London, 1971, pp.281 *ff*.

[8] Axelrod, 1980*a* & 1980*b*, p.21 & p.402 respectively.

An analogy with Darwinian evolutionary theory might be appropriate. Within a given species, genetic mutations give rise to phenotypic differences. Some phenotypes are more favoured by natural selection than are others. If one asked a Darwinian evolutionary theorist "What kind of phenotype is best-adapted?" for a given species, the theorist would almost certainly reply that there is **no** "best" adaptation independent of environment.

If one takes into account local environmental factors (such as climate, terrain, indigenous flora and fauna, species-specific habits, and so forth), then conjectures about the adaptiveness or non-adaptiveness of a particular phenotype become meaningful. And selective domestic breeding, enhanced by knowledge of genetic theory, enables the directed dispersion or suppression of a given phenotype's frequency in successive generations of a particular population in a controlled environment.

Similarly, by studying the interactions of different strategies under varying conditions, one may gradually identify properties which tend to make a given strategy more (or less) effective in a particular strategic population in a defined environment. Flexible strategies can be modified, until their performance is optimally effective in the context of their competitors and surroundings.

The key environmental factors in Axelrod's tournaments are: the payoffs to the players, the number of iterations in a game, the players' knowledge of this number, and the actual strategies in competition. Let the import of these be discussed in turn.

First, consider the iterated game matrix (game 7.1, overleaf). The transitivity of the payoffs in the iterated mode is identical to that in the static mode. But game 7.1 differs from game 4.1 with respect to the added constraint, $R > (1/2)(S+T)$. This constraint is necessary in iterated games. If $(S+T) > 2R$, the players would gain more by alternating choices (C,d) and (D,c) than by mutual co-operation (C,c). As Rapoport points out, the players would then have at their disposal a form of tacit collusion, which is not supposed to occur in a Prisoner's Dilemma.[9]

[9] Rapoport & Chammah, 1965, pp.34-35.

Game 7.1 - The Iterated Prisoner's Dilemma

			B	
		c		d
A	C	R,R		S,T
	D	T,S		P,P

where $T > R > P > S$

and $R > (1/2)(S+T)$

The actual payoffs used by Axelrod are:

Game 7.2 - Axelrod's Tournament Matrix

			B	
		c		d
A	C	3,3		0,5
	D	5,0		1,1

All competitors were aware of this payoff structure before submitting their strategies. In game 7.2, one can see explicitly how the constraint applies. If the players alternate choices of (C,d) and (D,c), they each gain an average of 2.5 utiles per move. If they both co-operate, they each gain 3 utiles per move. Thus the constraint $R > (1/2)(S+T)$ discourages tacit collusion, by making it less profitable than mutual co-operation.

Second, the number of iterations in a game is important because some strategies employ *a posteriori* probability considerations; in other words, they look at relative frequencies of past outcomes. If the game is not long enough, these frequencies may fail to attain sufficient closeness to their limiting values (where such values exist), and the accuracy of the probability calculus in question may be impaired.[10]

Third, the players' knowledge of the length of a game (rather than the length of a game itself) can give rise to the phenomenon of "reverse induction".[11] If a game is known to consist of j moves, then player *A* may be tempted to defect on the j^{th} move, since there will be no $(j+1)^{st}$ opportunity for player *B* to retaliate. If player *B* reasons similarly, mutual defection occurs as the j^{th} outcome. In that case, player *A* reasons that he may as well defect on his $(j-1)^{st}$ move as well, since player *B* will defect on the next move in any case. If player *B* reasons similarly, mutual defection will occur on the $(j-1)^{st}$ move. Then, by reverse or backward induction, the players will defect on all moves.

Axelrod found partial empirical confirmation of this phenomenon. In his first tournament, the length of each game was held constant at two hundred moves, and all players were informed of this in advance. Some players submitted strategies which, regardless of their respective decision rules operative throughout most of the game, defected unconditionally during the last several moves, in the hope of exploiting both intrinsically co-operative strategies and strategies too slow to retaliate. In his second tournament, Axelrod amended the ground rules such that

> ". . .the length of the games was determined probabilistically with a 0.00346 chance of ending with each given move. This parameter was chosen so that the expected median length of a game would be 200 moves. . . Since no one knew exactly when the last move would come, end-game effects were successfully avoided in the second round."[12]

---

[10] For example, in a different context: if one attempts to assess the "fairness" of a coin by tossing it only ten times, it might yield two heads and eight tails. This would not justify the conclusion that the coin is unfair. A fair coin will approach a limiting frequency of $n/2$ heads and $n/2$ tails per n tosses, as the number of trials increases. To obtain a result within a desired degree of closeness to this distribution, one must make a correspondingly large number of trials. E.g. see von Mises, 1981, *passim*.

[11] Rapoport & Chammah, 1965, pp.28-29.

[12] Axelrod, 1980*b*, p.383.

Fourth, the actual strategies in competition are a major determining factor of the effectiveness (or ineffectiveness) of a given strategy. For instance, both of Axelrod's tournaments were won by a strategy submitted by Rapoport, called "tit-for-tat" (acronym *TFT*). *TFT* is a simple and elegant decision rule: it co-operates on its first move, and plays next whatever its opponent played previously. *TFT* is the game-theoretic equivalent of *lex talionis*. But, notwithstanding *TFT*'s impressive performance (it defeated fourteen other entrants in the first tournament, and sixty-two others in the second), Axelrod is able to furnish several reasons why he considers *TFT* not to be the "best" strategy in iterated Prisoner's Dilemmas.

In the first tournament, relative success among the eventual top eight strategies turned out to be heavily influenced by the presence of two "kingmaker" strategies, so-called because they did not finish well themselves, but largely determined the order of finish among the top eight.[13] *TFT* fared better against these kingmakers than did any other strategy.

A strategic environment is shaped not only by the presence of certain strategies, but also in the absence of others. Again with respect to the first tournament, Axelrod cites three strategies, any of which would have won had it been submitted.

The first, somewhat ironically, was a "sample program" sent to prospective contestants, in order to illustrate the desired format for a submission. The strategy itself is "tit-for-two-tats" (acronym *TTT*). *TTT* co-operates on the first two moves, and defects only after its opponent defects on two consecutive moves. The sample program of *TTT*, says Axelrod,

> ". . . would in fact have won the tournament if anyone had simply clipped it and mailed it in! But no-one did."[14]

The second unsubmitted winning strategy was also available to most contestants, since it was included in a report of a preliminary tournament circulated for subsequent recruitment. The strategy used a look-ahead, tree-searching technique that is popular in artificial

---

[13] Idem., 1980*a*, pp.10–13.

[14] Ibid, p.20.

intelligence programs. It will not be considered in this enquiry's game-theoretic context.

A third strategy, which ranked only tenth among fifteen entrants, would have won the tournament with a slight modification. Called *Downing* (after its submittor), this strategy is none other than the maximization of expected utility. Downing, faced with the task of assessing the probability of each opposing strategy's co-operation or defection before the commencement of play, without knowing explicitly against which strategies he would compete, had recourse to the *a priori* principle of insufficient reason. He initially assigned equiprobable values of (1/2, 1/2) to the likelihood of an opposing strategy's co-operation or defection. His program then updated this arbitrary probability distribution after each move, according to the actual move made by the given opposing strategy.

Had Downing selected a more optimistic initial weighting; i.e. one that assumed a greater *a priori* likelihood of co-operation and a correspondingly lesser *a priori* likelihood of defection on the part of opposing strategies, then Axelrod asserts that Downing would have won by a large margin.[15]

Axelrod does not say which of these three hypothetical winning strategies would have prevailed had they all been submitted, but his point about the relativity of *TFT*'s success is well taken.

And in the second tournament, although the number of entrants more than quadrupled, *TFT* once again proved its relative superiority, prevailing against all competitors. Nevertheless, Axelrod steadfastly maintains that *TFT* is <u>not</u> the "best" decision rule in the iterated Prisoner's Dilemma, and gives three reasons why not.[16]

Firstly, Axelrod describes a hypothetical strategy that would have won the second tournament, had it been submitted. Such a strategy would have the necessary property (which no other submission manifested) of being able to identify and defect against a random strategy, while not mistaking any non-random strategy for a random one. This property is difficult to implement; Axelrod confesses that

---

[15] Ibid.

[16] Idem., 1980*b*, pp.401-402.

he attempted to write such a program himself, but the program did not perform ideally.[17]

Secondly, Axelrod observes that, had a third tournament been conducted, with a field of entries composed solely of the upper half of the second tournament standings, then *TFT* would have ranked only fourth. The first, second, and third places in this hypothetical third tournament would have been occupied by strategies which placed twenty-fifth, sixteenth, and eighth, respectively, in the second tournament.

This observation is an excellent illustration of the dependence of a given strategy's success on the constitution of the overall strategic population. In the second tournament, the twenty-fifth-ranked strategy evidently fared magnificently against the upper half, but this success was altogether marred by its exceedingly poor performance against the lower half. The standings of the second tournament completely disguise the fact that the twenty-fifth-ranked strategy is actually the best among the upper half of the whole, if one discards the lower half. This example shows the necessity for caution when drawing inferences about the strength or weakness of a given strategy.

Thirdly, Axelrod re-states his overriding conclusion: *TFT* cannot be the best strategy in the iterated Prisoner's Dilemma, because there is no "best" strategy independent of environment.[18]

Nonetheless, it is germane to speak of the "successfulness" of a particular strategy, as a relative indicator (rather than an absolute measure) of its performance in a given environment. Axelrod describes a "successful" strategy, in the context of his tournaments, in terms of three attributes: "niceness", "provocability", and "forgiveness".[19] A strategy is said to be *nice* if it never defects first; *provocable*, if it is able to defect in response to defection; *forgiving* if, after having been provoked, it is able to co-operate in response to renewed co-operation.

[17] Ibid, p.402.

[18] Ibid.

[19] Ibid, pp.389-95.

In the first tournament, the eight top-ranking strategies were nice; the bottom six, not nice.[20] In the second tournament, fourteen of the fifteen top-ranking strategies were nice; fourteen of the bottom fifteen, not nice.[21] Thus Axelrod found some correlation between niceness and success.

The other two attributes' effects on a strategy's success are more difficult to gauge. *TFT*, for example, is rapidly provocable yet quick to forgive. By playing next whatever its opponent played previously, it swiftly punishes defection, but harbours no grievance in the process. It judges each opponent's move on its individual merit, and disregards the history of the game. *TTT* is just as forgiving, but less provocable, since it defects only after two consecutive defections by an opponent. Recall that *TTT* would have won the first tournament, but was ironically not submitted. It was submitted in the second tournament, but ironically did not win.

Thus *TFT* won the first tournament because of the absence of *TTT*, and won the second tournament despite its presence. Again, this result reflects the dependence of a strategy's success upon the other types of strategies in the competing population. The extent to which a given degree of provocability or forgiveness conduces to success, is therefore also environment-dependent.

A strategy that manages to perform with relative success in a variety of environments is said to be "robust".[22] In Axelrod's two tournaments, *TFT* demonstrated greater robustness than any other strategy submitted. In sum, according to Axelrod's findings, a robust strategy should possess the property of niceness, and be imbued with a propitious combination of the qualities of provocability and forgiveness.

Axelrod, however, adds the following caveat:

"Being able to exploit the exploitable without paying too high a cost with the others is a task which was not

---

[20] Idem., 1980*a*, p.9.

[21] Idem., 1980*b*, pp.389-90.

[22] Ibid, pp.396-8.

> successfully accomplished by any of the entries in round two of the tournament."[23]

This remark gives pause to wonder whether a strategy's robustness might improve if it were imbued with the additional quality of "exploitiveness". An *exploitive* strategy, then, would be able to exploit the exploitable without becoming vulnerable to exploitation itself.

This completes a sketch of Axelrod's environment for the iterated Prisoner's Dilemma, in terms of four key factors (the payoffs to the players, the number of iterations in a game, the players' knowledge of this number, and the actual strategies in competition). Axelrod organizes and presents both tournaments' results in a highly interesting fashion, and this enquiry will continue to refer to, and aspire to develop, particular aspects of his findings.

It can be observed that Axelrod's tournaments are quintessentially game-theoretic in spirit, since they provide environments for strategic competition while eschewing involvement in the psychology of the strategists themselves. In an iterated Prisoner's Dilemma, a player adopts a particular strategy, which generates choices according to its decison rule. A game-theorist is interested in the relative robustness of the given strategy; he is not concerned with the strategist's psychological motives for adopting it.

And although it is assumed that each strategist prefers his strategy to fare as well (and not as poorly) as possible in competition,[24] an assessment of a particular strategy's success can be made quite independently of its strategist's rationality, or irrationality.

Suppose ten players compete in an iterated Prisoner's Dilemma. Let player *A*'s strategy be: defect on Mondays, and co-operate on other days of the week. Let the remaining nine players' strategies

---

[23] Ibid., p.403.

[24] A known exception is the thirteenth-ranked strategy in Axelrod's first tournament, submitted "out of scientific interest rather than an expectation that it would be a likely winner". See Axelrod, 1980*a*, pp.23-4.

be: co-operate on Mondays, and defect on other days of the week. The game-theorist who conducts this competition observes that player *A*'s strategy is the most successful on Mondays, and the least successful on other days of the week. This observation is independent of the rationality, or irrationality, of the players.

Another significant feature of Axelrod's experiments now bears mention. Axelrod regulated three of four key environmental factors (the payoffs to the players, the number of iterations in a game, and the players' knowledge of this number). He made no initial attempt to regulate the actual strategic types in competition. His strategic population can thus be labelled the "wild" variety. Note that this "wildness" led to a spate of late-game defections in the first tournament. In order to counteract that phenomenon, Axelrod replaced the fixed number of moves per game (in the first tournament) with a probabilistic number of moves per game (in the second tournament), and thus exerted indirect influence on the late-game behaviour of the second field of entries. The strategic population itself, however, remained of the "wild" variety, since Axelrod regulated neither the inclusion nor exclusion of particular strategic types in or from competition.

Now suppose a complementary experiment were conducted. The alternative to a "wild" population is, of course, a "domesticated" one. If the strategic types in competition are themselves regulated, the experimentor then exercises fuller control over the tournament environment. "Domesticated" strategies can be "bred" which incorporate, or lack, virtually any combination of niceness, provocability, forgiveness, and exploitiveness, among other qualities. And certain "wild" strategies, selected for their robustness, can be maintained "in captivity", and induced to compete against the "domesticated" strategies.

It was hypothesized that such an experiment, featuring competition among "wild" robust strategies and "domesticated" strategies of unknown robustness, would provide a complementary basis for comparison with Axelrod's results and conclusions. In order to test the hypothesis, an "interactive" tournament was conducted; so named because it involves interaction between "domesticated" and "wild" strategies, and also because all possible sub-tournaments (i.e. all possible combinations of strategies) of the main tournament are considered. This enquiry now proceeds to describe, and to discuss some results of, several facets of the interactive experiment.

As in the case of Axelrod's tournaments, the interactive tournament can be described in terms of four environmental factors. First, its payoff structure is identical to that of game 7.2. Second, the number of moves in each game is fixed and constant at one thousand. This number of moves allows slowly-developing strategies to attain their optimal performance levels (and does not affect the performance of quickly-developing strategies). Third, all strategies have the property of "integrity"; that is, each strategy adheres to its normal decision rule for the full one thousand moves per game. No strategy deviates from its normal decision rule by making late-game defections. Fourth, the twenty competing strategies are grouped into "families". The members of each strategic family share distinguishing characteristics.

The five families in the interactive tournament, and their respective members' acronyms and decision rules, are as follows:

(I) The Probabilistic Family

Members of this family co-operate and defect randomly, according to their individual probabilistic weightings. The two pure strategies (pure co-operation and pure defection) are included in this family because their program structure is identical to that of the other members. The members' decision rules thus differ by a sole parameter; namely, the probability of co-operation on a given move. This is the only family in the tournament whose members make their moves without taking their opponent's moves into account.

(a) *DDD*: This is the strategy of pure defection. On every move, *DDD* co-operates with a probability of zero, and defects with a probability of unity.

(b) *TQD*: This is the strategy of three-quarter random defection. On every move, *TQD* co-operates with a probability of 1/4, and defects with a probability of 3/4.

(c) *RAN*: This is the strategy of random equiprobability. On every move, *RAN* co-operates or defects with a probability of 1/2.

(d) *TQC*: This is the strategy of three-quarter random co-operation. On every move, *TQC* co-operates with a probability of 3/4, and defects with a probability of 1/4.

(e) *CCC*: This is the strategy of pure co-operation. On every move, *CCC* co-operates with a probability of unity, and defects with a probability of zero.

(II) The Tit-for-Tat Family

Members of this family are all related to tit-for-tat, and hence share a similar program structure. Small variations in members' decision rules can naturally result in large variations in competitive performance.

(a) *TFT*: Tit-For-Tat is the "founding member" of the family, and was the most robust strategy in Axelrod's tournaments. *TFT* co-operates on the first move, and plays next whatever its opponent played previously.

(b) *TTT*: Tit-for-Two-Tats is less provocable than *TFT*. *TTT* would have won Axelrod's first tournament (had it competed), but fared less well in the second. *TTT* co-operates on the first two moves, and defects only after two consecutive defections by its opponent.

(c) *BBE*: This strategy attempts to "burn both ends" of the strategic candle. It plays exactly as *TFT*, with one modification: *BBE* responds to an opponent's co-operative move by co-operating with a probability of 9/10. *BBE* thus attempts to out-perform *TFT* by being equally provocable but less reliably forgiving.

(d) *SHU*: This is Shubik's strategy, which ranked fifth in Axelrod's first tournament. It plays as *TFT*, with the following modification. *SHU* defects once following an opponent's first defection, then co-operates. If the opponent defects on a second occasion when *SHU* co-operates, *SHU* then defects twice before resuming co-operation. After each occasion on which the opponent defects when *SHU* co-operates, *SHU* increments its retaliatory defections by one. *SHU* thus becomes progressively less forgiving, in direct arithmetic relation to the number of occasions on which *SHU*'s co-operation meets with an opponent's defection.

(e) *TAT*: Tat-for-Tit is the binary complement of *TFT*. *TAT* defects on its first move, then plays next the opposite of whatever its opponent played previously. *TAT* thus defects in response to co-operation, and co-operates in response to defection. *TAT* has been bred to exhibit contrariness, and can be thought of as the "*bête noire*" of the *TFT* family.

(III) The Maximization Family

All members of this family maximize expected utilities, but do so with different initial probabilistic weightings. Each member plays randomly for one hundred moves (co-operating or defecting according to its particular weighting), and keeps track of all moves made by both itself and its opponent. After one hundred moves, an "event matrix" of joint outcome frequencies is used to assign *a posteriori* probabilities in the calculation of expected utilities for the one-hundred-and-first move, and all moves thereafter. The generalized event matrix takes this form:

Game 7.3 - Event Matrix for Maximization Strategy versus Opponent

		Opponent	
		c	*d*
Maximization Strategy	*C*	*W*	*X*
	D	*Y*	*Z*

---

where W = number of occasions on which outcome (C, c) obtained

X = number of occasions on which outcome (C, d) obtained

Y = number of occasions on which outcome (D, c) obtained

Z = number of occasions on which outcome (D, d) obtained

Now, recalling the generalized expressions introduced in Chapter Four, the maximization strategy finds its expected utilities as follows:

$$EU(C) = p(c/C)(R) + (1-p)(d/C)(S)$$
$$EU(D) = (1-p')(c/D)(T) + p'(d/D)(P)$$

Note that the maximization strategy avoids the difficulties latent in *a priori* probability formulations, by the expedient of evaluating *a posteriori* probabilities, or outcome frequencies, directly from the event matrix. Hence

$$p(c/C) = W/(W+X)$$
$$(1-p)(d/C) = X/(W+X)$$
$$(1-p')(c/D) = Y/(Y+Z)$$
$$p'(d/D) = Z/(Y+Z)$$

while the tournament payoffs are $T = 5$, $R = 3$, $P = 1$, $S = 0$. Then

$$EU(C) = 3W/(W+X)$$
$$EU(D) = (5Y+Z)/(Y+Z)$$

If $EU(C)$ is greater than or equal to $EU(D)$, the maximization strategy co-operates on move one-hundred-and-one; otherwise, it defects. The maximization strategy continues to record outcomes throughout the game, and thus updates the event matrix after every outcome. As the frequency distribution of outcomes changes, the maximization strategy's propensity toward co-operation or defection also changes accordingly.

The program structure is identical for every member of this family. The critical parameter, in whose value the members differ, is the weight accorded to the probability of a member's random co-operation during the first one hundred moves. It was hypothesized that the properties of the event matrix would be substantially affected by a combination of two factors: this initial choice of weight, and the type of opposing strategy encountered. Consequently, the maximization family was "bred" to represent a range of weights.

(a) *MEU*: Maximization of Expected Utility is the familial prototype, which appeared in Axelrod's tournaments under the name of its submittor, Downing. Downing ranked tenth among fifteen entries in the first tournament; fortieth among sixty-three in the second. Downing adopted the principle of insufficient reason, and assigned *a priori* probabilities of

$p(c/C) = p(d/C) = 1/2$, and $p'(c/D) = p'(d/D) = 1/2$

prior to its first move. It then updated the probabilities according the relative frequencies of actual outcomes. Recall that Downing would have finished first (in the first tournament) had its initial probabilistic outlook been more optimistic.

MEU randomly co-operates or defects with probability 1/2 for the first hundred moves. But in contrast to Downing, *MEU* assumes nothing about the play of its opponent. Instead, *MEU* notes its opponent's choice on each move, and records each joint outcome in the event matrix.

For example, suppose *MEU* encounters *TQC* (which randomly co-operates on every move with probability 3/4, and defects with probability 1/4). Then, after one hundred moves, the most probable event matrix is as follows:

Game 7.4 - *MEU* versus *TQC*, Event Matrix After 100 Moves

		TQC	
		c	*d*
MEU	*C*	38	12
	D	38	12

MEU's expected utilities are:

$$EU(C) = (38/50)\text{x}3 = 2.28$$
$$EU(D) = (38\text{x}5 + 12)/50 = 4.04$$

Thus *MEU* defects on its one-hundred-and-first move.

(b) *MAD*: This strategy maximizes expected utilities, with initial weighting at defection. *MAD* plays exactly as *MEU*, except that on each of its first one hundred moves, *MAD* defects with a probability of 9/10, and co-operates with a probability of 1/10.

Once again, for example, suppose *MAD* encounters *TQC*. Now the most probable event matrix, after one hundred moves, is:

<u>Game 7.5 - *MAD* versus *TQC*, Event Matrix After 100 Moves</u>

		TQC	
		c	*d*
MAD	*C*	7	3
	D	67	23

MAD's expected utilities are:

$$EU(C) = (7/10)\text{x}3 = 2.1$$
$$EU(D) = (67\text{x}5 + 23)/90 = 3.98$$

Thus *MAD* defects on its one-hundred-and-first move.

(c) *MAE*: This strategy maximizes expected utilities, with initial weighting at equal expectation. The actual values of this weighting are dependent upon the particular payoffs of the game. Most generally, if expectations are to be equal, then

$$p(c/C)R + p(d/C)S = p(c/D)T + p(d/D)P$$

Since *MAE* makes no *a priori* assumptions about conditional probabilities (i.e. makes no assumptions about an opponent's moves), it re-expresses this equality in terms of the probability distribution of its own moves:

$$xR + (1-x)S = (1-x)T + xP$$

where x is the probability that *MAE* co-operates on each of its first one hundred moves. Applying the payoffs of the interactive tournament,

$$3x = 5(1-x) + x$$
$$x = 5/7$$

So *MAE* plays exactly as *MEU*, except that on each of its first one hundred moves, *MAE* co-operates with a probability of 5/7, and defects with a probability of 2/7.

As in the preceeding examples, if *MAE* encounters *TQC*, the most probable event matrix after one hundred moves is:

Game 7.6 - *MAE* versus *TQC*, Event Matrix After 100 Moves

		TQC	
		c	*d*
MAE	*C*	54	18
	D	21	7

MAE's expected utilities are:

$$EU(C) = (54/72)\text{x}3 = 2.25$$
$$EU(D) = (21\text{x}5 + 7)/28 = 4$$

Thus *MAE* defects on its one-hundred-and-first move.

(d) *MAC*: This strategy maximizes expected utilities, with initial weighting at co-operation. *MAC* plays exactly as *MEU*, except that on each of its first one hundred moves, *MAC* co-operates with a probability of 9/10, and defects with a probability of 1/10.

Again, for example, if *MAC* encounters *TQC*, the most probable event matrix, after one hundred moves, is:

Game 7.7 - *MAC* versus *TQC*, Event Matrix After 100 Moves

		TQC c	*TQC* d
MAC	*C*	67	23
	D	7	3

MAC's expected utilities are:

$$EU(C) = (67/90)\times 3 = 2.23$$
$$EU(D) = (7\times 5 + 3)/10 = 3.8$$

Thus *MAC* defects on its one-hundred-and-first move.

These examples (games 7.4 - 7.7) are presented to illustrate the calculi of the maximization family, whose members employ fairly sophisticated decision rules. Although, in each of these examples, the expected utility of defection on the one-hundred-and-first move is greater than that of co-operation, one can see that the distributions of joint outcomes differ radically in the four event matrices. Since the opposing strategy has been held constant, and its play is insensitive to that of its opponents, the differences in distributions result solely from the different initial weights accorded to the members of the maximization family.

(IV) The Optimization Family

Unlike the preceeding strategic families, members of the optimization family are related neither by common program structures nor by variations on a common decision rule. The attribute shared by this family's members is their demonstrated success in previous competition(s), achieved by implementing decision rules which attempt to optimize future outcomes in light of past ones.

(a) *NYD*: This is Nydegger's strategy. It ranked third in Axelrod's first tournament, and thirty-first in the second. *NYD* is succinctly described by Axelrod:

> "The program begins with tit for tat for the first three moves, except that if it was the only one to cooperate on the first move and the only one to defect on the second move, it defects on the third move. After the third move, its choice is determined from the 3 preceeding outcomes in the following manner. Let *A* be the sum formed by counting the other's defection as 2 points and one's own as 1 point, and giving weights of 16, 4 and 1 to the preceeding three moves in chronological order. The choice can be described as defecting only when A equals 1, 6, 7, 17, 22, 23, 26, 29, 30, 31, 33, 38, 39, 45, 49, 54, 55, 58, or 61. Thus if all three preceeding moves are mutual defection, *A* = 63 and the rule cooperates. This rule was designed for use in laboratory experiments as a stooge which had a memory and appeared to be trustworthy, potentially cooperative, but not gullible."[25]

(b) *GRO*: This is Grofman's strategy. It ranked fourth in Axelrod's first tournament, and twenty-eighth in the second. *GRO* co-operates on the first move. After that, *GRO* cooperates with probability 2/7 following a dissimilar joint outcome [either (C,d) or (D,c)], and always co-operates following a similar joint outcome [either (C,c) or (D,d)].

(c) *CHA*: This is Champion's strategy. It ranked second in Axelrod's second tournament. *CHA* co-operates on the first ten moves, and plays tit-for-tat on the next fifteen moves. From move twenty-six onward, *CHA* co-operates unless <u>all</u> of the following conditions are true: the opponent defected on the previous move, the opponent's frequency of co-operation is less than 60%, and the random number between zero and one is greater than the opponent's frequency of co-operation.

(d) *ETH*: This is Eatherly's strategy. It ranked fourteenth in Axelrod's second tournament, but proved quite robust in a tournament conducted privately by Eatherly himself.[26] As Axelrod observes, *ETH* is an elegant rule.[27] *ETH* co-operates on the first move, and keeps a record of its opponent's moves. If its opponent defects, *ETH* then defects with a probability equal to the relative frequency of the opponent's defections.

---

[25] Ibid, 1980*a*, p.22.

[26] Idem., 1980*b*, p.392.

[27] Ibid.

(V) The Hybrid Family

The members of this family share the common attribute that their decision rules, as implied by the family name, are formed by the hybridizatión of other strategic pairs. This family consists of one "pure" hybrid (bred from two pure strategies), and one "mixed" hybrid (bred from two mixed strategies).

(a) *FRI*: This is Friedman's strategy, which ranked seventh in Axelrod's first tournament. *FRI* co-operates until its opponent defects, after which *FRI* defects for the rest of the game. Hence *FRI* is both nice and provocable, but completely unforgiving. Its properties in other contexts are elsewhere discussed.[28] In the context of the interactive tournament, *FRI* is interesting because its sequence of choices consists either in a string that is identical to *CCC*, or else in a string that is identical to *CCC* up to some move, and identical to *DDD* thereafter. Thus *FRI* is a pure strategic hybrid.

(b) *TES*: This is a strategy called "Tester", submitted by Gladstein. *TES* finished only forty-sixth in Axelrod's second tournament, but proved highly adept at exploiting potentially successful strategies,[29] thus compromising their would-be robustness. *TES* defects on the first move. If its opponent ever defects, *TES* "apologizes" by co-operating, and plays tit-for-tat thereafter. Until its opponent defects, *TES* defects with the maximum possible relative frequency that is less than 1/2, not counting its first defection. In other words,

> "This means that until the other player defects, *TESTER* defects on the first move, the fourth move, and every second move after that."[30]

TES appears somewhat "opportunistic" in character. On the one hand, it attempts to exploit co-operative strategies, without being excessively provocative. On the other, it attempts to appease provoc-

---

[28] E.g. see R. Harris, `Note on "optimal" policies for the Prisoner's Dilemma', *Psychological Review*, **76**, 1969, pp.373-5; & J. Friedman, `A Non-Cooperative Equilibrium for Supergames', *Review of Economic Studies*, **38**, 1971, pp.1-12.

[29] See Axelrod, 1980*b*, pp.391-3.

[30] Ibid, p.391.

able strategies, while retaining its capacity to retaliate. In sum, *TES* incorporates two mixed strategies: defection with relative frequency up to one-half, and *TFT*. Thus *TES* is a mixed strategic hybrid.

This completes a description of the twenty competing strategies in the interactive tournament, and their classification by common characteristics. It should be stressed that the familial organization employed herein is far from unique; any such collection of strategies can be grouped in a large number of ways.

One might group the strategies according to other attributes. For instance, *CCC* and *DDD* are pure; the others, mixed. But this distinction does not yield further information about the mixed group.

One might choose niceness (the property of never being the first to defect) as a criterion of distinction. *CCC, TFT, TTT, SHU, NYD, GRO, CHA, ETH,* and *FRI* are nice strategies; whereas *DDD, TAT,* and *TES*, can be termed *rude* strategies (where *rudeness* is the property of always being the first to defect). This leaves *TQD, RAN, TQC, BBE, MEU, MAD, MAE,* and *MAC* unqualified, for these strategies are neither nice nor rude. They might (after Goodman), be assigned the predicate of *nide*.

One might classify a strategy in terms of its functional calculus: a strategy either employs a probabilistic component at some stage, or utilizes a fully deterministic decision rule. *TQD, RAN,* and *TQC* are wholly probabilistic; *TFT, TTT, SHU, TAT, NYD, TES,* and *FRI* are wholly deterministic; *BBE, GRO, CHA,* and *ETH* are partly probabilistic and partly deterministic; *MEU, MAD, MAE,* and *MAC* are sequentially probabilistic and deterministic; while *DDD* and *CCC* are predetermined. But differences between the members of some of these groups seem to outweigh their respective common attribute.

Similarly, any system of classification is bound to admit of shortcomings. Given the potential variety of strategies in a collection, it seems difficult to develop a uniquely rigorous taxonomy.

But, given also the infinity of possible strategies in the iterated Prisoner's Dilemma,[31] and the concomitantly infinite ratio of strategies to choices, it does seem reasonable to associate these strategies in families which distinguish their overall program structures, or conceptual functions. Then, although a given family may contain an infinite number of members, one of three cases obtains.

In the first case, which associates identical program structures, the members are parametrically related, which is a very close relation indeed. In such families (the random family and the maximization family), members differ only by the value of a single parameter.

In the second case, which associates similar conceptual functions, the members are related either as variations on a common decision rule (the tit-for-tat family) or as multiple expressions of a common decision principle (the optimization family).

In the third case, which associates compounded strategies, the members are related by their capabilities of entering two (or conceivably more) distinct decision paths. This is the hybrid family.

A fourth case also exists, whose members belong to the meta-strategic family. In the context of the iterated Prisoner's Dilemma, an ideal meta-strategy would attempt to ascertain the identities of the other strategies against which it competes, so that it might adopt the most effective decision rule against each individual opponent. In Axelrod's second tournament, a meta-strategy (that tested for both random and highly non-co-operative opponents) ranked third overall.[32] Iterated meta-strategic considerations, while of undoubted complexity and interest, are deemed to lie beyond the scope of this enquiry.

It was decided not to mingle meta-strategies with strategies in the interactive tournament, because the development of effective

---

[31] Any strategy which employs a probabilistic component in its calculus assigns x and $(1-x)$, where $0 \leq x \leq 1$, as a probability distribution over the choices. Since x can take on an infinite number of real values, then any such strategy admits of an infinite number of possible variations.

[32] Axelrod, 1980*b*, p.395.

strategies is logically prior to that of effective meta-strategies. The latter makes use of the former. When more is learned about effective strategic choice in the iterated Prisoner's Dilemma, effective meta-strategies can be developed. Meanwhile, one can posit the existence of a family of meta-strategies, without actually describing any of its members.

In sum, while the five strategic families do not constitute a rigorous or exhaustive system of classification, they are useful as heuristic aids in a controlled experiment. A tournament whose population is of the wild variety has no express need of such groupings; as in Axelrod's experiments, the idea is to observe the competition of an unregulated population, and to see which strategies are successful in a "free-for-all" environment. In a tournament whose population is of the domesticated and captive varieties, however, these familial groupings allow the observation of the relative success of various strategic shadings, whether across the spectrum of a single parameter in a common program structure, or in terms of conceivable variations on a common functional theme.

The results of the interactive tournament are presented and discussed in several chapters to follow. For rapid reference, a glossary of strategic families and acronyms can be found in Appendix One (pages 258-260). A table of raw tournament scores is given in Appendix Two (page 261). Other pertinent data, to be discussed in the next chapter, is tabled in Appendix Three.

The main tournament, which pits twenty strategies against one another (and themselves), consists of one-hundred-and-sixty-five (fairly short) computer programs. This number is the difference between the two-hundred-and-ten programs theoretically necessary for a twenty-by-twenty competition (such that each strategy meets every other strategy plus its twin), and the forty-five programs in the nine-by-nine sub-competition of nice strategies (again, such that each strategy meets every other strategy plus its twin). Since a pair of nice strategies commences with and never deviates from mutual co-operation, their game scores are predictable. Thus forty-five programs did not have to be written (although some were written anyway, in order to verify the program logic of certain strategies.)

Combinatoric sub-tournaments, ecological scenarios, and maximization family analyses consist of fewer programs of greater length and complexity.

All programs are written in GW-BASIC, and documented samples are listed in Appendix Four (pages 271-290).

Chapter Eight
Analysis of Sub-Tournaments

Let the results of the main tournament be considered first; those of the other sub-tournaments, afterward. (One can draw an analogy with set theory, and consider the main tournament as a proper sub-tournament of itself.) The overall results are tabled as follows:

Table 8.1 - Main Tournament, Ranks and Scores

Rank (Offence)	Strategy	Points Scored	Average Score	Points Allowed	Average Allowed	Rank (Defence)
1	*MAC*	52901	2645	32054	1603	6
2	*MAE*	50058	2503	26891	1345	4
3	*SHU*	49844	2492	39699	1985	9
4	*FRI*	48823	2441	35403	1770	7
5	*CHA*	48719	2436	55874	2794	16
6	*ETH*	48484	2424	55270	2764	14
7	*MEU*	47235	2362	22607	1130	3
8	*TFT*	47210	2361	47240	2362	11
9	*TES*	46804	2340	41789	2089	10
10	*TTT*	45927	2296	54057	2703	13
11	*BBE*	42688	2134	37343	1867	8
12	*GRO*	42424	2121	60594	3030	17
13	*TQD*	41787	2089	31267	1563	5
14	*MAD*	41717	2086	15274	764	2
15	*DDD*	40024	2001	14994	750	1
16	*RAN*	40007	2000	47291	2365	12
17	*TAT*	38676	1934	55636	2782	15
18	*NYD*	37803	1890	72783	3639	19
19	*TQC*	37047	1852	62922	3146	18
20	*CCC*	36486	1824	75676	3784	20

The winner of the main tournament, by a comfortable margin, is *MAC*. *MAC* is the most co-operatively weighted member of the maximization family. Second place, by a narrower margin, goes to *MAC*'s closest relative, *MAE*. Third place is taken by the least-forgiving member of the tit-for-tat family, *SHU*. Fourth place belongs to the pure hybrid, *FRI*. Fifth and sixth places are occupied by members of the optimization family, *CHA* and *ETH*.

The upper twelve places are taken by members of four of the five competing families. No member of the probabilistic family finished higher than thirteenth place; and two of its members, *TQC* and *CCC*, finished nineteenth and twentieth respectively.

The average scores (per game) are distributed within the following limits. The length of a game is 1000 moves. Hence, the maximum achievable score in any game is 5000 points; the minimum, zero points. These extrema occur if one strategy defects 1000 times, while its opponent co-operates 1000 times. This extreme situation actually obtained in two cases: *DDD* and *TAT* both scored maximum points against *CCC*, which went scoreless against both. While these two dismal outings by *CCC* contributed to its last-place finish, neither of the two strategies that exploited *CCC* to the limit fared much better than their victim overall.

A useful bench mark is the 3000 point level, attained by both members of any strategic pair that practices mutual co-operation for an entire game. This occurred on eighty-one occasions, in all possible encounters between nice strategies (*CCC*, *TFT*, *TTT*, *SHU*, *NYD*, *GRO*, *CHA*, *ETH*, and *FRI*). Owing to the mixture of nice, rude, and nide strategies in the population, no strategy—nice or otherwise—was able to maintain an average score of 3000 points. *MAC* and *MAE*, which fared best with respective averages of 2645 and 2503 points per game, are neither nice nor rude, but nide. *SHU*, the best of the nice strategies, managed an average of 2492.

The tournament is, of course, zero-sum with respect to total points scored and total points allowed. The former is the sum of the sums of the rows of the raw score matrix (see Appendix Two); the latter, the sum of the sums of the columns. Although total points scored equal total points allowed (for all strategies combined), the distributions of points scored and points allowed are obviously quite different. Although no strategy averaged more than 2645 points scored per game, several strategies allowed, on average, in excess of three thousand points per game to be scored against them.

With respect to points allowed, *CCC*, *TQC*, *NYD* and *GRO* surpassed the 3000 point bench mark. On this side of the ledger, the accomplishment is of dubious merit. It indicates that these four strategies are the most exploitable. When the average score allowed by a strategy exceeds 3000 points per game, which is the level of constant mutual co-operation, then that strategy is being exploited by opponents which regularly defect while the strategy itself continues to co-operate. An exploitable strategy lacks the quality of provocability. In game-theoretic terms, an unprovocable strategy invites and encourages exploitation from all exploitive strategies in its environment.

Overall, in table 8.1, there does not appear to be a strong correlation between points scored and points allowed. The three strategies that allowed the fewest points (*DDD, MAD* and *MEU* respectively) ranked fifteenth, fourteenth and seventh (respectively) in points scored. Thus, extreme stinginess on defence did not conduce to copious success on offence. As well, one notes that the three most exploitable strategies (*CCC, NYD,* and *CTQ* respectively) also fared worst in points scored, though not in that order. Then again, the fourth highly exploitable strategy, *GRO,* scored enough points to finish twelfth. Thus, extreme generosity on defence did not necessarily conduce to copious failure on offence.

Correlations (and lack thereof) can be better observed in table 8.2, in which strategies are ranked not only according to their relative offensive and defensive performances, but also according to their relative differences between points scored and points allowed.

Table 8.2 illustrates that the relative differences between average points scored and allowed correlate fairly strongly with relative average points allowed.[1] But poor overall correlation obtains between relative average points scored and relative average points allowed.[2] The upper four strategies all had more points scored than allowed; the lower five, fewer. But from ranks five through fifteen inclusive, there appears to be no correlation between offensive and defensive performance.

Indeed, four of the top ten strategies (*CHA, ETH, TFT* and *TTT*) were out-scored, on average, by their opponents. But crude averages

---

[1] A linear regression program (least-squares method) computes their co-efficient of determination ("goodness of fit") as 0.952.

[2] Their co-efficient of determination is similarly computed as 0.187.

can be misleading. The relative success of these strategies lies in the precise distribution and magnitudes of their individual scores.

Table 8.2 – Offensive, Defensive, and Differential Rankings

Strategy	Average Score	Average Allowed	Difference A.S. - A.A.	Rank (Offence)	Rank (Defence)	Rank A.S.- A.A.
MAC	2645	1603	1042	1	6	5
MAE	2503	1345	1158	2	4	4
SHU	2492	1985	507	3	9	7
FRI	2441	1770	471	4	7	8
CHA	2436	2794	−358	5	16	13
ETH	2424	2764	−340	6	14	12
MEU	2362	1130	1232	7	3	3
TFT	2361	2362	−1	8	11	11
TES	2340	2089	251	9	10	10
TTT	2296	2703	−407	10	13	15
BBE	2134	1867	267	11	8	9
GRO	2121	3030	−909	12	17	17
TQD	2089	1563	526	13	5	6
MAD	2086	764	1322	14	2	1
DDD	2001	750	1251	15	1	2
RAN	2000	2365	−365	16	12	14
TAT	1934	2782	−848	17	15	16
NYD	1890	3639	−1749	18	19	19
TQC	1852	3146	−1294	19	18	18
CCC	1824	3784	−1960	20	20	20

For example, compare the records of *FRI* and *CHA*, which ranked fourth and fifth respectively. Offensively, *FRI* out-pointed *CHA* by a mere 5 points per game, on average; while defensively, *FRI* allowed an average of 1024 fewer points per game. Of its twenty tournament games, *FRI* won nine, lost one, and drew ten. *CHA*, on the other hand, won none of its games, lost eleven, and drew nine. Had these tournament game results been applied to a meta-tournament, with meta-payoffs (assessed by comparing game scores) of two points for a win, one point for a draw, and zero points for a loss, then *FRI*'s meta-tournament score would be 28 points; *CHA*'s, 9 points. These two meta-scores are clearly not in the same proximity as their strategies' offensive ranks.

After this procedure is applied to all tournament games, the strategies can be ranked according to their meta-tournament points:

Table 8.3 - Meta-Tournament Rankings

Rank	Strategy	Games	Wins	Losses	Draws	Points
1	*DDD*	20	19	0	1	39
2	*MAD*	20	17	2	1	35
3	*MEU*	20	15	3	2	32
3	*BBE*	20	15	3	2	32
5	*FRI*	20	9	1	10	28
6	*MAE*	20	13	6	1	27
7	*MAC*	20	11	8	1	23
8	*TQD*	20	10	8	2	22
8	*SHU*	20	6	4	10	22
10	*RAN*	20	9	9	2	20
11	*TAT*	20	8	10	2	18
12	*TES*	20	5	8	7	17
13	*TQC*	20	6	13	1	13
13	*TFT*	20	0	7	13	13
15	*GRO*	20	1	9	10	12
16	*NYD*	20	0	10	10	10
16	*ETH*	20	0	10	10	10
18	*CHA*	20	0	11	9	9
18	*TTT*	20	0	11	9	9
18	*CCC*	20	0	11	9	9

Total participations/wins/losses/draws : 400/144/144/112

Table 8.3 reveals a different dimension of strategic interaction. With respect to the prior discussion, one finds indeed that *FRI* and *CHA*, which ranked fourth and fifth in the tournament, now rank fifth and eighteenth in the meta-tournament. *DDD*, which ranked fifteenth, now ranks first. In fact, the upper three places in the meta-tournament are occupied by the strategies with the three best defensive records in the tournament, in that order. These strategies previously ranked fifteenth, fourteenth and seventh.

The maximization family still fares relatively well, occupying second, third, sixth and seventh places, but their members' order of placement is now reversed. Other curious results appear. For instance, one sees that *SHU*, which ranked third in the tournament, won only six of its twenty tournament games. And *TFT*, which placed eighth in the tournament, thus fared better than twelve other strategies without winning even a single tournament game. (It achieved the highest number of draws, thirteen, which is attributable to its mirror-image play.) *TFT*'s untrustworthy relative, *BBE*, jumped from eleventh place in the tournament to a tie for third in the meta-tournament. The only strategy to maintain the same rank in both the tournament and the meta-tournament is *CCC*, which finished last with unenviable consistency.

One may well ask: how significant are these meta-tournament results? In what way, if any, can they be interpreted relative to a given strategy's success, or lack thereof, in the normal tournament?

From one perspective, it might appear that rankings based upon meta-tournament points provide a more accurate reflection of overall performance than rankings based solely upon offensive tournament points. In game-theoretic terms, the points scored in a tournament game are the number of utiles accrued by the given strategy, or, equivalently, the net utility to the player who employs the given strategy. By the same token, the points allowed in a tournament game are the number of utiles accrued by the opposing strategy. One must now enquire whether a symmetric equivalence obtains. Can the points allowed by a given strategy be similarly regarded, as the net dis-utility to the player who employs the given strategy? In other words, are defensive considerations of any importance?

In the case of a lop-sided game score, such as 5000 to 0 for *DDD* versus *CCC*, one might be inclined to regard *CCC*'s points allowed as a definite dis-utility to the purely co-operative player. But in the case of a mutually co-operative game score, such as 3000 to 3000 for *TFT* versus *CCC*, one is inclined to regard the outcome as mutually beneficial. Since 3000 points is far more than any strategy averaged throughout the tournament, and is consistently achievable only by constant mutual co-operation, it would seem contradictory for player *A* to celebrate the utility of 3000 points scored against player *B*, while bemoaning the dis-utility of 3000 points allowed to player *B*.

Nevertheless, the argument against regarding player *B*'s share of a mutually high score as a dis-utility to player *A* may be inadmissible in the context of the interactive tournament. By definition, this game-theoretic tournament is concerned with strategic interplay, not with hypothetical players' motives. One seeks a fair way in which to assess the overall performance of the strategies, without presuming upon the psychology of the players who adopt them.

A ranking scheme based solely upon points scored does seem incomplete, for it disregards the significance of points allowed. On the other hand, a ranking scheme based upon meta-tournament points is clearly inappropriate, for the simple reason that strategies are both defined and designed to compete in a tournament, not a meta-tournament.

It is possible to reconcile the apparent incompleteness of a purely offensive ranking scheme for the tournament, as well as the interesting result but questionable applicability of a meta-tournament ranking scheme, by translating both the tournament and the meta-tournament into an allegorical social context.

Suppose twenty players are to compete in an apple-picking contest. The contest format is as follows. All possible player-pairs (including "clones") are to be formed. One pair at a time is sent into the orchard, each player in the pair carrying an identical empty basket (whose capacity is five thousand apples). Each pair is allowed an identical time-period during which its players may accumulate apples in their respective baskets. At the expiration of its time-period, the pair exits the orchard, the players empty their baskets, and their respective numbers of apples are counted and recorded. The next pair is then sent into the orchard. It is understood that, after all possible pairs will have competed, the player who accumulates the greatest number of apples wins the contest.

The players in a given pair are not prohibited from interfering with each other's picking. A player may adopt one of a range of strategies, from attempting to maximize his own pickings while ignoring the other player, to attempting to minimize the other player's pickings while possibly diminishing his own. A "nice" player is never the first to interfere with the other player's picking; a "rude" player, always the first to do so. A "provocable" player is one who responds to interference with interference. A "forgiving" player is one who, after having been provoked to interference, also desists from interfering after the other player desists. An "exploitable" player is not provocable. An "exploitive" player interferes with an exploitable one.

Two nice players, when paired, are able to pick about 3000 apples each during the allotted time. An exploitive player, when paired with an exploitable one, is able to pilfer the exploitable player's pickings, and thus fills his basket by partly emptying the other's. At the extreme of this situation, the highly exploitive player emerges from the orchard with 5000 apples; the highly exploitable player, with none. By contrast, when two highly exploitive players are paired, their mutual interference limits each player's pickings to about 1000 apples.

Several other types of strategies, which reflect different mixtures of attributes, are adopted by other players in the competing population. It is understood that every player chooses his strategy prior to the commencement of play, and that no player alters his strategy during the course of the contest.

Since the winner of this apple-picking contest is, by definition, the player who accumulates the most apples, then the most successful strategy in the contest is, *ceteris paribus*, that strategy adopted by the winning player.

Now let a second apple-picking contest be conducted, which is identical to the first in all aspects of play, but whose manner of determining the winner differs. After each pair of players emerges from the orchard, their baskets are emptied and their respective apples are counted, as before. But in this contest, the precise apple-count is not recorded. Instead, for every pair, the player with the greater apple-count receives two oranges; the player with the lesser apple-count, no oranges. If both players in the pair have the same apple-count, they each receive one orange. The winner of this contest is the player who accumulates the greatest number of oranges.

Now, to differentiate between the results of the tournament ranking scheme (table 8.1) and the meta-tournament ranking scheme (table 8.3), one need only ask the allegorical question: are the strategies competing for apples, or for oranges? In the first apple-picking contest, the most successful strategy is that which yields the greatest accumulation of apples to the player who adopts it. In the second apple-picking contest, the most successful strategy is that which yields the greatest accumulation of oranges to the player who adopts it, by means of yielding the smallest accumulations of apples to the players who compete against it.

