# A non-linear transmission of Euclid's *Elements* in a medieval Hebrew calendrical treatise

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ABSTRACT

In this article I present the unique partial transmission of Euclid's *Elements* in the medieval Hebrew calendrical treatise *Yesod 'Olam* (The Foundation of the World), which was composed by Isaac Israeli in fourteenth-century Toledo. After a short introduction of *Yesod 'Olam*, I shall discuss the role of mathematics in the study of astronomy and the Jewish calendar, as understood by Israeli. Then I will provide a mapping of the *Elements* found in *Yesod 'Olam* and demonstrate Israeli's peculiar rendition of this seminal Greek work via four examples. Finally, I will show that Israeli's transmission of the *Elements* is lexically independent of earlier known Hebrew versions thereof.

KEYWORDS

Euclid

Isaac Israeli

The *Elements*

The Jewish calendar

*Yesod 'Olam*

## INTRODUCTION TO *YESOD 'OLAM*

In the Jewish year 5070 A.M. (1309/10 CE), Isaac ben Joseph Israeli from Toledo, also known as Isaac Israeli the Younger or The Second,[2] composed a vast treatise on the Jewish calendar, *Yesod 'Olam*, comprising five books, which was of high scientific level for its time. *Yesod 'Olam* aims to provide the scientific knowledge required for a thorough understanding of all aspects of the Jewish calendar, but it

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[2] To distinguish him from the tenth-century physician and philosopher Isaac Israeli.

does not merely lay out the necessary basic calendrical principles and algorithms, rather, it proposes a quasi-encyclopaedia. *Yesod ʻOlam* cannot, however, be regarded as a proper encyclopaedia because it does not contain the entire mathematical, astronomical and other scientific body of knowledge known at the time.

*Yesod ʻOlam* provides scientific knowledge in fields related to the Jewish calendar, such as mathematics (Book 1), geography, cosmography, and astronomy (Books 2 & 3),[3] Jewish Chronology, as well as the structure of and the conversion from and into the Christian and Muslim calendars (Books 4 & 5). Visual elements serving didactic purposes are abundant throughout *Yesod ʻOlam*. They include numerous mathematical, geographical, astronomical and calendrical diagrams as well as tables.[4]

Regarding the mathematics in *Yesod ʻOlam,* arithmetic is one of the main mathematical fields one encounters in Book 1. One finds claims such as ‘1 is the basis of all number’, the definition of ratios and how to deal with three and four proportional numbers. The second, more predominant, subject is geometry: Euclidean planar geometry as well as solid geometry, or stereometry. The third domain is trigonometry: planar and spherical. In *Yesod ʻOlam* we find no discussion of basic arithmetic - only the more advanced rule of three and four proportional numbers, which is applicable in spherical trigonometry.[5] Israeli does not revert at all to the Hebrew arithmetical tradition of the twelfth century such as the one found in Abraham ibn Ezra's *Sefer ha-Mispar* (The Book of the Number), a rudimentary text on the five basic arithmetical operations: multiplication, division, addition, subtraction and extraction of roots, a work well-known to Toledan Jews in the fourteenth century. Although the first three of the aforementioned arithmetical operations are necessary for the calculation of the Jewish calendar, the absence of their teaching in *Yesod ʻOlam* probably means that Israeli took for granted that his readers possessed at least some rudimentary arithmetical skills.

As for the structure of Book 1, it includes two chapters. The first chapter contains introductory teachings, in which Israeli elaborates on the role of mathematics in the service of astronomy and the two types of study of mathematics. The second chapter is dedicated to mathematics and is subdivided into twelve sections, ending with the book's final section: Sections 1-3 on arithmetical and geometrical preliminaries, Section 4, the longest one, includes 42 geometrical lessons, Sections 5-6 deal with planar

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[3] In fact, it is probably most correct to consider *Yesod ʻOlam* mainly as an astronomical treatise, and for numerous reasons, among which we find the following: not only will we see that Israeli considers the Jewish calendar to be a branch of astronomy, but he also stresses that understanding the motion of the heavenly bodies is fundamental for reaching God. In other words, Israeli provides a strong religious legitimation to the study of astronomy, which will entice readers to read his treatise and pursue the knowledge of astronomy. Furthermore, even though he declares that he would only treat the heavenly bodies which are relevant to the calculation of the Jewish calendar, i.e. the moon and the sun, Israeli cannot help but occasionally smuggle the other five planets into the discussion.
[4] The graphical quality of diagrams and their scientific accuracy varies greatly among the different manuscripts. I am preparing an article on the complex transmission of the diagrams in *Yesod ʻOlam* throughout the centuries.
[5] For example, in application of the Sine Rule on a sphere in Book 1, Chapter 2, Section 9.

trigonometry, Sections 7-9 teach spherical trigonometry, and Sections 10-12 and the final section elaborate on the notions of a sphere, a cone and proportional line-segments. Most importantly, the great majority of the mathematical materials in Book 1, in particular, in Sections 1-4, stems from Euclid's *Elements*, forming a separate branch of a partial transmission thereof within the medieval Hebrew mathematical tradition, as we shall see.

*Yesod ʻOlam* is a rich, albeit mathematically imperfect, text, able to quench various types of contemporary intellectual thirsts, be they literary, calendrical, scientific, or linguistic. Its importance is also manifest in its multifarious transmission, which spans over half a millennium and is testified by fifty-three manuscripts as well as two printed editions from Berlin: the first one, by Rabbi Barukh Schick of Shklov,[6] was printed in 1777 and the second one, by Goldberg and Rosenkranz, was printed in 1848, with a summary of the contents of the book in German. There is, in addition, a tiny fragment, of Byzantine origin, which I discovered in the Cairo Genizah collection.[7] The chain of manuscript transmission of *Yesod ʻOlam* includes many a provenance and various hands: Sephardic, Ashkenazic, Oriental, Byzantine, and Italian.[8] Fifty-four surviving hand-written witnesses, full and incomplete, is a most impressive number for a Hebrew scientific treatise composed in the Iberian Peninsula before the expulsion of the Jews from Spain and Portugal in 1492 and 1497, respectively.

Regarding the complex transmission of *Yesod ʻOlam*, one can discern, grosso modo, four different versions of the text. In a nutshell: Version 1,[9] the most disseminated among the surviving manuscripts, includes manuscripts which are probably closest to the urtext. Another group, Version 2, consists of manuscripts which tend to render the language of the text in Version 1 more elegant and concise but occasionally create erroneous interpretations. A further Version, Version 4, sometimes agrees only with one version against the other and sometimes it synthesises Versions 1 and 2, with an obvious pedagogical agenda of rendering the text as clear as possible by removing unnecessary

[6] Rabbi Barukh Schick of Shklov (1744-1808) also translated the *Elements* into Hebrew. For an extensive discussion of him, his learning, and his agenda, see Fishman (1995).

[7] Besides being an unexpected identification of a Genizah fragment thanks to one trigonometric term for sine (בקע), typical of Isaac Israeli it is an important witness of a Byzantine transmission of *Yesod ʻOlam*, see Wartenberg (2012).

[8] On the Italian transmission of *Yesod ʻOlam* (i.e. in Italy and/or in Italian hand) see Wartenberg (2018). Twelve manuscripts belong to this group, containing anything between one chapter of *Yesod ʻOlam* to its entirety, spanning from the fourteenth century to 1770. Manuscript Moscow, Russian State Library, Günzburg 571, was copied in Torino in 1770 and can probably be crowned as the worst witness of *Yesod ʻOlam* due to its heavily-erroneous text and diagrams with apparent negligence in every possible aspect. Manuscript Warsaw, The Emanuel Ringelblum Jewish Historical Institute, rkps 189 shows collaboration between Sephardic and Italian scribes. The most exciting manuscript in the Italian group was copied by no other than Ezra ben Isaac Fano during his youth, before becoming the Rabbi of Mantua and a famous kabbalist. This manuscript was in fact part of the 'Venitian Project' of the patron Johan Jakob Fugger.

[9] Establishing the complicated stemma was carried out by Israel Sandman, and the details are due to appear in a volume planned to be edited by him on the context and transmission of *Yesod ʻOlam*.

information, adding explanatory phrases, cross-referencing, disambiguation etc. Another version, Version 3, is a subset of the twenty Ashkenazic manuscripts with unique errors but also unique textual and paratextual features such as labelled glosses, metric instructions and vocalisations. In general, the Ashkenazic manuscripts tend to be more contaminated and corrupt than those of other provenances. More often than others, the Ashkenazic witnesses do not even include Books 1-3, which constitute the bulk of the scientific information. This conspicuous absence is probably not always due to physical loss of these parts of the text, but rather, to disinterest in or lack of understanding of the mathematics and astronomy in these books. The first printed edition clearly derives solely from Ashkenazic sources. Even the second printed edition, although improved by some corrections, and consultation of other manuscripts, still follows the text of the first edition and thus carries along most its problems.[10]

For all the excerpts in this article I have chosen a precious witness of *Yesod ʻOlam*, MS Add. 15977 from the British Library, a member of Version 4. It was written in semi-cursive Sephardic hand in the fifteenth century. No manuscript is perfect but some are more perfect than others. This manuscript from the British Library is the best surviving testimony due to its high scientific level, manifesting an intelligent synthesis of Version 1 and Version 2.[11]

The influentiality of *Yesod ʻOlam* among Jews in the late medieval and early modern period can be further attested by the existence of dozens of compendia, commentaries, as well as other treatises related to or inspired by *Yesod ʻOlam*. Among these works we know of a compendium written originally in Arabic by Israeli's son Joseph (תקציר יסוד עולם), which survived in a Hebrew translation by Isaac ben Solomon ben Isaac Israeli. Furthermore, Solomon ben Abraham Corcos wrote in 1331 an exegesis (באור) on *Yesod ʻOlam*.[12] In the early modern period, the polymath Rabbi David Gans wrote a commentary on *Yesod ʻOlam.*[13]

## THE INTELLECTUAL AND RELIGIOUS CONTEXTS

Before delving into *Yesod ʻOlam* itself, I would like to shed a bit of light on the intellectual environment in which it was composed and its history. At the beginning of the fourteenth century, Toledo had already established itself as a centre of academic learning and translations of scientific and philosophical

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[10] Details are due to appear in a planned volume to be edited by Israel Sandman, as mentioned in the previous footnote. My own experience with many other Ashkenazic manuscripts on scientific matters seems to confirm almost time and again that the level of transmission of scientific knowledge by Ashkenazic scribes is on the whole significantly lower compared to others. A higher level is noted for Ashkenazic manuscripts scribed in Italy, where clearly the Italian intellectual environment enriched with Sephardic scholars and manuscripts influenced the Ashkenazic community in a positive way.

