Nuffield Foundation

Low attainment in mathematics:

an investigation focusing on Year 9 pupils in England

MAIN REPORT

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Note

This report is accompanied by a technical report which provides additional, detailed information about the methods and analysis used in the research project. It is available from www.ucl.ac.uk/ioe.

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Glossary of abbreviations

CAI	Computer-Aided Instruction
CEM	Centre for Evaluation and Monitoring
CSMS	Concepts in Secondary Mathematics and Science
DI	Direct Instruction
ESRC	Economic and Social Research Council
FSM	Free School Meals
GCSE	General Certificate of Secondary Education
ICCAMS	Increasing Competence and Confidence in Algebra and Multiplicative Structures
IDACI	Income Deprivation Affecting Children Index
IMAP	Investigating Mathematical Attainment and Progress
InCAS	Interactive Computer Adaptive System
ITS	Intelligent Tutoring Systems
KS1, KS2 etc	Key Stage 1, Key Stage 2 etc
LNRP	Leverhulme Numeracy Research Programme
MidYIS	Middle Years Information System
NPD	National Pupil Database
Ofqual	Office of Qualifications and Examinations Regulation
SES	Socio-Economic Status
Y9, Y5 etc	Year 9, Year 5 etc

Executive Summary

Aims and methods

This project investigated low attainment in mathematics by focusing on the lowest attaining 40% of pupils in Year 9 in England and addressing the following research questions:

- What mathematics do low attaining secondary pupils understand, and what are their particular strengths and weaknesses in number, multiplicative reasoning and algebra?
- Can low attainment be characterised simply as delay? If not, to what extent and in what ways do low attaining pupils understand mathematics in qualitatively different ways to high attaining pupils?
- To what extent do low attaining pupils' prior understandings of mathematics, and of particular mathematical topics, help to explain the existence of the attainment gap? What is the relative contribution of these mathematical understandings in comparison to socio-economic status and other demographic factors?
- What is currently known about the effectiveness of teaching strategies and approaches that address low attainment in secondary mathematics?
- To what extent is mathematics currently taught in appropriate ways for low attainers?

In order to address these questions, we used the following methods:

- Development of a new test of low attainers' mathematics knowledge which was administered to middle and high attainers in Year 5 (1050 pupils) and low attaining Year 9 pupils (2841 pupils), and matched to demographic and prior attainment data in DfE's National Pupil Database. Analysis focused on a subsample of these pupils with similar overall mathematics attainment.
- Analysis of a sample of 10,913 Year 7, 8 and 9 pupils who took part in the ICCAMS national survey in 2008 and 2009 which tested pupils' conceptual understanding of algebra, decimals, fractions and ratio with matched demographic and attainment data from the National Pupil Database (including these pupils' later performance at GCSE).
- A systematic review of the relevant literature on approaches to teaching low attaining secondary pupils.
- Interviews with 195 pupils and 12 teachers.

Findings

Attainment dominates

Prior attainment in mathematics is the strongest predictor of future attainment. All other factors (including gender, socioeconomic status, attitudes, etc) are very much second order. What pupils can learn appears to be largely predicted by what pupils already know.

Pupils whose attainment profile is mixed (ie low attainers on only one of the measures of prior attainment that we considered) have dramatically improved GCSE prospects over those who are consistently low.

No evidence of threshold concepts, but some evidence that low attaining pupils have practically important weaknesses in number and calculation

We searched for evidence that there are particular concepts and areas of mathematics that would be crucial determiners of future learning: particular subsets of mathematical knowledge or skill that unlock future progression. Despite several attempts to find such concepts, we have found no empirical evidence for any 'threshold' concepts that enable or prevent future learning.

We did identify some small, but statistically significant, differences between the two groups: the Year 9 low attaining pupils performed better on arrays and area, percentages and arithmetic recall, whereas the Year 5 middle and high attaining pupils were stronger on derived facts and selecting a calculation. Our findings about derived facts and selecting a calculation suggest some practically important weaknesses in the Year 9 pupils' calculational fluency, flexibility and application. Addressing these areas is important for these pupils' further mathematical development.

Despite these small differences in performance, our analysis is broadly consistent with a view of low attainment as largely characterised by delay rather than qualitatively differential performance. The Y9 low attainers seem to be broadly similar to matched middle and high attaining Y5 group in terms of the broad profile of things they know and can do; however their general mathematical progress is some four years behind.

Being in school matters

Two variables –exclusions and total absences – that capture school presence were found to be significantly related to GCSE outcomes, although the relationships were smaller than those for prior attainment. One additional day of exclusion from school, or nine additional days of absence over a school career, were each associated with a reduction of one grade in GCSE mathematics, after taking into account prior attainment in mathematics, English and science, SES and gender. We cannot say whether these are causal relationships, or whether schools can affect them, but they suggest fruitful areas for trying to develop interventions. In contrast, we found no effect for socio-economic status after controlling for prior attainment.

Some attitudes may matter a bit

Some of our questionnaire measures, for example self-efficacy and performance goal orientation, were associated with improved prospects at GCSE for low attainers. These relationships became much smaller, albeit still positive, when prior attainment and other factors were taken into account. We cannot say whether there is anything schools or other agents can do to influence these self-perceptions or attitudes and, if there is, whether it leads to better attainment. Nevertheless, we believe they may be worthy of future study.

Twelve effective and evidence-based strategies

Our systematic review of the literature found that most generally effective strategies are also effective for low attainers. We identified 12 evidence-based strategies that appear to be effective and of relevance for teaching low attaining pupils in England. All were found to have a moderate positive impact on attainment, except for prompts for self-instruction, which had a large impact, and computer-aided instruction, which had a small impact. However, the the evidence base for the different strategies varied considerably in terms of the number of original studies, the number and quality of the meta-analyses, the consistency of the effects as well as where, when and with whom the studies were carried out. We consider this to be more important than the precise size of the effects and, hence, the strategies are ordered from highest to lowest in terms of the security of the evidence base:

Strategy	Definition
Explicit teaching	A variety of teacher-led approaches consisting of crafted and/or partially scripted instruction together with structured practice, and usually involving feedback
Computer-Aided Instruction (CAI)	Computer-based systems designed to deliver all or part of the curriculum or to support the management of learning by providing assessment and feedback to pupils
Peer tutoring	Tutoring by same-age peers
Heuristics	Strategies for approaching a range of different problems
Manipulatives	Concrete materials that can be manipulated by pupils to aid understanding
Tutoring by adults	Group or one-to-one additional support provided by teachers or teaching assistants (TAs)
Feedback to pupils	Information provided to a pupil regarding performance or understanding
Representations	Diagrams, graphs and tools such as number lines (includes concrete-pictorial-abstract [CPA])
Feedback to teachers	Information on pupil performance or understanding provided to a teacher
Prompts for self-instruction	A set of prompts for pupils relating to a particular method or set of methods
Cooperative Learning	Pupils collaborate on a shared task in structured programmes, often in groups of mixed attainment
Student-centred learning	A range of pupil- (or student-) led or pupil-mediated approaches to learning, including guided learning approaches

We also found evidence to support the use of early intervention for pupils at risk of low attainment. In general, the effect of an intervention reduced as the duration increased, although frequency was associated with increased benefits. A further strategy, working memory training, was found to have no impact on mathematics attainment. We found only weak and inconsistent evidence for seven other commonly used strategies relating to their use with low attaining pupils. Our findings suggest that interventions directed exclusively at increasing motivation or improving attitudes are less likely to be effective than interventions focused more directly on improving attainment.

Teachers are focused on building positive relationships

Most teachers reported that they believed building positive relationships was an especially important strategy for low attaining pupils. It is, therefore, pleasing to report that we found that most low attaining pupils enjoyed their mathematics lessons and valued their mathematics teacher, even though they mostly reported finding mathematics difficult. One danger of this focus on building positive relationships is that some teachers felt it was important to building pupils' confidence by not allowing much, if any, struggle or failure. We emphasise, however, that several teachers recognised this danger and attempted to avoid it.

Wide support from pupils and teachers for scaffolding learning

There was a great deal of commonality in the teaching approaches that teachers described. All teachers stressed the importance of structuring and scaffolding learning and some teachers reported that this meant that planning a mathematics lesson for low attainers was substantially more work than planning a lesson for higher attainers. We found that pupils valued detailed explanations with methods broken down into small steps, which links to our finding about the efficacy of explicit and direct instruction, although this strategy generally emphasises understanding and conceptual coherence.

Teachers consider derived facts are poorly understood

Derived facts refers to the use of a known fact to solve a problem involving an unknown fact (for example, 8+8=16, so 8+9=16+1=17; 5×5=25, so 5×6=25+5=30; or, 19=90, so 5×18== 5×(2×9)=90). When asked about this during interviews, teachers endorsed the importance of being able to derive an unknown fact from a known one as an important aspect of fluency, though most teachers reported that they had not taught this skill explicitly and that their pupils rarely used it. Teachers also reported that pupils rarely estimated, although they perceived this to be the case for high attainers as much as low attainers.

Recommendations

Schools should focus directly on raising the attainment of low attainers

In order to raise pupils' later attainment, schools should primarily focus on raising their current attainment, rather than on other factors such as their attitudes, behavioural tendencies or home support. Even if these factors do causally influence attainment (and can be changed), their effects are likely to be much smaller than the impact of what those pupils have already learnt. Although learners' self-efficacy is important, there is no evidence that approaches focused solely on motivation, engagement or attitudes lead to improved attainment.

Teachers, training organisations and funders should develop scalable interventions and training based on promising evidence-based strategies

The 12 evidence-based strategies and approaches that we identified all have the potential to improve the teaching and learning of mathematics for low attaining pupils. In each case, there are important caveats. Most of the strategies had a moderate, rather than a strong, effect on attainment. The effectiveness of any strategy is highly dependent on the teaching; how teachers implement and use strategies makes a difference to their effectiveness. Moreover, none of these strategies should be used universally; time-limited interventions are more effective. Hence, we recommend that implementation is carried out thoughtfully and should include professional development opportunities for teachers to develop their pedagogic skills.

Specifically, we identify as most promising from this list:

Explicit teaching can be beneficial

There is particularly consistent evidence to support use of *explicit* teaching for low attaining pupils. *Explicit* teaching does not just mean logical explanations, or clear descriptions of step-by-step procedures, but includes something additional to what competent teachers in the UK normally do as part of lesson planning. *Explicit* teaching emphasises carefully constructed explanations and structured practice materials which have usually been designed and evaluated by expert teams, incorporating both conceptual and procedural aspects of knowledge. These approaches often take the form of at least partially scripted lessons and usually involve feedback. In the US, where many of these interventions originate, the approach is referred to either as 'direct instruction' or 'explicit instruction'. However, such *explicit* teaching is not a panacea and the effect on attainment was found to be only moderate. Indeed, a contrasting approach, *student-centred learning*, was also found to be effective and also had a moderate to large effect, albeit with a much weaker and less consistent evidence base. Further, research indicates that the strategy of *explicit* teaching should be employed alongside other approaches, including *problem-solving* and *collaborative work*.

Computer-aided instruction can be a valuable supplement to teaching

Although the effects on attainment are small, *computer-aided instruction* (CAI) has the potential to free up valuable teacher time, because the effects are similar to those for competent teaching. However, it should be used as a supplement to, rather than a replacement, for face-to-face teaching and appears to be most effective for the development of basic number and calculation skills and less effective for developing reasoning.

Tutoring by teaching assistants is more effective if structured and time-limited

Tutoring by teaching assistants is commonly used to support low attaining pupils. This is much more likely to be effective when structured and time-limited. Unstructured support by adults is not effective and can have negative effects.

Some 'low cost' strategies are likely to be effective

Some strategies offer potential benefits at little or no cost, and we suggest that these are worth experimenting with in the classroom. These include *heuristics*, methods for approaching and solving a range of different problems, and *prompts for self-instruction*, a set of prompts relating to a particular method or set of methods.

Feedback is a powerful strategy, but should be implemented carefully

Feedback, and the associated strategy of *formative assessment*, has received a great deal of attention in the UK and beyond over the past 20 years. Overall, there is moderate evidence to suggest that providing feedback has a moderate positive effect on attainment, although we found these effects to be smaller than claimed in previous reviews. There was less evidence available relating to *formative assessment*, but this limited evidence base suggested it was associated with only small effects. *Feedback* needs to be used carefully, because, when used inappropriately, feedback can have a strong negative effect on attainment. For low attaining pupils, it may be more important to use *feedback* simply to demonstrate and reinforce learning rather than additionally to communicate learning objectives or next steps. In contrast, feedback to teachers about pupil progress is likely to be more effective when feedback on pupils' progress is given alongside suggestions for appropriate goals and approaches.

A more consistent approach to using representations and manipulatives is needed

In general, teachers were positive about the value of *representations and manipulatives*, although many teachers appeared to lack a consistent approach to their use. Development of training, guidance and resources in this area could be particularly helpful.

Suggestions for further research

There is a striking need for further research in two areas:

- The teaching of specific topics in mathematics: This may be both surprising and disappointing to many teachers, because the issue of how best to teach different topics, particularly topics that pupils struggle with, is of significant practical concern.
- Effective, replicable interventions designed for the UK: There is a need to first develop and then evaluate specific, targeted and replicable interventions that are appropriate to the current UK educational context and can be implemented at scale, and to better understand effective ways of providing professional development to support teachers to implement such interventions. This should include adapting successful interventions from overseas.

Whilst we have highlighted research needs across all the strategies identified, we consider that, in the current context of significant shortages of mathematics teachers (e.g., ACME, 2018; Allen & Sims, 2018), the area of computer-aided instruction may have potentially strong benefits for raising attainment through freeing up valuable teacher time, and, hence, may contribute towards reducing teacher workload and addressing teacher shortages. But, as the Smith (2017) Review of Post-16 Mathematics recommends, research is urgently needed to examine how best to use such technology to support mathematics teaching.

Introduction: The need for this research

Low attainment is widely considered to be one of the most serious problems in education in England (e.g., Marshall, 2013). Low attainment in mathematics has been singled out as a particular issue, because of its importance to the economy and to individuals (e.g., Layard et al., 2002; Vignoles, De Coulon, & Marcenaro-Gutierrez, 2011) and has been a central concern of government policy for more than two decades (e.g., DfEE, 1997; DfES, 2007; Gibb, 2015; Moser, 1999). Yet, despite concerted efforts to address the problem, the problem of low attainment amongst secondary pupils appears to have got worse rather than better (OECD, 2013; see also Shayer & Ginsberg, 2009). Nationally representative data from the ESRC-funded project, ICCAMS: Increasing Competence and Confidence in Algebra and Multiplicative Structures), indicate that in England the proportion of very low attaining pupils at the end of Key Stage 3 (KS3) has roughly doubled since the 1970s. This group now constitutes around a sixth of the Year 9 cohort (Hodgen et al., 2010, 2012). These pupils have difficulty answering even basic questions about core ideas from the primary school mathematics curriculum. During Key Stage 3, the gap between the lowest and highest attaining pupils in mathematics increases and the low attaining pupils fall further behind other pupils. The problem goes beyond the very lowest attaining group in that a substantial proportion of pupils have difficulty with the basic concepts in secondary mathematics: approximately 40% in algebra and 65% in ratio. We believe this low attainment in mathematics to be one of the most serious and urgent problems currently facing our educational system.