The first contest is won by a strategy that seeks to maximize its expected gains, without necessarily minimizing the expected gains of its competitors. In other words, the first apple-picking contest is won by the player who picks the most apples. This seems eminently reasonable. The second contest is won by a strategy that seeks only to minimize the expected gains of its competitors. In other words, the second apple-picking contest is won by the player against whom other players pick the fewest apples. But on the whole, the winner of the second contest accumulates fewer apples than the majority of the other players. Thus, although this winner accumulates the most oranges, he fares relatively poorly at accumulating apples. It does not seem reasonable that an apple-picking contest be won by a player who is a poorer picker than a majority of the other competitors.

The first contest is an obvious allegory of the interactive tournament; the second, of the associated meta-tournament. The allegorical social context vindicates the tournament's offensive ranking scheme, and illustrates the inappropriateness of the meta-tournament ranking scheme.

There is also a compelling game-theoretic reason why it must do so. By definition, the Prisoner's Dilemma is a non-zero-sum game. Both the interactive tournament, and the first apple-picking contest, are Prisoner's Dilemmas. But the meta-tournament and the second apple-picking contest are both constant-sum games, whose constant sums are two points, and two oranges, respectively. Hence, contrary to appearances, neither example is a Prisoner's Dilemma, and the strategic considerations applicable to Prisoner's Dilemmas do not carry over to these examples.

Why, then, were these examples presented? Because they delineate a crucial strategic development in the conflicts thus far examined; namely, the potential failure of the dominance strategy in larger populations of strategies.

The dominance strategy, *DDD*, is simply the dominance principle iterated over the entire course of a game. Part Two of this enquiry was devoted to an exposition of the unresolved conflict between dominance and maximization of expected utility, observed in the static, two-person Prisoner's Dilemma. Early in Part Three, it was noted (both theoretically and empirically) that the conflict persists in iterated two-person games, with a long-term tendency toward one of two joint similar outcomes, either (C, c) or (D, d). But now one has an indication that dominance reasoning breaks down in the *N*-pair, two-person Prisoner's Dilemma.

Reconsider the first apple-picking contest, with a competing population of only two players. The rules remain the same, with one amendment: if both players accumulate 3000 apples, they both win; if both accumulate 1000 apples, they both lose. If player *X* adopts the apple-picking strategic equivalent of *DDD*, then player *Y* cannot possibly pick more apples than *X*, no matter what strategy *Y* adopts. If both player reason similarly, both will adopt *DDD*, and both will lose.

But if a third player, *Z*, enters the competition, the strategic balance of power shifts away from *DDD*. Two nice players will both fare better, overall, than a single rude exploitive player, providing that the nice players are both provocable. No player, of course, can predict what strategies the other two will adopt. Let player *X* contemplate adopting a rude and exploitive strategy. Then *X* knows that if both *Y* and *Z* are rude and exploitive, all will lose; if *Y* is rude and exploitive while *Z* is nice, *Z* will lose; if both *Y* and *Z* are nice and provocable, *X* will lose; if both *Y* and *Z* are nide, *X* will lose.

In general, a rude and exploitive player's *a priori* chances of winning diminish as the strategic population grows and varies. The pure dominance strategy cannot guarantee that a competitor will not win, because the strategy cannot dominate all the competitor's interactions. A strategy that wins oranges at apple-picking contests wins a chimerical victory. Thus player *X* would be well-advised to contemplate the adoption of a strategy that is not rude and exploitive. What strategy should he adopt?

Leaving the apple-orchard, and returning to the interactive tournament, the maximization family members *MAC*, *MAE* and *MEU* all fared better than *DDD*. *MAC* and *MAE*, in fact, fared better than all other strategies. This result marks a turning point in the strategic conflict: in the interactive tournament involving twenty strategies, pure defection is relegated to a position of relative obscurity, whereas co-operative members of the maximization family appear to be ascendant.

But, as noted at the beginning of this chapter, the results in table 8.1 represent only one element of a large set of possible sub-tournaments. Thus far in the chapter, two principal results are established: first, that a ranking scheme based upon (offensive) points scored is appropriate for this type of tournament; second, given said ranking scheme, *MAC* is the most successful strategy in the main tournament involving twenty strategies. Next, one must ask: how robust is *MAC* in the interactive environment? This question can be answered by examining the results of all possible sub-tournaments of the main tournament.

Briefly, this approach can be contrasted with Axelrod's. In both of Axelrod's tournaments—as in this interactive tournament—the criterion of a strategy's success is the number of points it scores. Axelrod does not discuss the relative merits of different ranking schemes. He simply assumes that strategies should be ranked in ascending order of total points scored, and his discussions of strategic success and robustness are predicated upon that assumption. This chapter's comparison of ranking schemes supports Axelrod's assumption; moreover, such support emanates from a game-theoretic perspective. Hence Axelrod's tournaments and the interactive tournament employ the same method for evaluating a given strategy's success. But, with respect to robustness, the methodologies differ.

TFT won Axelrod's first tournament; however, recall that Axelrod describes three other stategies which would have won if submitted.[3] Thus *TFT* may not have been the most robust strategy in that environment. It certainly had potential rivals. Axelrod's second

---

[3] Axelrod, 1980*a*, p.20.

tournament, also won by *TFT*, had a much larger competing population, and here Axelrod uses a rather elegant method to evaluate *TFT*'s robustness. Owing to the unwieldiness of the second tournament's matrix of raw scores, which contains 63x63 or 3969 entries, Axelrod uses step-wise regression to express the overall performance of any strategy in terms of its performance against just five other strategies, which he calls "representatives":

> "These five rules [i.e. decision rules, or strategies] can be thought of as *representatives* of the full set in the sense that the scores a given rule gets with them can be used to predict the average score the rule gets over the full set."[4]

Axelrod is able to use these representatives to assess *TFT*'s robustness, in the following way. Each of these five strategies can be thought of as representing a "constituency" of strategies. A sixth constituency is formed by the unrepresented "residuals". Axelrod then conducts six hypothetical tournaments, in each of which one of the six constituencies, in turn, is enlarged to five times its original size by weighting its representative accordingly.[5] Thus *TFT* must now compete in populations formed by distending six different segments of the original strategic distribution. *TFT* won five of these six hypothetical tournaments. Based on this result, Axelrod pronounces *TFT* robust.[6]

This enquiry adopts a different methodology, namely that of combinatoric analysis. Given a set of n elements, one can combine r elements from that set in $n!/r!(n-r)!$ different ways. This operation is commonly referred to as "n choose r", or $C(n,r)$. In the interactive tournament, the number of strategies (or elements) is twenty. The twenty strategies can be combined in just one way, since $C(20,20) = 20!/20!0! = 1$. (By definition, the factorial of zero is unity.) The results of this single sub-tournament, for $r = 20$, appear in table 8.1. But r can assume a range of theoretical values, from

[4] Idem., 1980*b*, p.386. The co-efficient of correlation between the scores predicted by the step-wise regression and the actual tournament scores is a respectable .979.

[5] Ibid, pp.396-398.

[6] Ibid.

$1 \leq r \leq n$. In practice, at least two strategies are required for a competition to take place, so the value $r = 1$ is not applicable here.

Note also that, if $a + b = n$, then $C(n,a) = C(n,b)$.[7] For example, the number of sub-tournaments that can be conducted with different combinations of eighteen strategies is the same as the number that can be conducted with different combinations of two strategies (since 2 + 18 = 20). This number is 190. But if all 190 combinations of eighteen strategies are formed, each individual strategy appears in 171 of these combinations; whereas if all 190 combinations of two strategies are formed, each individual strategy appears in only 19 of these combinations. In general, for $C(n,r)$, each individual element appears in

$$(r/n)\times[C(n,r)], \text{ or } (n-1)!/(r-1)!(n-r)!$$

different combinations.

For all applicable values of r, the numbers of possible sub-tournaments and the numbers of appearances of each strategy are given in table 8.4, overleaf.

The total number of possible sub-tournaments that can be conducted, from all combinations of strategies for each applicable value of r, is 616,666. The total number of sub-tournaments in which each strategy competes is 524,287. Thus each strategy competes in more than half a million different sub-tournaments, against all possible combinations of the other strategies in the population.

In order to evaluate the results of this large number of sub-tournaments, the following procedure is adopted. All sub-tournament combinations involving r strategies are conducted, one at a time, for each value of r.

---

[7] $C(n,a) = n!/a!(n-a!) = n!/a!b! = n!/(n-b)!b! = C(n,b)$

Table 8.4 - Sub-Tournaments Resulting From Combinations of r Strategies

Value of r (Number of strategies competing in sub-tournament)	Combinatoric formula $C(n,r) = n!/r!(n-r)!$	Number of different sub-tournaments	Number of sub-tournaments in which each stratgy appears $(r/n)\times[C(n,r)]$
20	20!/20!0!	1	1
19	20!/19!1!	20	19
18 or 2	20!/18!2!	190	171 or 19
17 or 3	20!/17!3!	1140	969 or 171
16 or 4	20!/16!4!	4845	3876 or 969
15 or 5	20!/15!5!	15504	11628 or 3876
14 or 6	20!/14!6!	38760	27132 or 11628
13 or 7	20!/13!7!	77520	50388 or 27132
12 or 6	20!/12!8!	125970	75582 or 50388
11 or 9	20!/11!9!	167960	92378 or 75582
10	20!/10!10!	184756	92378

Let r have a given value. Suppose strategy S_j ranks first in the first sub-tournament conducted (for that r). Then strategy S_j fared better than (r-1) other strategies in that particular combination. Hence, strategy S_j is awarded (r-1) points. Similarly, if strategy S_j ranks second, then strategy S_j fared better than (r-2) other strategies in that particular combination. Hence, strategy S_j is awarded (r-2) points. This procedure is applied to all strategies in that sub-tournament combination. In other words, each strategy in that particular combination is awarded a number of points, equal to the number of strategies it betters. Suppose strategy S_k ranks last. Since S_k betters no strategies, it is awarded no points.

The second sub-tournament combination involving r strategies (for the same value of r) is then tried. Once again, points are awarded to each strategy appearing in this combination, according to the number of other strategies it betters, from (r-1) points for the first-ranking strategy to zero points for the last-ranking strategy.

When a given sub-tournament combination consists of nice strategies only, they all achieve identical scores. In such cases, when r nice strategies draw, they each receive (r-1) points. And most generally, if any sub-tournament involving r strategies sees p of these strategies tied for q^{th} place, then each of the p strategies receives $(r-q)$ points.

After $C(20,r)$ different combinations are exhausted for the given r, each strategy will have appeared in $19!/(r-1)!(20-r)!$ different sub-tournaments. In order to determine which strategy is most successful for this value of r, the *efficiency* of each strategy's performance is calculated according to the following formula. If a strategy wins each and every sub-tournament for this value of r, its point-awards would total

$$(r-1) \times 19!/(r-1)!(20-r)!$$
$$\text{or } 19!/(r-2)!(20-r)!$$

This is the maximum number of points awardable to a strategy, for any given value of r. The relative efficiency of a strategy, then, is simply its actual point-award total divided by this maximum number. (The relative efficiency is then multiplied by one hundred for expression as an efficiency percentage.)

A specific example of the entire procedure is tabled overleaf (table 8.5), for the twenty different sub-tournaments conducted by forming all possible combinations of nineteen strategies.

For $r = 19$, there are twenty possible sub-tournaments. Each strategy appears in nineteen sub-tournaments, and can be awarded a maximum of 18 points in each appearance. Hence, the ideal point-award total is 19x18 = 342 total points.

Since *MAC* ranked first in all its appearances, it actually achieved this ideal; hence, its efficiency is (342/342)x100, or 100%, in sub-tournaments involving nineteen strategies.

MAE ranked second in twelve sub-tournaments; third, in three sub-tournaments; fourth, in two sub-tournaments; sixth, in two sub-tournaments. Hence, *MAE* bettered seventeen opponents on twelve occasions; sixteen, on three occasions; fifteen, on two occasions; and thirteen, on two occasions. This tally accounts for *MAE*'s nineteen appearances. *MAE*'s relative efficiency is therefore

$$[(12\times17) + (3\times16) + (2\times15) + (2\times13)]/342$$
$$= 308/342 = .901$$

Thus *MAE* is 90.1% efficient in sub-tournaments involving nineteen strategies.

Table 8.5 - 19 Appearances in 20 Sub-Tournaments Involving 19 Strategies

Rank:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	EFF%
MAC	19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100
MAE	0	12	3	2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	90.1
SHU	1	6	9	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	89.8
FRI	0	0	6	8	1	3	0	0	1	0	0	0	0	0	0	0	0	0	0	81.6
CHA	0	2	0	6	6	4	1	0	0	0	0	0	0	0	0	0	0	0	0	79.5
ETH	0	0	1	1	8	4	4	1	0	0	0	0	0	0	0	0	0	0	0	74.3
MEU	0	0	1	2	1	5	2	3	1	4	0	0	0	0	0	0	0	0	0	66.7
TFT	0	0	0	0	1	2	10	5	1	0	0	0	0	0	0	0	0	0	0	65.8
TES	0	0	0	0	1	0	3	8	7	0	0	0	0	0	0	0	0	0	0	60.8
TTT	0	0	0	0	0	0	0	3	10	6	0	0	0	0	0	0	0	0	0	54.7
BBE	0	0	0	0	0	0	0	0	0	4	8	5	2	0	0	0	0	0	0	43.0
GRO	0	0	0	0	0	0	0	0	0	3	5	5	5	1	0	0	0	0	0	40.1
MAD	0	0	0	0	0	0	0	0	0	3	3	4	3	5	0	1	0	0	0	37.4
TQD	0	0	0	0	0	0	0	0	0	0	4	6	9	0	0	0	0	0	0	36.4
RAN	0	0	0	0	0	0	0	0	0	0	0	0	1	7	10	1	0	0	0	24.6
DDD	0	0	0	0	0	0	0	0	0	0	0	0	0	6	10	2	0	0	1	22.2
TAT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	1	2	0	15.2
NYD	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	16	2	0	11.4
TQC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	15	2	5.6
CCC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	17	0.9

At the bottom of the list, *CCC* ranked seventeenth in one sub-tournament; eighteenth, in one sub-tournament; nineteenth and last, in seventeen sub-tournaments. Hence *CCC* bettered two opponents on one occasion, and one opponent on another occasion. Its relative efficiency is therefore 3/342, or 0.009. Thus CCC is only 0.9% efficient in sub-tournaments involving nineteen strategies.

In table 8.5, notice that the non-zero entries tend to be clustered along the main diagonal of the matrix. This general lack of dispersion throughout each row indicates that a given strategy tends to achieve the same rank, or else to perform within a narrow range of ranks, in each of its appearances. One extreme case is *MAC,* which ranked first in the nineteen sub-tournaments in which it appeared. At the other extreme is *MEU,* whose rankings are distributed across eight consecutive columns. In its nineteen appearances, *MEU* attained a range of ranks between third and tenth places inclusive.

The average rank-dispersion in table 8.5 (that is, the average number of different ranks attained by a given strategy), is 4.6 of a possible 19 ranks per strategy. Overall, the actual rank attainments are dispersed over less than 25% of the field of possible rank attainments. This denotes an expected result; namely, that in the twenty sub-tournaments involving different combinations of nineteen strategies, the absence of any particular strategy from a given sub-tournament does not drastically influence the relative success of the remaining competitors. In other words, slight variations in the constitution of a large population do not exert a pronounced effect on the bulk of its members' performances.

By the same token, one expects an increased dispersion of rankings as the number of strategies per sub-tournament diminishes (and the corresponding number of possible combinations increases). Consider the distribution of rankings at the next combinatoric stage, in table 8.6 (overleaf).

Although *MAC* still dominates the standings, it too begins to show a dispersion of rank. The average rank-dispersion in table 8.6 is now 6.75 of a possible 18 ranks per strategy, while actual rank attainments are dispersed over 37.5% of the field of possible rank attainments. The absence of one additional strategy per sub-tournament, and the increased number of combinations resulting therefrom, give rise to a corresponding increase in variations of strategic performance.

Table 8.6 - 171 Appearances in 190 Sub-Tournaments Involving 18 Strategies

Rank:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	EFF%
MAC	162	7	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	99.6
SHU	16	56	51	28	14	4	1	1	0	0	0	0	0	0	0	0	0	0	88.6
MAE	8	91	23	18	9	10	7	4	1	0	0	0	0	0	0	0	0	0	87.8
CHA	4	18	21	46	42	33	6	1	0	0	0	0	0	0	0	0	0	0	80.3
FRI	0	13	58	36	26	17	5	7	9	0	0	0	0	0	0	0	0	0	80.2
ETH	0	5	15	29	54	36	26	6	0	0	0	0	0	0	0	0	0	0	75.4
NEU	0	0	19	23	18	21	23	17	21	27	2	0	0	0	0	0	0	0	66.6
TFT	0	0	0	5	15	46	66	32	7	0	0	0	0	0	0	0	0	0	66.3
TES	0	0	1	5	12	20	41	61	31	0	0	0	0	0	0	0	0	0	62.6
TTT	0	0	0	0	0	4	14	58	77	18	0	0	0	0	0	0	0	0	55.7
BBE	0	0	0	0	0	0	0	3	7	44	41	51	22	2	0	1	0	0	39.8
GRO	0	0	0	0	0	0	0	0	4	48	63	22	27	4	3	0	0	0	39.7
MAD	0	0	0	0	0	0	0	0	28	33	24	30	34	10	3	3	6	0	37.7
TQD	0	0	0	0	0	0	0	0	5	17	49	55	36	9	0	0	0	0	36.8
RAN	0	0	0	0	0	0	0	0	0	0	0	10	30	102	29	0	0	0	24.3
DDD	0	0	0	0	0	0	0	0	0	3	9	21	33	31	31	24	9	10	22.4
TAT	0	0	0	0	0	0	0	0	0	0	0	0	2	27	91	23	21	7	15.8
NYD	0	0	0	0	0	0	0	0	0	0	2	1	6	5	25	96	34	2	12.8
TQC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	34	62	73	4.7
CCC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	9	58	98	3.2

A final example of this tendency, for the 184,756 combinations of ten strategies, is given in table 8.7 (overleaf).

At this combinatoric level, each strategy appears in 92,378 sub-tournaments, and is absent from a like number. The average rank-dispersion in table 8.7 is now 9.6 of a possible 10 ranks per strategy, while actual rank attainments are dispersed over 96% of the field of possible rank attainments. The large number of combinations of ten strategies allows great variation in relative performance. With the exception of *NYD*, every strategy is able to win at least one sub-tournament; most, many more.

Once again, *MAC* proves most successful, winning more than half the sub-tournaments in which it appears. Its rankings, however, are now dispersed over nine of ten places; but *MAC* ranks ninth (its worst performance) in only seven of 92,378 sub-tournaments. In fact, four of the upper six strategies (*MAC*, *SHU*, *CHA*, and *ETH*) never finish last in any of the sub-tournaments in which they appear. By contrast, *CCC* maintains a secure hold on last place: it is the only strategy to finish tenth in more than 50% of its appearances.

Table 8.7 - 92,378 Appearances in 184,756 Sub-Tournaments Involving 10 Strategies

Rank:	1	2	3	4	5	6	7	8	9	10	EFF%
MAC	48981	18468	9999	5672	5111	3022	949	169	7	0	88.2
MAE	35411	21570	10181	7020	6460	6017	3858	1493	339	29	81.3
SHU	24361	25207	15892	12170	9008	4307	1248	165	20	0	80.8
FRI	25997	18180	14466	12839	9383	6998	3150	1101	216	48	77.5
CHA	17082	16832	21127	19435	13000	4221	637	43	1	0	76.6
ETH	11538	18032	21656	21768	13837	4691	767	89	0	0	74.7
TES	6724	15254	16875	18740	18540	12157	3396	607	78	7	68.1
TFT	3231	10740	22751	25645	19456	8048	2061	411	34	1	67.9
NEU	7461	22688	16227	10020	9994	11019	8339	4572	1758	300	66.4
TTT	1775	6151	11125	16241	22881	21515	9204	2881	587	18	57.1
MAD	932	5385	8292	7327	8869	13181	13269	11694	13396	10033	39.3
GRO	222	1071	2353	5227	11132	20304	26684	19762	5587	36	38.5
TQD	370	1985	4270	6302	9458	16892	22413	17352	10310	3026	37.8
BBE	17	435	1447	4333	11066	18122	19621	16137	11785	9415	32.5
DDD	52	1311	5040	6514	6817	9984	12785	12779	12005	25091	28.3
RAN	171	406	904	1988	3735	9560	24650	29770	16869	4325	27.7
TAT	455	979	1884	2581	3407	6543	10551	20650	24282	21046	21.8
NYD	0	13	110	532	1637	4132	9526	18188	33868	24372	15.3
TQC	28	81	180	371	680	1827	5777	16309	32065	35060	11.7
CCC	1	0	0	27	260	2216	5863	10585	21597	51829	8.5

The results of all combinations of all sub-tournament groups (from two to twenty competitors) are tabled in Appendix Three. It can be seen that *MAC* dominates all group sizes from twenty down to seven competitors, inclusive. *MAE* dominates groups of six and five competitors, while *FRI* prevails in groups of four and three. In the 190 sub-tournaments involving two strategies, wherein each strategy makes 19 appearances, *FRI*, *SHU* and *TFT* are most efficient.

These results can be summarized as follows. A total of 616,666 different sub-tournaments have been conducted, by taking all combinations of the population of competing strategies, in all group sizes from twenty to two competitors. In all, each strategy appears in 524,287 sub-tournaments (the sum of its appearances in each group size), and the efficiency of each strategy's performance is tabled for each group. A relative measure of robustness can now be made by calculating each strategy's overall efficiency across the entire range of group sizes.

A strategy's overall efficiency is simply the weighted average of its relative efficiencies in all groups. Suppose a given strategy appears in N_i sub-tournaments for all combinations $C(i,20)$ of i competitors, and attains a relative efficiency of E_i in that group. Then the given strategy's overall efficiency, E_0 is found by

$$E_0 = \sum_{i=2}^{20} (E_i)(N_i) \div \sum_{i=2}^{20} N_i$$

(where the denominator = 524,287)

The results of this calculation, for all strategies, appear in table 8.8.

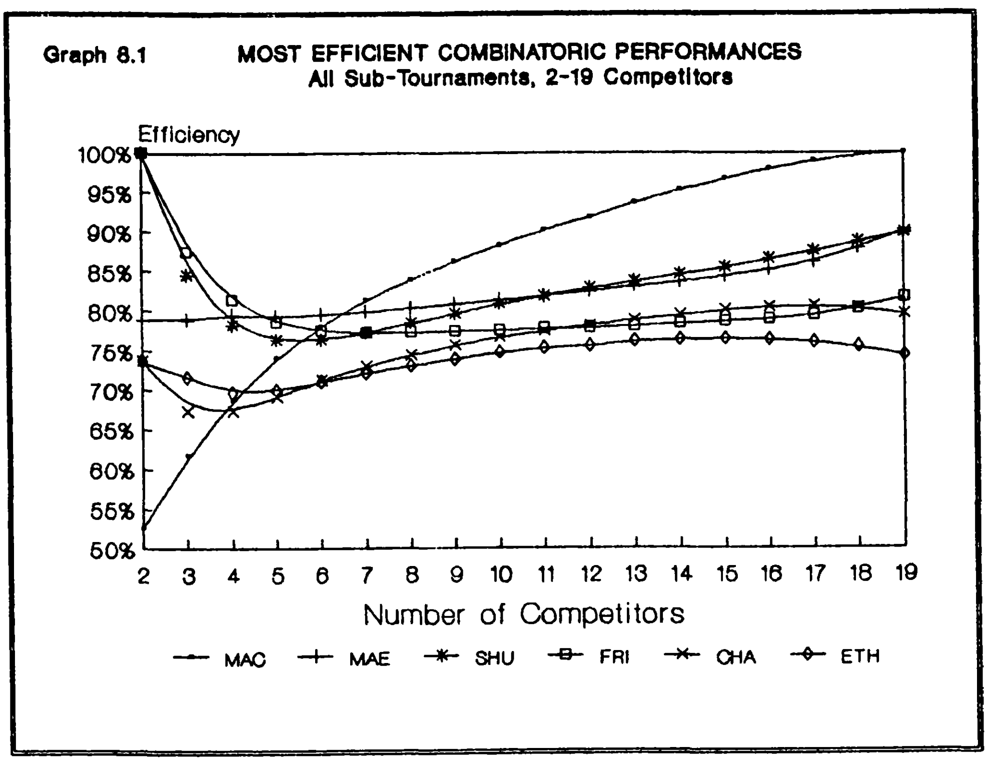
Table 8.8 - Overall Efficiencies: 524,287 Appearances in 616,666 Sub-Tournaments

Group Size:	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	E_0
MAC	52.6	61.7	68.7	74.0	78.0	81.3	83.9	86.2	88.2	90.1	91.8	93.6	95.2	96.6	97.9	98.9	99.6	100	100	88.8
MAE	78.9	78.9	79.5	79.3	79.5	79.8	80.3	80.8	81.3	81.8	82.4	83.0	83.6	84.3	85.1	86.1	87.8	90.1	90.0	81.6
SHU	100	84.5	78.2	76.3	76.3	77.2	78.4	79.6	80.8	81.8	82.8	83.6	84.5	85.4	86.4	87.4	88.6	89.8	85.0	81.2
FRI	100	87.4	81.3	78.5	77.5	77.2	77.3	77.4	77.5	77.7	77.8	78.0	78.3	78.6	78.9	79.4	80.2	81.6	80.0	77.7
CHA	73.7	67.3	67.3	69.1	71.2	73.0	74.4	75.6	76.6	77.4	78.1	78.8	79.4	80.0	80.4	80.5	80.3	79.5	75.0	76.7
ETH	73.7	71.6	69.6	70.0	71.0	72.1	73.1	73.9	74.7	75.3	75.7	76.1	76.3	76.4	76.3	76.0	75.4	74.3	70.0	74.7
TFT	100	80.4	74.3	70.7	68.9	68.2	67.9	67.9	67.9	68.0	67.9	67.8	67.5	67.2	66.8	66.5	66.3	65.8	60.0	67.9
TES	68.4	74.6	73.3	71.2	70.0	69.4	68.9	68.5	68.1	67.7	67.3	66.8	66.2	65.7	64.9	64.0	62.6	60.8	55.0	67.8
NEU	63.2	68.7	68.1	67.5	67.1	66.7	66.5	66.4	66.4	66.5	66.5	66.6	66.7	66.7	66.7	66.6	66.6	66.7	65.0	66.5
TTT	78.9	64.9	59.4	57.7	57.0	57.0	57.1	57.1	57.1	57.1	57.0	56.9	56.8	56.6	56.5	56.2	55.7	54.7	50.0	57.0
MAD	42.1	42.4	41.1	41.8	40.5	40.4	40.1	39.3	39.3	38.7	38.2	38.2	37.4	37.3	37.1	36.3	37.7	36.5	30.0	38.9
GRO	63.2	49.1	42.8	39.9	40.8	40.2	39.2	39.0	38.5	38.2	38.0	37.5	37.2	37.4	37.4	38.5	39.7	40.1	40.0	38.4
TQD	36.8	41.8	40.6	39.9	39.4	39.0	38.5	38.0	37.8	37.4	37.2	37.0	36.8	36.8	36.9	36.8	36.8	37.4	35.0	37.7
BBE	5.3	13.5	20.4	24.9	27.9	29.6	30.9	31.9	32.5	33.3	34.2	35.1	36.1	37.1	28.0	38.7	39.8	43.0	45.0	32.9
DDD	42.1	36.8	36.8	35.1	32.8	31.8	30.5	29.3	28.3	27.5	26.5	25.7	25.3	24.0	23.5	23.7	22.4	22.2	25.0	28.1
RAN	42.1	37.1	35.2	33.6	32.1	30.6	29.5	28.5	27.7	27.0	26.2	25.7	25.4	24.9	25.0	25.3	24.3	24.6	20.0	27.6
TAT	42.1	31.6	30.6	29.0	27.2	25.6	24.3	23.0	21.8	20.8	19.8	18.8	18.1	17.5	16.7	15.7	15.8	15.2	15.0	21.5
NYD	52.6	30.4	21.6	18.6	17.2	16.5	16.2	15.7	15.3	15.0	14.6	14.4	14.1	13.8	13.4	12.9	12.8	11.4	10.0	15.2
TQC	31.6	28.4	21.8	19.3	16.7	14.4	13.5	12.8	11.7	11.1	10.6	9.6	8.7	8.0	6.8	6.1	4.7	5.6	5.0	11.6
CCC	47.4	25.7	18.0	13.4	11.9	10.9	9.8	9.2	8.5	7.9	7.4	6.9	6.5	5.8	5.3	4.1	3.2	0.9	0.0	8.4

The overall efficiencies in table 8.8 can be fairly said to represent the relative robustness of the strategies. *MAC* is clearly the most robust strategy in the population of the interactive tournament. *MAC*'s "sibling" strategy, *MAE*, is the next most robust, followed closely by *SHU*, the least forgiving member of the tit-for-tat family.

Comparing the standings in tables 8.8 (overall efficiencies) and 8.1 (main tournament results), it seems significant that the upper six and lower six strategies maintain identical ranks in both cases. Given that table 8.1 is the result of the unique sub-tournament involving the single combination of twenty strategies, and that table 8.8 is the weighted result of 616,666 different sub-tournaments involving all combinations of all groups, then the upper and lower third of the compiled standings of more than six hundred thousand sub-tournaments are "determined", as it were, by the unique outcome featuring the largest group. It is a matter of speculation whether such determination would obtain anew, and to what degree, in different initial strategic populations.

One concludes the combinatoric analysis of sub-tournaments with a graph that illustrates how the efficiencies of the upper six strategies change as a function of group size:



MAC and *MAE* are the sole top strategies whose efficiencies increase uniformly with the size of the competing group. *SHU* and *FRI*, which rank third and fourth respectively, do so because their efficiencies increase after falling off sharply in smaller groups. *CHA* and *ETH*, which rank fifth and sixth respectively, experience a less sharp early decrease in smaller groups, a gradual increase in mid-sized groups, and a gradual falling-off in larger groups.

MAC, whose efficiency is lowest among the six top strategies at group sizes of two and three, experiences a much sharper rate of increase than *MAE*. Moreover, *MAC* continues to increase more sharply than *MAE*, *SHU* and *FRI*, even after assuming the lead at the group size of seven. The larger the competing population, the better *MAC* performs, relative both to its own increasing efficiency, and to the efficiencies of its competitiors.

That *MAC* and *MAE* are the most robust strategies in the population of the interactive tournament, is a matter that requires further investigation in the context of this enquiry. *MAC* and *MAE* are the two most closely-related, and most co-operatively weighted (in order of rank), members of the maximization family.[8] The final part of this enquiry attempts to account for their success, both in terms of their relatedness and co-operativeness.

Before that attempt is made, however, the strategic population is subjected to a different measure of robustness; namely, an ecological scenario.

---

[8] Recall that, during its first one hundred random moves, *MAC* co-operates with a probability of 9/10; *MAE*, 5/7; *MEU*, 1/2; *MAD*, 1/10.

Chapter Nine
An Ecological Scenario

The ecological scenario emerges from a consideration of evolutionary game theory, which itself developed from an application of game-theoretic concepts to certain types of conflicts in the sphere of biological evolution.[1] A distinction must be drawn, however, between Maynard Smith's evolutionary game-theoretic model and Axelrod's ecological scenario. It can be shown that Axelrod's tournaments, and the interactive tournament, are not susceptible to evolutionary modelling in the Maynard Smith sense. The ecological scenario, however, provides an interesting alternative perspective on strategic robustness.

The classic Maynard Smith evolutionary game models con-specific conflicts in the animal kingdom exclusive of humans.[2] Essentially, Maynard Smith hypothesizes that if two members of a species compete for a fitness-enhancing resource of expected utility V, each member may adopt either the "hawk" strategy (H), which consists in monopolizing the resource, or the dove strategy (D), which consists in sharing it. If both competitors adopt the "hawk" strategy, then a mutually-injurious conflict ensues, which reduces their fitnesses by a palpable quantity C. The game matrix (9.1) follows.[3]

[Note that in game 9.1, D denotes the "dove" strategy and C denotes the injurious effect of an (H,H) conflict. This is the familiar notation for the evolutionary model, and these symbols should not be confused with their signification in the Prisoner's Dilemma.]

---

[1] Lewontin seems to have been the first to conceive of a literal game against nature, in applying the minimax criterion to population genetics. See R. Lewontin, `Evolution and the Theory of Games', *Journal of Theoretical Biology*, 1, 1961, pp.382-403.

[2] E.g. see J. Maynard Smith, `The Theory of Games and the Evolution of Animal Conflicts', *Journal of Theoretical Biology*, 47, 1974, pp.209-21.

[3] Idem., *Evolution and the Theory of Games*, Cambridge at the University Press, 1982, p.12.

Game 9.1 - The Maynard Smith Evolutionary Model

		B	
		H	D
A	H	$(1/2)(V-C)$	$V, 0$
	D	$0, V$	$(1/2)V$

The payoffs of game 9.1 can take on three different transitive orderings, depending upon the relative values of V and C. Explicitly, the three cases are:

case (1): $V > C$

case (2): $V = C$

case (3): $V < C$

Let each of these cases be considered in turn.

In case (1), $V > C$. In other words, the fitness enhancement resulting from possession of the resource is greater than the fitness reduction resulting from the conflict over its acquisition. To both competitors, then, the expected utility of the (H,H) outcome is greater than zero. To either competitor, the "hawk" strategy is strongly dominant, since $(1/2)(V-C) > 0$ and $V > (1/2)V$. Either competitor's fitness is enhanced by his adoption of the "hawk" strategy, no matter what his opponent does. If both competitors adopt the "hawk" strategy, their fitnesses are enhanced by $(1/2)(V-C)$; whereas, if both adopt the "dove" strategy, their fitnesses are enhanced by $(1/2)V$. Although monopolization is strongly dominant, both competitors (if they play alike) gain more by sharing. Thus, this case of game 9.1 is a Prisoner's Dilemma (with transitive ordering of payoffs $T > R > P > S$).

In case (2), $V = C$. To both competitors, the expected utility of possessing the resource is just balanced by the expected dis-utility of acquiring it. Then, to competitor A, the "hawk" strategy is weakly dominant, since his payoff of outcome (H,H) equals that of outcome (D,H) [equals zero], and his payoff of outcome (H,D) is greater than that of outcome (D,D) [since $V > (1/2)V$]. Similarly, from competitor B's point of view, the "hawk" strategy weakly dominates the "dove" strategy. Hence monopolization is weakly dominant. However, the competitors gain nothing if both adopt the "hawk" strategy, while each gains $(1/2)V$ if both adopt the "dove" strategy. Thus, this case of game 9.1 is a weak Prisoner's Dilemma (with transitive ordering of payoffs $T > R > P = S$).

In cases (1) and (2), where $V \geq C$, the pure "hawk" strategy appears to prevail in nature.[4] It is not difficult to understand why it prevails in these cases. Maynard Smith's evolutionary game theory models con-specific conflicts in the neo-Darwinian paradigm. In neo-Darwinian terms, the "hawk" and "dove" strategies are phenotypic behaviour patterns mediated by genotypic attributes. Natural selection acts upon the individual at the phenotypic level, thereby indirectly favouring, or disfavouring, the genotypes that mediate a given behavioural pattern. A significant component of an animal's inclusive fitness is its ability to reproduce. Thus, an increase in fitness implies an increase in potential reproductivity.

Now, for $V \geq C$, suppose genome A mediates the phenotypic behaviour of pure "hawk"; genome B, that of pure "dove". Enounters between animals carrying these genomes are represented in the following matrix:

Game 9.2 - Encounters Between Pure Strategies

	A	B
A	$(1/2)(V-C)$	$V,0$
B	$0,V$	$(1/2)V$

While the outcomes of games 9.1 and 9.2 are identical, the players are not. Game 9.1 models a conflict between two members of a species; game 9.2, all conflicts within the population at large. An animal carrying genome A enhances its fitness regardless of which

[4] Ibid, p.15.

type of con-specific it encounters; whereas an animal carrying genome *B* enhances its fitness only when it encounters a con-specific carrying genome *B*. Animals playing "hawk" will thus enhance their fitnesses with greater relative frequency and, *ceteris paribus*, will produce more offspring, than animals playing "dove". In consequence, genome *A* is positively selected; *B*, negatively selected. In theory, after a sufficient number of generations, the population will consist predominantly of pure "hawks"; pure "doves" will have become marginalized.

This explanation for the natural prevalence of the pure "hawk" strategy, albeit advanced in a game-theoretic model that is arguably over-simplified, is nonetheless interesting in neo-Darwinian terms. The explanation becomes more compelling when one considers the case to which it does not apply; namely, case (3), in which $V < C$. When the expected enhancement of fitness resulting from the possession of a resource is less than the expected reduction in fitness resulting from the attempt to monopolize it, the competitor is confronted by a novel situation. In this case, Maynard Smith's evolutionary model embodies a problem hitherto unseen in previous Prisoner's Dilemmas, but copiously apparent in nature.

Many animal species are equipped with physical or chemical weapons, lethal not only to their predators or prey, but also to con-specifics. Since natural selection has favoured the evolution of such weapons, it must also have favoured behavioural patterns that prevent armed con-specifics from annihilating one another. Indeed, the phenomenon of limited or ritualized con-specific combat abounds in the arenas of nature. From the mantis shrimp which batter one another on their heavily-armoured tails, to the venomous serpents which wrestle one another instead of unsheathing their deadly fangs, to the wolves which expose their jugulars in combat-terminating gestures of appeasement, one observes a myriad of ways in which con-specific competition over fitness-enhancing resources is conducted in a strenuous yet neither fatal nor debilitating fashion.

The competitors in the game-theoretic model cannot reflect this behaviour by adopting pure strategies, be they "hawk" or "dove". It is here that Maynard Smith makes an ingenious contribution to evolutionary games, by introducing the concept of an *evolutionarily stable strategy* (*ESS*). Suppose that there exists a mixed strategy *I*, which consists in playing *H* with probability *p*, and in playing *D* with probability (1-*p*). Again, within the paradigm of neo-Darwinism, it is presumed that the phenotypic behavioural pattern giving rise to strategy *I* is mediated by a genome that determines the probability distribution. If *I* is optimally effective in terms of fitness-enhancement, then this genome will be positively selected. Such an optimal mixed strategy, for a given species, is called an *ESS*.

An *ESS* is defined as a strategy such that

> ". . .if most of the members of a population adopt it, there is no `mutant' strategy that would give higher reproductive fitness",[5]

or, alternatively, as a strategy such that

> ". . .if all the members of a population adopt it, then no mutant strategy could invade the population under the influence of natural selection."[6]

Explicitly, to find probability *p* such that *I* is an *ESS*, Maynard Smith makes use of the Bishop-Cannings theorem[7] and writes the following equation:

$$EU(H,I) = EU(D,I)$$

In other words, the expected utility of playing "hawk" against an *ESS* is the same as that of playing "dove" against it. If one finds the probability distribution that satisfies this equation, one has the probability distribution of the *ESS* itself. Maynard Smith then solves this equation for game 9.1:[8]

---

[5] J. Maynard Smith & G. Price, `The Logic of Animal Conflict', *Nature*, **246**, 1973, pp.15-18.

[6] Maynard Smith, 1982, p.10.

[7] T. Bishop & C. Cannings, `A Generalized War of Attrition', *Journal of Theoretical Biology*, **70**, 1978, pp.85-124. They prove that if *I* is a mixed *ESS* with component strategies $a,b,\ldots,z$ then $EU(a,I) = EU(b,I) = \ldots = EU(z,I) = EU(I,I)$.

[8] Maynard Smith, 1982, pp.15-16.

$$P[(1/2)(V-C)] + (1-p)V = p(0) + (1-p)(1/2)V$$

or

$$p = V/C$$

Thus, in the case when $V < C$, it is evolutionarily stable to adopt the "hawk" strategy with probability V/C, and the "dove" strategy with probability $(1-V/C)$.

Now, Axelrod and Hamilton attempt to apply the concept of evolutionarily stable strategy to the iterated Prisoner's Dilemma.[9] It would be useful indeed if evolutionary game theory could point to an optimally effective mixed strategy in the Prisoner's Dilemma. Unfortunately, the theory cannot do so, for the simple reason that no *ESS* exists in the Prisoner's Dilemma. A rigorous proof that no *ESS* can be found for the Prisoner's Dilemma is given elsewhere.[10] For the purposes of this enquiry, a brief demonstration can be made that the concept of *ESS* is inapplicable to the Prisoner's Dilemma.

The demonstration takes the form of a *reductio ad absurdum*. Let one assume that an *ESS* exists in the Prisoner's Dilemma, and let the Maynard Smith equation be applied to find its explicit probability distribution.

Game 9.3 - The Prisoner's Dilemma

		B	
		c	*d*
A	*C*	*R,R*	*S,T*
	D	*T,S*	*P,P*

where $T > R > P > S$

With respect to game 9.3, the Maynard Smith equation is written

---

[9] R. Axelrod & W. Hamilton, 'The Evolution of Cooperation', *Science*, **211**, 1981, pp.1390–6.

[10] L. Marinoff, 'The Inapplicability of Evolutionarily Stable Strategy to the Prisoner's Dilemma', *The British Journal for the Philosophy of Science*, **41**, 1990, pp.458–470 (pending).

$$EU(C,I) = EU(D,I)$$

In other words, the expected utility of co-operating against the *ESS* is equal to the expected utility of defecting against it. Explicitly,

$$p(R) + (1-p)S = p(T) + (1-p)(P)$$

or

$$p = (P-S)/[(R+P)-(S+T)] \qquad (9.1)$$

Since p is a (real) probability, its permissible values are $0 \leq p \leq 1$. Hence, these are also the permissible values for the right-hand side of equation (9.1).

Now, consider the quotient $(P-S)/[(R+P)-(S+T)]$. The numerator, $(P-S)$, is always greater than zero (since by definition, $P > S$). Thus, to satisfy the constraint on the permissible values of p, the denominator $(R+P)-(S+T)$ must be greater than or equal to the numerator. That is,

$$(R+P)-(S+T) \geq (P-S)$$

or

$$R \geq T \qquad (9.2)$$

But inequality (9.2) cannot be satisfied, since, by definition, $T > R$. Thus, $(P-S) > (R+P)-(S+T)$.

In consequence, the quotient $(P-S)/[(R+P)-(S+T)]$ is either greater than unity [if $0 < (R+P)-(S+T) < (P-S)$], or less than zero [if $(R+P)-(S+T) < 0$]. But these are precisely the values of p that fail to satisfy the constraint on equation (9.1). Since equation (9.1) has no solution such that p is a real probability ($0 \leq p \leq 1$), therefore no *ESS* exists in the Prisoner's Dilemma.

Structurally, it is not difficult to see why this is so. The transitive ordering of payoffs in the Prisoner's Dilemma is $T > R > P > S$ (or, in the case of weak dominance, $T > R > P = S$). These orderings correspond to cases (1) and (2) (where $V > C$ and $V = C$, respectively), of Maynard Smith's evolutionary model. But it is case (3) of the evolutionary model (where $V < C$) that prompts Maynard Smith's search for an *ESS*. But in case (3), the transitive ordering of payoffs is $T > R > S > P$. This is not a Prisoner's Dilemma. So the existence of an *ESS* in case (3) cannot and does not imply the existence of an *ESS* in the Prisoner's Dilemma. The two games belong to utterly different classes, according to the respective orderings of their payoffs.

However, the inapplicability of the concept of *ESS* to the Prisoner's Dilemma does not preclude ecological modelling. Axelrod develops a very interesting scenario in his second tournament, based upon an ecological perspective.[11] The principal assumptions in the model are as follows.

Suppose that the total payoffs accrued (that is, points scored) by some strategy *A*, in competition against other strategies, represent the initial population of *A*-strategists in the first generation of the tournament. The relative population of *A*-strategists in that generation is therefore the ratio of strategy *A*'s total points scored to the sum of total points scored by all strategies. Similarly, each competing strategy represents a unique population of strategists, whose relative frequency is the ratio of that strategy's total points scored to the sum of total points scored by all strategies.

Next, one simulates future generations of the tournament. The ratio of total points scored by strategy *A* to the sum of total points scored by all strategies in the n^{th} generation, represents the population of *A*-strategist offspring, descended from *A*-strategists in the $(n-1)^{st}$ generation, presently competing in the n^{th} generation. Depending on how they fare against other strategists in the overall population, these *A*-strategists will produce a relative number of offspring who compete in the $(n+1)^{st}$ generation, and so forth.

Axelrod explains how his ecosystemic competition is conducted:

> "We simply have to interpret the average payoff received by an individual as proportional to that individual's expected number of offspring. For example, if one rule gets twice as high a tournament score in the initial round as another rule, then it will be twice as well-represented in the next round. This creates a simulated second generation of the tournament in which the average score achieved by a rule is the *weighted*

---

[11] Axelrod, 1980*b*, pp.398–401.

> average of its score with each of the rules, where the weights are proportional to the success of the other rules in the initial generation."[12]

As Axelrod indicates, this process simulates "survival of the fittest":

> "A rule which is successful on average with the current distribution other rules in the population will become an even larger proportion of the environment of the other rules in the next generation. At first a rule which is successful with all sorts of rules will proliferate, but later as the unsuccessful rules disappear, success requires good performance with other successful rules."[13]

When Axelrod conducts his ecological experiment with the sixty-three strategies of his second tournament, he finds that, after 500 generations, only eleven strategies have increased their relative sizes in the population, and that these strategies ranked uppermost in the parent generation. After 1000 generations, only six strategies continue to increase their relative numbers of offspring (and they ranked first, third, second, sixth, seventh and ninth originally). Of these, Axelrod finds that *TFT* has produced the greatest number of offspring, and that *TFT* continues to grow at the most rapid rate.[14]

Axelrod's ecological scenario is emulated in the environment of the interactive tournament. Axelrod does not explicitly state the algorithm he uses to simulate future generations, but the algorithm developed in this enquiry embodies the main precepts of Axelrod's model. When strategy *A* encounters strategy *B* in the initial generation, the ratio of their scores is interpreted as the ratio of their offspring produced in competition against one another. The likelihood with which these offspring encounter one another in the second generation is proportional to the relative numbers of *A*-strategists and *B*-strategists in the initial generation. This algorithm is applied to all strategies in the environment, and is iterated for a sufficient number of generations, until all rates of growth (and decline) subside to a quiescent state.

---

[12] Ibid, pp.398-9.

[13] Ibid, p.399.

[14] Ibid, pp.400-1.

A fairly straightforward mathematical notation is introduced, in order to show explicitly how this algorithm functions. (The actual documented computer program is listed in Appendix Four, program A4.12.)

$(AvB)_n$ means "the relative number of n^{th} generation offspring produced by *A*-strategists in competition against *B*-strategists." Thus $(AvB)_1$ is strategy *A*'s tournament score against strategy *B*.

$(TOT)_{A,n}$ means "the total relative number of n^{th} generation offspring produced by *A*-strategists in competition against all strategists";

i.e., $(TOT)_{A,n} = (AvA)_n + (AvB)_n + (AvC)_n + \ldots + (AvZ)_n$

for *Z* different strategies in the environment. Thus $(TOT)_{A,1}$ is strategy *A*'s total score in the tournament.

$(SUM)_n$ means "the total relative number of all strategists in the n^{th} generation";

i.e., $(SUM)_n = (TOT)_{A,n} + (TOT)_{B,n} + (TOT)_{C,n} + \ldots + (TOT)_{Z,n}$

for *Z* different strategies in the environment. Thus $(SUM)_1$ is the sum of all offensive scores in the tournament.

$(FRE)_{A,n}$ means "the relative frequency of *A*-strategists in the n^{th} generation";

i.e., $(FRE)_{A,n} = [(TOT)_{A,n}]/[(SUM)_n]$

Thus, the relative frequency of *A*-strategists in the initial generation, $[(FRE)_{A,1}]$, is the ratio of strategy *A*'s total offensive score $[(TOT)_{A,1}]$ to the sum of all strategies' total offensive scores $[(SUM)_1]$. Note that all such relative frequencies, in the initial generation, are computed directly from the tournament matrix of raw scores (see Appendix Two).

The raw scores for the second generation of *A*-strategists are then computed from the following recurrence relation:

$$(AvB)_2 = (AvB)_1[(TOT)_{A,1}]/[(TOT)_{A,1} + (TOT)_{B,1}]$$

An exhaustive implementation of this recurrence relation yields a second-generation matrix of raw scores, or relative numbers of offspring (for *Z* different strategies):

$$\begin{matrix} (AvA)_2 & (AvB)_2 & \ldots & (AvZ)_2 \\ (BvA)_2 & (BvB)_2 & \ldots & (BvZ)_2 \\ \vdots & \vdots & & \vdots \\ (ZvA)_2 & (ZvB)_2 & \ldots & (ZvZ)_2 \end{matrix}$$

Once this matrix has been computed, the above-described procedure for finding the relative frequencies is implemented for this second generation.

