Summary of contents

Introduction

Our Supplementary material is composed of two main elements, the present document and a VS2017 C++ project, containing the code used to run some of our evaluations (i.e. performance comparisons among multi-point solvers).

We start by providing an overview of the C++ code, and how this can be used to generate the data used in our evaluations, and then move onto the extended results and data that are not covered in the main paper.

1. Using our code:

We provided a VS2017 C++ solution, that contains the implementation of the solvers compared and all the main files used to generate the data used during our evaluations comparing performance.

The solution contains all the external libraries required to be compiled and run (i.e. Lapack and CImg), and was tested in several computers, so you should be able to directly open it from Visual Studio (2017 or newer) and run it.

The source code in the solution is structured in several folders, for your convenience:

- The *Helper* folder contains common and basic definitions, such as the parameters used for our simulations (frequency, speed of sound, particle sizes), basic sound-field computations (sound-field propagation, Gor'kov potential, stiffness computation), as well as a simple visualiser for the sound fields produced.
- The *Solvers* folder contains an implementation of the four multi-point algorithms evaluated.
- The *MainFiles* folder contains three different main programs, each used to test/show some of the aspects related to our paper.

You can run any of these main files, but you need to be sure that only one *main()* method is active at a time. In order to do this, you can simply select one of these main files from your *SolutionExplorer,* right click and select *Properties*. A menu will pop-up and the first option available should be *Excluded from build.* Use this option to enable the desired file and exclude the remaining 2 main files (a small *stop* icon should be displayed on these files).

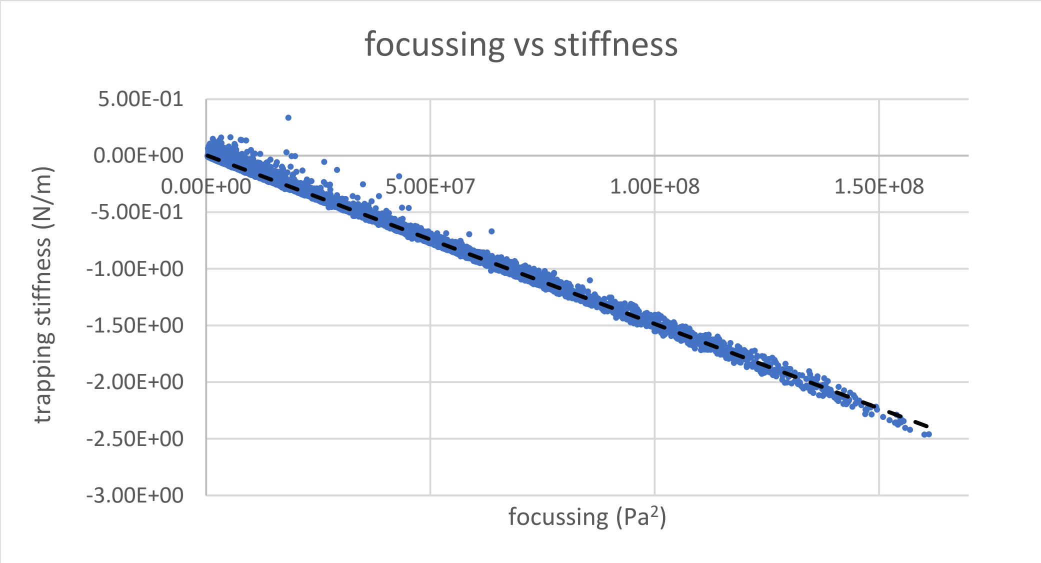
2. Extended results and data:

2.1. Correlation between focussing amplitude and trap stiffness:

As introduced in the paper, our paper tests techniques from two different domains: haptics and levitation. Haptics are intended to create high pressure focal points, with their intensity being measured in Pascals. The quality of levitation traps is however determined by the trap stiffness (i.e. Laplacian of Gor'kov potential).

While the relationship between focussing points and levitation traps has been somewhat understood for a few years (i.e. [Marzo 2014] showed that a focussing point can be transformed into a high quality levitation trap with the simple addition of a levitation signature), there is not an accepted single metric or established relationship. In order to be able to compare the performance of GS-PAT (and the rest of the solvers tested) across domains, we needed to first find a metric that related both parameters (pressure (Pa) and stiffness (N/m)).

In this section we show how the square of the acoustic pressure (Pa) before applying the levitation signature to the focus points strongly correlates (R^2>0.99) with trapping stiffnesses (N/m) after applying the signature (i.e. vertical twin trap, adding PI radians to the transducers in the top array). We reused the 5000 random geometries used for our test of reconstruction accuracy (Section 4.2. in the paper), as to provide a wide distribution of points, both in terms of their position in space (62000 points tested, randomly distributed in an area of 12x12x12cm), and in terms of amplitude (pressure in Pa and the focus point). We then applied our levitation signature (i.e. +PI to top array) and computed trapping stiffness and looked at the correlation between both parameters, which is shown below:



The data was generated using *0.mainStiffnessA2Correlation.cpp* which produces a set of CSV files (*ComparisonStiffA2_<num_points>.csv*). The results obtained can be linearly fit with b1= -1.49E-08 and b2= 4.96E-03, with an overall coefficient of determination R2 = 9.98E-01, confirming the relationship between these two parameters.

2.2. Avoiding destructive interference (Extended data):

The main paper contains a summary of the results obtained for our evaluation, but some of the data is only provided via figures (not the explicit value). This section contains such data in tabular format, as well as collected in terms of histograms as to better assess their distribution. Other relevant parameters and a description of how data was generated from our code can be found in Annex I.

The data summarized here was generated using *1. mainEvaluateFixedA.cpp* which produced a set of CSV files (*Comparison_<num_points>.csv*). Please note that the 5000 geometries were randomly generated, but the random seed was initialised to a fixed value (i.e. see line 137: `srand((unsigned) 1);`). This should allow you to generate exactly the same geometries that we used for our tests and, hence, obtain the exact same results (as long as no other changes are made to the code).

AMPLITUDE FOR ALL POINT AND GEOMETRIES (AVG(A))

|Z|	Naïve		IBP		Long		GS-PAT	
	AVG	STDEV	AVG	STDEV	AVG	STDEV	AVG	STDEV
2	12825	1273	12998	1216	12953	520	12967	774
4	9027	1112	9387	928	9322	411	9334	782
8	6383	940	6842	749	6726	319	6780	693
16	4512	955	5085	730	4886	302	5024	703
32	3205	944	3850	704	3544	307	3787	693

AMPLITUDE FOR WEAKEST POINT IN ALL GEOMETRIES (MIN(A))

|Z|	Naïve		IBP		Long		GS-PAT	
	AVG	STDEV	AVG	STDEV	AVG	STDEV	AVG	STDEV
2	11880	906	12086	875	12768	512	12466	602
4	7921	844	8378	524	8962	317	8561	450
8	5058	756	5841	352	6302	219	5909	310
16	2776	712	4018	235	4364	173	4018	241
32	1189	484	2722	191	2931	165	2631	208

AMPLITUDE FOR WEAKEST POINT IN DESTRUCTIVE CASES (MIN(A, P5))

|Z|	Naïve		IBP		Long		GS-PAT	
	AVG	STDEV	AVG	STDEV	AVG	STDEV	AVG	STDEV
2	9679	327	10302	900	11979	524	11498	616
4	5481	1267	8087	824	9250	485	8219	624
8	2889	811	5699	371	6456	238	5692	359
16	858	302	3915	253	4404	168	3952	225
32	236	77	2743	198	2880	191	2614	227

Robustness of phase retrieval methods against destructive interference:

It can be observed that while *Naïve* was subject to great losses in amplitude for cases showing destructive interference, the losses for the phase retrieval methods were much smaller. Here we provide a summary of these losses, computed as the ratio of Min(A)/Min(A,p5), for each solver and condition (Z=2,4,8,16,32).

DIFF MIN(A,P5)/MIN(A)

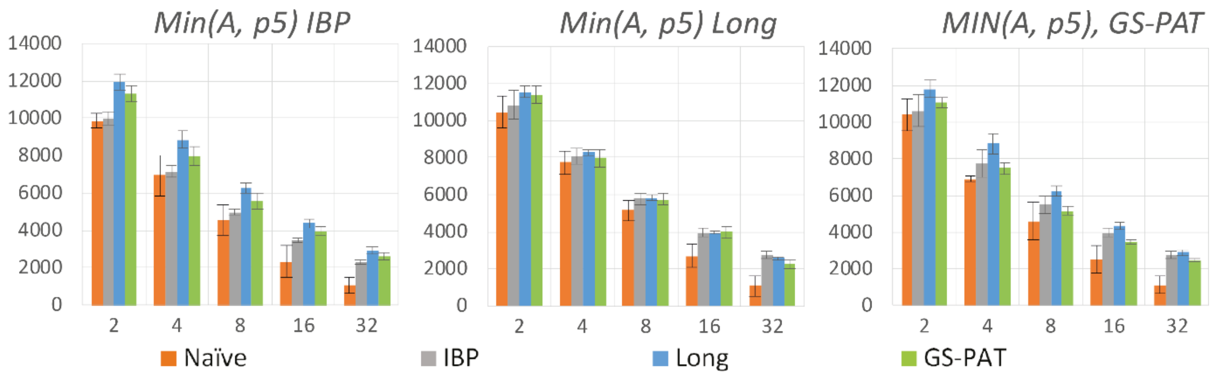
|Z|	Naïve	IBP	Long	GS-PAT
2	0.81	0.85	0.94	0.92
4	0.69	0.97	1.03	0.96
8	0.57	0.98	1.02	0.96
16	0.31	0.97	1.01	0.98
32	0.20	1.01	0.98	0.99
AVG±SD	0.51 ± 0.23	0.95 ± 0.05	0.99 ± 0.03	0.96 ± 0.02

It can be observed that, while Naïve suffered great losses, all other solvers managed to retain very similar levels of focussing amplitude than in a general case.

Double checking our selection criteria for destructive cases:

We identified cases (geometries) subject to destructive interference (i.e. to compute Min(A, p5)) by looking at the 5% of points producing lowest amplitude when using the *Naïve* solver (i.e. not optimizing phases to avoid destructive interference).

We also wanted to make sure that the much better performance of phase retrieval algorithms when compared with *Naïve* in terms of *Min(A,p5)* was due to interference, and not the result of our selection criteria (5% worst cases from *Naïve*). Thus, we repeated the same process, selecting the 5% weakest cases from each of the other solvers (5% worst for *IBP*, 5% worst for *Long* and 5% worst for *GS-PAT*), and summarize the results obtained here, both as a summary plot (similar to Figure 4c in the paper) and in tabular format below. In all cases, we can observe that phase retrieval algorithms maintain good amplitude reconstruction (not registering losses anywhere as large as *Naïve*), retaining similar performance among phase retrieval solvers and always performing better than *Naïve*.