This problem is exacerbated by the fact that the research evidence about mathematical low attainment is limited and fragmented. Indeed, there are relatively few studies in mathematics education that either investigate what and how low attaining secondary pupils understand in mathematics or demonstrate what approaches are effective at addressing these pupils' difficulties. Yet, such evidence is vital to informing interventions and policies that seek to address low attainment in mathematics.

Research aims and questions

Our overall aim is to understand the nature of low attainment in mathematics in lower secondary, to gather evidence on what mathematics pupils do know, to investigate how the attainment gap develops over time, and to review existing evidence on what teaching strategies and approaches are most likely to improve the problem.

Our focus is on the lowest attaining 40% of the Year 9 cohort who are likely not to achieve the new grade 4 at GCSE by age 16 (Ofqual, 2014). We limited the study to Year 9 as this is the year group on which we had already done significant analysis and because it is a critical year by which time secondary pupils' attainment has become more stable and which is strongly predictive of GCSE attainment, but before too much examination preparation has started.

Our specific research questions were:

- What mathematics do low attaining secondary pupils understand, and what are their particular strengths and weaknesses in number, multiplicative reasoning and algebra? [RQ1]
- Can low attainment be characterised simply as delay? If not, to what extent and in what ways do low attaining pupils understand mathematics in qualitatively different ways to high attaining pupils? [RQ2]

- To what extent do low attaining pupils' prior understandings of mathematics, and of particular mathematical topics, help to explain the existence of the attainment gap? What is the relative contribution of these mathematical understandings in comparison to socio-economic status and other demographic factors? [RQ3]
- What is currently known about the effectiveness of teaching strategies and approaches that address low attainment in secondary mathematics? [RQ4]
- To what extent is mathematics currently taught in appropriate ways for low attainers? [RQ5]

We address the first two research questions in Section 1, the third research question in Section 2, the fourth research question in Section 3, and the final research question in Section 4. In Section 5, Conclusion, we draw our findings together and consider the implications of our research.

1: Characterising low attainment

In this section, we address our first two research questions:

- What mathematics do low attaining Year 9 pupils understand and what are their particular strengths and weaknesses in number, multiplicative reasoning and algebra?
- Can low attainment be characterised simply as delay? If not, to what extent and in what ways do low attaining pupils understand mathematics in qualitatively different ways to high attaining pupils?

Background

Over the past decade, there have been a number of studies examining pupils' understanding of mathematics (e.g., Nunes et al., 2009; Ryan & Wiliams, 2007; Watson et al., 2013; Brown et al., 2008). However, with the exception of Finesilver's (2014) small scale study, research specifically investigating what low attaining pupils understand about mathematics, and the specific difficulties that they have, is either very dated (Denvir et al., 1982) or at primary (e.g., Dowker, 2009a, 2009b; Gifford & Rockliffe, 2012). It is not currently clear whether low mathematical attainment can be validly conceptualised simply as a form of global 'delay', whether low attainers have specific learning characteristics or difficulties which give rise to qualitative differences in their understanding of mathematics in comparison to middle and/or high attainers or whether there are particular 'threshold' concepts that appear to be particularly crucial for the secondary low attainers' mathematical progression (Meyer, Land & Bailie, 2010).

Based on a review of the literature on low attaining pupils' mathematical understanding, we hypothesised that the low attaining Year 9 pupils would have weaknesses in arithmetic (fact) retrieval (Geary, 2011), derived facts (Dowker, 2009a; Gray & Tall, 1994), number lines (Siegler et al., 2010; see also Bartelet et al., 2014; Schneider et al., 2018) and estimation (Dowker, 2015). Of these, the literature suggested derived facts and number lines to be the most likely candidates for threshold concepts, which are concepts that transform how a pupil thinks mathematically and key to further progression (Cousin, 2006; Meyer & Land, 2006).

Our focus was on the lowest attaining 40% of the Year 9 cohort, the group of pupils who are likely not to achieve the new grade 4 at GCSE mathematics by age 16. Our overall aim was to understand the nature of low attainment in mathematics in lower secondary and to gather evidence on what mathematics these pupils know. In doing this, we aimed to investigate whether low attainers' understandings of mathematics are qualitatively different from those of middle and high attaining pupils. We did this by comparing Year 9 low attainers to a group of middle-high attaining Year 5 pupils whose overall attainment was similar to the Year 9 low attainers.

Methods

Test development

In order to address this research question, we designed and validated a new computer-delivered test (the Investigating Mathematical Attainment and Progress [IMAP] test), designed specifically to assess secondary low attainers' knowledge of number, multiplicative reasoning and algebra.

The current study built upon our previous Increasing Competence and Confidence in Algebra and Multiplicative Structures (ICCAMS) research, which surveyed pupils using tests first developed in the Concepts in Secondary Mathematics and Science (CSMS) study and highlighted that the performance of the lowest attaining groups of Year 9 pupils had decreased over time in absolute terms (Hodgen et al., 2012). However, in the ICCAMS survey, the lowest attaining group of pupils was unable to answer most of the items, or questions, on the tests. As a result, the ICCAMS study provided much stronger evidence of what low attainers *could not* do than what they *could* do. Hence, a central aim of the current study was to design a test that would more fully capture what the broad group of low attaining Year 9 pupils *do* understand.

The IMAP test was designed to:

- be delivered via computers available in a wide range of schools;
- be completed by most pupils within a standard school lesson period (a maximum of 50 minutes);
- be recognisably related to the school mathematics curriculum as delivered to low attaining Year 9 pupils, in order to provide convincing, and easily interpretable, evidence for teachers and to provide the basis for a test that could be used more widely in schools;
- be able to discriminate, or distinguish different levels of attainment, within the broad group of low attaining Year 9 pupils (and include some items that all or most of the pupils would answer correctly). In practice, this meant that the test focused largely on the primary number curriculum and placed greater emphasis on whole numbers and pre-algebra than decimals, fractions and symbolic algebra;
- enable comparison with middle and high attaining Year 5 pupils;
- assess additional mathematical topics and concepts that might be key to further progression in mathematics: area/array (e.g., Barmby et al., 2009), place value (e.g., Brown et al., 2010) and ratio (e.g., Hart, 1984).

The test was trialled and validated through two main phases across five schools:

- *Trial 1* was conducted on Year 9 pupils across two secondary comprehensive schools (307 pupils), after which 20 items were dropped because of misfit, redundancy or difficulty.
- *Trial 2* was conducted on the Year 9 cohorts from two secondary comprehensive schools (370 pupils) and the Year 5 cohort from one primary school (56 pupils).

In addition, validation interviews were conducted with Year 9 pupils (N=20) and Year 5 pupils (N=6) in order to ensure that items tested the intended topics or concepts. This process indicated that all items were interpreted as intended and were aligned with the current curriculum. Some minor changes were made to wording in a small number of items.

We intended the test to be recognisably a test of the mathematics curriculum in general in order that the results would be accepted and easily interpretable by teachers, but also to enable comparison between groups of similar items within the test. In technical terms, this meant that we wanted the test to be broadly ‘unidimensional’. Rasch modelling confirmed the overall unidimensionality of the final test and good discrimination across the sample.[1]

The final test consisted of two broad elements:

- *The main Number, or mathematics, element [henceforth, IMAP Number]*: 61 items, focused largely on key aspects of the number, calculation and ‘pre-algebra’ curriculum. Items were presented online in a variety of formats (including multiple choice, free entry and sliders). We reported outcomes as a total score and as scores for subsets of items grouped by pre-designed topic (see below).
- *A separate timed Arithmetic fact retrieval, or recall, element [henceforth, Arithmetic recall]*: a speeded fact retrieval test, consisting of 30 items, with as many as possible to be completed in a total of two minutes. This is reported separately from the main IMAP score.

In addition, the test included two identical items assessing confidence, at the beginning and at the end of the test, in order to examine whether test anxiety might be a factor in low attaining pupils’ performance.

The mathematics items were mainly drawn and adapted from the previously validated Leverhulme Numeracy Research Programme (LNRP) Year 5 and Year 6 tests [40 items], together with 9 additional items adapted from items in the CSMS Algebra, Decimals, Fractions and Ratio tests[2] and 13 specially developed items. The items covered 10 topics: Areas and arrays, Derived facts, Estimation, Fractions, Integer calculation, Number lines, Percentage, Place value, Ratio, and Selecting an appropriate calculation, as summarised in Table 1. See the technical report (Appendices 1 and 2) for the text and origin of each item and a list of the 30 arithmetic recall items.

[1] Rasch modelling is a statistical approach to measurement that is commonly used to create and validate measures, or scales, and in the design and validation of tests.
[2] In addition, six items were used in both the LNRP and CSMS tests.

Table 1: An overview of the topics covered in the IMAP Number test

Topic	Number of items	Sample item
Arrays/area model	6	58: Here is a 12 by 10 array. How many dots are there?
Derived facts	7	19: Look at this calculation. 34+28=62. Find a quick way to work out the answer to 34+29
Estimation	2	45: Click on the number that is nearest in size to nought point one eight. [0.18: Decimals]
Fractions	11	34: Enter the missing number in the box. 1/3 = 2/?
Integer calculation	4	2: What is 12 more than 26?
Number lines	5	6: Look at the number line. What number is the arrow pointing to? [6230]
Percentage	4	14: 4 children out of the 100 children on a school trip forgot to bring their lunch. What percentage is this?
Place value	11	27: Enter a number that is larger than nought point six but smaller than nought point seven.
Ratio	6	47: A soup recipe for 3 people needs 8 carrots. How many carrots are needed for 9 people? [x3;=24]
Selecting an appropriate calculation	5	37: Pencils cost 18 pence each. What calculation would you do to work out how many you could buy for 90 pence? [Multiple choice]

Missing responses (i.e., no attempt) were coded as wrong for this analysis. A fuller analysis of the test can be found in the technical report (RQ 1 & 2).

Test administration

The test was administered during 2017 to 2841 Year 9 pupils from 25 secondary schools and to 1050 Year 5 pupils from 20 primary schools. Secondary schools were asked to administer the test to low attaining pupils, whilst primary schools were asked to administer the test to middle and high attaining pupils. The original intention had been to recruit secondary and primary schools that had used the Centre for Evaluation and Monitoring (CEM) MidYIS (Middle Years Information System) and InCAS (Interactive Computer Adaptive System) tests, respectively, in order to provide an independent measure of attainment. However, recruitment difficulties meant that this was not possible for all pupils.

In order to make valid comparisons between Year 9 low attainers and Year 5 middle to high attainers, we selected a matched subsample from each group, so that the two distributions of IMAP scores were the same. We carried out all of our analysis on these matched samples (which are referred to as the Y9 and Y5 matched samples, respectively). The matched samples comprised 759 pupils from each year group. Having matched the two groups on the basis of scores on the main IMAP test, the Y9s were slightly higher scoring on the Arithmetic fact retrieval, or recall, element of the test than the Y5s.

Results

We now report our findings under each of the two research questions that we addressed using the test.

What mathematics do low attaining Year 9 pupils understand and what are their particular strengths and weaknesses in number, multiplicative reasoning and algebra?

Table 2 gives an overview of the Y9 pupils' performance, overall and broken down by quintiles (within this low attaining sample).[3] In order to enable comparison across the topics, we present the percentage of items correct within each topic.

Table 2: An overview of the performance of the Y9 matched sample

Topic [Number of items]	Overall percentage correct	Percentage correct by quintile				
		Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
IMAP Number Overall [61]	46	34	56	67	79	90
Elements of the overall IMAP Number score						
Area & arrays [6]	44	34	53	62	71	86
Derived facts [7]	52	42	61	66	80	92
Estimation [2]	40	24	55	63	79	95
Fractions [11]	52	39	63	76	86	91
Integer calculation [4]	67	58	77	80	90	93
Number lines [5]	40	27	49	62	74	92
Percentages [4]	23	12	26	42	53	83
Place Value [11]	55	40	68	80	91	96
Ratio [6]	34	19	45	58	70	89
Select a calculation [5]	35	23	45	57	69	78
Arithmetic recall [30]	45	36	53	60	68	71

On average, pupils in the Y9 matched sample got just less than half the items on the IMAP test correct (46%). Relative to other topics, the hardest were the percentage items (23% correct). Aside from quintile 5, which is likely to be subject to ceiling effects, the profile for each quintile is broadly similar.

The matched samples enabled a comparison of the performance of a group of low attaining Y9 pupils with a group of middle and high attaining Y5 pupils, whose overall performance on the test was the same as the Y9 low attaining group. Figure 1 compares the two groups across the different topics.

[3] Quintile 1 is the lowest 20% of pupils within our sample, and Quintile 5 is the highest 20% of pupils within our low-attaining Year 9 sample.

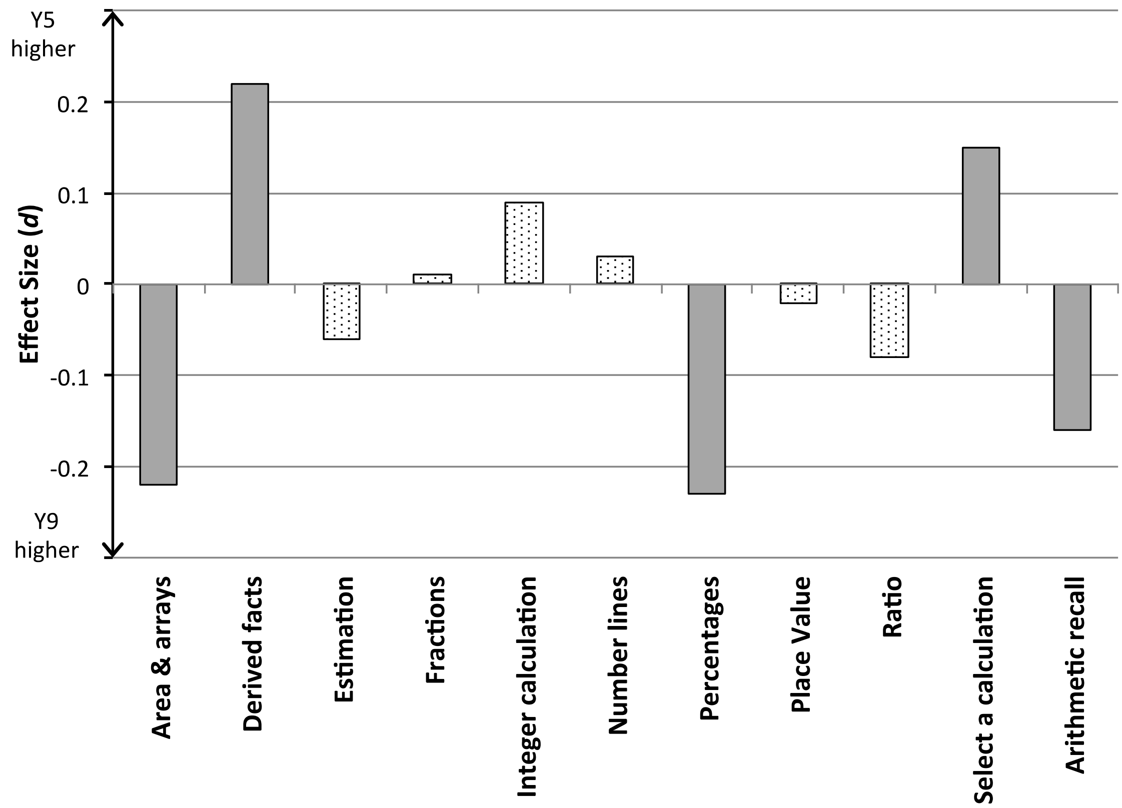


Figure 1: Comparison of Y5 and Y9 by topic showing effects sizes (Cohen's d). Effect sizes significantly different from zero are indicated with darker shading (p<0.05).