[11] For additional visual features of this manuscripts see Wartenberg (ForthcomingB).

[12] He was the disciple of Judah bar Asher, the Rosh's son. See details on the Rosh in the next section.

[13] See Stern (2016).

treatises from Arabic into Hebrew and Latin.[14] Jews had played an important role in the transmission of Greco-Arabic science in mathematics, astronomy, astrology and medicine.[15] Interreligious cooperation, which had its roots already in ninth-century Baghdad reached Christian Spain in the twelfth century. Astronomy, in particular, was considered a 'neutral zone', to quote B. R. Goldstein, i.e. a field in which members of one religion could borrow ideas from another religion without any difficulty.[16] Israeli's work is thus part of a well-established Hebrew scientific tradition in the Iberian Peninsula. It emerged as a result of complex historical, social and linguistic factors. One speaks of the *Hebrew Renaissance of the Twelfth Century*, in which Hebrew had become the language of science and philosophy among Jews in Christian-ruled areas. A salient factor in this process directly relates to the invasions of Berber tribes, the Almoravids and the Almohads, into Muslim Spain (Al-Andalus) in the eleventh and twelfth centuries, respectively. This led to the collapse of a rather tolerant and intellectual milieu, forcing non-Muslim to either convert to Islam or to flee.[17]

The encounter between Jewish scholars who had mastered the Arabic language and science and local Jewish communities thirsty for knowledge but possessing a relatively a low level of scientific knowledge and little or no knowledge of the Arabic language naturally raised the question: in which language should the newcomers write for the local community? De facto, only Hebrew was a common language to all, and thus the natural candidate to become a *lingua franca* within the scientific and philosophical context. But one should be aware of the fact that the existing Hebrew scientific vocabulary was meagre, in particular in mathematics.

Until the twelfth century, Jews in Muslim lands usually composed scientific treatises in their cultural language, Arabic. Hebrew served mainly as a religious and poetic language. Pre-medieval sources such as the Bible and the Rabbinic literature possessed a limited amount of mathematical terminology. Thus much linguistic ingenuity was called for. The evolution of the medieval Hebrew scientific language, in particular the mathematical one, was a lengthy and intricate process, resulting in hundreds of novel lexemes. Various techniques were implemented to coin new mathematical words. The two main methods, in a nutshell, were: (1) the extension of the semantic field of an existing word in Jewish sources, such as the Bible or Rabbinic literature, endowing it with a mathematical meaning and (2) creating calques from Arabic.[18]

[14] See Burnett (2001).
[15] See Gomez-Aranda (2008).

[16] See Goldstein (2009).
[17] Maimonides's family is a well-known example of a Jewish family who chose to convert, or rather, pretend to have done so. A few years later they decided to leave the Iberian Peninsula altogether and finally settled in Fustat (Old Cairo).
[18] A thorough discussion of this theme is well-beyond the scope of this article. For more insight into the medieval Hebrew mathematical language and texts See Lévy (1996a, 1996b), Sarfatti (1968) and Wartenberg (2014).

It is important to note that *Yesod ʻOlam* was not a mere coincidental fruit of intellectual endeavour by Isaac Israeli. A critical contextual element motivated its composition*:* the presence and influence of Rabbi Asher ben Yeḥiel (known as the *Rosh*, the Hebrew acronym for Rabbi Asher) in Toledo, to whom *Yesod ʿOlam* was officially dedicated. The Rosh had escaped persecutions in Ashkenaz, and was appointed the chief Rabbi of Toledo. *Yesod ʻOlam* seems to present Israeli's endeavour to consolidate the Greco-Arabic intellectual *Weltanschauung* under the growing threat of the contrasting Talmud-centric world view of the influential Rosh.[19] Israeli, being highly diplomatic, first praises the Rosh's Halakhic scholarship, but then gently insinuates the latter's lack of scientific knowledge. Furthermore, Israeli tells us that he has heard of the Rosh's will to learn astronomy. Whether this is true or not is unclear, but in any case, it served as an excellent excuse for Israeli to compose *Yesod ʿOlam*. Although Israeli refers to the Rosh in reverence as 'my teacher', this matter needs to be taken with a pinch of salt. A careful analysis of the introduction of *Yesod ʿOlam* indicates that the two had probably never met.[20] We only know that Isaac Israeli's brother, Israel, was in fact a student of the Rosh. By connecting himself to the revered Rosh, Israeli was probably hoping to entice more potential readers to read his treatise. This move was bound to create excellent public relations not only for Israeli, but also for the creators of both printed editions, helping to enhance their sales. They did not shy to state under Israeli's name the fact that he was a student of the Rosh, and this was printed in bold letters on the front page of the book, creating a myth that is still found in today's scholarship.

Except for *Yesod ʻOlam*, Isaac Israeli wrote two other works on astronomy, *Shaʿar ha-Shamayim* (The Gate of the Heavens) and *Shaʿar ha-Milu'im* (The Supplementary Gate) ten and twenty years later, respectively.[21] One learns about Israeli's retrospective perception of *Yesod ʿOlam* from the introduction to *Shaʿar ha-Shamayim*:[22]

> *...ועל שלא פירשתי בו תחלה ולא הודעתי כמה שמות זרות ומילות נכריות שאני עתיד להשתמש בהם בספר הזה כגון מלת נקודה וקו ושטח ועגולה וארכו וקשת ואלכסון ויתר וזוית וקוטב גלגל ואופן [...] מפני שהספר הזה לא יסדתי אלא לבני ושאר התלמידים שכבר למדו את כל זה והשכילו בו מחבורי הגדול שקראו יסוד עולם שבו נתבאר כל זה והדומה לו וכן לא נזקקתי בו ללמד בכאן ולברר שום דבר מאותם העיניינים הנפלאים והעצומים שנתבארו בספר ההוא מתכונת העולם וצורתו [...] וכמה עניינים אחרים שנתבארו לשם מאותות שני המאורות [...] וכמה עיקרים ויסודות מטעמי חכמות העיבור והלכותיו [...] ובאמת החבור הזה אינו אלא כמו החלוק האחרון מספר יסוד עולם וכאילו הוא מחזיק בשוליו ונלוה אליו*

[19] See Galinsky (2006).

[20] Israel Sandman has carefully analyzed the introduction and has succeeded to break the myth regarding the nature of the relationship, or rather, the lack thereof, between Israel and the Rosh. The details are due to appear in a planned volume to be edited by him on the context and transmission of *Yesod ʻOlam*, mentioned above.

[21] See Goldstein and Chabás (2017).

[22] My partial edition of the introduction here is based on Parma, Biblioteca Palatina MS 3167 and Paris, BnF MS héb. 1070.

*ולולא שכבר יצא הספר ההוא לידי כמה בני אדם הייתי מצרף ענייני הספר הזה אליו והייתי עושה הכל ספר אחד אעפ"י שלפעמים ובמקומות רבים יכרחני סדר החבור לסדר בכאן ולהביא הרבה עניינים וכמה יסודות ועיקרים שכבר ביארתים בספר יסוד עולם...*

*...and that I have not first explained and made known several foreign names and foreign words in it [i.e. in Shaʿar ha-Shamayim], which I am due to use in this book, such as the word 'point', 'line', 'area', 'circle', its 'circumference', 'arc', 'diameter', 'hypotenuse', 'angle', 'pole', 'sphere', 'orb' […] because I did not create this book but for my son and the rest of the students, who had already studied and intelligized all this from my great composition they(!) named Yesod ʿOlam, in which all this and similar matters had been explained. Furthermore, I have not needed to teach here and clarify any of those wondrous and great matters which had been explained in that book concerning the astronomy of the universe, its shape […] and several other matters explained there regarding the two luminaries […] and several principles and foundations regarding the reasons in the sciences of the calendar and its rules […] In truth, this composition is just like a latter part of Yesod ʿOlam, and as if it were holding its margins, accompanying it. Had that book [i.e. Yesod ʿOlam] not reached the hands of several people, I would have attached to it the matters of this one and made it all into one book - even though at times and in many places, the structure of the composition does force me to arrange and adduce many matters here, foundations and principles which I had already explained in the book Yesod ʿOlam...*[23]

## MATHEMATICS IN SERVICE OF ASTRONOMY AND THE JEWISH CALENDAR

In Book 1, Israeli emphasizes that knowledge of mathematics is a pre-requisite for the study of astronomy; astronomy, or more specifically, the lunar and solar components thereof, in turn, are necessary for a proper understanding of the luni-solar Jewish calendar. Interestingly, Israeli further claims that the study of the Jewish calendar is an integral part of astronomy.

*ידוע הוא וברור לכל משכיל ומבין שעיקרי יסוד חכמת התכונה ואותותיה שחכמת העיבור היא גזע מגזעיה לא הכירו בהם החכמים וידעו משפטיהם אלא על ידי העיון...*

*...regarding the laws of the fundamentals and foundations of the science of astronomy, the science of calculating the Jewish calendar being one of its trunks, it*

[23] This is my own translation.