MIN(A,P5), SELECTED FROM IBP

	Naïve		IBP		Long		GS-PAT	
	AVG	STDEV	AVG	STDEV	AVG	STDEV	AVG	STDEV
2	9861	397	9965	355	11945	404	11331	424
4	6926	1056	7136	327	8866	452	7958	494
8	4576	800	4978	190	6259	250	5570	428
16	2338	887	3476	111	4368	208	3959	245
32	1075	419	2299	79	2927	161	2590	206

MIN(A,P5), SELECTED FROM LONG

	Naïve		IBP		Long		GS-PAT	
	AVG	STDEV	AVG	STDEV	AVG	STDEV	AVG	STDEV
2	10459	860	10851	807	11552	303	11394	480
4	7739	636	8065	455	8290	175	7967	486
8	5150	550	5770	333	5828	136	5760	327
16	2677	631	3940	236	3962	93	3973	288
32	1095	553	2755	220	2567	87	2257	216

MIN(A,P5), SELECTED FROM GS-PAT

	Naïve		IBP		Long		GS-PAT	
	AVG	STDEV	AVG	STDEV	AVG	STDEV	AVG	STDEV
2	10418	871	10627	841	11818	450	11068	260
4	6914	150	7774	755	8830	531	7491	310
8	4586	1029	5494	459	6236	263	5166	210
16	2541	748	3965	237	4353	199	3463	143
32	1130	456	2741	203	2903	173	2460	91

2.3. Accurate amplitude reconstruction: Correction for *Long*

We observed that the basic method proposed by Long et al, has a tendency to produce results with slightly lower amplitudes than intended. This is easy to fix, and we added a simple correction as not to provide misleading conclusions (i.e. global optimization methods being less accurate than a heuristic method).

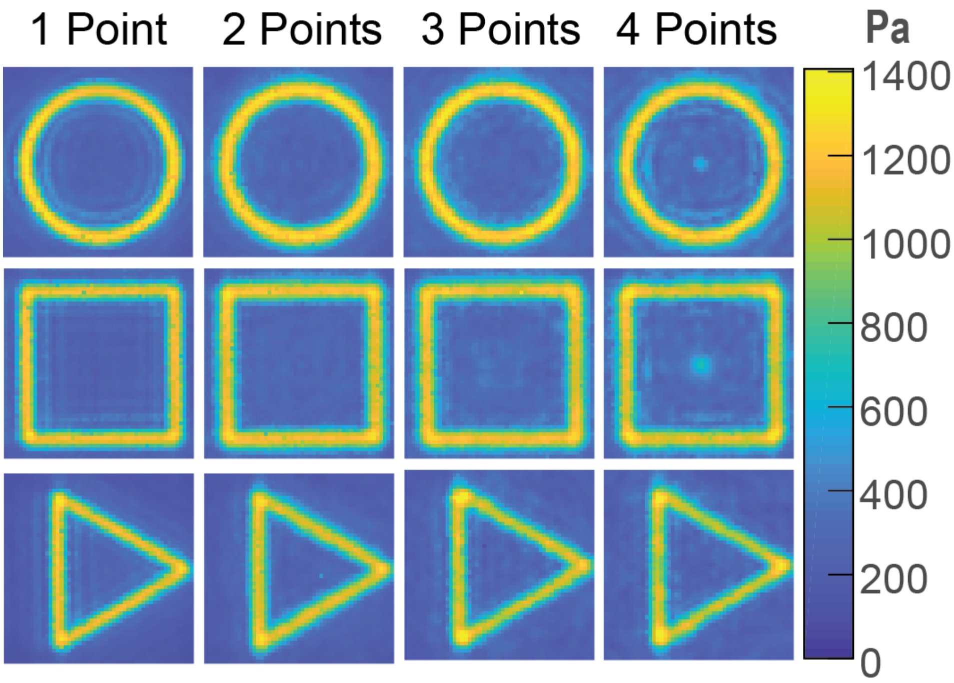
Let $\boldsymbol{\tau}^{\Omega}$ represent the final transducer activation computed and $\boldsymbol{\zeta}^{(0)}$ the complex representation for our intended target points. We simply forward propagate the transducer activation to achieve an estimate of the actual reconstruction intensities that will be achieved by $\boldsymbol{\tau}^{\Omega}$ and then use this to correct the transducer activation ($\boldsymbol{\tau}^{C}$):

$$\boldsymbol{\zeta}^{\Omega} = \boldsymbol{F} \cdot \boldsymbol{\tau}^{\Omega}$$

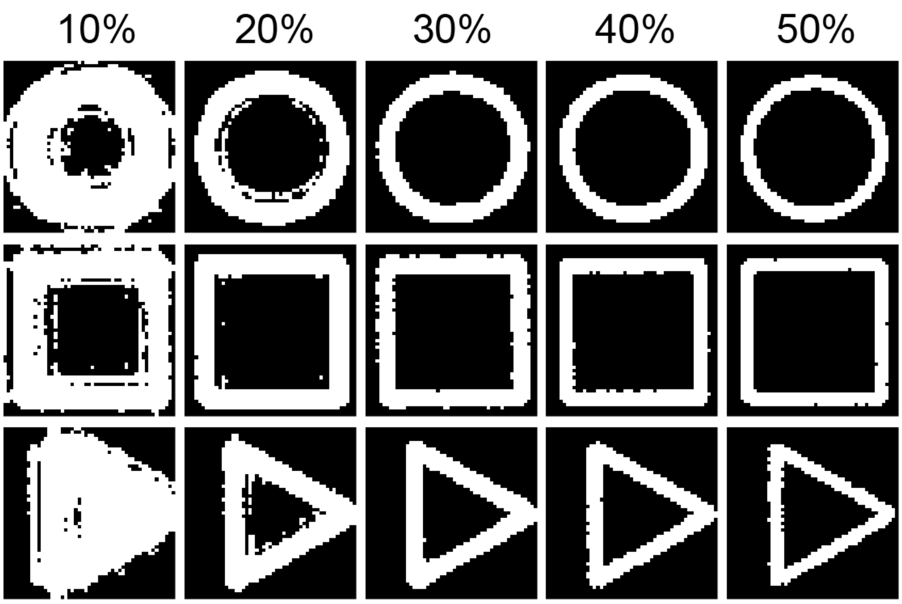
$$\boldsymbol{\tau}^{C} = \frac{|\boldsymbol{Z}|}{\sum_{z}^{\boldsymbol{\zeta}^{\Omega}} \frac{a_z{}^{\Omega}}{a_z{}^{(0)}}} \cdot \boldsymbol{\tau}^{\Omega}$$

2.4. Masks and Measurements

The main paper only shows the measurements retrieved for one of our shapes (i.e. square). This section contains the measurements of all the shapes reproduced, and for each number of points (1, 2, 3 and 4 tactile points):



The analysis of the pressure distributions for each of the shapes was performed according to two areas, the *stimulation* area and the *surrounding area.* We used the pressure distributions obtained for each shape when rendered by a single point to identify these areas (as explained in the paper, we then looked at how rendering the shape with more points changed pressure in these two areas). In order to identify these areas, we applied a binary threshold to the pressure distributions of each shape (always using the single point shape) using different levels of pressure (i.e. 10% of peak pressure, 20% of peak pressure, etc) as shown below. Based on this, we selected a threshold of 30% peak pressure to identify the *stimulation* and *surrounding areas*.



3. Annex I: Data generation and analysis

The data used for our evaluations was generated using `1. mainEvaluateFixedA.cpp` and `2. mainEvaluateVariableA.cpp` which produces a set of the required CSV files (`Comparison_<num_points>.csv` *and* `ComparisonVarA_<num_points>.csv`). Please note that the 5000 geometries used in each evaluation were randomly generated, but the random seed was initialised to a fixed value (i.e. see line 137: `srand((unsigned) 1);`). This should allow you to generate exactly the same geometries that we used for our tests and, hence, obtain the exact same results (as long as no other changes are made to the code).

The data obtained was analysed using SPSS v25, and you can find a template file to help you fill your data (i.e. simply copy the data from each of the CSV files to `data_template.sav`). The following SPSS syntax should help you reproduce our analysis:

```
GRAPH /HISTOGRAM=Amplitude /PANEL COLVAR=Solver COLOP=CROSS.
means Amplitude by Solver /cells count mean stddev.
ONEWAY Amplitude BY Solver /MISSING ANALYSIS.
UNIANOVA Amplitude BY Solver /METHOD=SSTYPE(3) /INTERCEPT=INCLUDE
/POSTHOC=Solver(TUKEY) /PRINT=ETASQ HOMOGENEITY
/CRITERIA=ALPHA(.05) /DESIGN=Solver.
```

While this syntax should help you identify percentile p5. Pay attention to the value of percentile 5 obtained from this table, as you will need it in the next step:

```
USE ALL.
COMPUTE filter_$=(Solver = 0).
VARIABLE LABELS filter_$ 'Solver = 0 (FILTER)'.
VALUE LABELS filter_$ 0 'Not Selected' 1 'Selected'.
FORMATS filter_$ (f1.0).
FILTER BY filter_$.
EXECUTE.
*1.2. compute percentiles using Naive cases only.
FREQUENCIES VARIABLES=Min_A
  /PERCENTILES=5.0 10.0 25.0 50.0 75.0 90.0 95.0
  /format notable.
```

And this syntax can be used to filter data according to the percentile identified:

```
DATASET ACTIVATE DataSet3.
DATASET COPY  NaiveCasesLowA.
DATASET ACTIVATE  NaiveCasesLowA.
FILTER OFF.
USE ALL.
COMPUTE filter_MinA=(Solver = 0 & Min_A < <REPLACE_ME> ).
EXECUTE.
SELECT IF (Solver = 0).
EXECUTE.
*Delete uneeded variables.
DELETE VARIABLES Solver Point Amplitude Phase AVG_A STDEV_A min_A
max_A ratioA.
*remove duplicates.
SORT CASES BY Geometry (A) .
IF (lag(Geometry)=Geometry) filter1 = 1 .
SELECT IF sysmis(FILTER1).
EXECUTE.
DELETE VARIABLES filter1.
EXECUTE.
```

```
MATCH FILES file = DataSet3 / table = NaiveCasesLowA / by
Geometry.
EXECUTE.
FILTER BY filter_MinA.
```

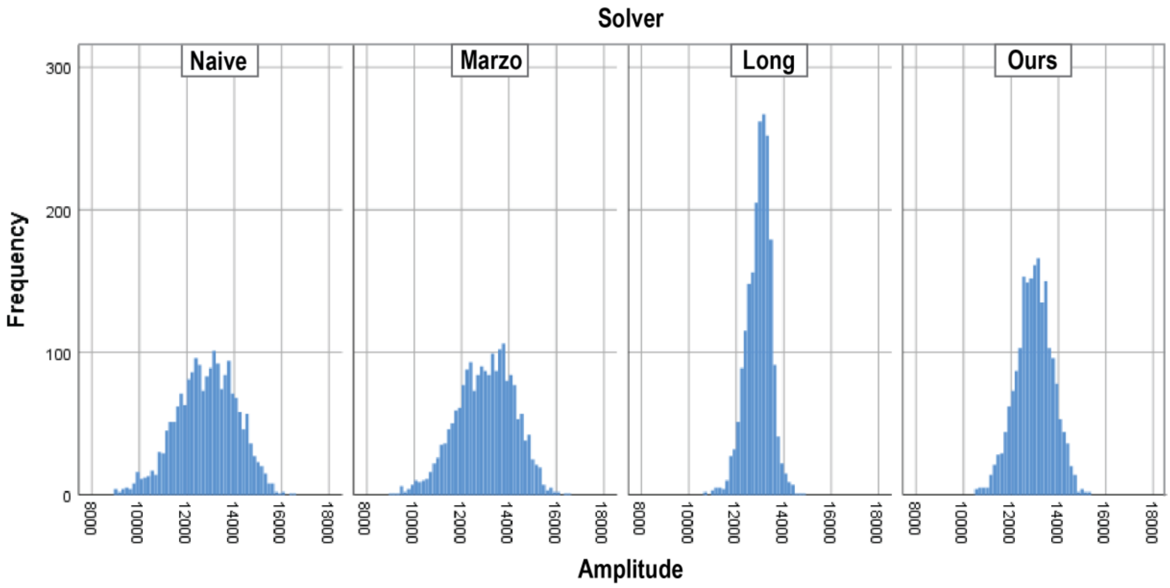
Please note that you should replace the text **<REPLACE_ME>** by the value you obtained for percentile p5 before. You can also replace the solver ID, to select geometries based on the results of worst points according to other solvers (i.e. as we did to test the correctness of *Min(A,p5)* for other solvers).