In general, then, for each subsequent generation, the recurrence relation

$$(AvB)_{n+1} = (AvB)_n[(TOT)_{A,n}]/[(TOT)_{A,n} + (TOT)_{B,n}]$$

is used to compute the new matrix of offspring, from which each strategy's relative frequency in that generation can be found.

Note that if the n^{th} generation ratio of offspring, $[(AvB)_n]/[(BvA)_n]$, has the numerical value a/b, then the $(n+1)^{st}$ generation ratio will be

$$[(AvB)_{n+1}]/[(BvA)_{n+1}] = (a/b)[(TOT)_{A,n}]/[(TOT)_{B,n}]$$

This satisfies the two principal requirements of Axelrod's model; namely, that the ratio of offspring between two competing strategies, in any future generation, be proportional to

(i) their ratio of offspring in the previous generation, and

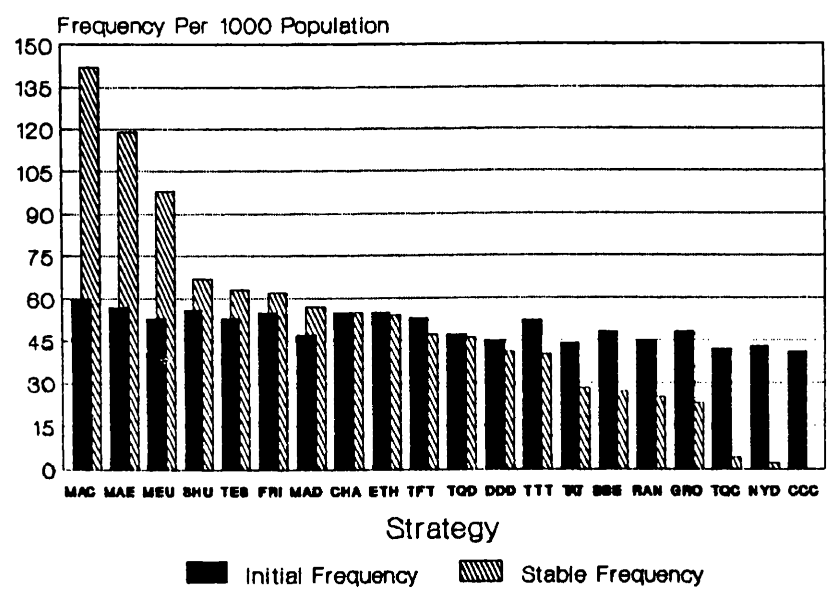
(ii) their relative frequencies in the previous generation.

The relative frequency of each strategy's progeny, in a given generation, is expressed in parts per thousand (*ppt*) of the overall population in that generation. The ecological scenario involving the twenty strategies of the interactive tournament attains a stable state after about 325 generations. That is to say, following the 325^{th} generation, the rate of change has slowed to the extent that all strategies' cumulative increases or decreases in relative frequency are less than one part per thousand over the next several generations. Although minor fluctuations continue to take place, in increments (or decrements) of parts per ten thousand per generation and less, these fluctuations are negligible on the scale of the scenario.

The results of the ecological scenario involving twenty strategies are displayed in the following bar-chart, which shows the initial (parent generation) and stable (325^{th} generation) frequencies for each strategy. The strategies appear, from left to right, in descending order of their stable frequencies.

It is clear from graph 9.1 that *MAC*, which has the largest initial frequency (60 *ppt*), experiences the greatest increase, to a stable frequency of 142 *ppt*. This represents an increase of 82 *ppt* over 325 generations, or an average growth rate of 0.25 *ppt* per generation. And *MAE*, which has the second largest initial frequency (57 *ppt*), experiences the second greatest increase, to a stable frequency of 119 *ppt*. *MAE*'s average rate of growth is thus 0.19 *ppt* per generation.

Graph 9.1 **ECOLOGY OF MAIN TOURNAMENT**
Initial vs. Stable States

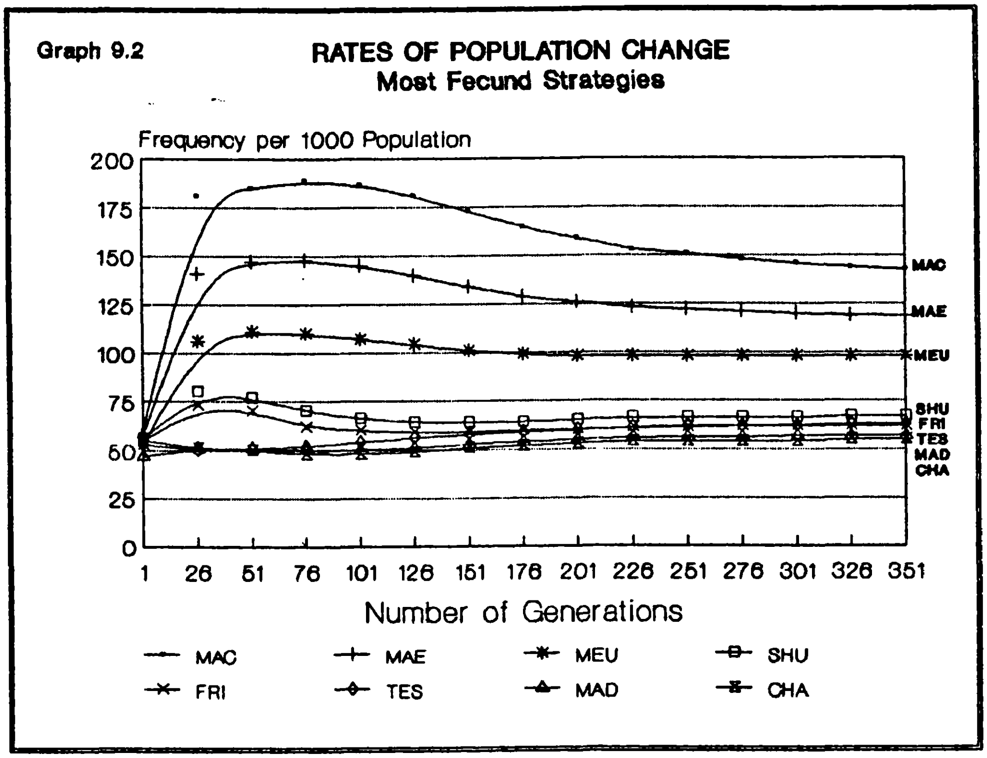


Obviously, the size-order of the initial frequencies is identical to the rank-order of the tournament, since a strategy's initial frequency is its total tournament score divided by the sum of all strategies' total tournament scores, and this dividend remains constant (for a given matrix of raw scores). However, the size-order of the stable frequencies does not necessarily correspond to that of the initial frequencies. For example, *SHU* ranks third in initial frequency (56 *ppt*), but slips to a distant fourth in stable frequency (67 *ppt*). *SHU* is overtaken by *MEU*, which ranks only seventh in initial frequency (53 *ppt*), but third in stable frequency (98 *ppt*). *SHU*'s growth rate is 0.034 *ppt* per generation; *MEU*'s, 0.14 *ppt* per generation.

That *MAC*, *MAE* and *MEU* produce the greatest relative numbers of progeny, respectively, is a testament not only to their individual fitnesses, but also to the overall fitness of the maximization family in this ecosystem.

At the other end of the spectrum, it it probably no coincidence that the greatest declines in frequency are experienced by *CCC* (−41 *ppt*), *NYD* (−41 *ppt*), and *TQC* (−38 *ppt*). Not only do these strategies have the lowest initial frequencies, but also, perhaps significantly, their order of ecological decline corresponds exactly to their order of points allowed in the interactive tournament. Moreover, *CCC* has apparently become "extinct"; since, from the tenth generation onward, its relative frequency is zero *ppt*.

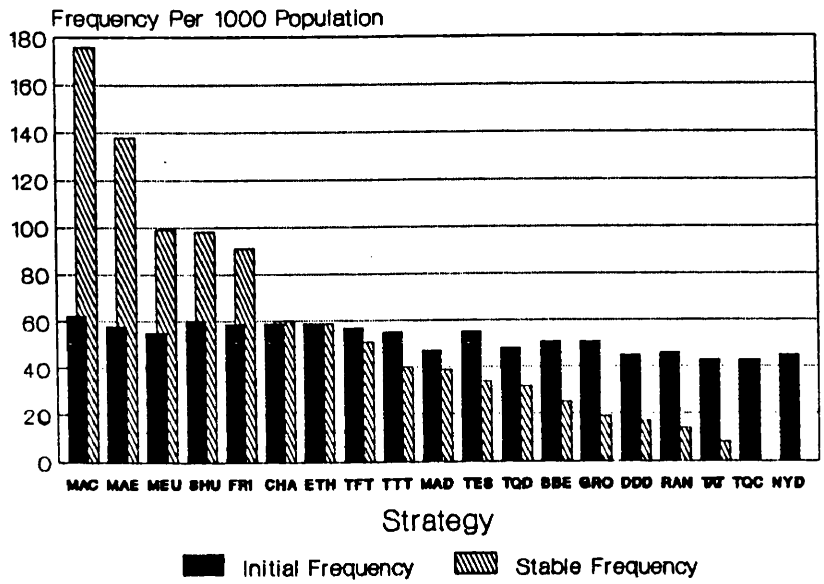
The reproductive fortunes of the eight most fecund strategies, in terms of their instantaneous rates of change, can be gauged from the following graph (overleaf):



Given that one of the strategies (*CCC*) has become extinct in this ecosystem, it seems reasonable to ask another question: what would happen if the nineteen surviving strategies were to re-establish themselves in a new ecological habitat, with corresponding initial conditions, and subject to the same generative algorithm, save that all *CCC*-strategists have disappeared from the ecosystem?

The scenario is thus regenerated in a new ecosystem of nineteen surviving strategies, with the following result (overleaf):

Graph 9.3 **ECOLOGY OF NINETEEN SURVIVORS**
Initial vs. Stable States



In this ecosystem, rates of growth and decline subside to negligibility after about 450 generations. Again, *MAC* has the largest initial frequency (62 *ppt*), and experiences the greatest increase, to a stable frequency of 176 *ppt*. *SHU*, with the second largest initial frequency (60 *ppt*), ranks fourth at stability (98 *ppt*). *MAE*, which has the sixth largest initial frequency (58 *ppt*), vaults past *FRI*, *CHA*, *ETH*, and *SHU*, to rank second at stability (138 *ppt*). And *MEU*, initially in a three-way tie for eighth place (55 *ppt*), finishes third at stability (99 *ppt*). The maximization family continues to exhibit reproductive fitness in this ecosystem.

This procreative model is clearly sensitive to perturbation (by the removal or, inversely, by the addition of a competing strategy). The term "ecology" seems well-chosen by Axelrod, in that the extinction of one strategy has palpable repercussions on the interactions among the nineteen survivors. In the original ecosystem, both *MAE* and *MAC* enjoy comparatively high reproductive success in competition with *CCC*. As soon as *CCC* becomes extinct, *MAE* falls from second to fourth place in initial frequency; *MEU*, from sole possession of seventh to a three-way tie for eighth. That *MAE* and *MEU* now overtake numerous competitors, in order to finish second and third behind *MAC*, illustrates their fitness in regaining lost reproductive ground.

The perturbation also results in the extinction of two more strategies: in this new ecosystem, *NYD*'s progeny vanish after the tenth generation; *TQC*'s, after the eleventh. Once again, the first strategy to become extinct in this ecosystem is the strategy with the lowest initial frequency (*NYD*, 41 *ppt*). But *TQC*, which disappears one generation later, shares the second-lowest initial frequency with *TAT* (43 *ppt*). Although *TAT* experiences a sharp decline, it manages to stabilize at 8 *ppt*.

The eliminatory process is continued by establishing another ecosystem, composed of the eighteen surviving strategies after the demise of *NYD*. This ecosystem is similarly procreated until stability is attained, whereupon another new ecosystem is formed, by deleting the next strategy to become extinct. This eliminatory process is repeated to its eventual conclusion. The results of all ecosystemic competitions are summarized in table 9.1 (overleaf). These results are revealing, and also somewhat intriguing.

Each column of table 9.1 (except the last) is headed by two numbers. The first is the number of strategies competing in a given ecosystem; the second is the number of generations required to attain approximate stability in that ecosystem. These numbers alone yield interesting information.

Table 9.1 - Initial and Stable Frequencies, in Parts per Thousand

#con: / #gen:	20 / 325	19 / 450	18 / 399	17 / 76	16 / 530	15 / 242	14 / 352	13 / 63	12 / 219	11 / 252	10 / 76	9 / 109	8 / 72	7 / 36	6 / 32	5 / 36	4 / 1	Rec. / Gen. / Avg.
TFT	53 / 47 (-6)	57 / 51 (-6)	61 / 52 (-9)	65 / 54 (-11)	71 / 102 (+31)	74 / 100 (+26)	78 / 99 (+21)	83 / 97 (+14)	89 / 86 (-3)	96 / 98 (+2)	106 / 65 (-41)	119 / 244 (+125)	128 / 194 (+66)	143 / 121 (-22)	167 / 235 (+68)	200 / 296 (+96)	250 / 250 (+0)	+351 / 3261 / +.108
SHU	56 / 67 (+11)	60 / 98 (+38)	65 / 152 (+87)	68 / 130 (+62)	69 / 93 (+24)	73 / 96 (+23)	76 / 115 (+39)	80 / 116 (+36)	84 / 59 (-25)	91 / 53 (-38)	99 / 43 (-56)	112 / 59 (-53)	128 / 194 (+66)	143 / 121 (-22)	167 / 235 (+68)	200 / 296 (+96)	250 / 250 (+0)	+356 / 3261 / +.109
GRO	48 / 23 (-25)	51 / 19 (-32)	54 / 20 (-34)	57 / 13 (-44)	64 / 25 (-39)	68 / 30 (-38)	72 / 44 (-28)	77 / 63 (-14)	83 / 27 (-56)	91 / 33 (-58)	102 / 136 (+34)	109 / 17 (-92)	128 / 193 (+65)	143 / 120 (-23)	167 / 120 (-47)	200 / 111 (-89)	250 / 250 (+0)	-520 / 3261 / -.159
ETH	55 / 54 (-1)	59 / 59 (±0)	63 / 50 (-13)	68 / 45 (-23)	75 / 122 (+47)	79 / 117 (+38)	83 / 119 (+36)	89 / 114 (+25)	97 / 193 (+96)	104 / 290 (+186)	110 / 204 (+94)	117 / 181 (+64)	128 / 194 (+66)	143 / 121 (-22)	167 / 235 (+68)	200 / 296 (+96)	250 / 250 (+0)	+757 / 3261 / +.232
TES	53 / 63 (+10)	55 / 34 (-21)	60 / 47 (-13)	64 / 50 (-14)	69 / 93 (+24)	72 / 91 (+19)	76 / 83 (+7)	80 / 80 (±0)	87 / 72 (-15)	93 / 67 (-26)	102 / 52 (-50)	115 / 175 (+60)	123 / 11 (-112)	150 / 397 (+247)	167 / 175 (+8)	200 / 0 (-200)	ext.	-76 / 3260 / -.023
CHA	55 / 55 (±0)	59 / 60 (+1)	63 / 48 (-15)	69 / 47 (-22)	76 / 125 (+49)	79 / 121 (+42)	84 / 137 (+53)	89 / 128 (+39)	97 / 266 (+169)	102 / 270 (+168)	109 / 199 (+90)	116 / 144 (+28)	128 / 192 (+64)	143 / 119 (-24)	167 / 0 (-167)	ext.	ext.	+475 / 3227 / +.143
TTT	52 / 40 (-12)	55 / 40 (-15)	59 / 35 (-24)	64 / 39 (-25)	70 / 95 (+25)	73 / 95 (+22)	77 / 74 (-3)	82 / 81 (-1)	89 / 96 (+7)	97 / 104 (+7)	106 / 183 (+77)	112 / 164 (+52)	120 / 21 (-99)	133 / 0 (-133)	ext.	ext.	ext.	-122 / 3179 / -.038
FRI	55 / 62 (+7)	59 / 91 (+32)	64 / 154 (+90)	66 / 136 (+70)	66 / 84 (+18)	69 / 84 (+15)	72 / 89 (+17)	75 / 84 (+9)	79 / 33 (-46)	84 / 13 (-71)	92 / 25 (-67)	103 / 16 (-87)	117 / 0 (-117)	ext.	ext.	ext.	ext.	-130 / 3106 / -.042
MAC	60 / 142 (+82)	62 / 176 (+114)	64 / 144 (+80)	67 / 129 (+62)	68 / 86 (+18)	72 / 91 (+19)	75 / 115 (+40)	79 / 117 (+38)	83 / 100 (+17)	88 / 64 (-24)	94 / 94 (±0)	97 / 0 (-97)	ext.	ext.	ext.	ext.	ext.	+349 / 2995 / +.117
BBE	48 / 27 (-21)	51 / 25 (-26)	54 / 25 (-29)	57 / 35 (-22)	61 / 57 (-4)	63 / 55 (-8)	66 / 33 (-33)	70 / 33 (-37)	73 / 8 (-65)	79 / 8 (-71)	79 / 0 (-79)	ext.	ext.	ext.	ext.	ext.	ext.	-395 / 2917 / -.135
MAE	57 / 119 (+62)	58 / 138 (+80)	60 / 114 (+54)	62 / 108 (+46)	62 / 66 (+4)	65 / 68 (+3)	68 / 66 (-2)	71 / 68 (-3)	74 / 61 (-13)	75 / 0 (-75)	ext.	ext.	ext.	ext.	ext.	ext.	ext.	+156 / 2666 / +.059
KEU	53 / 98 (+45)	55 / 99 (+44)	56 / 79 (+23)	57 / 86 (+29)	56 / 41 (-15)	59 / 41 (-18)	61 / 26 (-35)	63 / 18 (-45)	64 / 0 (-64)	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	-36 / 2447 / -.015
RAN	45 / 25 (-20)	46 / 14 (-32)	48 / 10 (-38)	49 / 12 (-37)	52 / 3 (-49)	55 / 4 (-51)	58 / 1 (-57)	62 / 0 (-62)	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	-346 / 2383 / -.145
TQD	47 / 46 (-1)	48 / 32 (-16)	49 / 21 (-28)	50 / 32 (-18)	50 / 7 (-43)	53 / 6 (-47)	55 / 0 (-55)	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	-208 / 2031 / -.102
MAD	47 / 57 (+10)	47 / 39 (-8)	47 / 25 (-22)	47 / 44 (-3)	45 / 1 (-44)	46 / 0 (-46)	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	-113 / 1789 / -.063
DDD	45 / 41 (-4)	45 / 17 (-28)	47 / 23 (-24)	47 / 40 (-7)	44 / 0 (-44)	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	-107 / 1259 / -.085
TAT	44 / 28 (-16)	43 / 8 (-35)	43 / 0 (-43)	43 / 0 (-43)	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	-137 / 795 / -.172
TQC	42 / 4 (-38)	43 / 0 (-43)	45 / 0 (-45)	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	-126 / 345 / -.365
NYD	43 / 2 (-41)	45 / 0 (-45)	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	-86 / 335 / -.257
CCC	41 / 0 (-41)	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	ext.	-41 / 9 / -4.6

(ext. ≡ extinct)

The number of generations required to stabilize an ecosystem is not (as might be expected) a smoothly decreasing function of the number of competing strategies. While such a trend is observable overall, many individual reversals of that trend are apparent. Since the number of generations required to attain stability diminishes only in tendency with the number of competing strategies, and seems to depend critically on the particular combination of strategies in competition, one can conclude that this eliminatory process is somewhat stochastic.

(An exponential curve fit, which gives the number of generations required to stabilize the population frequencies as a function of the number of strategies in competition, yields the result $y = 16.9e^{.169x}$ with the poor correlation $r = .65$. This low correlation is expected, since all available (x,y) data are used. A better fit is obtained by selecting fewer and more convenient (x,y) data: $y = 26e^{.138x}$ with the improved correlation $r = .82$. These equations offer one possible explanation as to why Axelrod's ecosystem does not attain stability after 1000 generations. With 63 competitors, the second equation predicts that 155,000 generations are required to attain stability. Of course, any such extrapolation remains highly conjectural.)

Each cell of table 9.1 contains three numbers: a given strategy's initial frequency, its stable frequency, and its change in frequency; all in parts per thousand of the population for the given ecosystemic competition.

One might use table 9.1 to follow the fortunes of the maximization family, which dominates the stable populations of ecosystems involving twenty and nineteen strategies. In the ecosystem involving 18 strategies (following the extinction of *NYD*), *SHU* holds the greatest initial frequency (65 *ppt*), while *MAC* and *FRI* are tied with the second greatest (64 *ppt*). *FRI* experiences the largest increase, however, and realizes the greatest stable frequency (154 *ppt*), followed by *SHU* (152 *ppt*) and *MAC* (144 *ppt*). *MAE* initially ranks seventh (60 *ppt*), but climbs to fourth at stability (114 *ppt*), while *MEU* initially ranks ninth (56 *ppt*) but finishes fifth (79 *ppt*). Thus *MAC*, *MAE*, and *MEU* continue to perform quite well, but they slip to third, fourth and fifth places with respect to magnitudes of stable frequencies.

A glance at the tournament matrix of raw scores (Appendix Two) affords an explanation for what is taking place. In the context of the tournament, the maximization family fared extremely well against *CCC* and *NYD*. In fact, each member of the maximization family realizes its two highest scores against these very strategies. But in the ecological context, this large margin of success not only contributes to the rapid extinction of the weaker strategies, but also proves detrimental to the exploitive ones.

In the tournament, for example, *MAC* out-scored *CCC* by 4824 to 264. So in the ecological scenario, their parent generation ratio is thus 4824:264, or about 18:1 in favour of *MAC*. And in the parent generation of the twenty-strategy ecosystem, their respective initial frequencies are 60 and 41 *ppt* of the overall population. Thus the ratio of their second-generation offspring is (4824x60):(264x41), or about 27:1 in favour of *MAC*. In the tournament context, *MAC* exploits *CCC* rather heavily (as do many other strategies) with no dire consequences to itself. But in the ecological context, *MAC*'s heavy exploitation of *CCC* has a three-fold result.

First, *MAC* benefits from a proportionately large increase in progeny. Second, *CCC*, which experiences a generally poor differential procreative rate in the ecosystem as a whole, is unable to stave off elimination. Third, in subsequent ecosystems, *MAC* no longer benefits from its high procreative rate in competition against *CCC*, since *CCC* is extinct. In future ecosystems, *MAC* must compete more frequently against strategies with greater procreative fitness than *CCC*, strategies which *MAC* cannot exploit as readily.

This is a classic instance of over-exploitation of a resource, to the eventual detriment of the exploiters. All strategies that over-exploit *CCC* (such as *DDD*, *TQC*, *TAT*, *TES*, and the maximization family) abet *CCC*'s rapid extinction, and in so doing deprive themselves of a competitor which allows them to create large relative numbers of progeny. When a new ecosystem is established, with *CCC* absent from the environment, the population frequencies undergo an ecological shift, such that those strategies which over-exploited the extinct competitor now experience corresponding declines in their procreative rates. In future ecosystems, former exploiters may themselves become the victims of exploitation.

In the ecosystem with 17 strategies (following the extinction of *TQC*), the stable order is once again *FRI* (136 *ppt*), *SHU* (130 *ppt*), *MAC* (129 *ppt*), *MAE* (108 *ppt*), and *MEU* (86 *ppt*). The population gaps between these upper five strategies have closed, compared with the previous ecosystem. And now, with *TAT*'s extinction, one observes that, in the first four ecosystems, the lower four strategies of the tournament have become extinct, in reverse-order of their tournament ranks, from twentieth to seventeenth (*CCC, TQC, NYD, TAT*).

TAT's extinction (combined with the previous extinctions) results in a re-ordering of initial frequencies in the next ecosystem, which precipitates new stable standings. In the ecosystem with sixteen strategies, *CHA* (76 *ppt*), *ETH* (75 *ppt*), and *TFT* (71 *ppt*) are most successful, both initially and at stability, realizing eventual frequencies of 125, 122, and 102 *ppt* respectively. *TTT* places fourth at stability (95 *ppt*), while *SHU* manages a tie for fifth with *TES* (93 *ppt*). Evidently, *TAT*'s extinction results in a complete upheaval in the environment, with new strategies in the ascendancy, and previously successful strategies in decline. *MAC* slips to seventh at stability (86 *ppt*); *MAE*, ninth (*66* ppt). Moreover, this ecosystemic competition requires the greatest number of generations (more than five hundred) to settle down. In addition, the precedent for extinction is broken. *DDD* (which ranks ahead of *RAN* in the tournament) now vanishes from the ecology.

In ecosystems involving from fifteen to ten competitors, *CHA* and *ETH* continue to predominate at stability, while *MAC, SHU, TFT* and *TTT* also tend to flourish. In ecosystems involving from nine to five competitiors, *TFT* ranks first four times and second once. The ecosystemic competition of seven strategies is won by *TES*. In this competition, *TES* experiences the greatest increase of any strategy in any ecosystem, from an initial frequency of 150 *ppt* to a stable frequency, after 36 generations, of 397 *ppt*. But *TES* becomes extinct in the ecosystem of five competitors.

The final ecosystem is composed of four nice strategies: *TFT*, *SHU*, *GRO*, and *ETH*. In such a system, all future generations of progeny maintain respective ratios of 1:1. Thus, initial frequencies and stable frequencies are identical and equal to one another, and stability is attained in the parent generation. This situation would, of course, obtain in an ecosystem of any size, providing that it were composed exclusively of nice strategies. The other nice strategies, however (namely *CHA*, *TTT*, *FRI*, *NYD* and *CCC*), are already extinct, because their respective combinations of attributes were disfavoured in previous ecosystemic competitions.

The last column of table 9.1 contains three numbers for each strategy. The first is that strategy's aggregate increase (or decrease) in frequency, cumulative over the entire ecological scenario; in other words, its overall fecundity. The second is the total number of generations survived by that strategy in all ecosystemic competitions (conducted to stability); in other words, its longevity. The third number is the quotient of the first two; in other words, is that strategy's average rate of increase (or decrease) in frequency, in parts per thousand per generation extant.

The entries show that, while the nine earliest-extinct strategies have aggregate decreases in frequency, three of the last four to become extinct, as well as one of the survivors, also have have aggregate decreases. Thus, while an aggregate increase in frequency indicates that a competitor does not face early extinction, neither is it a passport to ultimate survival.

One might find the survival of *GRO* perplexing. *GRO* experiences an increase in only two of the sixteen ecosystemic competitions that result in an extinction; nonetheless *GRO* survives to the final ecosystem. *GRO* does not excel in any of these competitions, and ranks near the bottom in all of them. Yet *GRO* is tenacious enough to survive them all, apparently by dint of consistent mediocrity. Since *GRO* is never highly successful, it cannot be said to depend on any particular strategies for its success. Hence *GRO* is not subject to the vicissitudes of over-exploitation, which cause the rise and fall of many of its more successful, and later extinct competitors.

Similarly, *TES* is the last strategy to become extinct. Despite its superlative performance in the one ecosystemic competition, *TES* also has an aggregate decrease in frequency.

On the other side of the coin, one finds that *ETH* has the largest aggregate increase in frequency, yet *ETH* won only two of the eliminatory ecosystemic competitions. Moreover, *CHA* has a larger aggregate increase than three of the four survivors, yet *CHA* eventually succumbs to extinction.

Table 9.2 illustrates the waxing and waning fortunes of the top five ranking strategies, at stability, for each of the ecosystemic competitions.

Table 9.2 - Top Five Strategies, With Respect to Stable Frequency

Competitors	First Place	Second Place	Third Place	Fourth Place	Fifth Place
20	*MAC*	*MAE*	*MEU*	*SHU*	*TES*
19	*MAC*	*MAE*	*MEU*	*SHU*	*FRI*
18	*FRI*	*SHU*	*MAC*	*MAE*	*MEU*
17	*FRI*	*SHU*	*MAC*	*MAE*	*MEU*
16	*CHA*	*ETH*	*TFT*	*TTT*	*SHU*
15	*CHA*	*ETH*	*TFT*	*SHU*	*TTT*
14	*CHA*	*ETH*	*MAC, SHU*	-	*TFT*
13	*CHA*	*MAC*	*SHU*	*ETH*	*TFT*
12	*CHA*	*ETH*	*MAC*	*TTT*	*TFT*
11	*ETH*	*CHA*	*TTT*	*TFT*	*TES*
10	*ETH*	*CHA*	*TTT*	*GRO*	*MAC*
9	*TFT*	*ETH*	*TES*	*TTT*	*CHA*
8	*TFT, ETH, SHU*	-	-	*GRO*	*CHA*
7	*TES*	*TFT, SHU, ETH*	-	-	*GRO*
6	*TFT, SHU, ETH*	-	-	*TES*	*GRO*
5	*TFT, SHU, ETH*	-	-	*GRO*	*TES*

Table 9.2 shows that, in general, the maximization family is most successful in the larger ecosystems; the optimization family, in the medium-sized ecosystems; the tit-for-tat family, in the smaller ecosystems. But no single strategy emerges as most robust overall, if the sole criterion of robustness is stable frequency. Indeed, though several strategies claim varying degrees of success in different sizes of ecosystem, it does not seem possible to ascribe a coherent order of robustness from one criterion alone.

One criterion suffices for Axelrod, who conducts a single ecosystemic competition among sixty-three strategies. Based solely on its magnitude of relative frequency in the population, *TFT* wins that particular competition. However, given what transpires in eliminatory ecosystemic competitions in the environment of the interactive tournament, it seems feasible to speculate that, if a similar range of competitions were conducted in Axelrod's environment, no single strategy would win them all. It seems rather more likely that one would observe a similar waxing and waning of strategic procreativity in different ecosystems.

Be that as it may, the question remains: how can one assess robustness across the range of ecosystemic competitions? Clearly, there is no unique way to accomplish this task. One possible method consists in a parametric approach, where relative robustness can be quantified according to certain parameters. The parameters themselves are quantifications of vital attributes of robustness in the ecological context. In other words, the above question is answered in three stages. First, vital properties of an ideal ecologically-robust strategy are posited. Second, the varying extents to which the competing strategies embody these properties are quantified according to appropriate ranking schemes. Third, these quantifications serve as parameters which reflect each strategy's combined embodiment of vital properties, and which permit a corresponding overall index of robustness to be assigned.

This enquiry utilizes four parameters, drawn from the ecological scenario. Four vital properties of an ecologically-robust strategy are posited, and their corresponding parameters defined, as follows:

(1) The ideal ecologically-robust strategy's progeny are able to avoid extinction. Hence the first parameter is survival, or ecosystemic longevity. Each strategy is ranked in ascending order of the total number of generations during which its progeny avoid extinction (regardless of their relative frequencies, if non-zero).

(2) The ideal ecologically-robust strategy is reproductively fit; i.e., its number of progeny increases in future generations. Hence the second parameter is overall average increase in relative

population frequency, between initial and stable states of every ecosystem. Each strategy is ranked in ascending order of the quotient of its aggregate frequency and the number of generations its progeny survive. This quotient is thus a measure of a strategy's average increase in relative frequency, in parts per thousand of the population per generation extant. (A negative increase, of course, indicates a decrease.)

(3) The ideal ecologically-robust strategy maintains a consistently high stable frequency, from one ecosystemic competition to another. Hence the third parameter is overall stable efficiency. A strategy's stable efficiency is computed in the following way. Suppose that strategy *A* has the j^{th}-largest stable frequency in an ecosystemic competition involving *k* competitors (including itself). Thus, strategy *A* achieves a higher stable frequency than $(k-j)$ other competitors. Its best possible performance (if it finishes first) entails achieving a higher stable frequency than $(k-1)$ other competitors (excluding itself). Hence, strategy *A*'s relative stable efficiency in this competition is $(k-j)/(k-1)$. Strategy *A*'s overall relative stable efficiency, in *n* ecosystemic competitions, is therefore

$$[(k_1-j_1) + (k_2-j_2) + \ldots + (k_n-j_n)]/[(k_1-1) + (k_2-1) + \ldots + (k_n-1)]$$

which is the net ratio of the number of competitors it betters to the number of competitors it faces. Each strategy is ranked in ascending order of its overall stable efficiency.

(4) The ideal ecologically-robust strategy shows adaptivity across the range of ecosystemic competitions, by means of consistent improvement within them. That is, it consistently increases its frequency, relative to other competitors, thereby tending to improve its position in a given competition. Hence the fourth parameter is the sum of the fractions of competitors overtaken in each competition, divided by the total number of competitions. If a strategy overtakes j_1/k_1 competitors in its first competition, j_2/k_2 competitors in its second competition, and so on, up to and including j_n/k_n competitors in its n^{th} competition, then that strategy's average

adaptivity is:

$$(1/n) \sum_{i=1}^{n} j_i/k_i$$

Each strategy is ranked in ascending order of the signed magnitude of its adaptivity, whose dimensions are: average fraction of competitors overtaken, per competition. (A negative adaptivity obtains when a strategy is overtaken by more competitors than it overtakes.)

The rankings of the strategies according to parameters (1) and (2), net longevity and average fecundity, are determined from table 9.1. The rankings according to parameters (3) and (4), stable efficiency and adaptivity, are determined from table 9.3 (overleaf).

Each cell of table 9.3 displays the initial and stable rankings for a given strategy in a given competition, according to the strategy's initial and stable relative frequencies. The given cell then displays that strategy's point-award in that competition (to be used in the calculation of its overall stable efficiency) and its change in frequency rank (to be used in the calculation of its adaptivity). For example, in the competition involving twenty strategies, *MEU*'s initial and stable frequencies are seventh and third-largest, respectively; whence the entry "7-3". So, at stability, *MEU* betters 20 minus 3, or 17 competitors; and *MEU*'s change in rank is 7 minus 3, or +4, whence the entry "+4,17".

Once again, a strategy's overall relative stable efficiency is the ratio of the sum of its stable point-awards (the total number of competitors it betters) to the total number of competitors it faces. *MEU*, for example, faced a total of (19+18+17+16+15+14+13+12+11) or 135 competitors, in consecutive ecosystemic competitions, before becoming extinct. *MEU* bettered a total of (17+16+13+12+5+4+2+1±0) or 70 competitors, in terms of stable frequency rankings in these competitions. Hence, *MEU*'s overall stable efficiency is 70/135, or 51.9 percent. By contrast, for example, *TIT* faced a total of 175 competitors (it survived more competitions than did *MEU*), of which it bettered a total of 94. *TIT*'s overall stable efficiency is thus

94/175, or 53.7 percent.

Table 9.3 – Fecundity Rankings at Initial and Stable Frequencies

	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	S.E.% Adap.
TFT	7-10	7-8	6-6	6-6	3-3	3-3	3-5	3-5	3-5	4-4	3-6	1-1	1-1	2-2	1-1	1-1	1-1	72.7
	-3,10	-1,11	±0,12	±0,11	±0,13	±0,12	-2,9	-2,8	-2,7	±0,7	-3,4	±0,8	±0,7	±0,5	±0,5	±0,4	±0,3	-.062
SHU	3-4	2-4	1-2	2-2	5-5	4-4	5-3	5-3	6-8	6-7	7-8	5-6	1-1	2-2	1-1	1-1	1-1	75.9
	-1,16	-2,15	-1,16	±0,15	±0,11	±0,11	+2,11	+2,10	-2,4	-1,4	-1,2	-1,3	±0,7	±0,5	±0,5	±0,4	±0,3	-.025
GRO	11-17	11-14	11-15	10-15	9-12	9-12	8-10	8-10	7-10	6-8	5-4	7-7	1-4	2-5	1-5	1-4	1-1	27.3
	-6,3	-3,5	-4,3	-5,2	-3,4	-3,3	-2,4	-2,3	-3,2	-2,3	+1,6	±0,2	-3,4	-3,2	-4,1	-3,1	±0,3	-.271
ETH	4-9	3-7	4-7	2-9	2-2	1-2	2-2	1-4	1-2	1-1	1-1	2-2	1-1	2-2	1-1	1-1	1-1	80.2
	-5,11	-4,12	-3,11	-7,8	±0,14	-1,13	±0,12	-3,9	-1,10	±0,10	±0,9	±0,7	±0,7	±0,5	±0,5	±0,4	±0,3	-.089
TES	7-5	8-11	7-9	7-7	5-5	6-6	5-7	5-8	5-6	5-5	5-7	4-3	6-7	1-1	1-4	1-5	-	56.5
	+2,15	-3,8	-2,9	±0,10	±0,11	±0,9	-2,7	-3,5	-1,6	±0,6	-2,3	+1,6	-1,1	±0,6	-3,2	-4,0		-.157
CHA	4-8	3-6	4-8	1-8	1-1	1-1	1-1	1-1	1-1	2-2	2-2	3-5	1-5	2-6	1-6	-	-	74.4
	-4,12	-3,13	-4,10	-7,9	±0,15	±0,14	±0,13	±0,12	±0,11	±0,9	±0,8	-2,4	-4,3	-4,1	-5,0			-.236
TTT	10-13	8-9	9-10	7-12	4-4	4-5	4-8	4-7	3-4	3-3	3-3	5-4	7-6	7-7	-	-	-	53.7
	-3,7	-1,10	-1,8	-5,5	±0,12	-1,10	-4,6	-3,6	-1,8	±0,8	±0,7	+1,5	+1,2	±0,0				-.074
FRI	4-6	3-5	2-1	5-1	8-8	8-8	8-6	9-6	9-9	9-9	9-9	8-8	8-8	-	-	-	-	58.0
	-2,14	-2,14	+1,17	+4,16	±0,8	±0,7	+2,8	+3,7	±0,3	±0,2	±0,1	±0,1	±0,0					+.038
MAC	1-1	1-1	2-3	4-3	7-7	6-6	7-3	7-2	7-3	8-6	8-5	9-9	-	-	-	-	-	77.2
	±0,19	±0,18	-1,15	+1,14	±0,9	±0,9	+4,11	+5,11	+4,9	+2,5	+3,5	±0,0						+.135
BBE	11-15	11-13	11-11	10-13	11-10	11-10	11-11	11-11	11-11	10-10	10-10	-	-	-	-	-	-	26.0
	-4,5	-2,6	±0,7	-3,4	+1,6	+1,5	±0,3	±0,2	±0,1	±0,1	±0,0							-.034
MAE	2-2	6-2	7-4	9-4	10-9	10-9	10-9	10-9	10-7	11-11	-	-	-	-	-	-	-	61.4
	±0,18	+4,17	+3,14	+5,13	+1,7	+1,6	+1,5	+1,4	+3,5	±0,0								+.128
MEU	7-3	8-3	10-5	10-5	12-11	12-11	12-12	12-12	12-12	-	-	-	-	-	-	-	-	51.9
	+4,17	+5,16	+5,13	+5,12	+1,5	+1,4	±0,2	±0,1	±0,0									+.137
RAN	15-16	15-16	14-16	14-16	13-14	13-14	13-13	13-13	-	-	-	-	-	-	-	-	-	11.3
	-1,4	-1,3	-2,2	-2,1	-1,2	-1,1	±0,1	±0,0										-.061
TQD	13-11	13-12	13-14	13-14	14-13	14-13	14-14	-	-	-	-	-	-	-	-	-	-	25.0
	+2,9	+1,7	-1,4	-1,3	+1,3	+1,2	±0,0											+.025
MAD	13-7	14-10	15-11	15-10	15-15	15-15	-	-	-	-	-	-	-	-	-	-	-	37.4
	+6,13	+4,9	+4,7	+5,7	±0,1	±0,0												+.181
DDD	15-12	16-15	15-13	15-11	16-16	-	-	-	-	-	-	-	-	-	-	-	-	27.1
	+3,8	+1,4	+2,5	+4,6	±0,0													+.116
TAT	17-14	18-17	18-17	17-17	-	-	-	-	-	-	-	-	-	-	-	-	-	12.9
	+3,6	+1,2	+1,1	±0,0														+.068
TQC	19-18	18-18	17-17	-	-	-	-	-	-	-	-	-	-	-	-	-	-	7.4
	+1,2	±0,1	±0,1															+.018
NYD	18-19	16-18	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5.4
	-1,1	-2,1																-.082
CCC	20-20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0
	±0,0																	±0.0

And *MEU*'s average adaptivity, for example, is found from the sum (4/19 + 5/18 + 5/17 + 5/16 + 1/15 + 1/14 ± 0/13 ± 0/12 ± 0/11) [the fractions of competitors it overtakes in each competition], divided by 9 [the total number of competitions in which *MEU* competes]. *MEU*'s adaptivity is therefore +.137. By contrast, *TIT*'s adaptivity is (−3/19 − 1/18 − 1/17 − 5/16 − 0/15 − 1/14 − 4/13 − 3/12 − 1/11 ± 0/10 ± 0/9 + 1/8 + 1/7 ± 0/6) divided by 14, or −.074. So, in terms of difference between stable and initial population frequency, *MEU* overtakes, on average, .137 of its competitors per competition; while *TIT* is overtaken, on average, by .074 of its competitors per competition.

Thus, while *TIT* is slightly more efficient than *MEU*, it is also considerably less adaptive. All strategies' overall stable efficiencies, and adaptivities, are displayed in the last column of table 9.3.

Now, from tables 9.1 and 9.3, one has four different ranking schemes, which order the strategies in terms of the four parameters: longevity, fecundity, stability, and adaptivity. These rank parameters are abbreviated, respectively, as R_l, R_f, R_s, and R_a. With each strategy is then associated a unique set of four rank numbers, which correspond to that strategy's particular values for $\{R_l, R_f, R_s, R_a\}$.

A given strategy's index of robustness, I_r, is evaluated in the following way. Each of its four rank numbers is subtracted from twenty, to give the number of competitors it betters according to each parameter. These four new numbers are then added, and their sum is divided by 76 (which is the total number of competitors it could have bettered overall; i.e., nineteen competitors in each of four schemes). This quotient is the given strategy's index of robustness. That is,

$$I_r = [(20-R_l) + (20-R_f) + (20-R_s) + (20-R_a)]/76$$

or

$$I_r = [80 - (R_l + R_f + R_s + R_a)]/76$$

The ideal ecologically-robust strategy would rank first in each scheme, and its index of robustness would then attain the maximum value of unity. An utterly non-robust strategy would rank twentieth in each scheme, and its index of robustness would take on the minimum value of zero.

The magnitudes of the four parameters, their corresponding rank numbers, and the resulting indices of robustness are displayed in table 9.4.

According to this parametric approach, *MAC* is the most ecologically-robust strategy, followed by *SHU, ETH, TFT* and *MAE*, to round out the top five. Although *MAC* became extinct earlier than its most robust rivals (which rank first in longevity compared to *MAC*'s ninth), these rivals prove comparatively less adaptive. In fact, *SHU, ETH* and *TFT* are all negatively-adapted; that is, they are surpassed, on average, by a larger fraction of competitors than they surpass.

These parameters are quite revealing with respect to the competitive performance of a given strategy, as indeed they must be if they are to provide a reasonable quantification of robustness.

Table 9.4 - Four-Parameter Quantification of Ecological Robustness

	Longe-vity	R_l	Avg Fe-cundity	R_f	Stable Eff%	R_s	Adapti-vity	R_a	I_r
MAC	2995	9	+.117	3	77.2	2	+.135	3	.829
SHU	3261	1	+.109	4	75.9	3	-.025	11	.803
ETH	3261	1	+.232	1	80.2	1	-.089	17	.789
TFT	3261	1	+.108	5	72.7	5	-.062	14	.724
MAE	2666	11	+.059	6	61.4	6	+.128	4	.697
MEU	2447	12	-.015	7	51.9	10	+.137	2	.645
CHA	3227	6	+.150	2	74.4	4	-.236	19	.645
FRI	3106	8	-.042	10	58.0	7	+.038	7	.632
MAD	1789	15	-.063	11	37.4	11	+.181	1	.553
TES	3260	5	-.023	8	56.5	8	-.157	18	.539
TTT	3179	7	-.038	9	53.7	9	-.074	15	.526
DDD	1259	16	-.085	12	27.1	13	+.116	5	.447
GRO	3261	1	-.159	16	27.3	12	-.271	20	.408
TQD	2031	14	-.102	13	25.0	15	+.025	8	.395
BBE	2917	10	-.135	14	26.0	14	-.034	12	.395
TAT	795	17	-.172	17	12.9	16	+.068	6	.316
RAN	2383	13	-.145	15	11.3	17	-.061	13	.289
TQC	345	18	-.365	19	7.4	18	+.018	9	.211
CCC	9	20	-4.60	20	0.0	20	±.000	10	.132
NYD	335	19	-.257	18	5.4	19	-.082	16	.105

Examine the case of *ETH*, for example. *ETH* shares the greatest longevity, produces the largest average number of offspring per generation, and is most efficient in overall stable frequency rankings. Given this outstanding combination of attributes, one might expect *ETH* to win a substantial number of ecosystemic competitions. Yet a glance at table 9.2 shows that *ETH* is the outright winner in only two of the competitions. Moreover, those competitons do not involve a relatively large number of strategies (11 and 10 strategies, respectively). In fact, in table 9.2, *ETH* is conspicuously absent from the top rankings in competitions involving 20, 19, 18 and 17 strategies. Why does *ETH* not fare better?

The fourth parameter provides an explanation. *ETH* turns out to be one of the least-adaptive strategies. *ETH*'s great longevity, prodigious fecundity, and high efficiency do not reveal its principal weakness: in larger groups, *ETH* is readily overtaken by a substantial fraction of competitors. These competitors, which produce fewer progeny on average than *ETH*, and which better fewer strategies overall than *ETH*, are nevertheless more reproductively fit than *ETH* when the competitive traffic is heaviest (as table 9.3 reveals). Thus, notwithstanding *ETH*'s fortitude with respect to three attributes, *ETH*'s robustness is compromised by an acute lack of adaptivity in large groups.

No single attribute, however outstanding, suffices for great robustness in eliminatory competitions. *GRO*, for example, has a share of the greatest longevity, but it experiences a considerable average decrease in fecundity, a middling stable efficiency, and the poorest adaptivity in the scenario. These results sink *GRO* to thirteenth place in robustness. Thus, while *GRO* endures, it neither thrives nor prospers. Similarly, *MAD* is the most adaptive strategy, surpassing a larger average fraction of its competitors than any other strategy; but *MAD* is fairly short-lived, negatively-fecund, and not very efficient. In sum, *MAD* ranks ninth in robustness.

The seven most robust strategies, not surprisingly, are also the seven most fecund (though not in that order). The three most robust strategies are also the most efficient (though again, not in that order). Overall, fecundity and efficiency are the most closely correlated pair of attributes. But the two most robust strategies, *MAC* and *SHU*, show respective improvements in rank with respect to this attribute pair. *MAC* ranks third in fecundity and second in efficiency; *SHU*, fourth in fecundity and third in efficiency. This type of improvement, however slight, denotes an interesting performance characteristic; namely, an effective frequency distribution of progeny across the range of ecosystemic competitions.

Average relative frequencies, by definition, do not take instantaneous changes in frequency (from one competition to another) into account. *MAC* experiences an increase in progeny in nine of its twelve competitions; *SHU*, an increase in eleven of its seventeen competitions. Both *MAC* and *SHU* achieve frequency distributions which, in terms of rank efficiency, enable these strategies to realize the beneficial potential of their increases and to minimize the detrimental effects of their decreases.

In contrast, *CHA* ranks second in fecundity, but slips to fourth in efficiency. Although *CHA*'s average increase in progeny is greater than that of *MAC* and *SHU*, *CHA*'s distribution of instantaneous increases is less effective. *CHA* experiences an increase in progeny in ten of its fifteen competitions (and no change in one competition), but its largest increases occur in competitions in which a smaller increase would confer the same efficiency rank. In other words, *CHA* produces more offspring than it requires in some situations, and not enough in others. *CHA* is nonetheless relatively robust, although its robustness is severely compromised by its poor adaptivity.

The point to be made here is that, notwithstanding instances of pair-wise correspondence between R_f and R_s these two rank parameters reflect quite distinct attributes. A given strategy's difference in rank between these parameters (or lack thereof) is indicative of a particular performance characteristic.

Finally, one can compare strategic robustness in the combinatoric sub-tournaments of the previous chapter with strategic robustness in this ecological scenario. The order of overall robustness is determined by taking the average of each strategy's rank with respect to combinatoric and ecological robustness. Since *MAC* ranks first in both categories, it is obviously most robust overall in the interactive environment. *SHU* is deserving of second overall, while *MAE* retains third overall despite its decline in the ecological scenario.

Table 9.5 - Comparison of Strategic Robustness

Rank	Combinatoric Robustness	Ecological Robustness	Overall Robustness
1	*MAC*	*MAC*	*MAC*
2	*MAE*	*SHU*	*SHU*
3	*SHU*	*ETH*	*MAE*
4	*FRI*	*TFT*	*ETH*
5	*CHA*	*MAE*	*TFT, CHA*
6	*ETH*	*MEU, CHA*	-
7	*TFT*	-	*FRI*
8	*TES*	*FRI*	*MEU*
9	*MEU*	*MAD*	*TES*
10	*TTT*	*TES*	*MAD*
11	*MAD*	*TTT*	*TTT*
12	*GRO*	*DDD*	*GRO*
13	*TQD*	*GRO*	*DDD, TQD*
14	*BBE*	*TQD*	-
15	*DDD*	*BBE*	*BBE*
16	*RAN*	*TAT*	*RAN, TAT*
17	*TAT*	*RAN*	-
18	*NYD*	*TQC*	*TQC*
19	*TQC*	*CCC*	*NYD*
20	*CCC*	*NYD*	*CCC*

Once again, it must be stressed that this parametric approach to the evaluation of ecological robustness is by no means a unique determinant; many other schemes could be conceived and applied. The addition or deletion of a single parameter can alter the standings, either mildly or drastically. One might hypothesize that a parametric approximation of ecological robustness would improve as the number of parameters increases. While more (or fewer) than four parameters could be used, the result in this case seems reasonably unbiased. At the least, an attempt has been made to neutralise or otherwise balance any bias that inheres in such a quantification.