*is known and clear to every intellectual and understanding person that the scientists discerned and knew these laws by means of observation…*

Through the following excerpt from the Introduction to Book 1, one gains a good understanding of Israeli's scientific *Weltanschauung*. He describes the status of Ptolemy's *Almagest*, underlining how vital it is to master arithmetic and geometry in order to understand its contents. Yet, says Israeli, the scientific teachings in *Yesod ʻOlam* will suffice for the one who is too lazy to study the *Almagest* but who does wish to learn enough astronomy to understand the Jewish calendar. However, as Israeli describes in the following beautiful parable, this type of student will be like someone who has tasted the delicious fruit of a garden without ever having entered it:

*ואחרי חורבן בית שני בכמו שבעים שנה עמד באומות בטלמיוס החכם ובזמן מועט היה לפני רבנו הקדוש ז"ל והוא השכיל מאד בחכמת התכונה. והוציא לאור תעלומה. וחיבר בה ספרו הגדול הנקרא מג'סטי קלע בו באבן שכלו אל אחד אחד מעניני זאת החכמה ולא החטיא ומאז ועד עתה כל שבא אחריו למד מספרו כל אדם חזו בו. אבל לא יוכל להבין את דבריו. ולבא לחדרי מצפוניו. אלא מי שנשתדל ולמד תחלה והשכיל בחכמת התשבורת ובחכמת החשבון. אמנם מי שנתעצל ולא זכה ללמוד ולהשכיל בשתי החכמות האלו ונשאו לבו לידע מחכמת התכונה כדי מה שיספיק לו להבין ולהשכיל בטעמי יסודי העיבור וסודותיו ימלא כיום את ידו ויכין את לבבו ללמוד ולהשכיל במה שאציע ואסדר בספר הזה ובו ימצא כל צרכו ויצליח דרכו אכן יהיה כמו האיש ההוא שהיה טועה בשדה רעב וצמא וכה וכה מתהלך עד שהגיע לגן המלך ויהי הוא נבהל לבא לאכול ולמלא את בטנו מפירות הגן ומעדניו קמו השומרים והשוערים אליו ויסגרו שערי הגן לפניו ויבהילוהו. ובאף ובחמה משם הדפוהו כי לא הכירם ולא הכירוהו. ואפי' לראות לא הניחוהו. ויסב משם עצב וזעף ויגע ויעף. וירץ אליו אחד מאנשי הגן מהרה ויאמר שלום לך אל תירא. עמד על עמדך אני אתן מעדנים לנפשך. וישב לגן וימלא את חצנו מכל פרי מגדים חדשים גם ישנים ויתן לו ויאכל ויבא לו יין וישת ולא סר מלקט מפירות הגן לו ומטעימו ומאכילו עד שהשביע נפשו השוקקה ותחשב לו לצדקה. ועתה ראה שהאיש ההוא אע"פ שסר רעבונו ומלא את בטנו איך תנוח דעתו עליו והוא לגן לא בא ולא אכל בטובה ולא שלטו ידיו באילניו. ולא ארה מראש בשמיו. ולא שתה מהיין המשומר בענביו. וכן המשל בלומד מהספר הזה. רצוני לומ' שממנו ישכיל בטעמי יסודי העבור ויעלו בידו וממנו יבין נפלאות סודו. אבל לא יכיר ולא ידע מאין ואיך באו אליו. ולמי אלה לפניו.*

*Now, approximately 70 years after the destruction of the Second Temple, the sage Ptolemy arose amongst the nations. For a short time, he was before our holy Rabbi [Judah the Prince], in blessed memory, and he intelligized much in the science of astronomy, bringing to light its hidden aspect. On it he composed his magnum opus, entitled al-Magesti, in which he slings, with the stone of his intellect, at each and every one of the topics of this discipline and does not miss. From then to now, all who came after him learned from his book. Each man beholds within it, but is not able to understand its words, to enter the chambers of its hidden matters – except for he who has endeavoured and first learned and intelligized the science of* *geometry and the science of arithmetic. However, he who was lazy, and did not merit to learn and intelligize these two sciences, yet his heart has moved him to know about the science of astronomy, enough to suffice him to understand and intelligize the reasons of the foundations of the calendar and its esoterica, let him first of all dedicate himself and focus his heart to learn and intelligize what I shall set forth and arrange in this book. In it he will find all his need and will prosper in his way. Surely, he will be as that man who was wandering in the field hungry and thirsty. Hither and thither he walks about, until he reached the king's garden. Now, he is agitated to arrive to eat and fill his stomach with the fruit of the garden and its dainties. But the watchmen and gatekeepers rose up against him and closed the garden's gates before him. They disquieted him; and in anger and wrath they drove him from there – for he was not acquainted with them, and they were not acquainted with him. They did not give him leave even to look. So, he turned from there sad and dejected, worn out and weary. When one of the men of the garden ran to him quickly, saying: Peace unto you; have no fear! Maintain your station and I shall provide dainties for your soul. So, he returned to the garden and filled his bosom with all excellent fruit, both new and old, which he gave to him, whereupon he ate. He brought to him wine, and he drank. He did not turn from gathering the fruit of the garden and taking them out to him, giving him to taste and giving him to eat, until he sated his longing soul, and that was counted to him an act of righteousness. Now, see regarding that wandering man – although his hunger did depart, and he did fill his stomach – how can his mind be at rest within him, since he did not enter the garden, did not eat any of its good, his hands did not touch its trees, he did not gather its chief spices, and neither did he drink of the wine preserved in its grapes. Thus, is the parable about the learner from this book. I mean to say that from it he will intelligize the reasons of the foundations of the calendar and he will attain them; and from it he will understand the wonders of its esoterica. However, he will neither be acquainted with nor know how and whence they came to him, and unto whom are these which are before him.* [24]

## EUCLID'S *ELEMENTS* AND ITS RENDITION IN *YESOD ʿOLAM*

Euclid's *Elements* (Στοιχεῖα) is a pivotal Greek mathematical treatise composed around the year 300 BCE and it has a long and complex history. The *Elements* became a mathematical 'best-seller' in the ancient world and in the Middle Ages as well as in the Early Modern and Modern periods. It was used

[24] The English excerpts in this article from *Yesod ʿOlam* are based on Israel Sandman's translation, with my contribution, in particular, to the scientific vocabulary.

in the study of geometry and arithmetic in medieval universities, two of the four study subjects among the seven liberal arts, which formed the upper division, the *Quadrivium*, together with music, and astronomy. The *Elements* was translated into Arabic, Latin, Hebrew, and numerous other languages. Interestingly, during the fourteenth and fifteenth centuries, the study of at least two of the books of the *Elements* was mandatory for all students at Cambridge and Oxford universities. Much of its contents are still being taught in schools today.

One divides the thirteen books of the *Elements* into three categories: Books I-IV on planar geometry: points, lines, angles, triangles, quadrilaterals, parallelograms etc.; Books V-X on ratios and proportions between numbers and lines, and number theory; Books XI-XIII on spatial geometry and the analysis of three-dimensional figures such as the Platonic solids. What are the building blocks of the *Elements*? There are first principles, which constitute of definitions (e.g. a point is that which has no part),[25] postulates (e.g. to draw a straight line from any point to any point), common notions (e.g. things which are equal to the same thing are also equal to each other). The deductions from the first principles are divided into problems (e.g. on a given finite straight line to construct an equilateral triangle)[26] and theorems (e.g. If a straight line touch a circle, and a straight line be joined from the centre to the point of contact, the straight line so joined will be perpendicular to the tangent).[27]

Before presenting the detailed mapping of Euclid's *Elements* in *Yesod 'Olam* in Tables 1 and 2, I will first present the kernel: in *Yesod 'Olam* we find preliminaries, which include some of Euclid's definitions, postulates, and common notions. There are lessons, which contain one or more problems or theorems, or part thereof, and sometimes even a definition or a numerical example. Out of the thirteen books of the *Elements*, it is mainly Book I on planar geometry that was used by Israeli as a mathematical source in the composition of Book 1 of *Yesod 'Olam*, but also a fraction of Books III, IV, VI, VII, XI and XII. Israeli's focus is on triangles and circles, the basis for spherical trigonometry, which is at the foundation of mathematical astronomy. In fact, Israeli lists geometrical objects relevant for his treatise, such as the straight line, the circle, the sphere, the triangle, and the square. He also explicitly mentions those geometrical objects he has no business with, for example, the curved line, and others he has only little business with, such as the parallelogram. Tables 1 and 2 present the mapping of Euclid's *Elements* within *Yesod 'Olam*, demonstrating well Israeli's selectivity.

[25] See Example 1.
[26] See Example 4.
[27] See Example 3.

**TABLE 1: Definitions, postulates, and common notions**

| **Euclid's *Elements*** | **Heath's translation from the Greek**[28] |
|---|---|
| Book I Definition 1 | A point is that which has no part. |
| Book I Definition 2 | A line is breadthless length. |
| Book I Definition 3 | The extremities of a line are points. |
| Book I Definition 4 | A straight line is a line which lies evenly with the points on itself. |
| Book I Definition 5 | A surface is that which has length and breadth only. |
| Book I Definition 6 | The extremities of a surface are lines. |
| Book I Definition 7 | A plane surface is a surface which lies evenly with the straight lines on itself. |
| Book XI Definition 1 | A solid is that which has length, breadth, and depth. |
| Book I Postulate 1 | To draw a straight line from any point to any point. |
| Book I Definition 8 | A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line. |
| Book I Definition 10 | When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right, and the straight line standing on the other is called a perpendicular to that on which it stands. |
| Book I Definition 11 | An obtuse angle is an angle greater than a right angle. |
| Book I Definition 12 | An acute angle is an angle less than a right angle. |
| Book I Definition 23 | Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction. |
| Book I Definition 15 | A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another. |
| Book I Definition 16 | And the point is called the centre of the circle. |

[28] The entries in the table correspond to their order of appearance in *Yesod 'Olam*. All English translations of Euclid's *Elements* in this article derive from Heath (1956). The parts set in parentheses do not appear in Yesod 'Olam. I bring Heath's translation as is, including the original emphasis. It contains several archaic forms that are no longer, or only rarely used in Modern English, such as the subjunctive forms (e.g. "If a parallelogram have…").

| | |
|---|---|
| Book I Definition 17 | A diameter of the circle is any straight line drawn through the centre and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle. |
| Book I Definition 18 | A semicircle is the figure contained by the diameter and the circumference cut off by it. (And the centre of the semicircle is the same as that of the circle.) |
| Book I Definition 19 | Rectilineal figures are those which are contained by straight lines, trilateral figures being those contained by three, quadrilateral those contained by four, and multi-lateral those contained by more than four straight lines |
| Book I Definition 20 | Of trilateral figures, an equilateral triangle is that which has its three sides equal, an isosceles triangle that which has two of its sides alone equal, and a scalene triangle that which has its three sides unequal. |
| Book I Definition 21 | Further, of trilateral figures, a right-angled triangle is that which has a right angle, an obtuse-angled triangle that which has an obtuse angle, and an acute-angled triangle that which has its three angles acute. |
| Book I Definition 22 | Of quadrilateral figures, a square is that which is both equilateral and right-angled; an oblong that which is right-angled but not equilateral; a rhombus that which is equilateral but not right-angled; and a rhomboid that which has its opposite sides and angles equal to one another but is neither equilateral not right-angled. (And let quadrilaterals other than these be called trapezia.) |
| Book I Common Notion 1[29] | Things which are equal to the same thing are also equal to one another. |
| Book VII Definition 2 | A *number* is a multitude composed of units. |
| Book VII Definition 11 | A *prime number* is that which is measured by an unit alone. |
| Book VII Definition 16 | And, when two numbers having multiplied one another make some number, the number so produced be called *plane*, and its *sides* are the numbers which have multiplied one another. |
| Book VII Definition 18 | A *square number* is equal multiplied by equal, or a number which is contained by two equal numbers. |