The following subsections only contains a summary of the data generated (and yet this is extensive). More specifically, histograms are included as they provide a good tool to assess the distribution of pressures obtained visually. We then include the summary of relevant statistics (Means and SD), and a summary of homogeneous groups identified in each test, to assess significant differences between pairs of solvers, for each condition (Z=2,4,8,16,32) and parameter (*AVG(A)*, *Min(A)*, *Min(A,p5)*).

The sections are structured according to the evaluations in the paper, starting with the tests performed for *Avoiding destructive interference* and then those for *Accurate Amplitude Reconstruction.* Within each of these, results are presented from lower to higher number of points (i.e. {2,4,8,16,32} points).

3.1. Avoiding destructive interference (Z=2 Points)

Amplitude per point



Report

Amplitude

Solver	N	Mean	Std. Deviation
Naive	2000	12825.31790423	1273.747359378
Marzo	2000	12998.33728332	1216.391713200
Long	2000	12952.72264885	520.394448658
Ours	2000	12967.83416776	774.154451088
Total	8000	12936.05300104	998.512465527

Amplitude

Tukey HSD[a,b]

Solver	N	Subset 1	Subset 2
Naive	2000	12825.31790423	
Long	2000		12952.72264885
Ours	2000		12967.83416776
Marzo	2000		12998.33728332
Sig.		1.000	.470

Means for groups in homogeneous subsets are displayed.

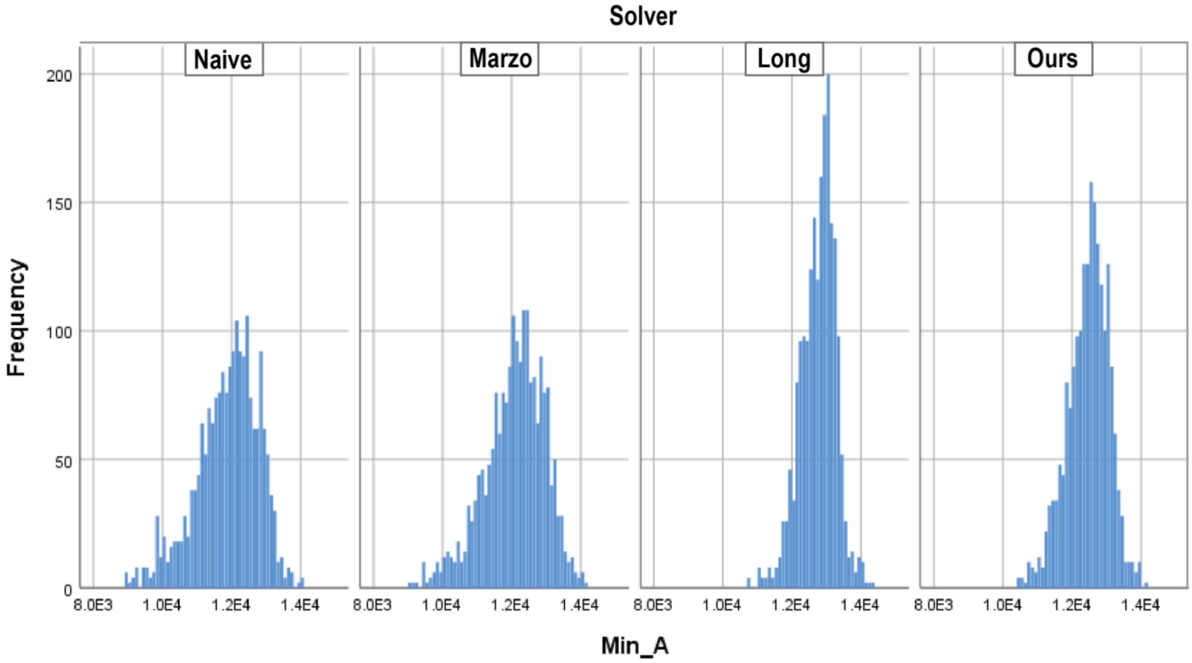
Based on observed means.

The error term is Mean Square(Error) = 993041.658.

a. Uses Harmonic Mean Sample Size = 2000.000.

b. Alpha = .05.

Amplitude of the weakest point (all cases)



Report

Min_A

Solver	N	Mean	Std. Deviation
Naive	2000	1.18804506E+004	9.055105939E+002
Marzo	2000	1.20858242E+004	8.747332659E+002
Long	2000	1.27679250E+004	5.122887405E+002
Ours	2000	1.24657624E+004	6.028761020E+002
Total	8000	1.22999906E+004	8.183153888E+002

Min_A

Tukey HSD[a,b]

		Subset			
Solver	N	1	2	3	4
Naive	2000	1.18804506E+004			
Marzo	2000		1.20858242E+004		
Ours	2000			1.24657624E+004	
Long	2000				1.27679250E+004
Sig.		1.000	1.000	1.000	1.000

Means for groups in homogeneous subsets are displayed.

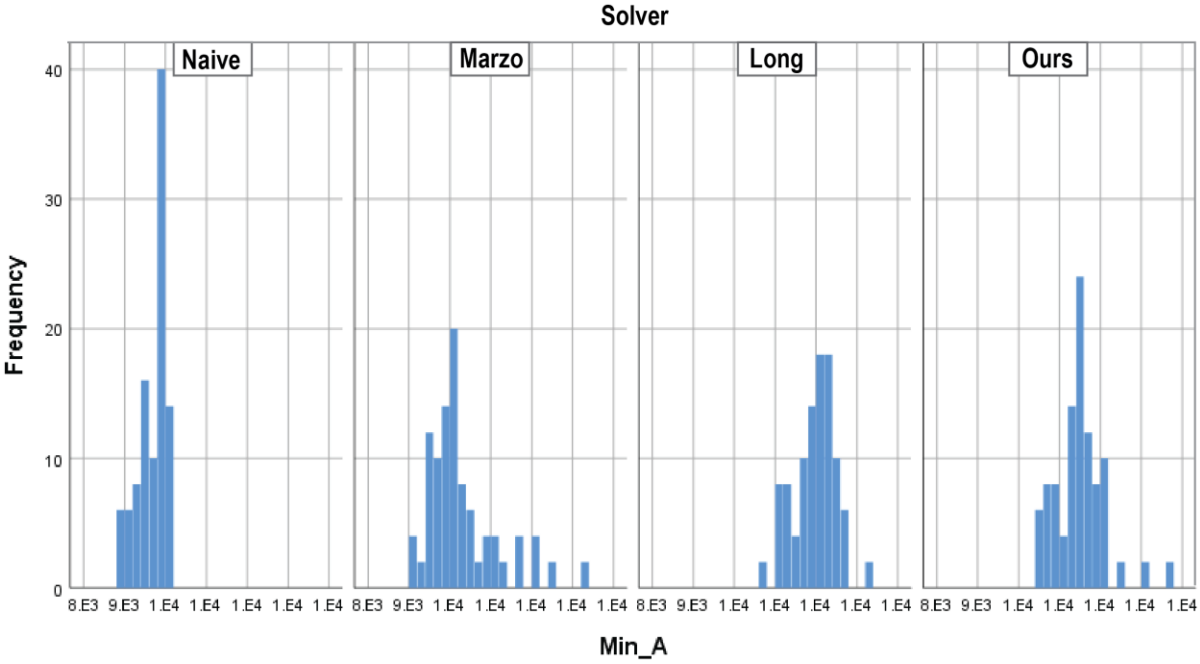
Based on observed means.

The error term is Mean Square(Error) = 552751.768.

a. Uses Harmonic Mean Sample Size = 2000.000.

b. Alpha = .05.

Amplitude of weakest point (p5 selected from Naive)



Report

Min_A

Solver	N	Mean	Std. Deviation
Naive	100	9.67943740E+003	3.274971913E+002
Marzo	100	1.03023071E+004	8.999379469E+002
Long	100	1.19793447E+004	5.238469266E+002
Ours	100	1.14981243E+004	6.157882173E+002
Total	400	1.08648034E+004	1.110424314E+003

Min_A

Tukey HSD[a,b]

		Subset			
Solver	N	1	2	3	4
Naive	100	9.67943740E+003			
Marzo	100		1.03023071E+004		
Ours	100			1.14981243E+004	
Long	100				1.19793447E+004
Sig.		1.000	1.000	1.000	1.000

Means for groups in homogeneous subsets are displayed.

Based on observed means.

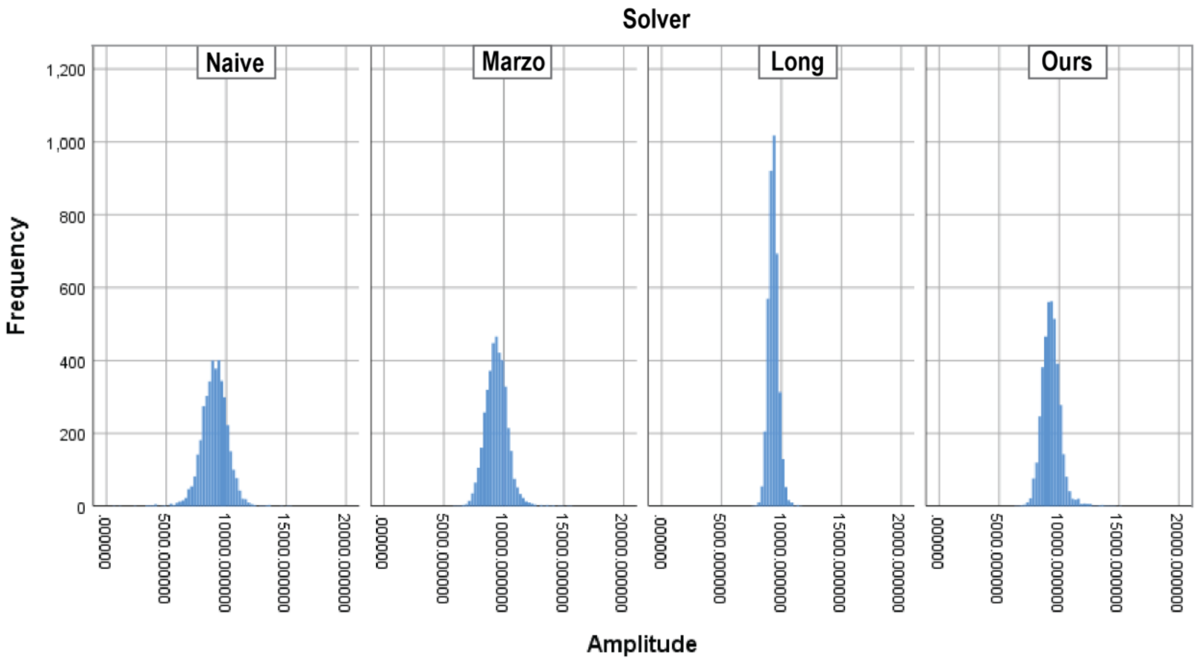
The error term is Mean Square(Error) = 392688.362.

a. Uses Harmonic Mean Sample Size = 100.000.

b. Alpha = .05.