The ecological scenario is clearly rich in interactions and implications, and many more such models can and should be developed within the evolutionary paradigm. The main difference between ecological and evolutionary modelling is, as Axelrod points out, that the former does not admit of any "mutational" influences.[15] In other words, the ecology unfolds strictly from initial conditions, with no behavioural modifications made to the strategies involved. However, it is evident that, on the basis of strategic interaction alone, and in the absence of strategic modification, the complexities of eliminatory ecosystemic competition necessitate correspondingly complex methods of assessing robusteness.

Having found *MAC* to be the most robust strategy overall in the interactive environment, according to combinatoric and ecological criteria that are admittedly not unique but also not necessarily unfair, this enquiry now seeks to answer some questions raised by these findings. Why is *MAC* most robust? Why are *MAC*'s maximization family members less robust? Given that these family members differ only by an initial probabilistic weighting factor, why does their familial order of robustness increase with the co-operativeness of their respective weightings? What are *MAC*'s principal weaknesses, and can they be improved?

It is the task of the next section to address these and other pertinent questions.

---

[15] Ibid, p.399.

PART FOUR:

THE ETHIC OF COLLECTIVE RATIONALITY

Chapter Ten

The Maximization Family Versus Others

Thus far, analyses of the interactive tournament have taken completed game scores as a departure point for the manipulation of data. A strategy's game scores have been treated as "finished products", from which desired "by-products" (such as combinatoric and ecological robustness) are obtained. This part of the enquiry is devoted to an examination of the actual process by which the maximization family "manufactures" its game scores. So, while Part Three can be said to have adopted a macroscopic view of the interactive tournament, the first two chapters of Part Four will adopt a microscopic view. This higher resolution of analysis should enable a better understanding of the mechanics of the maximization family, and of *MAC*'s particular robustness in the interactive environment.

Axelrod concludes the analysis of his second tournament with a salient observation:

> "Being able to exploit the exploitable without paying too high a cost with the others is a task which was not successfully accomplished by any of the entries in round two of the tournament."[1]

The main implication of his observation is that a strategy capable of accomplishing this task could have won the second tournament.

The only maximization strategy to have participated in that tournament was *Downing*, an equivalent of *MEU*. *Downing* is fully able to exploit the exploitable, but *Downing* ranked fortieth among sixty-three strategies. This undistinguished performance is attributable to *Downing* having paid too high a price with the others.

In the environment of the interactive tournament, the task defined by Axelrod is accomplished in large measure by *MAC*, and to a lesser extent by *MAE*. Although *MEU* and *MAD* are also able to exploit the exploitable, they (like *Downing*) pay too high a price with certain others. Indeed, it has been quite apparent that the performance of the maximization family members improves with the co-operativeness of their weightings. To illustrate the performances of the

---

[1] Axelrod, 1980*b*, pp.403.

maximization family in the context of Axelrod's task, one examines their games against two respresentative strategies: *CCC* (representing "the exploitable"), and *TFT* (representing "the others").[2]

Recall that the maximization family members begin their games by co-operating or defecting randomly for one hundred moves, while recording all joint outcomes in an event matrix. During these one hundred moves, *MAC* co-operates with probability 9/10; *MAE*, with probability 5/7; *MEU*, with probability 1/2; *MAD*, with probability 1/10. From the one-hundred-and-first move onward, members of this family maximize their expected utilities, and continue to update the event matrix after each joint outcome. The frequency distribution of outcomes in the event matrix constitutes the *a posteriori* probability distribution in the calculation of expected utilities.

Recall also that the respective payoffs of the possible outcomes are:
$(C,c) = 3,3$; $(C,d) = 0,5$; $(D,c) = 5,0$; $(D,d) = 1,1$. Thus, if outcome (C,c) occurs on W occasions, (C,d) on X occasions, (D,c) on Y occasions, and (D,d) on Z occasions, then the expected utilities are

$$EUC = 3W/(W+X)$$
$$EUD = (5Y+Z)/(Y+Z)$$

First, consider how the maximization family exploits *CCC*, beginning with *MAD*. By definition, *CCC* co-operates unconditionally, while *MAD* co-operates with probability 1/10 during the first one hundred moves. In game 10.1, seven outcomes of (C,c) and ninety-three outcomes of (D,c) have occurred. The score is correspondingly lopsided.

---

[2] Note: the final scores of these sample games may differ slightly from the game scores between identical opponents in Appendix Two. Differences are due to expected fluctuations in the computer's pseudo-random generator, which is re-seeded on each occasion that a program containing a probabilistic component is run.

Game 10.1 - *MAD* versus *CCC*, Event Matrix After 100 Moves

		CCC	
		c	*d*
MAD	*C*	7	0
	D	93	0

EUC = (3x7)/(7+0) = 3

EUD = (5x93 + 1x0)/(93+0) = 5

Score after 100 moves: *MAD* 486, *CCC* 21

Note that the expected utility of co-operation is at its theoretical maximum (which is 3 for this particular payoff structure), since no instances of (*C,d*) have occurred. A (*C,d*) outcome would reduce the expected utility of co-operation by its presence in the denominator of the *EUC* equation. Similarly, the expected utility of defection is also at its theoretical maximum (which is 5 for this particular payoff structure), since no instances of (*D,d*) have occurred. Although both *EU*'s are at their respective maxima, *EUD* > *EUC*. Hence *MAD* defects on move one-hundred-and-one.

After two hundred moves, the situation is as follows:

Game 10.2 - *MAD* versus *CCC*, Event Matrix After 200 Moves

		CCC	
		c	*d*
MAD	*C*	7	0
	D	193	0

EUC = (3x7)/(7+0) = 3

EUD = (5x193 + 1x0)/(193+0) = 5

Score after 200 moves: *MAD* 986, *CCC* 21

Between moves 101 and 200, there have been 100 successive outcomes of (*D,c*), with *MAD* having out-scored *CCC* by 500 to zero. *CCC* continues to co-operate, while *MAD*'s expected utilities remain unchanged. As on the previous one hundred moves, defection is prescribed for *MAD* on move two-hundred-and-one.

By induction, it is obvious that the outcome (*D*, *c*) obtains for the duration of the game. Indeed, after 1000 moves, one finds:

Game 10.3 - *MAD* versus *CCC*, Event Matrix After 1000 Moves

		CCC *c*	*d*	
MAD	*C*	7	0	*EUC* = (3x7)/(7+0) = 3
	D	993	0	*EUD* = (5x993 + 1x0)/(993+0) = 5

Score after 1000 moves: *MAD* 4986, *CCC* 21

CCC is thoroughly exploited by *MAD*. Again by induction (following the one hundred probabilistic moves), one can show that the other maximization family members also exploit *CCC*. Consider *MEU*'s performance, after 100 and 1000 moves:

Game 10.4 - *MEU* versus *CCC*, Event Matrices (100 & 1000 Moves)

100 moves:

		CCC *c*	*d*
MEU	*C*	50	0
	D	50	0

EUC = 3, *EUD* = 5

Score: *MEU* 400, *CCC* 150

1000 moves:

		CCC *c*	*d*
MEU	*C*	50	0
	D	950	0

EUC = 3, *EUD* = 5

Score: *MEU* 4900, *CCC* 150

The only difference between *MAD*'s and *MEU*'s performances against *CCC* lies in the frequency distribution of outcomes after 100 moves. Since *MEU* is probabilistically more co-operative than *MAD*, more instances of (*C*, *c*) obtain during *MEU*'s initial hundred moves. As a result, *CCC* garners more points. But observe that *MEU*'s expected utilities are identical to *MAD*'s, and that they too remain constant throughout. Hence *MEU* defects from move 101 to the end of the game, and ultimately exploits *CCC* almost as thoroughly as does *MAD*.

Similarly, *MAE* exploits *CCC*, but not quite as thoroughly as does *MEU*:

Game 10.5 - *MAE* versus *CCC*, Event Matrices (100 & 1000 Moves)

100 moves: *CCC*

		c	*d*
MAE	*C*	76	0
	D	24	0

EUC = 3, *EUD* = 5

Score: *MAE* 348, *CCC* 228

1000 moves: *CCC*

		c	*d*
MAE	*C*	76	0
	D	924	0

EUC = 3, *EUD* = 5

Score: *MAE* 4848, *CCC* 228

Once again, with the exception of the first hundred moves, *MAE*'s performance against *CCC* is identical to that of *MAD* and of *MEU*. Finally, consider the performance of *MAC*. As expected, *MAC* also exploits *CCC*, but does so to a slightly lesser extent than *MAE*:

Game 10.6 - *MAC* versus *CCC*, Event Matrices (100 & 1000 Moves)

100 moves: *CCC*

		c	*d*
MAC	*C*	87	0
	D	13	0

EUC = 3, *EUD* = 5

Score: *MAC* 326, *CCC* 261

1000 moves: *CCC*

		c	*d*
MAC	*C*	87	0
	D	913	0

EUC = 3, *EUD* = 5

Score: *MAC* 4826, *CCC* 261

All maximization family members achieve scores against *CCC* that are well in excess of their respective average scores in the main tournament. (Their average scores, from table 8.1, are: *MAD*, 2086; *MEU*, 2362; *MAE*, 2503; and *MAC*, 2645 points per game.) *CCC*, on the other hand, achieves scores against this family that are well below its tournament average (which is 1824 points per game).

Completely different situations obtain, however, when the maximization family competes against *TFT*. Because it is highly provocable, *TFT* is not exploitable. First, consider the performance of *MAD* after 100 and 1000 moves:

Game 10.7 - *MAD* versus *TFT*, Event Matrices (100 & 1000 Moves)

100 moves:

		TFT	
		c	*d*
MAD	*C*	2	11
	D	12	75

EUC = .462, *EUD* = 1.55

Score: *MAD* 141, *TFT* 136

1000 moves:

		TFT	
		c	*d*
MAD	*C*	2	11
	D	12	975

EUC = .462, *EUD* = 1.05

Score: *MAD* 1041, *TFT* 1036

During the first hundred moves, *MAD* randomly defected on 87 occasions and co-operated on 13 occasions. Since *TFT* responded in kind after its initial co-operation, it defected on 86 occasions and co-operated on 14 occasions. Thus, after one hundred moves, *MAD* leads *TFT* by only five points, which *MAD* accrued on the first move's (*D*,*c*) outcome. And, after one hundred moves, *EUD* well exceeds *EUC*. Hence *MAD* defects on move 101, and *TFT* follows suit.

It transpires that both strategies become locked into mutual defection for the duration of the game. The event martrix for 1000 moves differs from that for 100 moves only with respect to the number of (*D*,*d*) outcomes, which has increased by 900 in the 900 subsequent moves. The game score stands at *MAD* 1041, *CCC* 1036; *MAD*'s narrow margin of victory having been gained on the very first move of the game.

That no outcome other than (D,d) occurred in the last 900 moves shows that *EUD* remained greater than *EUC* from move 101 onward. However, an interesting and important phenomenon becomes manifest in this game. Note that the value of *EUD* actually diminishes between 100 and 1000 moves (from 1.55 to 1.05). In fact, *EUD* decreases uniformly from move 101 onward, and it is the very occurrence of a (D,d) outcome that forces the decrease. In other words, the occurrence of a mutual defection has the effect of lowering the expected utility of further defection.

In the case of *MAD* versus *TFT*, however, the manifestation of this phenomenon does not result in eventual mutual co-operation, since the value of the expected utility of co-operation shows no increase at all during the course of the game. One has already observed that the maximum possible values of *EUC* and *EUD* (for the current payoff structure) are 3 and 5, respectively. One is now interested in the minimum possible value of *EUD*. If this minimum is smaller than 3, then a sufficient decrease in *EUD* combined with a sufficient increase in *EUC* can result in the alteration of a maximization family member's play, from defection to co-operation.

Recall the equation for the expected utility of defection:

$$EUD = (5Y+Z)/(Y+Z)$$

This is a two-variable function, which takes on its maximum of 5 when Z equals zero (as in the case of *MAD* vesus *CCC*), and takes on its minimum of unity when Y equals zero. When both Y and Z are non-zero (as in the case of *MAD* versus *TFT*), the function approaches these extrema only in the limit, with one variable held constant and the other increasing without bound. For the minimum value:

$$EUD_{min} = \lim_{Z\to\infty}(5Y+Z)/(Y+Z) = 1$$

Thus, if the number of mutual defections increases without bound, the expected utility of defection is driven toward its minimum value of unity. This value is well below the maximum value of *EUC*, so it is indeed possible for a maximization family member to alter its play during the course of a game, from defection to co-operation.

In the case of *MAD* versus *TFT*, one can now appreciate that 1000 moves sufficed to drive the value of *EUD* quite close to its lower limit. But since the value of *EUC* did not increase at all, *MAD* continued to defect.

Now consider the performance of *MEU* against *TFT*:

<u>Game 10.8 – *MEU* versus *TFT*, Event Matrices (100 & 200 Moves)</u>

100 moves:

		TFT c	*TFT* d
MEU	*C*	21	27
	D	27	25

EUC = 1.31, *EUD* = 3.08

Score: *MEU* 223, *TFT* 223

200 moves:

		TFT c	*TFT* d
MEU	*C*	21	27
	D	28	124

EUC = 1.31, *EUD* = 1.73

Score: *MEU* 327, *TFT* 322

During its first hundred moves, *MEU* has randomly co-operated on 48 occasions, and defected on 52 occasions. The score is tied after 100 moves. Since *EUD* is greater than *EUC*, *MEU* defects on move 101. *MEU* must have co-operated on move 100, since *TFT* co-operates on move 101. Hence, at move 101, the (D,c) entry in the event matrix is incremented. Mutal defection ensues for the next 99 moves, during which *EUD* decreases from 3.08 to 1.73. After 200 moves, *MEU* leads *TFT* by the five points it accrued on move 101.

Between moves 201 and 300, another 100 mutual defections occur. *EUC* remains constant at 1.31, while *EUD* drops to 1.44. *MEU* retains its five point advantage, and leads *TFT* by 427 to 422 after 300 moves. During the next two hundred moves, the following events take place.

Between moves 301 and 400, another 100 mutual defections have occurred. But the value of *EUD* has been driven very close to that of

EUC. Following seven more mutual defections, at move 407, the value of *EUD* falls below that of *EUC* (1.311 to 1.312, respectively). In consequence, at move 408, *MEU* co-operates. But *TFT* defects at move 408, in response to *MEU*'s previous defection at move 407. Thus the event matrix outcome of (*C,d*) is incremented (from 27 to 28) at move 408. But this increment results in a decrease in the value of the expected utility of co-operation, from 1.312 to 1.286.

Game 10.9 - *MEU* versus *TFT*, Event Matrices (400 & 500 Moves)

400 moves: *TFT*

		c	*d*
MEU	*C*	21	27
	D	28	324

EUC = 1.312, *EUD* = 1.318

Score: *MEU* 527, *TFT* 522

500 moves: *TFT*

		c	*d*
MEU	*C*	21	29
	D	30	420

EUC = 1.26, *EUD* = 1.267

Score: *MEU* 633, *TFT* 628

Naturally, from *MEU*'s point of view, a co-operative move on its part in tandem with a defection by its opponent is detrimental to *MEU*'s expected utility of co-operation. (In fact, *TFT* has gained five points on this move, and has tied the score.) Hence, at move 409, *EUD* is once again greater than *EUC*, by 1.311 to 1.286, and *MEU* defects anew. But *TFT* co-operates on move 409, in response to *MEU*'s previous co-operation. Thus the event matrix outcome of (*D,c*) is incremented (from 28 to 29) on move 409, and *MEU* regains a five-point lead.

But this increment, not surprisingly, results in an increase in *MEU*'s expected utility of defection, from 1.311 to 1.322. Another sequence of mutual defections follows, until the value of *EUD* is once again driven below that of *EUC*. Then the above process repeats itself. At this juncture, the strategies have reached the half-way mark of their game.

Although *EUC* has superseded *EUD* on two separate occasions, no instances of mutual co-operation have occurred. Clearly, *EUC* must remain greater than *EUD* on two consecutive occasions for mutual co-operation to occur. But the existing distribution of outcomes for these strategies, which eventually gives rise to the sequence (D,d), (C,d), (D,c), (D,d), in fact precludes the possibility of (C,c) occurring, because the distribution is re-enforced by the very sequence it produces. Consider the final one hundred moves of the game:

Game 10.10 - *MEU* versus *TFT*, Event Matrices (900 & 1000 Moves)

900 moves:

		TFT c	*TFT* d
MEU	C	21	34
	D	35	810

$EUC = 1.145$, $EUD = 1.657$

Score: *MEU* 1048, *TFT* 1043

1000 moves:

		TFT c	*TFT* d
MEU	C	21	34
	D	35	910

$EUC = 1.145$, $EUD = 1.148$

Score: *MEU* 1148, *TFT* 1143

In sum, a string of mutual defections periodically drives the value of *EUD* below that of *EUC*. Then, an outcome of (C,d) ensues, which in turn depresses the value of *EUC*. An outcome of (D,c) follows, which temporarily inflates the value of *EUD*. Another string of mutual defections ensues, and the pattern repeats until the game ends. Both expected utilities are driven toward their minimum limiting value of unity. *MEU* defeats *TFT* by the five points it accrues on the ultimate occurrence of (D,c).

It transpires that *MAE*'s performance against *TFT* unfolds in a similar fashion, with one significant difference. During its first hundred moves, *MAE* co-operates randomly with a probability of 5/7, as compared with *MEU*'s 1/2. This results in a comparatively more co-operative distribution of outcomes for those one hundred moves, which in turn increases the frequency of the movement away from mutual defection. Consider the first two hundred moves of *MAE*'s performance against *TFT*.

Game 10.11 – *MAE* versus *TFT*, Event Matrices (100 & 200 Moves)

100 moves:

		TFT	
		c	*d*
MAE	*C*	44	22
	D	22	12

EUC = 2.00, *EUD* = 3.588

Score: *MAE* 254, *TFT* 254

200 moves:

		TFT	
		c	*d*
MAE	*C*	44	27
	D	28	101

EUC = 1.859, *EUD* = 1.868

Score: *MAE* 373, *TFT* 368

Observe that the sequence of outcomes (*D,d*), (*C,d*), (*D,c*), (*D,d*), which did not occur until after the four hundredth move in the game between *MEU* and *TFT*, occurs five times before the two hundredth move in the game between *MAE* and *TFT*. This pattern repeats, with comparatively greater frequency, for the duration of the game. Consider the final one hundred moves:

Game 10.12 – *MAE* versus *TFT*, Event Matrices (900 & 1000 Moves)

900 moves:

		TFT	
		c	*d*
MAE	*C*	44	58
	D	59	739

EUC = 1.294, *EUD* = 1.295

Score: *MAE* 1166, *TFT* 1161

1000 moves:

		TFT	
		c	*d*
MAE	*C*	44	60
	D	61	835

EUC = 1.269, *EUD* = 1.272

Score: *MAE* 1272, *TFT* 1267

From move 101 onward, a total of 77 departures from mutual defection occurred between *MAE* and *TFT*, as compared with none between *MAD* and *TFT*, and only 15 between *MEU* and *TFT*. Nonetheless, *MAE* is unable to make the two consecutive co-operative moves necessary to engender an occurrence of mutual co-operation. In consequence, the *MAE-TFT* pair attains slightly higher scores than the *MAD-TFT* and *MEU-TFT* pairs, but all these scores remain well below the average main tournament scores of the strategies involved.

When *MAC* competes against *TFT*, a rather different picture emerges. Consider the first two hundred moves of their encounter:

Game 10.13 - *MAC* versus *TFT*, Event Matrices (100 & 200 Moves)

100 moves:

		TFT *c*	*TFT* *d*
MAC	*C*	84	7
	D	7	2

EUC = 2.769, *EUD* = 4.111

Score: *MAC* 289, *TFT* 289

200 moves:

		TFT *c*	*TFT* *d*
MAC	*C*	173	8
	D	8	11

EUC = 2.867, *EUD* = 2.684

Score: *MAC* 570, *TFT* 570

In Game 10.13, novel circumstances arise. During its first hundred moves, *MAC* randomly co-operates on 91 occasions, and defects on 9 occasions. *TFT*, of course, replies in kind. But owing to the preponderance of *MAC*'s co-operations over its defections, *MAC* is bound to co-operate on a fair number of consecutive occasions. Combined with *TFT*'s play, the result is a relatively large number of (*C*, *c*) outcomes during the first hundred moves. Indeed, although *MAC*'s initial co-operative weighting is only 18% higher than *MAE*'s, the *MAC-TFT* pair realizes almost twice as many mutually co-operative outcomes as the *MAE-TFT* pair (84 to 44, during the first hundred moves).

Although *MAC*'s expected utility of defection is greater than its expected utility of co-operation at move 101, the distribution of outcomes after 100 moves is co-operative enough for the pair to lock into mutual co-operation within a dozen further moves. They do so in the following way.

MAC defects at move 101. Since *MAC* co-operated at move 100, *TFT* co-operates at move 101. The *(D, c)* outcome gives *MAC* a five-point lead, and temporarily inflates the value of the expected utility of defection, from 4.11 to 4.2. A string of mutual defections ensues, which forces the value of the expected utility of defection steadily downward, while the value of the expected utility of co-operation remains unchanged (at 2.769). At move 111, *EUD* is finally less than *EUC* (2.684 to 2.769), so *MAC* co-operates. Owing to *MAC*'s defection at move 110, *TFT* defects at move 111. The *(C, d)* outcome allows *TFT* to tie the score, and depresses the value of the expected utility of co-operation, from 2.769 to 2.739.

In the games involving *MAD*, *MEU* and *MAE* against *TFT*, this cyclical depression forced the value of *EUC* below that of *EUD*, resulting in a *(D, c)* outcome on the subsequent move, followed by another string of mutual defections. But in this game, given the overwhelmingly co-operative distribution of outcomes after the initial 100 moves, the value of the expected utility of co-operation remains greater than that of defection, by 2.739 to 2.684. Thus *MAC* co-operates at move 112. Finally, two consecutive co-operative moves have been prescribed by the maximization calculus. Since *MAC* co-operated at move 111, *TFT* co-operates at move 112. The outcome *(C, c)* results.

This mutually co-operative outcome drives the value of *EUC* upward, from 2.739 to 2.741, while the value of *EUD* remains unchanged at 2.684. Hence *MAC* co-operates at move 113. And since *MAC* co-operated at move 112, *TFT* also co-operates at move 113. This *(C, c)* outcome further increases the value of *EUC*, and so forth. The strategic pair is locked into mutual co-operation, which continues for the duration of the game.

Note that the value of *EUC* increases steadily owing to the lengthy string of mutual co-operations. By the end of the game, the expected utility of co-operation is quite close to its maximum limiting value of three. Note also that both *MAC* and *TFT* attain scores in this game (a draw at 2970 points) which are comparable to scores attained by a pair of nice strategies (a draw at 3000 points). The *MAC-TFT* pair realizes scores well above either of their main tournament averages.

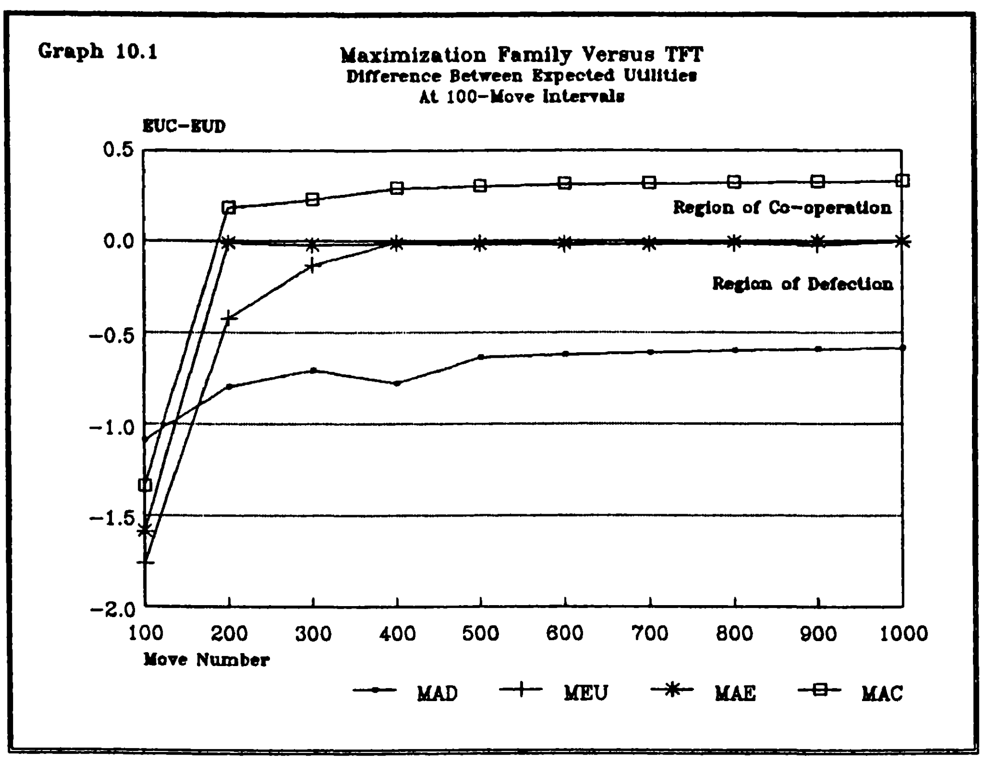
Game 10.14 - *MAC* versus *TFT*, Event Matrix After 1000 Moves

		TFT c	*TFT* d
MAC	*C*	973	8
	D	8	11

EUC = 2.976, *EUD* = 2.684

Score after 1000 moves: *MAC* 2970, *TFT* 2970

The move-by-move interactions of the maximization family members with *TFT* are summarily displayed in the following graph, which plots the algebraic difference in expected utilities (*EUC* minus *EUD*) as a function of the number of moves made (in increments of 100 moves), for each of the four members versus *TFT*:



Obviously, when the difference *EUC* minus *EUD* is greater than zero, a maximization family member co-operates; when less than zero, it defects. Thus, in graph 10.1, the abscissa marks a boundary between regions of co-operation and defection. After 100 initial moves, all four strategies find themselves in the region in which the maximization of expected utility prescribes defection. Whether a strategy proceeds to the boundary or not, and whether a strategy crosses the boundary or not, depends upon the initial distribution of outcomes in its event matrix, which in turn depends upon its initial probabilistic weighting.

The *MAD-TFT* pair is mired in perpetual mutual defection. *MAD*'s *EUC* remains fixed at .462 (for the distribution that obtains in game 10.7), while *MAD*'s *EUD* approaches its asymptotic minimum of unity. Thus the difference approaches -0.538 as the number of moves increases. *MAD* will never be able to venture within .538 utiles of the co-operative border.

The *MEU-TFT* and *MAE-TFT* pairs both manage to approach the border, and even to straddle it on occasion. *MAE*, being initially more co-operative than *MEU*, approaches it more quickly and straddles it more frequently. However, while both strategies' expected utilities converge toward their asymptotic values of unity, *EUD* converges more slowly, in the mean, than does *EUC*. Thus the differences between *EUC* and *EUD*, for both *MAE* and *MEU*, remain mostly negative. Although both strategies manage occasional co-operative moves, neither strategy is able to extricate itself from the region of defection.

The *MAC-TFT* pair quickly traverses the border, and remains thereafter in the co-operative region. *MAC*'s *EUC* approaches its limiting maximum value of 3, while *MAC*'s *EUD* remains constant (after move 110) at 2.684 (for the distribution that obtains in game 10.13). Thus the difference between *EUC* and *EUD* increases toward the asymptote +0.316. The longer the game continues, the more deeply *MAC* moves into the region of co-operation, subject to its limit of 0.316 utiles from the border.

Now, to place the results of the maximization family's performances against *CCC* and *TFT* into perspective, consider the total and average scores that each member obtained against them, compared with each member's average main tournament score:

Table 10.1 - Comparison of Scores

	vs. *CCC*	vs. *TFT*	Average ½(*CCC*+*TFT*)	Tournament Average
MAC	4826	2970	3898	2645
MAE	4848	1272	3060	2503
MEU	4900	1148	3024	2362
MAD	4986	1041	3014	2086

It is evident from table 10.1 that the maximization family members realize substantial average gains when they compete against exploitable and non-exploitable strategies in a one-to-one ratio. Their relative gains against *CCC* far outweigh their relative losses against *TFT*. And note that the average scores, ½(*CCC*+*TFT*), increase with the co-operativeness of the maximization members' weightings. *MAD* is the most exploitive member of its family; *MAC*, the least exploitive. Against the exploitable *CCC*, *MAD* scores 160 points more than *MAC*. But against the non-exploitable *TFT*, *MAD* scores 1929 fewer points than *MAC*. Thus *MAC*'s initial co-operativeness does not greatly impair *MAC*'s ability to exploit *CCC*, while it greatly enhances *MAC*'s performance against *TFT*.

But do the maximization family members satisfy Axelrod's hypothetical criterion of success? Are they able to exploit the exploitable without paying too high a cost with the others? If the maximization family's performances against *CCC* and *TFT* are fair indicators, then the answer to this question seems to depend upon the ratio of "exploitable" to "other" strategies in the environment. If the ratio is one-to-one, then the answer is in the affirmative for the whole family, and especially so for *MAC*.

However, a glance at the last two columns of table 10.1 shows that the maximization family members all realize lower average scores against the twenty strategies of the main tournament than against *CCC* and *TFT* only. This indicates that the ratio of exploitable to non-exploitable strategies is less than one-to-one in the main tournament. Given that *MAC* won the main tournament handily, and that *MAE* placed second, it does appear that both these strategies satisfy

Axelrod's criterion; whereas both *MEU* and *MAD* pay far too high a price for their slightly greater exploitiveness.

In light of this microscopic examination of the maximization family's mechanics, the reasons for *MAC*'s success in the interactive tournament (and the graduated performances of *MAC*'s siblings) become clearer. But *MAC*'s success also entails a certain cost. Ironically, that cost is exacted not by "others" belonging to different strategic families, but by *MAC*'s siblings and its own twin. Consider how the maximization family members fare against one other, compared with their average main tournament scores:

Table 10.2 - The Maximization Family Versus Itself

	MAC	*MAE*	*MEU*	*MAD*	Family Avg.	Tour. Avg.
MAC	1807	1849	1741	971	1592	2645
MAE	2123	2594	2356	987	2015	2503
MEU	1887	2396	2384	1003	1918	2362
MAD	1332	1266	1181	1029	1202	2086

As table 10.2 reveals, the maximization family members' average scores against one another are considerably below their average main tournament scores. In particular, *MAC*'s is 1053 points lower. In intra-family competition, *MAC* ranks third among four siblings. And in auto-competition, the *MAC-MAC* pair also ranks third behind *MAE-MAE* and *MEU-MEU*.

Thus Axelrod's dictum, that there is no "best" strategy independent of environment, continues to ring true. *MAC* proved its robustness in hundreds of thousands of combinatoric sub-tournaments, and in thousands of generations of ecosystemic competition. But in an environment consisting solely of its family members, *MAC* loses every competition against its siblings and fares poorly against itself.

This result suggests that if Axelrod's hypothetical criterion of success is to have broader applicability, then it should be amended. If a strategy could be devised which exploits the exploitable without paying too high a cost with the others, and which emerges victorious in a sub-tournament against the maximization siblings, then that strategy would win the interactive tournament, and would probably win many other tournaments as well. Such a feat, however, may be more easily articulated than accomplished.

MAC still remains the most robust strategy in the interactive environment, but it is somewhat surprising to find that *MAC*'s success is most jeopardized by its siblings and its twin. The task of the next chapter is to discover why the members of the maximization family encounter their greatest difficulties in competition against one another.

Chapter Eleven
The Maximization Family Versus Itself

In order to understand what takes place when a maximization family member encounters a sibling, or its twin, one must recognize a unique property of this family; namely, its members' sequential and mutually exclusive use of probabilistic, then deterministic algorithms. To clarify the meaning of this property, one can first qualify the algorithms used by other strategic families.

The probabilistic family members function by random methods. *TQC, RAN* and *TQD* do so exclusively. As has been mentioned, an alternative interpretation can be made for *CCC* and *DDD*. From the viewpoint of program logic, they co-operate with probabilities unity and zero, respectively. From the viewpoint of ends rather than means, they are also pure strategies, whose play is therefore not deterministic; rather, pre-determined.

The *TFT* family members function deterministically, with the exception of *BBE*. *BBE* employs a deterministic rule with a probabilistic condition attached, and thus mixes two kinds of algorithm.

In the optimization family, *NYD* is strictly deterministic, while *CHA*, *GRO* and *ETH* make simultaneous use of both determinism and probabilism.

The strategic hybrids, *FRI* and *TES*, function according to strictly deterministic rules.

Maximization family members, however, can be regarded as algorithmic hybrids. They employ a purely probabilistic rule for their first hundred moves, then shift to a strictly deterministic calculus for the duration of the game. But unlike *BBE, CHA, GRO* and *ETH*, the maximization strategies never mix these two kinds of algorithm; their use of the two is always sequential and mutually exclusive.

This property naturally gives rise to two discernibly different phases in a maximization strategy's play: first, its construction of the initial event matrix for 100 moves; second, its calculation of expected utilities, and updating of the matrix, for the subsequent 900 moves. These phases were observed in the previous chapter, during encounters between maximization strategies, *CCC* and *TFT*. But when maximization family members encounter one another, the phasing of their play takes on a dual aspect, wherein certain symmetries, as well as anti-symmetries, become apparent. New and interesting properties of the event matrix are thereby revealed, and a deeper understanding of the results of these intra-familial encounters is achieved.

In sum, one can identify five different kinds of algorithmic function in the interactive environment: pre-determined, probabilistic, deterministic, mixed probabilistic and deterministic, and sequential probabilistic and deterministic. The reason for this identification is quite important.

If two pre-determined and/or deterministic strategies are paired in a sequence of games, the scores of a given pair will obviously not vary from one game to another. For example, if *DDD* plays *TFT*, their score is always the same: *DDD* 1004, *TFT* 999.

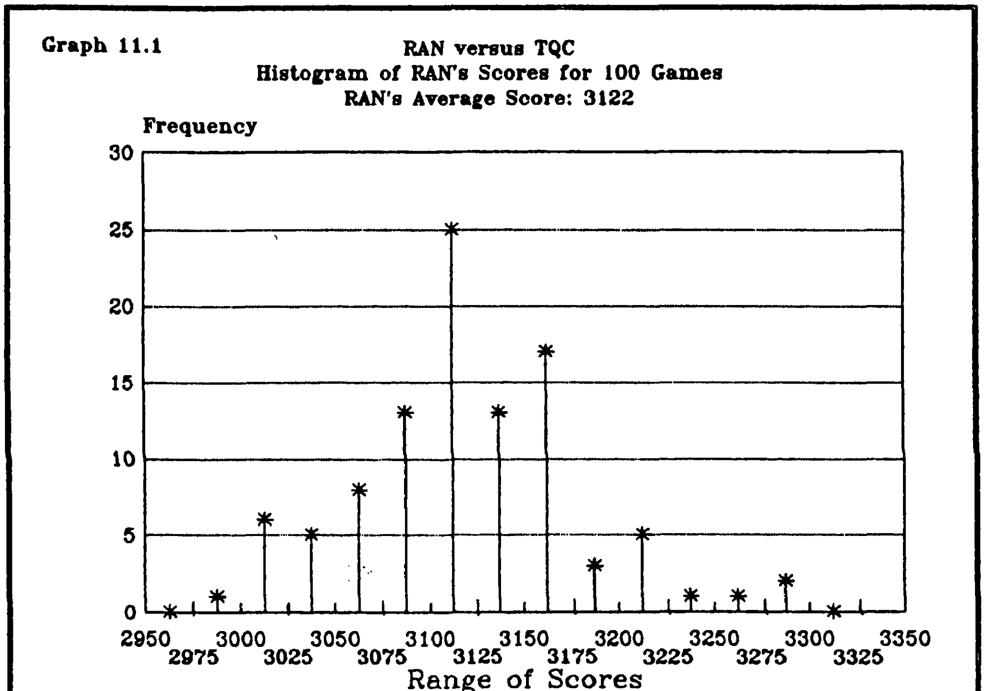
If a probabilistic (or mixed probabilistic and deterministic) strategy is paired with any strategy <u>other than</u> a sequential strategy in a sequence of games, the scores of the given pair will vary according to a normal distribution, in which the mean score tends toward the most probable score, as the number of games increases.

For a simple example, consider two probabilistic strategies. If *RAN* plays *TQC*, then the *a priori* probabilities of the outcomes are as follows:

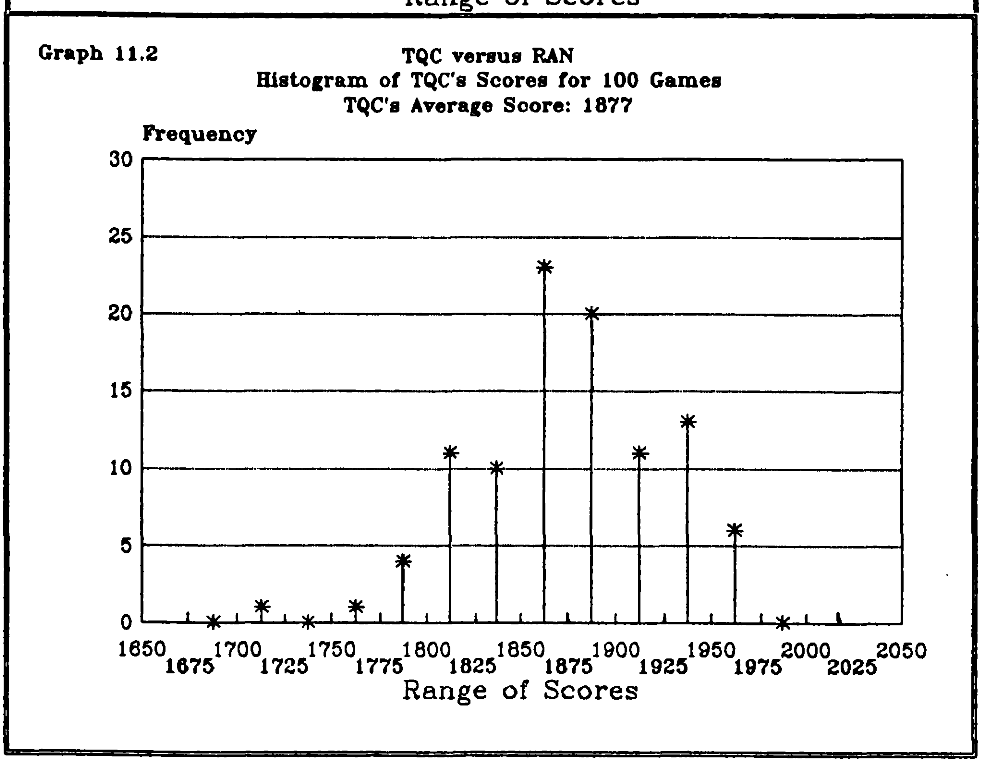
<u>Game 11.1 - Probability Matrix for *RAN* versus *TQC*</u>

		TQC	
		$p(c)=3/4$	$p(d)=1/4$
RAN	$p(C)=1/2$	3/8	1/8
	$p(D)=1/2$	3/8	1/8

In a game of 1000 moves, the most probable distribution of outcomes is therefore: (*C*,*c*) and (*D*,*c*), 375 occasions each; (*C*,*d*) and (*D*,*d*), 125 occasions each. Hence *RAN*'s most probable score is 3x375 + 5x375 + 0x125 + 1x125 = 3125. *TQC*'s most probable score is 3x375 + 0x375 + 5x125 + 1x125 = 1875. The actual score obtained in their main tournament game is *RAN* 3139, *TQC* 1914. But a sequence of 100 games produces the mean score *RAN* 3122, *TQC* 1877, with both sets of scores distributed fairly normally about their respective means:



Graph 11.1 **RAN versus TQC**
Histogram of RAN's Scores for 100 Games
RAN's Average Score: 3122

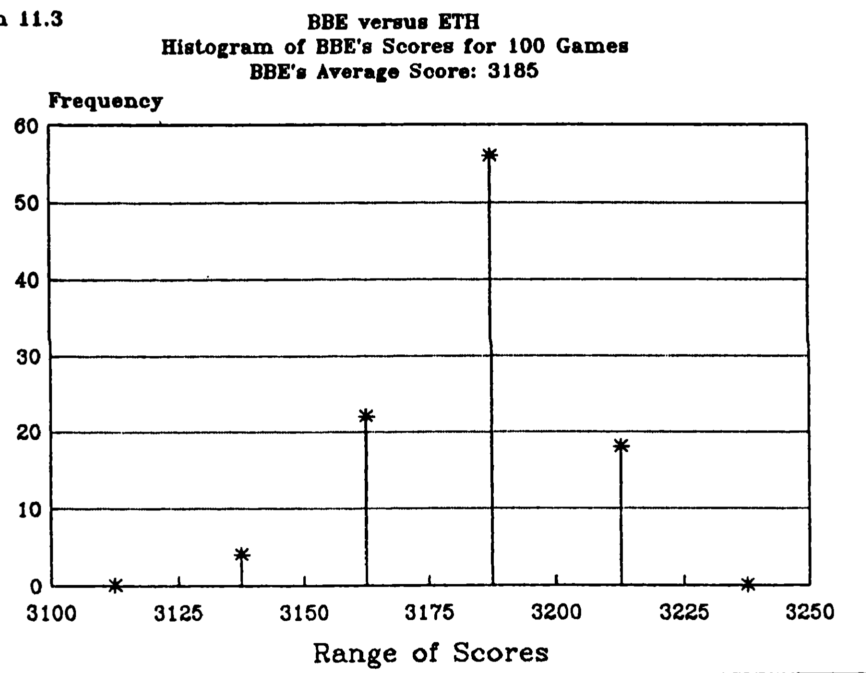


Graph 11.2 **TQC versus RAN**
Histogram of TQC's Scores for 100 Games
TQC's Average Score: 1877

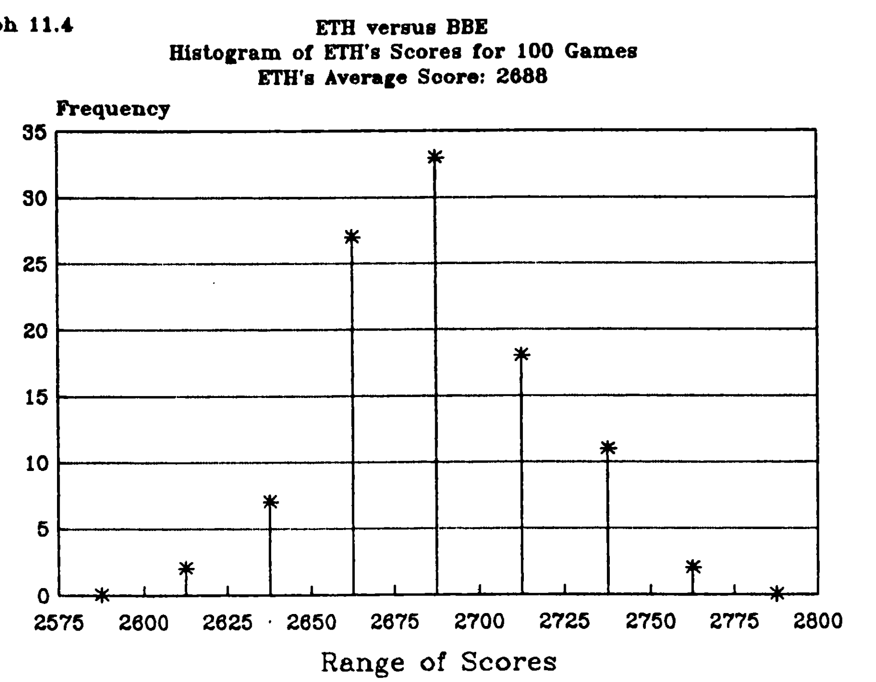
Again, given a normal distribution, the difference between the most probable score and the mean score tends to decrease with the number of games played.

For a more complex example, consider a pair of strategies that use mixed deterministic and probabilistic algorithms, for instance *BBE* and *ETH*. In such a case, one cannot readily construct a matrix of *a priori* probable outcomes, but one can make an empirical test to see whether a normal distribution of scores obtains. The following histograms show the distributions of scores for 100 games between *BBE* and *ETH*:

Graph 11.3



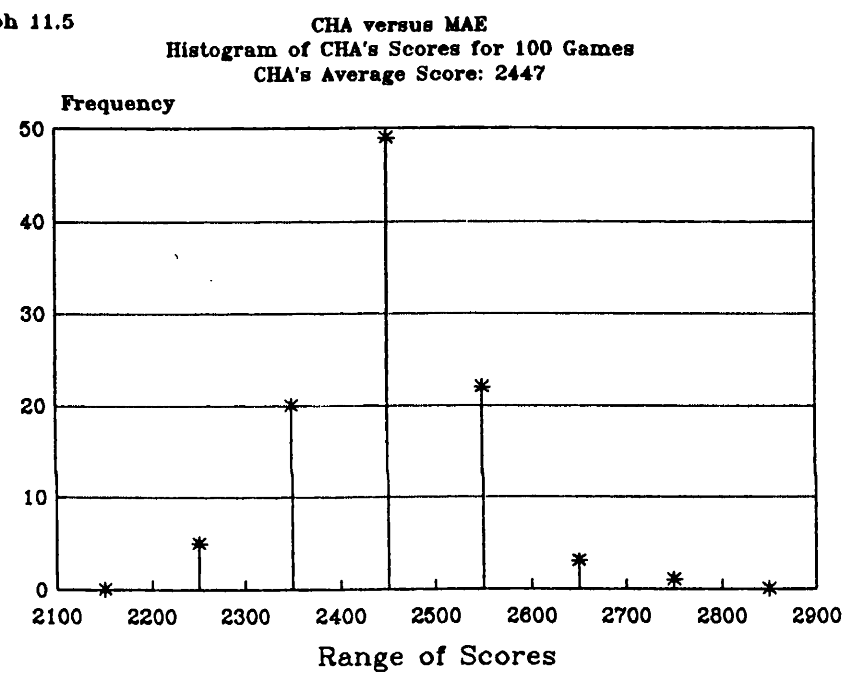
Graph 11.4



Once again, these scores appear to be distributed normally.

Now, consider what takes place between a maximization strategy and a strategy that uses a mixed probabilistic and deterministic algorithm, for instance *MAE* and *CHA*. Again, an empirical test is conducted, and the following histogram shows *CHA*'s distribution of scores for 100 games against *MAE*:

Graph 11.5



One finds *CHA*'s scores to be normally distributed.

MAE's scores against *CHA* are comparatively highly-concentrated. Ninety-nine scores lie in the 2850–3000 point range; one, in the 3000–3100 point range.

The maximization family members' scores against one another, however, are neither concentrated nor distributed normally, with one noteworthy exception. In consequence, their average scores do not, as a rule, approach their most probable scores as the number of games increases.