[29] This common notion is used in the proof of Proposition 1 in Lesson 1 without ever having been mentioned by Israeli, see Example 4.

| | |
|---|---|
| Book VII Definition 20 | Numbers are *proportional* when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth. |
| Book VI Definition 1 | *Similar* rectilineal figures are such as have their angles severally equal and the sides about the equal angles proportional. |

**TABLE 2: Theorems**

| **Lesson in *Yesod ‘Olam*** | **Proposition(s) in Euclid’s *Elements*** | **Heath’s translation** |
|---|---|---|
| 1 | I.1 | On a given finite straight line to construct an equilateral triangle. |
| 2 | Not in the *Elements*[30] | |
| 3 | I.2 | To place at a given point (as an extremity) a straight line equal to a given straight line. |
| 4 | I.3 | Given two unequal straight lines, to cut off from the greater a straight line equal to the less. |
| 5 | I.4 & I.8 | (I.4) If two triangles have the two sides equal to two sides, respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend.<br><br>(I.8) If two triangles have the two sides equal to two sides respectively, and also have the base equal to the base, they will also have the angles equal which are contained by the equal straight lines. |
| 6 | I.5 | In isosceles triangles the angles at the base are equal to one another, and, if the equal straight lines be produced further, the angles under the base will be equal to one another. |
| 7 | I.6 | If in a triangle two angles be equal to one another, the sides which subtend the equal angles will also be equal to one another. |
| 8 | I.9 | To bisect a given rectilineal angle. |
| 9 | I.10 | To bisect a given finite straight line. |

[30] The lesson concerns Proclus’ teaching of how to draw an isosceles triangle. For an interesting analysis of this teaching in the medieval Hebrew tradition see Elior (2018b).

| | | |
|---|---|---|
| 10 | I.11 | To draw a straight line at right angles to a given straight line from a given point on it. |
| 11 | I.16 | In any triangle, if one of the sides be produced, the exterior angle is greater than either of the interior and opposite angles. |
| 12 | I.17 | In any triangle two angles taken together in any manner are less than two right angles. |
| 13 | I.18 | In any triangle the greater side subtends the greater angle. |
| 14 | I.19 | In any triangle the greater angle is subtended by the greater side. |
| 15 | I.20 | In any triangle two sides taken together in any manner are greater than the remaining one. |
| 16 | I.22 | Out of three straight lines, which are equal to three given straight lines, to construct a triangle: thus it is necessary that two of the straight lines taken together in any manner should be greater than the remaining one. |
| 17 | I.23 | On a given straight line and at a point on it to construct a rectilineal angle equal to a given rectilineal angle. |
| 18 | I.27-I.30 | (I.27) If a straight line falling on two straight lines make the alternate angles equal to one another, the straight lines will be parallel to one another.<br>(I.28) If a straight line falling on two straight lines makes the exterior angle equal to the interior and opposite angle on the same side, or the interior angles on the same side equal to two right angles, the straight lines will be parallel to one another.<br>(I.29) A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the interior angles on the same side equal to two right angles.<br>(I.30) Straight lines parallel to the same straight line are also parallel to one another. |
| 19 | I.31 | Through a given point to draw a straight line parallel to a given straight line. |
| 20 | I.32 | In any triangle, if one of the sides be produced, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of the triangle are equal to two right angles. |
| 21 | I.34 | In parallelogrammic areas the opposite sides and angles are equal to one another, and the diameter bisects the areas. |

| | | |
|---|---|---|
| 22 | I.35 | Parallelograms which are on the same base and in the same parallels are equal to one another. |
| 23 | I.36 | Parallelograms which are on equal bases and in the same parallels are equal to one another. |
| 24 | I.37 | Triangles which are on the same base and in the same parallels are equal to one another. |
| 25 | I.38 | Triangles which are on equal bases and in the same parallels are equal to one another. |
| 26 | I.41 | If a parallelogram have the same base with a triangle and be in the same parallels, the parallelogram is double of the triangle. |
| 27 | I.46 | On a given straight line to describe a square. |
| 28[31] | I.47 | In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle. |
| 29 | I.48 | If in a triangle the square on one of the sides be equal to the squares on the remaining two sides of the triangle, the angle contained by the remaining two sides of the triangle is right. |
| 30 | VI.1 | Triangles and parallelograms which are under the same height are to one another as their bases. |
| 31 | VI.2 | If a straight line be drawn parallel to one of the sides of a triangle, it will cut the sides of the triangle proportionally (and, if the sides of the triangle be cut proportionally, the line joining the points of section will be parallel to the remaining side of the triangle). |
| 32 | VI.4 | In equiangular triangles the sides about the equal angles are proportional, and those are corresponding sides which subtend the equal angles. |
| 33 | VI.5 | If two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend. |
| 34 | III.1 | To find the centre of a given circle. |
| 35 | III.3 | If in a circle a straight line through the centre bisect a straight line not through the centre, it also cuts it at right angles; and if it cut it at right angles, it also bisects it. |

[31] This is Pythagoras' Theorem, which is useful in the solution of spherical triangles, the basis for astronomical models. Israeli emphasizes its importance: 'Lesson 28. Know that every right-angled triangle, such as this triangle ABC, in which angle A is right, has a wondrous property. Pay attention to it, for you will need it much in the science of astronomy…'

| | | |
|---|---|---|
| 36[32] | III.18, 19 | (III.18) If a straight line touch a circle, and a straight line be joined from the centre to the point of contact, the straight line so joined will be perpendicular to the tangent.<br>(III.19) If a straight line touch a circle, and from the point of contact a straight line be drawn at right angles to the tangent, the centre of the circle will be on the straight line so drawn. |
| 37 | III.20 | In a circle the angle at the centre is double of the angle at the circumference, when the angles have the same circumference as base. |
| 38 | III.21 | In a circle the angles in the same segment are equal to one another. |
| 39 | III.26, 27 | (III.26) In equal circles equal angles stand [i.e. lean] on equal circumferences, whether they stand at the centres or at the circumferences.<br>(III.27) In equal circles angles standing on equal circumferences are equal to one another, whether they stand at the centres or at the circumferences. |
| 40 | III.31 | In a circle the angle in the semicircle is right,[33] that in a greater segment less than a right angle, and that in a less segment greater than a right angle; and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle. |
| 41 | III.30 | To bisect a given circumference. |
| 42[34] | Related to IV.15 | In a given circle to inscribe an equilateral and equiangular hexagon. |
| Section 10 (Post-Lessons) | XII.18 | Spheres are to one another in triplicate ratio of their respective diameters.[35] |

[32] Lesson 36 is explicitly referred to in Book 3 on astronomy, in the discussion of the discrepancy between the true and mean location of the moon.
[33] This is Thales's Theorem.
[34] In *Yesod 'Olam*: Know that the chord of one-sixth of the circumference of a circle is equal to one-half of its diameter.
[35] This proposition is applied in Book 2 on astronomy, accompanied by numerical data, when discussing the Ptolemaic analysis of the ratios of the volumes of the sun, moon and earth.

## EXEMPLIFYING THE UNTRADITIONAL TRANSMISSION OF THE EUCLIDEAN ELEMENTS IN YESOD 'OLAM

In this section I analyse four examples from *Yesod 'Olam* which demonstrate various aspects of Israeli's unusual transmission of Euclidean elements with some interesting linguistic additions to it. Example 1 concerns the definition of a point (Definition 1 in Book I of the *Elements*), to which Israeli adduces parts of a later definition of a line. The rearrangement is intentional and is spiced up with some humour. Example 2 regards various types of triangles, and there we find a novel concept, *pinnah* (פנה), unknown in other Hebrew or non-Hebrew mathematical sources. This word exists in the Bible, where its meaning 'corner' but in Israeli's text it acquires a new, mathematical, meaning: one of the six components of every triangle, three sides and three angles. Example 3 manifests methodological laxity in the transmission of the theorem that the line from the centre of circle to the point of tangency is perpendicular to the tangent (III.18), and its converse (III.19). The proof of III.18 contains 'holes' and that of III.19 is almost non-existent. Israeli trusts the readers to be able to fill the gaps themselves by study. Example 4 concerns the construction of an equilateral triangle (I.1). Although it follows Euclid rather faithfully, there are several methodological gaps. Example 4 also serves to demonstrate lexical independence from earlier known Hebrew transmissions of the *Elements*.

### EXAMPLE 1: DEFINITION OF A POINT

In this section I wish to show Israeli’s unique presentation of Euclid’s definition of a point..[36]

> *ואתחיל מהנקודה מפני שהיא לזה הענין כמו היסוד לבנין וכמו שהוא האחד שרש ועקר המנין. שער דע כי הנקודה היא דבר שאינו מתחלק כלל. ואפי' במחשבה. לא מפני דקותה בלבד אלא מפני שאינה ראויה לכך הואיל ואין לה שום מדה ולא שיעור ולא שום התפשטות ולא ממשות. אמנם היא תכלית הקו וקצהו. ואינה חלק ממנו כמו שחשבו הטפשים באמרם כי הקו הוא מחובר מנקודות רצופות דבוקות זו לזו. וזה אינו אמת אלא הוי יודע כי אלף אלפי אלפים נקודות אם תחשוב אותן מחוברות כאחת אינן אלא נקודה אחת בלבד. כי איך יתחבר ממה שאין לו שום מדה ולא שיעור דבר כמו הקו שהוא בעל מדה ושיעור והתפשטות.*
>
> *I shall begin from the point because, for this matter, it is as the foundation of the edifice, as ‘one’ is the root and fundament of counting. A Section: Know that the point is a thing that is not divisible at all, even in thought – not merely on account of its rarefaction, but because it is not fit for this [i.e. divisibility], since it does not have any measure, neither amount, nor any extension, nor substance. Indeed, it is the terminus and extremity of the line and it is not a part of it – as the fools thought, in their saying that the line is composed of points that are contiguous, conjoined to*

[36] Book 1, Chapter 2, Section 1. Following Euclid, Israeli provides the definition of a point first.