3.2. Avoiding destructive interference (4 points)

Amplitud per point



Report

Amplitude

Solver	N	Mean	Std. Deviation
Naive	4000	9027.31101346	1112.083551699
Marzo	4000	9386.87509051	927.946714757
Long	4000	9321.70205974	410.930811070
Ours	4000	9333.75171325	782.215415350
Total	16000	9267.40996924	859.836248266

Amplitude

Tukey HSD[a,b]

		Subset		
Solver	N	1	2	3
Naive	4000	9027.31101346		
Long	4000		9321.70205974	
Ours	4000		9333.75171325	
Marzo	4000			9386.87509051
Sig.		1.000	.921	1.000

Means for groups in homogeneous subsets are displayed.

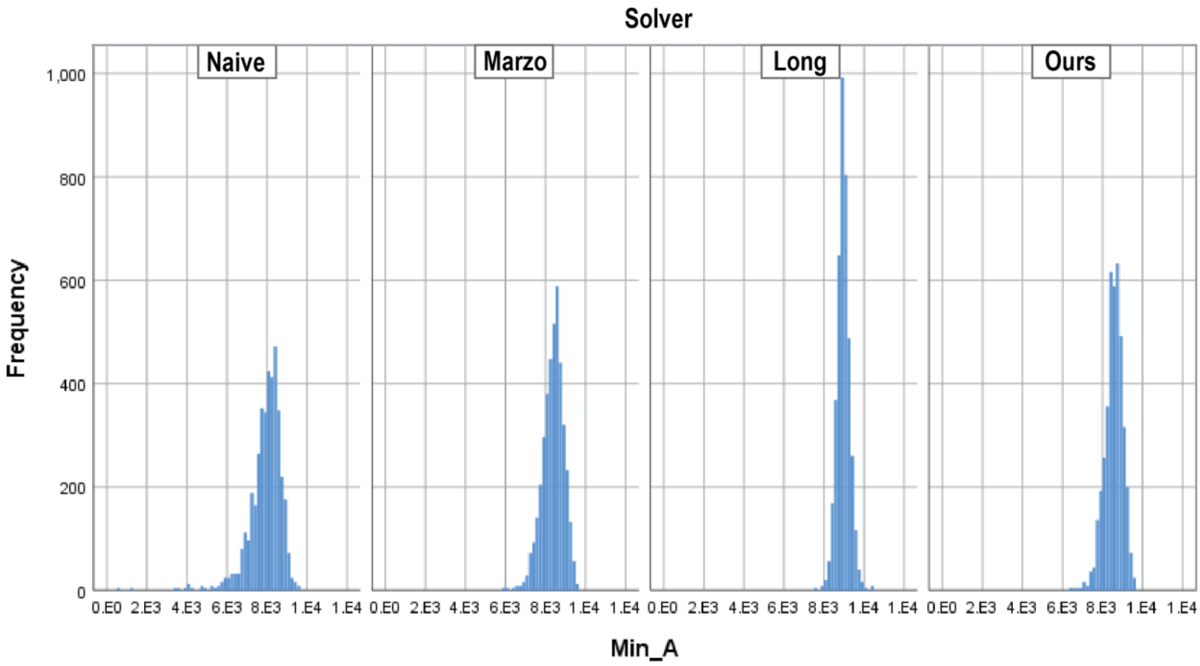
Based on observed means.

The error term is Mean Square(Error) = 719635.005.

a. Uses Harmonic Mean Sample Size = 4000.000.

b. Alpha = .05.

Amplitude of the weakest point (all cases)



Report

Min_A

Solver	N	Mean	Std. Deviation
Naive	4000	7.92112421E+003	8.447870720E+002
Marzo	4000	8.37828830E+003	5.243610298E+002
Long	4000	8.96261215E+003	3.156529177E+002
Ours	4000	8.56074143E+003	4.495270018E+002
Total	16000	8.45569152E+003	6.800517407E+002

Min_A

Tukey HSD[a,b]

		Subset			
Solver	N	1	2	3	4
Naive	4000	7.92112421E+003			
Marzo	4000		8.37828830E+003		
Ours	4000			8.56074143E+003	
Long	4000				8.96261215E+003
Sig.		1.000	1.000	1.000	1.000

Means for groups in homogeneous subsets are displayed.

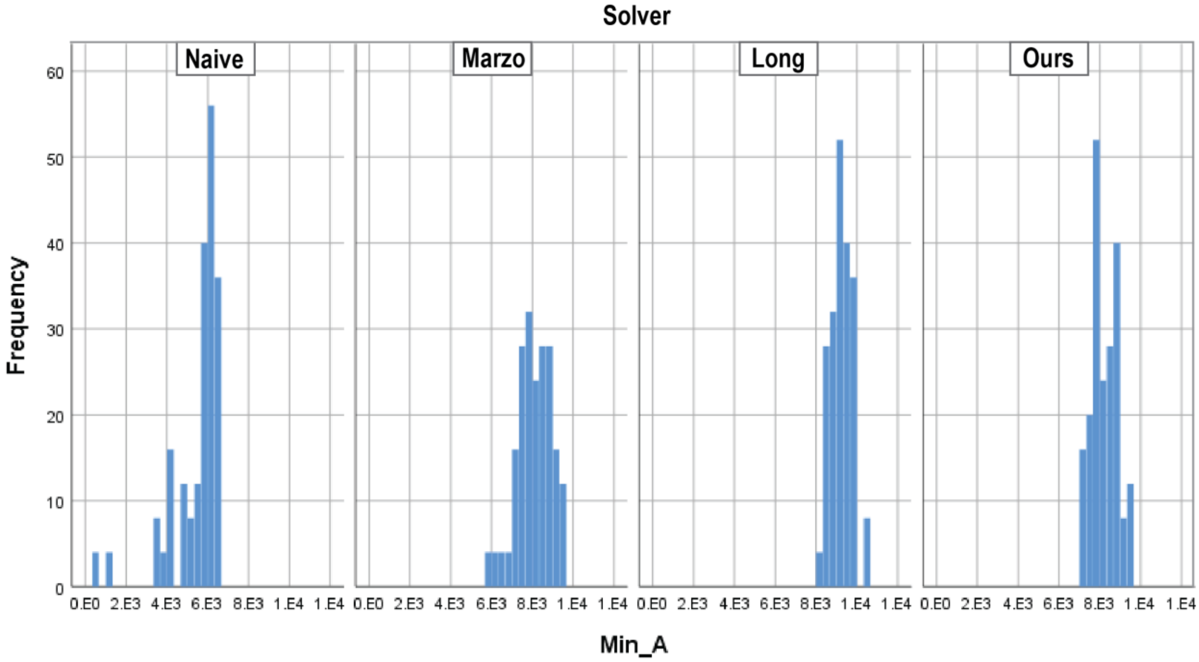
Based on observed means.

The error term is Mean Square(Error) = 322582.744.

a. Uses Harmonic Mean Sample Size = 4000.000.

b. Alpha = .05.

Amplitude of weakest point (p5 selected from Naive)



Report

Min_A

Solver	N	Mean	Std. Deviation
Naive	200	5.48071313E+003	1.266668390E+003
Marzo	200	8.08681058E+003	8.241851724E+002
Long	200	9.24685336E+003	4.856599949E+002
Ours	200	8.21857567E+003	6.243204431E+002
Total	800	7.75823818E+003	1.630266817E+003

Min_A

Tukey HSD[a,b]

		Subset		
Solver	N	1	2	3
Naive	200	5.48071313E+003		
Marzo	200		8.08681058E+003	
Ours	200		8.21857567E+003	
Long	200			9.24685336E+003
Sig.		1.000	.411	1.000

Means for groups in homogeneous subsets are displayed.

Based on observed means.

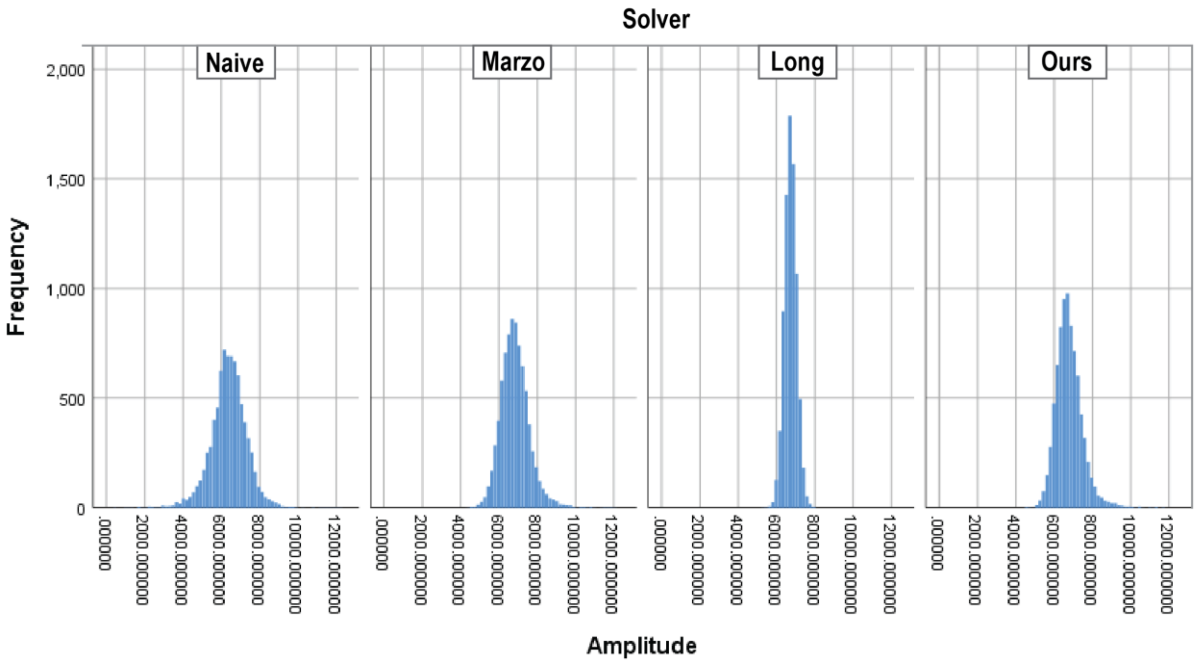
The error term is Mean Square(Error) = 727342.914.

a. Uses Harmonic Mean Sample Size = 200.000.

b. Alpha = .05.

3.3. Avoiding destructive interference (Z=8 points)

Amplitude per point



Report

Amplitude

Solver	N	Mean	Std. Deviation
Naive	8000	6383.23420178	940.277947765
Marzo	8000	6841.88157766	749.169832205
Long	8000	6725.81096748	319.087982449
Ours	8000	6779.28541201	692.790930909
Total	32000	6682.55303973	733.684897167

Amplitude

Tukey HSD[a,b]

		Subset			
Solver	N	1	2	3	4
Naive	8000	6383.23420178			
Long	8000		6725.81096748		
Ours	8000			6779.28541201	
Marzo	8000				6841.88157766
Sig.		1.000	1.000	1.000	1.000

Means for groups in homogeneous subsets are displayed.