Let the exception to the rule, which occurs in games involving *MAD*, be considered first. The extreme case of this exception obtains when *MAD* plays itself. After the first one hundred moves, the most probable event matrix is as follows:

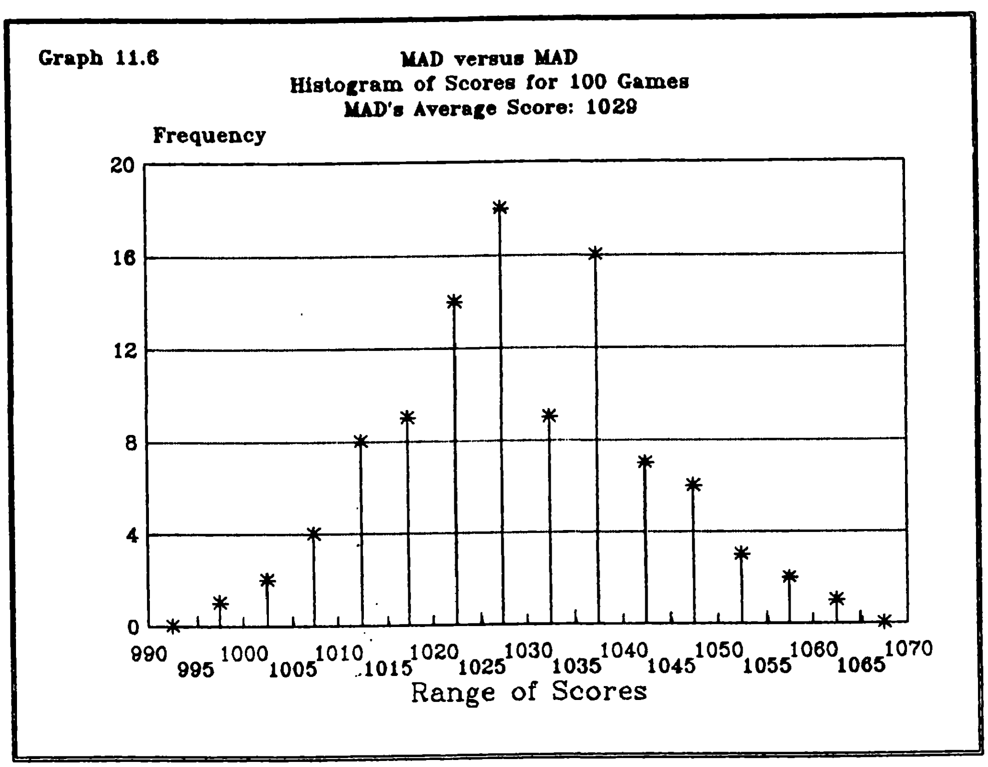
Game 11.2 - Most Probable Event Matrix for *MAD* versus *MAD* (100 Moves)

		MAD	
		$p(c)$=1/10	$p(d)$=9/10
	$p(C)$=1/10	1	9
MAD			
	$p(D)$=9/10	9	81

EUC = 0.3, *EUD* = 1.4

Score tied at 129

The deterministic play that ensues from this matrix, from moves 101 to 1000, consists of 900 consecutive mutual defections. The game ends with the score tied at 1029. Since this score is a deterministic end-product of the most probable event matrix, it is the most probable score. Empirically, after five hundred games, *MAD*'s average score is found to be 1029. The scores themselves appear to be distributed normally, as the following histogram reveals:



Next, consider the most probable event matrix for *MEU* versus *MEU*, after 100 moves:

Game 11.3 – Most Probable Event Matrix for *MEU* versus *MEU* (100 Moves)

		MEU	
		$p(c)$=1/2	$p(d)$=1/2
MEU	$p(C)$=1/2	25	25
	$p(D)$=1/2	25	25

EUC = 1.5, *EUD* = 3

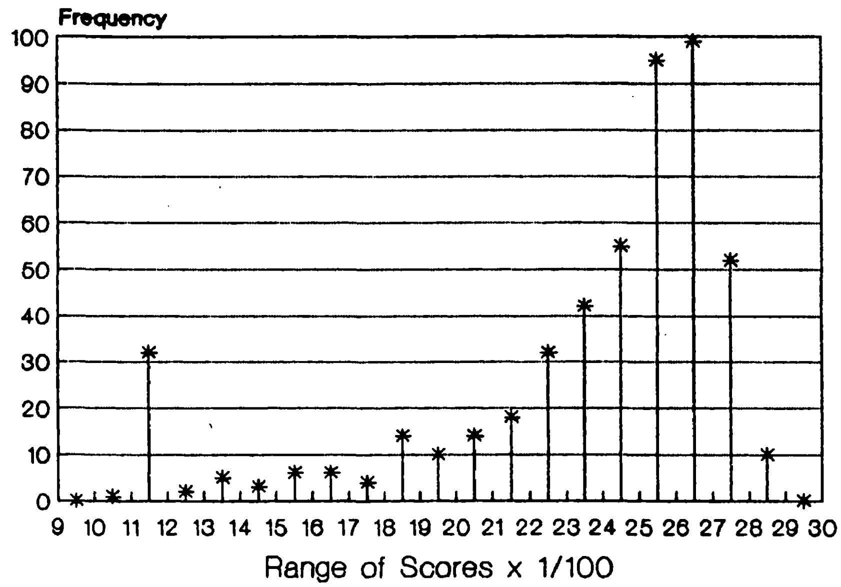
Score tied at 225

The deterministic phase of game 11.3 proceeds as follows. One-hundred-and-fifty consecutive mutual defections obtain between moves 101 to 250, with a concomitant steady decrease in the value of *EUD*. By move 251, the value of *EUD* is forced below that of *EUC*, and 750 consecutive mutual co-operations ensue. After 1000 moves, the score is tied at 2625. Again, it is the most probable score.

Empirically, however, after 500 games of *MEU* versus *MEU*, the average score is found to be 2384. This is substantially less than the most probable value. The cause of the discrepancy is revealed in a histogram showing the distribution of scores for 500 games of *MEU* versus *MEU*.

Graph 11.7 displays a bi-modal distribution, with a minor prominence in the 1100–1200 point range, and a skewed distribution across the middle and upper ranges. The peak of the skewed distribution indeed coincides with the most probable (*a priori*) score, in the 2600–2700 point range. But the minor feature at the low end of the range, along with the overall skewness, diminishes the average score.

Graph 11.7 MEU versus MEU
Histogram of Scores for 500 Games
MEU's Average Score: 2384



Next, consider the most probable event matrix for *MAE* versus *MAE*, after 100 moves:

<u>Game 11.4 - Most Probable Event Matrix for *MAE* versus *MAE* (100 Moves)</u>

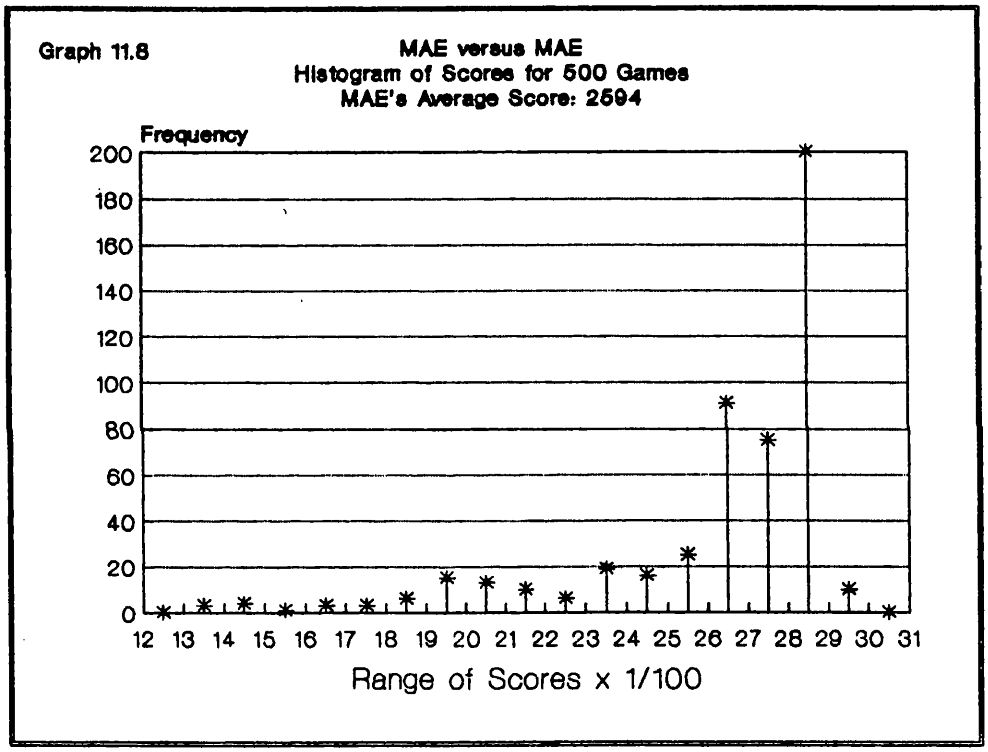
		MAE	
		$p(c)$=5/7	$p(d)$=2/7
MAE	$p(C)$=5/7	52	20
	$p(D)$=2/7	20	8

EUC = 2.17, *EUD* = 3.86

Score tied at 264

Game 11.4 proceeds in this way. Forty-one mutual defections take place between moves 101 and 141, followed by 859 mutual co-operations. After 1000 moves, the score is tied at 2882 points. Again, this represents the most probable score.

Empirically, however, after 500 games of *MAE* versus *MAE*, the average score is found to be 2594 points. Again, a histogram reveals the cause of the discrepancy between the most probable and the average scores:



Graph 11.8 displays a skewed distribution. While the most frequent scores by far occur in the 2800-2900 point range, which is the range of the most probable score, the skew of the distribution toward the lower ranges diminishes the average score by some 250 points.

A closer look at the histogram affords a more detailed interpretation. The minor prominence from the previous histogram (located in the 1000-1300 point range of graph 11.7) may have experienced a radical decrease, and migrated to the 1300-1600 point range. Indeed, other features of increasing prominence appear in the 1900-2000, 2300-2400, and 2600-2700 point ranges.

It is not yet possible to judge whether these features merely denote statistical irregularities in the profile of a badly-skewed distribution, or whether they indicate that the distribution itself is beginning to become fragmented.

Finally, consider the most probable event matrix for *MAC* versus *MAC*, after 100 moves:

<u>Game 11.5 - Most Probable Event Matrix for *MAC* versus *MAC* (100 Moves)</u>

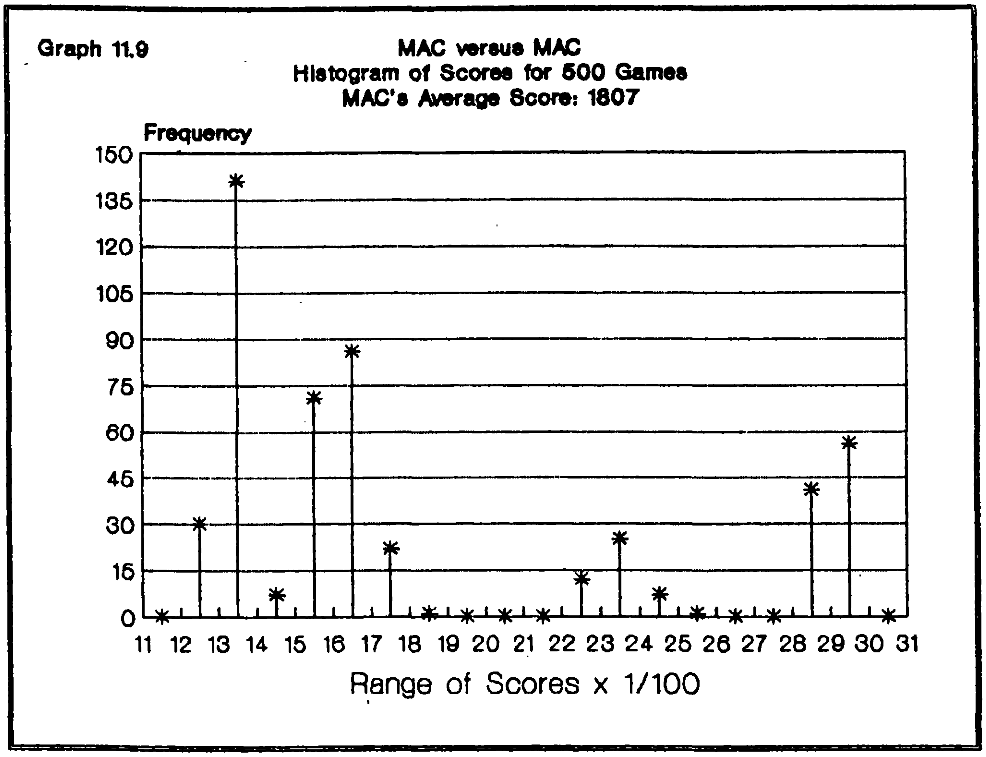
		MAC	
		$p(c)$=9/10	$p(d)$=1/10
MAC	$p(C)$=9/10	81	9
	$p(D)$=1/10	9	1

EUC = 2.7, *EUD* = 4.6

Score tied at 289

In game 11.5, mutual co-operation commences on move 113, after only twelve consecutive mutual defections. The deterministic string of 888 mutual co-operations between moves 113 and 1000, in addition to the 81 probabilistic mutual co-operations during the first 100 moves, yields a total of 969 instances of mutual co-operation in a game of 1000 moves. The resultant score, which again represents the most probable score, is tied at 2965 points.

MAC versus *MAC*, however, yields the largest empirical deviation in its family. After 500 games of *MAC* versus *MAC*, the average score is found to be 1807 points, a remarkable difference of 1158 points between most probable and average scores. Again, a histogram reveals the cause of this large discrepancy.



Graph 11.9 shows a multi-modal distribution of scores, with prominent features in the 1300-1400, 1600-1700, 2300-2400, and 2900-3000 point ranges. In addition, troughs appear between 1900-2200 and 2600-2700 points, from which ranges scores seem to be excluded. The histogram clearly illustrates why the average score for *MAC* versus *MAC* is well below the most probable *a priori* score. And in retrospect, it seems that the previous histogram (graph 11.8) shows signs of the impending fragmentation. But this illustration merely begs the question: why does the distribution become so fragmented?

Indeed, this is one of a number of questions raised by an examination of the distribution of scores among members of the maximization family. In the four cases considered, in increasing order of initial co-operative weighting, one finds: first, a concentration of scores at the low end of the scale; second, a skewed bi-modal distribution with a minor prominence at the low end; third, a skewed distribution which may be in the preliminary stages of fragmentation; and fourth, a multi-modal distribution which has fragmented into several distinct features. One may ask why these differences occur, given that each distribution represents a range of deterministic results stemming from a domain of probabilistic initial conditions. What causes such pronounced changes in the profiles of the distributions?

Answers can be found in deeper analysis of the event matrix. There are 176,851 different combinations of 100 trials of the four possible outcomes; in other words, for the first 100 moves in the iterated prisoner's dilemma, there are 176,851 possible event matrices. To facilitate analysis, one seeks to formulate a few general principles that extend to the many different cases.

First, consider those matrices which are symmetric across their major diagonals; that is, event matrices in which the numbers of (C,d) and (D,c) outcomes are identical after 100 moves. Such matrices obtain from *a priori* probabilistic encounters between maximization family twins. As a most general example, suppose that any maximization strategy *MAX*, with an initial co-operative weighting of p, meets its twin. Then, during their first hundred moves, both strategies co-operate randomly with probability p, and defect with probability $(1-p)$. The most probable event matrix is:

Game 11.6 – Most Probable Event Matrix for *MAX* versus *MAX* (100 Moves)

		MAX	
		$p(c)=p$	$p(d)=1-p$
MAX	$p(C)=p$	$100p^2$	$100p(1-p)$
	$p(D)=1-p$	$100p(1-p)$	$100(1-p)^2$

$EUC = 3p$, $EUD = 4p+1$

Score tied at $100(1+3p-p^2)$

Particular members of this class of event matrix have already been encountered in games 11.2 through 11.5, inclusively. The significance of symmetry across the major diagonal is as follows. When the number of (C,d) outcomes equals the number of (D,c) outcomes, then both competitors have the same expected utility of co-operation, and the same expected utility of defection. In consequence, from move 101 onward, their joint play is identical, with outcomes of either (D,d) or (C,c).

Precisely this process unfolds in the *a priori* evaluations of most probable scores for *MAD* versus *MAD*, *MEU* versus *MEU*, *MAE* versus *MAE*, and *MAC* versus *MAC*. One naturally observes increasing scores (1029, 2625, 2882, and 2965 points respectively) as the co-operative weighting increases. *MAD*'s most probable score against its twin is far lower than *MAD*'s siblings' most probable scores against their respective twins because, unlike *MAD*, the other siblings sooner or later attain mutual co-operation with their respective twins.

Empirically, it is found that the threshold weighting for the eventual attainment of mutual co-operation is $p = 37/100$ (in a game of 1000 moves with the payoffs of game 7.2). This is not a highly co-operative weighting; nevertheless, it does result in mutual co-operation from move 719 onward. The initial and final event matrices are as follows:

Game 11.7 – *MAX* versus *MAX* (p=37/100), Initial & Final Event Matrices

100 moves:

		MAX	
		c	*d*
MAX	*C*	14	23
	D	23	40

EUC = 1.14, *EUD* = 2.46

Score tied at 197

1000 moves:

		MAX	
		c	*d*
MAX	*C*	296	23
	D	23	658

EUC = 2.78, *EUD* = 1.14

Score tied at 1661

Now, compare this result with that of a game in which the initial co-operative weighting of the competitors is 36/100, or just under the threshold value:

Game 11.8 – *MAX* versus *MAX* (p=36/100), Initial & Final Event Matrices

100 moves:

		MAX c	*MAX* d
MAX	*C*	13	23
	D	23	41

EUC = 1.08, *EUD* = 2.44

Score tied at 195

1000 moves:

		MAX c	*MAX* d
MAX	*C*	13	23
	D	23	941

EUC = 1.083, *EUD* = 1.095

Score tied at 1095

While the initial conditions of games 11.7 and 11.8 scarcely differ, the final results admit of considerable difference. Having established that the minimum threshold weighting of p = 37/100 leads to the eventual attainment of mutual co-operation at move 719, one might next find the maximum rapidity with which such co-operation can be attained.

The highest admissible value of p, to the nearest 1/100, is p = 99/100. (If p equals unity, then *EUD* is undefined owing to division by zero). At this maximum value of p, the following matrices obtain:

Game 11.9 – *MAX* versus *MAX* (p=99/100), Initial & Final Event Matrices

100 moves:

		MAX c	*MAX* d
MAX	*C*	98	1
	D	1	0

EUC = 2.97, *EUD* = 5

Score tied at 299

1000 moves:

		MAX c	*MAX* d
MAX	*C*	996	1
	D	1	2

EUC = 2.996, *EUD* = 2.333

Score tied at 2995

In game 11.9, only two mutual defections, at moves 101 and 102, suffice to initiate perpetual mutual co-operation.

Evidently, the number of mutual defections required to bring on mutual co-operation is a decreasing exponential function of initial co-operative weighting. An exponential curve-fit yields the following equation:

$$n = f(p) = 7093e^{-7.164p} \text{ for } 37/100 \leq p < 1$$

where n is the number of mutual defections between move 101 and the onset of perpetual mutual co-operation and p is initial co-operative weighting. The co-efficient of determination for this exponential equation is $r = .985$.

Similarly, the final scores that result from these initial distributions can be fitted to a second exponential curve:

$$s = g[f(p)] = 3007e^{-.000958f(p)}$$

where s is the score after 1000 moves. The coefficient of determination for this expression is $r = .9997$.

Needless to say, the numerical coefficients of both curves depend upon the particular payoff structure and the length of the game, but the form of the curves is independent of these variables. In general, then, both the play that ensues from event matrices exhibiting symmetry across their major diagonals, and the scores which result from this play, conform to simple mathematical expressions. This class of event matrix gives rise to regular and readily comprehensible outcomes.

The broader class of event matrices, whose members do not exhibit symmetry across their main diagonals, is unfortunately (from the viewpoint of simplicity) the far larger of the two classes. The event matrices in this class give rise to the non-normal distributions displayed in graphs 11.7 through 11.9. It is possible (and desirable) to gain an understanding of how these distributions arise without having to analyze tens of thousands, nor even thousands, of such matrices. Fortuitously, the process can be well-represented by the tabling of results of a few dozen small probabilistic fluctuations about the most probable outcome, for each of the strategic pairs.

Recall the notation for entries in the generalized event matrix: *W, X, Y* and *Z* are the respective numbers of *(C,c)*, *(C,d)*, *(D,c)* and *(D,d)* outcomes. One first considers the case of *MEU* versus *MEU*:

Table 11.1 – *MEU* versus *MEU*, Varying Event Matrices and Scores

Initial *W,X,Y,Z*	Perpetual *(C,d)*	Final Score	Initial *W,X,Y,Z*	Perpetual *(C,d)*	Final Score	Initial *W,X,Y,Z*	Perpetual *(C,d)*	Final Score
20,33,32,15	none	1139 - 1139	25,30,30,15	move 386	2370 - 2370	30,28,27,15	move 262	2622 - 2622
20,34,31,15	none	1138 - 1143	25,31,29,15	move 423	2299 - 2299	30,29,26,15	move 281	2587 - 2587
20,35,30,15	none	1137 - 1147	25,32,28,15	move 464	2220 - 2220	30,30,25,15	move 301	2550 - 2550
20,36,29,15	none	1136 - 1151	25,33,27,15	move 510	2131 - 2131	30,31,24,15	move 323	2509 - 2509
20,37,28,15	none	1131 - 1156	25,34,26,15	move 562	2030 - 2030	30,32,23,15	move 347	2464 - 2464
20,30,30,20	move 651	1830 - 1830	25,28,27,20	move 324	2488 - 2488	30,25,25,20	move 213	2709 - 2709
20,31,29,20	move 755	1625 - 1625	25,29,26,20	move 354	2431 - 2431	30,26,24,20	move 229	2682 - 2682
20,32,28,20	move 885	1368 - 1368	25,30,25,20	move 386	2370 - 2370	30,27,23,20	move 245	2653 - 2653
20,33,27,20	none	1139 - 1139	25,31,24,20	move 423	2299 - 2299	30,28,22,20	move 262	2622 - 2622
20,34,26,20	none	1138 - 1143	25,32,23,20	move 464	2220 - 2220	30,29,21,20	move 281	2587 - 2587
20,28,27,25	move 497	2132 - 2132	25,25,25,25	move 251	2625 - 2625	30,23,22,25	move 185	2759 - 2759
20,29,26,25	move 567	1995 - 1995	25,26,24,25	move 273	2584 - 2584	30,24,21,25	move 199	2736 - 2736
20,30,25,25	move 651	1830 - 1830	25,27,23,25	move 298	2537 - 2537	30,25,20,25	move 214	2709 - 2709
20,31,24,25	move 755	1625 - 1625	25,28,22,25	move 324	2488 - 2488	30,26,19,25	move 229	2682 - 2682
20,32,23,25	move 885	1368 - 1368	25,29,21,25	move 354	2431 - 2431	30,27,18,25	move 245	2653 - 2653
20,25,25,30	move 346	2425 - 2425	25,23,22,30	move 213	2695 - 2695	30,20,20,30	move 151	2820 - 2820
20,26,24,30	move 389	2342 - 2342	25,24,21,30	move 231	2662 - 2662	30,21,19,30	move 162	2801 - 2801
20,27,23,30	move 439	2245 - 2245	25,25,20,30	move 251	2625 - 2625	30,22,18,30	move 168	2788 - 2788
20,28,22,30	move 497	2132 - 2132	25,26,19,30	move 273	2584 - 2584	30,23,17,30	move 186	2759 - 2759
20,29,21,30	move 567	1995 - 1995	25,27,18,30	move 298	2537 - 2537	30,24,16,30	move 199	2736 - 2736
20,23,22,35	move 277	2557 - 2557	25,20,20,35	move 166	2780 - 2780	30,18,17,35	move 127	2858 - 2863
20,24,21,35	move 309	2496 - 2496	25,21,19,35	move 181	2753 - 2753	30,19,16,35	move 127	2853 - 2868
20,25,20,35	move 346	2425 - 2425	25,22,18,35	move 196	2726 - 2726	30,20,15,35	move 126	2850 - 2875
20,26,19,35	move 389	2342 - 2342	25,23,17,35	move 213	2695 - 2695	30,21,14,35	move 126	2845 - 2880
20,27,18,35	move 439	2245 - 2245	25,24,16,35	move 231	2662 - 2662	30,22,13,35	move 125	2842 - 2887

Columns labled "Initial *W,X,Y,Z*" contain differing values of these variables after the first 100 moves; i.e. contain different probabilistically-generated event matrices. With each initial event matrix, described by a set $\{W, X, Y, Z\}$, is associated the move number on which perpetual mutual co-operation commences [column labled "Perpetual (C,c)"] in the deterministic phase of the game (moves 101–1000) arising from that set. If no mutual co-operation occurs between moves 101–1000, the entry for that set reads "none". The column labled "Final Score" associates the score (after 1000 moves) which results from the given initial set $\{W, X, Y, Z\}$.

The sets of values $\{W, X, Y, Z\}$ are arranged in blocks. Within each block, the values of W and Z are held constant, while the difference between X and Y increases. Each column of blocks holds the value of W constant, while the value of Z increases from block to block. Similarly, each row of blocks holds the value of Z constant, while the value of W increases from block to block. Thus table 11.1 can be read both vertically and horizontally.

Reading down a column shows the effect of increasing initial difference in anti-symmetric outcomes (within blocks), and of increasing initial mutual defections (between blocks), upon the attainment of perpetual mutual co-operation and upon the game score. Reading across a row shows the effect of increasing initial mutual co-operation upon the attainment of perpetual mutual co-operation and upon the game score, with the number of initial mutual defections held constant and the variance in difference between anti-symmetric outcomes held to one.

Recall that, for *MEU* versus *MEU*, the most probable $\{W, X, Y, Z\}$ is {25,25,25,25}. In table 11.1, the sets of initial event matrices are representative of the probabilistic fluctuations in these values that would naturally occur in empirical trials. Three main tendencies, and one interesting exception to them, are quickly made apparent by this table.

First, within each block, the onset of perpetual mutual co-operation (when it occurs) is increasingly delayed by increases in the difference between X and Y. For a given number of mutual co-operations, a given number of mutual defections, and an initial unequal number of (C,d) and (D,c) outcomes, the *MEU* pair proceeds to equalize the number of (C,d) and (D,c) outcomes. Once that happens, their expected utilities become equal, and the pair defects until the value of *EUD* is driven below that of *EUC*. Perpetual mutual co-operation then ensues. A tied final score is indicative of this process. The greater the initial difference between *X* and *Y*, the greater number of moves are required for their equalization, and the still greater number of moves must be made before mutual co-operation is attained. Thus, for a given *W* and *Z*, the smaller the initial difference between *X* and *Y*, the larger the final score.

Second, reading down the columns, one perceives that for a constant value of *W*, the onset of perpetual mutual co-operation is actually hastened as the initial number of mutual defections increases. Within certain probabilistic limits, which vary according to their initial weightings, the maximization strategies demonstrate the capacity of enlisting mutual defections in the service of perpetual mutual co-operation. While one wishes to refrain from lapsing into trite moralization, this counter-intuitive capacity suggests that, in certain instances, the game-theoretic end may justify the game-theoretic means.

Third, reading across the rows, one perceives that for a constant value of *Z*, the onset of perpetual mutual co-operation is hastened as the initial number of mutual co-operations increases. This tendency is not surprising, but re-assuring in terms of the integrity of the maximization strategy.

In general, table 11.1 shows that perpetual mutual co-operation between *MEU* pairs, and thus their game scores, depend upon three factors. The scores tend to increase as *W* increases with *Z* fixed, as *Z* increases with *W* fixed, and as the difference between *X* and *Y* decreases with both *W* and *Z* fixed. One can amalgamate the first two tendencies, and observe that the scores tend to increase as the sum of symmetric outcomes, $W + Z$, increases; or, equivalently, as the sum of anti-symmetric outcomes, $X + Y$, decreases. This observation, however, leads to the aforementioned exception.

The {30,*X*,*Y*,35} block boasts the largest *W* and *Z* values in table 11.1, yet the results that stem from this block are not altogether consistent with the tendencies so uniformly prevalent in the rest of the table. To begin with, the onset of perpetual mutual co-operation is hastened (albeit only slightly) as the difference between *X* and *Y* increases, not decreases. And, as evidenced by the absence of tied final scores, the *MEU* pairs in this block attain perpetual mutual co-operation without having first equalized *X* and *Y* values, and without ever equalizing them. The scores themselves are the highest in the table, in keeping with this block's highest $W + Z$ sum. The significance of this unusual block will be brought to light in subsequent tables.

Meanwhile, table 11.1 does account for the distribution of scores in graph 11.7. One can observe the contributions toward skewness, with a majority of scores occurring in the 2400–2700 point range, and none exceeding 2900 points. Contributions to the minor prominence in the 1100–1200 point range occur when the sum of $W + Z$ falls below a certain threshold, making mutual co-operation unattainable within 1000 moves; or when the sum of $W + Z$ is theoretically sufficient for perpetual mutual co-operation, but the difference between *X* and *Y* is large enough to prevent its onset. These latter conditions prevail in the {20,*X*,*Y*,15} and {20,*X*,*Y*,20} blocks, respectively.

Next, a similar table is considered for *MAE* versus *MAE*. Recall that the most probable {*W*,*X*,*Y*,*Z*} for *MAE* versus *MAE* is {52,20,20,8}. Table 11.2 (overleaf) displays corresponding fluctuations about these most probable values, and the results to which they give rise.

Reading down the first column of table 11.2, one observes that the two previous tendencies hold until the {40,*X*,*Y*,14} block; that is, the onset of perpetual mutual co-operation is hastened as the difference $X - Y$ decreases within blocks, and as the sum $X + Y$ decreases between blocks. The {40,23,23,14} matrix of the {40,*X*,*Y*,14} block also conforms to these tendencies. But the other matrices in that block yield results comparable to those of the {30,*X*,*Y*,35} block in table 11.1; that is, they give rise to perpetual mutual co-operation without first equalizing *X* and *Y* values, and the onset of mutual co-operation is hastened slightly as the difference $X - Y$ increases.

Table 11.2 - *MAE* versus *MAE*, Varying Event Matrices and Scores

Initial *W,X,Y,Z*	Perpetual (*C,C*)	Final Score	Initial *W,X,Y,Z*	Perpetual (*C,C*)	Final Score	Initial *W,X,Y,Z*	Perpetual (*C,C*)	Final Score
40,29,29,2	move 227	2715 - 2715	50,24,24,2	move 169	2836 - 2836	60,19,19,2	move 140	2899 - 2899
40,30,28,2	move 239	2694 - 2694	50,25,23,2	move 203	2754 - 2809	60,20,18,2	move 257	2622 - 2622
40,31,27,2	move 252	2671 - 2671	50,26,22,2	move 206	2742 - 2812	60,21,17,2	move 623	2059 - 2059
40,32,26,2	move 265	2648 - 2648	50,27,21,2	move 209	2730 - 2815	60,22,16,2	move 668	1975 - 1975
40,33,25,2	move 280	2621 - 2621	50,28,20,2	move 211	2720 - 2820	60,23,15,2	move 717	1883 - 1883
40,28,27,5	move 216	2734 - 2734	50,23,22,5	move 158	2851 - 2856	60,18,17,5	move 623	2059 - 2059
40,29,26,5	move 227	2715 - 2715	50,24,21,5	move 209	2730 - 2815	60,19,16,5	move 668	1975 - 1975
40,30,25,5	move 239	2694 - 2694	50,25,20,5	move 211	2720 - 2820	60,20,15,5	move 717	1883 - 1883
40,31,24,5	move 252	2671 - 2671	50,26,19,5	move 210	2717 - 2827	60,21,14,5	move 263	2585 - 2850
40,32,23,5	move 265	2648 - 2648	50,27,18,5	move 212	2707 - 2832	60,22,13,5	move 859	1614 - 1614
40,26,26,8	move 195	2770 - 2770	50,21,21,8	move 148	2869 - 2869	60,16,16,8	move 124	2922 - 2922
40,27,25,8	move 205	2753 - 2753	50,22,20,8	move 211	2720 - 2820	60,17,15,8	move 717	1883 - 1883
40,28,24,8	move 216	2734 - 2734	50,23,19,8	move 210	2717 - 2827	60,18,14,8	move 263	2585 - 2850
40,29,23,8	move 227	2715 - 2715	50,24,18,8	move 212	2707 - 2832	60,19,13,8	move 859	1614 - 1614
40,30,25,8	move 239	2694 - 2694	50,25,17,8	move 214	2697 - 2837	60,20,12,8	move 265	2568 - 2868
40,25,24,11	move 180	2793 - 2798	50,20,19,11	move 210	2717 - 2827	60,15,14,11	move 263	2585 - 2850
40,26,23,11	move 195	2770 - 2770	50,21,18,11	move 212	2707 - 2832	60,16,13,11	move 859	1614 - 1614
40,27,22,11	move 205	2753 - 2753	50,22,17,11	move 214	2697 - 2837	60,17,12,11	move 265	2568 - 2868
40,28,21,11	move 216	2734 - 2734	50,23,16,11	move 443	2357 - 2357	60,18,11,11	move 268	2555 - 2875
40,29,20,11	move 227	2715 - 2715	50,24,15,11	move 213	2688 - 2853	60,19,10,11	none	1334 - 1359
40,23,23,14	move 140	2819 - 2819	50,18,18,14	move 129	2898 - 2898	60,13,13,14	move 110	2941 - 2941
40,24,22,14	move 166	2814 - 2824	50,19,17,14	move 214	2697 - 2837	60,14,12,14	move 265	2568 - 2868
40,25,21,14	move 166	2809 - 2929	50,20,16,14	move 443	2357 - 2357	60,15,11,14	move 268	2555 - 2875
40,26,20,14	move 165	2806 - 2836	50,21,15,14	move 213	2688 - 2853	60,16,10,14	none	1334 - 1359
40,27,19,14	move 165	2801 - 2841	50,22,14,14	move 523	2209 - 2209	60,17,9,14	none	1327 - 1372

Reading down the second column, one observes that this departure from precedent tendency now becomes the norm itself. With the obvious exception of matrices in which *X* equals *Y* initially, the second column of blocks behaves as the last block in the first column. Note that, within each block except the first, the order of the onset of perpetual mutual co-operation is increasingly jumbled.

The most important overall effect of this departure, exemplified in the first three blocks of column two, is reflected in the final scores. Because the *X* and *Y* values are not equalized prior to perpetual mutual co-operation, the gap between the final scores increases as the initial difference between *X* and *Y* increases. Owing to the vicissitudes of chance during the first hundred moves, one member of the *MAE* pair finds that joint occurrences of its co-operation and its twin's defection outnumber joint occurrences of its defection and its twin's co-operation. In the {*W*,*X*,*Y*,*Z*} region under consideration, this member's final score decreases, while its twin's increases, as the initial difference *X* − *Y* becomes larger.

Then, suddenly, in the {50,*X*,*Y*,11} block, a new phenomenon is manifest. Four of five sets in this block give rise to perpetual mutual co-operation between moves 210-214, with respective final scores within the 2688-2853 point range. But the {50,23,16,11} matrix, which contains neither the largest nor the smallest (*X*,*Y*) difference in the block, gives rise to an unexpectedly large number of mutual defections, with the onset of perpetual mutual co-operation delayed until move 443. The resultant final score, tied at 2357 points, indicates that *X* and *Y* values are once again equalized during the game.

This phenomenon is increasingly more frequent, and more drastic, through the balance of column two, and throughout column three. For instance, consider what takes place in the {60,*X*,*Y*,8} block. The first matrix, {60,16,16,8}, gives rise to early perpetual mutual co-operation, commencing on move 124, and the *MAE* twins attain a correspondingly high score, tied at 2922 points. But the second matrix, {60,17,15,8}, leads to comparative disaster: perpetual mutual co-operation does not commence until move 717, and the pair attains a correspondingly low final score, tied at 1883 points. Hence, a small increment in the difference between *X* and *Y* produces a momentous delay in the onset of perpetual mutual co-operation, with a correspondingly large decrement in the final scores.

The third matrix in the block, {60,18,14,8}, reverses the previous disaster. Perpetual mutual co-operation begins at move 263, which is not unreasonable in light of the initial (*X*,*Y*) difference. No equalization of (*X*,*Y*) values takes place, and the final scores are therefore fairly high but disparate, at 2585-2850 points. But the fourth matrix, {60,19,13,8}, leads to renewed disaster, with perpetual mutual co-operation commencing only on move 859, and a resultant low tied score of 1614 points.

The culmination of these alternating radical changes appears in the last two blocks of column three. The combination of a sufficiently large $W + Z$ sum and a sufficiently large $X - Y$ difference can result in perpetual mutual defection from move 101 to the end of the game. In such cases, the *MAE* pair attains scores of less than 1400 points.

Evidently, the event matrix becomes increasingly unstable as the sum of symmetric outcomes begins to exceed that of anti-symmetric outcomes. The expected utilities associated with these outcomes begin to reverse their prescriptions with each increment of the (X, Y) difference, and the pendulum of joint outcomes swings steadily away from perpetual mutual co-operation, and toward perpetual mutual defection, as $W + Z$ grows and $X + Y$ diminishes.

Table 11.2 does account for the distribution of scores in graph 11.8, albeit in an unexpected fashion. When random fluctuations about the most probable event matrix, {52,20,20,8}, are relatively small, the scores attained are fairly high. Larger fluctuations which reduce the sum $W + Z$ do not substantially reduce the final scores. But larger fluctuations which increase the sum $W + Z$ produce both the highest scores in the distribution (when $X = Y$), as well as the lowest scores (when $X - Y$ is sufficiently large).

Next, a similar table is considered for *MAC* versus *MAC*. The process leading to the fragmented distribution of scores for 500 games of *MAC* versus *MAC*, as displayed in graph 11.9, is well-depicted in table 11.3 (overleaf). Table 11.3 shows a continuation of the new tendency observed in table 11.2; namely, a transition to increasingly less stable event matrices. Recall that the most probable event matrix for *MAC* versus *MAC* is {81,9,9,1}. This set of values evidently lies in a highly unstable region of the $\{W, X, Y, Z\}$ spectrum, in which probabilistic fluctuation gives rise to one of three situations. Together, the three situations account for the fragmentation of the *MAC* pair's distribution.

Table 11.3 - *MAC* versus *MAC*, Varying Event Matrices and Scores

Initial *W,X,Y,Z*	Perpetual (*C,c*)	Final Score	Initial *W,X,Y,Z*	Perpetual (*C,c*)	Final Score	Initial *W,X,Y,Z*	Perpetual (*C,c*)	Final Score
79,10,10,1	move 115	2960 - 2960	81,9,9,1	move 113	2965 - 2965	83,8,8,1	move 111	2970 - 2970
79,11,9,1	move 364	2352 - 2887	81,10,8,1	none	1330 - 1585	83,9,7,1	none	1318 - 1623
79,12,8,1	none	1330 - 1565	81,11,7,1	none	1318 - 1603	83,10,6,1	none	1306 - 1641
79,13,7,1	none	1318 - 1583	81,12,6,1	none	1306 - 1621	83,11,5,1	move 396	2256 - 2931
79,14,6,1	none	1306 - 1601	81,13,5,1	move 385	2278 - 2933	83,12,4,1	none	1285 - 1680
79,10,9,2	move 364	2352 - 2960	81,9,8,2	none	1330 - 1585	83,8,7,2	none	1318 - 1623
79,11,8,2	none	1330 - 1565	81,10,7,2	none	1318 - 1603	83,9,6,2	none	1306 - 1641
79,12,7,2	none	1318 - 1583	81,11,6,2	none	1306 - 1621	83,10,5,2	move 396	2256 - 2931
79,13,6,2	none	1306 - 1601	81,12,5,2	move 385	2278 - 2933	83,11,4,2	none	1285 - 1680
79,14,5,2	move 375	2298 - 2933	81,13,4,2	none	1285 - 1660	83,12,3,2	none	1272 - 1702
79,9,9,3	move 111	2965 - 2965	81,8,8,3	move 109	2970 - 2970	83,7,7,3	move 107	2975 - 2975
79,10,8,3	none	1330 - 1565	81,9,7,3	none	1318 - 1603	83,8,6,3	none	1306 - 1641
79,11,7,3	none	1318 - 1583	81,10,6,3	none	1306 - 1621	83,9,5,3	move 396	2256 - 2931
79,12,6,3	none	1306 - 1601	81,11,5,3	move 385	2278 - 2933	83,10,4,3	none	1285 - 1680
79,13,5,3	move 375	2298 - 2933	81,12,4,3	none	1285 - 1660	83,11,3,3	none	1272 - 1702
79,8,8,5	move 107	2970 - 2970	81,7,7,5	move 105	2975 - 2975	83,6,6,5	move 104	2978 - 2978
79,9,7,5	none	1318 - 1583	81,8,6,5	none	1306 - 1621	83,7,5,5	move 396	2256 - 2931
79,10,6,5	none	1306 - 1601	81,9,5,5	move 385	2278 - 2933	83,8,4,5	none	1285 - 1680
79,11,5,5	move 375	2298 - 2933	81,10,4,5	none	1285 - 1660	83,9,3,5	none	1272 - 1702
79,12,4,5	none	1285 - 1640	81,11,3,5	none	1272 - 1682	83,10,2,5	none	1259 - 1724
79,7,6,8	none	1306 - 1601	81,6,5,8	move 101	2976 - 2981	83,5,4,8	move 101	2977 - 2982
79,8,5,8	move 375	2298 - 2933	81,7,4,8	none	1285 - 1660	83,6,3,8	move 101	2972 - 2987
79,9,4,8	none	1285 - 1640	81,8,3,8	none	1272 - 1682	83,7,2,8	move 101	2967 - 2992
9,10,3,8	none	1272 - 1662	81,9,2,8	none	1259 - 1704	83,8,1,8	none	1245 - 1750
79,11,2,8	none	1259 - 1684	81,10,1,8	none	1245 - 1730	83,9,0,8	none	1231 - 1776

First, perpetual mutual co-operation can be attained very rapidly (as on move 115 in the {79,*X*,*Y*,1} block), or even immediately (as on move 101 in the {83,*X*,*Y*,8} block). The onset of rapid perpetual mutual co-operation, when it occurs, is hastened as the sum *W* + *Z* increases. And when it does occur, it results in very high (though not necessarily equal) scores for both twins, in the 2960-2992 point range. This situation contributes to the prominence at the high end of the scale in graph 11.9.

Second, the onset of perpetual mutual co-operation can be noticeably retarded, occurring anywhere between move 364 and move 396 in table 11.3. The delay increases with the sum of $W + Z$. And the delay, when it occurs, marks a disparity in the final scores. One pair-member attains roughly 2800–2950 points; the other, roughly 2200–2500 points. This situation thus contributes to the high-range prominence, and it forms the prominence in the next-lowest point range in graph 11.9. The trough from 2600–2800 points occurs, self-evidently, because no probabilistic event matrix in this region of the $\{W, X, Y, Z\}$ spectrum can give rise to a deterministic score in that range.

Third, there may be no onset of perpetual mutual co-operation. Such cases give rise to disparate, low final scores. The range of the disparity varies roughly from 250 points to 550 points. This range increases, between blocks, with the sum $W + Z$; and it increases, within blocks, with the difference $X - Y$. A typical score is 1621–1306 points. This situation contributes to the two other prominences, in the 1500–1700 and 1300 point ranges of graph 11.9. Again, troughs occur in the 1900–2200 and 1000–1200 point ranges because such scores are deterministically inaccessible from the event matrices in this probabilistic region of the $\{W, X, Y, Z\}$ spectrum.

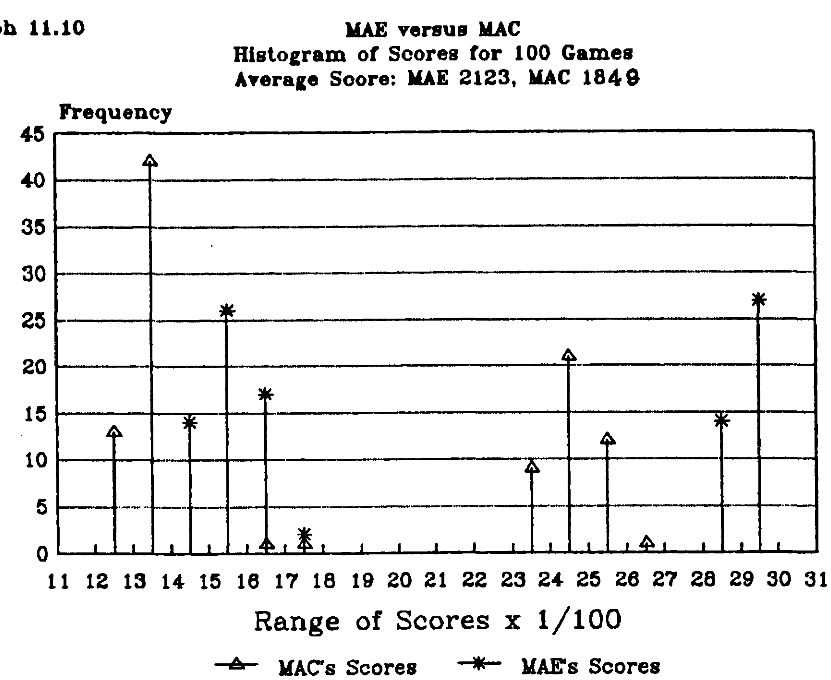
These three respective situations occur consecutively in the $\{83, X, Y, 5\}$ block of table 11.3. The instability of the event matrix is well evidenced in this block. The matrix {83,6,6,5} gives rise to perpetual mutual co-operation on move 104, and results in a final score tied at 2978 points. When the (X, Y) values fluctuate from (6,6) to (7,5), perpetual mutual co-operation does not begin until move 396, with a resultant score of 2256–2931. One further fluctuation in (X, Y) values, from (7,5) to (8,4), debars further perpetual mutual co-operation from this block, and results in scores such as 1285–1680. Thus, in this block, an initial (X, Y) difference of only 4 causes severe decrements, of 1693 and 1298 points, to the final scores of the *MAC* twins.

In sum, tables 11.1, 11.2 and 11.3 account for the different non-normal distributions of final scores in repeated encounters between *MEU–MEU*, *MAE–MAE* and *MAC–MAC* pairs. Moreover, these tables reveal some unexpected, interesting and shifting tendencies across the spectrum of possible event matrices. These tendencies convey an understanding of the general nature of the relationship between the probabilistic and deterministic phases of the maximization family's play.

This understanding extends to cases in which siblings, rather than twins, are paired. One need not resort to further tedious analyses of numerous representative probabilistic fluctuations, but one might outline just one case to illustrate how the understanding can be applied. One hundred games of *MAC* versus *MAE* give the distributions of final scores displayed in graph 11.10.

The most probable event matrix for *MAC* versus *MAE* is {64,26,7,3}, which gives rise to perpetual mutual co-operation on move 295, and thence to the most probable score of *MAC* 2473, *MAE* 2913. But the average score for one hundred games is found to be *MAC* 1849, *MAE* 2123. Again, the distributions explain the discrepancy. But what gives rise to the distributions?

Graph 11.10



In the initial event matrix, let *W* and *Z* be held constant at their most probable respective values of 64 and 3, and let (*X*, *Y*) fluctuate from (25,8) to (29,4). Then the following results obtain:

Table 11.4 - *MAC* versus *MAE*, Varying Event Matrices and Scores

Initial {*W*,*X*,*Y*,*Z*}	Perpetual (*C*,*c*)	Final Score
64,25,8,3	none	1320 - 1425
64,26,7,3	move 295	2473 - 2913
64,27,6,3	move 299	2457 - 2922
64,28,5,3	move 302	2443 - 2933
64,29,4,3	none	1280 - 1495

Table 11.4 shows how the distributions in graph 11.10 arise. The probabilistic event matrices for *MAC* versus *MAE* lie in an unstable region of the {*W*,*X*,*Y*,*Z*} spectrum, from which two main deterministic states are accessible. Perpetual mutual co-operation either commences around move 300, or it does not commence at all. The first state contributes to the higher point-range features in the respective distributions; the second, to the lower. In the first situation, *MAE* defeats *MAC* by a typical score of 2900-2500; in the second situation, by a typical score of 1600-1350.

Similar outlines could naturally be drawn to account for the results of other encounters between maximization family siblings. But the foregoing analyses convey an appreciation of the reason for *MAC*'s relatively poor performances against its twin and its siblings, as displayed in table 10.2. *MAC*'s initially high co-operative weighting, which stands *MAC* in better stead than its siblings in competition against other strategic families, militates against *MAC* in intra-familial competition. *MAC*'s probabilistic event matrices span an unstable region of the {*W*,*X*,*Y*,*Z*} spectrum, and the instability causes moderate to extreme discrepancies between *MAC*'s most probable and average scores.

MAC's less co-operatively weighted siblings, *MAE* and *MEU*, are also afflicted by this intra-familial syndrome, but to correspondingly lesser extents. *MAD* is immune to it; hence *MAD*'s most probable and average scores coincide. But *MAD*'s immunity is conferred by a property which entails far worse consequences in the interactive environment; namely, the inability to cross the threshold of perpetual mutual co-operation. Hence, *MAD*'s prophylactic measure is more debilitating than the syndrome which it prevents.

Given the broad range of possible final scores of the more co-operative maximization strategies' intra-familial play, it seems justifiable to have recorded their average scores against one another in repeated encounters in the main tournament table, as opposed to scores resulting from single encounters. Since the scores between other strategies (as well as between maximization and other strategies) have either pre-determined, deterministic, or normally-distributed values, such scores can be more confidently recorded from single enounters.

It must be fairly observed that the main tournament standings could be affected by replacing the existing intra-familial maximization strategies' scores, which are averaged for five hundred encounters between twins and one hundred encounters between siblings, with scores resulting from single encounters. Specifically, if *MAE* were to realize its lowest probabilistic scores in all intra-familial pairings, it would slip to third place, behind *SHU*, in the main tournament standings. Similarly, *MEU* would slip from seventh to eighth, and possibly to ninth place in the standings. Then again, if *MEU* were to realize its highest probabilistic scores in all intra-familial pairings, it might climb past *ETH* in the standings. Any such changes could affect the order of overall robustness.

But significantly, the combination of a propitious intra-familial showing by *MAE*, and a poor one by *MAC*, would still not allow *MAE* to overtake *MAC* in the standings. And because *MAC* would thus maintain its hold on first place, one can hypothesize that *MAC* would continue to remain most robust overall in the interactive environment.

In sum, this averaging procedure yields a main tournament order which is not necessarily absolute, but which seems to be fair. And now that a deeper understanding of the performances of the maximization strategies has been reached, one can proceed to the summary and conclusions of this enquiry.

Chapter Twelve
Summary and Conclusions

Before conclusions are drawn from this study of strategic interaction in the Prisoner's Dilemma, its perspective and principal findings are summarized. Parts One and Two of the enquiry are theoretical in nature; Parts Three and Four, experimental. At an appropriate stage, one must ask whether some form of continuity obtains between theory and experiment.