*one another. This is not true. Rather, you should know that thousands upon thousands upon thousands of points, if you reckon them attached as one, are nothing other than a single point only. Indeed, how could something such as a line, which possesses measure, amount, and extension, be composed of that which has neither any measure nor amount?!*

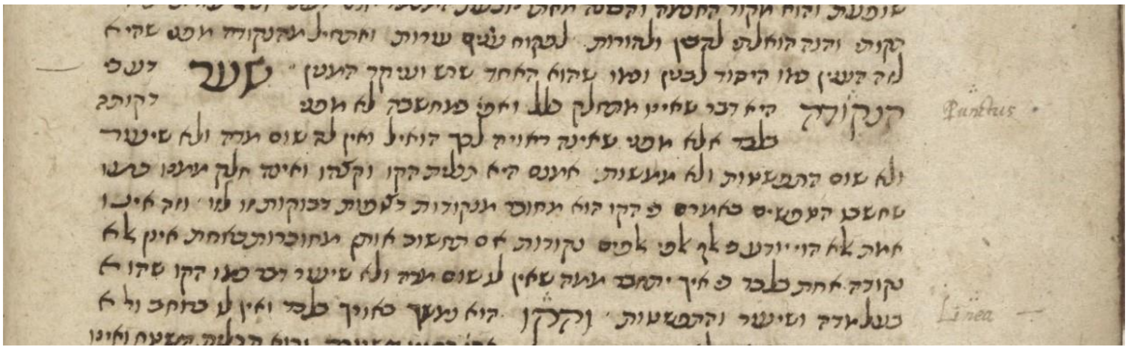


FIGURE 1
Definition of a point in *Yesod ʻOlam*


The point is the first geometrical entity defined by Euclid. Israeli includes Euclid's Definitions 1 and 3 from Book I in the *Elements*:

Definition 1: The point is that which has no part.

Definition 3: The extremities of a line are points.

In Euclid's *Elements*, Book I Definition 3 forms part of a definition of a line, the extremities of which are points, whereas in *Yesod ʻOlam* the definition are part of the description of a point and the emphasis is on the point forming the extremities of a line. When Israeli teaches the definition of a line he does not refer to its extremities again. This seemingly intentional rearrangement of the Euclidean definitions seems to have served his aim of proving the absurdity of the claim that the line is composed of points contiguous to one another. The use of the derogatory term 'fools' to depict the claimants was perhaps an expression of Israeli's sense of humour, his way to criticize ignorant people in general, or specific figures in his intellectual milieu. Perhaps he was referring to the contemporary debate on the Aristotelian concept of the continuous. Alternatively, this may have been a provocative writing style to entice readers to continue reading and get to the later, more complex, parts of the text on trigonometry.

Finally, one notes that before introducing his definition of the point, Israeli compares the point, depicted as 'the foundation of an edifice', and 1, the root of counting, or the building block of all numbers (i.e. ‘A number is a multitude composed of units’, the *Elements*, Book VII Definition 2). It is important not to understand the analogy as identity: unlike 1, the atom which creates every number, the grouping of points does not create new and greater geometrical entities of higher dimension, such as a line. What Israeli seems to have had in mind is that both 1 and the point are not defined by other arithmetical or geometrical objects. In any case, it is clear, that throughout the text Israeli sometimes goes beyond merely stating Euclidean enunciations. However, alongside his occasional verbosity on some geometrical issues, in most cases he uses Euclidean materials rather selectively.

EXAMPLE 2: THE TYPES OF TRIANGLES

In the *Elements*, Book I, Definitions 20 and 21, we find Euclid’s categorization of triangles, first, according to their sides, and then according to their angles:

> *[20] Of trilateral figures, an equilateral triangle is that which has its three sides equal, an isosceles triangle that which has two of its sides alone equal, and a scalene triangle that which has its three sides unequal.*
>
> *[21] Further, of trilateral figures, a right-angled triangle is that which has a right angle, and obtuse-angled triangle that which has an obtuse angle, and an acute-angled triangle that which has its three angles acute.*

In *Yesod ‘Olam*, Chapter 1, Section 3, Israeli elaborates on the types of triangles, in a similar way found in the *Elements*. However, in his definitions, we find an interesting linguistic addition, the Hebrew term *pinnah* (פנה), meaning ‘element’ or ‘component’, six of which every triangle possesses, (three sides and three angles). It is also a novel mathematical concept in the medieval Hebrew mathematical language:[37]

> *ואשוב למשולש ואומר כי כל משולש שבעולם ידוע הוא וברור שהוא בעל [שש] פנות והם שלשת צלעיו ושלש זויותיו. והמשולש מצד צלעותיו הוא על שלש מדות על דרך כלל. יש משולש ששלשת צלעיו הם שוים זה לזה. ויש שהשנים מהם בלבד הם שוים זה לזה. ויש ששלשת צלעיו הם מתחלפים זה מזה במדתם. וכן הוא עוד מצד זויותיו על שלש מדות כמו כן: יש משולש שהוא בעל זוית נצבת כמו א'ב'ג' שזוית ב' ממנו היא נצבת. ויש שהוא בעל זוית מרווחת כגון משולש ד'ה'ז' שזוית ה' היא מרווח.[38] ויש שכל אחד מזויותיו היא צרה כגון משולש כ'ל'מ'.*

---

[37] The word *pinnah* per se is not new but was devoid of mathematical meaning until Israeli endowed it with one. In the Bible, it means ‘corner’ (e.g. 2Chronicles 28:24). Israeli, however, extended the semantic field of the Biblical word *pinnah* to include a new, geometric, meaning: ‘element’ or ‘component’, being a side or an angle of a triangle, altogether six for each triangle. As far as I have been able to verify, there is no term in Arabic or Greek which refers to either a side or an angle of a triangle. Israeli seems to have been inspired by the Arabic term *rukn* ركن (corner), and in its abstract sense of 'element' or 'foundation' in expressions such as *arkān al-islam* (أركان الإسلام), i.e. the pillars/foundations of Islam, or *arkān al-dawla* (أركان الدولة) i.e. the important people, pillars, of the empire. For a thorough discussion of this term and its evolution see Sarfatti (1968), pp. 18-19 & 215-220 and Wartenberg (forthcomingA).

[38] Adherence to the correct gender in Hebrew was rather lax in the Middle Ages.

*I shall revert to the triangle and say that it is known and clear that every triangle in the world possesses six*[39] *components/elements, namely, its three sides and its three angles. In general, from the perspective of its sides, the triangle usually exists in three measures. It can be that the three sides are equal to one another [i.e. an equilateral triangle]; it can be that only the two of them are equal to one another [i.e. an isosceles triangle]; and it can be that all three of them differ from one another in their measurement [i.e. a scalene triangle]. Furthermore, also from the perspective of its angles, the triangle usually exists in three measures. It can possess a right angle, as in the case of triangle ABC, the angle B of which is right; it can possess an obtuse angle, as in the case of triangle DEF, the angle E of which is obtuse;*[40] *and it can be that each of its angles is acute, as in the case of triangle KLM.*

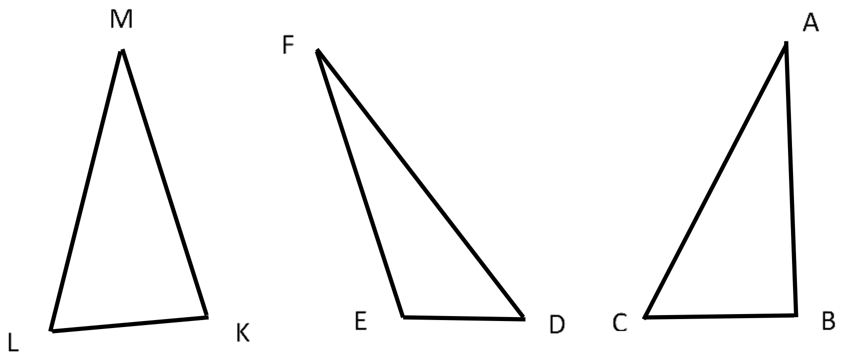


FIGURE 2

Types of triangles according to angles in the Preliminaries of *Yesod ʻOlam*

[39] We find 'two' in the manuscript, but with a sign for deletion, but the correct 'six' cannot be discerned.
[40] This is one of the rare diagrammatical errors in the otherwise excellent manuscript Add. 15977 from the British Library, as mentioned earlier. The vast majority of the manuscripts present erroneous diagrams when it comes to the obtuse triangle and it is often depicted as a right triangle. The few 'righteous' mss. include (i) Vatican, Biblioteca Apostolica, ebr. 380 (Sephardic hand, 15c.) (ii) Florence, Biblioteca Nazionale Centrale Magl. II. VI. 26 (Italian hand, 1421) (iii) New York, Jewish Theological Seminary Ms. 9830 (Ashkenazic hand, 16-17c.) (iv) Munich, Bayerische Staatsbibliothek, Cod. hebr. 35 (Italian hand, Venice 1551), and (v) Prague, Jewish Museum 33 (Ashkenazic hand, 17c.), in which the incorrect diagram was erased from the body of the text and the correct diagram was inserted in the margins. The second printed edition includes the correct diagram.

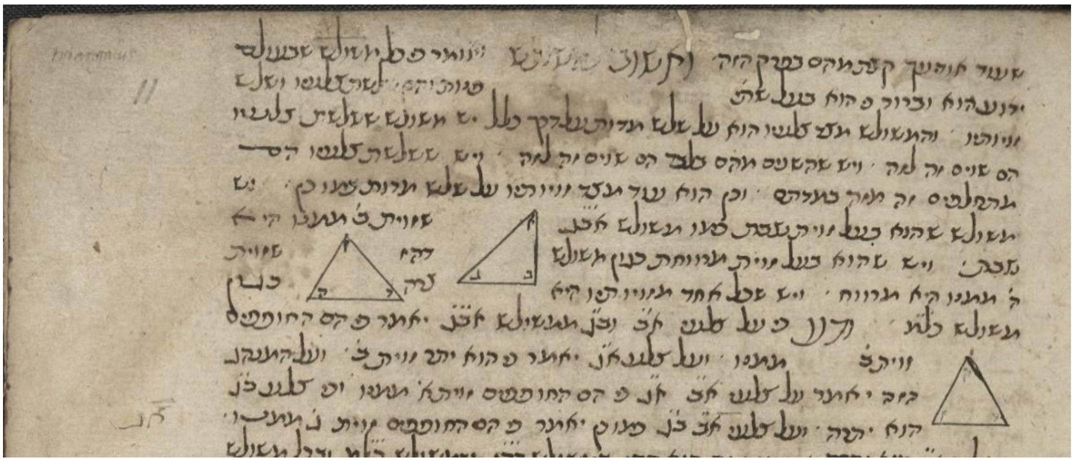

FIGURE 3

Types of triangles in *Yesod ʻOlam*

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EXAMPLE 3: THEOREMS REGARDING THE TANGENT TO A CIRCLE

Lesson 36 contains two Euclidean theorems:

(III.18) If a straight line touch a circle, and a straight line be joined from the centre to the point of contact, the straight line so joined will be perpendicular to the tangent.