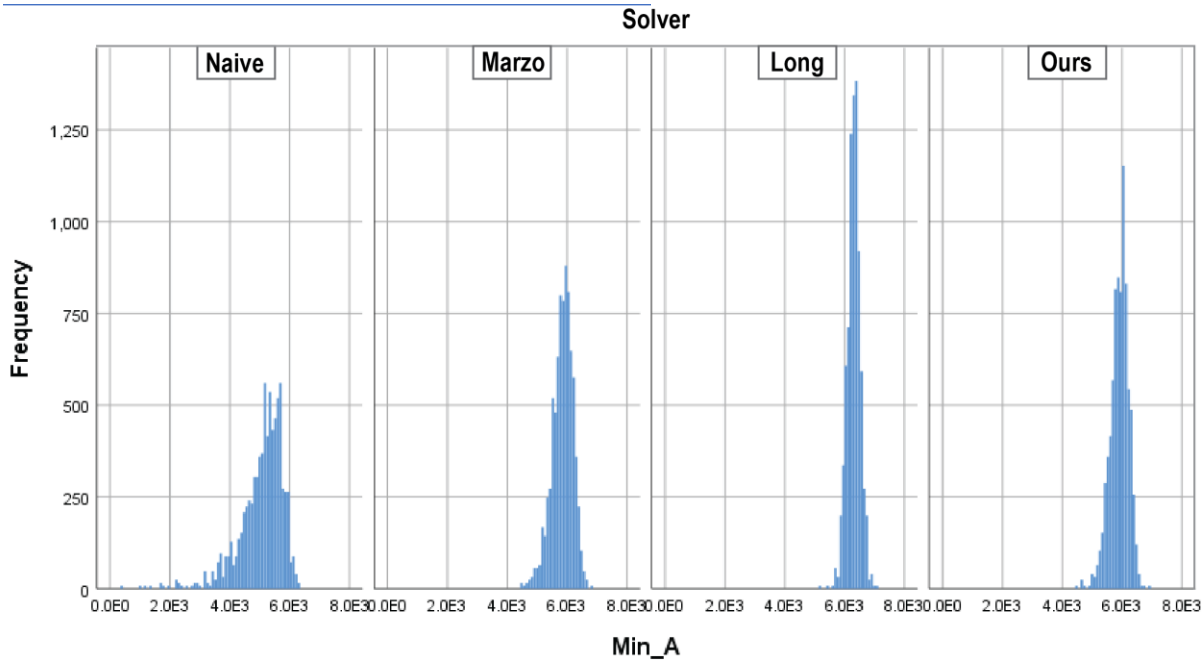
Based on observed means.

The error term is Mean Square(Error) = 506788.618.

a. Uses Harmonic Mean Sample Size = 8000.000.

b. Alpha = .05.

Amplitude of the weakest point (all cases)



Report

Min_A

Solver	N	Mean	Std. Deviation
Naive	8000	5.05751210E+003	7.562373337E+002
Marzo	8000	5.84114779E+003	3.518459699E+002
Long	8000	6.30321334E+003	2.165797293E+002
Ours	8000	5.90913767E+003	3.097224691E+002
Total	32000	5.77775272E+003	6.431579345E+002

Min_A

Tukey HSD[a,b]

Solver	N	Subset 1	Subset 2	Subset 3	Subset 4
Naive	8000	5.05751210E+003			
Marzo	8000		5.84114779E+003		
Ours	8000			5.90913767E+003	
Long	8000				6.30321334E+003
Sig.		1.000	1.000	1.000	1.000

Means for groups in homogeneous subsets are displayed.

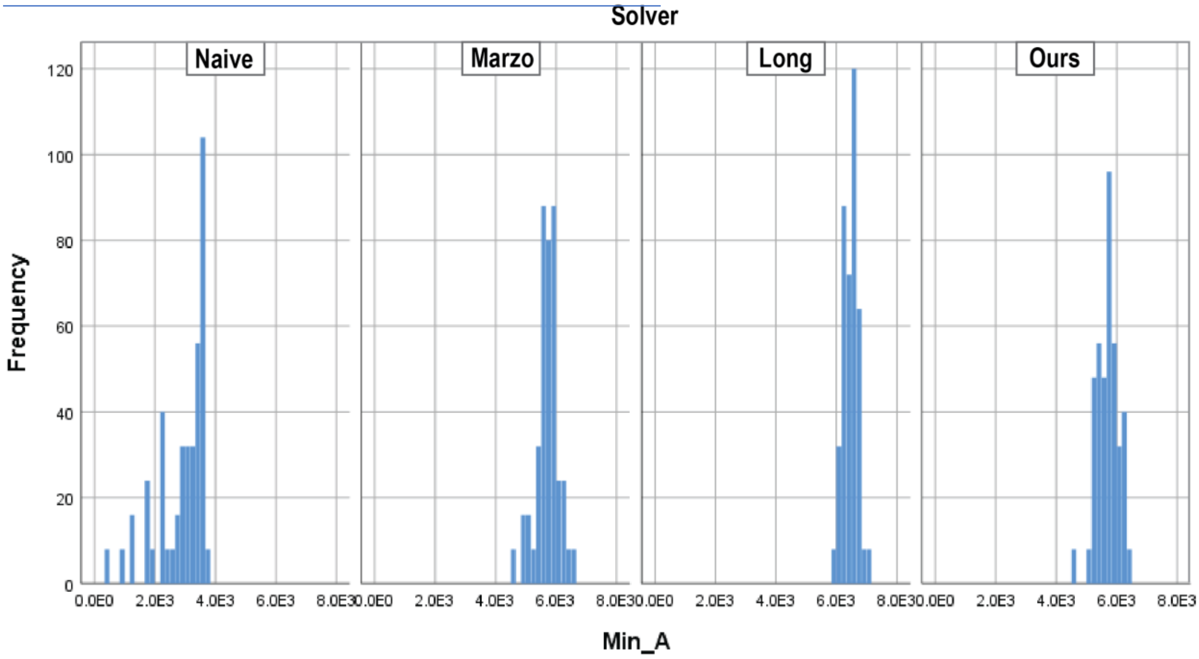
Based on observed means.

The error term is Mean Square(Error) = 209631.320.

a. Uses Harmonic Mean Sample Size = 8000.000.

b. Alpha = .05.

Amplitude of weakest point (p5 selected from Naive)



Report

Min_A

Solver	N	Mean	Std. Deviation
Naive	400	2.88936392E+003	8.107567820E+002
Marzo	400	5.69921768E+003	3.708764909E+002
Long	400	6.45562791E+003	2.318383781E+002
Ours	400	5.69196775E+003	3.588716320E+002
Total	1600	5.18404432E+003	1.447936838E+003

Min_A

Tukey HSD[a,b]

		Subset		
Solver	N	1	2	3
Naïve	400	2.88936392E+003		
Ours	400		5.69196775E+003	
Marzo	400		5.69921768E+003	
Long	400			6.45562791E+003
Sig.		1.000	.997	1.000

Means for groups in homogeneous subsets are displayed.

Based on observed means.

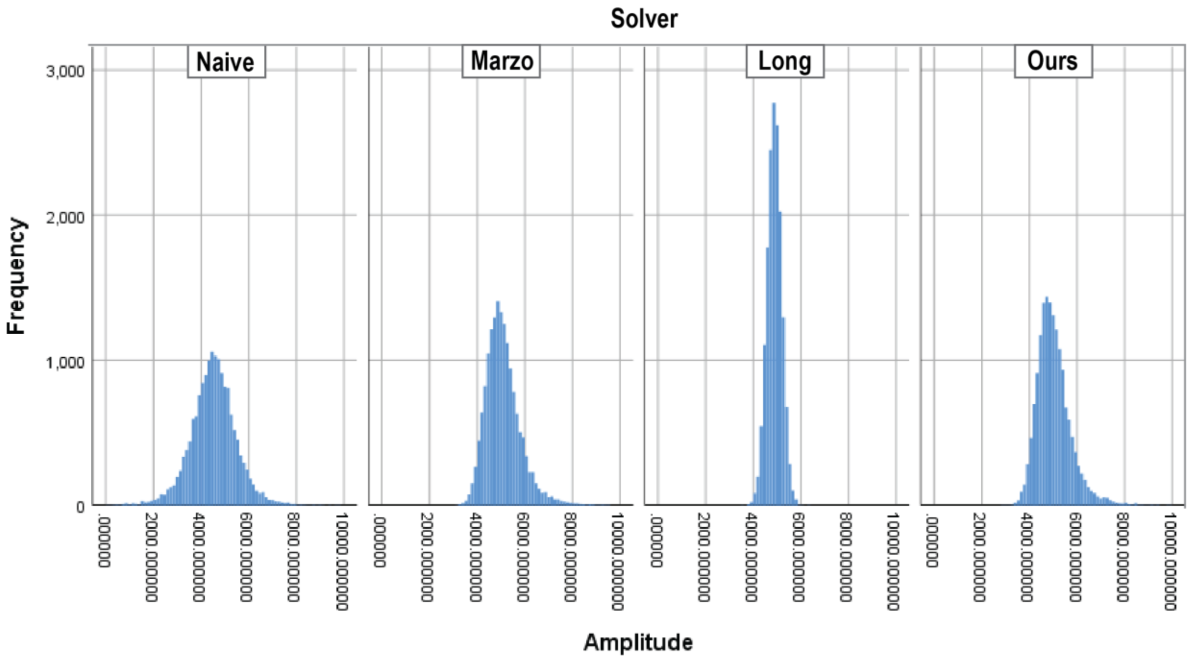
The error term is Mean Square (Error) = 244353.453.

a. Uses Harmonic Mean Sample Size = 400.000.

b. Alpha = .05.

3.4. Avoiding destructive interference (16 points)

Amplitude per point



Report

Amplitude

Solver	N	Mean	Std. Deviation
Naive	16000	4512.13827066	955.152625251
Marzo	16000	5085.17568874	730.477130939
Long	16000	4887.15367760	301.740848343
Ours	16000	5024.05295504	703.039148519
Total	64000	4877.13014801	746.545651947

Amplitude

Tukey HSD[a,b]

		Subset			
Solver	N	1	2	3	4
Naïve	16000	4512.13827066			
Long	16000		4887.15367760		
Ours	16000			5024.05295504	
Marzo	16000				5085.17568874
Sig.		1.000	1.000	1.000	1.000

Means for groups in homogeneous subsets are displayed.