This comparison indicates that the matched Y9 sample performed better on arrays and area, percentages and arithmetic recall, whereas the matched Y5 sample was stronger on derived facts and select a calculation, although these differences were all small.[4] Effect sizes were not significantly different from zero for estimation, fractions, integer calculation, number lines, place value or ratio. An inspection of the six Area and arrays items indicated that the Y9 superiority resulted from the two numeric (dimensionless) area items and, in fact, the matched Y5 sample performed better on the remaining four array items, which are related also to derived facts (see Figure 2).

[4] We standardised these differences using an effect size known as Cohen's *d,* which is calculated by dividing the difference in means by the pooled standard deviation. Cohen (1988) categorises effects of d=0.2 as small. The effect sizes were as follows: in favour of the Y9 low attainers, arrays and area (d=0.22), percentages (d=0.23) and arithmetic recall (d=0.16); and, in favour of the Y5 middle and high attainers, derived facts (d=0.22) and select a calculation (d=0.15)

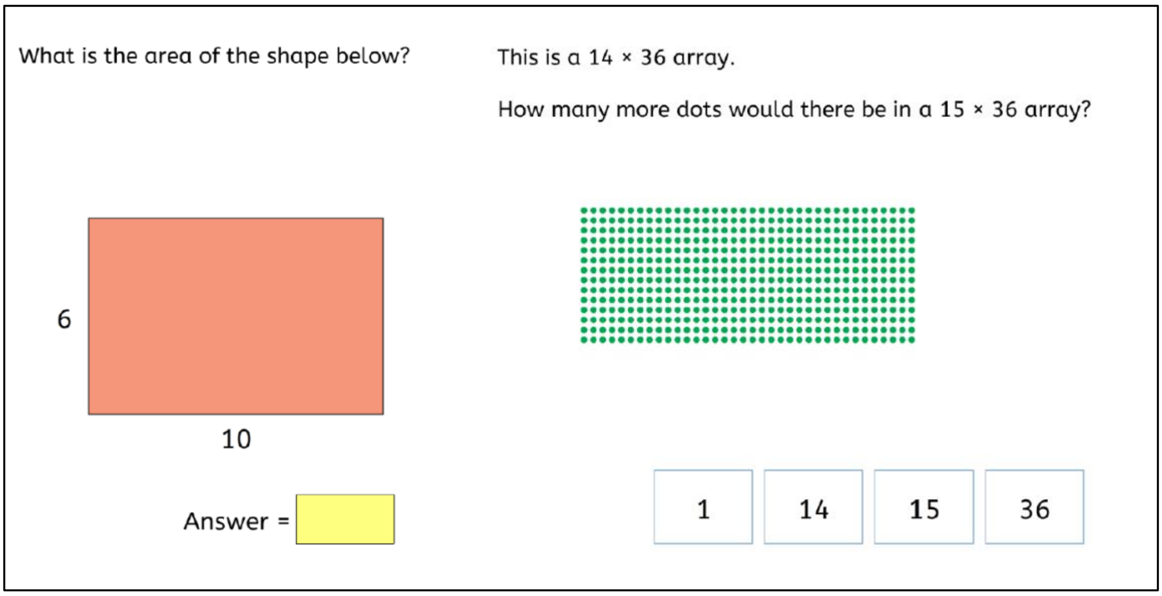


Figure 2: Examples of the area and array items. The item on the left (What is the area of a 6 by 10 rectangle) was taken from the CSMS Algebra test and assessed numeric, or dimensionless area. The item on the right was a multiple choice item specially developed for the IMAP test.

It is important to appreciate that in our design, in which the two groups were matched on the score on the IMAP Number element of the test, there is a compensatory effect (i.e., relative strengths on a topic are compensated by relative weaknesses elsewhere within the test). This does not apply to the Arithmetic recall element of the test, which was not used for matching.

The magnitude of the differences identified is probably too small to provide substantial evidence of potential threshold concepts, which are concepts that transform how a learner thinks mathematically (Cousin, 2006). In the next section, we consider this issue in more depth.

Can low attainment be characterised simply as delay? If not, to what extent and in what ways do low attaining pupils understand mathematics in qualitatively different ways to high attaining pupils?

In this analysis, we compared the Y9 and Y5 matched samples in order to investigate the extent to which the mathematical profiles of the two groups are similar or different, and whether any differences are sufficient to be classed as potential threshold concepts.

Threshold concepts (Meyer & Land, 2006) are:

- *transformative*, completely altering your outlook on a subject;
- usually *irreversible;*
- *integrative*, exposing connections between concepts;
- at the *boundary* with other concepts;
- '*troublesome*'; i.e., counter-intuitive, inhibited by 'common sense' understandings.

In short, threshold concepts could be key to understanding why some pupils make slow progress in mathematics.

To explore whether we had evidence for different profiles of relative strengths and weaknesses for the Y9s and the Y5s we used several different analytic approaches as described in the technical report (RQ1 & 2).

We found that the items in the test split statistically into the six components, or factors, as shown in Table 3.[5]

Table 3: The six-component structure of the IMAP Number test

Component	Examples of items
Ratio	47: A soup recipe for 3 people needs 8 carrots. How many carrots are needed for 9 people? 49: A soup recipe for 3 people needs 2 onions. How many onions are needed for 9 people? 56: These two ticks are exactly the same shape. Find the length of the red part.
Area	54: What is the area of the shape below? (3 by 4 gridded rectangle) 55: What is the area of the shape below? (6 by 10 un-gridded rectangle) 34: Enter the missing number in the box. 1/3 = 2/?
Number Lines	6: Look at the number line. What number is the arrow pointing to? 8: Look at the number line. Click and drag the arrow so that it points to the number six thousand and twenty-five. 9: Look at the number line. Click and drag the arrow so that it points to the number six thousand one hundred and eighty.
Derived Facts	20: Look at this calculation. 86+57=143. Find a quick way to work out the answer to 57+86 21: Look at this calculation. 86+57=143. Find a quick way to work out the answer to 860+570 19: Look at this calculation. 34+28=62. Find a quick way to work out the answer to 34+29?
General number	11: What is 1 less than 200? 51: What is half of 16? 1: A shirt costs £20. Alex buys 3 shirts. How much does this cost?
Whole number bias[6]	43: Click on the larger fraction. 3/7, 5/7 42: Click on the larger fraction. 1/4, 3/4 44: Click on the larger fraction. 3/5, 3/4 [Negative loading]

Crucially for our investigation, we found very similar component structures for the Y5 (high attaining) and Y9 (low attaining) matched comparison groups. We did not find any further evidence of substantive differences in the relative strengths of the two groups in their responses to a range of mathematical topic areas and skills within the IMAP Number test.

As noted above, the Y9 low attaining group performed better than the matched Y5 group on arithmetic fact retrieval. This was somewhat surprising, since Geary's (2011) review suggests that difficulties with factual recall differ across the attainment range. We investigated this by

5 We carried out a principal components analysis (PCA). We found 34 of the 61 IMAP Number items to be associated with these six components. The remaining items were not sufficiently strongly associated with any one component. PCA is closely related to factor analysis and is a method for describing variability in terms of a number of components (or factors). See the Technical Report for further details.

6 Whole number bias refers to the tendency to focus on the separate parts of the fraction, numerator and denominator, rather than the magnitude (Braithwaite & Siegler, 2018). For example, items 42 and 43 can be answered correctly simply by comparing the numerators of the fractions. Item 44, 3/5 compared to 3/4, where the denominators of the fractions are different, was negatively associated with the whole number bias factor. In other words, an *incorrect* response to this item was more strongly associated with the whole number bias factor, whereas *correct* responses to items 42 and 43, where the denominators of the fractions are the same, were more strongly associated with the whole number bias factor.

comparing the performance on the main IMAP test (on which the Y5 and Y9 samples were matched) and the arithmetic fact recall test. The relationship between scores on these two tests is similar for both year groups. There is some, rather weak, evidence to suggest that some of the highest attainers within the low attaining Y9 group are relatively strong on arithmetic recall. It may be that pupils categorised as low attaining on entry to secondary school are provided with a mathematical diet strongly weighted towards arithmetic, leading to some general improvement on arithmetic, possibly at the expense of other mathematical areas.

We also investigated the relationship between specific subsections of the IMAP Number test, including the Derived facts items, the Number lines items and the Estimation items , and either the total on the rest of the IMAP Number test or the score on the Arithmetic recall test, in order to examine whether there were any differences within the two groups. For most of the comparisons that we made, we found that the relationships between different sections and subsections were very similar for Y5 (middle and high attaining, relative to their cohort) and Y9 (low attaining, relative to their cohort) pupils.

Summary: Understanding low attainment

Overall, we did not find evidence that performance in any particular topic had a special place in explaining performance in other areas of mathematics. Moreover, this held when each of the two year groups were analysed separately. Our hypothesis was that weaknesses in those key threshold concepts might explain why low attainers had failed to progress. If we had discovered such differences, they could well have led to clear recommendations for curriculum and teaching: ‘address those blockages and progress will be improved’. However, we have been unable to identify evidence for any such ‘threshold concepts’.

Nevertheless, we did identify some small, but statistically significant, differences between the two groups: the Y9 low attaining pupils performed better on arrays and area, percentages and arithmetic recall, whereas the Y5 middle and high attaining pupils were stronger on derived facts and select a calculation.

We have failed to find evidence for a special place in mathematics learning progressions for the understanding of concepts such as number lines or derived facts as suggested by previous research (Dowker, 2009a; Gray & Tall, 1994; Siegler et al., 2010).

2: Understanding the factors associated with low attainment

In this section, we address our third research question:

- To what extent do low attaining pupils' prior understandings of mathematics, and of particular mathematical topics, help to explain the existence of the attainment gap? What is the relative contribution of these mathematical understandings in comparison to socio-economic status and other demographic factors?

For this analysis, these broad research questions were reframed into sub-questions as follows:

- What factors determine success in GCSE mathematics?
- Are some areas of mathematics particularly crucial?
- Can low attainment be overcome?

Background

There are a number of studies that demonstrate an association between attainment and socio-economic status (SES) (e.g., Cooper & Dunne, 2000; Gorard et al., 2012) and between later progression and early mathematical attainment (e.g., Duncan et al., 2007; see also Sylva et al., 2014), but the relative contributions of prior mathematical attainment in comparison to SES and other demographic factors is poorly understood. Morever, few studies have used measures of mathematical attainment that are sufficiently fine-grained to distinguish the impact of particular mathematical topics. One notable exception is Siegler et al.'s (2012) analysis of the 1970 British Cohort Study dataset, which finds performance at age 10 in fractions and division to be associated with later performance in mathematics overall at age 16. Siegler et al.'s study is limited, however, because the instrument used to assess mathematical performance at age 10 is focused on mathematics procedures rather than conceptual understanding.

Our study addressed this gap by drawing on data from a large nationally representative survey data carried out as part of the Increasing Competence and Confidence in Algebra and Multiplicative Structures (ICCAMS) study and linking this to demographic, prior attainment and further attainment data held in the National Pupil Database.

Methods

The sample analysed here is the 10,913 pupils who took one or more ICCAMS tests in Y7, 8 or 9 in 2008 or 2009 as part of the ICCAMS study. Pupils were randomly allocated to take two of the three ICCAMS tests (Algebra, Number, Ratio) and an attitude questionnaire.

These pupils were matched with data held in the National Pupil Database, including their KS1 levels, KS2 levels, KS3 teacher assessments and KS4 (GCSE) results. We also included several variables captured in the annual school census, with data for each term of their school career: Free School Meals (FSM) entitlement, Income Deprivation Affecting Children Index (IDACI), recorded absences, and number of days excluded. Other variables held in the NPD were not available to us for this analysis, including English as an Additional Language (EAL), Special Educational Needs (SEN) status and ethnicity.

Items from the ICCAMS attitudes questionnaire were grouped statistically to produce the following three scales:

- Self-efficacy: a person's confidence and belief in their own ability in mathematics, not finding mathematics too hard or feeling too anxious about it.
- Intrinsic enjoyment: enjoyment of mathematics for its own sake, particularly enjoying being challenged and having to work.
- Performance goals: the extent to which a person is motivated by manifest achievement, particularly in comparison to others.

We examined first the relationships between attainment at GCSE mathematics (KS4 result) and each of the variables for prior attainment, demographic factors and attitude. However, viewed individually, these relationships can be misleading, because we had not controlled for any inter-relationships between the different variables. Hence, we then conducted regression analysis.

Full descriptive statistics for these variables, their interactions and how the analysis was conducted can be found in the technical report (RQ3).

Results

What determines success in GCSE mathematics?

One way to explain and understand the attainment gap is to find predictors of attainment. If we can predict an attainment outcome, then we can say what characteristics are associated with doing well – or less well – on it. Factors that strongly predict who does well may help to explain why some do better than others.

The strongest single predictors of attainment in GCSE mathematics are KS3 mathematics teacher assessment level, ICCAMS test scores and KS2 mathematics level. Other prior attainment measures (KS3 and KS2 levels in science and English, KS1 average level) are also strong predictors, but less so.

The following had moderate predictive power: the number of recorded absences (totalAbsence), self-efficacy (seffic), number of half-days excluded from school, (exclusions), number of terms eligible for free school meals (fsmCount), performance goal orientation (perfgo) and IDACI (idaciMean: an index of deprivation based on pupil's home postcode, recorded each term and averaged over all recorded values).

Other variables, including intrinsic motivation, the number of schools attended, the amount of variation in recorded IDACI scores and gender, had no correlation with GCSE mathematics grade.