(III.19, the converse theorem to III.18) If a straight line touch a circle, and from the point of contact a straight line be drawn at right angles to the tangent, the centre of the circle will be on the straight line so drawn.

Lesson 36 is a salient example of Israeli's general tendency to simplify and informalize Euclidean enunciations, in particular in its second part. In the first part of the lesson, he states and proves III.18, being overall loyal to the Euclidean text, with some short-cuts.[41] However, when it comes to demonstrating Proposition 19, he does not even bother to prove it,[42] only formulate it. Israeli says that the reader can easily do it himself!

***למוד ל"ו*** *נחוג העגולה הזאת סביב מרכז ה' ונציע שקו א"ב הוא פוגש אותה ונוגע בה על נקודת ג' ממנה*

*וזה כי אי אפשר שיפול מנקודת ה' על קו א"ב שום קו שיהיה עמוד עליו זולתי קו ה"ג זה. ואם תאמר כי הוא איפשר נציע שיהיה קו ה"ד החותך את העגולה על נקודת ז' ממנה ויהיה אז לפי זה צלע ה"ג ממשולש ה'ג'ד' גדול מצלע ה'ד' הואיל וזוית ד' ממנו היא הגדולה. וזה שקר גדול מפני שקו ה"ז ממנו הוא שוה לקו ה"ג הילכך קו ה"ג הוא בהכרח עמוד על קו א'ג'ב' ואין אחר עמוד עליו. ובמעט עיון יתבאר לך ותדע הפך*

[41] See my annotations to the English translation of Lesson 36 below.

[42] Unlike the diagram he provides for the first part (III.18), which is the same as Euclid's, one can find no diagram to prove the reverse theorem (III.19) in any of the surviving manuscripts.

*המשפט הזה רצו' לומ' כי היות קו ה"ג עמוד על קו א"ב הנוגע בעגולה ופוגש אותה על נקודת ג' יגרום שיהיה העמוד הזה הנופל על קו א'ג'ב' עובר בהכרח על מרכז העגולה.*

***Lesson 36****: We draw this circle around centre E. We posit that line AB meets the circle and touches it at its point C [i.e. it is tangent to it]. It is impossible for any line to reach line AB from point E and be perpendicular to it, except for this line EC. If you say that it is possible, then we posit that it be line ED, which intersects the circle at point F, which reaches perpendicularly from point E to line AB. Then, accordingly, side EC of triangle ECD will be greater than side ED, since its angle D is the greater one*[43] *but this is big lie, for its line EF [which is a mere segment of line ED] is equal to line EC. Therefore, line EC is necessarily perpendicular to line ACB, and no other is perpendicular to it. With a little scrutiny, the converse of this proposition will become clear to you and you will know it. I mean to say that since line EC is perpendicular to line AB, which touches the circle and meets it at point C, causes that this perpendicular, which reaches line ACB, to necessarily pass through the centre of the circle.*

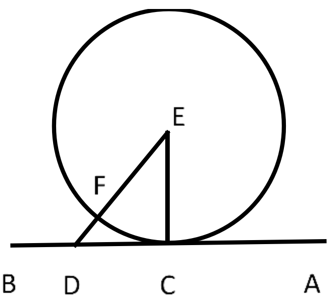


FIGURE 4: Lesson 36 in *Yesod ʻOlam*

EXAMPLE 4: THE CONSTRUCTION OF AN EQUILATERAL TRIANGLE AND ISRAELI'S LEXICAL INDEPENDENCE FROM EARLIER HEBREW SOURCES

Lesson 1 is an example of a proposition which teaches how to construct an equilateral triangle, and it corresponds to Theorem 1 in Book I of the *Elements*:

*למוד א' נרצה להקים על קו א'ב' זה הישר [המוצע] ולעשות משולש שיהיו שלשת צלעיו שוים זה לזה במדתם וזה יאות להיותו על הדרך הזאת. נתכוון לקו א'ב' זה המוצע. ונשים נקודת א' ממנו מרכז. ונרחיק עד נקודת ב' ממנו ונחוג עגולת ד' הימנית. ונשים עוד נקודת ב' ממנו מרכז. ונרחיק עד נקודת א'. ונחוג עגולת ה' השמאלית. והנה שתי העגולות האלה נפגשות על נקודת ג' מהן. ונמשיך עתה קוי א'ג' ג'ב' כמו*

[43] This is due to I.19, taught in Lesson 14: In any triangle the greater angle is subtended by the greater side. Israeli, like in many other cases, does not follow Euclid entirely by providing the previous, necessary, statement, that since angle EDC is right, angle ECD is acute, based on I.17 and taught in Lesson 12: In any triangle two angles taken together in any manner are less than two right angles.

*שהם בצורה. ונאמר כי הנה נעשה משולש א'ב'ג' זה מהצורה שוה הצלעים כמו שרצינו לעשותו. והמופת על זה הוא שקוי א'ג' א'ב' הם שוים זה לזה הואיל ושניהם יצאו ממרכז עגולת ד' ועד ההקף שלה. וכן הם קוי ב'ג' ב'א' שוים כמו כן זה לזה הואיל ושניהם יצאו ממרכז עגולת ה' ועד ההקף שלה. הרי שקו א'ב' המוצע הוא שוה לכל אחד מקוי א'ג' ג'ב'. הילכך צלעי משולש א'ב'ג' השלשה הם שוים זה לזה כמו שאמרנו.*

*Lesson 1***:** *Upon this given straight line AB we wish to erect and make a triangle whose three sides will be equal to one another in their measurement. It is appropriate that it be according to this method: focusing on this given line AB, we set point A on it as the centre, moving as far its point B, and we draw the right circle D. Likewise, we further set point B of it as the centre, moving as far as point A, and we draw the left circle E. Behold, these two circles intersect at point C on them. Now we draw line AC and line CB, as they are in the image. And we say that – Behold! – from the image, this triangle ABC has been made equilateral, as we wanted to make it. The demonstration of this is that lines AC AB are equal to one another, since both of them emerged from the centre of circle D up to its circumference; and so are lines BC and BA likewise equal to one another, since both of them likewise emerged from the centre of circle H up to its circumference. Behold, the given line AB is equal to each one of the lines AC AB. Therefore, the three sides of triangle ABC are equal to one another, as we stated.*[44]

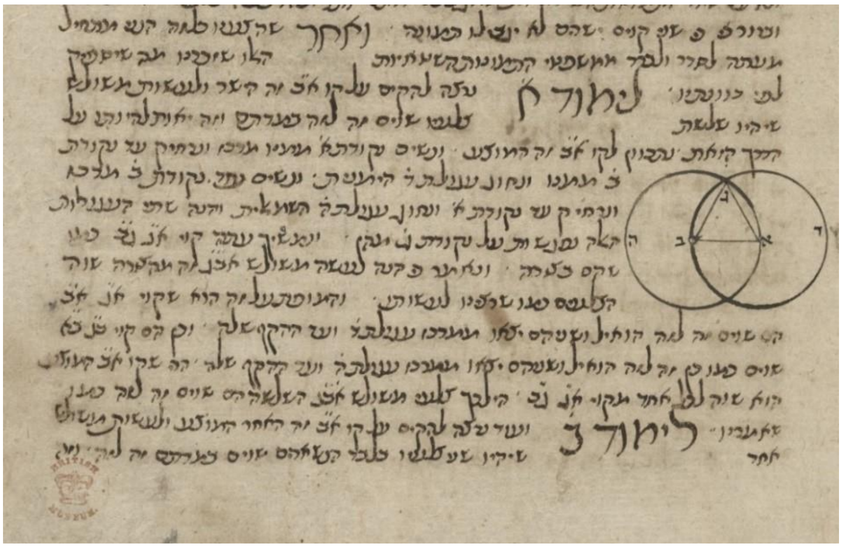

FIGURE 5

The construction of an equilateral triangle in *Yesod 'Olam*

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[44] Note that the orientation is a mirror image of what is found in Greek and Latin diagrams, this is the orientation common in Hebrew and Arabic sources.

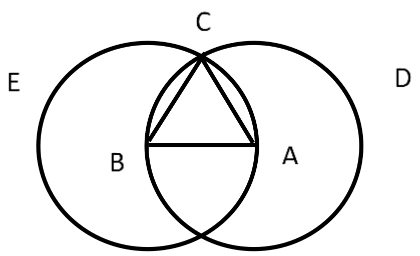


FIGURE 6

Lesson 1 in *Yesod 'Olam*

I wish to raise awareness to the following point: Israeli's rendition of I.1 in the *Elements* is in itself faithful content-wise. However, within the proof, Israeli implicitly uses preliminaries without having made them explicit in *Yesod 'Olam* in the way Euclid had done. For example, towards the end of the proof of Lesson 1, Israeli uses Euclid's Common Notion I: 'Things which are equal to the same thing are also equal to one another', but this had never been mentioned in *Yesod 'Olam*! In this context, it is important to remember that Euclid's text is not perfect, either. The British logician Bertrand Russell (1872-1970) criticized the *Elements* and pointed to some of the methodological as well as logical flaws in Book I, showing the exaggeration in the perception of the *Elements* as a masterpiece of logic. In particular, says Russell, in the proof of I.1, Euclid assumes, due to his reliance on the diagram, that the two circles constructed in the proof actually intersect, but the latter should have been stated as a Postulate, but had never been.[45]

ISRAELI'S LEXICAL INDEPENDENCE FROM EARLIER HEBREW SOURCES

I shall now illustrate the lexical independence of Israeli's rendition of the *Elements* in *Yesod 'Olam* of other earlier known medieval Hebrew treatises, which transmit the *Elements* in its entirety or in part. For the comparison, I have chosen Lesson 1 in Example 4 above.