Part One outlines a game-theoretic background against which the main problem of the enquiry is configured.

Chapter One reviews fundamental taxonomic criteria of the theory of games, and introduces some received terminology in the process. One of the main strengths of the theory lies in its ability to classify games, but the intent of such classification is not restricted to the establishment of a reliable taxonomy. Because game theory embraces an awareness of its own limitations, the very act of classifying a given game is also a means of determining whether the theory is prescriptive, or merely descriptive, of the actual play. For those (relatively few) classes of games over which the theory holds normative sway; i.e., up to and including two-person, zero-sum games whose matrices contain saddle-points, the theory can prescribe the "best" moves according to minimax and maximin criteria. For those (relatively many) classes of games which are neither zero-sum nor strictly determined, the theory can still prove useful in a descriptive capacity.

Chapter Two discusses one of the main weaknesses of game theory; namely, its necessary but problematic incorporation of utility theory. With the enlistment of the utility function, game theory inherits a wealth of problems. These range from conflicting interpretations in the philosophy of probability to practical difficulties latent in probabilistic calculi, and from value-ordering scales in the intra-personal comparison of utilities to lack of value-equivalence in the inter-personal comparison of utilities. The utile is posited as a unit of utility, and is assumed to be a pure, conserved quantity. The utile enables game-theoretic modelling of qualitative situations of risk and conflict of interest by mapping players' preferences to the real numbers. The utile is thus an indispensable but also largely hypothetical unit of measure.

Chapter Three discusses a second area of game-theoretic contention; namely, the vexed question of rationality. It appears that no definition of game-theoretic rationality has been articulated that is universally acceptable or satisfactory. Examples are cited to support the argument that the rationality or irrationality of a player cannot be reliably assessed from his play alone. A move that appears tactically disadvantageous in an isolated context, such as a bluff in poker or the sacrifice of a piece in chess, can be quite sound from a strategic point of view. Depending on the nature of the game, a losing tactic can form part of a winning strategy. Similarly, a game itself can be lost in order that an associated meta-game be won.

This enquiry posits a criterion of rationality (and irrationality) which depends not upon winning (or losing) *per se*; rather, in maintaining consistency between one's preference in a game and such play as conduces to the realization of that preference. If a player prefers to win a game, he is said to be rational if he plays according to the best of his ability to win. By the same token, if a player prefers to lose a game, he is said to be rational if he plays according to the best of his ability to lose. This criterion is quintessentially game-theoretic in character, for it adheres to Rapoport's precept of appealing to the logical, as opposed to the psychological, aspects of play.[1] One does not enquire into the motives underlying a player's preference; one merely seeks consistency between that preference and the principle or strategy which the player chooses to implement.

Part Two examines the Prisoner's Dilemma in the static mode, with the object of exposing several levels at which the dilemma persists, despite ingenious attempts to resolve it. As a two-person, non-zero-sum, non-co-operative game, the Prisoner's Dilemma has no infallible, prescriptive resolution.

---

[1] Rapoport, 1966, p.103.

Chapter Four presents the paradigmatic case of the Prisoner's Dilemma, and shows how the dilemma can be represented as a fundamental conflict between two principles of choice: dominance versus maximization of expected utility. The dominance principle dictates that prisoner *A* fares better by defecting than by co-operating, regardless of what prisoner *B* elects to do. But if both prisoners adopt the dominance principle, they attain a mutually-detrimental outcome. Maximization of expected utility prescribes unequivocal co-operation in the event of complete probabilistic dependence; i.e., when the probability of a joint similar outcome, either (C,c) or (D,d), is unity. In the event of partial probabilistic dependence, the maximization principle prescribes either co-operation or defection, depending upon the relative values of the probability of a joint similar outcome and the particular payoff structure of the game. If both players co-operate, they attain a mutually-beneficial outcome.

This fundamental conflict of principle can be viewed as a conflict of rationality. In Rapoport's terms, it is individually rational to defect, but collectively rational to co-operate.[2] If two individual rationalists are caught in the dilemma, they both defect and reap detrimental desserts. If two collective rationalists are caught in the dilemma, they both co-operate and thereby extricate themselves. But if one prisoner defects while the other co-operates, the defector receives the largest reward while the co-operator sustains the worst punishment. How then can a collective rationalist avoid being exploited by an individual rationalist? This enquiry suggests that he can do so by adopting the principle of the maximization of expected utility, and taking as $p(c/C)$ the probability that the other prisoner is collectively rational.

Naturally, the suggestion is highly theoretical, and difficult to implement. Were it not so, the Prisoner's Dilemma would be resolved. In the static mode, this suggestion merely transposes the problem from one kind of dilemma to another. To wit: neither prisoner has at his disposal an objective method for ascertaining the probabi-

[2] Ibid, p.146.

lity that the other is collectively rational. Nonetheless, the persistence of the dilemma does not detract from the acuteness of Rapoport's distinction between individual and collective rationality.

Chapter Five examines another attempted resolution of the dilemma, this time in decision-theoretic terms. The resolution proceeds from a reformulation of Newcomb's Paradox. Newcomb's Paradox is a game against a state of nature, in which the player faces divergent dictates of the same two principles of choice encountered in the Prisoner's Dilemma: dominance versus maximization of expected utility. The reformulation of the paradox effectively eliminates the dominance principle from contention. Owing to the insights of Brams and Lewis, it is possible to view the Prisoner's Dilemma as a dual Newcomb's Paradox.[3] When the Prisoner's Dilemma is reformulated in a similar fashion, the dominance principle is again eliminated. This leaves maximization of expected utility as the remaining decision-theoretic principle, with mutual co-operation as a possible outcome.

As a result of this reformulation, a given prisoner's deliberation shifts from "What will the other prisoner choose?" to "Will the other prisoner correctly predict what I choose?" If the answer to the second question is in the affirmative, then the given prisoner co-operates; if in the negative, then he defects. But, notwithstanding this reformulation, the dilemma persists. It does so because, once again, a prisoner has no objective means by which to answer the new question reliably. Hence, even two collectively rational prisoners might still both defect, owing to mutual errors in judgement.

Chapter Six examines Howard's meta-game resolution of the Prisoner's Dilemma. A second-level meta-game of conditional strategies generates a matrix in which defection is neither strongly nor weakly dominant. But another kind of dominance emerges from the individually rational prisoner's consideration of the meta-strategic possibilities involved; namely, set-theoretic dominance. In the meta-game situation, set-theoretic dominance converges with the maximization of expected utility in prescribing co-operation. Thus, the meta-game formulation of the Prisoner's Dilemma marks a reconciliation

---

[3] Respectively: Brams, 1975; and Lewis, 1979.

between individual and collective rationality.

Nonetheless, the dilemma is still not resolved in a legislative sense. Even if both prisoners were made aware of the existence of the meta-game resolution, they would not be compelled to implement it. Indeed, such awareness could actually lead to mutual defection, if each prisoner sought to take advantage of the other's meta-game justification for co-operation.

Part Three examines the Prisoner's Dilemma in the iterated mode, by means of a simulated tournament of interacting strategies. Twenty different strategies compete against one another (and their twins) in games of one thousand moves in length. The experiment asks two main questions: which strategy (or strategies) are most robust in the given environment, and why?

Chapter Seven summarizes the salient results of Axelrod's two previous tournaments, and shows how the interactive tournament is intended to complement his experiments. These complemetary tournaments differ primarily in terms of the constitution of their strategic populations. Axelrod draws upon a population of "wild" strategies, by soliciting unrestricted contributions from diverse sources. The interactive tournament draws upon a population of "captive" and "domesticated" strategies, both by "capturing" interesting wild types and by selectively "breeding" a range of experimental traits.

The interactive population is heuristically grouped into five "families" of strategies: the probabilistic family, the tit-for-tat family, the maximization family, the optimization family, and the hybrid family. Each family's members are related either closely, by program structure, or more distantly, by conceptual function.

The interactive tournament's controlled environment facilitates comparison of the effectiveness of closely-related strategies, whose programs differ by the value of a single parameter. This type of control, where applicable, thus enables a parametric assessment of performance. It applies herein to the probabilistic family and to the maximization family. The relative effectiveness of these two groups of strategies, both intra-familially and in the overall environment, is of particular relevance to this enquiry, since these families contain the strategic equivalents of the divergent principles of choice encountered in the static mode. It is desirable to know how these principles fare as dynamic strategies, in the iterated mode.

Chapter Eight examines the scores of the main tournament. The winner is *MAC*, the most co-operatively weighted member of the maximization family. An analysis of all possible sub-tournaments, formed by exhausting all combinations of the twenty strategies in groups of two to twenty competitors, again shows *MAC* to be the most efficient strategy in the environment. The measure of a strategy's efficiency is taken to be the ratio of the number of strategies it betters to the number of strategies it encounters.

A strategy which is successful in a number of different situations is said to be robust. As *MAC* is the most efficient strategy in the greatest number of sub-tournaments, *MAC* is the most robust strategy with respect to combinatoric criteria.

Chapter Nine generates a series of ecosystemic competitions, based on Axelrod's ecological scenario.[4] In the ecological scenario, the ratio of two strategies' scores is assumed to represent the ratio of their populations in direct competition against one another. The ratio of their offspring in the next generation is assumed to be proportional to the ratio of their directly competing populations in the previous generation, and to their respective fractional representations in the overall population.

After a certain number of generations, the rate of population change becomes negligibly small for all competing strategies. One or more strategies may become "extinct", while each survivor attains a stable population level in his particular niche. After stability is attained, the scores of the first strategy to have become extinct are withdrawn from the pool, and a new ecosystemic competition is generated among the survivors. The process is repeated until no further extinctions take place.

Robustness in the ecological scenario is evaluated according to four parameters: longevity, average fecundity, stable efficiency, and adaptivity. Longevity is the total number of generations survived by a strategy's progeny, over all ecosystemic competitions. Average

---

[4] Axelrod, 1980*b*, pp.398-401.

fecundity is the net increase (or decrease) in a strategy's progeny, divided by its longevity. Stable efficiency is a strategy's overall efficiency at stability, in all ecosystemic competitions in which it figures. Adaptivity is the average fraction of competitors that a strategy overtakes (in transition from initial to stable population frequency), per ecosystemic competition in which it figures. Based on this four-parameter approach, *MAC* is the most robust strategy in the ecological scenario.

Chapters Eight and Nine together answer the first experimental question: *MAC* is the most robust strategy in the interactive environment, according to the criteria employed. While these criteria are neither unique nor absolute, they are combinatorically exhaustive and ecologically variegated, and arguably appropriate and fair.

Part Four, in its first two chapters, seeks to answer the second experimental question: why is *MAC* most robust in the given environment? To find an answer, a somewhat detailed examination is made of the maximization family members' performances, against other strategies and against their own siblings and twins.

Chapter Ten accounts for *MAC*'s robustness in light of *MAC*'s fulfilment of Axelrod's criterion of success: the ability to exploit the exploitable strategies without paying too high a price against the others.[5] Move-by-move analyses of the maximization family's games against representative exploitable and non-exploitable strategies reveal two important facets of their play. First, the maximization family members become slightly more exploitive as their weightings become pronouncedly less co-operative. Second, the onset of perpetual mutual co-operation with non-exploitable strategies is increasingly retarded as the maximization strategies' weightings become less co-operative.

The drastic differences between final scores resulting from early perpetual mutual co-operation with non-exploitable strategies, and final scores resulting from late or no perpetual mutual co-operation with such strategies, far overshadow the small differences between final scores against exploitable strategies, which result

---

[5] Ibid, p.403.

from large graduations in co-operative weighting. In other words, while *MAC* and *MAE* are only slightly less able to exploit the exploitable than are *MEU* and *MAD*, *MAC* and *MAE* fare far better against the non-exploitable than do *MEU* and *MAD*.

These two performance characteristics are sufficient to account for the maximization family's robustness, which increases as a function of its members' co-operative weighting. However, it transpires that the maximization strategies' average intra-familial scores are appreciably lower than their average inter-familial scores.

Chapter Eleven examines the performances of the maximization strategies against their siblings and twins, in order to ascertain the causes of their relatively poor intra-familial scores. The examination reveals that, with the exception of *MAD* versus *MAD*, large discrepancies exist between the most probable scores and the average scores attained by twins. This is indicative of non-normal distributions. The reasons for the non-normalcy naturally lie in the initial, probabilistic event matrices, of which the final scores are deterministic end-products.

The investigation then turns to the event matrix, whose intricate properties undergo marked transitions as the distribution of outcomes within the matrix changes. Four distinct types of deterministic interaction are discernible from all possible $\{W,X,Y,Z\}$ outcomes, where W is the number of instances of (C,c); X, of (C,d); Y, of (D,c); Z, of (D,d) during the initial one hundred moves.

First, below a certain threshold value of W, no perpetual mutual co-operation can occur between moves 101-1000, regardless of the (X,Y,Z) values. *MAD*'s probabilistic fluctuations about its most probable event matrix, {1,9,9,81}, lie well below this threshold. In consequence, *MAD*'s scores against its twin are normally distributed, and so *MAD*'s average score tends toward its most probable score as the number of games increases. But the scores in this region are very low.

Second, above the threshold value of W, perpetual mutual co-operation can occur when the sum $W + Z$ is sufficiently large. Empirically, it is found that (W,Z) values of (20,15) do not give rise to perpetual mutual co-operation, whereas values of (20,20) do so. The overall trend in this region is that the onset of perpetual mutual co-operation is hastened both as the sum $W + Z$ increases, and as the difference $X - Y$ decreases.

Third, as the the sum $W + Z$ becomes too large (more than 60, empirically), the matrix becomes increasingly less stable, giving rise to oscillating final scores. In this region, perpetual mutual co-operation can occur very shortly after the one hundredth move, or can occur after a delay of several hundred moves, or cannot occur at all, depending upon particular values of $\{W, X, Y, Z\}$. The oscillations exhibit some periodicity.

Fourth, above upper threshold values of (W, Z) [empirically, (83,8)], perpetual mutual co-operation either commences immediately on move 101 (if the difference between X and Y is small), or else never commences at all. At W values of 84 and higher, immediate perpetual mutual co-operation occurs only when $X = Y$. Otherwise, perpetual mutual defection occurs after move 100.

All *MAD-MAD* interactions take place in the first region. The *MEU-MEU* pair straddles the first and second regions; the *MAE-MAE* pair, the second and third regions; the *MAC-MAC* pair, the third and fourth regions. The twins' respective distributions of scores are explained by the respective spectra of scores accessible to them via probabilistic fluctuations of the event matrix in the regions in which their interactions take place. The non-normal distributions of scores between siblings are similarly explicable.

This completes the summary of the enquiry's perspective and findings. Next, an attempt is made to draw pertinent conclusions, with some attention paid to a comparison between the static and iterated modes of the Prisoner's Dilemma. The conclusions are grouped into five sets, which pertain to the following topics: first, an articulation of the relation between probabilism and collective rationality; second, a comparison between results of Axelrod's tournaments and those of the interactive tournament; third, observations about the strategic population of the interactive tournament itself; fourth, matters arising from the performance of the maximization family of strategies; and fifth, general remarks concerning the game-theoretic approach to conflict research.

That portion of the enquiry devoted to game-theoretic background discusses two principal areas in which the theory's efficacy is disputed: utility theory and rationality. The Prisoner's Dilemma is an interesting conflict model partly because it links these two areas, and in so doing gives rise to numerous complications that engender mathematical, philosophical, and social scientific enquiry. The implementation of the utile, the hypothetical pure unit of utility, allows the model to side-step the difficulties associated with intra-personal and inter-personal comparison of utilities. But the model does not seek to avoid the complexities associated with the probabilistic aspect of utility theory; rather, it creates a direct and intimate relation between probabilism and collective rationality.

A first set of conclusions pertains to the articulation of that relation, and to the way in which it is expressed in different modes.

In the static mode, the dilemma is theoretically resolved by eliminating strong and weak dominance as contending principles of choice. This invariably leaves the maximization of expected utility as the remaining principle of choice; not necessarily by preference, but certainly by default. The dilemma is then re-cast in light of the probabilistic component of the calculus of expected utility. In order to apply the maximization principle, a given prisoner is obliged to ask questions such as "What degree of probabilistic dependence, if any, obtains between both prisoners' choices?", or "What is the probability that the other prisoner is collectively rational?", or "With what probability will the other prisoner correctly predict my choice?". The dilemma persists because these questions have no definitive answer.

While attempted resolutions of the dilemma eliminate unconditional defection as a collectively rational choice, they do not prescribe unconditional co-operation to the collectively rational prisoner. The maximization of expected utility allows the possibility of defection, in order that the collectively rational prisoner protect himself, as best he can, against an individually rational or irrational fellow-prisoner. The given prisoner's assessment of the other's rationality is reflected in the given prisoner's assignment of probability values. Thus probabilism and collective rationality are inextricably linked. But in the static mode, the nature of the dilemma imposes an *a priori* probabilistic calculus, with all its attendant difficulties, upon the collectively rational prisoner.

In the iterated mode, a principle of choice becomes a strategy, which generates a sequence of choices. The iterated dominance principle is the strategy of pure defection. The soundness of not adopting pure defection, and of maximizing expected utility, is reflected in the results of the interactive tournament. In the iterated mode, it becomes possible to make use of an *a posteriori*, or frequentist interpretation of probabilism. This permits an objective assessment of the probability component in the calculation of expected utilities, which in turn reveals the capacity of the co-operatively weighted maximization strategy to perform effectively against a variety of other strategies.

But a novel weakness becomes apparent in the iterated mode. The degree of co-operative weighting necessary to extract optimum inter-familial performance from the maximization algorithm ironically condemns the maximization family members to relatively poor intra-familial performances. The maximization strategies, as formulated in this study, are unable to achieve consistent mutual co-operation with one another primarily because they do not recognize one another in competition.

Thus one perceives a dual continuity between the static and iterated modes. In both modes, maximizing expected utility is favoured as a decision rule. This is an encouraging continuity. But in both modes, the rule admits of weaknesses. These weaknesses are probabilistic complements. In the static mode, the maximization principle cannot unerringly identify its twin, because of the subjective nature of *a priori* probabilism. In the iterated mode, the maximization rule cannot consistently recognize its twin, in spite of the objective nature of *a posteriori* probabilism. When it confronts itself in either mode, the maximization rule is hoist with its own probabilistic petard. This is a discouraging continuity.

A second set of conclusions pertains to a comparison of salient results of Axelrod's tournaments with those of the interactive tournament.

Overall, the results of the interactive tournament corroborate Axelrod's most general conclusions from both of his tournaments. Respectively, these are:

> "The effectiveness of a particular strategy depends not only on its own characteristics, but also on the nature of the other strategies with which it must interact",[6]

and

> "There is no best rule [strategy] independent of the environment."[7]

Although *MAC* proved to be the most robust strategy in the interactive environment (at least according to the criteria of robustness adopted herein), *MAC* is by no means the "best" strategy for all iterated Prisoner's Dilemmas.

Without much difficulty, one can create environments in which *MAC* is not the most successful strategy. As seen in Chapter Eleven, one such environment consists of the maximization family members themselves: *MAC*, *MAE*, *MEU* and *MAD*. In the environment of these family members, *MAC* ranks third among four in points scored. And although the criteria of combinatoric and ecological robusteness have not been applied to this group in isolation, one can, with an eye on their raw scores, speculate that *MAC* would not prove most robust in these familial confines.

The interactive tournament corroborates most of Axelrod's findings with respect to particular properties of successful strategies. *TFT*'s combination of niceness, provocability and forgiveness stood it in most robust stead in Axelrod's two tournaments.[8] Similarly, *MAC* is both provocable and forgiving (except against its twin, when the instability of the event matrix can pre-empt its capacity to forgive). But *MAC* is neither nice nor rude; rather, nide. (Recall, a nice strategy is never the first to defect; a rude strategy, always the first to defect; a nide strategy, indeterminate with respect to primacy of defection.) The property of nideness is not absolutely

---

[6] Idem., 1980*a*, p.21.

[7] Idem., 1980*b*, p.402.

[8] Ibid, p.403.

preferable to that of niceness; it merely supersedes niceness in certain environments.

Of the five most robust strategies overall in the interactive tournament, *MAC* and *MAE* are nide, while *SHU*, *ETH* and *FRI* are nice. All these strategies are provocable. But *SHU* becomes incrementally less forgiving following each provocation, and *FRI* is not forgiving at all. That the quality of mercy can be strained and lacking in two fairly successful strategies is perhaps indicative of the harshness of the interactive environment.

The results of the interactive tournament also corroborate two of Axelrod's corollary conclusions.

As previously mentioned, after his first tournament Axelrod concludes that if the strategy submitted by Downing had been given a higher initial co-operative weighting, then that strategy would have won the tournament, "and won by a large margin."[9] Downing's strategy was none other than the maximization of expected utility, with the same weighting as *MEU*. The increasingly co-operative weightings of *MAE* and *MAC*, and their respective performances in the interactive tournament, support Axelrod's finding.

And as previously discussed, Axelrod's criterion of hypothetical success in his second tournament is the ability to exploit the exploitable strategies without paying too high a price against the non-exploitable strategies.[10] This criterion is not satisfied by any of the sixty-three competitors in that tournament, but it is satisfied by *MAC* (and, to a lesser extent, by *MAE*). Axelrod's criterion thus applies fully to the interactive tournament. This applicability does not confer the authority to declare that *MAC* would have won Axelrod's second tournament, but it would be interesting to re-conduct that tournament with *MAC* and *MAE* in the competing population.[11] This enquiry predicts that *MAC* would win that tournament as well.

---

[9] Idem., 1980*a*, p.20.

[10] Idem., 1980*b*, p.403.

[11] *MAC*'s and *MAE*'s algorithms would have to be modified to accommodate Axelrod's tournament games, which consist of fewer moves.

A third set of conclusions pertains to certain findings within the interactive environment itself. Since the interactive population is a regulated population, one might ask what conclusions can be drawn from the particular ways in which this experiment is regulated. Let the competing strategies be considered relative to their respective family members, and relative to their respective families' performances.

The probabilistic family fares least well overall. It is clear that neither the pure strategies (*DDD* and *CCC*) nor the mixed random strategies (*TQD*, *RAN* and *TQC*) are particularly viable in a population containing more sophisticated decision rules. *CCC* is the least robust member of this family, and the least robust strategy in the tournament, because it is utterly non-provocable and therefore thoroughly exploitable. The most successful members of this family, *DDD* and *TQD*, are also the least co-operative. *DDD* is radically exploitive, but is also radically provocative to strategies that can retaliate. *TQD* is highly exploitive, and is just co-operative enough to gull the less-provocable segment of the population. But *DDD* and *TQD* rank only thirteenth in overall robustness.

The tit-for-tat family spans an interesting range of performance. The robustness of its three most successful members, *SHU*, *TFT* and *TTT*, increases as their provocability increases and as their forgiveness decreases. It is noteworthy that their order of finish in this tournament is precisely the reverse of that in Axelrod's first tournament, which *TTT* would have won (had it been submitted) and in which *TFT* out-ranks *SHU*. Once again, this reversal seems to indicate that the environment of Axelrod's first tournament is less harsh than that of the interactive tournament. In a friendlier environment, one would naturally expect decreasing provocability and increasing forgiveness to conduce to increasing success.

The other two members of this family, *TAT* and *BBE*, do not fare well in the interactive environment. *TAT*'s contrariness (recall, *TAT* is the binary complement of *TFT*) allows it to exploit the least provocable strategies. For example, *TAT* defeats *CCC* by 5000 to 0, and *NYD* by 4984 to 14. But the same contrariness results in *TAT* itself being heavily exploited by other exploitive or unforgiving strategies. For example, *DDD* defeats *TAT* by 4996 to 1, while *FRI* defeats *TAT* by 4991 to 6. Thus *TAT*'s contrariness can lead to extremes, but does not conduce to overall success.

And note the difference between *TFT*'s and *BBE*'s performances. *TFT* is fifth most robust overall; *BBE*, fifteenth. Yet *BBE* is identical to *TFT*, save that it defects randomly with probability 1/10 following an opponent's co-operation. *BBE*'s poor showing indicates that its unprovoked defections are not tolerated by the many provocable strategies in the environment. It seems possible to observe, without trite moralization, that *BBE* is a strategy which cheats but does not prosper.

The optimization family members, whose program structures are not closely related, also span a range of performance. Respectively, *ETH* and *CHA* are fourth and fifth most robust overall. *ETH*'s decision rule can be regarded as a partial but incomplete exercise in statistical optimization. While *ETH* defects, on a give move, with a probability equal to the current frequency of an opponent's defection, *ETH* disregards joint outcomes and payoff structures alike. *CHA* is a relatively complex strategy, which does not defect lightly (recall, three conditions must be simultaneously fulfilled to provoke a defection from *CHA*).

Observe that *CHA*'s complexity does not always guarantee appreciably more effectiveness than *ETH*'s simplicity. In Axelrod's second tournament, *CHA* ranks second, offensively, among sixty-three strategies; *ETH*, fourteenth. In the main interactive tournament, their respective offensive ranks are fifth and sixth. And *ETH*, according to the criteria of this study, is actually more robust than *CHA*. But on the whole, both rules are fairly successful.

GRO, its great longevity notwithstanding, is not provocable enough to be successful in the interactive environment. And *NYD*, owing partly to its magnanimity (*NYD* always co-operates following three mutual defections) is thoroughly exploited in this environment.

The hybrid family's performance shows that strategies with alternative decision paths can be relatively effective. *FRI*, a hybrid of the two pure strategies (*CCC* and *DDD*), is seventh most robust in the interactive tournament. *FRI*'s performance is thus incomparably more successful than that of *CCC* or *DDD*. It seems reasonable to observe that, as a strategic whole, *FRI* is certainly greater than the sum of its parts. Interestingly, although *TES* is compounded from *TFT* and an exploitive strategy, *TES* does not fare as well as *FRI*, perhaps because both of its decision paths entail compromises. *FRI* either co-operates consistently or defects permanently, whereas *TES* pursues a deterministic but not pre-determined course along either path. But in general, a potential seems to exist for effective hybrids.

The maximization family's performance has been evaluated in some detail. One observes that, against a range of other strategies, a maximization strategy's robustness increases strictly with its initial co-operative weighting. Against a range of siblings and twins, however, a co-operative weighting that is too high begins to lose effectiveness. Thus, just as there is no best strategy independent of environment, there seems to be no best co-operative weighting independent of maximization family representation in the environment.

In sum, four of the five competing families place members among the seven most robust strategies in the interactive environment. *MAC* and *MAE*, from the maximization family, rank first and third; *SHU* and *TFT*, from the tit-for-tat family, second and fifth; *ETH* and *CHA*, from the optimization family, fourth and fifth; *FRI*, from the hybrid family, seventh. The only family not to be represented among the most robust strategies is the probabilistic family. One can conclude that many types of strategic program structure and/or conceptual function are capable of respectable performance in the interactive environment, with the exception of the pure and the purely random strategies.

A fourth set of conclusions pertains to the performance of the maximization family. Given Axelrod's findings and conclusions, it is not surprising that *MAC* (and *MAE*) fare so well, that *MEU* fares indifferently, and that *MAD* fares poorly. However, prior to the analysis of the inner workings of the event matrix, the results of inter-familial competition were quite surprising, with respect both to the scores and to their distributions.

It is possible to criticize the maximization family's performance on several grounds.

To begin with, it is clear that the game must be of sufficient length to allow the *a posteriori* probabilistic calculus to become maximally effective. The onset of perpetual mutual co-operation sometimes requires several hundred moves, whether in intra-familial or inter-familial competition. It must be admitted that *MAC* and *MAE*, as formulated herein, would be increasingly disadvantaged in games of correspondingly fewer moves. By the same token, of course, their performances ought to become more effective in games of correspondingly greater length.

It might be instructive to conduct a further series of experiments, in which the maximization strategies would be obliged to calculate expected utilities after varying numbers of weighted probabilistic encounters. A change in the initial number of such encounters results in a change in the quantity (and therefore in the quality) of statistical information upon which these strategies begin to act. Ultimately, one could learn how the properties of the event matrix change with respect both to alterations of the initial sum $W+X+Y+Z$, and to alterations of the length of the game itself.

If a degree of anthropomorphic latitude were permitted, one could make different kinds of judgements about *MAC*. As formulated in the interactive tournament, *MAC* virtually sacrifices the first ten percent of its moves, prior to taking any decisions, in order to obtain some idea of its opponents' play. One might be inclined to admire *MAC*'s "confidence" or "courage". One might equally well be inclined to reprobate *MAC*'s "boldness" or "bravado".

However, it can be objected that anthropomorphisms are inappropriate in formal game-theoretic contexts. As earlier observed, Rapoport excludes the psychological orientations of the players from such contexts.[12] Rapoport refrains from imputing psychological motives to a player's choice of strategy, in order to consider the question of strategic rationality strictly from the viewpoint of game theory. If one follows Rapoport's lead, then one would refrain from imputing human qualities to the strategies themselves. Whether such qualities be admirable or distasteful in a social context is arguably

[12] Rapoport, 1966, p.103.

irrelevant to a formal game-theoretic discussion of strategic effectiveness.

However, a more serious objection to the maximization family's performance can be made on game-theoretic grounds. In this enquiry's treatment of the Prisoner's Dilemma in the static mode, the maximization of expected utility is defined as a collectively rational strategy. Recall that an important attribute of a collectively rational strategy is its ability to contribute to the attainment of a Pareto-optimal outcome. In the Prisoner's Dilemma, Pareto-optimality is congruent with mutual co-operation. And in the static mode, the maximization of expected utility indeed attempts to realize this joint outcome (at least in theory), by prescribing that a strategist co-operate with the probability that the other strategist is collectively rational. Notwithstanding the difficulties associated with assigning a value to the foregoing probability statement, the strategic intent is clearly collectively rational.

The objection arises in the iterated mode, in which the maximization family exploits nice strategies that are not provocable (such as *CCC*). Is exploitiveness an attribute of collective rationality? At first blush, it appears not to be so. One reflexively associates exploitation with individual rationality, since the exploiter's gain is the exploitee's loss. When a non-exploitive strategy such as *TFT* competes with *CCC*, the pair attains immediate and perpetual mutual co-operation, and hence realizes an iterated Pareto-optimal outcome. Is this not collectively rational? When a maximization strategy such as *MAC* competes with *CCC*, the outcomes from move 101 onward are (D, c), with associated payoffs favouring the exploiter alone. Is this not individually rational?

Thus, the objection can be raised that even the most co-operatively weighted members of the maximization family are "wolves in sheep's clothing". They appear to subscribe to collective rationality in the static mode, yet they exploit non-provocable strategies in the iterated mode. This objection, of course, is not made on moral grounds. However opposed one may be to the exploitation of the weak by the strong, for example in economic or military contexts, one neither approves nor disapproves of exploitation in a Prisoner's

Dilemma computer tournament. One merely studies it as a phenomenon. No moral judgement is rendered about simulated game-theoretic exploitation; the issue is whether an exploitive strategy can claim to be collectively rational.

The objection can be met with a logical argument. Simply stated, it amounts to this: the co-operatively weighted maximization strategies can indeed claim to be collectively rational, because their performances are consistent with the concept of collective rationality supported in this enquiry. By the working definition employed herein, a strategy is collectively rational if it maximizes its expected utility, and co-operates with the probability that its opponent is collectively rational. *CCC*, albeit a nice strategy, is not a collectively rational strategy (since it lacks the capacity to defend itself by retaliatory defections).[13] Thus *MAC*'s exploitation of *CCC* is not inconsistent with *MAC*'s claim to collective rationality.

The same objection can be countered if levelled against *MAC*'s exploitation of its twin. If *MAC* is collectively rational, then why does it often fail to recognize its twin? It fails to do so because of its ironic blind-spot. That there is no best strategy independent of environment means that every strategy admits of some weakness. In *MAC*'s case, its strength against others causes its weakness against itself. Since the *MAC-MAC* pair begins its game by playing one hundred weighted random moves, certain initial distributions of outcomes cause the twins to react as if they were competing against members of the probabilistic family. Thus *MAC* is susceptible to unfortunate but understandable occurrences of mistaken identity. *MAC* is collectively rational, but prone to err in assessing the probability of its twin's collective rationality.

A strategy that is consistently nice (like *CCC*) may arouse more sympathy than a strategy that is consistently rude (like *DDD*); nonetheless, *MAC*'s perpetual defection against both indicates that, in *MAC*'s estimation, neither pure strategy is collectively rational.

---

[13] Recall that a collectively rational player, while desirous of mutual co-operation, must be able to protect himself against individually rational and irrational players.

At the same time, *MAC*'s perpetual co-operation with *TFT* indicates that *MAC* deems *TFT* to be a collectively rational strategy.

And this leads to a vital conclusion. *TFT* is indeed a collectively rational strategy, although not an exploitive one. Collective rationality is not uniquely defined. In consequence, one can conclude that there is no "most collectively rational" strategy independent of environment.

If an environment is universally "friendly", i.e. if it is populated exclusively by nice strategies, then iterated competition ceases to exist. Immediate perpetual mutual co-operation is attained by all pairs, and all scores are tied. All strategies win; none lose.

But let one rude strategy be introduced into the environment, and equality of outcome vanishes. Nice strategies that are non-provocable are exploited by the rude one, while provocable strategies retaliate against it. Winners and losers emerge.

Now let a nide strategy be introduced. If the nide strategy co-operates with the provocable, retaliates against the rude, and exploits the non-provocable, then the nide strategy wins.

If an environment is overwhelmingly "hostile", i.e. if it is populated solely by rude and unforgiving strategies, then iterated competition also tends to cease. All pairs lock into perpetual mutual defection. All strategies lose; none wins.

A collectively rational strategy exhibits different performance characteristics in differently-constituted environments. In a friendly environment, collective rationality should not manifest exploitiveness. But if the environment contains individually rational and/or irrational strategies, then a collectively rational strategy must be exploitive to be successful.

One final property of the maximization family bears mention anew. It is a remarkable dual-aspect property, unique to this family, which provides partial compensation for the maximization strategies' intermittent lack of recognition of their siblings and twins. Consider consecutive event matrices of a game between any maximization family member (*MAX*) and any opponent (*OPP*), after any number of moves have been made, wherein the latest outcome is a mutual defection:

Game 12.1 - *MAX* versus *OPP*, Consecutive Event Matrices

$W+X+Y+Z$ moves:		OPP		$W+X+Y+Z+1$ moves:		OPP	
		c	d			c	d
MAX	C	W	X	MAX	C	W	X
	D	Y	Z		D	Y	$Z+1$

$EUC = (WR + XS)/(W+X)$ | $EUC = (WR + XS)/(W+X)$

$EUD = (YT + ZP)/(Y+Z)$ | $EUD = [YT + (Z+1)P]/(Y+Z+1)$

In game 12.1, it is understood that $T > R > P > S$ and $R > 1/2(S+T)$ and $W,X,Y,Z > 0$.)

It can readily be shown, algebraically, that

$$[YT + (Z+1)P]/(Y+Z+1) < (YT + ZP)/(Y+Z)$$

The expression for *EUD* is a decreasing monotonic function of Z. It is bounded below by the value of P, which it approaches in the limit as Z becomes very large. In other words, in any iterated Prisoner's Dilemma, the expected utility of defection is actually decreased by mutual defection. This decrease, in turn, increases the maximization strategy's propensity to co-operate. Thus mutual defection contributes to co-operativeness.

Now consider consecutive event matrices of a game wherein the latest move is mutual co-operation. (In game 12.2, as in game 12.1, it is understood that $T > R > P > S$ and $R > 1/2(S+T)$ and $W,X,Y,Z > 0$.)

Game 12.2 - *MAX* versus *OPP*, Consecutive Event Matrices

$W+X+Y+Z$ moves:

		OPP c	OPP d
MAX	C	W	X
	D	Y	Z

$EUC = (WR + XS)/(W+X)$

$EUD = (YT + ZP)/(Y+Z)$

$W+X+Y+Z+1$ moves:

		OPP c	OPP d
MAX	C	$W+1$	X
	D	Y	Z

$EUC = [(W+1)R + XS)]/(W+1+X)$

$EUD = (YT + ZP)/(Y+Z)$

Similarly, it can be shown that

$$[(W+1)R + XS)]/(W+1+X) > (WR + XS)/(W+X)$$

The expression for *EUC* is an increasing monotonic function of *W*. It is bounded above by the value of *R*, which it approaches in the limit as *W* becomes very large. In other words, in any iterated Prisoner's Dilemma, the expected utility of co-operation is increased by mutual co-operation. In consequence, once a maximization strategy participates in mutual co-operation by virtue of expected utility, it becomes nice instead of nide. It will never be the first to defect following such an outcome.

In sum, maximization strategies possess a property that increasingly favours mutual co-operation as the length of the game increases, by dint of either mutually defective or mutually co-operative outcomes.

A fifth and final set of conclusions pertains to the game-theoretic approach to conflict research. One can place this enquiry in perspective by envisaging two very large sets, side by side. One set is that of actual human conflicts; the other, that of conflict models.

Human conflict, interpreted in its broadest sense, encompasses a host of phenomena that range from intra-personal ethical quandaries to international warfare. Any number of individuals and/or groups can be involved in situations of conflict, whether of principle, of interest, of ideology, of nationality, and so forth. Conflicts can be expressed verbally, violently, symbolically, structurally, and in numerous other ways.

Models are as diverse as the conflicts they represent. And models, like conflicts themselves, are susceptible to change. Game-theoretic models (as well as other types of conflict models) exhibit variation and alteration, if not amelioration, through research and development. An effective conflict model sheds coherent light on some aspect or aspects of a given conflict, and may thus contribute to the formulation of a resolution. But actual conflicts are resolved by people, not models. The existence of a resolution to a given conflict does not guarantee its implementation. Nonetheless, the existence of a resolution, or the belief that a resolution exists, can profoundly influence the will to resolve a conflict.

A sub-set of the set of conflict models is game-theoretic in character. A sub-set of this sub-set deals with Prisoner's Dilemmas. The sub-sub-set of Prisoner's Dilemmas can be further partitioned according to the number of prisoners involved. This study treats that partition which contains two-person Prisoner's Dilemmas, in both static and iterated modes. Although twenty strategies are involved in the interactive tournament, the competitions take place pair-wise; hence the tournament is a multiple, iterated two-person Prisoner's Dilemma.

Prisoner's Dilemmas are rife in the set of actual human conflicts. Hobbesian wars of *omnia contra omnes,*[14] the nuclear arms

[14] E.g. see T. Hobbes (1651), *Leviathan,* (*ed.* M. Oakeshott), Basil Blackwell, Oxford, 1957, Part I, Chapter XIII: "Hereby it is manifest, that during the time men live without a common purpose to keep them all in awe, they are in that condition which is called war; and such a war, as is of every man, against every man." Among other modern scholars, Rawls identifies this Hobbesian "state of nature" as an *N*-person Prisoner's Dilemma. See J. Rawls, *A Theory of Justice,* Clarendon Press, Oxford, 1972, p.269.

race,[15] the manufacture and sale of conventional arms,[16] among many other conflicts, can be modelled as Prisoner's Dilemmas that involve varying numbers of people, groups and nation-states.[17]

In perspective, then, this enquiry belongs to a small sub-sub-sub-set of a large universe of conflict models, which corresponds with some sub-set of a large universe of actual human conflicts. This enquiry is concerned with strategic interaction in the Prisoner's Dilemma as a game-theoretic dimension of conflict research; it does not attempt to relate its findings to human conflicts *per se*. The existence of such relations is naturally implied, since a dimension of strategic interaction is superimposed upon many—if not most—human conflicts.

However, an articulation of such relations is a task that lies far beyond the scope of the study at hand. Any such articulation must eventually re-introduce vexed questions associated with intra-personal and inter-personal comparisons of utilities, and cannot avoid confronting further paradoxes of human rationality.

Rapoport makes a cogent observation about the potential applicability, and inapplicability, of game-theoretic conflict models:

> "At present game theory has, in my opinion, two important uses, neither of them related to games nor to conflict *directly*. First, game theory stimulates us to think *about* conflict in a novel way. Second, game theory leads us to

---

[15] The nuclear arms race between the U.S.A. and the U.S.S.R. is a classic two-nation Prisoner's Dilemma. E.g. see J. Wiesner & H. York, `National Security and the Nuclear Test-Ban', *Scientific American*, **211**, October 1964, pp.27-35. For a game-theoretic treatment of the allied doctrine of "brinkmanship", see e.g. M. Deutch & R. Lewicki, ` "Locking-in" Effects During a Game of Chicken', *Journal of Conflict Resolution*, **14**, 1970, pp.367-379.

[16] The creation, maintenance and exploitation of global conventional arms markets, by individuals and governments, can be viewed as a "tragedy of the commons". The generalized socio-economic model is developed and presented by G. Hardin, `The Tragedy of the Commons', in A. Baer (ed.), *Heredity and Society*, The Macmillan Company, N.Y., 1973, pp.226-239. It can be observed that the tragedy of the commons is itself an *N*-person Prisoner's Dilemma.

[17] Aron, for example, recognizes the general usefulness of game-theory to political science, in that the theory permits "abstract formulation of the dialectic of antagonism". See R. Aron, *Peace and War*, Weidenfeld and Nicolson, London, 1966, p.772.

> some genuine impasses, that is, to situations where its axiomatic base is shown to be insufficient for dealing even theoretically with with certain types of conflict situations. These impasses set up tensions in the minds of people who care. They must therefore look around for other frameworks into which conflict situations can be cast. Thus, the impact is made on our thinking processes themselves, rather than on the actual content of our knowledge."[18]

Rapoport's observation is borne out in ensuing literature on conflict research. One can perceive an impact of game-theoretic conflict models generally, and of Prisoner's Dilemmas particularly, on contemporary philosophical conceptions of rationality.

For example, contemplation of the Prisoner's Dilemma leads the philosopher Davis to conclude that

> ". . . co-operation between individuals with clashing interests may be more rationally defensible than has been widely thought."[19]

This enquiry is intended as a modest contribution to the understanding of the Prisoner's Dilemma conflict model. It reveals, perhaps above all, that much work remains to be done to further develop this understanding. The emergence of a clearer and more detailed picture of strategic interaction could lend some impetus, in turn, to the demanding task of implementing effective strategies in flesh-and-blood Prisoner's Dilemmas, in order that actual conflicts be resolved and future ones circumvented.

Although this enquiry in no way purports to treat such momentous human problems, it strives, at least, to ponder an associated game-theoretic conflict model in a rigorous fashion. Let it conclude with the oft-cited, perhaps prescient words of Braithwaite:

> "And if anyone is inclined to doubt whether any serious enlightenment can come from the discreetly shaded candles of the card-room, I would remind him that, three hundred years ago this year (1954), that most serious of men, Blaise Pascal, laid the foundations of the mathematical theory of probability in a correspondence with Fermat about a question asked him by the Chevalier de Méré, who had found that he was losing at a game of dice more often

---

[18] Rapoport, 1960, p.242. See also K. Boulding, *Conflict and Defense*, Harper & Row, N.Y., 1963, pp.56-57.

[19] L. Davis, 'Prisoners, Paradox, and Rationality', *American Philosophical Quarterly*, 14, 1977, pp.319-327.

than he had expected. No one today will doubt the intensity, though he may dislike the colour, of the (shall I say) sodium light cast by statistical mathematics, direct descendant of theory of games of chance, upon the social sciences. Perhaps in another three hundred years' time economic and political and other branches of moral philosophy will bask in radiation from a source—theory of games of strategy—whose prototype was kindled round the poker tables of Princeton."[20]

---

[20] R. Braithwaite, *Theory of Games as Tool for the Moral Philosopher*, Cambridge at the University Press, 1955, pp.54–55. For an elucidation of the Chevalier de Méré's problem, and its solution, see e.g. Rapoport, 1960, pp.113–115.

APPENDICES

Appendix One - Glossary of Strategies

The Probabilistic Family

DDD: The strategy of pure defection. Defects unconditionally (equivalently, with probability equal to unity).

TQD: The strategy of three-quarter random defection. Co-operates and defects randomly, with respective probabilities 1/4 and 3/4.

RAN: The equiprobable random strategy. Co-operates and defects randomly with probability 1/2.

TQC: The strategy of three-quarter random co-operation. Co-operates and defects randomly, with respective probabilities 3/4 and 1/4.

CCC: The strategy of pure co-operation. Co-operates unconditionally (equivalently, with probability equal to unity).

The Tit-for-Tat Family

TFT: The strategy of tit-for-tat, the familial prototype. It co-operates on the first move, and plays next whatever its opponent played previously.

TTT: The strategy of tit-for-two-tats. It co-operates on the first two moves, and defects only after two consecutive defections by its opponent.

BBE: The strategy that "burns both ends". It plays exactly as *TFT*, but also defects randomly with a probability of 1/10 following each mutual co-operation.

SHU: Shubik's strategy. It plays exactly as *TFT*, but increments its retaliatory defections. It defects once following its opponent's first departure from mutual co-operation, defects twice consecutively following its opponent's second departure, and defects n times consecutively following its opponent's n^{th} departure.

TAT: The strategy of tat-for-tit. It defects on the first move, and plays next the opposite of whatever its opponent played previously.

The Maximization Family

MEU: The strategy of maximizing expected utilities. It plays randomly during the first 100 moves, with equal random probability of co-operating or defecting on each move. It then calculates its expected utilities of co-operation and defection, using the frequencies of past outcomes as the (*a posteriori*) probabilistic component. It then plays according to the greater expected utility, and updates the appropriate outcome frequency after each move.

MAD: The strategy of maximizing expected utilities, weighted at defection. It plays exactly as *MEU*, but its initial probabilistic weighting is 1/10 random co-operation and 9/10 random defection.

MAE: The strategy of maximizing expected utilities, weighted at equiprobable expectation. It plays exactly as *MEU*, but its initial probabilistic weighting is 5/7 random co-operation and 2/7 random defection.

MAC: The strategy of maximizing expected utilities, weighted at co-operation. It plays exactly as *MEU*, but its initial probabilistic weighting is 9/10 random co-operation and 1/10 random defection.

The Optimization Family

NYD: Nydegger's strategy. It plays tit-for-tat for the first three moves, save that if it was the only one to co-operate on the first move and the only one to defect on the second move, it defects on the third move. After that, its choice is determined from the 3 preceeding outcomes in the following manner. Let *A* be the sum formed by counting the other's defection as 2 points and one's own as 1 point, and giving weights of 16, 4 and 1 to the preceeding three moves in chronological order. The choice can be described as defecting only when *A* equals 1, 6, 7, 17, 22, 23, 26, 29, 30, 31, 33, 38, 39, 45, 49, 54, 55, 58, or 61.

GRO: Grofman's strategy. It co-operates on the first move. After that, it cooperates with probability 2/7 following a dissimilar joint outcome, and always co-operates following a similar joint outcome.

CHA: Champion's strategy. It co-operates on the first ten moves, and plays tit-for-tat on the next fifteen moves. From move twenty-six onward, it co-operates unless all of the following conditions are true: the opponent defected on the previous move, the opponent's frequency of co-operation is less than 60%, and the random number between zero and one is greater than the opponent's frequency of co-operation.

ETH: Eatherly's strategy. It co-operates on the first move, and keeps a record of its opponent's moves. If its opponent defects, it then defects with a probability equal to the relative frequency of the opponent's defections.

The Hybrid Family

FRI: Friedman's strategy. It co-operates until its opponent defects, after which it defects for the rest of the game.

TES: Gladstein's strategy, called "tester". It defects on the first move. If its opponent ever defects, it "apologizes" by co-operating, and plays tit-for-tat thereafter. Otherwise, it defects with the maximum possible relative frequency that is less than 1/2, not counting its first defection. In other words, until its opponent defects, it defects on the first move, the fourth move, and every second move after that.