Many previous studies find socio-economic status (SES) variables to be a predictor of academic outcomes, and our findings confirm this relationship, albeit with a lower correlation than sometimes found. However, once we controlled for prior achievement using regression analysis, the additional predictive power of SES dropped to close to zero. This suggests that the impact of SES is already captured in those prior attainment measures, but also that the additional progress made from KS2 to GCSE is typically not related to socio-economic status.

Similarly, we found gender to be only very weakly related to either outcomes or progress to GCSE.

We found that a combination of a subset of prior attainment variables, KS3 mathematics teacher assessment, ICCAMS test score and KS2 mathematics level, provided a very strong prediction of GCSE grade in mathematics. KS1 level had almost no predictive power once KS2 was included.

Among the remaining, non-cognitive, variables that retain substantial predictive power once the attainment variables are included are exclusions and absences. We found that, on average, one additional day of exclusion from school, or nine additional days of absence over a school career, were each associated with a reduction of one grade in GCSE mathematics. This was after taking into account other factors, such as prior attainment, SES and gender. However, it is important to stress that we have no evidence that these are causal relationships: factors that we have not measured could have caused both poorer GCSE performance and a propensity for exclusion or absence. Nevertheless, the strength of the relationship suggests a need for further research to investigate whether there are things schools could do to reduce exclusions and absences, and, if so, whether increased GCSE results might ensue.

Somewhat smaller relationships, but still above the $p=0.05$ threshold for statistical significance, were found with two of the attitudinal variables, self-efficacy and intrinsic motivation, and one further demographic factor, the number of schools attended.

Are some areas of mathematics particularly crucial?

To answer our second sub-question, we assessed the strength of relationship between each individual question in the ICCAMS tests and pupils' subsequent GCSE mathematics grade.

We looked for any items or groups of items which correlated more strongly than we would expect with GCSE score, but we did not find any sufficiently large differences between correlations to draw conclusions from. From this, it is not clear that there are particular mathematical topics, skills or items that punch above their weight in predicting subsequent GCSE performance. This means that we do not have any evidence of any specific items having particular predictive power for GCSE outcomes, over and above what is already captured in their prior attainment, attitudes, gender, absence and exclusions.

This is probably not surprising, given the weight of information in the combined predictors compared with that in a single item. However, our hypothesis was that there might be certain crucial elements of mathematical knowledge or competence that were a barrier to or an enabler of further learning. If this was the case, we might expect to see an additional effect in the form of different outcomes for those pupils who had mastered that learning in KS3 from those who had not. Controlling for all the other predictors is necessary here in order to rule out the competing explanation that those differences are just a reflection of general mathematical attainment, since higher attaining pupils will both be more likely to get an item correct and to be successful at GCSE. However, it does make this a tough test for the existence of threshold concepts, particularly when, as here, those concepts are represented by a single item in the test.

Can low attainment be overcome?

Our third sub-question focuses specifically on low attainers. We looked at pupils who were low attaining in mathematics at different points in their school career and estimated their chances of achieving a grade C or above at GCSE. We then investigated how these chances may be different for pupils who differed on any of our additional variables.

Pupils in the lowest 10% of attainment, according to their KS3 teacher assessment in mathematics, had only a 7% chance of achieving C or above at GCSE, whereas, for those in the top 30%, over 99% achieved C or above. The prospects for pupils who were low attaining at KS1 (i.e. aged 7) were not quite as firmly determined, though the relationship was still strong: 23% of those in the bottom 10% went on to achieve C or above, rising to 98% for those in the top 10%. This shows that low attainment in the early years of school can be, and was, overcome by some pupils.

We now address the question of whether our predictions of which pupils will go on to achieve A*-C in GCSE mathematics can be improved by including other relevant information, such as their score on another assessment, or demographic data.

We found that being in the top half on some measures offered a very different prospect from being in the bottom half. For example, pupils whose scores on any of the ICCAMS tests put them in the top half had a better than 80% chance of going on to achieve C or above at GCSE, even though their KS1 levels were in the bottom 26% of the cohort. This compares with chances of around 30% of gaining C or above for those whose ICCAMS scores were in the bottom half. A similar difference was observed for performance in mathematics at KS2 and KS3, and a slightly smaller effect for performance in science and English: again those in the top half on these measures had around a 70-80% chance of success, while for those in the bottom half it remained around 30%.

For the demographic and attitude variables, the picture was more mixed. First of all, none of these variables made as much difference to the chances of getting C or above as the attainment variables did. Nevertheless, for some of these variables, there was a significant difference. In particular, a pupil’s self-efficacy and the number of absences were associated with differences of 25 and 20 percentage points, respectively, in the chances of getting C or above between those in top and bottom halves on these measures. The number of terms that pupils were eligible for FSM, the number of half-days excluded and the performance goal orientation (explained above) were each associated with differences of around 10 percentage points.

Summary: Understanding the factors associated with low attainment

We found that a combination of a subset of prior attainment variables, KS3 mathematics teacher assessment, ICCAMS test score and KS2 mathematics level, provided a very strong prediction of GCSE grade in mathematics. KS1 level had almost no predictive power once KS2 was included.

Among the remaining, non-cognitive, variables that retained substantial predictive power after taking into account prior attainment, SES and gender were exclusions and absences. We found that, on average, one additional day of exclusion from school, or nine additional days of absence over a school career, were each associated with a reduction of one grade in GCSE mathematics. We also found smaller, but still significant, relationships with two of the attitudinal variables, self-efficacy and intrinsic motivation, and one further demographic factor, the number of schools attended.

We found no effects for SES meaning that SES was not associated with progress from KS2 to GCSE. Similarly, we found only very weak effect for gender.

Whilst the ICCAMS overall test score, and each of the individual tests (Algebra, Decimals and Ratio) were found to be associated with GCSE, we found no evidence to suggest that any individual items had strong predictive power for GCSE outcomes in mathematics.

We found evidence indicating that some pupils, who were low attaining at KS1, did overcome low attainment. However, whilst we found some small associations between GCSE and self-efficacy and number of absences, the most significant factor in this achievement was subsequent performance in mathematics and to a lesser extent English and science.

3: Systematic review of teaching approaches

In this section, we address the following research question:

- What is currently known about the effectiveness of teaching strategies and approaches that address low attainment in secondary mathematics?

Background

There have been a number of relatively recent 'best evidence' reviews examining the teaching and learning of mathematics in general (e.g., Nunes et al., 2009; see also Conway, 2005; Kilpatrick et al., 2001). More recently, there have been systematic reviews examining effective interventions for raising the attainment of disadvantaged pupils in general (Dietrichson et al., 2017), and, in mathematics, pupils across the attainment range in primary (Simms et al., Forthcoming), upper primary / lower secondary (Hodgen et al., 2018), or with special educational needs (e.g., Kroesbergen & Van Luit, 2003). Additionally, some systematic reviews have examined the effectiveness of particular strategies or approaches (e.g., Carbonneau et al., 2013). But, there has not been a systematic review examining the evidence as a whole on the effectiveness of teaching interventions and strategies aimed at raising the mathematical attainment of low attaining secondary pupils.

We addressed this gap by conducting a systematic review of the literature focusing on the evidence about the effectiveness of different teaching strategies and approaches appropriate for the teaching of mathematics to low attaining pupils in lower secondary (ages 11-14). Our aim was to summarise the current evidence on teaching mathematics, drawing mainly on evidence from experimental studies, in order to make recommendations about effective ways of teaching mathematics to low attaining secondary pupils, as well as to identify areas to which future research might be profitably directed.

Methods

Our data set consists of a total of 107 items: 76 meta-analyses and 31 other relevant papers (mainly systematic reviews), written in English and published between 1970 and August 2018. These were identified using searches of electronic databases, the reference lists of the literature obtained and our own and colleagues' knowledge of the research literature. The original search was conducted in 2016, then updated in February 2017 and again in September 2018. This constitutes a substantial database summarising an extensive set of experimental studies (n≈3000).

Methods of synthesising the literature

The literature on effective strategies and approaches in mathematics is extensive and it would not be possible to directly review all of this literature. Instead, we focused on existing systematic reviews, which enabled us to build on existing summaries and syntheses of the primary evidence base, and, thus, synthesise a much more extensive range of literature than would otherwise be possible. Specifically, we conducted a second-order meta-analysis (synthesis of a collection of meta-analyses) by reviewing meta-analyses of the existing literature relevant to the teaching of low attainers in secondary mathematics.

Meta-analysis is a technique for combining the results from a set of related experimental studies in order to arrive at an overall conclusion that is more secure than that possible from any individual study. For example, two well-known practitioner-focused syntheses of the research evidence in education, John Hattie's (2011) *Visible Learning* and the Education Endowment Foundation's (2019) *Teaching and Learning Toolkit*, are based on second-order meta-analyses. For further information, see Higgins (2016; 2018).

Where possible, we have aggregated, or combined, the effect sizes from the meta-analyses to produce an overall estimate of the effect of a strategy. Effect sizes are a method of quantifying the difference between two groups (Coe, 2000). In particular, effect sizes can be used in experimental studies to calculate the impact, or effect, of an intervention by comparing the results of the treatment and control groups. In educational experiments, the most commonly used effect size measure is Cohen's d, which is calculated by dividing the difference in the means of the two groups by the standard deviation.[7] This process of standardisation enables effect sizes to be compared across different studies, although this must be done with caution.

Some researchers and research-informed tools convert standardised effect sizes such as these into more easily understood values (e.g., see Hattie, 2011). For example, the Education Endowment Foundation toolkit equates an effect size of d=1 to approximately a year's normal progress in learning (see Higgins & Katsipataki, 2016, for a discussion). Whilst these rules of thumb can be valuable in communicating statistical results to a non-technical audience, they need to be treated with some caution, because they are based on an extrapolation from the settings where research is conducted to schools and classrooms more generally. However, effect sizes are better regarded as indicative of the relative impacts of different strategies rather than as precise estimates of actual impact. In this report, we categorise effects as negligible, small, moderate or large.[8]

Coding

Each paper was coded and details were recorded, including the content area, strategies addressed, research questions, effect sizes and associated statistics, and any additional pedagogic or methodological factors considered.[9] We also assessed the methodological quality of each meta-analyses using an approach developed by ourselves.[10]

For each of the strategies identified in the review, we assessed the security of evidence supporting its use and effectiveness using a judgement-based approach.[11] Two members of the

[7] Another similar, and commonly used, effect size measure is Hedges' g, which is similar to Cohen's d, but includes a correction for bias in studies with small samples.
[8] For these categories we broadly follow Cohen's (1988) rule of thumb and we classify effects of below d=0.05 as negligible, up to d=0.25 as small, of 0.25< d <0.75 as moderate and of d=0.75 or greater as large.
[9] We also recorded the year of publication, author key words, abstract, key definitions, number of studies and number of pupils, age range, countries in which studies were conducted, study inclusion dates, and the inclusion/exclusion criteria.
[10] Our methodological quality criteria was informed by the PRISMA framework for rating the methodological quality of meta-analyses (http://www.prisma-statement.org/) and the AMSTAR criteria (Shea et al., 2009).
[11] Our strength of evidence judgments were based on the GRADE system in medicine (Guyatt et al., 2008), which is an expert-judgment-based approach that is informed, but not driven, by quantitative metrics and takes account of the number of original studies and meta-analyses, where, when and with whom the studies were carried out, the methodological quality of the original meta-analyses, consistency of results and any reporting, or publication, bias. Since some of the original research was either dated or conducted outside England, we also assessed any threats to the directness of, application of the results, to the teaching of low attaining pupils in England.

research team independently made a judgment for each strategy. Differences in ratings were resolved through discussion. See the Technical

A serious problem that we faced was that there is little consistency in the definition of low attainment in the literature. Indeed, we found only one meta-analysis which specifically referred to "low attaining" pupils in its title (Baker et al., 2002). Hence, our study includes meta-analyses (and systematic reviews) relating to "strugglers", pupils (or students) with mathematical learning difficulties or learning difficulties generally, "persistent low attainers", pupils "at risk" of underachievement and disadvantaged pupils. In order to reflect this, we adopted an "onion" approach (Coffield et al., 2004), whereby literature was coded into three categories: directly and highly relevant to the teaching and learning of mathematics for low attaining pupils (35 meta-analyses), highly relevant (13 meta-analyses), or relevant as background or supplementary evidence (59 studies). Only the first two categories were considered suitable for inclusion in any quantitative meta-analysis.

Further details on the searches, the inclusion and exclusion criteria and the methods of analysis are available in the technical report together with the full dataset and examples of excluded meta-analyses (RQ4 and Appendix 5).

Limitations

Whilst our second-order meta-analytic approach has several advantages, there are also limitations. We are dependent on the theoretical and methodological decisions that underpin the existing meta-analyses. In examining a strategy such as explicit instruction, meta-analyses generally group together similar, but not identical, approaches with similar, but not identical, measures of effect (or impact). Inevitably some nuance is lost in the focus on the "big picture". We have attempted to mitigate this limitation by providing a narrative as well as a quantitative synthesis of the literature. However, we note that there is an active debate on the validity of first- and second-order meta-analytic techniques in education (e.g., Coe, 2019; Higgins, 2018; Simpson, 2017).

Our approach is also reliant on the academic interests and choices of other researchers both in conducting primary studies and in synthesising these in meta-analyses. There may be evidence from primary studies in an area that has not yet been synthesised in a meta-analysis. We attempted to mitigate this limitation by searching for and including additional relevant and recent systematic reviews to supplement the meta-analyses.

Effect sizes are influenced by many factors, including research design, the nature of outcome measures or tests, and whether a teaching approach was implemented by the researchers who designed it or by teachers. Meta-analyses of the highest quality use techniques such as moderator analysis to examine whether these and other factors affect the magnitude of the effect sizes. We attempted to mitigate this limitation through our judgements about the strength of evidence and the methodological quality of the original meta-analyses.

Results

We found evidence relating to the effectiveness of a total of 20 broad teaching strategies and approaches. These are detailed in Table 4. For the 12 of these strategies, the evidence was judged sufficient to conduct a secondary meta-analysis by calculating an overall effect sizes (indicated in bold in Table 4). A further strategy, working memory training, was examined by two meta-analyses, which found that working memory training had no impact on mathematics attainment (Melby-Lervåg & Hulme, 2013; Schwaighofer et al., 2015). For the seven remaining strategies, the evidence was weak and/or inconsistent.

The overall effect for the 12 strategies with sufficient evidence are presented in Table 5, together with our judgements about the strength of the evidence and its relevance to UK mathematics classrooms. We found positive effects for all 12 of these strategies, ranging from a small effect for computer-aided instruction to a large effect for self-instruction, although most strategies were found to have a moderate effect. In our judgment, these effects are all of practically important magnitude and, with some caveats, all the strategies have potential benefits to raise the mathematical achievement of low attaining secondary pupils.

Before considering the 12 strategies in detail, we discuss the evidence on whether specific approaches are of particular benefit for low attaining pupils. We then discuss the significant "gaps" in the meta-analysis literature. Finally, we summarise our findings and recommendations.