Judging from the number of surviving Hebrew manuscripts, Euclid's *Elements* was probably one of, if not the most, popular mathematical works on the medieval Hebrew mathematical bookshelf. During the thirteenth century, it was translated in Provence from Arabic into Hebrew by Moses ibn Tibbon, Jacob ben Makhir ibn Tibbon, and a certain Rabbi Jacob, possibly Jacob Anatoli. Pedagogically similar to Israeli, but in a much more rigorous, broad, orderly and thorough manner, Judah ben Solomon ha-Cohen, also from Toledo, incorporated some of the Euclidean books (I-VI and IX-XIII) in the introduction to astronomy in his scientific encyclopaedia *Midrash ha-Ḥokhma*, originally composed in Arabic and then

45 Russell (1902).

translated into Hebrew by the author himself.[46] Table 3 includes a sample of the comparison between the lexemes used by Israeli and those by Moses ibn Tibbon, Jacob ben Machir ibn Tibbon and Judah ben Solomon ha-Cohen in the transmission of I.1:

**TABLE 3: Lexical comparison between *Yesod 'Olam* and earlier Hebrew translations of Euclid's *Elements***

| **The mathematical idea** | **Isaac Israeli's *Yesod 'Olam* [British Library, MS Add. 15977, Sephardic hand, 15c.]** | **Moses ibn Tibbon's translation of the *Elements*[47] [Mantua, Comunità Israelitica, MS ebr. 1, Italian hand 15c.]** | **Jacob ben Machir ibn Tibbon's translation of the *Elements* [Bodleian Library, MS Hunt. 16, Sephardic hand, 15c.]** | **Judah ben Solomon *ha-Cohen's Midrash ha-Ḥokhma* [Bodleian Library, MS Mich. 400, Byzantine hand, 15c.]** |
|---|---|---|---|---|
| To construct a triangle | להקים ולעשות משולש<br><br>*Lit*. to establish and make a triangle | להעמיד משולש<br><br>*Lit*. to erect a triangle | נעמיד משולש<br><br>*Lit*. we will erect a triangle | נעשה משולש<br><br>*Lit*. we will make a triangle |
| An equilateral triangle | משולש שיהיו שלשת צלעיו שוים זה לזה במדתם<br><br>*Lit*. a triangle whose three sides are equal to each | משלש שוה הצלעות<br><br>*Lit.* triangle of equal sides | משלש שוה הצלעות<br><br>*Lit.* triangle of equal sides | משולש שוה הצלעות<br><br>*Lit.* triangle of equal sides |

[46] For the complicated and not yet fully understood history of the Hebrew translations of the *Elements*, see Elior (2018a, 2018b, 2019) and Lévy (1996a, 1996b, 1997a, 1997b, 2000, 2005).

[47] Rabbi Jacob (Anatoli?) only lists the theorems, using identical terms to Jacob ben Makhir *sans plus*. Ofer Elior kindly informed me that creating lists of Euclidean Theorems was considered a memory exercise.

| | | | | |
|---|---|---|---|---|
| | other in measure [48] | | | |
| The given segment | הישר המוצע<br><br>*Lit*. the given line | קו ישר בעל תכלית מונח<br><br>*Lit*. a straight line with a laid end | קו ישר בעל תכלית מונח<br><br>*Lit*. a straight line with a laid end | קו ישר שיעורו ידוע<br><br>*Lit*. a straight line whose rate is known |
| We will draw circle D | נחוג עגולת ד'<br><br>*Lit*. we will create circle D[49] | נקיף על נקודת ג' וברוחק א'ב' עגולה יהיה עגולת ג'ד'ב'<br><br>*Lit*. we will set around point C at distance [radius] AB circle CDB | נקיף עגלת ג'ד'ב'<br><br>*Lit*. we will set around [the centre of] circle CDB | נחוק עגולת ג'ד'ב'<br><br>*Lit*. we will engrave/create the shape of circle CDB |
| Point C upon which the two circles intersect | והנה שתי העגולות האלה נפגשות על נקודת ג' מהן.<br><br>*Lit*. then these two circles meet at point C [which is] on [both of] them | נקודת ג' אשר עליה נתחתכו העגולים<br><br>*Lit*. point C upon which the two circles cut [past tense] each other | נקודת ג' אשר יתחתכו עליה שתי העגלות<br><br>*Lit*. point C upon which the two circles cut [present tense] each other | נקודת החציבות והיא ג'<br><br>*Lit*. the point of quarry/ intersection which is C |

**Comparison of Israeli's terminology with those of earlier Hebrew transmitters of the *Elements***

Table 3 shows great lexical differences between Israeli and the other authors in the case of more complex mathematical ideas. The few terms which are identical for all authors (משלש or משולש = triangle, עגלה or עגולה = circle, קו ישר = a [straight] line, מרכז = centre, מופת = demonstration) are rather basic and well-established scientific terms in the medieval Hebrew mathematical literature right across the board[50] so they cannot point to any textual dependence, only the more advanced notions can. This sample of comparative study shows beyond doubt that Israeli's rendition of Euclid's *Elements* is

[48] Later in the excerpt we find משולש שוה הצלעים *lit*. a triangle of equal sides.

[49] The verb לחוג already encompasses the meaning 'to create a circle' but requires a noun in the accusative.

[50] The exception being Abraham ibn Ezra from the twelfth century, who insisted upon calling centre מוצק.

lexically independent of the earlier Hebrew versions. Furthermore, Israeli's style, syntax and word order, as well as the glaring lack of order and adherence to the *Elements*, in contrast to the other sources, makes it safe to conclude that Israeli was unfamiliar with earlier Hebrew transmissions of the *Elements*. Judging from Israeli's language and structure of his sentences, Israeli's source must have been an Arabic one, but we do not know at this point which one it is, or even whether his source was a full Arabic translation of the *Elements* or an abbreviation thereof.[51] Israeli occasionally refers to the Arabic language. For example, in his teaching of a definition of a circle,[52] he says '…that point which we have discussed, directed in its middle [of the circle] will be called in the language of Hagar [Arabic] مركز [pronounced *markaz*, Hebrew מרכז pronounced *merkaz*] and it will be said about it that it is its centre [מרכזה *merkaza*]'.[53]

Israeli's lexical independence from earlier Hebrew translations of Euclid from Provence corroborates Ruth Glasner's findings regarding the lack of familiarity of Sephardic authors with Provencal Hebrew translations of the thirteenth century at least until the middle of the fourteenth century. Glasner analysed a Hebrew translation of *The Measurement of a Circle* by Archimedes, probably by Abner of Burgos/Alfonso of Valladolid, a Jew who converted to Christianity.[54]

## EVALUATING ISRAELI'S PARTIAL RENDITION OF THE *ELEMENTS* AND ISRAELI'S MATHEMATICAL AGENDA

As becomes evident throughout this article, Israeli does not transmit the Euclidean text in its entirety, or in a faithful manner. The order of appearance of Euclidean elements in *Yesod 'Olam* is not always consistent with the order found in the *Elements*, as can be discerned in the Tables 1 and 2. Occasionally,

---

[51] Ofer Elior has corroborated the results of my own research regarding Israeli's lexical independence from earlier Hebrew sources. In his current study of the Hebrew and Arabic transmission of the *Elements*, he has recently started to examine possible connections between Israeli's rendition of Book I of the *Elements* to the extremely intricate Arabic transmission thereof.

[52] As found in the *Elements*, Book 1, Definition 16.

[53] For more lexical examples of 'Arabicized' Hebrew mathematical terminology in Book 1 of *Yesod 'Olam* see Gad ben Ami Sarfatti, *Mathematical Terminology in Hebrew Scientific Literature of the Middle Ages* (Jerusalem, 1968), p. 216, as well as the lexicon in the book Israel Sandman and I are writing. It is important to note that Israeli did use Hebrew literary sources when he coined new mathematical terms, for example בקע (sine), see Sarfatti (1968), pp. 218–220 and Wartenberg (2012, ForthcomingA).

[54] See Glasner (2013). Further evidence for the lack of acquaintance with the Provencal Hebrew translations of the thirteenth century by a Sephardic author outside the Iberian Peninsula even as late as the end of the fourteenth century, can be found in the first known Hebrew treatise on algebra in Hebrew, *The Epistle of the Number*, written in Sicily at the end of the fourteenth century by the Castilian polymath Isaac ben Solomon ibn al-Aḥdab. He adduces two Euclidean common notions, and their formulation clearly manifests lexical independence from the Provencal Hebrew translations, see Wartenberg (2015), pp. 12, 218-219, 386-387.

Israeli adds some information to Euclidean enunciations, as in Examples 1 and 2, but most of the time he tends to incorporate abridged, simplified and less rigorous forms thereof, as seen in Examples 3 and 4. It is a priori easy to criticise Israeli's transmission of the *Elements*, for its partiality and even more so, for its lack of rigour. However, one must know that unlike other medieval Jewish (and other) transmitters of the *Elements*, Israeli did not declare any intention of transmitting the *Elements* as a solid body of mathematical knowledge and in a rigorous manner at all.[55] Furthermore, Israeli did not plan a meticulous and thorough mathematical teaching, as can be learned from the following excerpt from the introduction to Book 1. In it, he covers the two possible ways of learning mathematics, either by descriptive study or by explanatory study. Israeli clarifies that his teaching strategy is to deliver most of his mathematical teaching in a descriptive manner, which requires no proof. Only occasionally does he revert to explanatory study:

> *אמרו המשכילים כי לכל חכמה יש שני לימודים לימוד הראשון מהם הוא כגון שישאל התלמיד לרב לאמר מה הדבר הזה הנקרא כדור. אז יאות לרב ללמדו ולומ' לו כי היא התמונה הגופנית שתבניתה כך וכך ותשובתו זאת הוא הלמוד הראשון. ויקרא לימוד ציורי. וזה לפי שעל ידו יבין התלמיד וידע צורת הדבר ההוא ששאל עליו. ובזה הלימוד יש על התלמיד לקבל מן הרב ואין לו רשות לטעון ולחלוק עליו ולא לבקש ממנו שיביא לו ראיה וחיזוק לדבריו. והלימוד השני הוא כגון שיאמר לו הרב אחרי כן דע כי לזה הכדור ששאלת עליו ולעגולות החקוקות על גבו יש כך וכך משפטים ואותות. ובזה הלימוד השני אין על התלמיד לקבל מן הרב עד שיביא לו ראיות ברורות ויתן טעם לדבריו והבאת הראיות ההן וסידור הסברות הוא הלימוד השני הזה ויקרא לימוד ביאורי. וזה לפי שעל ידו תתבאר לתלמיד אותן האותות ותסור מלבו הספקות. עתה לפי שכוונתי בכאן כמו שאמרתי היא לחנך את התלמיד ולהרגילו ראיתי להביא רוב הלמוד והביאורים בספר הזה על דרך הלימוד הראשון ובכמה מקומות ממנו אביאנו גם לפי הלימוד השני ואחרי כן אם תשתוקק נפשו לרוות צמאונה ולמלאות חסרונה הנה לפניו ספרי הקדמונים אליו נתונים ישתדל ויקרב אל המלאכה וילמוד הכל משם כהלכה אע"פ שבכאן ימצא מזה כדי מה שיספיק לו עד שתנוח דעתו עליו.*
>
> *The erudites have said that every discipline has two types of studies. A first one of those studies is, for instance, when the student asks his master, 'What is this thing called a 'sphere'?' Then it is fitting for the master to teach and tell him that it is a corporeal figure the structure of which is thus and such. This answer of his is the first study; it is called 'descriptive study'. This is because by means of it, the student understands and knows the form of that thing about which he asked. In this study, it is incumbent on the student to accept what the master says; he neither has*

[55] In fact, neither the *Elements* nor Euclid are mentioned by Israeli, whereas he does mention other sources and authors throughout *Yesod 'Olam*. This can perhaps be explained by the fact that the *Elements* was considered such a rudimentary mathematical text in the Middle Age, that every learned person must have been familiar with.