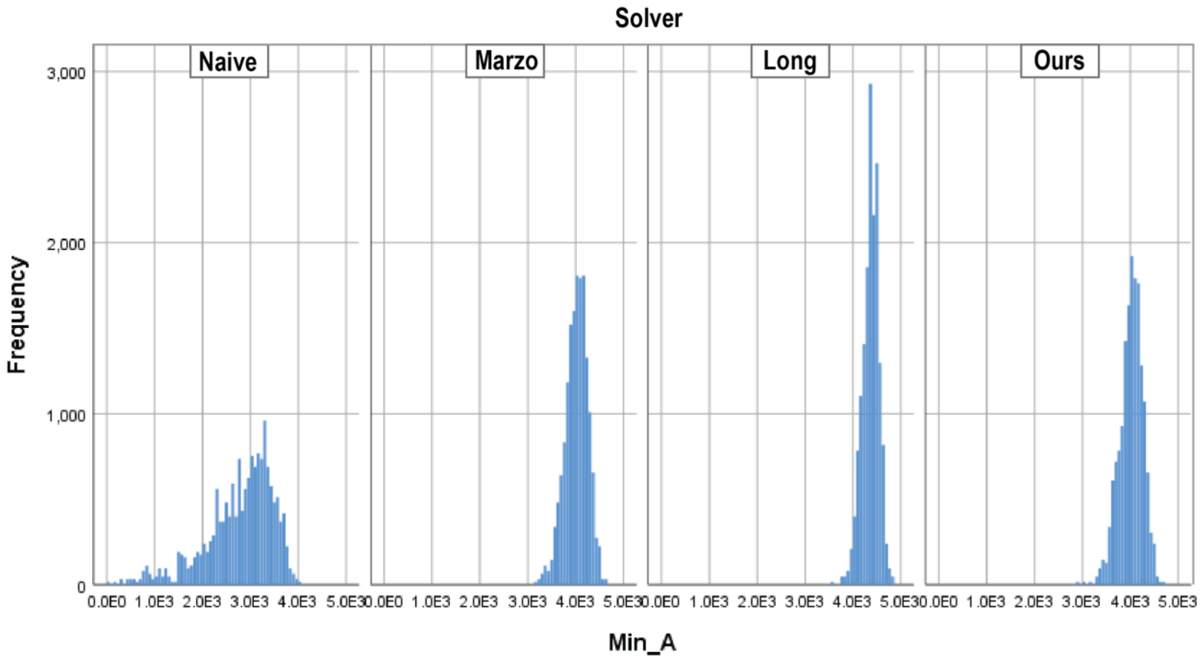
Based on observed means.

The error term is Mean Square(Error) = 507806.240.

a. Uses Harmonic Mean Sample Size = 16000.000.

b. Alpha = .05.

Amplitude of weakest point



Report

Min_A

Solver	N	Mean	Std. Deviation
Naive	16000	2.77554160E+003	7.117768765E+002
Marzo	16000	4.01773096E+003	2.351954346E+002
Long	16000	4.36833694E+003	1.731814029E+002
Ours	16000	4.01833449E+003	2.414501256E+002
Total	64000	3.79498600E+003	7.276200380E+002

Min_A

Tukey HSD[a,b]

Solver	N	Subset 1	Subset 2	Subset 3
Naive	16000	2.77554160E+003		
Marzo	16000		4.01773096E+003	
Ours	16000		4.01833449E+003	
Long	16000			4.36833694E+003
Sig.		1.000	.999	1.000

Means for groups in homogeneous subsets are displayed.

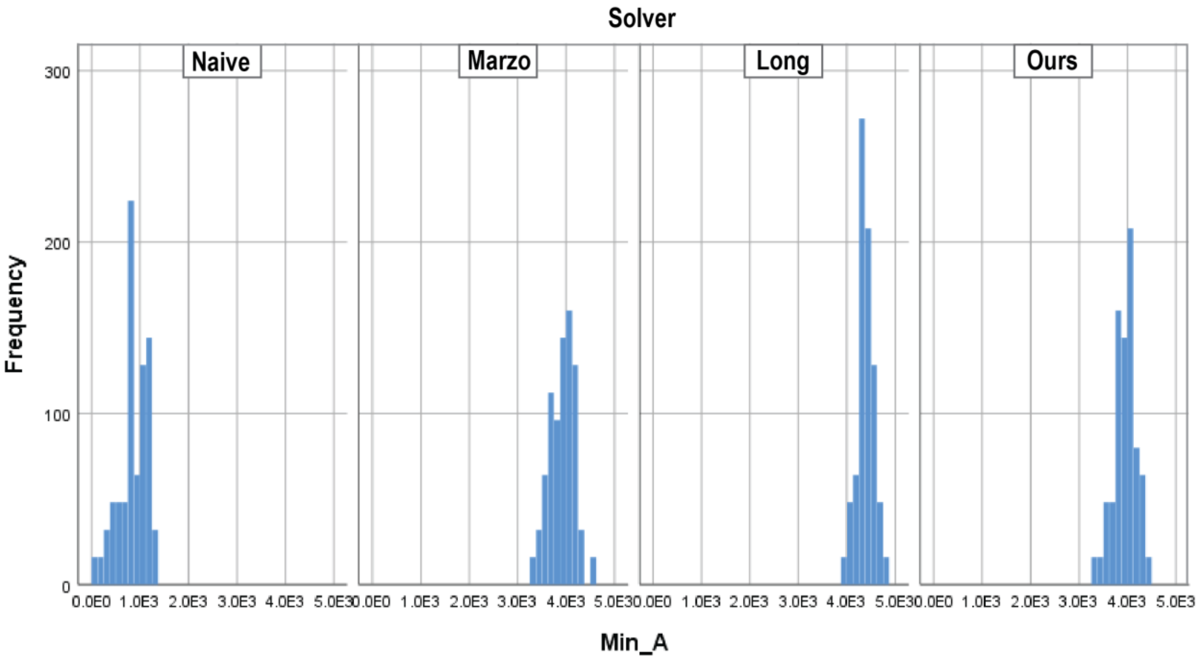
Based on observed means.

The error term is Mean Square(Error) = 162558.294.

a. Uses Harmonic Mean Sample Size = 16000.000.

b. Alpha = .05.

Amplitude of weakest point (p5 selected from Naive)



Report

Min_A

Solver	N	Mean	Std. Deviation
Naive	800	8.57611662E+002	3.016393410E+002
Marzo	800	3.91494941E+003	2.525738366E+002
Long	800	4.38771902E+003	1.731389748E+002
Ours	800	3.95242895E+003	2.250310067E+002
Total	3200	3.27817726E+003	1.430731914E+003

Min_A

Tukey HSD[a,b]

		Subset			
Solver	N	1	2	3	4
Naive	800	8.57611662E+002			
Marzo	800		3.91494941E+003		
Ours	800			3.95242895E+003	
Long	800				4.38771902E+003
Sig.		1.000	1.000	1.000	1.000

Means for groups in homogeneous subsets are displayed.

Based on observed means.

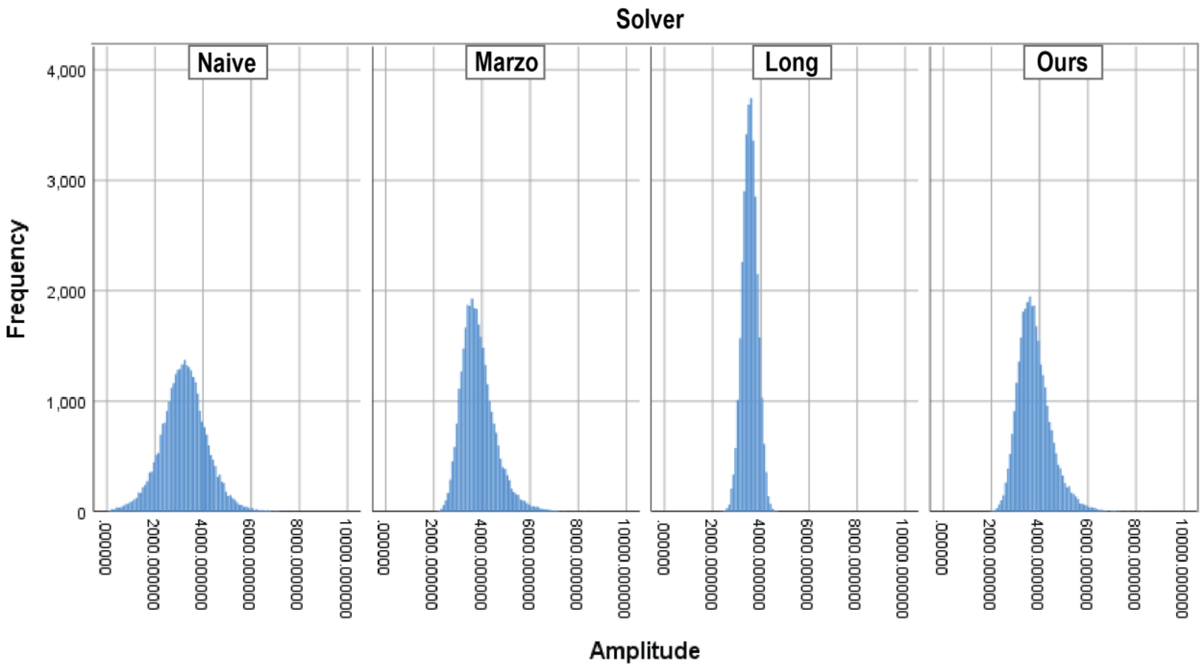
The error term is Mean Square(Error) = 58848.973.

a. Uses Harmonic Mean Sample Size = 800.000.

b. Alpha = .05.

3.5. Avoiding destructive interference (32 points)

Amplitude per point



Report

Amplitude

Solver	N	Mean	Std. Deviation
Naive	32000	3204.98836547	943.966554248
Marzo	32000	3850.48991083	704.429470877
Long	32000	3543.71441486	307.490556559
Ours	32000	3786.80195741	693.581723346
Total	128000	3596.49866214	744.924353957

Amplitude

Tukey HSD[a,b]

		Subset			
Solver	N	1	2	3	4
Naïve	32000	3204.98836547			
Long	32000		3543.71441486		
Ours	32000			3786.80195741	
Marzo	32000				3850.48991083
Sig.		1.000	1.000	1.000	1.000

Means for groups in homogeneous subsets are displayed.

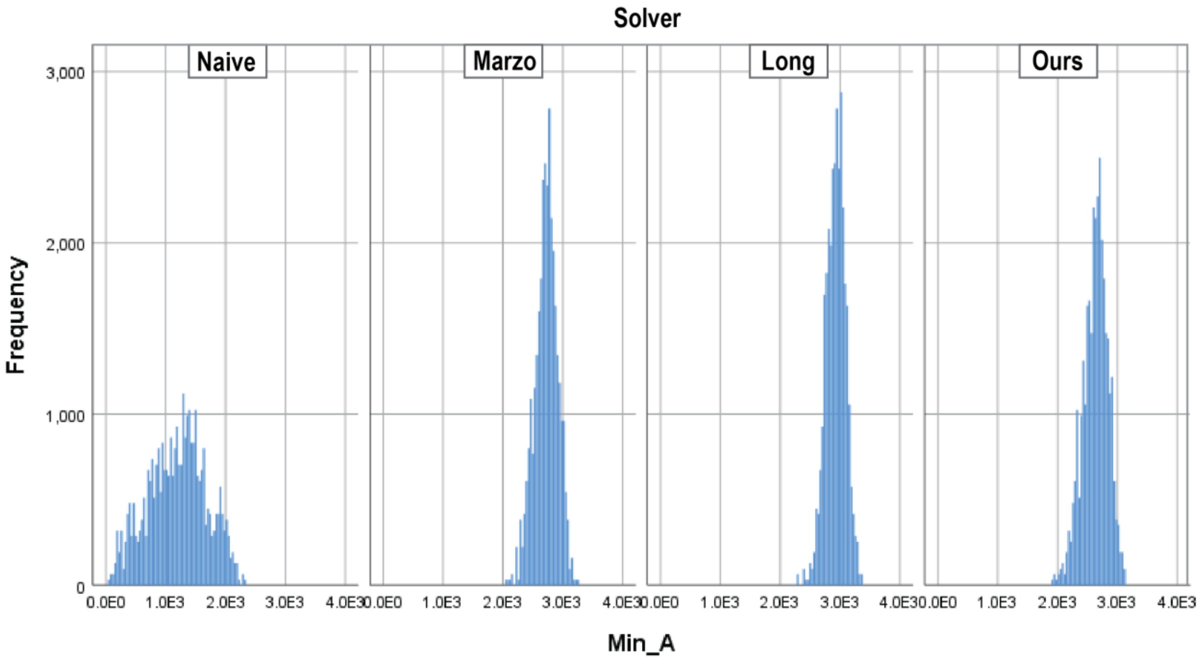
Based on observed means.