Table 4: Definitions of teaching strategies and approaches identified, with the 12 evidence-based strategies highlighted

Strategy	Definition
Attitude and behaviour	Interventions directed at improving attitudes to mathematics, reducing maths anxiety, improving pupil self-regulation or metacognition and behavioural interventions
Computer-Aided Instruction (CAI)	**Computer-based systems designed to deliver all or part of the curriculum or to support the management of learning by providing assessment and feedback to pupils**
Cooperative Learning	**Pupils collaborate on a shared task in structured programmes, often in groups of mixed attainment**
Cross-age tutoring	Older pupils tutor younger pupils
Explicit teaching	**A variety of teacher-led approaches consisting of crafted and/or partially scripted instruction together with structured practice, and usually involving feedback**
Feedback to pupils	**Information provided to a pupil regarding performance or understanding**
Feedback to teachers	**Information on pupil performance or understanding provided to a teacher**
Heuristics	**Strategies for approaching a range of different problems**
Individualised instruction schemes	Schemes designed to be followed by pupils individually under the management and direction of the teacher (e.g. SMILE)
Instructional components	A diverse range of strategies, including teacher modelling and advanced organisers, often implemented as part of a broader strategy
Manipulatives	**Concrete materials that can be manipulated by pupils to aid understanding**
Peer tutoring	**Tutoring by same-age peers**
Providing information to parents	Parents are provided with information on their child's progress in class
Representations	**Diagrams, graphs and tools such as number lines (includes concrete-pictorial-abstract [CPA])**
Prompts for self-instruction	**A set of prompts for pupils relating to a particular method or set of methods**
Student-centred learning	**A range of pupil- (or student-) led or pupil-mediated approaches to learning, including guided learning approaches**
Technology tools	A diverse range of hardware and software, including digital technologies, dynamic geometry software (DGS) and information and communications technology (ICT)
Textbooks	Evidence related to US (not UK) textbooks, often referred to as ‘curricula’ in the literature
Tutoring by adults	**Group or one-to-one additional support provided by teachers or teaching assistants (TAs)**
Working memory training	Training directed towards improving pupil working memory (WM). Increase in WM is associated with increased mathematics attainment

Table 5: Second-order meta-analysis results for the 12 strategies that had a sufficiently high quantity and quality of evidence to aggregate, ordered by the security of evidence

Strategy	**Number of meta-analyses**	**Number of original effects**	**Effect size (Cohen's *d*)**		**Security of Evidence**
Explicit teaching	9	283	0.59	Moderate	Very Strong
Computer-Aided Instruction (CAI)	9	207	0.18	Small	Strong
Peer tutoring	5	94	0.66	Moderate	Moderate
Heuristics	6	55	0.62	Moderate	Moderate
Manipulatives	5	119	0.39	Moderate	Moderate
Tutoring by adults	3	77	0.36	Moderate	Moderate
Feedback to pupils	5	42	0.51	Moderate	Moderate/Weak
Representations	6	39	0.45	Moderate	Moderate/Weak
Feedback to teachers	4	19	0.39	Moderate	Moderate/Weak
Prompts for self-instruction	4	32	1.02	Large	Moderate/Weak
Cooperative Learning	6	68	0.29	Moderate	Moderate/Weak
Student-centred approaches	5	47	0.73	Moderate/High	Weak

Do low attaining pupils benefit more than other pupils from specific approaches and interventions?

Several of the meta-analyses examined the extent to which strategies have a greater or lesser effect for low attaining pupils. Most, but not all, suggest that low attaining pupils do benefit more than middle or high attaining pupils, although the findings are somewhat nuanced. Dennis et al. (2016) found that the lowest attaining third of pupils made greater gains than middle attainers. Similarly, Hartley (1977) found a larger effect for low attaining pupils than for middle and high attaining pupils, but also found higher variability for the effect on low attainers. Examining sustained interventions of 12 weeks or more, Pellegrini et al. (2018) identified effects for low achievers that were twice as large as those for other pupils, although this difference was not statistically significant. However, these differences were not found in all the meta-analyses; Jacobse & Harskamp (2011) found no significant differences between the effects of interventions directed at low attainers and those directed at groups with a range of attainment.

The findings of this review are broadly similar to the findings of our review of the teaching of mathematics for all pupils in Key Stages 2 and 3 (Hodgen et al., 2018). In other words, the majority of generally effective teaching strategies and approaches are also effective for low attaining pupils. However, few of the meta-analyses directly compare the effects of different approaches in order to examine which strategies and approaches may be *particularly* effective for low attainers. Dietrichson et al. (2017) and Gertsten, Chard et al. (2009) are notable exceptions. Gertsen et al. found *explicit* teaching and *heuristics* to be particularly effective strategies for low attaining pupils, whilst Dietrichson et al. highlighted the benefits of *tutoring by adults*, *feedback*, *co-operative learning* and small-group instruction on the attainment of disadvantaged pupils. As noted below under the *Twelve evidence-based strategies*, we consider the evidence supporting *some* use of explicit and direct instruction to be particularly strong.

When and for how long should an intervention or strategy be used?

Chodura et al.'s (2018) meta-analysis suggests that early intervention may be advantageous. They found that interventions targeted at pupils at risk of dyscalculia have a greater effect than those more specifically targeted at the narrower group of pupils assessed as having dyscalculia.

Several meta-analyses investigated the effects of length of interventions. This a complex question, involving not only duration (or overall delivery period) of the intervention, but also the frequency and length of sessions ('dosage'). In general, longer interventions (lasting more than around 10 weeks) were found to be associated with smaller effects (e.g., Carbonneau et al., 2013; Dietrichson et al., 2017; Domino, 2010; Haas, 2005; Jacobse & Harskamp, 2011; Lee, 2000), although, in contrast, Dietrichson et al. also found greater frequency to be associated with larger effects. Both Carbonneau et al. (2013) and Rakes et al. (2010) found positive effects for relatively short interventions of around two weeks. The reasons for these apparently contradictory relationships is not well understood. It is possible that the drop off is due to fatigue effects for pupils or teachers, or that the initial effects are more significant, particularly if delivered frequently, or, that pupils require learning to be reinforced using another strategy, or, as Carbonneau et al. observe, to limitations or inconsistencies in differentiating duration, frequency and dosage.

How relevant are these strategies for the teaching of low attaining pupils in English mathematics classrooms?

The meta-analyses review studies that were carried out in the US. Although the US is a different educational system to the English educational systems, there are many similarities in the content of the curriculum and the approaches used by teachers. In most cases, in our judgement, the broad findings of the meta-analyses are relevant for the teaching of low attaining pupils in English mathematics classrooms. Nevertheless, some caution needs to be exercised. First, many of the interventions were implemented through programmes that were explicitly designed with the US context in mind. As a result, they are not explicitly aligned to the mathematics curriculum in England and sometimes use language and examples that is likely to be unfamiliar to teachers and pupils in England. Moreover, many of these programmes are not commercially available in England and would thus be difficult to obtain. Second, there are some differences between the US and English educational systems that might affect the impact of a particular intervention. For example, as we observe below, cooperative learning may not be as effective in English classrooms, because there may already be more use of groupwork than is generally the case.

Twelve evidence-based strategies

In the following section, we consider the 12 strategies judged to have sufficient evidence to calculate a meaningful aggregated effect size. We address these in the order presented in Table 4.2 with the strongest and most relevant evidence presented first. We pay particular attention to explicit and direct instruction, for which the evidence is particularly strong. For ease of presentation, manipulatives and representations are considered together.

Explicit teaching

Explicit teaching does not just mean logical explanations, or clear descriptions of step-by-step procedures, but includes something additional to what competent teachers in the UK normally do as part of lesson planning. *Explicit* teaching emphasises carefully constructed explanations and structured practice materials which have usually been designed and evaluated by expert teams, incorporating both conceptual and procedural aspects of knowledge. Most *explicit* teaching interventions also involve feedback.

In the US, where many *explicit* teaching interventions originate, the approach is referred to either 'explicit instruction' (e.g., Gersten, Chard et al., 2009), 'direct instruction' (e.g., Kroesbergen & Van Luit, 2003) or 'mastery learning' (e.g., Kulik et al., 1990). This covers a broad range of approaches ranging from highly regimented and scripted (e.g., Engelman's *Direct Instruction* programme ,Stockhard et al., 2017) to much looser guidance on teaching such as 'teacher think alouds' (e.g., Gersten, Beckman et al., 2009).[12] At its core, DI stresses the modelling of a limited set of methods, explaining how and when they are used, followed by extensive structured practice aimed at mastery. There are many similarities between DI and Bloom's (1971) approach to mastery learning and, as a result, we have aggregated the effects of mastery learning within explicit teaching.[13]

The evidence underlying *explicit* teaching is very strong and consistent, although the effect on attainment is moderate. Based on an aggregation of nine meta-analyses (seven of which were judged to be of high quality), which incorporated over 200 effects from studies conducted over several decades, we found *explicit* teaching to be associated with a moderate effect. However, the use of *explicit* teaching is a hotly contested topic in debates around the perceived benefits of traditional and progressive teaching approaches (e.g., Ashman, 2018; Rosenshine, 2008; McMullen & Madelaine, 2014).

Explicit teaching is quite distinct from the more general teacher-led approaches that are often referred to as *transmissionist* teaching in the UK and which have been found to be less effective than alternative connectionist teaching approaches (e.g., Askew et al., 1997; Pampaka et al., 2012; Swan, 2006). *Transmissionist* teaching is a 'catch-all' term that refers to any teacher-led instruction usually supported by a set of beliefs about the efficacy of a naïve model of teacher-led instruction simply as 'telling' and often associated simplistic 'behaviourist' beliefs about pupil learning emphasising 'rote learning' (Askew et al., 1997). In contrast, *explicit* teaching consists of carefully crafted and (partially or fully) pre-designed teaching, together with

[12] A 'think aloud' provides a teacher with guidance to demonstrate and verbalise how a problem might be approached and solved (Gersten, Beckman et al., 2009).

[13] Bloom's mastery learning differs very significantly from current interpretations of mastery in England. One key difference is that, whereas Bloom adopted a structured approach to teaching that required pupils to already have *mastery of* the prerequisite knowledge prior to teaching, current approaches in England focus on teaching directed at ensuring pupils achieve mastery as a result of teaching (e.g., NCETM, 2016).

structured practice materials, usually including some feedback mechanism and specifically designed to make connections to previous knowledge. Gersten, Chard et al. (2009) observe that, whilst early approaches did draw on a broadly behaviourist tradition, more recent approaches to *explicit* teaching have drawn more on insights from research in developmental psychology and mathematics education. Whilst some direct instruction approaches are designed as a 'complete curriculum', the evidence overall suggests that *explicit* teaching should be viewed as one instructional strategy among many rather than a complete approach. So, for example, Gersten, Beckman et al.'s (2009) *What Works Clearinghouse* guidance for teaching learners struggling with mathematics recommends that *explicit* teaching should be used alongside opportunities for problem solving, collaborative work and discussion.

Several meta-analyses indicate that *explicit* teaching is particularly effective with low attainers (e.g., Chen, 2004; Gersten, Chard et al., 2009; Haas, 2005). However, it is unlikely that, even for low attaining pupils, *explicit* teaching will be effective across all mathematics topics at all times. For example, in Kroesbergen & Van Luit's (2003) meta-analysis, *explicit* teaching is more effective for learning basic facts, but self-instruction (see below) was more effective for other aspects of arithmetic. Carbonneau et al. (2013) investigated the effects of *explicit* teaching, or high instructional guidance, when using concrete manipulatives and found that *explicit* teaching was very effective for initial knowledge retention and enabling pupils to use manipulatives for problem-solving, but was much less effective for helping learners transfer knowledge from one topic to another.

Many of the primary studies included in the nine meta-analyses were conducted in the US, where commercially developed structured explicit or direct instruction programmes are available (although many of the studies are focused on programmes which are now dated). In addition to being difficult to obtain, these replicable and manualised programmes would require some adaptation to the language and tasks. Nevertheless, the US system has many similarities to that in the UK and US guidance on struggling learners is publicly available and relevant to UK classrooms (e.g., Gersten, Beckman et al., 2009). However, the literature does little to address the question of how to balance explicit teaching with other less "direct" or less explicit teaching strategies and with independent work by pupils.

In summary, there is secure evidence indicating that *explicit* teaching has a moderate impact on attainment and that it is especially effective for low attaining pupils. *Explicit* teaching appears to be more effective for specific aspects of mathematics, such as learning facts or procedures, but less effective for other aspects, such as transferring knowledge from one context to another or aspects of problem-solving. There is much for mathematics education in the UK to learn from the broad range of *explicit* teaching programmes that have been developed in the US, particularly relating to the careful articulation of procedures and the use of structured practice. We recommend that teachers and schools should consider ways of creating and/or making use of *explicit* teaching to support low attaining pupils, although this should not be the only teaching approach used. More research is needed on how to balance *explicit* teaching with other teaching strategies, in order not only to raise attainment, but also to improve attitudes to and engagement with mathematics, and on how such approaches could be adapted for implementation in the UK.

Computer aided instruction (CAI)

CAI encompasses a broad range of computer-based systems designed to deliver all or part of the curriculum or to support the management of learning by providing assessment and feedback to pupils (e.g., see Cheung & Slavin, 2013). Some CAI is designed to supplement regular teaching, whilst other CAI is comprehensive. In general, CAI is intended to be adaptive to the needs of individual learners. CAI systems with such adaptability are sometimes referred to as Intelligent Tutoring Systems (ITS), which are designed to have 'enhanced adaptability' and attempt to replicate to some extent human tutoring (Steenbergen-Hu & Cooper, 2013).

Overall, the evidence indicates that, in general, CAI has at most a small effect on attainment and may be a valuable supplement to, but not a replacement for, the teacher. CAI may be most effective for the development of basic number and calculation skills and less effective for developing reasoning. We consider that CAI has potential to benefit low attaining pupils in mathematics, particularly by freeing up valuable teacher time. However, it is possible that ITS may have a smaller effect than CAI more generally; one high quality meta-analysis (Steenbergen-Hu & Cooper, 2013) found only a small effect for ITS in general and a small *negative* effect for low attainers. The evidence base is judged to be only of moderate relevance, since none of the CAI programmes that have been rigorously evaluated are available in the UK. Various CAI systems are available in the UK, and some are in widespread use, although they have not been independently and rigorously evaluated. More research is needed to evaluate the use of CAI in the UK and to examine how best to use CAI as a supplement to teaching.

Manipulatives and representations

The evidence indicates that the use of *manipulatives and representations* can have a moderate positive effect on attainment, based on moderately strong evidence for *manipulatives* and slightly weaker evidence for *representations*. However, there was some evidence of variation in the effects across the meta-analyses, possibly due to differences in how *manipulatives and representations* are used. Initially, pupils appear to benefit from explicit teaching on how to use manipulatives (and representations) in order to establish connections to the intended mathematical ideas (e.g., Carbonneau et al., 2013). However, explicit teaching appears to *hinder* pupils' ability to use the manipulatives to transfer mathematical ideas to other areas of mathematics. Low attaining pupils benefit from using *representations* rather than simply observing a teacher demonstrate using *representations* (Gersten, Chard et al., 2009).