Appendix Two - Table of Raw Scores

Table A2.1 - Matrix of Raw Scores, Main Tournament

	DDD	TQD	RAN	TQC	CCC	TFT	TTT	BBE	SHU	TAT	NEU	NAD	NAE	NAC	NYD	GRO	CHA	ETH	FRI	TES
DDD	1000	1952	2992	3996	5000	1004	1008	1004	1176	4996	1212	1024	1272	1380	3664	3292	1040	1004	1004	1004
TQD	762	1727	2634	3550	4470	1673	2324	1520	948	3580	920	777	1025	1098	4484	3023	2426	2405	748	1693
RAN	502	1354	2243	3139	3972	2193	3129	2098	713	2299	659	529	743	825	3968	2681	3200	3076	523	2161
TQC	251	1095	1914	2685	3472	2706	3295	2436	537	1124	405	307	479	570	3460	2586	3476	3280	248	2721
CCC	0	795	1542	2292	3000	3000	3000	2700	3000	0	120	30	243	264	3000	3000	3000	3000	3000	1500
TFT	999	1673	2193	2701	3000	3000	3000	1036	3000	2250	1108	1019	1267	2965	3000	3000	3000	3000	3000	2999
TTT	998	1444	1874	2365	3000	3000	3000	2662	3000	1800	1143	1002	1204	2935	3000	3000	3000	3000	3000	1500
BBE	999	1690	2433	2766	3200	1041	3197	1033	1174	2367	1148	1018	2989	3140	3242	2735	3215	3225	1027	1049
SHU	956	1878	2913	3877	3000	3000	3000	974	3000	4529	1125	1008	1263	1322	3000	3000	3000	3000	3000	2999
TAT	1	1055	2219	3534	5000	2250	2800	2132	384	2000	230	42	368	466	4984	3266	2837	2851	6	2251
NEU	947	1995	2899	3940	4920	1113	1538	1133	1180	4750	2384	1003	2396	1887	4875	3241	2610	2294	955	1175
NAD	994	2037	3004	3912	4980	1024	1147	1013	1168	4972	1181	1029	1266	1332	4940	3273	1193	1243	996	1013
NAE	932	1970	2903	3899	4838	1272	1624	2544	1183	4628	2356	987	2594	2123	4852	3232	2870	3000	939	1312
NAC	905	1878	2900	3750	4824	2965	2995	2665	1237	4556	1741	971	1849	1807	4814	3270	3000	2893	955	2926
NYD	334	764	1543	2305	3000	3000	3000	2637	3000	14	165	45	217	279	3000	3000	3000	3000	3000	2500
GRO	427	1233	2111	2721	3000	3000	3000	2385	3000	1156	586	468	672	670	3000	3000	3000	3000	3000	2995
CHA	990	1541	2010	2281	3000	3000	3000	2660	3000	1697	2140	988	2215	2210	3000	3000	3000	3000	3000	2987
ETH	999	1475	1875	2445	3000	3000	3000	2640	3000	1681	1614	988	2263	2505	3000	3000	3000	3000	3000	2999
FRI	999	2023	2933	4033	3000	3000	3000	1027	3000	4991	1195	1031	1259	1325	3000	3000	3000	3000	3000	1007
TES	999	1688	2156	2731	4000	2999	4000	1044	2999	2246	1175	1008	1307	2951	2500	2995	3007	2999	1002	2998

Appendix Three - Efficiency Tables

Table A3.1 - 19 Appearances in 190 Sub-Tournaments Involving 2 Strategies

	1	2	EFF%
TFT	19	0	100
SHU	19	0	100
FRI	19	0	100
TTT	15	4	78.9
MAE	15	4	78.9
CHA	14	5	73.7
ETH	14	5	73.7
TES	13	6	68.4
MEU	12	7	63.2
GRO	12	7	63.2
MAC	10	9	52.6
NYD	10	9	52.6
CCC	9	10	47.4
DDD	8	11	42.1
RAN	8	11	42.1
TAT	8	11	42.1
MAD	8	11	42.1
TQD	7	12	36.8
TQC	6	13	31.6
BBE	1	18	5.3

Table A3.2 - 171 Appearances in 1,140 Sub-Tournaments Involving 3 Strategies

	1	2	3	EFF%
FRI	132	35	4	87.4
SHU	120	49	2	84.5
TFT	105	65	1	80.4
MAE	122	26	23	78.9
TES	100	55	16	74.6
ETH	85	75	11	71.6
MEU	90	55	26	68.7
CHA	78	74	19	67.3
TTT	75	72	24	64.9
MAC	72	67	32	61.7
GRO	44	80	47	49.1
MAD	47	51	73	42.4
TQD	40	63	68	41.8
RAN	35	57	79	37.1
DDD	40	46	85	36.8
TAT	32	44	95	31.6
NYD	35	34	102	30.4
TQC	27	43	101	28.4
CCC	36	16	119	25.7
BBE	1	44	126	5.3

Table A3.3 - 969 Appearances in 4,845 Sub-Tournaments Involving 4 Strategies

	1	2	3	4	EFF%
FRI	604	213	125	27	81.3
MAE	639	129	135	66	79.5
SHU	492	333	132	12	78.2
TFT	371	450	146	2	74.3
TES	414	365	158	32	73.3
ETH	330	403	226	10	69.6
MAC	386	317	205	61	68.7
MEU	420	274	173	102	68.1
CHA	313	398	220	38	67.3
TTT	241	346	311	71	59.4
GRO	113	220	466	170	42.8
MAD	182	198	254	335	41.1
TQD	151	189	349	280	40.6
DDD	142	198	247	382	36.8
RAN	97	201	329	342	35.2
TAT	100	175	240	454	30.6
TQC	80	93	209	587	21.8
NYD	87	60	247	575	21.6
BBE	5	108	362	494	20.4
CCC	97	30	173	669	18.0

Table A3.4 - 3,876 Appearances in 15,504 Sub-Tournaments Involving 5 Strategies

	1	2	3	4	5	EFF%
MAE	2290	624	425	406	131	79.3
FRI	2050	797	640	299	90	78.5
SHU	1570	1261	749	268	28	76.3
MAC	1590	1059	786	364	77	74.0
TES	1229	1280	976	338	53	71.2
TFT	952	1564	1114	229	17	70.7
ETH	1039	1334	1190	310	3	70.0
CHA	1075	1239	1177	343	42	69.1
MEU	1359	1071	635	553	258	67.5
TTT	622	980	1334	800	140	57.4
MAD	519	661	726	962	1008	41.8
TQD	332	638	855	1241	810	39.9
GRO	267	468	1048	1612	481	39.9
DDD	331	643	663	867	1372	35.1
RAN	219	446	806	1378	1027	33.6
TAT	225	470	494	1200	1487	29.0
BBE	22	268	835	1305	1446	24.9
TQC	141	246	378	935	2176	19.3
NYD	129	189	330	1140	2088	18.6
CCC	155	91	180	826	2624	13.4

Table A3.5 - 11,628 Appearances in 38,760 Sub-Tournaments Involving 6 Strategies

	1	2	3	4	5	6	EFF%
MAE	6253	2081	1161	1186	750	197	79.5
MAC	4996	2852	1976	1324	403	77	78.0
FRI	5341	2354	1922	1292	570	149	77.5
SHU	4085	3516	2284	1328	365	50	76.3
CHA	2978	3035	3476	1800	312	27	71.2
ETH	2651	3328	3652	1753	239	5	71.0
TES	2826	3219	3166	1848	507	62	70.0
TFT	1887	3848	3839	1671	357	26	68.9
MEU	3217	3341	1646	1665	1283	476	67.1
TTT	1299	2220	3307	3255	1338	209	57.0
GRO	399	1097	1899	3983	3664	586	40.8
MAD	1041	1787	1500	2116	2460	2724	40.5
TQD	615	1386	1702	2886	3422	1617	39.4
DDD	544	1549	1454	1929	1924	4228	32.8
RAN	333	728	1338	2983	4086	2160	32.1
BBE	42	446	1718	2986	3118	3318	27.9
TAT	366	1009	1034	1545	3733	3941	27.2
NYD	118	308	595	1222	3972	5413	17.2
TQC	210	340	590	1121	3303	6064	16.7
CCC	147	175	342	782	2865	7317	11.9

Table A3.6 - 27,132 Appearances in 77,520 Sub-Tournaments Involving 7 Strategies

	1	2	3	4	5	6	7	EFF%
MAC	12293	6222	3839	2934	1461	333	50	81.3
MAE	13365	5308	2644	2447	2174	980	214	79.8
FRI	11032	5342	4606	3157	2076	742	177	77.2
SHU	8603	7867	5079	3642	1516	389	36	77.2
CHA	6490	6222	7352	5386	1497	175	10	73.0
ETH	5333	6820	7951	5496	1395	135	2	72.1
TES	5021	6766	6580	5758	2422	533	52	69.4
TFT	3025	7134	8897	5811	1886	343	36	68.2
MEU	5754	7830	3907	3399	3522	2113	607	66.7
TTT	2112	4108	5949	7935	5160	1665	203	57.0
MAD	1560	3543	3238	3652	4807	5240	5092	40.4
GRO	520	1559	3007	5714	9516	6402	414	40.2
TQD	808	2326	3241	4443	7203	6249	2862	39.0
DDD	608	2574	3144	3031	4467	4614	8694	31.8
RAN	426	896	1817	3356	8324	8785	3528	30.6
BBE	58	703	2292	5328	6627	5957	6167	29.6
TAT	547	1416	1933	2189	4085	8775	8187	25.6
NYD	72	304	810	1573	3778	9441	11154	16.5
TQC	189	393	709	1225	3063	7656	13897	14.4
CCC	90	94	440	988	2503	6951	16066	10.9

Table A3.7 - 50,388 Appearances in 125,970 Sub-Tournaments Involving 8 Strategies

	1	2	3	4	5	6	7	8	EFF%
MAC	24118	11002	6177	4618	3146	1128	180	19	83.9
MAE	22822	10531	5072	4109	4015	2681	992	166	80.3
SHU	14803	14207	9027	7098	3768	1218	253	14	78.4
FRI	18226	9825	8418	6356	4318	2416	677	152	77.3
CHA	11166	10566	12612	10669	4594	720	60	1	74.4
ETH	8613	11497	13544	11382	4490	810	50	2	73.1
TES	7066	11089	11207	11065	7302	2209	415	35	68.9
TFT	3840	10310	15743	12739	5841	1604	284	27	67.9
NEU	8033	14164	7650	5792	6401	5032	2646	670	66.5
TTT	2604	6093	8916	12794	12552	5748	1551	130	57.1
MAD	1805	5332	5582	5330	7810	7762	8689	8078	40.1
GRO	522	1669	3841	6646	12486	16522	8449	253	39.2
TQD	783	2939	4547	5954	10290	13061	9267	3547	38.5
BBE	68	775	2417	6944	10962	10819	9609	8794	30.9
DDD	451	3029	5037	4588	6501	7718	7598	15466	30.5
RAN	420	821	2015	3573	8291	16768	13404	5096	29.5
TAT	627	1515	2654	2860	4688	9030	15441	13573	24.3
NYD	29	197	705	1513	3459	8712	18274	17449	16.2
TQC	143	340	576	1113	2628	6937	15365	23286	13.5
CCC	36	25	184	828	2398	5069	12750	29098	9.8

Table A3.8 - 75,582 Appearances in 167,960 Sub-Tournaments Involving 9 Strategies

	1	2	3	4	5	6	7	8	9	EFF%
MAC	38131	15736	8597	5573	4708	2206	572	56	3	86.2
MAE	31536	16673	7967	5859	5626	4769	2347	717	88	80.8
SHU	20881	21017	13060	10510	6751	2639	622	100	2	79.6
FRI	24281	14599	12413	9957	7354	4497	1913	479	89	77.4
CHA	15374	14734	17942	15995	9178	2100	246	13	0	75.6
ETH	11107	15962	18819	17679	9408	2265	332	10	0	73.9
TES	7777	14673	15121	16171	13968	6246	1381	228	17	68.5
TFT	3943	11754	21262	21043	12227	4245	990	109	9	67.9
NEU	8746	20204	12365	8228	8873	8630	5643	2396	497	66.4
TTT	2452	6994	11196	15725	20251	13200	4575	1122	67	57.1
MAD	1536	6068	7700	6602	9043	11614	10279	12160	10580	39.3
GRO	406	1450	3357	7047	13746	21195	20340	7923	118	39.0
TQD	630	2755	5083	6913	10454	17595	17422	11133	3597	38.0
BBE	42	702	2042	6426	13416	16030	14712	11927	10285	31.9
DDD	212	2424	5897	6094	6901	11203	11005	9673	22173	29.3
RAN	295	647	1640	2863	5898	16244	25914	17014	5067	28.5
TAT	597	1308	2660	3108	4564	7837	15252	21339	18917	23.0
NYD	5	77	398	1214	2666	6555	13720	28139	22808	15.7
TQC	89	180	397	753	1388	4353	12364	25710	30348	12.8
CCC	9	0	39	224	1513	4520	8328	17737	43212	9.2

Table A3.9 - 92,378 Appearances in 184,756 Sub-Tournaments Involving 10 Strategies

	1	2	3	4	5	6	7	8	9	10	EFF%
MAC	48981	18468	9999	5672	5111	3022	949	169	7	0	88.2
MAE	35411	21570	10181	7020	6460	6017	3858	1493	339	29	81.3
SHU	24361	25207	15892	12170	9008	4307	1248	165	20	0	80.8
FRI	25997	18180	14466	12839	9383	6998	3150	1101	216	48	77.5
CHA	17082	16832	21127	19435	13000	4221	637	43	1	0	76.6
ETH	11538	18032	21656	21768	13837	4691	767	89	0	0	74.7
TES	6724	15254	16875	18740	18540	12157	3396	607	78	7	68.1
TFT	3231	10740	22751	25645	19456	8048	2061	411	34	1	67.9
NEU	7461	22688	16227	10020	9994	11019	8339	4572	1758	300	66.4
TTT	1775	6151	11125	16241	22881	21515	9204	2881	587	18	57.1
NAD	932	5385	8292	7327	8869	13181	13269	11694	13396	10033	39.3
GRO	222	1071	2353	5227	11132	20304	26684	19762	5587	36	38.5
TQD	370	1985	4270	6302	9458	16892	22413	17352	10310	3026	37.8
BBE	17	435	1447	4333	11066	18122	19621	16137	11785	9415	32.5
DDD	52	1311	5040	6514	6817	9984	12785	12779	12005	25091	28.3
RAN	171	406	904	1988	3735	9560	24650	29770	16869	4325	27.7
TAT	455	979	1884	2581	3407	6543	10551	20650	24282	21046	21.8
NYD	0	13	110	532	1637	4132	9526	18188	33868	24372	15.3
TQC	28	81	180	371	680	1827	5777	16309	32065	35060	11.7
CCC	1	0	0	27	260	2216	5863	10585	21597	51829	8.5

Table A3.10 - 92,378 Appearances in 167,960 Sub-Tournaments Involving 11 Strategies

	1	2	3	4	5	6	7	8	9	10	11	EFF%
MAC	51491	17887	9448	5181	3996	3084	1048	222	21	0	0	90.1
MAE	32357	22884	10656	6935	6248	5775	4670	2102	648	99	4	81.8
SHU	23552	24482	16053	11800	8963	5393	1809	312	13	1	0	81.8
FRI	22190	19110	13934	12783	9988	7808	4519	1514	447	70	15	77.7
CHA	15369	15934	20411	19263	14272	5931	1064	130	4	0	0	77.4
ETH	9639	16562	20590	21494	15896	6611	1380	197	9	0	0	75.3
TFT	2024	7913	19333	24429	22358	12229	3220	744	126	2	0	68.0
TES	4472	12623	15148	17808	18866	15712	6381	1193	164	11	0	67.7
NEU	4988	19940	17480	10283	9151	10729	9757	6167	2861	894	128	66.5
TTT	987	4113	8577	13631	20009	24285	14557	4581	1425	209	4	57.1
NAD	390	3427	6900	6643	7115	11081	13985	11422	9922	12679	8814	38.7
GRO	59	547	1475	2912	7404	15317	22087	24071	15288	3210	8	38.2
TQD	190	1140	2742	4434	6644	11672	20162	21513	14680	7384	1817	37.4
BBE	4	187	906	2479	6590	13170	20889	18887	13387	9193	6686	33.3
DDD	11	436	2856	5102	5555	7716	11781	12424	12165	10859	23473	27.5
RAN	59	174	399	950	1891	4357	12771	28750	26979	12940	3108	27.0
TAT	223	600	1006	1593	2196	3801	7833	11652	22142	22269	19063	20.8
NYD	0	0	15	85	538	2212	5303	9866	18791	34771	20797	15.0
TQC	10	23	56	126	261	573	1994	6420	18236	32438	32241	11.1
CCC	0	0	0	0	15	499	2754	5797	10655	20979	51679	7.9

Table A3.11 - 75,582 Appearances in 125,970 Sub-Tournaments Involving 12 Strategies

	1	2	3	4	5	6	7	8	9	10	11	12	EFF%
MAC	44390	14442	7149	3852	2582	2037	968	138	24	0	0	0	91.8
SHU	18720	19300	13502	9578	6995	5060	1973	436	18	0	0	0	82.8
MAE	23880	19982	9158	5771	4884	4538	4097	2310	780	163	19	0	82.4
CHA	11212	12350	16130	15787	12457	6163	1311	151	21	0	0	0	78.1
FRI	15039	16367	11673	9968	8559	6734	4717	1918	477	116	12	2	77.8
ETH	6416	12300	16000	17325	14543	6934	1789	243	32	0	0	0	75.7
TFT	945	4513	12881	18665	19306	13829	4382	821	216	24	0	0	67.9
TES	2171	8249	10946	13523	15264	14742	8477	1908	285	17	0	0	67.3
NEU	2546	13781	15169	8534	7336	8247	8778	6298	3264	1262	322	45	66.5
TTT	395	2020	5058	9136	13921	19812	16762	6066	1864	479	69	0	57.0
MAD	125	1592	4314	4904	4651	7277	10328	10493	9228	7462	9656	5552	38.2
GRO	6	159	622	1385	3286	8595	15700	18683	16999	8747	1400	0	38.0
TQD	62	461	1331	2289	3837	6912	12962	18194	14896	9320	4343	975	37.2
BBE	0	53	456	1173	2998	6829	14556	18091	13507	8583	5474	3862	34.2
DDD	0	82	1103	2864	3477	4361	7233	9939	11219	9088	8394	17822	26.5
RAN	15	48	119	337	688	1493	4454	13838	24482	19852	8511	1745	26.2
TAT	83	294	385	791	1087	1782	3895	6750	10365	19415	16372	14363	19.8
NYD	0	0	0	7	36	448	2464	5539	8171	15305	28803	14809	14.6
TQC	0	1	10	41	62	158	505	1712	5510	17465	25323	24795	10.6
CCC	0	0	0	0	0	30	614	2438	4616	8709	17266	41909	7.4

Table A3.12 - 50,388 Appearances in 77,520 Sub-Tournaments Involving 13 Strategies

	1	2	3	4	5	6	7	8	9	10	11	12	13	EFF%
MAC	31377	9561	4329	2169	1340	963	563	80	6	0	0	0	0	93.6
SHU	12032	12451	9457	6382	4491	3460	1697	406	30	0	0	0	0	83.6
MAE	14086	14423	6323	4039	3114	2935	2640	1980	658	171	19	0	0	83.0
CHA	6583	7748	10509	10601	8702	4893	1194	132	22	4	0	0	0	78.8
FRI	7993	11068	8433	6600	5556	4780	3534	1823	499	91	10	1	0	78.0
ETH	3320	7400	9958	11594	10395	5738	1710	243	29	1	0	0	0	76.1
TFT	316	1916	6481	11581	12453	11699	4893	841	158	49	1	0	0	67.8
TES	737	4065	6293	8153	9963	10382	7978	2441	330	46	0	0	0	66.8
NEU	937	7439	10485	5758	5050	4883	6155	5096	2777	1308	404	90	6	66.6
TTT	99	699	2158	4690	7849	12043	13628	6711	1738	653	116	4	0	56.9
MAD	17	473	1896	2732	2685	4014	6252	7597	6253	5677	4538	5545	2709	38.2
GRO	0	14	182	428	1159	3022	7523	11845	12314	9418	4078	405	0	37.5
TQD	14	154	516	902	1531	2831	6078	10604	12693	8539	4372	1830	324	37.0
BBE	0	8	175	515	971	2638	6696	12507	10904	7323	4210	2762	1679	35.1
DDD	0	0	193	1040	1707	2154	3873	5540	6815	6591	5871	5497	11107	25.7
RAN	1	15	32	77	195	389	1236	3672	11330	17693	10953	4118	677	25.7
TAT	29	103	130	216	361	629	1305	3091	4388	7937	13612	9954	8633	18.8
NYD	0	0	0	0	1	18	446	2069	3921	5676	11142	19075	8040	14.4
TQC	0	0	0	2	11	30	82	302	963	3513	12297	16747	16441	9.6
CCC	0	0	0	0	0	0	67	539	1702	2823	5917	11497	27843	6.9

Table A3.13 – 27,132 Appearances in 38,760 Sub-Tournaments Involving 14 Strategies

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	EFF%
MAC	18087	5033	1992	952	509	325	209	25	0	0	0	0	0	0	95.2
SHU	6111	6656	5368	3541	2349	1780	1025	275	27	0	0	0	0	0	84.5
MAE	6550	8493	3461	2404	1617	1447	1395	1164	478	115	8	0	0	0	83.6
CHA	3037	3927	5529	5853	4908	3003	775	91	7	2	0	0	0	0	79.4
FRI	3206	5796	5197	3682	2873	2590	2120	1193	399	67	9	0	0	0	78.3
ETH	1288	3543	4907	6304	5893	3681	1312	188	16	0	0	0	0	0	76.3
TFT	70	556	2437	5462	6423	7075	4175	805	89	35	5	0	0	0	67.5
NEU	240	3007	5710	3185	2814	2425	3230	3232	2010	879	325	64	11	0	66.7
TES	164	1466	2743	3819	5175	5756	5281	2371	322	32	3	0	0	0	66.2
TTT	11	148	662	1665	3394	5692	7891	5533	1568	406	142	20	0	0	56.8
MAD	0	82	556	1089	1182	1549	2702	4002	3883	3338	2653	2161	2776	1159	37.4
GRO	0	0	14	98	240	813	2566	5466	6377	5788	4244	1459	67	0	37.2
TQD	5	40	125	215	520	889	2160	4333	6582	6123	3789	1625	627	99	36.8
BBE	0	0	45	174	259	778	2048	5509	7193	4864	3160	1529	1035	538	36.1
RAN	0	0	1	12	26	71	205	781	2564	7396	9899	4463	1524	190	25.4
DDD	0	0	3	260	506	738	1291	2465	3609	3637	3753	3068	2483	5319	25.3
TAT	3	14	31	30	69	157	325	814	1775	2678	4910	7389	4870	4067	18.1
NYD	0	0	0	0	0	0	20	404	1341	2076	2709	6757	10375	3450	14.1
TQC	0	0	0	0	1	5	11	40	141	522	1786	7074	8124	9428	8.7
CCC	0	0	0	0	0	0	0	71	382	799	1376	3158	6858	14488	6.5

Table A3.14 – 11,628 Appearances in 15,504 Sub-Tournaments Involving 15 Strategies

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	EFF%
MAC	8396	1976	714	283	136	78	40	5	0	0	0	0	0	0	0	96.6
SHU	2372	2972	2370	1677	951	707	437	130	12	0	0	0	0	0	0	85.4
MAE	2324	3998	1523	1152	681	570	577	474	277	51	1	0	0	0	0	84.3
CHA	1081	1573	2300	2588	2233	1409	410	33	1	0	0	0	0	0	0	80.0
FRI	923	2307	2606	1691	1218	1045	930	677	186	37	8	0	0	0	0	78.6
ETH	345	1308	1894	2713	2650	1823	765	124	6	0	0	0	0	0	0	76.4
TFT	7	101	626	1899	2646	3129	2452	693	56	12	7	0	0	0	0	67.2
NEU	37	868	2364	1417	1250	1078	1242	1508	1156	445	218	36	9	0	0	66.7
TES	23	374	859	1320	2093	2487	2562	1629	268	13	0	0	0	0	0	65.7
TTT	0	12	122	417	987	2020	3330	3245	1208	195	79	13	0	0	0	56.6
GRO	0	0	0	5	31	117	472	1756	2829	2818	1889	1380	331	0	0	37.4
MAD	0	6	108	261	379	468	988	1506	1807	1555	1448	996	768	1021	317	37.3
BBE	0	0	9	34	48	169	454	1660	3047	2682	1743	950	416	286	130	37.1
TQD	0	6	15	33	89	196	493	1209	2618	3182	2116	1063	433	156	19	36.8
RAN	0	0	0	1	2	8	24	74	375	1307	3758	4181	1509	351	38	24.9
DDD	0	0	0	7	103	175	278	623	1136	1578	1473	1812	1288	1204	1951	24.0
TAT	0	1	4	3	2	28	54	128	277	860	1422	2402	2971	2011	1465	17.5
NYD	0	0	0	0	0	0	0	24	212	576	841	1352	3267	4260	1096	13.8
TQC	0	0	0	0	0	0	0	2	9	20	205	800	3228	3103	4261	8.0
CCC	0	0	0	0	0	0	0	0	28	165	304	520	1277	3111	6223	5.8

Table A3.15 - 3,876 Appearances in 4,845 Sub-Tournaments Involving 16 Strategies

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	EFF%
MAC	3053	552	187	45	25	10	4	0	0	0	0	0	0	0	0	0	97.9
SHU	672	1076	821	634	299	199	130	42	3	0	0	0	0	0	0	0	86.4
MAE	597	1488	509	417	248	179	168	159	100	11	0	0	0	0	0	0	85.1
CHA	287	478	751	878	808	516	146	12	0	0	0	0	0	0	0	0	80.4
FRI	176	672	1012	613	428	325	288	258	94	10	0	0	0	0	0	0	78.9
ETH	58	341	566	895	926	679	344	67	0	0	0	0	0	0	0	0	76.3
TFT	0	9	98	460	783	1077	1006	402	34	6	0	1	0	0	0	0	66.8
NEU	3	164	714	485	443	409	360	489	494	233	64	16	2	0	0	0	66.7
TES	1	64	179	324	594	816	936	772	182	8	0	0	0	0	0	0	64.9
TTT	0	0	8	61	192	456	1025	1310	700	109	8	7	0	0	0	0	56.5
BBE	0	0	1	4	2	22	84	298	868	1036	747	414	245	96	31	28	38.0
GRO	0	0	0	0	0	6	31	262	791	1097	880	436	316	57	0	0	37.4
NAD	0	0	1	24	91	108	192	449	618	588	509	477	281	241	246	51	37.1
TQD	0	0	0	4	5	26	84	218	593	1012	1033	603	207	70	20	1	36.9
RAN	0	0	0	0	0	0	5	5	38	158	648	1313	1342	322	45	0	25.0
DDD	0	0	0	0	1	14	40	89	292	401	495	702	541	389	288	624	23.5
TAT	0	0	0	0	0	3	3	10	21	71	274	474	1101	863	677	379	16.7
NYD	0	0	0	0	0	0	0	0	18	100	143	259	397	1403	1333	223	13.4
TQC	0	0	0	0	0	0	0	0	0	0	7	47	243	995	1032	1552	6.8
CCC	0	0	0	0	0	0	0	0	0	5	38	99	167	411	1169	1987	5.3

Table A3.16 - 969 Appearances in 1,140 Sub-Tournaments Involving 17 Strategies

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	EFF%
MAC	838	96	30	2	2	1	0	0	0	0	0	0	0	0	0	0	0	98.9
SHU	131	297	224	177	70	39	22	9	0	0	0	0	0	0	0	0	0	87.4
MAE	101	429	121	113	63	42	43	36	21	0	0	0	0	0	0	0	0	86.1
CHA	49	107	174	227	222	153	32	5	0	0	0	0	0	0	0	0	0	80.5
FRI	16	134	290	167	126	82	55	60	39	0	0	0	0	0	0	0	0	79.4
ETH	5	55	124	209	252	191	106	27	0	0	0	0	0	0	0	0	0	76.0
NEU	0	16	149	123	115	106	102	106	143	88	18	3	0	0	0	0	0	66.6
TFT	0	0	8	65	157	277	288	155	18	1	0	0	0	0	0	0	0	66.5
TES	0	6	20	55	109	179	251	256	92	0	1	0	0	0	0	0	0	64.0
TTT	0	0	0	2	24	50	200	355	291	44	2	1	0	0	0	0	0	56.2
BBE	0	0	0	0	0	2	6	31	140	266	232	174	71	32	9	2	4	38.7
GRO	0	0	0	0	0	0	0	13	166	260	238	160	87	38	7	0	0	38.5
TQD	0	0	0	0	0	1	4	19	67	228	298	217	98	22	13	2	0	36.8
NAD	0	0	0	0	0	18	30	61	134	184	114	142	120	79	38	40	9	36.3
RAN	0	0	0	0	0	0	0	0	1	5	46	234	415	237	31	0	0	25.3
DDD	0	0	0	0	0	0	0	7	28	60	165	138	144	168	73	65	121	23.7
TAT	0	0	0	0	0	0	0	0	0	1	6	31	111	438	169	146	67	15.7
NYD	0	0	0	0	0	0	0	0	0	4	20	37	66	78	446	284	34	12.9
TQC	0	0	0	0	0	0	0	0	0	0	0	0	13	21	260	305	370	6.1
CCC	0	0	0	0	0	0	0	0	0	0	0	3	15	26	95	296	534	4.1

Table A3.17 – 171 Appearances in 190 Sub-Tournaments Involving 18 Strategies

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	EFF%
MAC	162	7	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	99.6
SHU	16	56	51	28	14	4	1	1	0	0	0	0	0	0	0	0	0	0	88.6
MAE	8	91	23	18	9	10	7	4	1	0	0	0	0	0	0	0	0	0	87.8
CHA	4	18	21	46	42	33	6	1	0	0	0	0	0	0	0	0	0	0	80.3
FRI	0	13	58	36	26	17	5	7	9	0	0	0	0	0	0	0	0	0	80.2
ETH	0	5	15	29	54	36	26	6	0	0	0	0	0	0	0	0	0	0	75.4
MEU	0	0	19	23	18	21	23	17	21	27	2	0	0	0	0	0	0	0	66.6
TFT	0	0	0	5	15	46	66	32	7	0	0	0	0	0	0	0	0	0	66.3
TES	0	0	1	5	12	20	41	61	31	0	0	0	0	0	0	0	0	0	62.6
TTT	0	0	0	0	0	4	14	58	77	18	0	0	0	0	0	0	0	0	55.7
BBE	0	0	0	0	0	0	0	3	7	44	41	51	22	2	0	1	0	0	39.8
GRO	0	0	0	0	0	0	0	0	4	48	63	22	27	4	3	0	0	0	39.7
MAD	0	0	0	0	0	0	0	0	28	33	24	30	34	10	3	3	6	0	37.7
TQD	0	0	0	0	0	0	0	0	5	17	49	55	36	9	0	0	0	0	36.8
RAN	0	0	0	0	0	0	0	0	0	0	0	10	30	102	29	0	0	0	24.3
DDD	0	0	0	0	0	0	0	0	0	3	9	21	33	31	31	24	9	10	22.4
TAT	0	0	0	0	0	0	0	0	0	0	0	0	2	27	91	23	21	7	15.8
NYD	0	0	0	0	0	0	0	0	0	0	2	1	6	5	25	96	34	2	12.8
TQC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	34	62	73	4.7
CCC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	9	58	98	3.2

Table A3.18 – 19 Appearances in 20 Sub-Tournaments Involving 19 Strategies

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	EFF%
MAC	19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100
MAE	0	12	3	2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	90.1
SHU	1	6	9	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	89.8
FRI	0	0	6	8	1	3	0	0	1	0	0	0	0	0	0	0	0	0	0	81.6
CHA	0	2	0	6	6	4	1	0	0	0	0	0	0	0	0	0	0	0	0	79.5
ETH	0	0	1	1	8	4	4	1	0	0	0	0	0	0	0	0	0	0	0	74.3
MEU	0	0	1	2	1	5	2	3	1	4	0	0	0	0	0	0	0	0	0	66.7
TFT	0	0	0	0	1	2	10	5	1	0	0	0	0	0	0	0	0	0	0	65.8
TES	0	0	0	0	1	0	3	8	7	0	0	0	0	0	0	0	0	0	0	60.8
TTT	0	0	0	0	0	0	0	3	10	6	0	0	0	0	0	0	0	0	0	54.7
BBE	0	0	0	0	0	0	0	0	0	4	8	5	2	0	0	0	0	0	0	43.0
GRO	0	0	0	0	0	0	0	0	0	3	5	5	5	1	0	0	0	0	0	40.1
MAD	0	0	0	0	0	0	0	0	0	3	3	4	3	5	0	1	0	0	0	37.4
TQD	0	0	0	0	0	0	0	0	0	0	4	6	9	0	0	0	0	0	0	36.4
RAN	0	0	0	0	0	0	0	0	0	0	0	0	1	7	10	1	0	0	0	24.6
DDD	0	0	0	0	0	0	0	0	0	0	0	0	0	6	10	2	0	0	1	22.2
TAT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	1	2	0	15.2
NYD	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	16	2	0	11.4
TQC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	15	2	5.6
CCC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	17	0.9

Appendix Four - Sample Programs

This appendix contains fifteen sample programs from the interactive tournament. All programs are listed in GW-BASIC. These listings were saved as ASCII files, and imported into WordPerfect. To accelerate actual data processing, the longer programs were compiled in TURBO BASIC and saved as DOS-executable files.

All programs in this appendix are documented according to GW-BASIC syntax. Remarks are therefore inserted in one of two permissible ways: either following an "REM:" statement on a separately-enumerated line, or following a single quotation mark, " ' ", after an executable instruction.

The first ten programs consist of games between different strategic pairs. Thus, the algorithms of all twenty strategies are represented. Each program is named according to the particular strategies in competition; e.g. "*TFTvDDD*" (tit-for-tat versus the strategy of pure defection). In all games, the strategies' moves are represented in binary form; that is, "0" means a defection; "1", a co-operation.

The eleventh sample program, *COMTOU*, generates the data used in the combinatoric analyses of Chapter Eight. It accepts input of the number of strategies to be involved in a given set of sub-tournaments, then outputs the efficiency table for that set.

The twelfth sample program, *ECOSYS*, generates the data for the first ecosystemic competition of Chapter Nine, involving all twenty strategies.

The remaining three sample programs are used for analyzing various aspects of the maximization strategies' intra-familial performance.

TESTMAT accepts input of any values of $\{W,X,Y,Z\}$ (such that $W + X + Y + Z = 100$) for the initial probabilistic event matrix, and outputs the subsequently determined event matrices and their associated expected utilities and scores, at one hundred move intervals, for the remainder of the game.

MAXvMAX accepts input of initial co-operative weightings and the number of games to be run, and outputs the resulting average scores and distribution of scores.

MAXrMAX accepts input of *W* and *Z* values for the probabilistic event matrix, and the input of the domain of the difference between *X* and *Y*, over which the corresponding range of initial event matrices is generated. *MAXrMAX* outputs the $\{W,X,Y,Z\}$ values for each initial event matrix in the selected range, the final score resulting from each event matrix in the range, and the number of moves required for the onset of perpetual mutual co-operation (if it occurs).

<u>Program A4.1 - *TFTvDDD*</u>

```
100 DIM DDD(1000)            'ARRAY OF DDD's MOVES
110 DIM TFT(1000)            'ARRAY OF TFT's MOVES
120 TFT(1) = 1               'TFT CO-OPERATES ON MOVE 1
130 RANDOMIZE TIMER          'SEEDS PSEUDO-RANDOM GENERATOR
140 FOR I = 1 TO 1000        'GAME OF 1000 MOVES
150 IF RND(I) < 1 THEN DDD(I)= 0 ELSE DDD(I)= 1
155 REM: LINE 150: DDD DEFECTS WITH PROBABILITY OF UNITY
160 IF I<=999 THEN TFT(I+1) = DDD(I)      'TFT's DECISION RULE
165 REM: LINES 170-200 ASSIGN PAYOFFS TO OUTCOMES
170 IF DDD(I)=1 AND TFT(I)=1 THEN R=R+3: T=T+3
180 IF DDD(I)=1 AND TFT(I)=0 THEN T=T+5
190 IF TFT(I)=1 AND DDD(I)=0 THEN R=R+5
200 IF DDD(I)=0 AND TFT(I)=0 THEN R=R+1: T=T+1
210  IF I MOD  100=0 THEN  PRINT I;R;T   'DISPLAY  SCORE AT  100 MOVE
INTERVALS
220 NEXT I                             'NEXT MOVE
230 PRINT "DDD's score is" R           'PRINT FINAL SCORES
240 PRINT "TFT's score is" T
```

<u>Program A4.2 - *CCCvTAT*</u>

```
100 DIM CCC(1000)
110 DIM TAT(1000)
120 FOR I = 1 TO 1000
130 TAT(1) = 0           'TAT DEFECTS ON MOVE 1
140 RANDOMIZE TIMER
150 IF RND(I) < 0 THEN CCC(I)= 0 ELSE CCC(I)= 1
165 REM: CCC CO-OPERATES WITH PROBABILITY OF UNITY
160 IF I<1000 THEN IF CCC(I) = 0 THEN TAT(I+1) = 1 ELSE TAT(I+1) = 0
165 REM: LINE 160: TAT's DECISION RULE
170 IF CCC(I)=1 AND TAT(I)=1 THEN R=R+3: T=T+3
180 IF CCC(I)=1 AND TAT(I)=0 THEN T=T+5
190 IF TAT(I)=1 AND CCC(I)=0 THEN R=R+5
200 IF CCC(I)=0 AND TAT(I)=0 THEN R=R+1: T=T+1
210 IF I MOD 100=0 THEN PRINT I;R;T
220 NEXT I
230 PRINT "CCC's score is" R
240 PRINT "TAT's score is" T
```

Program A4.3 - *TQCvTTT*

```
100 DIM TQC(1000)
110 DIM TTT(1000)
120 RANDOMIZE TIMER
130 FOR J=1 TO 1000
140 IF RND(J) < .75 THEN TQC(J) = 1 ELSE TQC(J) = 0
145 REM: LINE 140: TQC CO-OPERATES RANDOMLY WITH PROBABILITY 3/4
150 NEXT J
160 TTT(1) = 1      'TTT CO-OPERATES ON MOVES 1 AND 2
170 TTT(2) = 1
180 FOR I=1 TO 1000
190  IF  I>2 THEN  IF TQC(I-2)=0  AND  TQC(I-1)=0 THEN  TTT(I)=0 ELSE
TTT(I)=1
195 REM: LINE 190: TTT's DECISION RULE
200 IF TQC(I)=1 AND TTT(I)=1 THEN R=R+3: T=T+3
210 IF TQC(I)=1 AND TTT(I)=0 THEN T=T+5
220 IF TTT(I)=1 AND TQC(I)=0 THEN R=R+5
230 IF TQC(I)=0 AND TTT(I)=0 THEN R=R+1: T=T+1
240 NEXT I
250 PRINT "TQC's score is" R
260 PRINT "TTT's score is" T
```

Program A4.4 - *TQDvCHA*

```
100 DIM TQD(1000)
110 DIM CHA(1000)
120 FOR I = 1 TO 1000
130 IF I<=10 THEN CHA(I)=1   'CHA CO-OPERATES ON FIRST 10 MOVES
140 RANDOMIZE TIMER
150 IF RND(I) < .75 THEN TQD(I)= 0 ELSE TQD(I)= 1
155 REM: LINE 150: TQD DEFECTS RANDOMLY WITH PROBABILITY 3/4
160 IF I>10 AND I<=25 THEN IF TQD(I-1)=1 THEN CHA(I)=1 ELSE CHA(I)=0
165 REM: CHA PLAYS TIT-FOR-TAT BETWEEN MOVES 11 AND 25
170 IF TQD(I)=1 THEN C=C+1 ELSE D=D+1
175 REM: LINE 170:  CHA INCREMENTS THE NUMBER OF  TQD's CO-OPERATIONS
OR DEFECTIONS
180 IF I>25 THEN RANDOMIZE TIMER
190 IF I>25 THEN IF TQD(I-1)=0 AND C/(C+D)<.6 AND RND(J)>C/(C+D) THEN
CHA(I)=0 ELSE CHA(I)=1     'CHA'S DECISION RULE
200 IF TQD(I)=1 AND CHA(I)=1 THEN R=R+3: T=T+3
210 IF TQD(I)=1 AND CHA(I)=0 THEN T=T+5
220 IF CHA(I)=1 AND TQD(I)=0 THEN R=R+5
230 IF TQD(I)=0 AND CHA(I)=0 THEN R=R+1: T=T+1
240 IF I MOD 100=0 THEN PRINT I;R;T
250 NEXT I
260 PRINT "TQD's score is" R
270 PRINT "CHA's score is" T
```

Program A4.5 - *RANvTES*

```
100 DIM RAN(1001)
110 DIM TES(1001)
120 Z=0: W=0        'FLAGS WHICH, WHEN  SET, DETERMINE TES's  DECISION
PATH
130 TES(1)=0       'TES DEFECTS ON MOVE 1
140 RANDOMIZE TIMER
150 FOR I = 1 TO 1000
160 IF RND(I)< .5 THEN RAN(I)=0 ELSE RAN(I)=1
165 REM: LINE 160: RAN CO-OPERATES  OR DEFECTS RANDOMLY WITH PROBABI-
LITY 1/2
170 IF I=1 THEN GOTO 230     'ASSIGN PAYOFFS TO OUTCOME OF MOVE 1
180 IF W=1 THEN GOSUB 390    'IF W FLAG IS SET THEN TES PLAYS TIT-FOR-
TAT
190 IF RAN(I-1)=0 THEN Z=1  'IF RAN DEFECTS THEN Z FLAG IS SET
200 IF Z=1 THEN GOSUB 320    'IF Z FLAG IS SET THEN TES APOLOGIZES AND
SETS W FLAG
210 IF RAN(I-1)=1 THEN Z=2  'TES's DECISION RULE UNTIL RAN DEFECTS
220 IF Z=2 THEN GOSUB 350
230 IF RAN(I)=1 AND TES(I)=1 THEN R=R+3: T=T+3
240 IF RAN(I)=1 AND TES(I)=0 THEN T=T+5
250 IF TES(I)=1 AND RAN(I)=0 THEN R=R+5
260 IF RAN(I)=0 AND TES(I)=0 THEN R=R+1: T=T+1
270 IF I MOD 100=0 THEN PRINT I;R;T
280 NEXT I
290 PRINT "RAN's score is" R
300 PRINT "TES's score is" T
310 END
320 TES(I)=1
330 W=1
340 RETURN 230
350 IF I=2 THEN TES(I)=1
360 IF I=3 THEN TES(I)=1
370 IF I>3 THEN IF I MOD 2=0 THEN TES(I)=0 ELSE TES(I)=1
380 RETURN 230
390 IF RAN(I-1)=0 THEN TES(I)=0 ELSE TES(I)=1
400 RETURN 230
```

Program A4.6 - *SHUvBBE*

```
100 DIM BBE(3000)
110 DIM SHU(3000)
115  REM: EXPANDED SCORE ARRAY SAFELY ACCOMMODATES SHU's INCREMENTING
RETALIATORY DEFECTIONS
120 SHU(1)=1     'BOTH SHU AND BBE CO-OPERATE ON MOVE 1
130 BBE(1)=1
140 RANDOMIZE TIMER
150 FOR I=1 TO 1000+Q
155 REM:  LINE  150: Q  IS THE  NUMBER OF  MOVES ADDED  DUE TO  SHU's
INCREMENTING DEFECTIONS. HOWEVER, THE GAME SCORE IS COMPUTED FROM THE
FIRST 1000 MOVES.
160 IF I=1 THEN GOTO 260  'FIRST JOINT OUTCOME IS RECORDED. MAKE NEXT
MOVES.
170 IF SHU(I-1)=1 AND RND(I)<.9 THEN BBE(I)=1 ELSE BBE(I)=0
175 REM: LINE 170: BBE's DECISION RULE
180 IF  BBE(I-1)=1 THEN SHU(I)=1    'SHU's  DECISION RULE IF  BBE CO-
OPERATES
190 IF BBE(I-1)=0 THEN Q=Q+1 ELSE GOTO 260
195 REM: LINE 190: SHU's DECISION RULE IF BBE DEFECTS
200 FOR K=I TO I+Q-1  'LINES 200-220: SHU's RETALIATORY DEFECTION(S)
210 SHU(K)=0
220 NEXT K
230 SHU(I+Q)=1
240 GOSUB 370    'BBE's RESPONSE TO SHU's RETALIATORY DEFECTIONS
250 I=I+Q
255  REM: LINE  250: GAME  MOVE ADJUSTED  TO ACCOMMODATE  RETALIATORY
DEFECTIONS. SHU RESUMES PLAYING TIT-FOR-TAT AT MOVE I+Q
260 NEXT I
270 FOR I=1 TO 1000      'ASSIGN PAYOFFS TO FIRST 1000 OUTCOMES
280 IF BBE(I)=1 AND SHU(I)=1 THEN R=R+3: T=T+3
290 IF BBE(I)=1 AND SHU(I)=0 THEN T=T+5
300 IF SHU(I)=1 AND BBE(I)=0 THEN R=R+5
310 IF BBE(I)=0 AND SHU(I)=0 THEN R=R+1: T=T+1
320 IF I MOD 100=0 THEN PRINT I;R;T
330 NEXT I
340 PRINT "BBE's score is" R
350 PRINT "SHU's score is" T
360 END
370 FOR J=I+1 TO I+Q
380 IF SHU(J-1)=1 AND RND(I)<.9 THEN BBE(J)=1 ELSE BBE(J)=0
390 NEXT J
400 RETURN
```

Program A4.7 - *ETHvMEU*

```
100 DIM ETH(1000)
110 DIM MEU(1001)
120 RANDOMIZE TIMER
130 FOR J=1 TO 100
140 IF RND(J)<.5 THEN MEU(J)=0 ELSE MEU(J)=1
150 NEXT J
155 REM: LINES 130-150: MEU MAKES FIRST 100 RANDOM MOVES, CO-OPERAT-
ING OR DEFECTING WITH EQUAL PROBABILITY
160 ETH(1)=1   'ETH CO-OPERATES ON FIRST MOVE
170 RANDOMIZE TIMER
180 FOR I=1 TO 1000  'MOVES OF THE GAME
190 IF MEU(I)=1 THEN X=X+1 ELSE Y=Y+1
195 REM:  LINE 190:  ETH UPDATES  NUMBER OF  MEU's CO-OPERATIONS  AND
DEFECTIONS
200 IF I>1  THEN IF MEU(I-1)=0  AND RND(I)<= (Y/(X+Y)) THEN  ETH(I)=0
ELSE ETH(I)=1   'ETH's DECISION RULE
210 IF MEU(I)=1 AND ETH(I)=1 THEN C=C+1  'MEU UPDATES EVENT MATRIX
220 IF MEU(I)=1 AND ETH(I)=0 THEN D=D+1  'MEU UPDATES EVENT MATRIX
230 IF MEU(I)=0 AND ETH(I)=1 THEN E=E+1  'MEU UPDATES EVENT MATRIX
240 IF MEU(I)=0 AND ETH(I)=0 THEN F=F+1  'MEU UPDATES EVENT MATRIX
250 IF I>=100 THEN UC=3*C/(C+D)   'MEU FINDS EXPECTED UTILITY OF  CO-
OPERATION
260  IF I>=100 THEN  UD=(5*E+F)/(E+F) 'MEU FINDS  EXPECTED UTILITY OF
DEFECTION
270 IF I>=100 THEN IF UC>=UD  THEN MEU(I+1)=1 ELSE MEU(I+1)=0  'MEU's
DECISION RULE
275 REM: LINES 280-310 UPDATE GAME SCORES
280 IF MEU(I)=1 AND ETH(I)=1 THEN M=M+3: S=S+3
290 IF MEU(I)=1 AND ETH(I)=0 THEN S=S+5
300 IF MEU(I)=0 AND ETH(I)=1 THEN M=M+5
310 IF MEU(I)=0 AND ETH(I)=0 THEN M=M+1: S=S+1
320 IF I  MOD 100=0 THEN GOSUB  350  'PRINTS OUTCOMES,  UTILITIES AND
SCORES AT 100-MOVE INTERVALS
330 NEXT I
340 END
350 PRINT C+D+E+F "MOVES"
360 PRINT C,D,UC
370 PRINT E,F,UD
380 PRINT "ETH's SCORE IS" S
390 PRINT "MEU's SCORE IS" M
400 PRINT
410 RETURN 330
```

Program A4.8 - *GROvMAD*

```
100 DIM GRO(1000)
110 DIM MAD(1001)
120 RANDOMIZE TIMER
130 FOR J=1 TO 100
140 IF RND(J)<.9 THEN MAD(J)=0 ELSE MAD(J)=1
150 NEXT J
155 REM: LINES 130-150: MAD MAKES 100 RANDOM MOVES, DEFECTING WITH
PROBABILITY 9/10
160 GRO(1)=1  'GRO CO-OPERATES ON MOVE 1
170 FOR I=1 TO 1000
180 IF I>1 THEN IF MAD(I-1)=GRO(I-1) THEN Q=1 ELSE Q=2   'Q IS GRO's
DECISION FLAG
190 IF Q=1 THEN GRO(I)=1     'GRO CO-OPERATES FOLLOWING SYMMETRIC
OUTCOME
200 IF Q=2 THEN RANDOMIZE TIMER
210 IF Q=2 THEN IF RND(I)<=(2/7) THEN GRO(I)=1 ELSE GRO(I)=0 'GRO
CO-OPERATES WITH PROBABILITY 2/7 FOLLOWING ASYMMETRIC OUTCOME
220 IF MAD(I)=1 AND GRO(I)=1 THEN C=C+1   'MAD UPDATES EVENT MATRIX
230 IF MAD(I)=1 AND GRO(I)=0 THEN D=D+1   'MAD UPDATES EVENT MATRIX
240 IF MAD(I)=0 AND GRO(I)=1 THEN E=E+1   'MAD UPDATES EVENT MATRIX
250 IF MAD(I)=0 AND GRO(I)=0 THEN F=F+1   'MAD UPDATES EVENT MATRIX
260 IF I>=100 THEN UC=3*C/(C+D) 'MAD FINDS EXPECTED UTILITY OF CO-
OPERATION
270 IF I>=100 THEN UD=(5*E+F)/(E+F) 'MAD FINDS EXPECTED UTILITY OF
DEFECTION
280 IF I>=100 THEN IF UC>=UD THEN MAD(I+1)=1 ELSE MAD(I+1)=0 'MAD'S
DECISION RULE
290 IF MAD(I)=1 AND GRO(I)=1 THEN M=M+3: S=S+3
300 IF MAD(I)=1 AND GRO(I)=0 THEN S=S+5
310 IF MAD(I)=0 AND GRO(I)=1 THEN M=M+5
320 IF MAD(I)=0 AND GRO(I)=0 THEN M=M+1: S=S+1
330 IF I MOD 100=0 THEN GOSUB 360
340 NEXT I
350 END
360 PRINT C+D+E+F "MOVES"
370 PRINT C,D,UC
380 PRINT E,F,UD
390 PRINT "GRO's SCORE IS" S
400 PRINT "MAD's SCORE IS" M
410 PRINT
420 RETURN 340
```