The error term is Mean Square(Error) = 490724.946.

a. Uses Harmonic Mean Sample Size = 32000.000.

b. Alpha = .05.

Amplitude of the weakest point (all cases)



Report

Min_A

Solver	N	Mean	Std. Deviation
Naive	32000	1.18919999E+003	4.842807245E+002
Marzo	32000	2.72177369E+003	1.911002922E+002
Long	32000	2.92752521E+003	1.616667110E+002
Ours	32000	2.63081717E+003	2.079791487E+002
Total	128000	2.36732901E+003	7.478820502E+002

Min_A

Tukey HSD[a,b]

		Subset			
Solver	N	1	2	3	4
Naïve	32000	1.18919999E+003			
Ours	32000		2.63081717E+003		
Marzo	32000			2.72177369E+003	
Long	32000				2.92752521E+003
Sig.		1.000	1.000	1.000	1.000

Means for groups in homogeneous subsets are displayed.

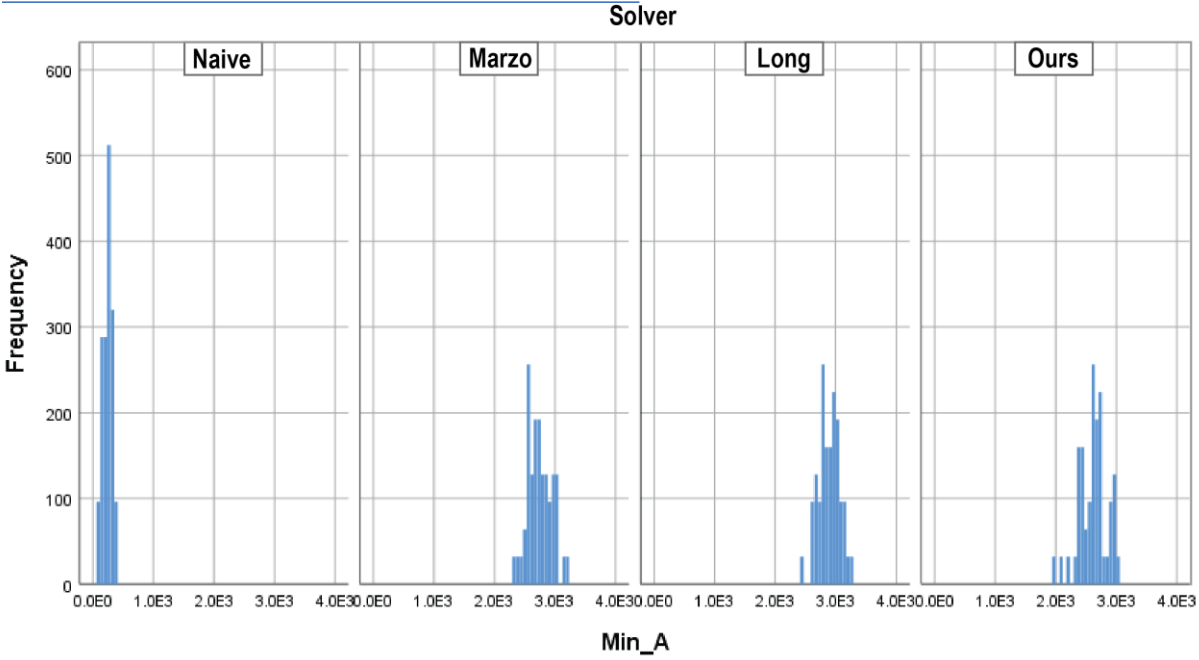
Based on observed means.

The error term is Mean Square(Error) = 85109.648.

a. Uses Harmonic Mean Sample Size = 32000.000.

b. Alpha = .05.

Amplitude worst points (p5 selected from Naïve)



Report

Min_A

Solver	N	Mean	Std. Deviation
Naive	800	8.57611662E+002	3.016393410E+002
Marzo	800	3.91494941E+003	2.525738366E+002
Long	800	4.38771902E+003	1.731389748E+002
Ours	800	3.95242895E+003	2.250310067E+002
Total	3200	3.27817726E+003	1.430731914E+003

Min_A

Tukey HSD[a,b]

		Subset			
Solver	N	1	2	3	4
Naive	800	8.57611662E+002			
Marzo	800		3.91494941E+003		
Ours	800			3.95242895E+003	
Long	800				4.38771902E+003
Sig.		1.000	1.000	1.000	1.000

Means for groups in homogeneous subsets are displayed.

Based on observed means.

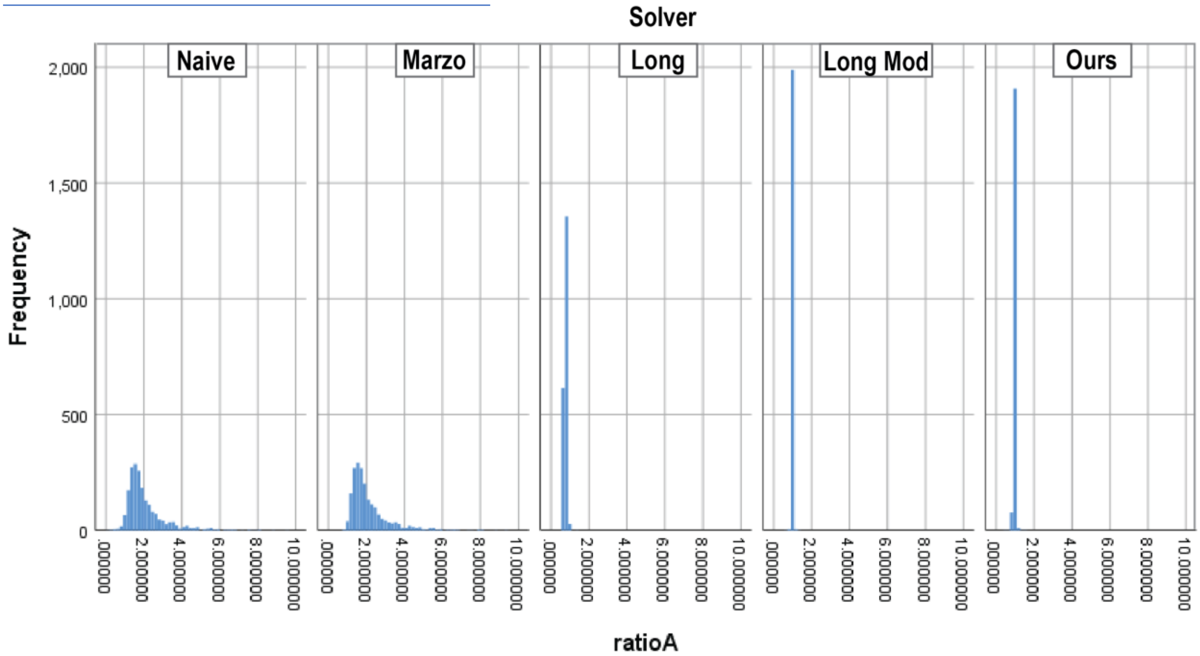
The error term is Mean Square(Error) = 58848.973.

a. Uses Harmonic Mean Sample Size = 800.000.

b. Alpha = .05.

3.6. Accurate amplitude reconstruction (Extended data)

2 points



Report

ratioA

Solver	N	Mean	Std. Deviation
Naive	2000	2.05955531	1.032213145
Marzo	2000	2.11930313	1.030467399
Long	2000	.76623170	.066683716
Long Mod	2000	.99934987	.028145356
Ours	2000	.98060030	.039018818
Total	10000	1.38500806	.874387663

ratioA

Tukey HSD[a,b]

		Subset			
Solver	N	1	2	3	4
Long	2000	.76623170			
Ours	2000		.98060030		
Long Mod	2000		.99934987		
Naïve	2000			2.05955531	
Marzo	2000				2.11930313
Sig.		1.000	.894	1.000	1.000

Means for groups in homogeneous subsets are displayed.

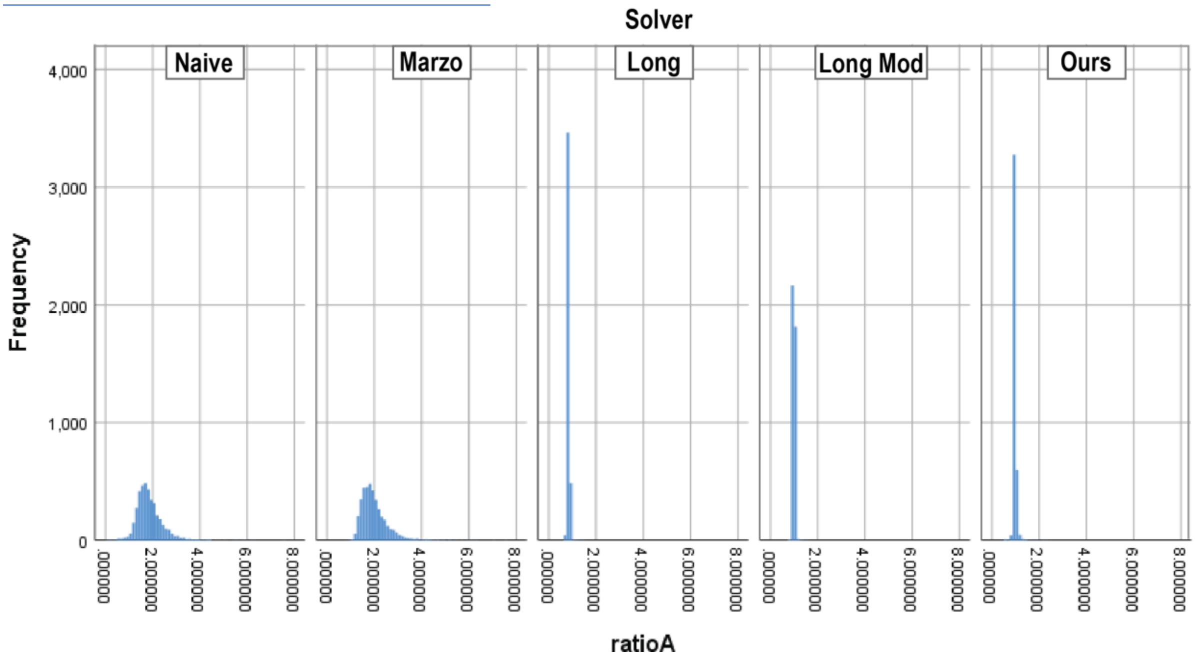
Based on observed means.