Tutoring by adults

Tutoring by adults, particularly Teaching Assistants (TAs), currently plays a major role in education in England, particularly in the support of low attaining pupils (Sharples et al., 2015; Warhurst et al., 2013). There is a great deal of evidence indicating that in general the receipt of TA support has no, or negative, effects on pupil attainment (e.g., Blatchford et al., 2009). However, where structured time-limited programmes are utilised, *tutoring by adults* can be an effective approach (Dietrichson et al., 2017). Moreover, Pellegrini et al. (2018) found that programmes involving tutoring by TAs were as effective as those involving tutoring by qualified teachers, and included several UK-based programmes. Somewhat counter-intuitively, it does not appear that one-to-one support is necessarily better than support delivered in small groups. (see, Dennis et al., 2016; Pellegrini et al., 2018).

Peer tutoring

There was a great deal of evidence directly applicable to low attaining pupils concerning tutoring by same-age peers, indicating a moderate positive effect on attainment, although there was evidence of variation in the effect sizes reported. Several meta-analyses indicated that the impact of *peer-tutoring* may be dependent on training of tutors (Baker et al., 2002; Gersten, Chard et al., 2009).

Feedback to pupils

Feedback, and the associated strategy of *formative assessment*, has received a great deal of attention in the UK and beyond since the publication of Black & Wiliam's (1998) seminal review. *Feedback* can be defined as "information provided by an agent (e.g. teacher, peer, book, parent, self, experience) regarding aspects of one's performance or understanding" (Hattie & Timperley, 2007, p. 81). *Formative assessment* refers to a broader set of strategies, including feedback alongside adapting teaching, *peer assessment* and *self-assessment* (Black & Wiliam, 1998; Thompson & Wiliam, 2008). There was less evidence available relating to *formative assessment*, but this limited evidence base suggested only very small effects for *formative assessment* (Kingston & Nash, 2011).

Overall, there is moderate evidence to suggest that providing *feedback* has a moderate positive effect on attainment, although we found these effects to be smaller than claimed in previous secondary meta-analyses. *Feedback* needs to be used carefully, because, when used inappropriately, feedback can have a negative effect on attainment (e.g., Hattie & Timperley, 2007; Kluger & DeNisi, 1996). For low attaining pupils, it may be more important to use feedback simply to demonstrate and reinforce learning rather than to accompany the feedback with learning goals (Gersten, Chard et al., 2009).

Feedback to teachers

We found moderate positive effects supported by moderate evidence for *feedback to teachers*. In contrast to the findings for low attaining pupils, where setting goals is associated with smaller effects, *feedback to teachers* about pupil progress is likely to be more effective when feedback on pupils' progress is given alongside suggestions for appropriate learning goals and teaching approaches (Gersten, Chard et al., 2009).

Heuristics

A *heuristic* is "a method or strategy that exemplifies a generic approach for solving a problem" … [such as] … "Read the problem. Highlight the key words. Solve the problem. Check your work" (Gertsen, Chard et al., 2009, p. 1210). *Heuristics* are not problem-specific and can be applied to different types of problems, and may involve more structured approaches to analysing and representing a problem. We found the use of *heuristics* to be associated with a moderate positive effect on attainment, based on a moderately strong evidence base. Gertsen et al. used meta-regression to compare the impacts of several strategies for low attaining pupils and found *heuristics* to have a larger effect on attainment than *explicit instruction*, *feedback*, *representations* or *peer-tutoring*.

Prompts for self-instruction

Prompts for self-instruction involves providing pupils with a set of "prompts to remind them of what they are doing" (Goldman, 1989, p. 45). The use of *prompts for self-instruction* has a large effect on attainment, the largest effect of all the evidence-based strategies aggregated. However, this is based on a relatively small number of original studies (32) and the evidence base is judged moderate to weak. These studies were largely carried out in the US. Nevertheless, whilst more research is needed into how to support UK teachers to use *prompts for self-instruction*, our judgment is that the use of *prompts for self-instruction* is likely to be an effective approach in the UK and is worthy of further study.

Cooperative learning

Cooperative learning ranges from the non-specific working with peers within group settings (Lee, 2000) through definitions built on Slavin's studies (e.g. Slavin, 1980, 1984, 1999), where pupils of mixed levels of attainment collaborate in small groups with a shared goal. *Cooperative learning* is commonly associated (particularly in the US) with specific programmes and approaches, such as 'Student Teams Achievement Divisions' (STAD), 'Team-Assisted Individualization' (TAI), and dyadic methods (such as peer-tutoring). The five meta-analyses central to this evidence focused on one or more of these programmes/approaches, although the majority of studies synthesised involved one particular programme, STAD.

Our analysis suggests that *cooperative learning* has a moderate positive effect on attainment, although the evidence underlying this was weak to moderately strong. Much of this evidence comes from studies that were carried out in the US and may not transfer easily to the UK context, because group work may already be more prevalent in mathematics classrooms. *Cooperative learning* is likely to be more successful where pupils are explicitly taught how to collaborate (e.g., Baines et al., 2014).

Student-centred learning

Student-centred learning encompasses a range of teaching strategies that "begin with the student's representation of a given task and attempt to alter it in the direction of task representation that is more 'expert'" (Goldman, 1989, p.45). These include *problem-based learning*, *inquiry learning*, *guided instruction* and a variety of *constructivist* approaches. The evidence suggests that student-centred approaches have a moderate effect on mathematical attainment. In addition, this was based on five meta-analyses, none of which was judged to be of high quality. Hence, the security of the evidence base is judged weak. One key problem is that the approach is defined in very different ways across the studies. Indeed, Jacobse & Harskamp (2011) used the term 'indirect instruction' to emphasise the contrast with explicit and direct instruction.

There is considerable interest in such approaches in the UK and elsewhere (e.g., Blair, 2014), although the benefits of this approach are somewhat contested. Proponents of the approach argue that it enables pupils to learn mathematics that is more relevant to the pupils' needs (Wake, 2015) and also to learn, and retain, this knowledge more effectively (e.g., Hmelo-Silver, Duncan, & Chinn, 2007). However, others argue that student-centred approaches are less effective than explicit teaching, because they expect novice learners to behave like experts, although they do not have the necessary bank of knowledge to do this (Kirschner, Sweller, & Clark, 2006).

Given the widespread interest in *student-centred learning*, it is somewhat surprising that the evidence is so limited. Since the evidence suggests *student-centred learning* to have a moderate to high effect but to be poorly understood, there is a need for more original well-constructed and robust studies examining different aspects or approaches to *student-centred learning* together with an up-to-date high-quality meta-analysis in this area that addresses potential differences between the various approaches and examines when these approaches are likely to be effective.

The seven strategies with limited or inconsistent evidence

As noted above, we identified some evidence focused on low attaining pupils relating to a further seven strategies, although we judged the evidence to be too weak or inconsistent to calculate an overall effect size. It is important to note that these strategies are not necessarily *ineffective*. For some, but not all, of these strategies, there was some, albeit weak, evidence of an impact on attainment. These strategies are discussed briefly below.

Cross-age tutoring

Gersten, Chard et al. (2009) found *cross-age peer tutoring* to have a larger effect for low attaining pupils than same-age *peer-tutoring*, although this finding was based on only two primary studies.

Individualised instruction schemes

The research in this area is dated and covers *individualised instruction schemes* that are no longer available and developed for the US market.

Instructional components

We identified a diverse range of strategies, often implemented as components of another intervention. The efficacy of these strategies is clearly of interest to teachers, because many would be relatively straightforward to implement. Codding et al.'s (2011) meta-analysis of single-case designs suggests that an over-reliance on a small set of strategies is problematic. They found that interventions involving a combination of component approaches (such as *modeling*, *prompting*, *error correction*, *performance feedback*, *reinforcement*, *drill*, *timed practice*, and *manipulatives*) were more effective than those involving just one or two components.

Motivation, behavioural and attitudes

Although the evidence is weak, our findings suggest that interventions focused on raising attainment, and which lead to low-attaining learners realising that they can be successful in mathematics, are likely to be a better route to generating motivation than approaches focused solely on motivation and engagement (e.g., Dietrichson, 2017; Lee, 2000). An exception to this is the issue of maths anxiety. For some pupils, maths anxiety has a large detrimental impact on performance by disrupting working memory and through avoidance of mathematical activities (Dowker et al., 2016). Yet, the issue of maths anxiety, and its prevalence, is very poorly understood and there is only as yet a very limited understanding of how to reduce maths anxiety.

Technology tools

The use of *technology tools* (including *dynamic geometry software* and *computer algebra systems*) has received a great deal of attention within mathematics education research over more than four decades (e.g., Hoyles, 2018). The relevant meta-analyses find a range of effects, some of which are large, but these may be inflated due to novelty or researcher-delivery effects. This evidence is somewhat disappointing, because we consider that there is evidence from small and innovative studies indicating that technological tools have the potential for large effects at least in favourable settings (e.g., Hoyles, 2018). However, more substantial research is needed before assuming that tools such as *dynamic geometry software* will be transformative in classrooms generally.

Textbooks

The impact of adopting one textbook scheme over another appears to be very small. Only three meta-analyses addressed the use of textbooks, and all were focused on the US, where textbooks are used to structure the content in the absence of a National Curriculum (Cheung & Slavin, 2013; Pellegrini et al., 2018; Slavin et al., 2009). Nevertheless, we do consider high-quality textbooks to have a key role in providing tasks and examples and might have a positive impact in reducing teacher workload. Hence, the choice about textbook adoption is an important decision for a school. But high quality teaching has a much larger effect on pupil attainment. Introducing a new textbook alongside a programme of professional development for the mathematics department might well provide a catalyst for improvements in teaching.

Gaps in the evidence base

We identified two significant, and striking, gaps in the literature relating to the teaching of specific mathematical topics and to evidence-based, effective and replicable interventions designed for the UK context.

The teaching of specific topics in mathematics

The literature on the effectiveness of pedagogical approaches to teach specific mathematical topics is severely limited. As Nunes et al. (2009) observe, there is little research in general on didactics, or the “technicalities of teaching”; i.e., how to teach pupils in specific topics. Several meta-analyses did compare the impact of particular strategies on different mathematical topics, but, although a few differences were identified, these differences were generally not statistically significant and the results were not consistent across the studies.

We found some evidence relating to the teaching of algebra, fractions, number and calculation, and problem solving, but found little evidence relating to the teaching of geometry, measures, probability or statistics. The evidence largely echoes our findings about effective strategies and finds evidence to support the use of explicit and direct instruction (for number and calculation), for the use of representations (for algebra, problem-solving and fractions and rational numbers) and heuristics and prompts (in the form of worked examples for algebra and number and calculation). Several meta-analyses indicate that algebra and problem-solving are accessible and appropriate topics for low attaining pupils (e.g., Dennis et al., 2016; Hughes et al., 2014) and Hembree (1992) found that reading difficulties do not appear to be a critical barrier to problem solving.

Effective, replicable interventions designed for the UK context

The vast majority of the meta-analyses and systematic reviews in our database synthesise the results of exploratory studies, many of which are small-scale. Aside from *Direct Instruction*, which Stockhard et al. (2017) focused on exclusively, we identified only three meta-analyses examining the effects of 'manualised' or replicable interventions designed, and ready, to be delivered at scale.[14] These meta-analyses were all carried out by Slavin and colleagues (Cheung & Slavin, 2013; Pellegrini et al., 2018; Slavin et al., 2009). In addition, the bulk of these studies, including all the *Direct Instruction* studies, were carried out in the US. Manualised, replicable interventions designed to raise mathematics attainment for low attainers and appropriate to the UK context do exist (see, e.g., Cohen Kadosh et al., 2013), and one positive sign is that the most recent of the meta-analyses (Pellegrini et al., 2018) includes several large-scale trials conducted in the UK. Nevertheless, there is a need for research developing and evaluating specific, targeted and replicable interventions that are appropriate to the current UK educational context. Professional development is likely to be crucial to the successful implementation of such interventions. However, the evidence relating to effective approaches to the professional development of teachers is also limited.

[14] By 'manualised', we mean interventions that are described clearly enough to be implemented by others (as opposed to the original developer or designer).

Summary: Systematic review of teaching approaches

We found sufficient evidence to identify the efficacy of 12 strategies and approaches for low attaining pupils. These are listed from highest to lowest in order of the security of the evidence base:

- explicit teaching,
- computer-aided instruction (CAI),
- peer tutoring,
- heuristics,
- manipulatives,
- tutoring by adults
- feedback to pupils,
- representations,
- feedback to teachers,
- self-instruction,
- cooperative learning, and
- student-centred learning.

Although the strength of evidence varied, all were found to have a moderate positive impact on attainment, except for prompts for self-instruction, which had a large impact, and computer-aided instruction, which had a small impact.

This review indicates that strategies that are generally effective are also effective for low attaining pupils, although low attainers may gain particular benefits from the consistent and effective use of these strategies. There is particularly consistent evidence to support use of *explicit* teaching for low attaining pupils. We also found evidence to support the use of early intervention for pupils at risk of low attainment. In general, the effect of an intervention reduced as the duration increased, although frequency was associated with increased benefits.

We found only weak and inconsistent evidence for seven other strategies, including the use of technological tools (such as dynamic geometry software). Our findings also suggest that interventions directed exclusively at increasing motivation or improving attitudes are less likely to be effective than interventions focused more directly on improving attainment. But again, the evidence was weak.

We identified two very significant ‘gaps’ in the literature, First, we found very little evidence about the specifics of teaching different mathematical topics, either within the meta-analytic literature or within the wider systematic reviews that we included. Of particular importance for low attainers, the literature on the teaching of number and calculation is limited. Second, the bulk of the literature that we reviewed was focused on small, experimental studies largely carried out by researchers. Few of the meta-analyses interventions designed are ready to be delivered at scale, and most of these were developed for the US context and not appropriate to the current UK educational context.

4: Current approaches to the teaching and learning of mathematically low attaining pupils

In this section, we address the following research question:

- To what extent is mathematics currently taught in appropriate ways for low attainers?

Background

Studies of classroom mathematics show that lower sets are characterised by low expectations, a restricted curriculum and a slow pace (e.g., Boaler et al., 2000; Gellert et al., 2011; Straehler-Pohl et al., 2013). However, evidence about how low attaining pupils in England are currently taught, particularly relating to specific mathematical topics, is limited. In this part of the study, we sought to address this gap and to consider whether, and how, the findings of the other parts of the study could be implemented in schools.