> *permission to argue against or disagree with him, nor to request of him that he adduce for him a proof and a strengthening of his words. The second study is, for instance, when the master subsequently tells him, 'Know that this sphere about which you queried, and the circles inscribed upon it have thus and such laws and foundations.' In this second study, it is not incumbent on the student to accept what the master says until he adduces clear proofs for him, giving reason for his words. The adducing of those proofs and the arranging of the theories is this second study; it is called 'explanatory study'. This is because by means of it those items of foundation are explained to the student, and the doubts are removed from his heart. Now, since my intention here, as I stated, is to train the student and to habituate him, I saw fit to bring most of the study and explanations in this book in the method of the first kind of study; but in a several places of this book I bring it also according to the second kind of study. Subsequently, if his soul yearns to slake its thirst and to fill its lack, behold, before him are the books of the ancients given unto him.*[56] *Let him endeavour and draw close to the work [cf. Exodus 36:2], and learn everything from there according to its rules – although here he will find enough of it to suffice for him until his knowledge becomes settled within him.*

It is clear that Israeli's aim was to provide basic mathematical knowledge, mainly Euclidean geometry, that will enable the student to understand spherical trigonometry, and later, astronomy, which is at the centre of the work.

## CONCLUSION

In this article I have analysed Isaac Israeli's rendition of Euclid's *Elements* in *Yesod 'Olam,* illustrating some of its peculiarities through four examples, while highlighting novel linguistic findings within. The first two examples show Israeli's elaboration or re-arrangement of Euclidean elements whereas the third and fourth examples include a reductive, simplified and less rigorous rendition of Euclidean theorems. The comparison between the more specialized mathematical vocabulary in *Yesod 'Olam* and that found in earlier Hebrew transmissions of the *Elements* clearly shows lexical independence, indicating that Israeli was probably completely unaware of these earlier transmissions. This adds further evidence to the finding by Ruth Glasner regarding the ignorance of the Provencal Hebrew translations by Sephardic authors on the Iberian Peninsula until the middle of the fourteenth century. Israeli's source must have been Arabic – he often refers to the Arabic language and the syntactical structure of his phrases

[56] By 'the books of the ancients' Israeli may also refer to Euclid's *Elements*.

resembles Arabic. However, it is not yet known which Arabic source he used. In any case, Israeli seems to have used his sources selectively, choosing Euclidean elements suitable for his own pedagogical purpose of leading the reader to understanding spherical trigonometry en route to understanding astronomy. Israeli's partial and rather untraditional rendition of the *Elements* is, in spite of, or perhaps thanks to its methodological, structural and linguistic imperfections, a variegated and interesting Hebrew rendition of the *Elements*. Furthermore, it creates an additional layer in the history of Hebrew Euclid, whose entire story is yet to be told.

BIBLIOGRAPHY

Burnett, C. (2001). The Coherence of the Arabic-Latin Translation Program in Toledo in the Twelfth Century, *Science in Context* Volume 14(1) (pp. 249–288). Cambridge.

Elior, O. (2019). The Hebrew Translations of Euclid's *Elements* by Moses Ibn Tibbon and by Jacob Ben Makhir–A Study of Differences in Book I, *Journal of Semitic Studies*, Vol. 64, Issue 2 (pp. 481–506). Oxford.

Elior, O. (2018b). How to Construct an Isosceles Triangle: Variations of a Case Added to Euclid's Elements, in the Medieval Hebrew Tradition, *Aleph:Historical Studies in Science & Judaism*, Vol. 18(2) (pp. 159-180)/ Bloomington.

Elior, O. (2018a). The Arabic tradition of Euclid's Elements preserved in the Latin translation by Adelard of Bath and the Hebrew translation by Rabbi Jacob, *Historia Mathematica*, Volume 45(2) (pp. 111-130). Amsterdam.

Fishman, D. E. (1995). *Russia's First Modern Jews: The Jews of Shklov*. New York.

Galinsky J. D. (2006). Ashkenazim in Sefarad: The Rosh and the Tur on the Codification of Jewish Law, *Jewish Law Annual*, Vol. 16 (pp. 3-23). Jerusalem.

Glasner R. (2013). Hebrew Translations in Medieval Christian Spain: Alfonso of Valladolid Translating Archimedes? *Aleph: Historical Studies in Science & Judaism*, Vol. 13(2) (pp. 185-199). Bloomington.

Goldstein, B. R. and Chabás J. (2017). The Astronomical Tables of Isaac ben Joseph Israeli of Toledo, *Aleph: Historical Studies in Science & Judaism*, Vol. 18(2) (pp. 357-370). Bloomington.

Goldstein, B. R. (2009). Astronomy as a "Neutral Zone": Interreligious Cooperation in Medieval Spain, *Medieval Encounters*, Vol. 15 (pp. 159-174). Leiden.

Gomez-Aranda, M. (2008). The Contribution of the Jews of Spain to the Transmission of Science in the Middle Ages, *European Review*, Vol. 16(2) (pp. 169-181). Cambridge.

Heath, T. L. (1956). *Euclid, The Thirteen Books of the Elements*, Volumes 1-3. New York.
(an unabridged and unaltered republication of the second edition (Oxford, 1926)).

Lévy, T. (2005). 'Le manuscrit hébreu Munich 36 et ses marginalia : un témoin de l'histoire textuelle des Eléments d'Euclide au Moyen Age ', in D. Jacquart et C. Burnett (Eds.), *Scientia in margine.*

*Etudes sur les marginalia dans les manuscrits scientifiques du Moyen Age à la Renaissance*, (pp. 103-116). Geneva.

Lévy, T. (2000). 'Mathematics in the "Midrash ha-Hokhmah" of Judah ben Solomon ha-Cohen', in S. Harvey (Ed.), *The Medieval Hebrew Encyclopedias of Science and Philosophy* (pp. 300-312). Amsterdam.

Lévy, T. (1997b). Les *Eléments* d'Euclide en hébreu (XIIIe–XVIe siècles). In A. Hasnawi, A. Elamrani-Jamal, M. Aouad (Eds.), *Perspectives arabes et médiévales sur la tradition scientifique et philosophique grecque* (pp. 79–94). Leuven and Paris.

Lévy, T. (1997a). Une version hébraïque inédite des *Eléments* d'Euclide. In D. Jacquart (ed.), *Les voies de la science grecque* (pp. 181–239). Geneva.

Lévy, T. (1996b). La littérature mathématique hébraïque en Europe (XIe-XVIe siècles). In C. Goldstein, J. Gray, J. Ritter (Eds.), *L'Europe mathématique* (pp. 83-99). Paris.

Lévy, T. (1996a). Hebrew mathematics in the Middle Ages: an Assessment. In F. Rajep and S. Rajep (Eds.), *Tradition, Transmission, Transformation* (pp. 71-88). Leiden/New York/Köln.

Russell B. (1902). The Teaching of Euclid. *The Mathematical Gazette 2* (pp.165-167). Cambridge (UK).

Sarfatti, G. B. A. (1968). *Mathematical Terminology in Hebrew Scientific Literature of the Middle Ages*. Jerusalem (in Hebrew).

Stern, S. (2016). David Gans on the Gregorian Reform, Modern Astronomy, and the Jewish Calendar, *Judaica Bohemiae*, LI/2016 (1) (pp. 127-147). Prague.

Wartenberg, I. (ForthcomingA). The mathematical Hebrew language in treatises on the Jewish calendar in the Middle Ages. In E. Bar-Asher Siegal, D. Yaakov (Eds.), *Collections and Introductions in the Language* (The Academy of the Hebrew language). Jerusalem (in Hebrew).

Wartenberg, I. (forthcomingB). Mudejar decorations within calendrical, geographical, and mathematical tables and diagrams in a fifteenth-century Sephardic manuscript of Isaac Israeli's *Yesod Olam* from the British Library (Add. 15977). In Luís Urbano Afonso, Tiago Moita (Eds.), Sephardic Book Art of the fifteenth-century. Turnhout.

Wartenberg, I. (2018). La trasmissione italiana dello *Yesod 'Olam* (1309/10 e.v.) di Isaac Ha-Israeli. In M. Perani (Ed.), *Materia giudaica XXIII* (pp. 331-348). Florence.

Wartenberg, I. (2015). *The Epistle of the Number by Ibn al-Aḥdab; the transmission of Arabic mathematics to Hebrew circles in medieval Sicily*, in the series Perspectives on Society and Culture, Piscataway.

Wartenberg, I. (2014). 'The Naissance of the Medieval Hebrew Mathematical Language as Manifest in Ibn al-Aḥdab's Iggeret ha-Mispar'. In A Universal Art: Hebrew grammar across disciplines and faiths, Eds. Nadia Vidro, Irene Zwiep, Judith Olszowy-Schlanger (pp. 117-131). Leiden.

Wartenberg, I. (2012). The discovery of a fragment of Isaac ha-Israeli's *Yesod 'Olam* in the Cairo Genizah. *Zutot: Perspective on Jewish Culture* (pp. 51-58). Leiden.