The error term is Mean Square(Error) = .427.

a. Uses Harmonic Mean Sample Size = 2000.000.

b. Alpha = .05.

4 points



Report

ratioA

Solver	N	Mean	Std. Deviation
Naive	4000	1.86255493	.581815389
Marzo	4000	2.00076090	.595449612
Long	4000	.83036064	.043042436
Long Mod	4000	.99822604	.029570358
Ours	4000	.98384756	.065855880
Total	20000	1.33515001	.618545286

ratioA

Tukey HSD[a,b]

		Subset			
Solver	N	1	2	3	4
Long	4000	.83036064			
Ours	4000		.98384756		
Long Mod	4000		.99822604		
Naïve	4000			1.86255493	
Marzo	4000				2.00076090
Sig.		1.000	.423	1.000	1.000

Means for groups in homogeneous subsets are displayed.

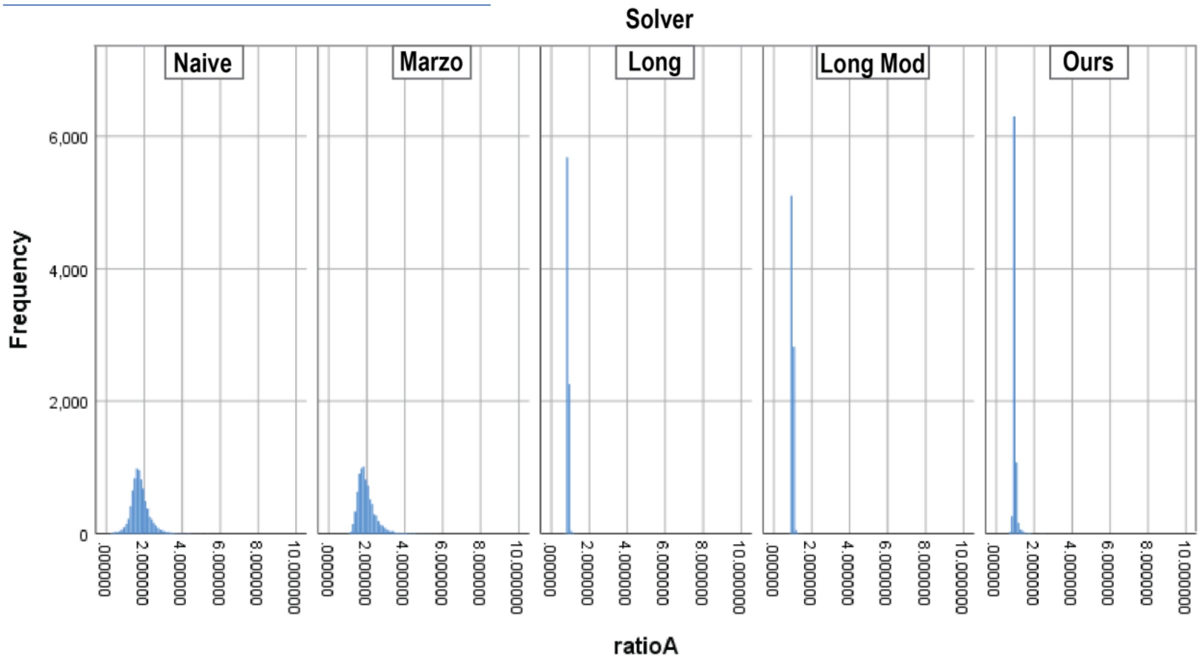
Based on observed means.

The error term is Mean Square(Error) = .140.

a. Uses Harmonic Mean Sample Size = 4000.000.

b. Alpha = .05.

8 points



Report

ratioA

Solver	N	Mean	Std. Deviation
Naive	8000	1.77984431	.521017160
Marzo	8000	1.99568027	.533076587
Long	8000	.87705530	.033126043
Long Mod	8000	.99614474	.030099664
Ours	8000	.98382869	.100424894
Total	40000	1.32651066	.574381468

ratioA

Tukey HSD[a,b]

		Subset			
Solver	N	1	2	3	4
Long	8000	.87705530			
Ours	8000		.98382869		
Long Mod	8000		.99614474		
Naïve	8000			1.77984431	
Marzo	8000				1.99568027
Sig.		1.000	.141	1.000	1.000

Means for groups in homogeneous subsets are displayed.

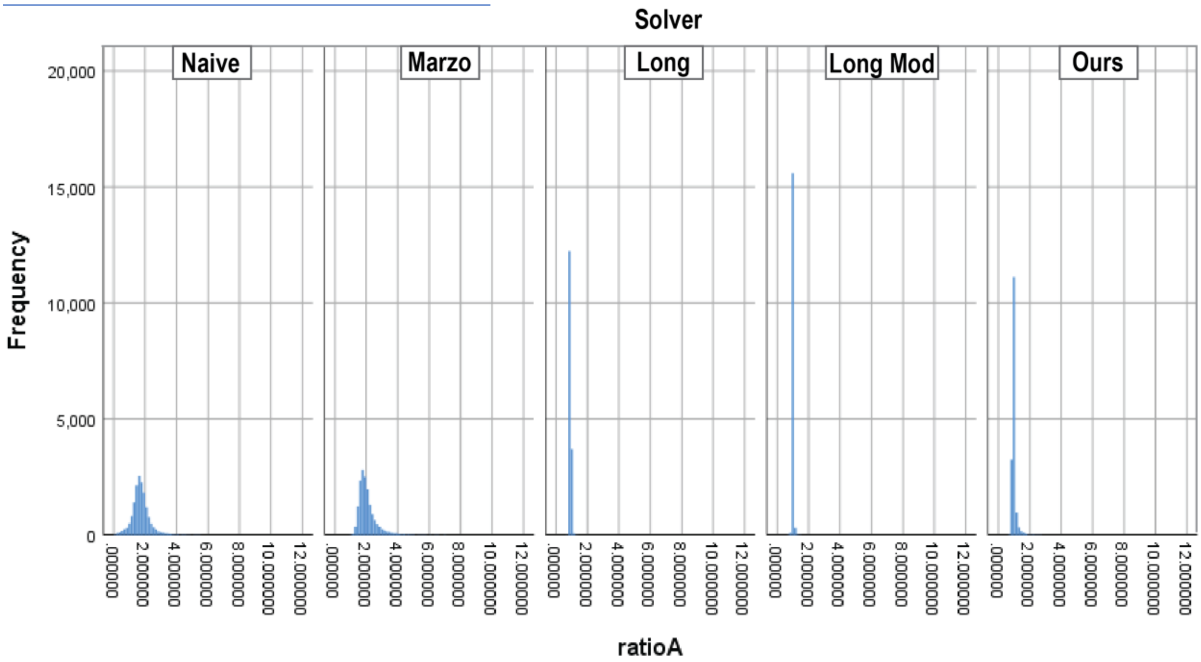
Based on observed means.

The error term is Mean Square(Error) = .114.

a. Uses Harmonic Mean Sample Size = 8000.000.

b. Alpha = .05.

16 points



Report

ratioA

Solver	N	Mean	Std. Deviation
Naive	16000	1.72537326	.596371611
Marzo	16000	2.05757741	.596081838
Long	16000	.91228878	.035754869
Long Mod	16000	.99438796	.036365170
Ours	16000	.98596026	.141185321
Total	80000	1.33511754	.604073365

ratioA

Tukey HSD[a,b]

		Subset			
Solver	N	1	2	3	4
Long	16000	.91228878			
Ours	16000		.98596026		
Long Mod	16000		.99438796		
Naïve	16000			1.72537326	
Marzo	16000				2.05757741
Sig.		1.000	.282	1.000	1.000

Means for groups in homogeneous subsets are displayed.

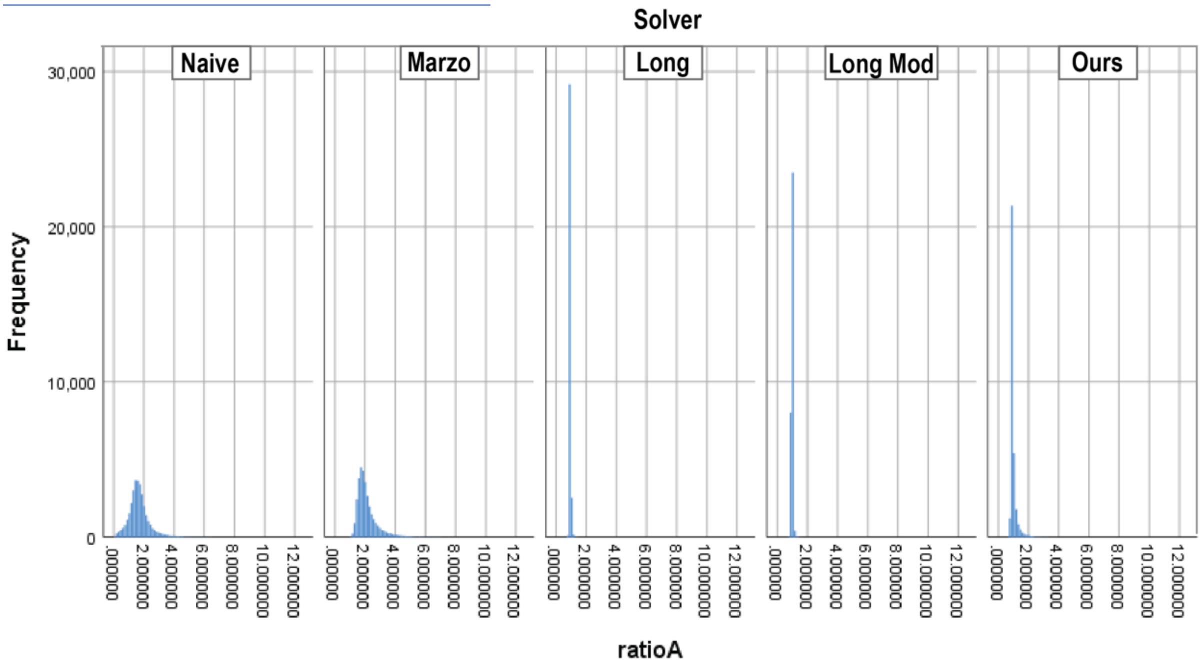
Based on observed means.

The error term is Mean Square(Error) = .147.

a. Uses Harmonic Mean Sample Size = 16000.000.

b. Alpha = .05.

32 points



Report

ratioA

Solver	N	Mean	Std. Deviation
Naive	32000	1.68858318	.739791575
Marzo	32000	2.14390986	.765360087
Long	32000	.93476977	.037320571
Long Mod	32000	.99071264	.039563690
Ours	32000	.98783338	.221307031
Total	160000	1.34916176	.687397470

ratioA

Tukey HSD[a,b]

		Subset			
Solver	N	1	2	3	4
Long	32000	.93476977			
Ours	32000		.98783338		
Long Mod	32000		.99071264		
Naïve	32000			1.68858318	
Marzo	32000				2.14390986
Sig.		1.000	.945	1.000	1.000

Means for groups in homogeneous subsets are displayed.

Based on observed means.

The error term is Mean Square(Error) = .237.

a. Uses Harmonic Mean Sample Size = 32000.000.

b. Alpha = .05.