Method

In order to address this question, we carried out three rounds of interviews: pilot interviews with 26 pupils (aimed at validating the IMAP test and developing an interview protocol), Round 1 interviews with another 99 pupils (aimed at further investigating pupils' difficulties on areas related to the IMAP test), and Round 2 interviews with 70 pupils and 12 teachers (focused on how mathematics is currently taught). In this report, we focus on the Round 2 interviews, because these interviews directly address the research question.

Interviewees were drawn from 5 schools. Pupils were interviewed individually for 25 minutes and teachers for 45-60 minutes. Pupils were selected by the schools (we asked schools for the "lowest 40% in attainment). We interviewed 12 mathematics teachers who were experienced in teaching low-attaining pupils in Year 9-10.

The content of the interviews was informed by the test content (see Section 1) and by the findings of the literature review (see Section 2), specifically to investigate and compare pupils' perceptions and teachers' reported practices about the use of representations and manipulatives (including number lines and arrays), derived facts and estimation and how these contribute to understanding. Additionally, we investigated whether pupils used these tools, strategies and approaches spontaneously.

Main findings from pupil interviews

General

Most pupils enjoyed mathematics lessons and valued their mathematics teacher

Many pupils had a generally positive outlook on their school mathematics lessons, although they mostly reported finding mathematics hard. Most liked their mathematics teacher and were particularly appreciative of the large quantity of one-to-one support available in small classes.

Pupils valued detailed explanations with methods broken into small steps

Many pupils said that the most helpful aspect of individual help was longer, more detailed explanations than those that the teacher had provided to the whole class. Many pupils said that they liked methods to be broken down into small steps and said that they wanted to be provided with lots of examples. This has some resonances with our findings about the effectiveness of explicit and direct instruction (see Section 2).

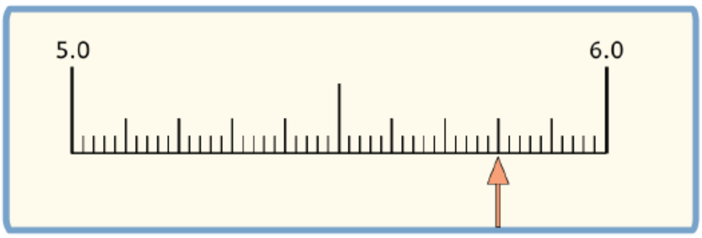
Representations and manipulatives

When asked for examples of diagrams or pictures that they might use in mathematics, many pupils suggested area representations for fractions, either using circles or rectangles. Several were positive in general about the value of representations. We specifically asked pupils about number lines and arrays, and we also report below our observations on pupils' use of fingers for calculation during the interviews.

Number lines

Most pupils reported that they did not use number lines very much. Many pupils said that they did not like them, either finding them too difficult and confusing, and too slow to draw, or else believing that they were too simple and that they no longer needed to use them. Many reported using number lines at primary school, and more recently only using them in the context of directed/negative numbers.

1. What number is the arrow pointing to?



2. What is 10 more than 3597?

Figure 3: Pupil interview questions 1 and 2

Many pupils struggled to answer Q1 (see Figure 3) or did so only with considerable assistance. Very few pupils approached Q2 by means of a number line and 'bridging through 10'[15] with most opting for a column addition method.

[15] By partitioning 10 into (3+7), a number line can be used to support the following 'bridging through 10' strategy:
3597 + 10 = (3597+3) + 7 = 3607

Arrays

Few pupils were familiar with the word ‘array’. On showing them an example, some said that they had not seen such things; others that they had used them “a really long time ago … in Year 3”, generally for division. Most said that they did not use arrays, other than for calculating areas by counting squares. Reasons given were that they take too long to draw and were perceived to be unnecessary, but it seemed that pupils were unfamiliar with potential uses of arrays for representing multiplication.

Fingers

Many pupils were observed to use their fingers during the interviews for various calculations, and when asked about this often stated that they preferred fingers to alternative representations, such as a number line. Generally, finger strategies were counting all, counting on or counting back. Some pupils were very reliant on fingers to carry out most of the calculations undertaken in the interview and saw fingers as a way to be sure of the correct answer.

Derived facts

When presented with Q3 (see Figure 4), most pupils proceeded to calculate 85 + 57, usually by using a standard column method, without reference to the given result to 86 + 57. When their attention was drawn to the instruction to ‘Find a quick way’, or pupils were asked explicitly whether they could use the given result to help them, some pupils were then able to do so, with increasing confidence as subsequent similar problems were presented. However, even with considerable prompting many pupils found it very difficult to see that they could modify the given result by subtracting 1. This appeared to be an unfamiliar style of question for most pupils, with pupils occasionally seeming unsure whether they could assume that the given statement was correct.

3.

Look at this calculation.

$$86 + 57 = 143$$

Find a quick way to work out the answer to

$$85 + 57 = ?$$

Figure 4: Pupil interview question 3

Q4 (see Figure 5), incorporates distributivity, and as expected was considerably more difficult for pupils than Q3. Very few pupils were able to succeed with this without considerable support.

4.

Look at this calculation.

$$15 \times 24 = 360$$

Find a quick way to work out the answer to

$$16 \times 24 = ?$$

Figure 5: Pupil interview question 4

When pupils were asked about the multiplication facts that they knew, and how they would work out ones that they did not know, most pupils resorted to repeated addition, counting up from zero in multiples, instead of starting from a nearer known fact. Overall, pupils showed a lack of flexibility, and their repeated addition methods were slow and sometimes inaccurate. Some pupils acknowledged this but stated that they had more confidence in their own methods.

Estimation

Pupils displayed different understandings of the term 'estimating'. Some equated this with rounding to a particular degree of accuracy, such as 1 decimal place. In many cases, pupils' difficulties with rounding and place value meant that the answers that they obtained from rough calculations were many orders of magnitude away from the correct value.

Other pupils saw estimating as making a wild guess or using everyday knowledge of typical sizes, checking an answer and having a gut feeling as to whether a value was too big or too small for a given calculation. Several pupils said that they did not estimate or did not like to, and those that said that they did estimate were sometimes apologetic about it. There was a strong preference among many pupils for attempting to calculate exactly rather than estimating, and many pupils did not seem to believe that the answer to a rough calculation should necessarily be at all close to the exact answer.

Main findings from teacher interviews

Teachers' perceptions of pupils' difficulties

Generic cognitive and literacy difficulties

All teachers highlighted difficulties that they did not regard as mathematics-specific, involving pupils' memory, concentration, processing speed and literacy. These were seen as across-the-board problems. Many teachers expressed their frustration with pupils' retention and processing difficulties when needing to bring together multiple aspects of a problem. An overriding concern for many of the teachers was pupils' literacy. This was sometimes, but not always, related to specific mathematical terminology.

Low self-efficacy

Teachers' perceptions of pupils' generic difficulties also included a lack of confidence and the expectation of not being able to succeed in school. Teachers saw low self-efficacy as leading to a reluctance to show initiative and 'have a go' at difficult problems.

Lack of number sense

A third difficulty, referred to by most of the teachers, was a lack of number sense and an overreliance on poorly understood procedures, meaning that pupils were not in a position to 'sense check' their answers. Teachers frequently commented on pupils' knowledge being compartmentalised by topic, making it hard for them to bring different ideas together or to cope when a problem demanded more than routine performance of a standard procedure. Many teachers felt that lack of facility with 'the basics', such as number bonds and multiplication tables, was a fundamental problem that undermined all of the later work. However, there was also a repeated view that many pupils could perform certain procedures quite reliably, and yet did so with very limited understanding of what they were doing, which limited the usefulness of those procedures. This kind of instrumental knowledge was perceived not to last long, and not to allow pupils to adapt their approach to even a slightly different scenario.

Difficulties with algebra and multiplicative reasoning

When asked which topics caused their low-attaining pupils the most difficulty, all teachers mentioned algebra. One teacher felt that this was more psychological than real. Many teachers also mentioned pupils' difficulties with proportional reasoning.

Teaching approaches

There was a lot of commonality in the teaching approaches that teachers described.

Build positive relationships with pupils

Several teachers stressed that relationships with pupils were critical to address pupil disaffection and low self-efficacy. Many teachers highlighted the benefits of small classes and individual attention, which allowed repeated and longer explanations.

Structure and scaffold learning carefully

All teachers stressed the importance of structuring and scaffolding learning when planning lessons. Several teachers were concerned with building pupils' confidence by not allowing much, if any, struggle or failure. However, several teachers recognised the dangers of making things too easy or allowing procedural teaching to dominate low-attainers' diets. For several teachers, the increased scaffolding meant that planning a mathematics lesson for low attainers was substantially more work than planning a lesson for higher attainers. Two teachers talked at great length about their perception of a lack of suitable resources for teaching low-attaining pupils, because the commercially produced materials to which they had access to were far too demanding.

Emphasise numeracy without losing sight of other aspects of mathematics

All teachers stressed that their prime focus with low-attaining pupils was numeracy, sometimes referred to as 'the basics', but one teacher felt strongly that low attainers should not be denied access to the full curriculum. Several teachers talked about strategies that they use to support numeracy alongside other teaching, such as multiplication table squares in pupils' books for them to refer to. All of the teachers had strategies in place to support pupils' development of number work in an ongoing fashion. This often took the form of starters and homework activities that revised aspects of numeracy.

Derived facts and estimation

Pupils rarely use derived facts

All of the teachers stated that their pupils would be unlikely to use derived facts to solve problems similar to the one we showed them (see Figure 3.2). Although this question prompts pupils to use derived facts by asking them to "find a quick way", teachers expected that their pupils would instead calculate the answer using a standard column algorithm. Most teachers stated that they had not taught derived facts explicitly, although they believed that it was important. One teacher felt that derived facts seemed almost too obvious to focus on: "I guess maybe we think it's so obvious and that's why we don't ever do a question like that."

Pupils rarely estimate

The words estimation, approximation and rounding were used in different ways during the interviews. Sometimes, teachers used 'estimation' to refer to using experience to state an approximately correct value (e.g. the mass of a bag of sweets), without performing any calculations. Teachers saw this as important for sense-checking the rough size of a final answer, which they complained that pupils rarely did. On other occasions, 'estimation' was used to refer to calculating with rounded values to obtain a rough final answer, as is commonly required in examination questions (e.g., GCSE). Some teachers felt that pupils' difficulties with estimation were partly a result of poor skill at rounding numbers. All teachers felt that estimation was very important and all teachers complained that pupils (even higher-attaining ones) rarely sense-checked their final answers or calculated an estimate, even when explicitly asked to do so. It may be that when pupils calculated exactly all the way through a question and then rounded the final answer they interpreted this as 'estimating'. Some teachers felt that estimation had become a formalised 'topic' with an associated style of assessment question, and that this detracted from bringing common sense into play. Several teachers wanted to see estimation as a natural part of number sense and not something that pupils should need to be explicitly asked to do.

Context, representations and manipulatives

Context may help or hinder

There was a tension in teachers' references to context. Sometimes, context was highlighted as a possible facilitator that can make the mathematical structure clearer. For example, using decimal currency for understanding decimals was perceived as helpful, because pupils value money and are interested in it. However, context was also seen as problematic for learners, in making additional cognitive and literary demands on top of the mathematics.

Representations and manipulatives are valued but rarely deployed in principled way

On the whole, teachers were positive about the value of representations and manipulatives for supporting pupils' learning. However certain representations and manipulatives were often perceived as too infantile to use with most older pupils, leaving teachers with the dilemma of whether to offer them to those pupils in a class who might still benefit from using them.

When asked for examples of representations and manipulatives, teachers mentioned number lines, factor trees, pretend money, blocks, cubes, counters, sweets and matchsticks for sequences. In some schools, manipulatives were described as scarce and troublesome to manage, and perhaps not worth the effort. Pupils took the opportunity to misbehave, and teachers felt that not all pupils really needed them. Several teachers reported that representations were used at the start of a topic to build the concepts, and then, when this was perceived to have been done, they were not used much subsequently. Alternatively, some representations were perceived as purely instrumental ways of obtaining answers (e.g., standard column algorithms).

Some teachers appeared to lack a consistent approach to their use of representations and manipulatives, without any clear principles regarding which to use and when, as outlined by this teacher:

> *I've picked up some of these number sense activities from some CPD that I did last year and I think at the minute I've got bits of using the bar model, bits of Singapore maths … But I think for me as a practitioner at the minute I've got lots of bits of different things that can work but the problem is that I'm not implementing them consistently because either it doesn't fit in with the scheme of learning or this one works better for that one and I think ultimately it's that the pupils need one model that they've always used that they can fall back on.*

When asked specifically about arrays, few teachers mentioned their use to conceptualise multiplicative structure. Instead, most referred to illustrating square numbers, triangle numbers and "dividing sweets", in which case they were generally seen as too simple.

There was a strong overall sense that number lines were a particularly valuable representation, especially in relation to negative numbers, and one teacher spoke about using a virtual vertical number line in space, with his hands representing the positions of the numbers, for addition and subtraction of directed numbers. He felt that this was highly effective, and yet he did not observe pupils using this approach themselves when not prompted to do so.

Summary: Current approaches to the teaching and learning of mathematically low attaining pupils

In summary, most teachers reported that they believed building positive relationships was an especially important strategy for low attaining pupils. It is, therefore, pleasing to report that we found that most low attaining pupils enjoyed their mathematics lessons and valued their mathematics teacher, even though they mostly reported finding mathematics difficult. One danger of this approach is that some teachers felt it was important to building pupils' confidence by not allowing much, if any, struggle or failure. We emphasise, however, that several teachers recognised this danger and attempted to avoid it.

There was a great deal of commonality in the teaching approaches that teachers described. All teachers stressed the importance of structuring and scaffolding learning and some teachers reported that this meant that planning a mathematics lesson for low attainers was substantially more work than planning a lesson for higher attainers. We found that pupils valued detailed explanations with methods broken into small steps, which links to our finding about the efficacy of explicit and direct instruction.

When asked about the use of derived facts, teachers endorsed their importance, though most teachers reported that they had not taught derived facts explicitly and that their pupils rarely use derived facts. Teachers also reported that pupils rarely estimate, although they perceived this to be an issue for high attainers as much as low attainers.

In general, teachers were positive about the value of representations and manipulatives, although many teachers appeared to lack a consistent approach to their use of representations and manipulatives. Many pupils reported finding number lines confusing and were unfamiliar with arrays. Some pupils did, however, make use of strategies involving the use of fingers.

5: Conclusion

In this section, we present our findings in terms of our five research questions followed by the implications of these findings for the teaching of mathematically low attaining pupils.

Findings

What mathematics do low attaining secondary pupils understand, and what are their particular strengths and weaknesses in number, multiplicative reasoning and algebra?
Can low attainment be characterised simply as delay? If not, to what extent and in what ways do low attaining pupils understand mathematics in qualitatively different ways to high attaining pupils?

Our analysis is broadly consistent with a view of low attainment as largely characterised by delay. We found no evidence of threshold concepts (i.e., particular subsets of mathematical knowledge or skill that unlock future progression). However, we found some evidence of practically important weaknesses in derived facts and selecting a calculation, both of which are related to fluency in number.