Program A4.9 - *FRIvMAE*

```
100 DIM FRI(1000)
110 DIM MAE(1001)
120 RANDOMIZE TIMER
130 FOR J=1 TO 100
140 IF RND(J)<.28 THEN MAE(J)=0 ELSE MAE(J)=1
150 NEXT J
155 REM: LINES 130-150: MAE MAKES  FIRST 100 RANDOM MOVES, CO-OPERAT-
ING WITH PROBABILITY 5/7
160 FRI(1)=1  'FRI CO-OPERATES ON MOVE 1
170 FOR I=1 TO 1000
180 IF Z=1 THEN GOTO  200  'Z IS A FLAG THAT IS SET IF FRI ENTERS THE
DECISION PATH OF PERPETUAL DEFECTION
190 IF I>1 THEN IF MAE(I-1)=1 THEN FRI(I)=1 ELSE GOSUB 410
195 REM: LINE  190: FRI  CO-OPERATES IF MAE  CO-OPERATED ON  PREVIOUS
MOVE. IF MAE DEFECTED, THEN FRI ENTERS PATH OF PERPETUAL DEFECTION.
200 IF MAE(I)=1 AND FRI(I)=1 THEN C=C+1
210 IF MAE(I)=1 AND FRI(I)=0 THEN D=D+1
220 IF MAE(I)=0 AND FRI(I)=1 THEN E=E+1
230 IF MAE(I)=0 AND FRI(I)=0 THEN F=F+1
240 IF I>=100 THEN UC=3*C/(C+D)
250 IF I>=100 THEN UD=(5*E+F)/(E+F)
260 IF I>=100 THEN IF UC>=UD THEN MAE(I+1)=1 ELSE MAE(I+1)=0   'MAE's
DECISION RULE
270 IF MAE(I)=1 AND FRI(I)=1 THEN M=M+3: S=S+3
280 IF MAE(I)=1 AND FRI(I)=0 THEN S=S+5
290 IF MAE(I)=0 AND FRI(I)=1 THEN M=M+5
300 IF MAE(I)=0 AND FRI(I)=0 THEN M=M+1: S=S+1
310 IF I MOD 100=0 THEN GOSUB 340
320 NEXT I
330 END
340 PRINT C+D+E+F "MOVES"
350 PRINT C,D,UC
360 PRINT E,F,UD
370 PRINT "FRI's SCORE IS" S
380 PRINT "MAE's SCORE IS" M
390 PRINT
400 RETURN 320
410 Z=1
420 FOR K=I TO 1000
430 FRI(K)=0
440 NEXT K
450 RETURN 200
```

Program A4.10 - *NYDvMAC*

```
100 DATA 1,6,7,17,22,23,26,29,30,31,33,38,39,45,49,54,55,58,61
105 REM: LINE 100: THESE ARE THE CRITICAL SUMS OF WEIGHTED VALUES FOR
THREE CONSECUTIVE MOVES WHICH ELICIT NYD's DEFECTION, ACCORDING TO
ITS DECISION RULE
110 DIM NYD(1000)
120 DIM MAC(1001)
130 DIM X(19)
140 FOR K=1 TO 19  'LINES 140-160 READ CRITICAL SUMS INTO ARRAY X
150 READ X(K)
160 NEXT K
170 RANDOMIZE TIMER
180 FOR J=1 TO 100
190 IF RND(J)<.1 THEN MAC(J)=0 ELSE MAC(J)=1
200 NEXT J
205 REM: LINES 180-200: MAC MAKES FIRST 100 RANDOM MOVES, CO-OPERAT-
ING WITH PROBABILITY 9/10
210 NYD(1)=1  'NYD CO-OPERATES ON MOVE 1
220 FOR I=1 TO 1000  'GAME BEGINS
230 IF I=1 THEN GOTO 390  'MAC UPDATES EVENT MATRIX AFTER FIRST
OUTCOME
240 IF I=2 THEN IF MAC(I-1)=1 THEN NYD(I)=1 ELSE NYD(I)=0
250 IF I=3 THEN IF (MAC(1)<>NYD(1) AND MAC(2)<>NYD(2)) OR MAC(2)=0
THEN NYD(3)=0 ELSE NYD(3)=1
255 REM: LINES 240-250: NYD's DECISION RULE FOR MOVES 2 AND 3
260 IF I<4 THEN GOTO 390  'MAC UPDATES EVENT MATRIX AFTER SECOND AND
THIRD OUTCOMES
270 FOR L=1 TO 3  'LINES 270-330: NYD FINDS SUM OF WEIGHTED VALUES
FOR THREE PREVIOUS CONSECUTIVE MOVES
280 IF MAC(I-L)=0 THEN P=2 ELSE P=0
290 IF NYD(I-L)=0 THEN Q=1 ELSE Q=0
300 IF L=3 THEN SUM=SUM+16*(P+Q)
310 IF L=2 THEN SUM=SUM+4*(P+Q)
320 IF L=1 THEN SUM=SUM+P+Q
330 NEXT L
340 FOR N=1 TO 19  'LINES 340-350: NYD DECIDES WHETHER SUM IS CRITI-
CAL
350 IF SUM=X(N) THEN GOSUB 600  'IF SUM IS CRITICAL THEN NYD DEFECTS
360 NEXT N
370 NYD(I)=1  'IF SUM IS NOT CRITICAL THEN NYD CO-OPERATES
380 SUM=0: P=0: Q=0  'RESETS SUM AND VALUE COUNTERS
390 IF MAC(I)=1 AND NYD(I)=1 THEN C=C+1
400 IF MAC(I)=1 AND NYD(I)=0 THEN D=D+1
410 IF MAC(I)=0 AND NYD(I)=1 THEN E=E+1
420 IF MAC(I)=0 AND NYD(I)=0 THEN F=F+1
430 IF I>=100 THEN UC=3*C/(C+D)
440 IF I>=100 THEN UD=(5*E+F)/(E+F)
450 IF I>=100 THEN IF UC>=UD THEN MAC(I+1)=1 ELSE MAC(I+1)=0  'MAC's
DECISION RULE
460 IF MAC(I)=1 AND NYD(I)=1 THEN M=M+3: S=S+3
470 IF MAC(I)=1 AND NYD(I)=0 THEN S=S+5
480 IF MAC(I)=0 AND NYD(I)=1 THEN M=M+5
490 IF MAC(I)=0 AND NYD(I)=0 THEN M=M+1: S=S+1
500 IF I MOD 100=0 THEN GOSUB 530
```

```
510 NEXT I
520 END
530 PRINT C+D+E+F "MOVES"
540 PRINT C,D,UC
550 PRINT E,F,UD
560 PRINT "NYD's SCORE IS" S
570 PRINT "MAC's SCORE IS" M
580 PRINT
590 RETURN 510
600 NYD(I)=0
610 RETURN 380
```

Program A4.11 - *COMTOU* and Chain Merge File Z2

```
5 REM: COMTOU COMPUTES EFFICIENCY TABLES FOR COMBINATORIC SUB-TOURNA-
MENTS INVOLVING Z COMPETITORS, WHERE Z CAN RANGE FROM 2 TO 19
7 REM: DATA STATEMENTS (LINES 10-105) ARE ROWS FROM TABLE OF RAW
SCORES
1 0 D A T A
1000,1952,2992,3996,5000,1004,1008,1004,1176,4996,1212,1024,1272,1380
,3664,3292,1040,1004,1004,1004
1 5 D A T A
762,1727,2634,3550,4470,1673,2324,1520,948,3580,920,777,1025,1098,448
4,3023,2426,2405,748,1693
2 0 D A T A
502,1354,2243,3139,3972,2193,3129,2098,713,2299,659,529,743,825,3968,
2681,3200,3076,523,2161
2 5 D A T A
251,1095,1914,2685,3472,2706,3295,2436,537,1124,405,307,479,570,3460,
2586,3476,3280,248,2721
3 0 D A T A
0,795,1542,2292,3000,3000,3000,2700,3000,0,120,30,243,264,3000,3000,3
000,3000,3000,1500
3 5 D A T A
999,1673,2193,2701,3000,3000,3000,1036,3000,2250,1108,1019,1267,2965,
3000,3000,3000,3000,3000,2999
4 0 D A T A
998,1444,1874,2365,3000,3000,3000,2662,3000,1800,1143,1002,1204,2935,
3000,3000,3000,3000,3000,1500
4 5 D A T A
999,1690,2433,2766,3200,1041,3197,1033,1174,2367,1148,1018,2989,3140,
3242,2735,3215,3225,1027,1049
5 0 D A T A
956,1878,2913,3877,3000,3000,3000,974,3000,4529,1125,1008,1263,1322,3
000,3000,3000,3000,3000,2999
5 5 D A T A
1,1055,2219,3534,5000,2250,2800,2132,384,2000,230,42,368,466,4984,326
6,2837,2851,6,2251
6 0 D A T A
947,1995,2899,3940,4920,1113,1538,1133,1180,4750,2384,1003,2396,1887,
4875,3241,2610,2294,955,1175
6 5 D A T A
994,2037,3004,3912,4980,1024,1147,1013,1168,4972,1181,1029,1266,1332,
4940,3273,1193,1243,996,1013
7 0 D A T A
932,1970,2903,3899,4838,1272,1624,2544,1183,4628,2356,987,2594,2123,4
852,3232,2870,3000,939,1312
7 5 D A T A
905,1878,2900,3750,4824,2965,2995,2665,1237,4556,1741,971,1849,1807,4
814,3270,3000,2893,955,2926
8 0 D A T A
334,764,1543,2305,3000,3000,3000,2637,3000,14,165,45,217,279,3000,300
0,3000,3000,3000,2500
8 5 D A T A
427,1233,2111,2721,3000,3000,3000,2385,3000,1156,586,468,672,670,3000
,3000,3000,3000,3000,2995
```

```
9              0   D              A              T              A
990,1541,2010,2281,3000,3000,3000,2660,3000,1697,2140,988,2215,2210,3
000,3000,3000,3000,3000,2987
9              5   D              A              T              A
999,1475,1875,2445,3000,3000,3000,2640,3000,1681,1614,988,2263,2505,3
000,3000,3000,3000,3000,2999
1          0          0 D          A          T          A
999,2023,2933,4033,3000,3000,3000,1027,3000,4991,1195,1031,1259,1325,
3000,3000,3000,3000,3000,1007
1          0          5 D          A          T          A
999,1688,2156,2731,4000,2999,4000,1044,2999,2246,1175,1008,1307,2951,
2500,2995,3007,2999,1002,2998
1          0          7 D          A          T          A
DDD,TQD,RAN,TQC,CCC,TFT,TTT,BBE,SHU,TAT,MEU,MAD,MAE,MAC,NYD,GRO,CHA,E
TH,FRI,TES
109 INPUT "ENTER THE NUMBER OF COMPETITORS (FROM 2-19)";Z
110 DIM RAW(20,20) 'ARRAY OF RAW SCORES FOR 20 VERSUS 20 STRATEGIES
120 DIM SCO(2,20)  'ARRAY THAT ASSOCIATES A STRATEGY'S SUB-TOURNAMENT
SCORE WITH ITS ACRONYM
125 DIM GAM(Z)     'ARRAY  THAT ENUMERATES RAW  SCORE ENTRIES USED  IN
SUB-TOURNAMENTS
130 DIM NA$(20)  'STRING ARRAY OF ACRONYMS
140 DIM PER(20)  'ARRAY OF EFFICIENCY PERCENTAGES
150 DIM TOU(20,Z+1) 'ARRAY THAT ASSOCIATES A STRATEGY'S NET APPEARAN-
CES AND NET RANKINGS WITH ITS ACRONYM, IN SUB-TOURNAMENTS FOR A GIVEN
Z
155 DIM OAN(20)  'ARRAY OF RANK NUMBERS
160 DIM PTF(2,20)   'ARRAY THAT ASSOCIATES A STRATEGY'S  ACRONYM WITH
ITS TOTAL SCORE IN A GIVEN SUB-TOURNAMENT
170 FOR J=1 TO  20  'LINES 170-230: READ  DATA (RAW SCORES) INTO  RAW
ARRAY
190 FOR K=1 TO 20
200 READ RAW(J,K)
210 NEXT K
230 NEXT J
240 FOR K=1 TO 20
250 READ NA$(K) 'READS ACRONYMS INTO NA$ STRING ARRAY
260  TOU(K,Z+1)=CVI(MID$(NA$(K),2,2))  'ASSOCIATES  A UNIQUE  INTEGER
WITH EACH  STRING ACRONYM. THUS  ENCODED, ACRONYMS CAN  BE ASSOCIATED
WITH NUMERIC DATA.
270 NEXT K
275 REM:  LINES 280-319: MERGE PROGRAM "Z*" WITH COMTOU, WHERE "*" IS
THE INPUT Z VALUE. THE Z* PROGRAMS SELECT ALL COMBINATIONS OF STRATE-
GIES  FOR EACH  GIVEN Z  VALUE. FOR  EXAMPLE, SEE  Z2 FOLLOWING  THIS
LISTING.
280 IF Z=2 THEN CHAIN MERGE "B:Z2",2000,ALL
282 IF Z=3 THEN CHAIN MERGE "B:Z3",3000,ALL
284 IF Z=4 THEN CHAIN MERGE "B:Z4",4000,ALL
285 IF Z=5 THEN CHAIN MERGE "B:Z5",5000,ALL
286 IF Z=6 THEN CHAIN MERGE "B:Z6",6000,ALL
288 IF Z=7 THEN CHAIN MERGE "B:Z7",7000,ALL
290 IF Z=8 THEN CHAIN MERGE "B:Z8",8000,ALL
300 IF Z=9 THEN CHAIN MERGE "B:Z9",9000,ALL
302 IF Z=10 THEN CHAIN MERGE "B:Z10",10000,ALL
304 IF Z=11 THEN CHAIN MERGE "B:Z11",11000,ALL
```

```
306 IF Z=12 THEN CHAIN MERGE "B:Z12",12000,ALL
308 IF Z=13 THEN CHAIN MERGE "B:Z13",13000,ALL
310 IF Z=14 THEN CHAIN MERGE "B:Z14",14000,ALL
312 IF Z=15 THEN CHAIN MERGE "B:Z15",15000,ALL
314 IF Z=16 THEN CHAIN MERGE "B:Z16",16000,ALL
316 IF Z=17 THEN CHAIN MERGE "B:Z17",17000,ALL
318 IF Z=18 THEN CHAIN MERGE "B:Z18",18000,ALL
319 IF Z=19 THEN CHAIN MERGE "B:Z19",19000,ALL
320 FOR X=1 TO Z 'LINES 320-350: INCREMENT APPEARANCE NUMBERS OF
STRATEGIES SELECTED FOR CURRENT SUB-TOURNAMENT
340 TOU(GAM(X),0)=TOU(GAM(X),0)+1
350 NEXT X
360 FOR X=1 TO Z 'LINES 360-420: ADDS SCORES OF STRATEGIES COMPETING
IN CURRENT SUB-TOURNAMENT
370 FOR Y=1 TO Z
380 SUM=SUM+RAW(GAM(X),GAM(Y))
390 NEXT Y
400 SCO(1,X)=SUM: PTF(1,X)=SUM
410 SUM=0
420 NEXT X
430 FOR Y=1 TO Z 'LINES 430-460: 'ASSOCIATES SCORES WITH ENCODED
ACRONYMS
440 SCO(2,Y)=CVI(MID$(NA$(GAM(Y)),2,2))
450 PTF(2,Y)=SCO(2,Y)
460 NEXT Y
470 SORT=1 'LINES 470-530: SORTS SCORES AND ASSOCIATED ACRONYMS
ACCORDING TO ORDER OF MAGNITUDE
480 WHILE SORT
490 SORT=0
500 FOR X=1 TO Z-1
510 IF PTF(1,X)<PTF(1,X+1) THEN SWAP PTF(1,X),PTF(1,X+1): SWAP
PTF(2,X),PTF(2,X+1): SORT=1
520 NEXT X
530 WEND
610 OAN(1)=1 'LINES 610-640: ASSOCIATE RANK NUMBERS WITH STRATEGIES
IN CURRENT SUB-TOURNAMENT, ACCORDING TO ORDERED SCORES
620 FOR Y=2 TO Z
630 IF PTF(1,Y)=PTF(1,Y-1) THEN OAN(Y)=OAN(Y-1) ELSE OAN(Y)=Y
640 NEXT Y
650 FOR X=1 TO 20 'LINES 650-690: INCREMENT RANK-COUNTERS FOR CURRENT
SUB-TOURNAMENT
660 FOR Y=1 TO Z
670 IF PTF(2,Y)=TOU(X,Z+1) THEN TOU(X,OAN(Y))=TOU(X,OAN(Y))+1
680 NEXT Y
690 NEXT X
695 REM: LINES 700-734: RETURN TO APPROPRIATE MERGED FILE, SELECT
COMPETITORS FOR NEXT SUB-TOURNAMENT
700 IF Z=2 THEN RETURN 2060
702 IF Z=3 THEN RETURN 3070
704 IF Z=4 THEN RETURN 4080
706 IF Z=5 THEN RETURN 5090
708 IF Z=6 THEN RETURN 6100
710 IF Z=7 THEN RETURN 7110
712 IF Z=8 THEN RETURN 8120
714 IF Z=9 THEN RETURN 9130
```

```
716 IF Z=10 THEN RETURN 10140
718 IF Z=11 THEN RETURN 11150
720 IF Z=12 THEN RETURN 12160
722 IF Z=13 THEN RETURN 13170
724 IF Z=14 THEN RETURN 14180
726 IF Z=15 THEN RETURN 15190
728 IF Z=16 THEN RETURN 16200
730 IF Z=17 THEN RETURN 17210
732 IF Z=18 THEN RETURN 18220
734 IF Z=19 THEN RETURN 19120
750  PRINT "INTERACTION TABLE FOR THE"W"SUB-TOURNAMENTS OF (20 CHOOSE
"Z") COMBINATIONS":PRINT   'OUTPUT TABLE HEADING
760 GOSUB 910 'COMPUTE EACH STRATEGY'S COMBINATORIC EFFICIENCY
770  PRINT"RULE    SEL%"SPC(2); 'PRINT  COLUMN  HEADINGS  (ACRONYM, %
APPEARANCES)
780 FOR J=1  TO Z          'LINES 780-800:  PRINT COLUMN HEADINGS (RANK
NUMBERS)
790 PRINT USING "##  ";J,
800 NEXT J
810 PRINT"EFF%":PRINT  'PRINT EFFICIENCY COLUMN HEADING
820 FOR J=1 TO 20 'LINES  820-890: OUTPUT COLUMN DATA FOR EACH  STRA-
TEGY
830 PRINT NA$(J)SPC(3);  'PRINT ACRONYM
840 PRINT USING "###  ";TOU(J,0)*100/W,  'PRINT % OF APPEARANCES
850  FOR K=1 TO  Z   'LINES 850-870: PRINT  RATIO OF  EACH RANKING TO
TOTAL APPEARANCES, FROM 1ST TO ZTH
860 PRINT USING ".## ";TOU(J,K)/TOU(J,0),
870 NEXT K
880 PRINT USING " ##.#";PER(J) 'PRINT EFFICIENCY
890 NEXT J
900 END
910 FOR J=1 TO 20
920 FOR K=1 TO Z-1
930 SUM=SUM+(Z-K)*TOU(J,K)
940 NEXT K
950 PER(J)=SUM*100/((Z-1)*TOU(J,0))
960 SUM=0
970 NEXT J
980 RETURN 770
990  REM: LINES 2000-2999 CONSTITUTE MERGED PROGRAM Z2, WHICH SELECTS
ALL POSSIBLE COMBINATIONS OF 2 STRATEGIES.
2000 W=0  'W COUNTS THE NUMBER OF COMBINATIONS
2005  REM: LINES 2010-2030:  SELECTS COMBINATIONS OF  TWO STRATEGIES,
ONE COMBINATION AT  A TIME. ENUMERATED  STRATEGIES ARE PASSED TO  GAM
ARRAY.
2010 FOR A=1 TO 19
2020 FOR B=A+1 TO 20
2030 GAM(1)=A: GAM(2)=B
2040 W=W+1
2050 GOSUB 320 'RETURNS TO MAIN PROGRAM WITH SELECTED COMBINATION
2060 NEXT B: NEXT A 'MAIN PROGRAM REQUESTS NEXT COMBINATION
2999 GOTO 750 'OUTPUT DATA AFTER ALL COMBINATIONS ARE EXHAUSTED
```

Program A4.12 - *ECOSYST*

```
5 REM: ECOSYST GENERATES THE ECOSYSTEMIC  COMPETITION FOR 20 STRATE-
GIES.
1                0  D                  A                   T                A
1000,1952,2992,3996,5000,1004,1008,1004,1176,4996,1212,1024,1272,1380
,3664,3292,1040,1004,1004,1004
1                5  D                  A                   T                A
762,1727,2634,3550,4470,1673,2324,1520,948,3580,920,777,1025,1098,448
4,3023,2426,2405,748,1693
2                0  D                  A                   T                A
502,1354,2243,3139,3972,2193,3129,2098,713,2299,659,529,743,825,3968,
2681,3200,3076,523,2161
2                5  D                  A                   T                A
251,1095,1914,2685,3472,2706,3295,2436,537,1124,405,307,479,570,3460,
2586,3476,3280,248,2721
3                0  D                  A                   T                A
0,795,1542,2292,3000,3000,3000,2700,3000,0,120,30,243,264,3000,3000,3
000,3000,3000,1500
3                5  D                  A                   T                A
999,1673,2193,2701,3000,3000,3000,1036,3000,2250,1108,1019,1267,2965,
3000,3000,3000,3000,3000,2999
4                0  D                  A                   T                A
998,1444,1874,2365,3000,3000,3000,2662,3000,1800,1143,1002,1204,2935,
3000,3000,3000,3000,3000,1500
4                5  D                  A                   T                A
999,1690,2433,2766,3200,1041,3197,1033,1174,2367,1148,1018,2989,3140,
3242,2735,3215,3225,1027,1049
5                0  D                  A                   T                A
956,1878,2913,3877,3000,3000,3000,974,3000,4529,1125,1008,1263,1322,3
000,3000,3000,3000,3000,2999
5                5  D                  A                   T                A
1,1055,2219,3534,5000,2250,2800,2132,384,2000,230,42,368,466,4984,326
6,2837,2851,6,2251
6                0  D                  A                   T                A
947,1995,2899,3940,4920,1113,1538,1133,1180,4750,2384,1003,2396,1887,
4875,3241,2610,2294,955,1175
6                5  D                  A                   T                A
994,2037,3004,3912,4980,1024,1147,1013,1168,4972,1181,1029,1266,1332,
4940,3273,1193,1243,996,1013
7                0  D                  A                   T                A
932,1970,2903,3899,4838,1272,1624,2544,1183,4628,2356,987,2594,2123,4
852,3232,2870,3000,939,1312
7                5  D                  A                   T                A
905,1878,2900,3750,4824,2965,2995,2665,1237,4556,1741,971,1849,1807,4
814,3270,3000,2893,955,2926
8                0  D                  A                   T                A
334,764,1543,2305,3000,3000,3000,2637,3000,14,165,45,217,279,3000,300
0,3000,3000,3000,2500
8                5  D                  A                   T                A
427,1233,2111,2721,3000,3000,3000,2385,3000,1156,586,468,672,670,3000
,3000,3000,3000,3000,2995
9                0  D                  A                   T                A
990,1541,2010,2281,3000,3000,3000,2660,3000,1697,2140,988,2215,2210,3
000,3000,3000,3000,3000,2987
```

```
9 5 D A T A
999,1475,1875,2445,3000,3000,3000,2640,3000,1681,1614,988,2263,2505,3
000,3000,3000,3000,3000,2999
1 0 0 D A T A
999,2023,2933,4033,3000,3000,3000,1027,3000,4991,1195,1031,1259,1325,
3000,3000,3000,3000,3000,1007
1 0 5 D A T A
999,1688,2156,2731,4000,2999,4000,1044,2999,2246,1175,1008,1307,2951,
2500,2995,3007,2999,1002,2998
1 0 7 D A T A
DDD,TQD,RAN,TQC,CCC,TFT,TTT,BBE,SHU,TAT,MEU,MAD,MAE,MAC,NYD,GRO,CHA,E
TH,FRI,TES
110 DIM NA$(20)     'ARRAY OF ACRONYMS
120 DIM  ECO(20,2)    'ARRAY OF  RELATIVE  POPULATIONS,  CURRENT  AND
PREVIOUS GENERATIONS
130 DIM  ALL(20,20,2)   'ARRAY OF  RAW SCORES,  CURRENT AND  PREVIOUS
GENERATIONS
140 DIM ROW(20)        'ARRAY OF TOTAL SCORES FOR EACH STRATEGY
160 FOR  I=1 TO 20         'LINES 160-190:  READ RAW TOURNAMENT  SCORES
(FIRST GENERATION)
170 FOR J=1 TO 20
180 READ ALL(I,J,1)
190 NEXT J: NEXT I
210 FOR  I=1 TO 20    'LINES  210-280: COMPUTE EACH  STRATEGY'S TOTAL
SCORE (FIRST GENERATION)
220 FOR J=1 TO 20
230 SUM=SUM+ALL(I,J,1)
240 NEXT J
250 ROW(I)=SUM
260 TOT=TOT+ROW(I)   'TOT  IS THE TOTAL  OF TOTAL  SCORES, FOR  FIRST
GENERATION
270 SUM=0
280 NEXT I
290 LPRINT "FRACTIONAL DISTRIBUTIONS,  PER 1000 OF POPULATION":LPRINT
'PRINT TABLE HEADING
300 FOR J=1 TO 20      'LINES 300-320: PRINT STRATEGY NAMES  AS COLUMN
HEADINGS
310 READ NA$(J)
315 LPRINT NA$(J)SPC(1);
320 NEXT J
325 PRINT
440 FOR Q=1  TO 20  'LINES  440-470: COMPUTE & PRINT  EACH STRATEGY'S
RELATIVE POPULATION FOR 1ST GENERATION
450 ECO(Q,1)=ROW(Q)/TOT
460 LPRINT USING "### ";1000*ECO(Q,1);
470 NEXT Q
473 TOT=0
475 N=2   'SETS NEXT GENERATION = 2
480 WHILE W<3      'STOP AFTER THREE CONSECUTIVE IDENTICAL GENERATIONS
OF POPULATION FREQUENCIES (THIS NEVER OCCURRED)
490 FOR I=1 TO 20  'LINES 490-530: COMPUTE SCORES FOR CURRENT GENERA-
TION
500 FOR J=1 TO 20
510 ALL(I,J,2)=ALL(I,J,1)*(ROW(I)/(ROW(I)+ROW(J)))
520 NEXT J
```

```
530 NEXT I
570  FOR P=1  TO 20   'LINES 570-640:  COMPUTE EACH  STRATEGY'S TOTAL
SCORE FOR CURRENT GENERATION
580 FOR Q=1 TO 20
590 SUM=SUM+ALL(P,Q,2)
600 NEXT Q
610 ROW(P)=SUM
620 TOT=TOT+ROW(P)      'COMPUTES TOTAL  OF TOTAL  SCORES FOR  CURRENT
GENERATION
630 SUM=0
640 NEXT P
650 FOR P=1  TO 20  'LINES  650-680: COMPUTE & PRINT  EACH STRATEGY'S
RELATIVE POPULATION FREQUENCY FOR CURRENT GENERATION
660 ECO(P,2)=ROW(P)/TOT
670 LPRINT USING "### ";1000*ECO(P,2);
680 NEXT P
685 TOT=0
730 M=0       'LINES 730-770: CHECK WHETHER  CURRENT GENERATION'S FRE-
QUENCIES ARE IDENTICAL TO PREVIOUS GENERATION'S FREQUENCIES
740 FOR J=1 TO 20
750 IF ECO(J,2)=ECO(J,1) THEN M=M+1
760 NEXT J
770 IF M=20 THEN W=W+1
780 FOR J=1 TO 20      'SETS PREVIOUS GENERATION'S ARRAYS  TO THOSE OF
CURRENT GENERATION
790 ECO(J,1)=ECO(J,2)
800 FOR K=1 TO 20
810 ALL(J,K,1)=ALL(J,K,2)
820 NEXT K: NEXT J
830 N=N+1   'INCREMENTS GENERATION
840 IF N MOD 50=0  THEN GOSUB 880  'REPRINTS COLUMN HEADINGS EVERY 50
GENERATIONS
850 WEND   'PROCESS NEXT GENERATION
860 LPRINT "STABILITY ATTAINED AFTER"N-W-1"GENERATIONS"
870 END
880 LPRINT:LPRINT N"GENERATIONS":LPRINT
890 FOR J=1 TO 20
900 LPRINT NA$(J)SPC(1);
910 NEXT J
920 LPRINT
930 RETURN 850
```

Program A4.13 - *TESTMAT*

```
90 REM: TESTMAT GENERATES A GAME BETWEEN TWO MAXIMIZATION STRATEGIES,
MX1 AND MX2. INPUT IS A 100-MOVE EVENT MATRIX. TESTMAT OUTPUTS
RESULTING EVENT MATRICES AND SCORES AT 100-MOVE INTERVALS
100 INPUT "INITIAL C,D,E,F VALUES";C,D,E,F
110 DIM MX1(100)  'ARRAY OF MX1's FIRST 100 MOVES
120 DIM MX2(100)  'ARRAY OF MX2's FIRST 100 MOVES
130  FOR L=1 TO 10  'CONDUCT GAME IN 10 BLOCKS OF 100 MOVES
140 IF L=1  THEN S1=(3*C+5*E+F): S2=(3*C+5*D+F): GOTO  210  'COMPUTES
INITIAL SCORE (AFTER 100 MOVES) FROM INPUT
150 FOR M=1 TO 100  'START NEXT BLOCK OF MOVES
160 IF (L>1) AND (M=1) THEN GOTO 260
165 REM: LINES 170-200: UPDATE EVENT MATRIX
170 IF MX1(M-1)=1 AND MX2(M-1)=1 THEN C=C+1
180 IF MX1(M-1)=1 AND MX2(M-1)=0 THEN D=D+1
190 IF MX1(M-1)=0 AND MX2(M-1)=1 THEN E=E+1
200 IF MX1(M-1)=0 AND MX2(M-1)=0 THEN F=F+1
205 REM: LINES 210-240: FIND EXPECTED UTILITIES
210 IF (C<>0) OR (D<>0) THEN U1C = 3*C/(C+D)
220 IF (E<>0) OR (F<>0) THEN U1D = (5*E+F)/(E+F)
230 IF (C<>0) OR (E<>0) THEN U2C = 3*C/(C+E)
240 IF (D<>0) OR (F<>0) THEN U2D = (5*D+F)/(D+F)
250 IF L=1 THEN GOTO 340
260 IF U1C >=U1D THEN MX1(M)=1 ELSE MX1(M)=0  'MX1's DECISION RULE
270 IF U2C >=U2D THEN MX2(M)=1 ELSE MX2(M)=0  'MX2's DECISION RULE
275 REM: LINES 280-310: UPDATE SCORES
280 IF MX1(M)=1 AND MX2(M)=1 THEN S1=S1+3: S2=S2+3
290 IF MX1(M)=1 AND MX2(M)=0 THEN S2=S2+5
300 IF MX1(M)=0 AND MX2(M)=1 THEN S1=S1+5
310 IF MX1(M)=0 AND MX2(M)=0 THEN S1=S1+1: S2=S2+1
320 IF L>1 AND M=100 THEN GOSUB 440
330 NEXT M   'MAKE NEXT MOVE
335  REM: LINES 340-410:  OUTPUT EXPECTED UTILITIES,  EVENT MATRICES,
AND SCORES
340 PRINT C,D,U1C
350 PRINT E,F,U1D
360 PRINT C+D+E+F; "MOVES"
370 PRINT C,E,U2C
380 PRINT D,F,U2D
390 PRINT "MX1's SCORE IS";S1
400 PRINT "MX2's SCORE IS";S2
410 PRINT
420 NEXT L  'START NEW BLOCK OF 100 MOVES
430 END
435 REM: LINES 440-500: UPDATE EVENT MATRIX AND SCORES ON  100TH MOVE
OF EACH BLOCK
440 IF MX1(100)=1 AND MX2(100)=1 THEN C=C+1
450 IF MX1(100)=1 AND MX2(100)=0 THEN D=D+1
460 IF MX1(100)=0 AND MX2(100)=1 THEN E=E+1
470 IF MX1(100)=0 AND MX2(100)=0 THEN F=F+1
480 U1C = 3*C/(C+D): U1D = (5*E+F)/(E+F)
500 U2C = 3*C/(C+E): U2D = (5*D+F)/(D+F)
520 RETURN
```

Program A4.14 - *MAXvMAX*

```
90 REM: MAXvMAX CONDUCTS N GAMES BETWEEN TWO MAXIMIZATION FAMILY
MEMBERS (MX1 & MX2), AND OUTPUTS THEIR AVERAGE SCORES AND A HISTOGRAM
OF MX1's SCORE DISTRIBUTION.
100 INPUT "ENTER THE NUMBER OF GAMES TO BE RUN";N
110 INPUT "THE VALUE OF RND(J)";L        'SET INITIAL CO-OPERATIVE
WEIGHTING FOR MX1
120 INPUT "THE VALUE OF RND(J+1)";M      'SET INITIAL CO-OPERATIVE
WEIGHTING FOR MX2
140 DEFINT A-F,I-K,P   'INTEGER VARIABLES DEFINED FOR MORE RAPID
EXECUTION
150 DIM S1(100)      'ARRAY OF MX1's SCORES
160 DIM S2(100)      'ARRAY OF MX2's SCORES
170 DIM MX1(1001)   'ARRAY OF MX1's MOVES
180 DIM MX2(1001)   'ARRAY OF MX2's MOVES
190 DEF FNSEL(X)=INT(X*.01)   'DEFINES FUNCTION THAT ALLOCATES EACH
GAME SCORE TO APPROPRIATE HISTOGRAM CHANNEL
200 DIM HIS(29)       'ARRAY OF 29 HISTOGRAM CHANNELS
210 FOR K=1 TO N     'PLAY FIRST GAME
220 RANDOMIZE TIMER
230 FOR J=1 TO 100  'LINES 230-260: MX1 & MX2 MAKE THEIR 100 RANDOM
MOVES
240 IF RND(J)< L THEN MX1(J)=0 ELSE MX1(J)=1
250 IF RND(J+1)< M THEN MX2(J)=0 ELSE MX2(J)=1
260 NEXT J
270 FOR I=1 TO 1000   'LINES 280-320: EVENT MATRIX AND SCORES ARE
UPDATED
280 IF MX1(I)=1 AND MX2(I)=1 THEN C=C+1: A=A+3: B=B+3
290 IF MX1(I)=1 AND MX2(I)=0 THEN D=D+1: B=B+5
300 IF MX1(I)=0 AND MX2(I)=1 THEN E=E+1: A=A+5
310 IF MX1(I)=0 AND MX2(I)=0 THEN F=F+1: A=A+1: B=B+1
320 IF I>=100 THEN IF D<>0 AND E<>0 THEN U1C=3*C/(C+D): U2C=3*C/(C+E)
'MX1 & MX2 FIND THEIR EXPECTED UTILITIES OF CO-OPERATION
330 IF I>=100 THEN IF D<>0 AND E<>0 THEN U1D=(5*E+F)/(E+F): U2D=
(5*D+F)/(D+F)  'MX1 & MX2 FIND THEIR EXPECTED UTILITIES OF DEFECTION
340 IF I>=100 THEN IF U1C>=U1D THEN MX1(I+1)=1 ELSE MX1(I+1)=0
'MX1's DECISION RULE
350 IF I>=100 THEN IF U2C>=U2D THEN MX2(I+1)=1 ELSE MX2(I+1)=0 'MX2's
DECISION RULE
360 NEXT I
370 SUM1=SUM1+A: SUM2=SUM2+B  'SCORES SUMMED FOR LATER AVERAGING
380 S1(K)=A: S2(K)=B  'GAME SCORES ENTERED IN SCORE ARRAYS
390 Q=FNSEL(A)    'MX1's SCORE IS ALLOCATED TO ITS APPROPRIATE HIS-
TOGRAM CHANNEL
400 HIS(Q)=HIS(Q)+1  'THAT HISTOGRAM CHANNEL'S CONTENTS ARE INCRE-
MENTED
410 A=0:B=0:C=0:D=0:E=0:F=0  'RESET SCORE AND OUTCOME COUNTERS
420 NEXT K     'NEXT GAME
430 PRINT "AFTER" K-1 "GAMES, THE MEAN SCORE IS
"SUM1/(K-1)"-"SUM2/(K-1)  'PRINT MEAN SCORES
440 FOR P=10 TO 29  'LINES 440-460: OUTPUT HISTOGRAM
450 PRINT "RANGE"P*100"-"P*100+99;"FREQUENCY"HIS(P)
460 NEXT P
```

Program A4.15 - *MAXrMAX*

```
90 REM: MAXrMAX ACCEPTS INITIAL "W" AND "Z" VALUES, AND A RANGE OF
DIFFERENCE BETWEEN "X" AND "Y", FOR THE 100-MOVE EVENT MATRIX.
MAXrMAX OUTPUTS FINAL GAME SCORES AND THE NUMBER OF MOVES REQUIRED
FOR PERPETUAL MUTUAL CO-OPERATION TO COMMENCE.
100 INPUT "W VALUE IS";W  'INITIAL INSTANCES OF (C,c)
110 INPUT "Z VALUE IS";Z  'INITIAL INSTANCES OF (D,d)
120 IF (W+Z) MOD 2 = 0 THEN GOSUB 410   'PROMPTS INPUT OF EVEN RANGE
IF W+Z IS EVEN
130 IF (W+Z) MOD 2 = 1 THEN GOSUB 440   'PROMPTS INPUT OF ODD RANGE
IF W+Z IS ODD
140 A=100-W-Z  'LINES 140-160: FINDS INITIAL VALUES OF X AND Y
150 IF A MOD 2=0 THEN X=A/2+1 ELSE X=INT(A/2)+1
160 IF A MOD 2=0 THEN Y=A/2-1 ELSE Y=INT(A/2)
170 WHILE R+2-(X-Y)  'GENERATES FIRST GAME WITHIN RANGE
180 C=W: D=X: E=Y: F=Z
190 LPRINT W;X;Y;Z;  'OUTPUTS INITIAL EVENT MATRIX
200 S1=3*W+5*Y+Z: S2=3*W+5*X+Z   'COMPUTES INITIAL SCORES
210 EUC1= 3*W/(W+X): EUD1= (5*Y+Z)/(Y+Z)  'FINDS EXPECTED UTILITIES
FOR 101ST MOVE
220 EUC2= 3*W/(W+Y): EUD2= (5*X+Z)/(X+Z)  'FINDS EXPECTED UTILITIES
FOR 101ST MOVE
225 REM: LINES 230-310: UPDATE EVENT MATRIX, SCORES AND EXPECTED
UTILITIES FOR REMAINDER OF GAME
230 FOR J=101 TO 1000
240 IF EUC1>=EUD1 AND EUC2>=EUD2 THEN W=W+1: S1=S1+3: S2=S2+3:
250 IF EUC1>=EUD1 AND EUC2>=EUD2 AND FLAG=0 THEN GOSUB 380   'NOTE
MOVE ON WHICH PERPETUAL MUTUAL CO-OPERATION COMMENCES
260 IF EUC1>=EUD1 AND EUC2<EUD2 THEN X=X+1: S2=S2+5
270 IF EUC1<EUD1 AND EUC2>=EUD2 THEN Y=Y+1: S1=S1+5
280 IF EUC1<EUD1 AND EUC2<EUD2 THEN Z=Z+1: S1=S1+1: S2=S2+1
290 EUC1= 3*W/(W+X): EUD1= (5*Y+Z)/(Y+Z)
300 EUC2= 3*W/(W+Y): EUD2= (5*X+Z)/(X+Z)
310 NEXT J
320 IF FLAG=0 THEN LPRINT "no (C,c) occurred ";
330 LPRINT S1;S2  'PRINT SCORES FOR THIS GAME
340 W=C: X=D+1: Y=E-1: Z=F   'INITIALIZE EVENT MATRIX FOR NEXT GAME
(INCREMENT X, DECREMENT Y)
350 S1=0: S2=0: EUC1=0: EUD1=0: EUC2=0: EUD2=0: FLAG=0   'RESET
COUNTERS
360 WEND  'PLAY NEXT GAME, IF IN RANGE
370 END
380 FLAG=1
390 LPRINT "(C,c) on move"J;
400 RETURN 310
410 P=0
420 INPUT "EVEN RANGE IS";R
430 RETURN 140
440 P=1
450 INPUT "ODD RANGE IS";R
460 RETURN 140
```

REFERENCES

Aron, R., *Peace and War*, Weidenfeld and Nicolson, London, 1966.

Arrow, K., *Social Choice and Individual Values*, Yale University Press, New Haven, 1970.

Axelrod, R., 'Effective Choice in the Prisoner's Dilemma', *Journal of Conflict Resolution*, **24**, 1980*a*, pp.3–25.

" " 'More Effective Choice in the Prisoner's Dilemma', *Journal of Conflict Resolution*, **24**, 1980*b*, pp.379–403.

" " 'The Emergence of Cooperation Among Egoists', *The American Political Science Review*, **75**, 1981, pp.306–318.

" " & Hamilton, W., 'The Evolution of Cooperation', *Science*, **211**, 1981, pp.1390–1396.

Baer, A. (*ed.*), *Heredity and Society*, The Macmillan Company, N.Y., 1973.

Bar-Hillel, M., & Margolit, A., 'Newcomb's Paradox Revisited', *The British Journal for the Philosophy of Science*, **23**, 1972, pp.295–305.

Bernoulli, D., 'Exposition of a New Theory on the Measurement of Risk', s.v. L. Sommer, *Econometrica*, **22**, 1954, pp.23–27.

Bishop, T., & Cannings, C., 'A Generalized War of Attrition', *Journal of Theoretical Biology*, **70**, 1978, pp.85–124.

Boulding, K., *Conflict and Defense*, Harper & Row, N.Y., 1963.

Braithwaite, R., *Theory of Games as a Tool for the Moral Philosopher*, Cambridge at the University Press, 1955.

Brams, S., 'Newcomb's Problem and Prisoner's Dilemma', *Journal of Conflict Resolution*, **19**, 1975, pp.596–612.

Campbell, R., & Sowden, L. (*eds.*), *Paradoxes of Rationality and Cooperation*, The University of British Columbia Press, Vancouver, 1985.

Clarke, R., *The Science of War and Peace*, Jonathan Cape, London, 1971.

Davis, L., 'Prisoners, Paradox, and Rationality', *American Philosophical Quarterly*, **14**, 1977, pp.319–327.

Davis, M., *Game Theory*, Basic Books, Inc., N.Y., 1970.

Deutch, M., 'Trust and Suspicion', *Journal of Conflict Resolution*, **2**, 1958, pp.267–279.

" " & Lewicki, R., ' "Locking-in" Effects During a Game of Chicken', *Journal of Conflict Resolution*, **14**, 1970, pp.367–379.

Flood, M., 'Some Experimental Games', *Research Memorandum RM*-789, The Rand Corporation, Santa Monica, 1952.

Frei, D., *Risks of Unintentional Nuclear War*, Published in Cooperation with the United Nations Institute for Disarmament Research, Allanheld, Osmun & Co. Inc., Totowa, N.J., 1983.

Friedman, J., 'A Non-Cooperative Equilibrium for Supergames', *Review of Economic Studies*, **38**, 1971, pp.1-12.

Grinyer, A., & Smoker, P., `It Couldn't Happen - Could It? An Assessment of the Probability of Accidental Nuclear War', *Richardson Institute for Conflict and Peace Research*, University of Lancaster, 1986.

Hardin, G., `The Tragedy of the Commons', in A. Baer (*ed.*), 1973, pp.226-239.

Harris, R., `Note on "Optimal" Policies for the Prisoner's Dilemma', *Psychological Review*, **76**, 1969, pp.373-375.

Hobbes, T., *Leviathan* (1651), (M. Oakeshott, *ed.*), Basil Blackwell, Oxford, 1957.

Howard, N., *Paradoxes of Rationality: Theory of Metagames and Political Behaviour*, The MIT Press, Cambridge, Mass., 1971.

" " `"General Metagames": An Extension of the Metagame Concept', in Rapoport (*ed.*), 1974, pp.260-283.

Jeffrey, R., *The Logic of Decision*, McGraw-Hill Book Company, N.Y., 1965.

Levi, I., `Newcomb's Many Problems', *Theory and Decision*, **6**, 1975, pp.161-175.

Lewis, D., `Prisoner's Dilemma is a Newcomb Paradox', *Philosophy and Public Affairs*, **8**, 1979, pp.235-240.

Lewontin, R., `Evolution and the Theory of Games', *Journal of Theoretical Biology*, **1**, 1961, pp.382-403.

Luce, R., & Raiffa, H., *Games and Decisions*, John Wiley & Sons Inc., N.Y., 1957.

Marinoff, L., `The Inapplicability of Evolutionarily Stable Strategy to the Prisoner's Dilemma', *The British Journal for the Philosophy of Science*, **41**, 1990, pp.458-470 (pending).

Maynard Smith, J., `The Theory of Games and the Evolution of Animal Conflicts', *Journal of Theoretical Biology*, **47**, 1974, pp.209-221.

" " " *Evolution and the Theory of Games*, Cambridge University Press, Cambridge, 1982.

" " " & Price, G., `The Logic of Animal Conflict', *Nature*, **246**, 1973, pp.15-18.

Mises, R. von, *Probability, Statistics and Truth* (1928), Dover Publications Inc., N.Y., 1981.

Nash, J., `The Bargaining Problem', *Econometrica*, **18**, 1950, pp.155-162.

Neumann, J. von, & Morgenstern, O., *Theory of Games and Economic Behaviour* (1946), John Wiley & Sons Inc., N.Y., 1964.

Nozick, R., `Newcomb's Paradox and Two Principles of Choice' in Rescher (*ed.*), 1969, pp.116-146.

O'D.Alexander, C., *Fischer V. Spassky: Reykjavik 1972*, Penguin Books Ltd., Harmondsworth, 1972.

Rapoport, A., *Fights, Games, and Debates*, The University of Michigan Press, Ann Arbor, 1960.
" " & Chammah, A., *Prisoner's Dilemma*, University of Michigan Press, Ann Arbor, 1965.
" " *Two-Person Game Theory*, The University of Michigan Press, Ann Arbor, 1966.
" " 'Escape from Paradox', *Scientific American*, July 1967, pp.50–56.
" " (*ed.*); *Game Theory as a Theory of Conflict Resolution*; D. Reidel Publishing Co., Dordrecht, 1974, p.20.
" " 'Interpersonal Comparison of Utilities', *Lecture Notes in Economics and Mathematical Systems*, **123**, 1975*a*, pp.17–43.
" " 'Comment on Brams's Discussion of Newcomb's Paradox', *Journal of Conflict Resolution*, **19**, 1975*b*, pp.613–619.
" " & Guyer, M. & Gordon, D., *The 2x2 Game*, The University of Michigan Press, Ann Arbor, 1976.
Rawls, J., *A Theory of Justice*, Clarendon Press, Oxford, 1972.
Rescher, N., (*ed.*), *Essays in Honour of Carl Hempel*, D. Reidel Publishing Co., Dordrecht, 1969, pp.116–146.

Savage, L., *The Foundations of Statistics*, John Wiley & Sons Inc., N.Y., 1954.
Schelling, T., 'The Strategy of Conflict: Prospectus for the Reorientation of Game Theory', *Journal of Conflict Resolution*, **2**, 1958, pp.203–264.
Seidenfeld, T., *Philosophical Problems of Statistical Inference*, D. Reidel Publishing Co., Dordrecht, 1979.
Shubik, M., *The Uses and Methods of Gaming*; Elsevier Scientific Publishing Company, N.Y., 1975.
Singleton, R., & Tyndall, W., *Games and Programs*, W.H. Freeman & Co., San Francisco, 1974.

Todhunter, I., *A History of the Mathematical Theory of Probability*, Macmillan & Co., London, 1865.

Vorob'ev, N., *Game Theory (Lectures for Economists and Systems Scientists)*, s.v. S. Kotz, Springer-Verlag, N.Y., 1977.

Weatherford, R., *Philosophical Foundations of Probability Theory*, Routledge & Kegan Paul, London, 1982.
Weber, R., 'Noncooperative Games', *Proceedings of Symposia in Applied Mathematics*, **24**, 1981, pp.83–125.
Wiesner, J., & York, H. 'National Security and the Nuclear Test-Ban', *Scientific American*, **211**, October 1964, pp.27–35.