As outlined in Sections 1 and 2 of this report, we searched for evidence that there are particular concepts, areas of mathematics or key ideas that would be crucial determinants of future learning: particular subsets of mathematical knowledge or skill that unlock future progression. If such ‘threshold concepts’ could be identified, we would expect to see a clear difference between pupils who can demonstrate having grasped them (by their performance on particular items or sections) and those who have failed to do so.

Overall, we did not find evidence that performance in any particular topic has a special place in explaining performance in other areas of mathematics. Moreover, this held when each of the two year groups was analysed separately. Our hypothesis was that weaknesses in some threshold concepts might explain why low attainers had failed to progress. If we had discovered such differences, they could well have led to clear recommendations for curriculum and teaching: ‘address those blockages and progress will be improved’. However, we have been unable to identify evidence for any such ‘threshold concepts’.

Nevertheless, we did identify some small, but statistically significant, differences between the two groups: the Y9 low attaining pupils performed better on arrays and area, percentages and arithmetic recall, whereas the Y5 middle and high attaining pupils were stronger on derived facts and select a calculation. These relative strengths and weaknesses in the mathematical understanding of the Year 9 low attaining pupils do have important messages for teaching. First, the relative strength in arithmetic recall is just that, a relative strength in comparison to pupils who were four year younger. Our findings should not be interpreted as implying that the arithmetic recall of the Year 9 pupils is satisfactory. It is weak in comparison to the remainder of the Year 9 cohort and we believe that, with appropriate teaching, this group of pupils can make (and would benefit from making) significant improvements in their understanding of basic facts. Second, the relative weaknesses in derived facts and selecting a calculation suggest some important weaknesses in these pupils’ fluency, flexibility and application with number and calculation (or number sense, see, e.g., McIntosh et al., 1992). We consider that addressing weaknesses in these areas to be very important for these pupils’ further mathematical development.

Despite these small qualitative differences in performance, our analysis is broadly consistent with a view of low attainment as largely characterised by delay rather than qualitatively differential performance. The Y9 low attainers seem to be broadly similar to matched middle and high attaining Y5 group in terms of the broad profile of things they know and can do; however, their general mathematical progress is some four years behind the average.

As a result, we conclude that learning appears to progress incrementally, on a broad front. Individuals can, and do, have specific strengths and weaknesses, but it is not clear that there are useful patterns that clearly distinguish low and high attainers. To make progress, pupils need to develop across the board, steadily. There are no obvious shortcuts: teach everything well, and make sure it is solidly learnt.

We have failed to replicate some previous research that claims a special place in mathematics learning progressions for the understanding of concepts such as number lines or derived facts (e.g., Siegler et al., 2010; Gray & Tall, 1994). The items that capture these concepts form a valid part of the overall Number measure, but they do not appear to have any particular priority in terms of their predictive power.

To what extent do low attaining pupils' prior understandings of mathematics, and of particular mathematical topics, help to explain the existence of the attainment gap? What is the relative contribution of these mathematical understandings in comparison to socio-economic status and other demographic factors?

Our analysis indicates that prior attainment is by far the strongest predictor of low attainment. We found small effects for exclusions and absences, and for self-efficacy and intrinsic motivation, and the number of schools attended. After controlling for prior attainment, we found no additional effects for SES meaning that SES is not associated with progress in secondary mathematics from KS2 to GCSE. Similarly, after controlling for prior attainment, we found only very weak effect for gender. We found evidence indicating that some pupils, who are low attaining at KS1, do overcome low attainment.

In Section 3 of this report, we outlined how we found that a combination of a subset of prior attainment variables provided a very strong prediction of GCSE grade in mathematics. All other factors were very much second order. It may seem like a truism to say that the strongest determinant of low attainment is low attainment, but some important implications lie behind this claim.

First, we have found that pupils who were low attainers on one measure but not low on another had dramatically improved prospects over those who are consistently low. This may be because the assessment on which they did poorly was not a good representation of their potential, or because they were on an upward trajectory (perhaps as a result of good teaching, for example), or it may just be that a statistical prediction is greatly improved by including two measures of the same thing. Further research would be needed to uncover the true explanation.

Second, this is an important message for schools, both because it is something they probably can influence, since developing pupils' attainment is one of the core aims of schools, and because it may prevent schools from focusing on less important determinants. In a classroom or school it can often feel like pupils' learning is held back by factors such as their attitudes, behavioural tendencies or home background. Our data suggest that even if these factors do causally influence attainment (and can be changed), their effects are much smaller than the impact of what those pupils have already learnt.

In addition, we found a robust relationship between GCSE outcomes and two variables that capture school presence: absences and exclusions. In our analyses, we found that pupils who missed or were excluded from school were significantly less likely to be successful. We cannot say whether these are causal relationships, or whether schools can even affect them, but the consistency and strength of the relationships suggest that it is worth trying to investigate whether there are strategies or interventions that can improve attainment through the mechanism of reducing absence and exclusion.

Scoring high on our questionnaire measures of self-efficacy and performance goal orientation was associated with improved prospects for low attainers. In our statistical models, after controlling for SES, gender and attainment, self-reported self-efficacy and intrinsic motivation were associated with small but significant improvements in GCSE mathematics grades. Of these variables, self-efficacy was both the largest and the most robust predictor. It seems likely that self-efficacy will be partly a result of high attainment rather than its cause, and that both may be a consequence of other characteristics. Nevertheless, it may also be that believing you are good at mathematics and feeling confident about it has value for future attainment, over and above actually being able to do it. Whether there is anything schools or other agents can do to influence these self-perceptions (other than actually getting pupils to succeed) may be worthy of future study.

What is currently known about the effectiveness of teaching strategies and approaches that address low attainment in secondary mathematics?

Our analysis indicates that strategies that are generally effective are also effective for low attaining pupils, although low attainers may gain particular benefits from the consistent and effective use of these strategies. We identified 12 effective and evidence-based strategies, which are beneficial specifically for low attaining secondary pupils. There is particularly strong evidence to support some use of explicit and direct instruction for low attaining pupils, although this should be used alongside other strategies such as problem-solving and collaborative work. Several of the effective and evidence-based strategies, such as heuristics, prompts, manipulatives and representations are ‘low cost’ and, thus, relatively straightforward to implement.

As outlined in Section 4 of this report, our review indicates that strategies that are generally effective are also effective for low attaining pupils, although low attainers may gain particular benefits from the consistent and effective use of these strategies. We also found evidence to support the use of early intervention for pupils at risk of low attainment. In general, the effect of an intervention reduced as the duration increased, although frequency was associated with increased benefits.

We found evidence to identify 12 strategies and approaches, which are particularly beneficial for low attaining secondary pupils. These are listed from highest to lowest in order of the security of the evidence base:

- explicit teaching,
- computer-aided instruction (CAI),
- peer tutoring,
- heuristics,
- manipulatives,
- tutoring by adults,
- feedback to pupils,
- representations,
- feedback to teachers,
- self-instruction,
- cooperative learning, and
- student-centred learning.

Although the security of evidence varied, all were found to have a moderate positive impact on attainment, except for prompts for self-instruction, which had a large impact, and computer-aided instruction, which had a small impact.

The 12 evidence-based strategies and approaches that we identified all have the potential to improve the teaching and learning of mathematics for low attaining pupils. In each case, there are important caveats. The effectiveness of any strategy is highly dependent on the teaching; how teachers implement and use strategies makes a difference to their effectiveness. Moreover, none of these strategies should be used universally; time-limited interventions are more effective. Hence, we recommend that implementation is carried out thoughtfully and should include professional development opportunities for teachers to develop their pedagogic skills.

Explicit teaching can be beneficial. There is particularly consistent evidence to support some use of *explicit* teaching for low attaining pupils. We caution that this should be employed alongside other approaches, including problem-solving and collaborative work, and that a contrasting approach, student-centred learning, was also found to be effective, albeit with much weaker evidence. It is important to emphasise that effective explicit instruction is very different from the everyday teacher-led approaches that are often referred in the UK as 'transmission' teaching. In particular, explicit and direct instruction emphasises pre-designed, crafted and carefully constructed explanations alongside structured practice.

Computer-aided instruction can be a valuable supplement to teaching. Although the effects on attainment are small, computer-aided instruction (CAI) has the potential to free up valuable teacher time. It should not be used as a replacement for teaching and appears to be most effective for the development of basic number and calculation skills and less effective for developing reasoning.

Tutoring by teaching assistants is more effective if structured and time-limited. Tutoring by teaching assistants is commonly used to support low attaining pupils. This is much more likely to be effective when structured and time-limited. Unstructured support by adults is not effective and can have negative effects.

Some ‘low cost’ strategies are likely to be effective. Some strategies, such as *heuristics*, *prompts for self-instruction*, and *manipulatives and representations*, offer potential benefits at little or no cost, and we suggest that these are worth experimenting with in the classroom.

Feedback is an effective strategy, although we found the evidence to be weaker than in previous reviews. *Feedback* should be implemented carefully, because, when used inappropriately, *feedback* can have a powerful negative effect on attainment. For low attaining pupils, it may be more important to use *feedback* simply to demonstrate and reinforce learning rather than to communicate learning objectives or next steps alongside this *feedback*.

We found only weak and inconsistent evidence for seven other strategies, including the use of *technological tools* (such as *dynamic geometry software*), and more research is needed to assess the efficacy of these strategies. Our findings suggest that interventions directed exclusively at increasing motivation or improving attitudes are less likely to be effective than interventions focused more directly on improving attainment.

We identified two very significant ‘gaps’ in the literature, First, we found very little evidence about the specifics of teaching different mathematical topics, either within the meta-analytic literature or within the wider systematic reviews that we included. Of particular importance for low attainers, the literature on the teaching of number and calculation is limited. Second, the bulk of the literature that we reviewed was focused on small, experimental studies largely carried out by researchers. Few of the meta-analyses interventions designed are ready to be delivered at scale, and most of these were developed for the US context and not appropriate to the current UK educational context.

To what extent is mathematics currently taught in appropriate ways for low attainers?

Our analysis indicates that teachers are focused on building positive relationships and that most low attaining pupils valued this. There was wide support from pupils and teachers for scaffolding learning and many low attaining pupils valued detailed explanations with methods broken into small steps. Teachers considered derived facts and estimation are poorly understood by low attaining pupils. In general, teachers were positive about the value of representations and manipulatives, although some appeared to lack a consistent approach to their use of representations and manipulatives.

As discussed in Section 5 of this report, most teachers reported that they believed building positive relationships was an especially important strategy for low attaining pupils. It is, therefore, pleasing to report that we found that most low attaining pupils enjoyed their mathematics lessons and valued their mathematics teacher, even though they mostly reported finding mathematics difficult. One danger of this approach is that some teachers felt it was important to building pupils’ confidence by not allowing much, if any, struggle or failure. We emphasise, however, that several teachers recognised this danger and attempted to avoid it.

There was a great deal of commonality in the teaching approaches that teachers described. All teachers stressed the importance of structuring and scaffolding learning and some teachers reported that this meant that planning a mathematics lesson for low attainers was substantially more work than planning a lesson for higher attainers. We found that many pupils valued detailed explanations with methods broken into small steps. This has some links to our finding about the efficacy of explicit and direct instruction, although this strategy generally emphasises understanding and conceptual coherence.

Teachers reported that pupils rarely use derived facts, although most teachers reported that they had not taught derived facts explicitly, despite reporting that they believed that it was important. Teachers also reported that pupils rarely estimate, although they perceived this to be an issue for high attainers as much as low attainers.

Teachers generally valued representations and manipulatives for supporting pupils' learning, although some could benefit from guidance and principles about how, and when, to use manipulatives and representations.

Implications

Schools should focus on raising the mathematical attainment of low attaining pupils

Prior attainment is the strongest predictor of future attainment. Nevertheless, we found that some pupils did overcome low attainment.

In order to raise pupils' later attainment, schools should primarily focus on raising their current attainment, rather than on other factors such as their attitudes, behavioural tendencies or home support. Even if these factors do causally influence attainment (and can be changed), their effects are likely to be much smaller than the impact of what those pupils have already learnt. Although pupils' self-efficacy is important, there is no evidence that approaches focused solely on motivation, engagement or attitudes lead to improved attainment.

Whilst our evidence is associational rather than causational, this nevertheless strongly suggests that, if we want to improve pupils' future attainment, schools need to focus directly on – and improve – theses pupils' current attainment. Although we found no evidence for the existence of particular threshold concepts, we did identify some small, but practically significant, relative weaknesses in derived facts and selecting a calculation. Addressing these weaknesses is likely to improve pupils' fluency and flexibility with number and calculation.

Some teaching strategies are particularly promising for mathematically low attaining pupils

In order to raise attainment, our systematic review of the literature indicates that there is particularly strong evidence to support some use of explicit and direct instruction for low attaining pupils, although the effect on attainment was moderate. We caution that this should be employed alongside other approaches, including problem-solving and collaborative work, and that a contrasting approach, student-centred learning, was also found to be effective, albeit with much weaker evidence. It is important to emphasise that effective explicit instruction is very different to the everyday teacher-led approaches that are often referred in the UK as 'transmission' teaching. In particular, explicit and direct instruction emphasises pre-designed, crafted and carefully constructed explanations and practice together with structured practice.

Although the effects of computer-aided instruction (CAI) on attainment are small, it has the potential to free up valuable teacher time. It should not be used as a replacement for teaching and appears to be most effective for the development of basic number and calculation skills and less effective for developing reasoning. Tutoring by teaching assistants is commonly used to support low attaining pupils. This is much more likely to be effective when structured and time-limited. Unstructured support by adults is not effective and can have negative effects.

Some strategies, such as heuristics, and prompts, offer potential benefits at little or no cost, and we suggest that these are worth experimenting with in the classroom.

Feedback is an effective strategy, although we found the evidence to be weaker than in previous reviews. Feedback should be implemented carefully, because, when used inappropriately, feedback can have a powerful negative effect on attainment. For low attaining pupils, it may be more important to use feedback to demonstrate and reinforce learning rather than to communicate learning objectives or next steps.

A more consistent approach to using representations and manipulatives is needed. Our systematic review indicated that, when used well, manipulatives and representations could help raise mathematical attainment. Development of training, guidance and resources in this area could be particularly helpful.

Teachers, training organisations and funders should develop scalable implementations and training based on the promising evidence-based strategies

The 12 evidence-based strategies and approaches that we identified all have the potential to improve the teaching and learning of mathematics for low attaining pupils. In each case, there are important caveats. The effectiveness of any strategy is highly dependent on the teaching; how teachers implement and use strategies makes a difference to their effectiveness. Moreover, none of these strategies should be used universally; time-limited interventions are more effective. Hence, we recommend that implementation is carried out thoughtfully and should include professional development opportunities for teachers to develop their pedagogic skills.

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